Packing and Covering with Non-piercing Regions

(pack and cover but don't pierce)

Sathish Govindarajan, Rajiv Raman

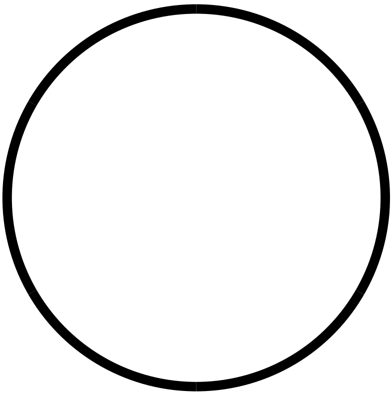
Saurabh Ray, Aniket Basu Roy

Set Cover
Dominating Set

Packing Regions
and Points

Local Search

parameter k



Local Search

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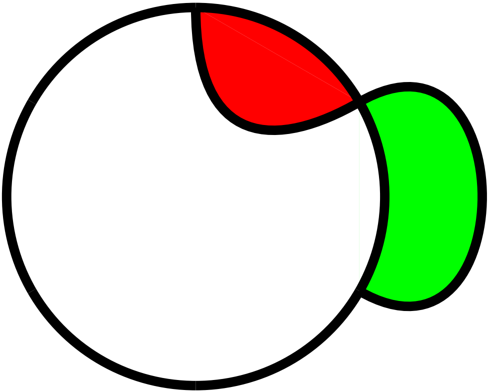

Local Search

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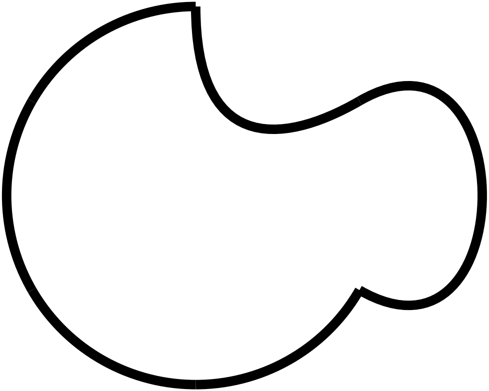

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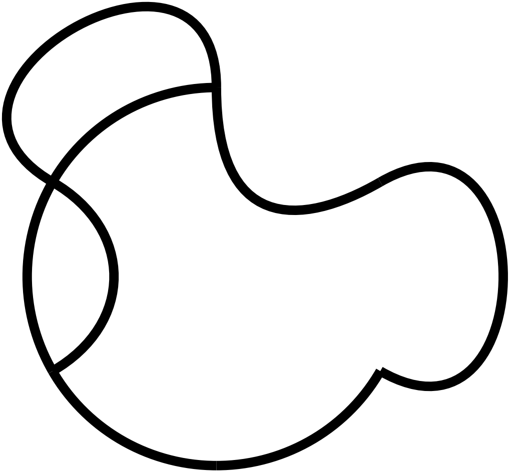

Local Search

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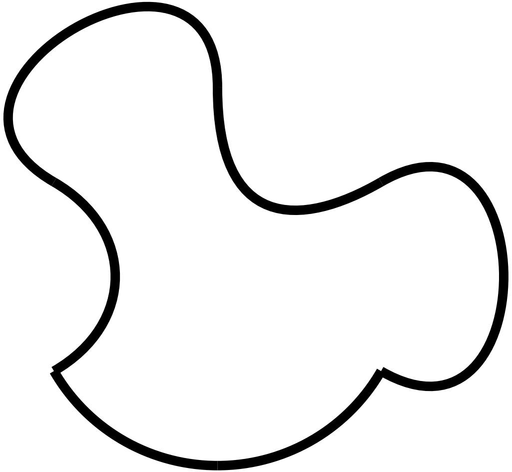

Local Search

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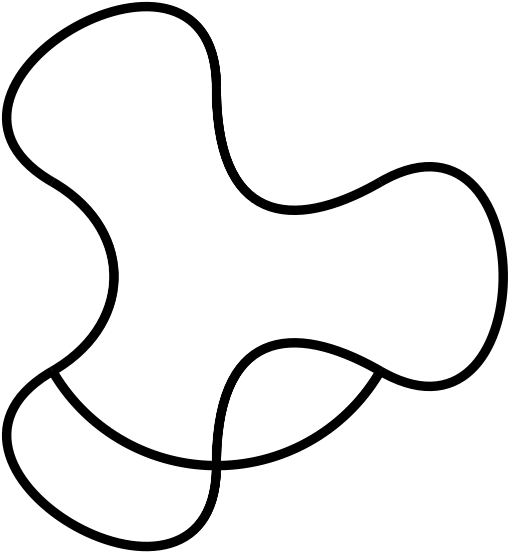

Local Search

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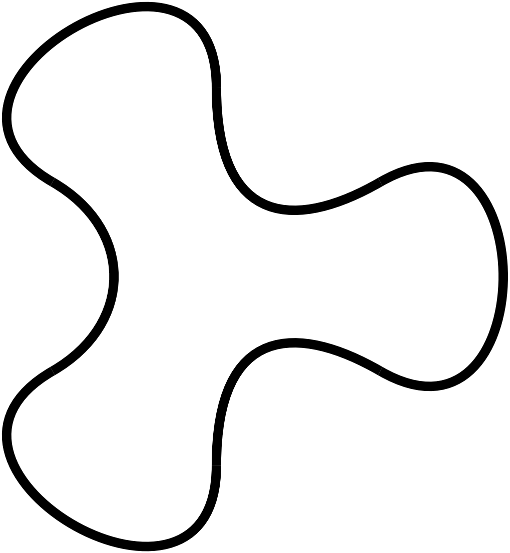
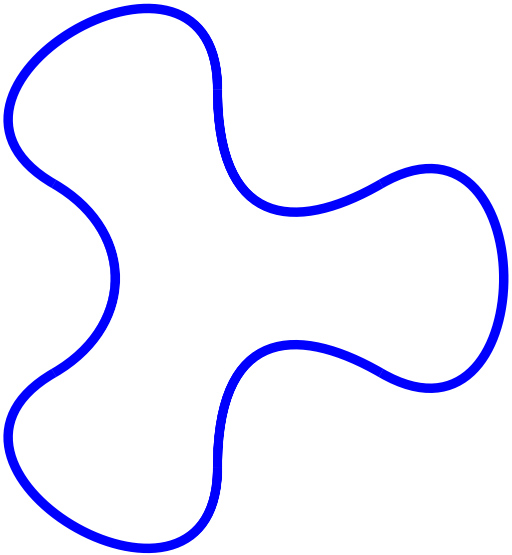


