

**END SEMESTER ASSESSMENT (ESA) B.Tech. III SEMESTER- Dec. 2016**

**UE15CS205 – Discrete Mathematics and Logic**

Time: 3 Hrs		Answer All Questions	Max Marks: 100
1.	a)	An RTO (Regional Transport Office) in India issues license plates for motor vehicles and the RTO has a unique code. <b>KA05</b> , for example, is the code for the <b>RTO at Jayanagar, Bengaluru</b> . Suppose the license plates issued by the RTO at Jayanagar starts with the RTO code (KA05) followed by one or two letters from the English alphabet, and then a four digit number. (i) <b>How many</b> license plates the RTO at Jayanagar can issue? (ii) <b>How many</b> of them have a palindrome in their last four digits? (A <b>palindrome</b> is a string whose reversal is identical to the string)	06
	b)	How many ordered pairs of integers ( <b>a, b</b> ) are needed to guarantee that there are two ordered pairs ( <b>a<sub>1</sub>, b<sub>1</sub></b> ) and ( <b>a<sub>2</sub>, b<sub>2</sub></b> ) such that <b>a<sub>1</sub> mod 10 = a<sub>2</sub> mod 10</b> , and <b>b<sub>1</sub> mod 10 = b<sub>2</sub> mod 10</b> ?	06
	c)	How many ways are there to distribute <b>10</b> indistinguishable chocolates to <b>3</b> children where every child gets at least a chocolate? What if there is no restriction of at least a chocolate for every child, that is, some children may not get any chocolates?	08
2.	a)	Let <b>p</b> be “you have a lot of black money”, and <b>q</b> be “you like demonetization”. Express the following statements using <b>p, q</b> and <b>logical operators</b> . (i) You having a lot of black money is a sufficient reason for you disliking demonetization. (ii) You like demonetization unless you have a lot of black money. (iii) Either you have a lot of black money or you like demonetization, but not both.	06
	b)	Identify the <b>rule of inference</b> is used in each of these arguments. (i) You can enter the campus only if you wear your id card. You did not bring your id card. Therefore, you cannot enter the campus. (ii) She is happy whenever it rains. She takes a selfie whenever she is happy. Therefore, she takes a selfie whenever it rains. (iii) You have watched “3 Idiots” or you are an idiot. You like Rancho or you have not watched “3 Idiots”. Therefore, you are an idiot or you like Rancho.	06
	c)	Prove the validity of the following argument. <b>Premises:</b> <ul style="list-style-type: none"> <li>“If God were unable to prevent evil, then he would be powerless”</li> <li>“If God were unwilling to prevent evil, then he would be evil-minded”</li> <li>“If God were able and willing to prevent evil, then he would prevent evil”</li> <li>“If God exists, then he is neither powerless nor evil-minded”</li> <li>“God does not prevent evil”</li> </ul> <b>Conclusion: “God does not exist”.</b>	08

3.	a)	Let <b>A</b> be set {1, 2, 3, 4}. (i) What is the cardinality of the Cartesian product <b>A X A</b> ? (ii) How many functions can be drawn from the set <b>A to A</b> ? (iii) How many binary relations can be drawn from the set <b>A to A</b> ?	06
	b)	Give a formal definition of an <b>equivalence relation</b> and a <b>partial ordering</b> . Give an example for each of them on the set of integers.	06
	c)	Find a recurrence relation and give initial conditions for the number of bit strings of length <b>n</b> that have <b>three</b> consecutive <b>0s</b> . How many such bit strings are there of length <b>eight</b> ?	08
4.	a)	Determine the number of edges in a complete graph <b>K<sub>10</sub></b> , complete bipartite graph <b>K<sub>10,12</sub></b> , and cube graph <b>Q<sub>4</sub></b> .	06
	b)	Give a formal definition of <b>edge-complement</b> of a simple graph. What is the number of edges in the edge-complement of the cube graph <b>Q<sub>4</sub></b> ?	06
	c)	What are <b>cut vertices</b> , <b>cut edges</b> , and <b>cut sets</b> in an undirected graph? Draw a graph of your institute campus where junctions and dead-ends are the vertices and the road segments between the identified vertices are the edges. Identify cut vertices, cut edges and cut sets in the graph.	08
5.	a)	Define <b>Euler circuit</b> and <b>Hamilton circuit</b> in a graph. Draw a wheel graph <b>W<sub>6</sub></b> and identify an Euler circuit and a Hamilton circuit in that.	06
	b)	Provide a proof for the theorem: Let G be a <b>connected planar simple graph</b> with e edges and v vertices. Let r be the number of regions in a planar representation of G. Then <b>r = e - v + 2</b> .	06
	c)	What is a regular polyhedron? Prove that there are only five <b>regular polyhedra</b> .	08