ME681 Assignment 2

Due Date: 24/01/17 (Next Tuesday)

January 16, 2017

Question 1. Which of the following subsets of R^3 are actually subspaces?
(a) The plane of vectors (b_1, b_2, b_3) with first component $b_1 = 0$.
(b) The plane of vectors b with $b_1 = 1$.
(c) The vectors b with $b_2 b_3 = 0$ (this is the union of two subspaces, the plane $b_2 = 0$ and the plane $b_3 = 0$).
(d) All combinations of two given vectors (1,1,0) and (2,0,1).
(e) The plane of vectors (b_1, b_2, b_3) that satisfy $b_3 - b_2 + 3b_1 = 0$.

Question 2. (a) Under what conditions on scalars ξ and η are the vectors $(1, \xi)$ and $(1, \eta)$ in R^2 linearly dependent.
(b) Under what conditions on scalars ξ, η, and ζ are the vectors $(1, \xi, \xi^2)$, $(1, \eta, \eta^2)$, and $(1, \zeta, \zeta^2)$ in R^3 linearly dependent.
(c) Guess a generalization of the above to R^n.

Question 3. The four types of subspaces of R^3 are planes, lines, R^3 itself, or Z containing only (0,0,0).
(a) Describe the three types of subspaces of R^2.
(b) Describe the five types of subspaces of R^4.

Question 4. The matrix $A = \begin{bmatrix} 2 & -2 \\ 2 & -2 \end{bmatrix}$ is a "vector" in the space M of all 2 by 2 matrices. Write the zero vector in this space, the vector $\frac{1}{2}A$, and the vector -A. What matrices are in the smallest subspace containing A?

Question 5. Decide the linear dependence or independence of
(a) the vectors (1,3,2), (2,1,3), and (3,2,1).
(b) the vectors (1,-3,2), (2,1,-3), and (-3,2,1).

Question 6. Show that v_1, v_2, v_3 are independent but v_1, v_2, v_3, v_4 are dependent:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, v_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, v_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, v_4 = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}.$$

Question 7. Find the dimensions of (a) the column space of A, (b) the column space of U, (c) the row space of A, (d) the row space of U. Which two of the spaces are the same?

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 3 & 1 \\ 3 & 1 & -1 \end{bmatrix} \text{ and } U = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Question 8. Find a basis for each of these subspaces of R^4:
(a) All vectors whose components are equal.
(b) All vectors whose components add to zero.
(c) All vectors that are perpendicular to (1,1,0,0) and (1,0,1,1).
(d) The column space (in R^2) of $U = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$.

Question 9. Find a basis for each of these subspaces of 3 by 3 matrices:
(a) All diagonal matrices.
(b) All symmetric matrices ($A^T = A$).
(c) All skew-symmetric matrices ($A^T = -A$).

Question 10. Find a basis for the space of polynomials $p(x)$ of degree ≤ 3. Find a basis for the subspace with $p(1) = 0$.