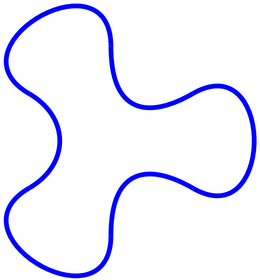
Local Search Solution



$(1+\epsilon)-$approximation

$\epsilon = c/\sqrt{k}$

Local Search Solution



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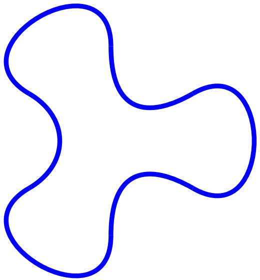
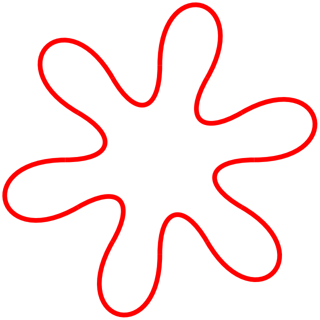


Optimum Solution



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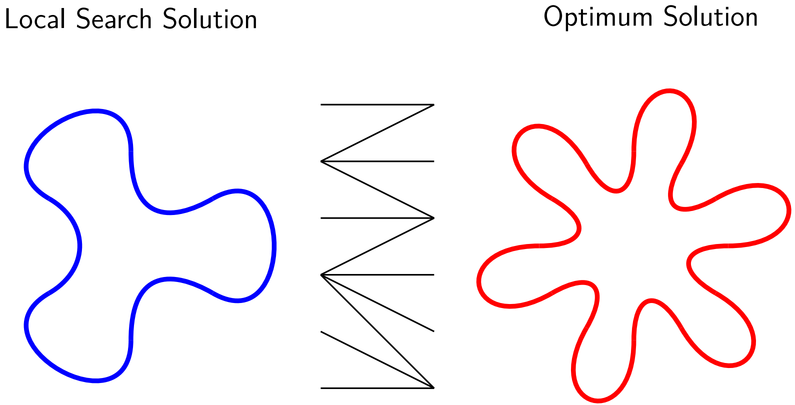


Properties of the Bipartite Graph

- Planar Bipartite Graph.
-

Properties of the Bipartite Graph

- A balanced vertex separator of sublinear size.
-

Properties of the Bipartite Graph

- A balanced vertex separator of sublinear size.
- Small Set Expansion

Properties of the Bipartite Graph

- A balanced vertex separator of sublinear size.
- Every small subset of the LARGER SET expands in the smaller set.

Properties of the Bipartite Graph

- A balanced vertex separator of sublinear size.
- Every $\leq k$ sized subset of the LARGER SET expands in the smaller set.

Properties of the Bipartite Graph

Minimization Problem

- A balanced vertex separator of sublinear size.
- Every $\leq k$ sized subset of the LOCAL SEARCH SOLUTION expands in the optimum solution.

Properties of the Bipartite Graph

Maximization Problem

- A balanced vertex separator of sublinear size.
- Every $\leq k$ sized subset of the OPTIMUM SOLUTION expands in the local search solution.

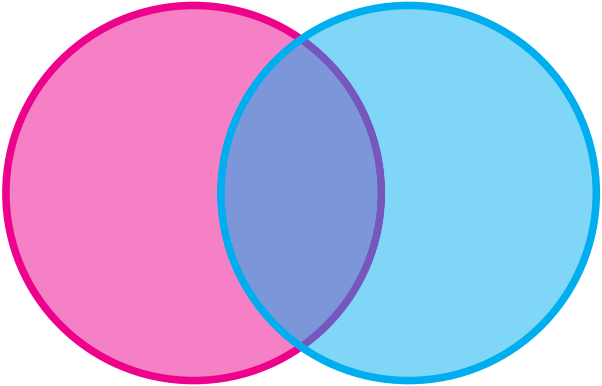
Properties of the Bipartite Graph

- A balanced vertex separator of sublinear size.
- Every $\leq k$ sized subset of the LARGER SET expands in the smaller set.

Take Away!

Non-piercing Regions

$A \setminus B$ is connected.
$B \setminus A$ is connected.



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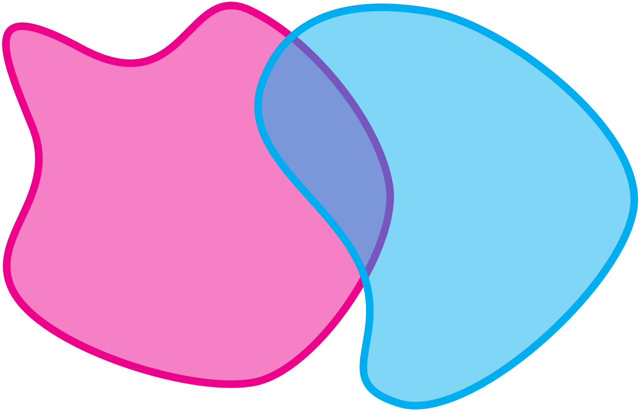

