Packing and Covering with Geometric Objects

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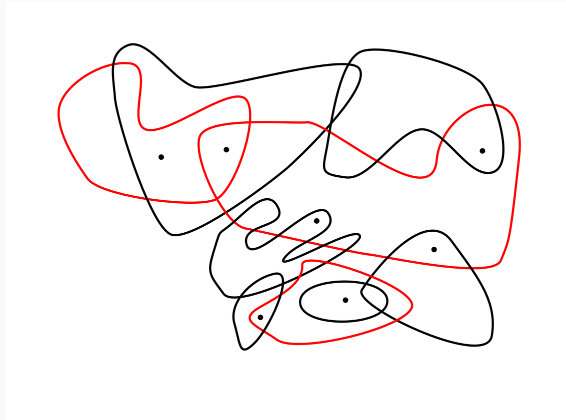
Abstract

We consider Geometric **Packing** and **Covering** problems that are NP-hard and design efficient approximation algorithms to solve them. We show that these problems admit Polynomial Time Approximation Schemes (PTAS) using **Local Search** algorithms.

A PTAS is an algorithm that takes an instance of the given optimization problem and a parameter ϵ, and outputs an $(1+\epsilon)$-approximate solution in $(n^{f(1/\epsilon)})$ time.

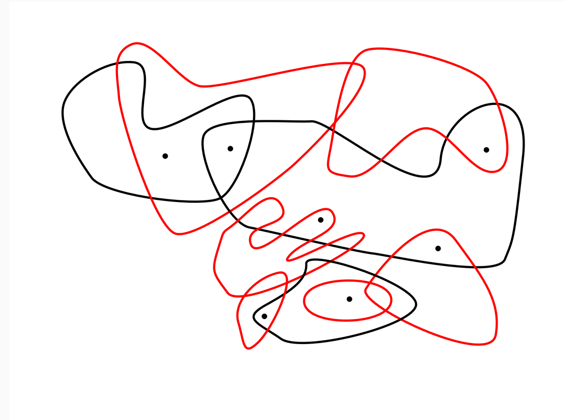
Set Cover Problem

Given a set of regions $\mathcal{R}$ and a point set P, compute the minimum sized subset $\mathcal{R}' \subseteq \mathcal{R}$ such that every point in P is contained in at least one object in $\mathcal{R}'$.



Set Packing Problem

Given a set of regions $\mathcal{R}$ and a point set P, compute the maximum sized subset $\mathcal{R}' \subseteq \mathcal{R}$ such that every point in P is contained in at most one object in $\mathcal{R}'$.



Local Search Algorithm

Input: $\mathcal{R}$, P, ϵ (error parameter).

- Start with some feasible solution.
- Change the current solution by making local changes (spending $n^{O(1/\epsilon^2)}$ time) if it improves the objective function.
- Return the current solution if local changes can no longer improve the solution.

Output: $(1+\epsilon)$-approximate solution in $n^{O(1/\epsilon^2)}$ time.

Analysis of Approximation factor

The Local Search algorithm yields an approximation factor of $(1+\epsilon)$ if the following properties hold.

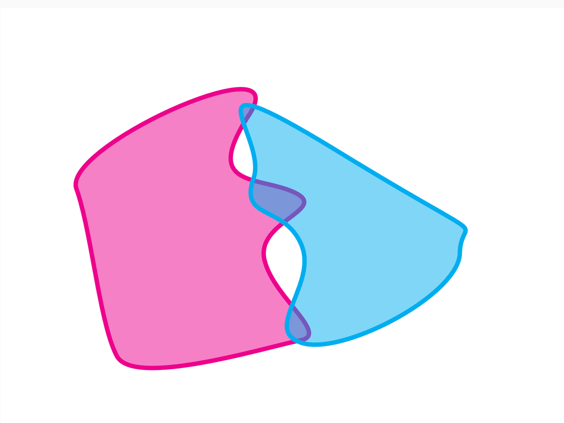
- A balanced vertex separator of sublinear size, e.g., Planar graphs.
- Small Set Expansion, i.e., Every small subset (size $O(1/\epsilon^2)$) of the LARGER SET expands in the smaller set.

Our Problems

1. Shallow Packing
2. Point Packing
3. Runaway Rectangle Escape problem
4. Unique Coverage
5. Multi-Covering problem
6. Prize Collecting Set Cover
7. Art Gallery problems

Our Geometric Objects

The Geometric Objects we have mostly studied are set of Non-Piercing Regions. A set of objects $\mathcal{R}$ are said to be non-piercing if for every $A, B \in \mathcal{R}$ the following holds.



- $A \setminus B$ and $B \setminus A$ are connected regions.
- The boundaries of A and B intersect at most k times, i.e., $|\partial A \cap \partial B| \leq k$ where k is some constant.

Shallow Packing

Definition. Given $\mathcal{R}$, P and an integer capacity, at most some constant, for every point in P, compute the maximum sized subset $\mathcal{R}' \subseteq \mathcal{R}$ such that every point in P is contained in at most as many objects in $\mathcal{R}'$ as its capacity.

