

**JAN – MAY- 2020: END SEMESTER ASSESSMENT (ESA) B.TECH. IV SEMESTER  
UE18MA251 – LINEAR ALGEBRA**

**Instructions:** You may bring only Calculators.

Time: 3 Hr

Answer All Questions.

Max Marks: 100

1.		Let $A = \begin{bmatrix} 1 & 2 & 0 & 1 \\ 2 & 4 & 1 & 4 \\ 3 & 6 & 3 & 9 \end{bmatrix}$	
	a)	Find a lower triangular L and an upper triangular U so that $A = LU$ .	10
	b)	Find the reduced row echelon form $R = \text{rref}(A)$ . How many independent columns in A?	10
2.	a)	Find a basis for the nullspace of A.	10
	b)	If the vector b is the sum of the four columns of A, write down the complete solution to $Ax = b$ .	10
3.	a)	Consider a $120^\circ$ rotation around the axis $x = y = z$ . Show that the vector $i = (1, 0, 0)$ is rotated to the vector $j = (0, 1, 0)$ . (Similarly $j$ is rotated to $k = (0, 0, 1)$ and $k$ is rotated to $i$ .) How is $j - i$ related to the vector $(1, 1, 1)$ along the axis?	10
	b)	This problem finds the curve $y = C + D2^t$ which gives the best least squares fit to the points $(t,y) = (0, 6), (1, 4), (2, 0)$ . Write down the 3 equations that would be satisfied if the curve went through all 3 points.	10
4	a)	Let $A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \\ 3 & 6 \end{bmatrix}$  Find the eigenvalues of $A^T A$ and also of $AA^T$ . For both matrices find a complete set of orthonormal eigenvectors.	10
	b)	If you apply the Gram-Schmidt process (orthonormalization) to the columns of this matrix A, what is the resulting output?	10

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5	a)	If A is any m by n matrix with $m > n$ , tell me why $AA^T$ cannot be positive definite. Is $A^T A$ always positive definite? (If not, what is the test on A?)	10
	b)	If a 3 by 3 matrix P projects every vector onto the plane $x+2y+z=0$ , find three eigen values and three independent eigenvectors of P. No need to compute P.	10