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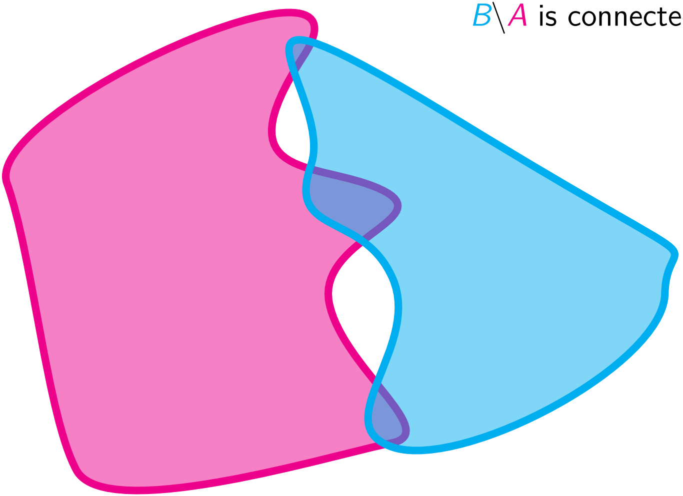
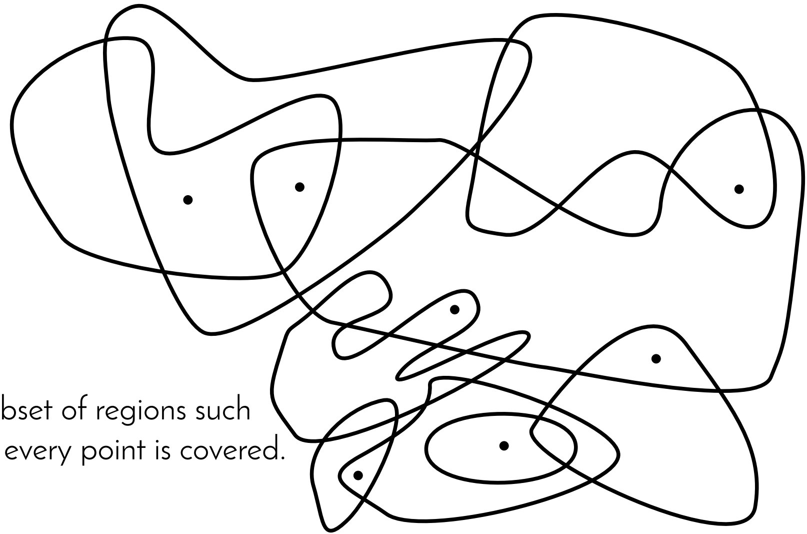


Set Cover
Dominating Set

Packing Regions
and Points

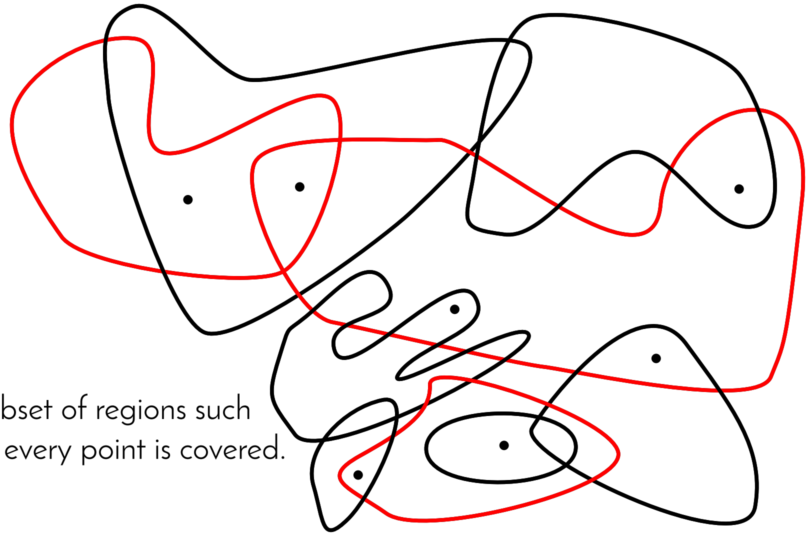
Set Cover
Dominating Set

Set Cover



A subset of regions such that every point is covered.

Set Cover



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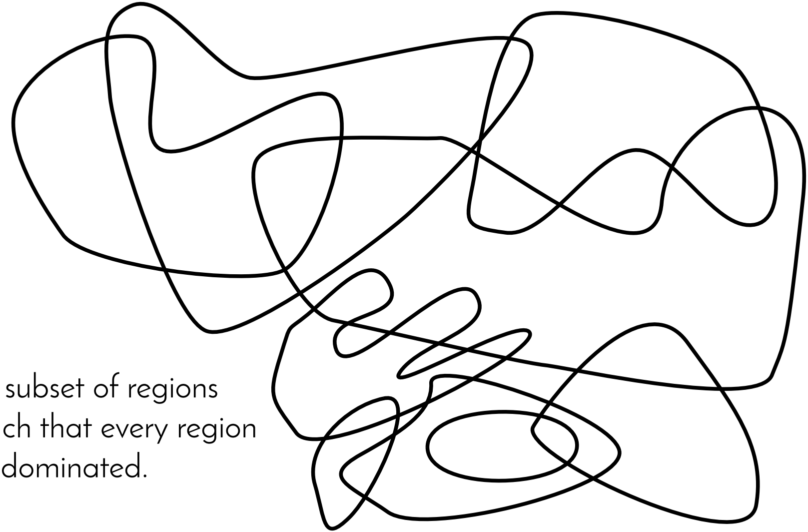
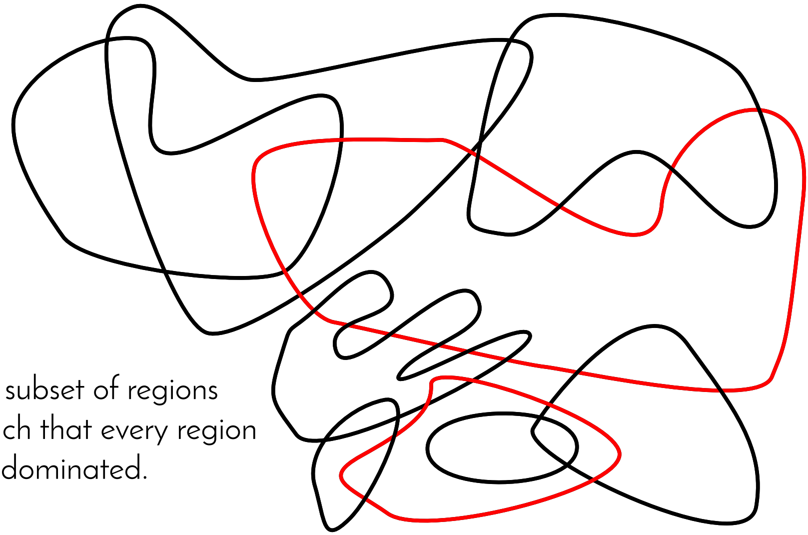
Dominating Set



