

# Introduction to Parallel Prefix

October 27, 2019

# The Problem

## Parallel Prefix Problem

Given  $n$  inputs  $i_0 \dots i_{n-1}$  and  $n$  outputs  $o_0 \dots o_{n-1}$ , compute:

$$o_0 = i_0$$

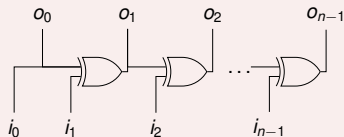
$$o_1 = i_0 \oplus i_1$$

$$o_2 = i_0 \oplus i_1 \oplus i_2$$

$$\vdots$$

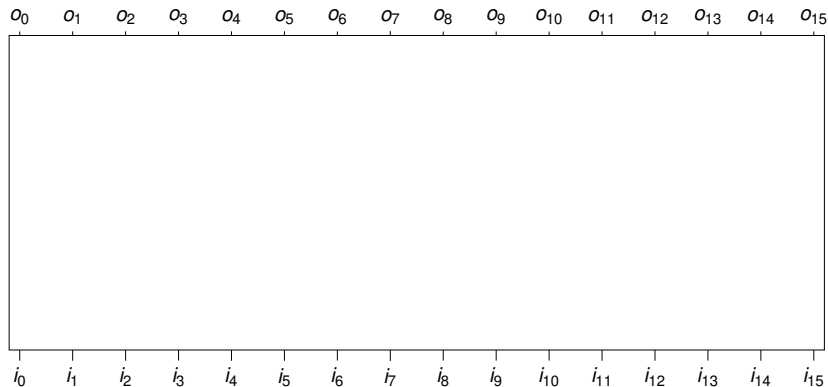
$$o_{n-1} = i_0 \oplus i_1 \oplus \dots \oplus i_{n-1}$$

