SRN | | | | | | | | | | | | |

PES UNIVERSITY, BENGALURU-560100
(Estd. Under Karnataka Act 10 of 2013)

UE18CS254

April 2020: (ESA) Model paper - BTech 4th Sem

UE18CS254 – Theory of Computation

Time: 3 Hrs **Answer All Questions** Max Marks: 100

SRN

1.	a)	What is alphabet and strings and language? Explain with examples. Explain following functions on strings with example: • Length • Concatenation • Replication • Reversal	8

<u>Ans:</u>

Alphabet:

- An alphabet is a non-empty, finite set of characters/symbols
- Use Σ to denote an alphabet
- Examples
  - Σ= { a, b }
  - Σ= { 0, 1, 2 }
  - Σ= { a, b, c,…z, A, B, … Z }
  - Σ= { #, $, *, @, & }

***<u>String</u>*: finite sequence of symbols from Σ, such as v = aba and w = abaaa**

- **Empty string (λ)**
- **Substring, prefix, suffix**

Language:

- A *language* is a (finite or infinite) set of strings over a (finite) alphabet Σ
- Examples: Let Σ = {a, b}
- Some languages over Σ:
- ∅ = { }   // the empty language, no strings
- {ε}   // language contains only the empty string
- {a, b}
- {ε, a, aa, aaa, aaaa, aaaaa}

Functions on strings:

Length:

- $|s|$ is the length of string s
- $|s|$ is the number of characters in string s.
- $|\varepsilon| = 0$
- $|1001101| = 7$
- $\#_c(s)$ is defined as the number of times that c occurs in s.
  $\#_a(abbaaa) = 4$.

***Concatenation*:**

- the *concatenation* of 2 strings s and t is the string formed by appending t to s; written as s||t or more commonly, *st*
- Example:
  If x = good and y = bye, then xy = goodbye and yx = byegood

Replication:

- For each string w and each natural number k, the string w^k is,

$$w^0 = \varepsilon$$
$$w^{k+1} = w^k w$$

Reverse:

- For each string w, w^R is defined as:
  if $|w| = 0$ then $w^R = w = \varepsilon$
  if $|w| = 1$ then $w^R = w$
  if $|w| > 1$ then:
  $\exists a \in \Sigma\ (\exists u \in \Sigma^*\ (w = ua))$
  So define $w^R = a\, u^R$

SRN

b) Define Deterministic Finite Automata . Design DFA for the following languages: 3+4

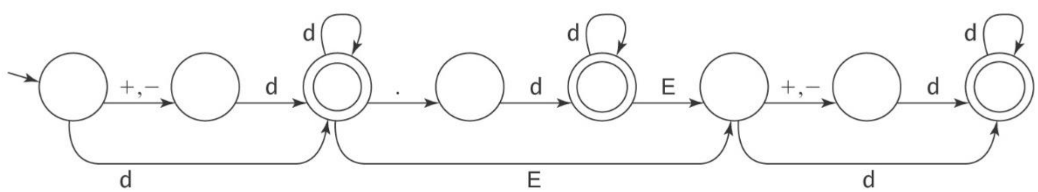
1) L={ w: w is the string representation of Floating Point numbers}
   a. FP no is optional sign, followed by decimal no, followed by optional exponent.
   b. Decimal no of the form x or x.y (33 or 33.54)
   c. Exponent begins with E , followed by optional sign and integer.
   d. Integer is nonempty string of decimal digits.

Solution: Sol:- A *deterministic finite accepter* is defined by
Q: a finite set of *internal states*
Σ: a set of symbols called the *input alphabet*
δ: a *transition function* from Q X Σ to Q
q0: the *initial state*
F: a subset of Q representing the *final states*

L={ w: w is the string representation of Floating Point numbers}

e. FP no is optional sign, followed by decimal no, followed by optional exponent.
f. Decimal no of the form x or x.y (33 or 33.54)
g. Exponent begins with E , followed by optional sign and integer.
h. Integer is nonempty string of decimal digits.

