

PES UNIVERSITY, BANGALORE-85

(Established under Karnataka Act. No. 16 of 2013)

ESA

UE18MA251-LINEAR ALGEBRA & ITS APPLICATIONS

Q. No.	Answer the following questions	Marks
1a	Propane is a common gas used for cooking and home heating. Each molecule of propane is comprised of 3 atoms of carbon, and 8 atoms of hydrogen written as C_3H_8 . When propane burns, it combines with oxygen gas O_2 to form carbon dioxide CO_2 and water H_2O . Balance the chemical equation $C_3H_8 + O_2 \rightarrow CO_2 + H_2O$ that describes this process.	6
b	Use the Gauss – Jordan method to invert the following matrices $A = \begin{bmatrix} 1 & 2 & -1 \\ -1 & 1 & 2 \\ 2 & -1 & 1 \end{bmatrix}$	7
c	Write down the elementary matrices E , F , G associated with the system of equations $2u + v + 3w = -1$, $4u + v + 7w = 5$, $-6u - 2v - 12w = -2$. Also find the LU decomposition of A .	7
2a	Reduce these matrices to their echelon form to find their rank. Also find a special solution to each of the free variables. $A = \begin{bmatrix} 1 & 2 & 2 & 4 & 6 \\ 1 & 2 & 3 & 6 & 9 \\ 0 & 0 & 1 & 2 & 3 \end{bmatrix}$	6
b	For every c , find R and the special solutions to $Ax = 0$: $A = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 2 & 4 \\ 1 & c & 2 \end{bmatrix}$	7
c	If the column space of A is spanned by the vectors $(1, 4, 2)$, $(2, 5, 1)$ and $(3, 6, 0)$ find all those vectors that span the left null space of A . Determine whether or not the vector $b = (4, -2, 2)$ is in that subspace. What are the dimensions of $C(A^T)$ and $N(A^T)$?	7
3a	On the space P_3 of cubic polynomials, what matrix represents $\frac{d^2}{dt^2}$? Find its null space and column space. What do they mean in terms of polynomials?	6

b	What multiple of $a = (1, 1, 1)$ is closest to $b = (2, 4, 4)$? Find also the point on the line through b that is closest to a .	7												
c	<p>An ice- cream vendor records the number of hours of sun shine (x) versus the number of ice- creams sold in an hour (y) at his shop from Monday to Friday and found the following data :</p> <table><tr><td>x</td><td>2</td><td>3</td><td>5</td><td>7</td><td>9</td></tr><tr><td>y</td><td>4</td><td>5</td><td>7</td><td>10</td><td>15</td></tr></table> <p>Find the best values of m and c that suit the equation $y = mx + c$. If there is a weather forecast that says there would be 8 hours of sun shine the next day, estimate the number of ice- creams that he expects to sell on that day.</p>	x	2	3	5	7	9	y	4	5	7	10	15	7
x	2	3	5	7	9									
y	4	5	7	10	15									
4a	Find the largest Eigen value and the corresponding Eigen vector of a matrix $A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$ by using the initial vector $x_0=(1, 1, 1)$.	6												
b	Find orthogonal vectors q_1, q_2, q_3 by Gram- Schmidt method from $a = (1, 1, 0), b = (1, 0, 1)$ and $c = (0, 1, 1)$.	7												
c	Factor the matrix $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ in to SAS^{-1} . Also find SAS^{-1} .	7												
5a	Let A be an $n \times d$ matrix with right singular vectors v_1, v_2, \dots, v_r , left singular vectors u_1, u_2, \dots, u_r , and corresponding singular values $\sigma_1, \sigma_2, \dots, \sigma_r$. Then $A = \sum_{i=1}^r \sigma_i u_i v_i^T$.	6												
b	Test the following matrices for positive or semi definite $A = \begin{bmatrix} 5 & 2 & 1 \\ 2 & 2 & 2 \\ 1 & 2 & 5 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}$	7												
c	Find SVD for Matrix $A = \begin{bmatrix} 1 & -1 \\ -2 & 2 \\ 2 & 2 \end{bmatrix}$.	7												