## One Solution

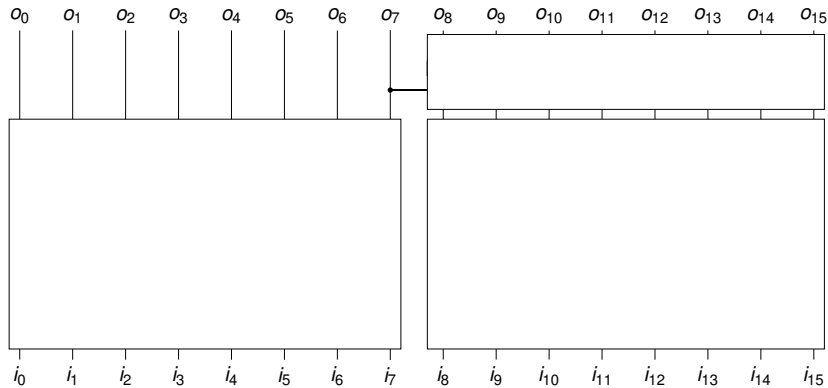


Gates =  $n - 1$ , Time =  $(n - 1)t_{xor}$

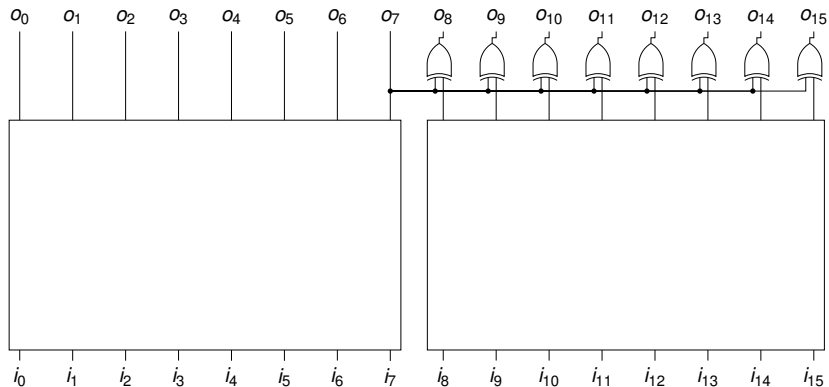
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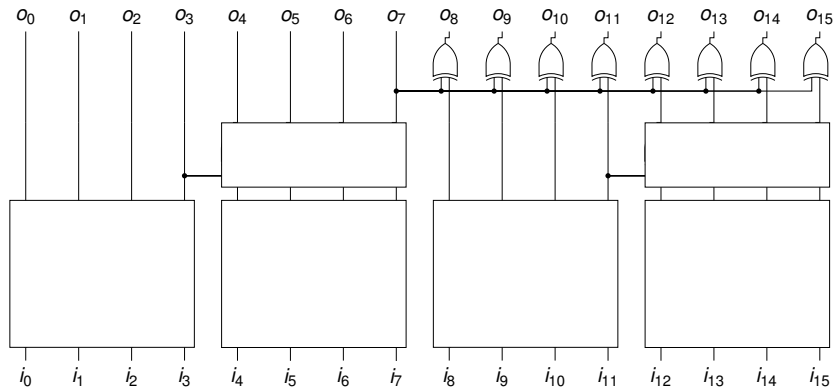
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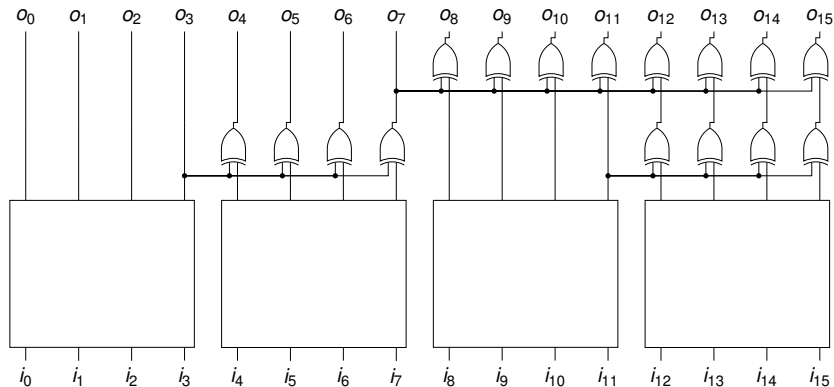
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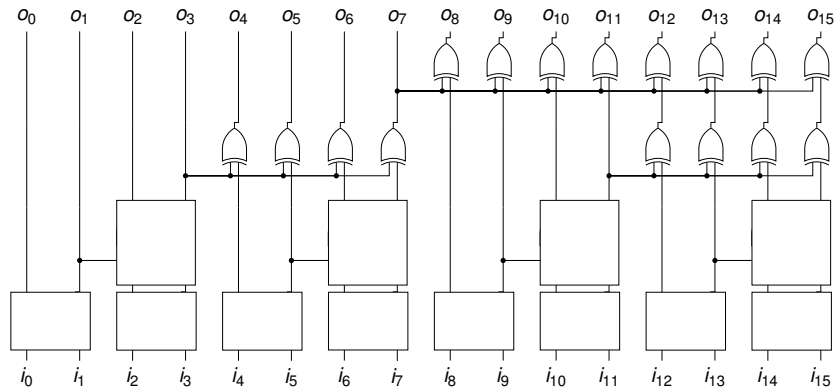
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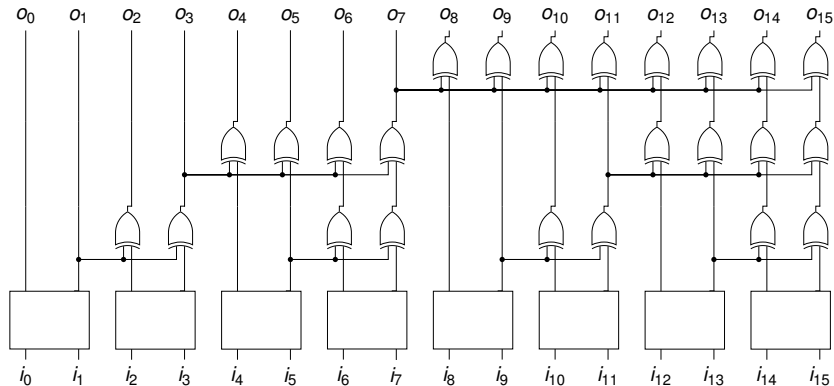


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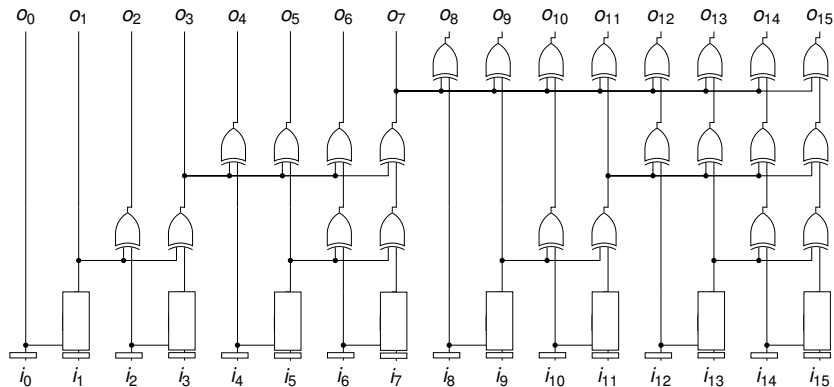