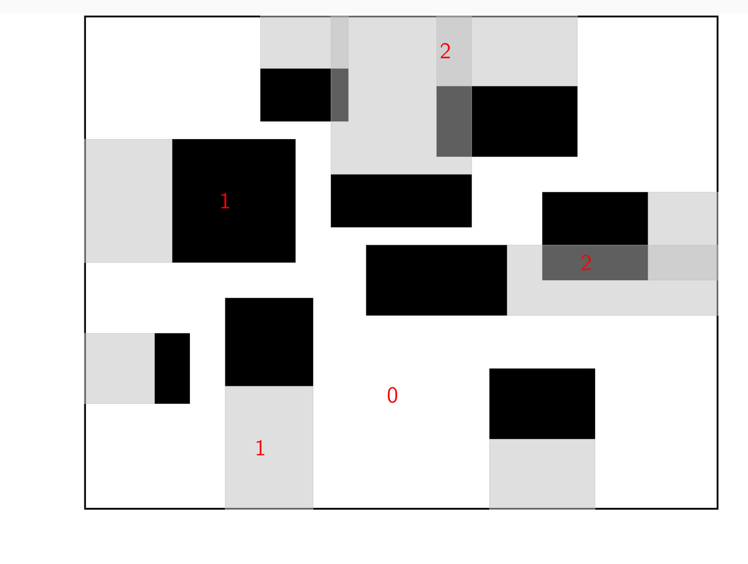
Our Contribution. We show the existence of appropriate graphs that have balanced separators of sublinear size which are not planar. We further extend the result for an even broader class of objects that have **sub-quadratic union complexity**. Existence of such a graph implies that the local search algorithm yields an $(1+\epsilon)$-approximation algorithm.

Intersection Graphs of Shallow Arrangements

- Given $\mathcal{R}$ and P such that the depth of every point in P is at most some constant ℓ, we define a graph G over $\mathcal{R}$ and put an edge between R_i and R_j if $R_i \cap R_j \cap P \neq \emptyset$.
- Observe that for $\ell \geq 5$, G need not be planar because K_5 can possibly exist as a subgraph of G.
- We prove that still the graph G has a balanced separator of sublinear size using some appropriate planar graphs. This proof works for non-piercing regions, and in the continuous setting (when $P = \mathbb{R}^2$) it even works for the class of objects with sub-quadratic union complexity.

Runaway Rectangle Escape problem

Definition. Given a set of rectangles $\mathcal{R}$ and the maximum allowed density (number of layers) d, maximize the number of rectangles that can escape in one of the 4 directions.



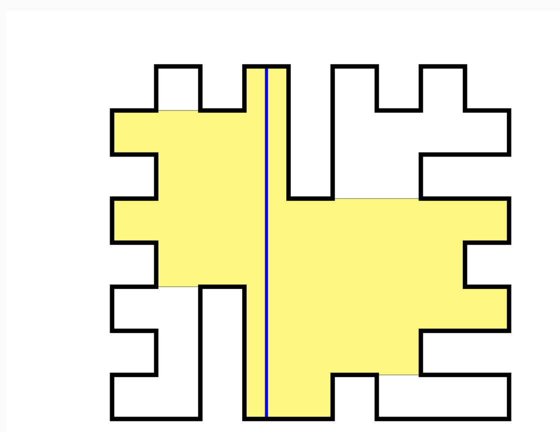
Our Contribution. We give a $(2+\epsilon)$-approximation algorithm.

Covering Problems

- **Unique Coverage.** Given $\mathcal{R}$ and P compute a subset $\mathcal{R}' \subseteq \mathcal{R}$ such that the number of points in P that are contained by exactly one object from $\mathcal{R}'$ is maximized.
- **Multi-Covering Problem.** Given $\mathcal{R}$, P and an integer demand of every point in P, compute the minimum sized subset $\mathcal{R}' \subseteq \mathcal{R}$ such that every point in P is contained in at least as many objects from $\mathcal{R}'$ as it demand.
- **Prize Collecting Set Cover.** Given $\mathcal{R}$, P, a weight for every $R \in \mathcal{R}$ and a penalty for every point $p \in P$, compute a subset $\mathcal{R}' \subseteq \mathcal{R}$ such that the sum of the weights of objects in $\mathcal{R}'$ plus the sum of the penalties for the points not covered by $\mathcal{R}'$ is minimized.

Our Contribution. We give an $(1+\epsilon)$-approximation algorithm with sparsity assumptions on the input.

Art Gallery Problems



We consider a variant of the art gallery problem where the art gallery is orthogonal in shape and sliding cameras (along orthogonal axis) are planned to be installed to guard the gallery. In the adjacent figure, the floor plan of an art gallery is shown which is guarded by a single vertical sliding camera (shown in blue). The region in yellow is its visibility area and thus there are places that remain unguarded. The objective is to compute the minimum number of sliding cameras such that the entire region is guarded.

Our Contribution. We give an $(1+\epsilon)$-approximation algorithm.

Publication

1. The Runaway Rectangle Escape Problem (with S. Govindarajan, A. Maheshwari, N. Misra, S. C. Nandy, S. Shetty) CCCG '14, arXiv 1603.04210
2. Packing and Covering with Non-Piercing Regions (with S. Govindarajan, R. Raman, S. Ray) ESA '16
3. Local Search strikes again: PTAS for variants of Geometric Covering and Packing (with P. Ashok, S. Govindarajan) under review
4. Effectiveness of Local Search for Art Gallery and Prize Collecting Problems (with S. Bandyapadhyay) under review