A subset of regions such that every region is dominated.

Dominating Set



A subset of regions such that every region is dominated.

Properties of the Bipartite Graph

Minimization Problem

- A balance vertex separator of sublinear size.
- Every $\leq k$ sized subset of the LOCAL SEARCH SOLUTION expands in the optimum solution.

Minimization Problem

Aschner et al. '13, Mustafa and Ray '09

Suppose Π is a minimization problem. If there exists a bipartite graph $H = (\mathcal{R} \cup \mathcal{B}, E)$, that belongs to a family of graphs having the following 2 properties, then the Local Search algorithm is a PTAS for Π.

- A balanced vertex separator of sub-linear size.
- For any subset $\mathcal{B}' \subseteq \mathcal{B}$, $(\mathcal{B} \setminus \mathcal{B}') \cup N(\mathcal{B}')$ is a feasible solution. Here, $N(\mathcal{B}')$ denotes the set of neighbors of $\mathcal{B}'$ in H.

Set Cover Problem

Suppose Π is the Set Cover problem. If there exists a bipartite graph $H = (\mathcal{R} \cup \mathcal{B}, E)$, that belongs to a family of graphs having the following 2 properties, then the Local Search algorithm is a PTAS for Π.

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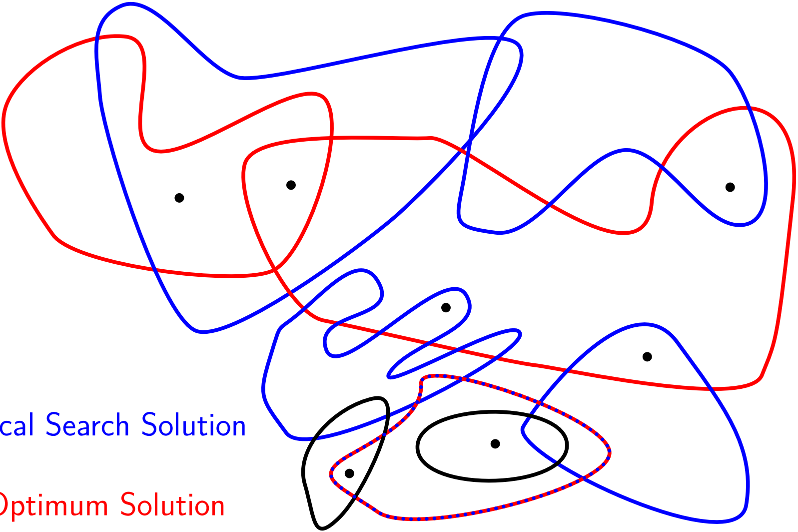
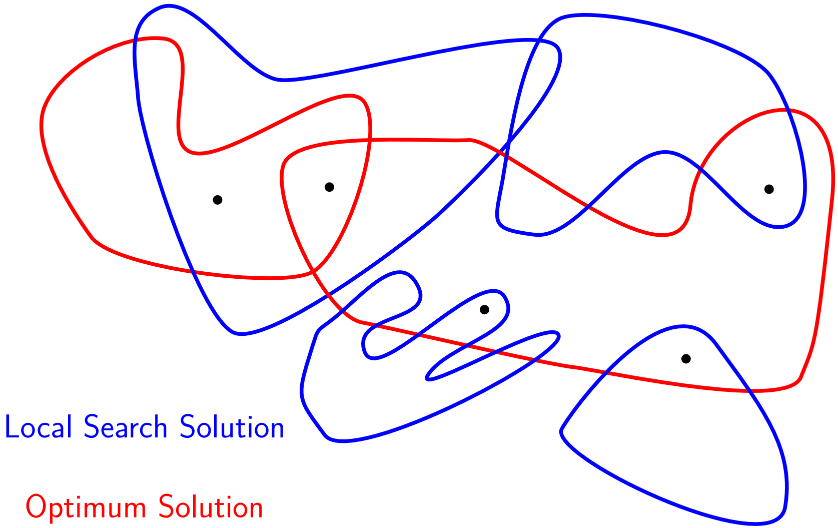
Set Cover Problem



