MT210 Midterm 1 Sample 2

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Question 1. SYSTEMS OF LINEAR EQUATIONS

The augmented matrix of a linear system has the form

$$\begin{bmatrix} a & 1 & 1 \\ 2 & a-1 & 1 \end{bmatrix}$$

Determine the values of a for which the linear system is consistent.

Question 2. ROW REDUCTION AND ECHELON FORMS

Write the augmented matrix corresponding the system below:

$$
\begin{array}{rrrrrrr}
x_1 & - & 6x_2 & - & 4x_3 & = & -5 \\
2x_1 & - & 10x_2 & - & 9x_3 & = & -4 \\
-x_1 & + & 6x_2 & + & 5x_3 & = & 3.
\end{array}
$$

Solve the system by applying the row reduction algorithm. If the system is consistent, find the general solution set.

Determine if $\mathbf{b}$ is a linear combination of the vectors $\mathbf{a}_1$, $\mathbf{a}_2$, and $\mathbf{a}_3$ where

$$\mathbf{a}_1 = \begin{bmatrix} 2 \\ -3 \\ 4 \\ 1 \end{bmatrix}, \mathbf{a}_2 = \begin{bmatrix} 1 \\ 6 \\ -1 \\ 2 \end{bmatrix}, \mathbf{a}_3 = \begin{bmatrix} -1 \\ -1 \\ 2 \\ 3 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 3 \\ -17 \\ 17 \\ 7 \end{bmatrix}.$$

If $\mathbf{b}$ is a linear combination of the vectors $\mathbf{a}_1$, $\mathbf{a}_2$, and $\mathbf{a}_3$, express $\mathbf{b}$ as a linear combination of the vectors $\mathbf{a}_1$, $\mathbf{a}_1$, and $\mathbf{a}_1$.

QUESTION 4. THE MATRIX EQUATION $A\mathbf{x}=\mathbf{b}$

A.) Write the given matrix equation below as system of linear equations:

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & -2 \\ 2 & 0 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -5 \\ 5 \end{bmatrix}.$$

B.) Solve the system and write the general solution.

Question 5. SOLUTION SETS OF LINEAR SYSTEMS

A. Solve the nonhomogeneous system $A\mathbf{x}=\mathbf{b}$ and write the solution in parametric vector form where

$$A = \begin{bmatrix} 2 & 1 & -1 \\ 1 & 2 & -3 \\ -1 & 2 & -4 \end{bmatrix} \text{ and } \mathbf{b} = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}.$$

B. Using the parametric vector form of the solution set in part **A.**, determine a particular solution **p**.

C. Write the general solution for the system $A\mathbf{x} = \mathbf{0}$ in parametric vector form.

QUESTION 6. LINEAR INDEPENDENCE

Find the value(s) of h for which the following set of vectors

$$\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} h \\ 1 \\ -h \end{bmatrix}, \begin{bmatrix} 1 \\ 2h \\ 3h+1 \end{bmatrix} \right\}$$

is **linearly independent**.