ME681 Assignment 1

Due Date: 17/01/17 (Next Tuesday)

January 9, 2017

Note: Please read Chapter 1 of the text book carefully. In fact read Section 1.7, which we have not covered in class. All the following problems are from the exercises in the text.

Question 1. Which number q makes this system singular and which right hand side t gives it infinitely many solutions? Find the solution that has $z = 1$. Answer using Gauss elimination.

$$\begin{aligned} x + 4y - 2z &= 1 \\ x + 7y - 6z &= 6 \\ 3y + qz &= t. \end{aligned}$$

Question 2. For which three numbers a will elimination fail to give three pivots?

$$\begin{aligned} ax + 2y + 3z &= b_1 \\ ax + ay + 4z &= b_2 \\ ax + ay + az &= b_3. \end{aligned}$$

Question 3. Which three matrices E_{21}, E_{31}, E_{32} put A into a triangular form U?

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 4 & 6 & 1 \\ -2 & 2 & 0 \end{bmatrix} \text{ and } E_{32}E_{31}E_{21}A = U.$$

Multiply the E's to get one matrix M that does elimination: $MA = U$.

Question 4. Decide whether the following systems are singular or non singular and whether they have no solution, one solution or infinitely many solutions:

$$\begin{aligned} v - w &= 2 \\ u - v &= 2 \\ u - w &= 2 \end{aligned}$$

$$\begin{aligned} v - w &= 0 \\ u - v &= 0 \\ u - w &= 0 \end{aligned}$$

$$\begin{aligned} v + w &= 1 \\ u + v &= 1 \\ u + w &= 1. \end{aligned}$$

Question 5. Compute L and U for the symmetric matrix

$$A = \begin{bmatrix} a & a & a & a \\ a & b & b & b \\ a & b & c & c \\ a & b & c & d \end{bmatrix}.$$

Find four conditions on a, b, c, d to get $A = LU$ with four pivots.

Question 6. What are L and D for the following matrix A? What is U in $A = LU$ and what is the new U in $A = LDU$?

$$A = \begin{bmatrix} 2 & 4 & 8 \\ 0 & 3 & 9 \\ 0 & 0 & 7 \end{bmatrix}.$$

Question 7. Use Gauss-Jordan elimination on $[A \quad I]$ to solve $AA^{-1} = I$:

$$\begin{bmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{x_1} & \mathbf{x_2} & \mathbf{x_3} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Question 8. The following matrix has a remarkable inverse. Find A^{-1} by elimination on $[A \quad I]$. Extend to a 5 by 5 "alternating matrix" and guess its inverse

$$A = \begin{bmatrix} 1 & -1 & 1 & -1 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Question 9. Solve $Ax = b$ by solving the triangular systems $Lc = b$ and $Ux = c$:

$$A = LU = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 2 & 4 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}, b = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

What part of A^{-1} have you found with this particular b?

Question 10. Solve by elimination or show that there is no solution:

$$\begin{aligned} u + v + w &= 0 \\ u + 2v + 3w &= 0 \\ 3u + 5v + 7w &= 1 \end{aligned} \qquad\qquad \begin{aligned} u + v + w &= 0 \\ u + v + 3w &= 0 \\ 3u + 5v + 7w &= 1. \end{aligned}$$