

DAA  
Assignment - 2

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CST-SPL-1  
49

①

$$\begin{aligned} j &= 1 \\ j &= 2 \\ j &= 3 \\ &\vdots \\ j &= k \end{aligned}$$

$$\begin{aligned} i &= 1 \\ i &= 1 + 2 = 3 \\ i &= 3 + 3 = 1 + 2 + 3 \\ &\vdots \\ i &= 1 + 2 + 3 + \dots + k \end{aligned}$$

as  $i < n$

Sum of  $k$  consecutive integers  $= \frac{k(k+1)}{2}$

$$\therefore \frac{k(k+1)}{2} < n$$

$$\frac{k^2 + k}{2} < n$$

after removing constant

$$k^2 < n$$

$$k < \sqrt{n}$$

$$\therefore T(n) = O(\sqrt{n}) \underline{\text{Ans}}$$

②  $T(n) = 2T(n/2) + n^2$

using Master's method  $T(n) = aT(n/b) + f(n)$

$$a \geq 1, b > 1, c = \log_b a$$

$$c = \log_2 2 = 1$$

$$f(n) > n^c$$

$$T(n) = f(n)$$

$$O(n^2) \underline{\underline{Ans}}$$

⑤

①

1  
2  
3  
⋮  
n

②

1, 2, 3, ..., n times  
1, 3, 5, 7, ...,  $n/2$  times  
1, 4, 7, ...,  $n/3$  times  
⋮  
j=1 — 1 times

$$T(n) = n + \frac{n}{2} + \frac{n}{3} + \frac{n}{4} + \dots + 1$$

$$= n \left( 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{n} \right)$$

$$T(n) = n(\log n) \underline{\underline{A}}$$

⑥  $T(n) = 2, 2^k, 2^{k^2}, 2^{k^4}, \dots, 2^{k^{\log k (\log n)}}$

as we know,  $2^{k^{\log k (\log n)}} = 2^{\log n} = n$

$\therefore$  total iteration =  $\log k (\log n)$

$\therefore T(n) = O(\log k (\log n)) \underline{\underline{B}}$

⑧ (a)  $100 < \log(\log n) < \log(n) < \log^2 n < \sqrt{\log(n)} < n < n \log n < n^2 < 2^n < 4^n < 2^{2^n} < \log(n!) < n!$

(b)  $1 < \log(\log(n)) < \sqrt{\log n} < \log n < \log^2 n < 2 \log n < n < 2n < 4n < n \log n < n^2 < \log(n!) < n! < 2(2^n)$

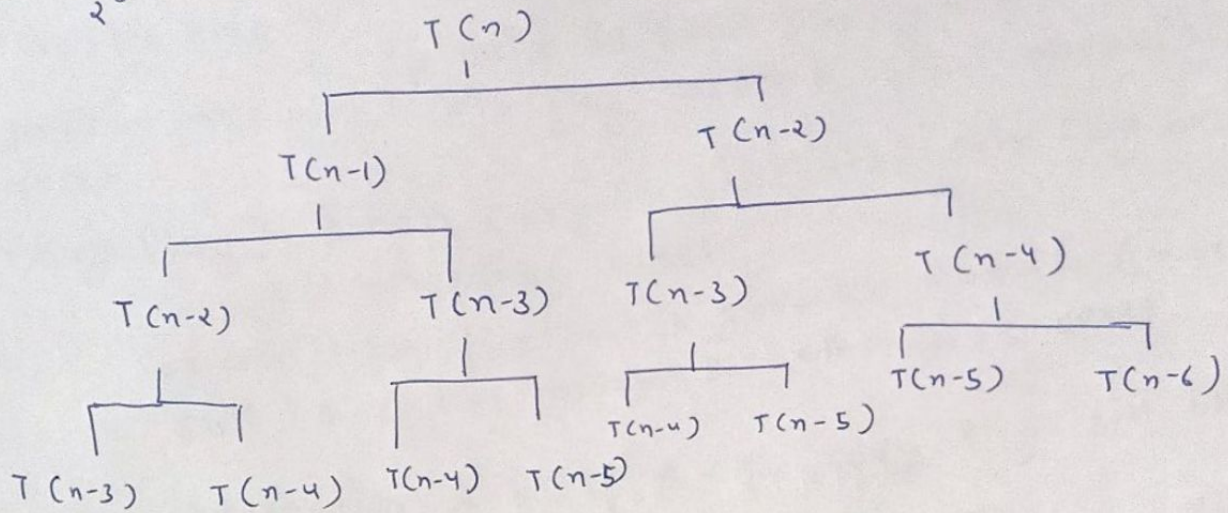
(c)  $ab < \log_8(n) < \log(n) < 5n < n \log_6 n < n \log_2 n < n! < \log n! < 8^{2n}$



② Recurrence relation of Fibonacci series:

$$T(n) = T(n-1) + T(n-2) + 1$$

$2^0$



$2^n$

$$T(n) = 2^0 + 2^1 + 2^2 + 2^3 + 2^4 + \dots + 2^n$$

now, sum of GP =  $\frac{a(r^n - 1)}{r - 1}$

$a=1, r=2$

$$1 \cdot \frac{(2^n - 1)}{2 - 1} = 2^n - 1 = (2^n - 1)$$

$$T(n) = O(2^n) \underline{\underline{A}}$$

space complexity depends on the depth of the tree

$\therefore$  space comp =  $O(n)$  M