Local Search Solution

Optimum Solution

Set Cover Problem



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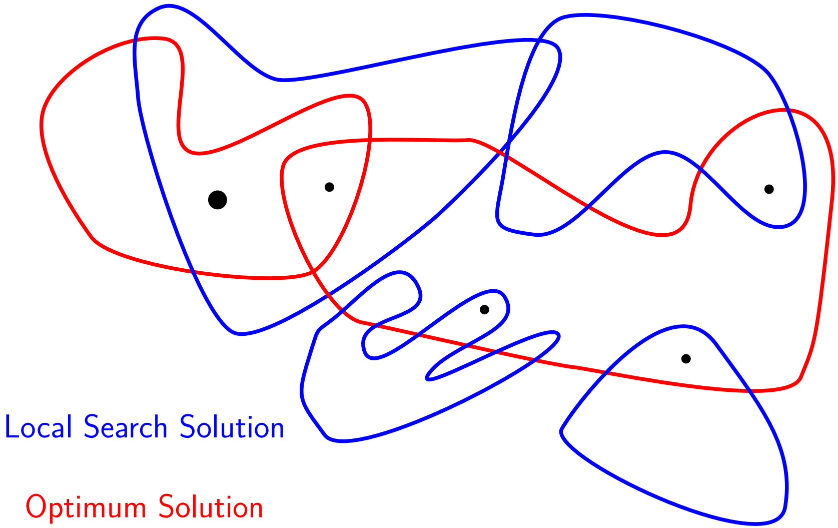

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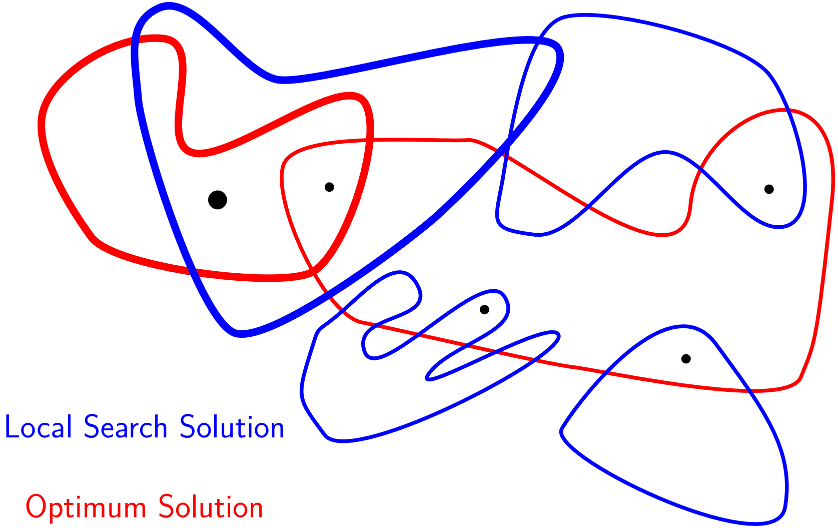

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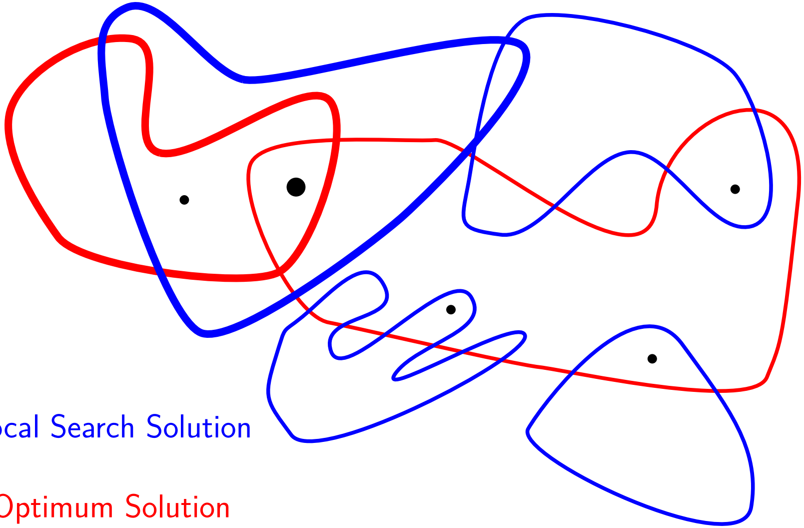

Local Search Solution

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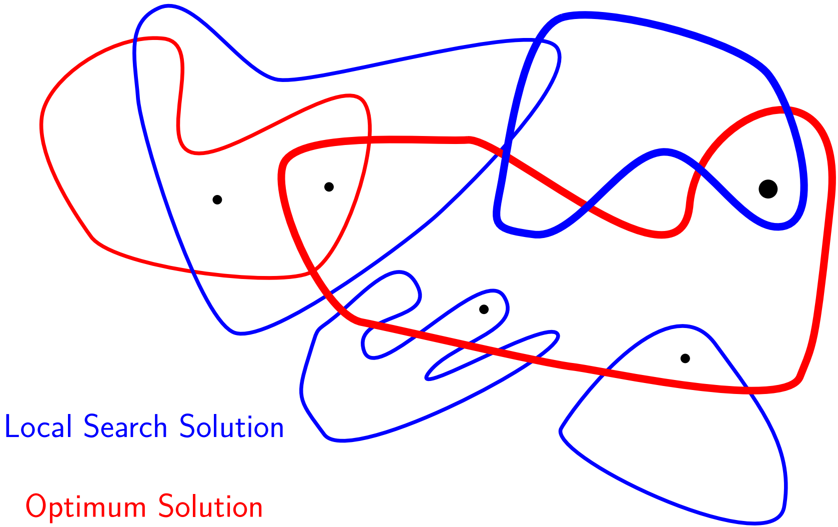
Set Cover Problem