Example strings:
+3.0, 3.0, 0.3E1, 0.3E+1, -0.3E+1, -3E8

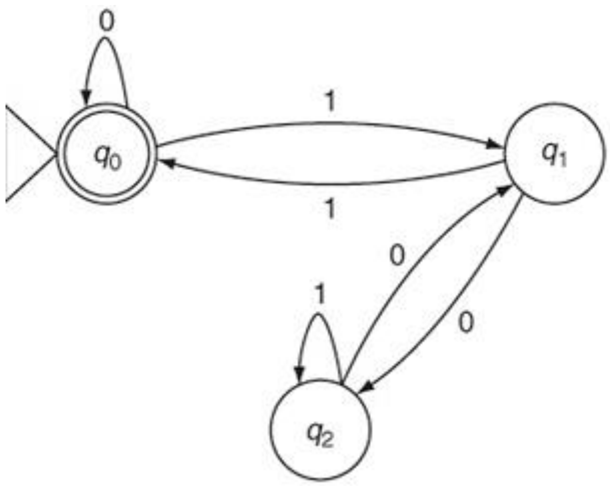
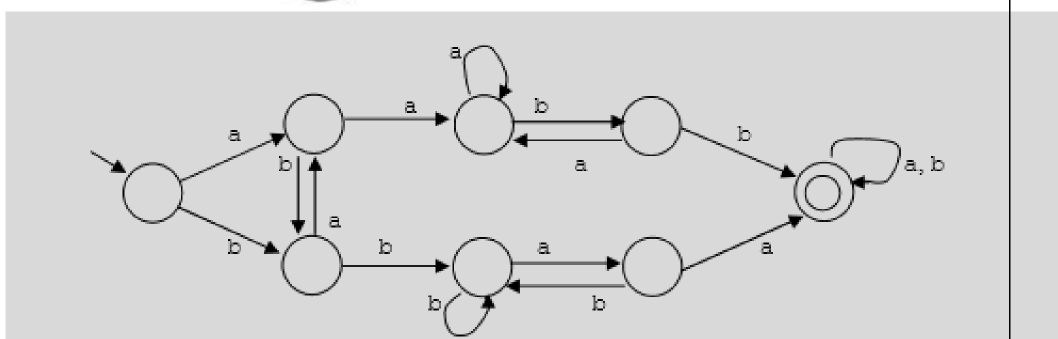


c Prove, If w and x are strings then $(wx)^R = x^R w^R$ 5

Proof: By induction on $|x|$:
$|x| = 0$: Then $x = \varepsilon$, and $(wx)^R = (w\,\varepsilon)^R = (w)^R = \varepsilon\, w^R = \varepsilon^R\, w^R = x^R\, w^R$.
$\forall n \geq 0\ (((|x| = n) \rightarrow ((w\,x)^R = x^R\, w^R)) \rightarrow$
$((|x| = n + 1) \rightarrow ((w\,x)^R = x^R\, w^R)))$:
Consider any string x, where $|x| = n + 1$. Then $x = u\,a$ for some character a and $|u| = n$. So:

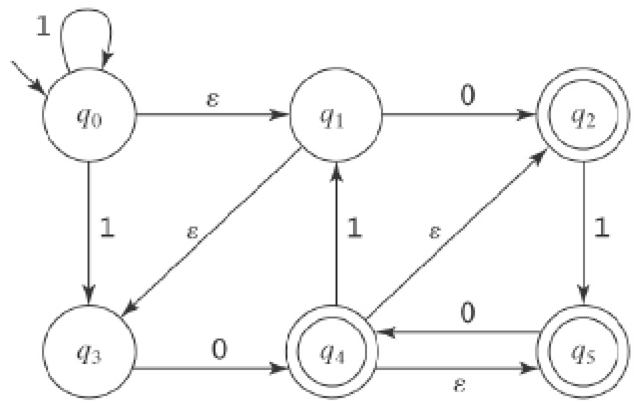
$(w\,x)^R = (w\,(u\,a))^R$	rewrite x as ua
$= ((w\,u)\,a)^R$	associativity of concatenation
$= a\,(w\,u)^R$	definition of reversal
$= a\,(u^R\, w^R)$	induction hypothesis
$= (a\,u^R)\, w^R$	associativity of concatenation
$= (ua)^R\, w^R$	definition of reversal
$= x^R\, w^R$	rewrite ua as x

