MODEL QUESTION PAPER 1

Theory of Computation: A Problem-Solving Approach

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Time: 3 Hrs | Answer All Questions | Max Marks: 100

1.		Old McDonald had a farm… And on that farm he had some chickens… and some sheep… and some horses… and some dogs. On his farm, there must always be twice as many chickens as sheep; twice as many sheep as horses; and twice as many horses as dogs. If we represent the animals on this farm using the alphabet $\sum$ = {c, s, h, d} standing for chicken, sheep, horse and dog, respectively, then a string such as ccccccccssshhd represents 8 chickens, 4 sheep, 2 horses and 1 dog (always in that order). Consider the formal language of all such strings with the above constraints.	
	a)	What class of formal languages does this language belong to if there can be at most 1000 animals on the farm? Prove your answer.	06
	b)	What class does it belong to if there is no limit on the number of animals? Prove your answer. (E.g., if you claim that the language is context-free, you must prove that it is context-free AND you must prove that it is not regular.)	14
2.	a)	Explain precisely why the Modified Post Correspondence Problem is undecidable.	06
	b)	Construct a standard Turing Machine that takes as input a string $w = w_1w_2w_3w_4\ldots w_n$ where each w_i is a or b and n is an even number, and halts with the contents of the tape being $w_{n-1}w_{n-3}\ldots w_5w_3w_1$.	08
	c)	Prove that there exists a recursively enumerable language whose complement is not recursively enumerable.	06
3.	a)	Write a regular expression for all strings over {0, 1} in which the third and the fourth symbols are the same as the first and the second (respectively), the 7^{th} and 8^{th} symbols are the same as the 5^{th} and 6^{th}, and so on. Strings may be of any length. For example, 01011010101 is in the language.	08
	b)	Construct a DFA that is equivalent to the above regular expression.	08
	c)	Construct a right-linear grammar for the above language.	04
4.	a)	State precisely and prove the Pumping Lemma for context-free languages.	10
	b)	We know that the concatenation of two regular languages is a regular language. However, consider the language L of all even palindromes over {0, 1}; L is not regular. Now consider the languages $L_1 = \{w \mid w.w^R \varepsilon L\}$ and $L_2 = \{ w^R \mid w.w^R \varepsilon L\}$. Show that L_1 and L_2 are regular. Explain why although L_1 and L_2 are regular, L is not regular.	10
5.	a)	Construct a context-free grammar for $L = \{a^nb^mc^k \mid 2m = n \text{ and } k \geq 2\}$.	08
	b)	Construct a deterministic pushdown automaton for the above language.	06
	c)	Convert the grammar S $\rightarrow$ YXY, Y $\rightarrow$ bba | λ, X $\rightarrow$ Y | Yc to Chomsky Normal Form.	06