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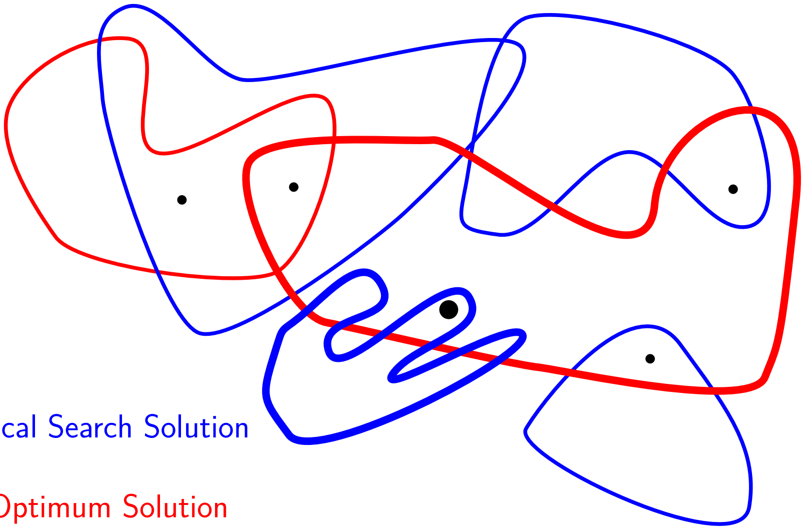
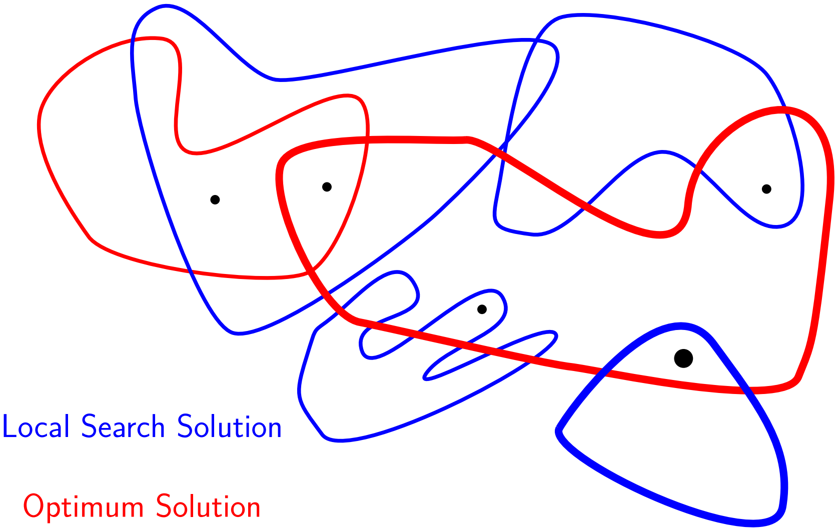


Local Search Solution

Optimum Solution

Set Cover Problem



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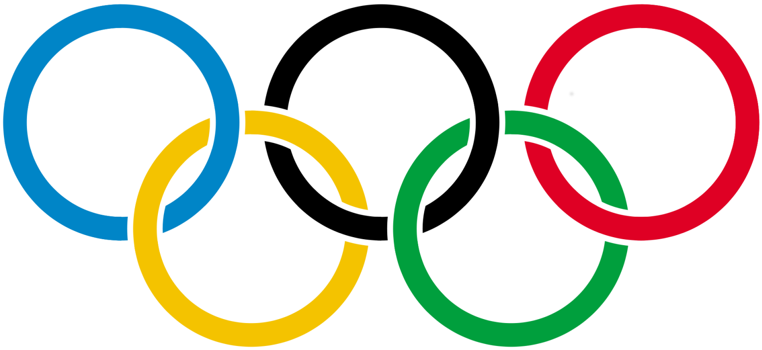
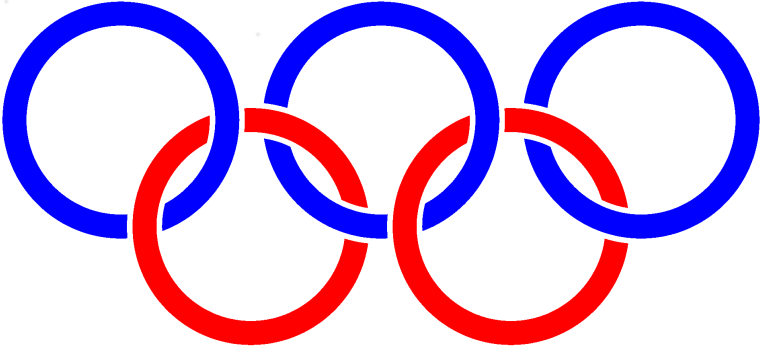
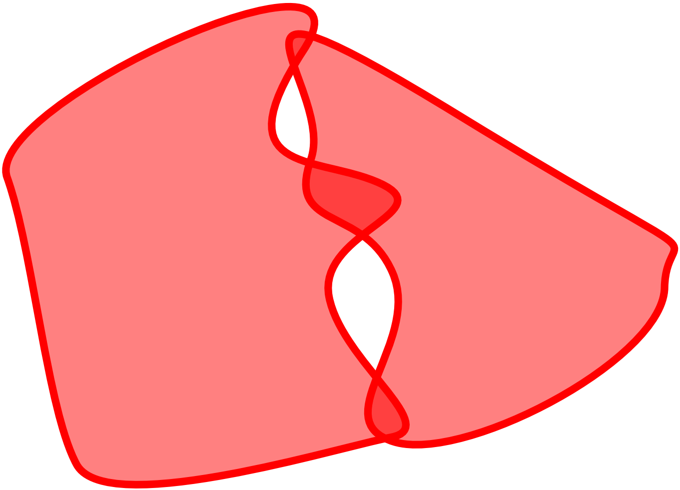


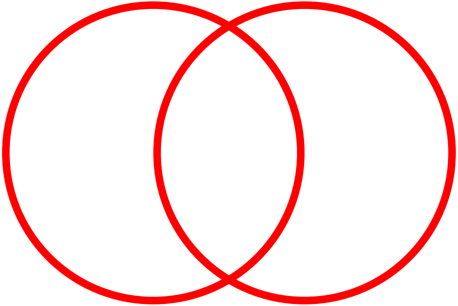


Planar Bipartite Graph

Minimal Lens



Bypassing Minimal Lens



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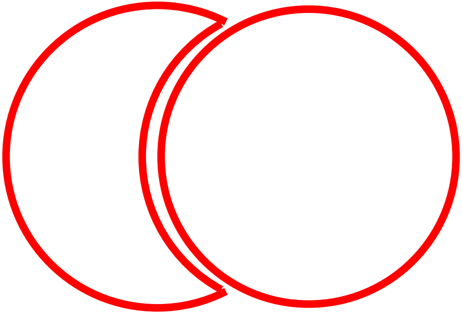