SRN

2.	a)	Design DFSM for the following languages and write all the five tuples. i) L = { w contains {0, 1}* , accepting Binary number divisible by 3} ii) L = {w contains {a, b}* : w has both aa and bb as a substrings}.  i)  ii)	8

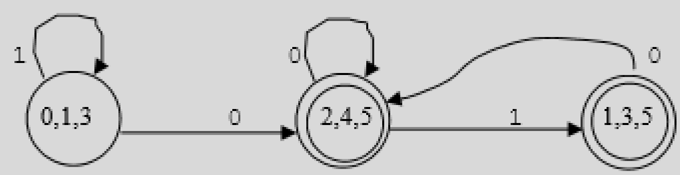
SRN

b) Convert the given NFA to equivalent DFA — 8



Solution:

$\{q_0, q_1, q_3\}$	0	$\{q_2, q_4, q_5\}$
	1	$\{q_0, q_1, q_3\}$
$\{q_2, q_4, q_5\}$	0	$\{q_2, q_4, q_5\}$
	1	$\{q_1, q_3, q_5\}$
$\{q_1, q_3, q_5\}$	0	$\{q_2, q_4, q_5\}$
	1	{ }



c) Write regular expression for — 4

$L = \{w\ \varepsilon\ \{a,b\}^* | w = a^{2n}b^{2m} | n >= 0, m >= 0\}$

$L = \{w\ \varepsilon\ \{a,b\}^* |$ w does not end in ba $\}$

Regular expression = $(aa)^* (bb)^*$

Regular expression = (a+b)* (a+b+aa+ab+bb+ε)

SRN

3.	a)	What is Context Free Grammar? Write CFG for the balanced parenthesis language.	5+5

Ans:

In CFG the rule must:

– Have a left hand side that is single NT
– Have a right hand side with no constraint
– More flexible, more powerful

S -> aSb , S -> ϵ, T -> T, S -> aSbbTT

A context-free grammar G is a quadruple,

(V, Σ, R, S), where:

● V is the rule alphabet, which contains nonterminals and terminals.
● Σ (the set of terminals) is a subset of V,
● R (the set of rules) is a finite subset of $(V - \Sigma) \times V^*$,
● S (the start symbol) is an element of $V - \Sigma$.

Example:

$(\{S, a, b\}, \{a, b\}, \{S \rightarrow a S b, S \rightarrow \varepsilon\}, S)$

Consider L = { w: w contains balanced parenthesis}

Grammar is G = {{ S,),(} , {),(} , R, S } where R is:

$S \rightarrow \varepsilon$

$S \rightarrow SS$

$S \rightarrow (S)$

SRN

b) Consider the following grammar G:

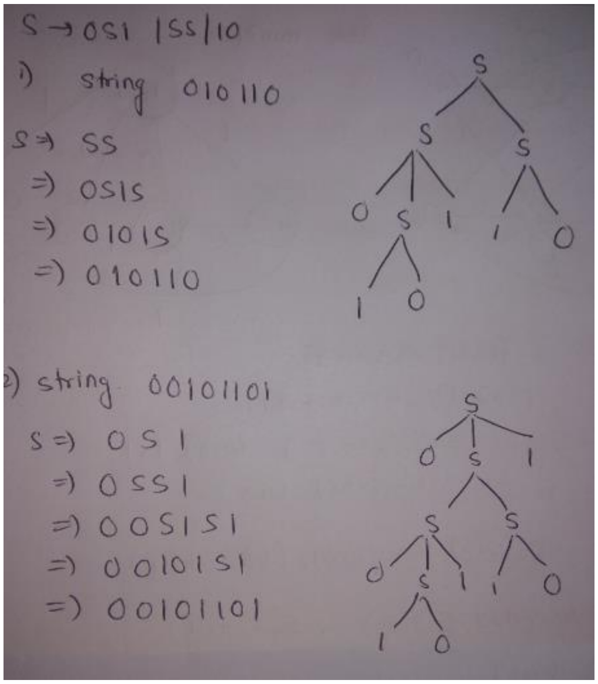
$S \rightarrow 0S1 \mid SS \mid 10$

Show the parse tree produced by G for each of the following strings:

1) 010110
2) 00101101

What is ambiguous grammar?