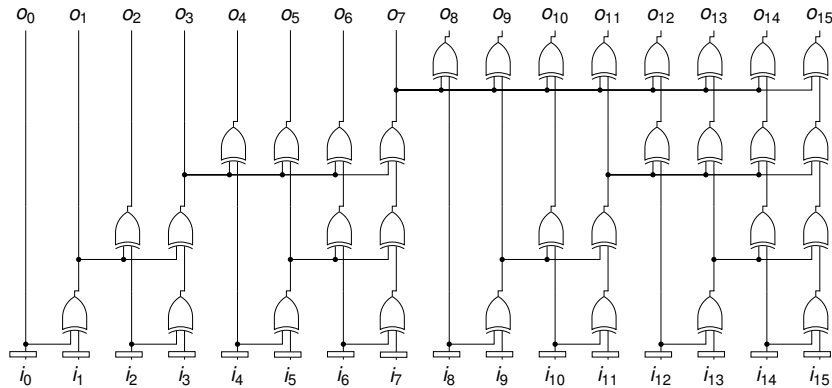
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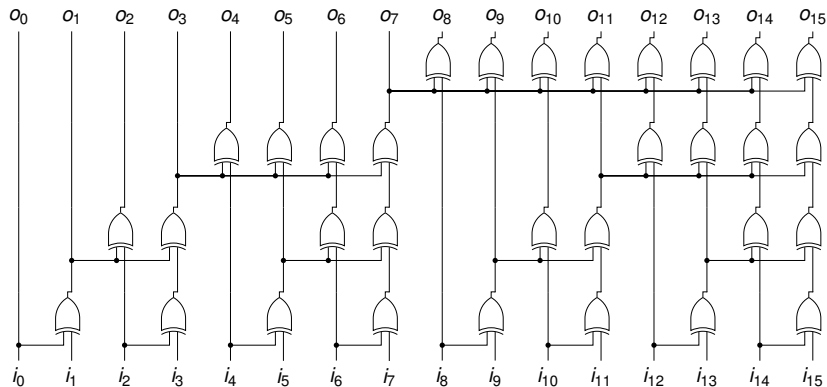
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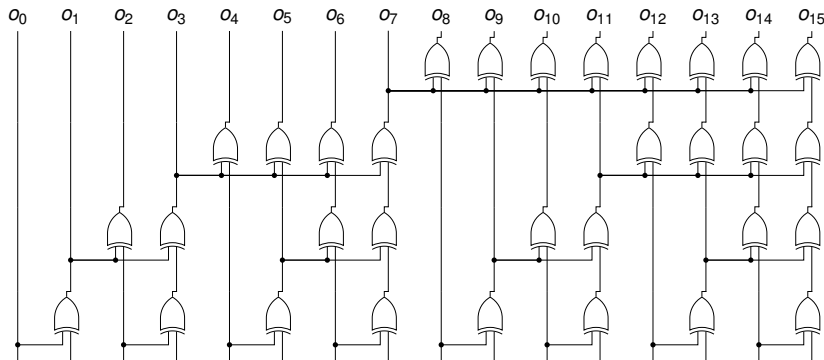
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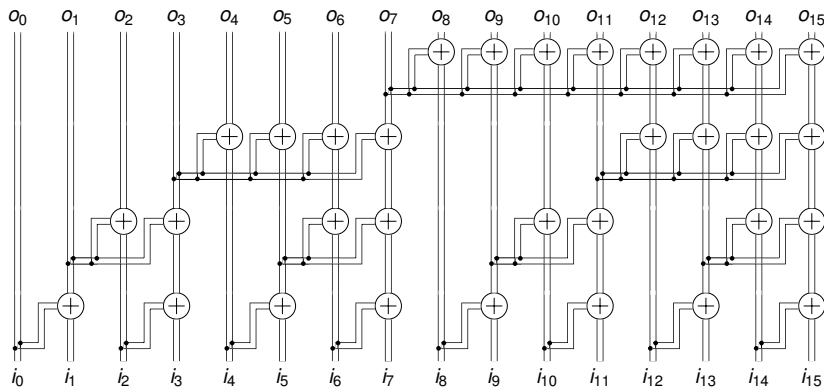


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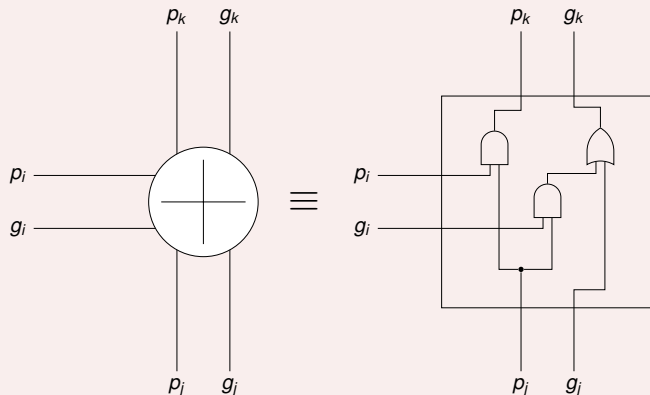
$$\text{Gates} = \frac{n}{2} \log_2 n, \text{ Time} = (\log_2 n) t_{xor}$$

# Adapting to Carry Computation



# Carry Logic

## Carry Logic (Associative)



- The generate outputs used as carry
- Computed in time  $(\log_2 n) t_{\text{carry\_logic}}$