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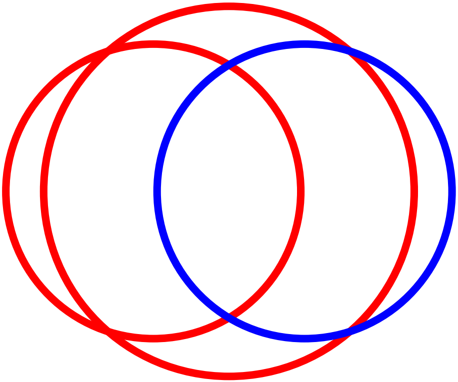

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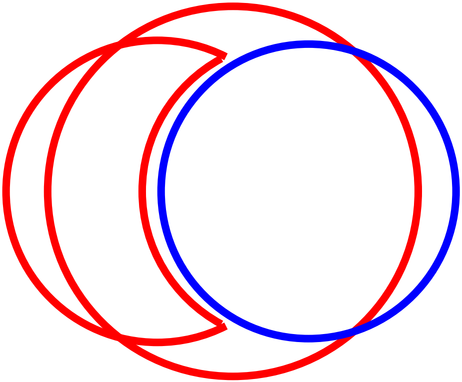

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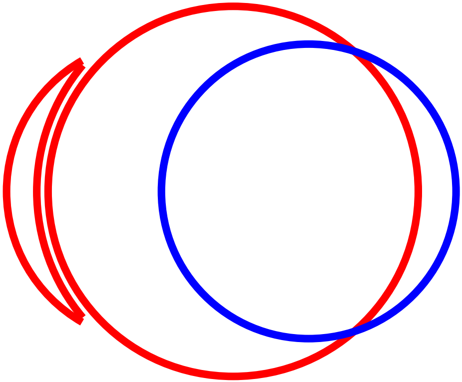
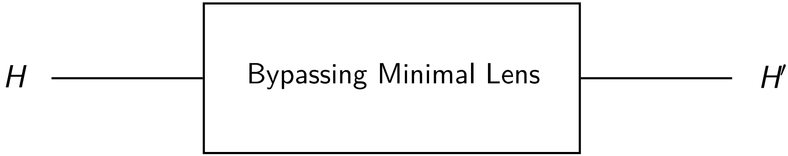




H' satisfies the *nice* properties $\Rightarrow$ H satisfies the *nice* properties.
$H \to H' \to \cdots \to$ Reds are disjoint and Blues are disjoint.

Set Cover
Dominating Set

Packing Regions
and Points

Packing Regions and Points

Maximization Problem

Aschner et al. '13

Suppose Π is a maximization problem. If there exists a bipartite graph $H = (\mathcal{R} \cup \mathcal{B}, E)$ that belongs to a family of graphs having the following 2 properties, then the Local Search algorithm is a PTAS for Π.

- A balanced vertex separator of sub-linear size.
- For any $\mathcal{R}' \subseteq \mathcal{R}$, $(\mathcal{B} \cup \mathcal{R}') \setminus N(\mathcal{R}')$ is a feasible solution. Here, $N(\mathcal{R}')$ denotes the set of neighbors of $\mathcal{R}'$ in H.

Packing Problem

Suppose Π is the Packing problem. If there exists a bipartite graph $H = (\mathcal{R} \cup \mathcal{B}, E)$ that belongs to a family of graphs having the following 2 properties, then the Local Search algorithm is a PTAS for Π.

- A balanced vertex separator of sub-linear size.
- For any $\mathcal{R}' \subseteq \mathcal{R}$, $(\mathcal{B} \cup \mathcal{R}') \setminus N(\mathcal{R}')$ is a feasible solution. Here, $N(\mathcal{R}')$ denotes the set of neighbors of $\mathcal{R}'$ in H.

Packing Problem

Suppose Π is the Packing problem. If there exists a bipartite graph $H = (\mathcal{R} \cup \mathcal{B}, E)$ that belongs to a family of graphs having the following 2 properties, then the Local Search algorithm is a PTAS for Π.

- A balanced vertex separator of sub-linear size.
- For any $\mathcal{R}' \subseteq \mathcal{R}$, $(\mathcal{B} \cup \mathcal{R}') \setminus N(\mathcal{R}')$ is a feasible solution. Here, $N(\mathcal{R}')$ denotes the set of neighbors of $\mathcal{R}'$ in H.

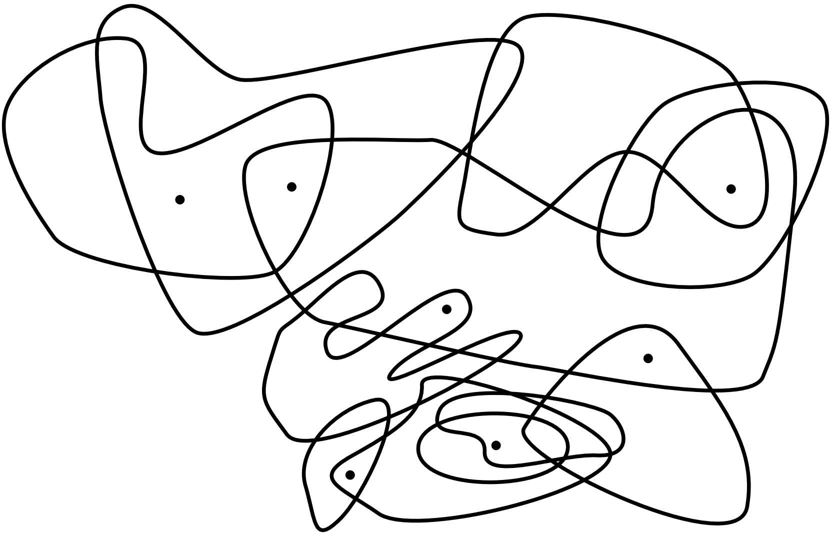
The intersection graph of $\mathcal{R} \cup \mathcal{B}$ is such a graph.