Ans:



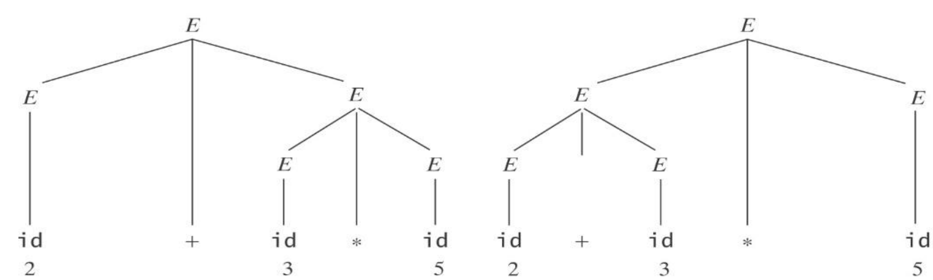
A grammar is ***ambiguous*** iff there is at least one string in $L(G)$ for which G produces more than one parse tree.

(for a string, ***more than 1 leftmost*** or ***more than 1 rightmost*** derivations exist)

Show that following grammar is ambiguous

$E \rightarrow E+E$, $E \rightarrow E*E$, $E \rightarrow (E)$, $E \rightarrow id$

String "id+id*id" has 2 leftmost derivations / parse trees:



5+5

SRN

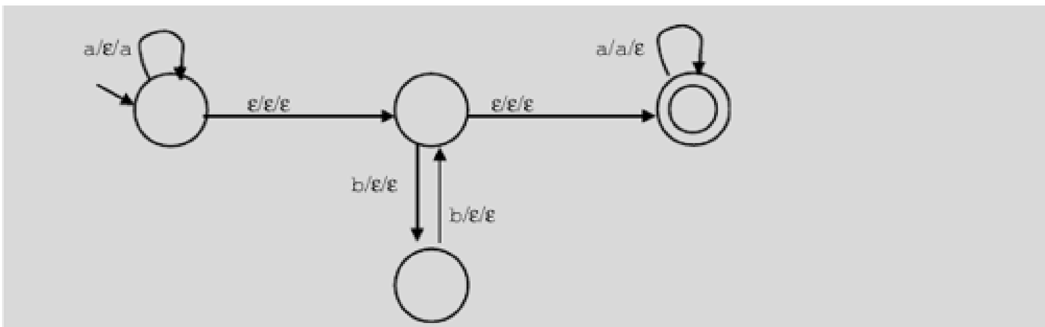
4. a) Define PDA. Build a PDA to accept the following language (2+6)

L = { $a^n b^m a^n$: n,m ≥ 0 and m is even }

- A pushdown automaton (PDA) is a seven-tuple:

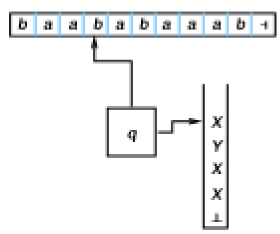
  M = (Q, Σ, Γ, δ, q_0, z_0, F)

  Q A finite set of states
  Σ A finite input alphabet
  Γ A finite stack alphabet
  q_0 The initial/starting state, q_0 is in Q
  z_0 A starting stack symbol, is in Γ // need not always remain at the bottom of stack
  F A set of final/accepting states, which is a subset of Q
  δ A transition function, where
  δ: Q x (Σ U {ε}) x Γ –> finite subsets of Q x Γ*



b) Define a deterministic PDA. Build a PDA to accept the following language (2+4)

$\{a^m b^n : m \leq n \leq 2m\}$.



A PDA with restrictions that:

- At most one move possible in any configuration.
  - For any state p, $a \in A$, and $X \in \Gamma$: at most one move of the form $(p, a, X) \to (q, \gamma)$ or $(p, \epsilon, X) \to (q, \gamma)$.
  - Effectively, a DPDA must see the current state, and top of stack, and decide whether to make an ϵ-move or read input and move.
- Accepts by final state.
- We need a right-end marker "⊣" for the input.

PDA for $\{a^m b^n : m \leq n \leq 2m\}$.