Capacitated Region Packing

Input:
$\mathcal{X}$ — A set of Non-piercing Regions*
P — A point set
ℓ — A constant positive integer

Output:
A maximum sized subset $\mathcal{X}' \subseteq \mathcal{X}$ such that every point $p \in P$ is contained in at most ℓ regions in $\mathcal{X}'$.

$\ell = 2$



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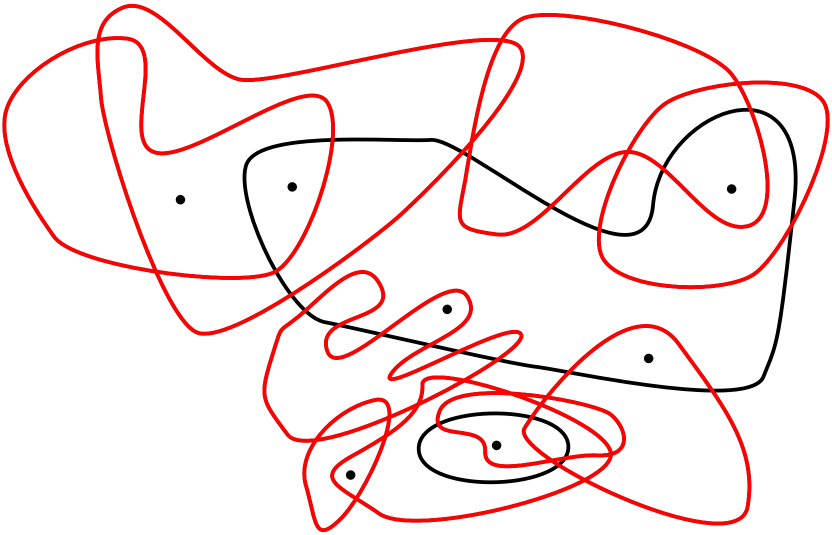


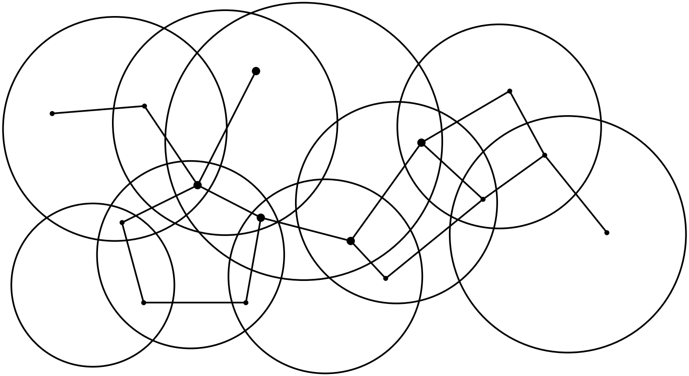
Capacitated Packing Problem

Suppose Π is the Capacitated Packing problem for non-piercing regions. If there exists a bipartite graph $H = (\mathcal{R} \cup \mathcal{B}, E)$ that belongs to a family of graphs having the following 2 properties, then the Local Search algorithm is a PTAS for Π.

- A balanced vertex separator of sub-linear size.
- For any $\mathcal{R}' \subseteq \mathcal{R}$, $(\mathcal{B} \cup \mathcal{R}') \setminus N(\mathcal{R}')$ is a feasible solution. Here, $N(\mathcal{R}')$ denotes the set of neighbors of $\mathcal{R}'$ in H.

The intersection graph of $\mathcal{R} \cup \mathcal{B}$ is such a graph.

Pyrga-Ray Graph ’08



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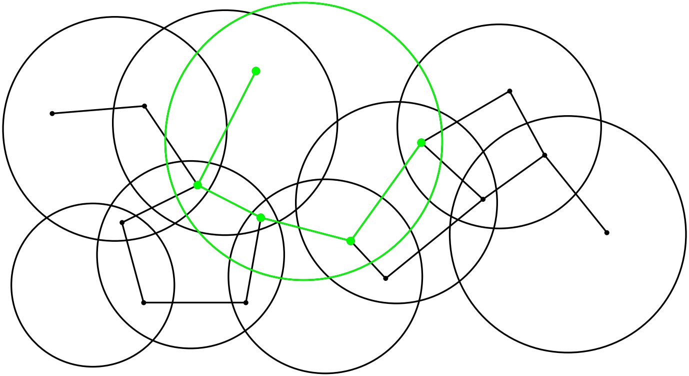

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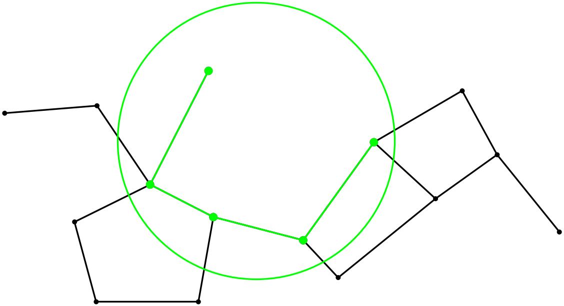

Packing Points

Input:
$\mathcal{X}$ — A set of Non-piercing Regions
P — A point set

Output:
A maximum sized subset $P' \subseteq P$ such that every region $X \in \mathcal{X}$ contains at most 1 point in P'.

Summary

The following problems admit PTAS for Non-piercing Regions using Local Search.

- Set Cover
- Dominating Set
- Capacitated Region Packing
- Point Packing

Mange Tak