M = ({1, 2}, {a, b}, {a}, Δ, 1, {1, 2}), where Δ =
{ ((1, a, ε), (1, a)),
((1, ε, ε), (2, ε)),
((2, b, a), (2, ε)),
((2, b, aa), (2, ε)) }.

	c	State Pumping Lemma and Prove L = { $a^nb^nc^n$: n>=0 } is not a Context Free language	6

Pumping Lemma

For every CFL L there is a constant $k \geq 0$ such that for any word z in L of length at least k, there are strings u, v, w, x, y such that

- $z = uvwxy$,
- $vx \neq \epsilon$,
- $|vwx| \leq k$, and
- for each $i \geq 0$, the string uv^iwx^iy belongs to L.

Suppose L_{anbncn} is context-free. *Let p be the pumping length.*

- Consider $z = a^pb^pc^p \in L_{anbncn}$.
- *Since $|z| > p$, there are u, v, w, x, y such that $z = uvwxy$, $|vwx| \leq p$, $|vx| > 0$ and $uv^iwx^iy \in L$ for all $i \geq 0$.*
- Since $|vwx| \leq p$, vwx cannot contain all three of the symbols a, b, c, because there are p bs. So vwx either does not have any as or does not have any bs or does not have any cs. Suppose, (wlog) vwx does have any as. Then $uv^0wx^0y = uwy$ contains more as than either bs or cs. Hence $uwy \notin L$. □

5.	a)	Define Turing Machine. Design a Turing Machine that accepts the language denoted by Regular expression 00*.	4+ 4

- A *Turing Machine* is defined by (Q, Σ, Γ, δ, q_0, B, F)
  - A finite set of internal states Q
  - An input alphabet Σ
  - A tape alphabet Γ
  - A transition function δ
  - An initial state q_0 Ɛ Q
  - A special symbol B from Γ called the blank
  - A set of final states F

  δ: Q X T -> Q X Γ X {L, R}
- The defined values of the transition function for the given language

$\delta(q_0, 0) = (q_1, 0, R)$

$\delta(q_1, 0) = (q_1, 0, R)$

$\delta(q_1, B) = (q_2, B, R)$

SRN

	b)	Design and draw a Turing Machine for L = { $a^nb^nc^n$: n>=0 }. Write a note on "CHURCH- TURING Thesis". L = {L = $a^nb^nc^n$ | n≥ 1 } • The transition function are $\delta(q_0, a) = (q_1, X, R)$ $\delta(q_1, a) = (q_1, a, R)$ $\delta(q_1, b) = (q_2, Y, R)$ $\delta(q_2, b) = (q_2, b, R)$ $\delta(q_2, c) = (q_3, Z, L)$ $\delta(q_3, b) = (q_3, b, L)$ $\delta(q_3, Y) = (q_3, Y, L)$ $\delta(q_3, a) = (q_3, a, L)$ $\delta(q_3, X) = (q_0, X, R)$ $\delta(q_1, Y) = (q_1, Y, R)$ $\delta(q_2, Z) = (q_2, Z, R)$ $\delta(q_3, Z) = (q_3, Z, L)$ $\delta(q_0, Y) = (q_4, Y, R)$ $\delta(q_4, Y) = (q_4, Y, R)$ $\delta(q_4, Z) = (q_5, Z, R)$ $\delta(q_5, Z) = (q_5, Z, R)$ $\delta(q_5, B) = (q_6, B, R)$ • Turing and Church showed that anything that can be computed can be computed by Turing Machine. • No one so far has been able to find a computing problem that Turing machines can not compute but some other machine, mechanism can. • This claim is known as **The Church-Turing Thesis.**	4+ 2
	C	Define PCP and Obtain the solution for the following system of PCP A = {1, 10111, 10} and B = {111, 10, 0}. **PCP** - Given two sequences of n strings on some alphabet Σ, for instance $A = w_1, w_2, \ldots, w_n$ and $B = v_1, v_2, \ldots, v_n$ there is a Post correspondence solution (PC solution) for the pair (A, B) if there is a nonempty sequence of integers i, j, …, k, such that $w_iw_j\ldots w_k = v_iv_j\ldots v_k$ The solution for the following system of PCP A = {1, 10111, 10} and B = {111, 10, 0}. is 2113.	2+ 4