MT210 Midterm 1 Sample 3

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Question 1. SYSTEMS OF LINEAR EQUATIONS

Verify that if $ad - bc \neq 0$, then the system of equations

$$\begin{array}{ccccc} ax_1 & + & bx_2 & = & r \\ cx_1 & + & dx_2 & = & s \end{array}$$

has a unique solution.

Question 2. ROW REDUCTION AND ECHELON FORMS

Write the augmented matrix corresponding the system below:

$$\begin{array}{rcrcrcrcr} 3x_1 & - & x_2 & + & x_3 & + & 2x_4 & = & -2 \\ x_1 & + & 2x_2 & - & x_3 & + & x_4 & = & 1 \\ -x_1 & - & 3x_2 & + & 2x_3 & - & 4x_4 & = & -6. \end{array}$$

Apply row reduction algorithm and solve the system. If the system is consistent, find the general solution set.

Question 3. VECTOR EQUATIONS

Let $A = \begin{bmatrix} 3 & -1 & 1 & 2 \\ 1 & 2 & -1 & 1 \\ -1 & -3 & 2 & -4 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} -2 \\ 3 \\ 6 \end{bmatrix}$. Denote the columns of A by $\mathbf{a}_1$, $\mathbf{a}_2$, $\mathbf{a}_3$, $\mathbf{a}_4$. Let $W = Span\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3, \mathbf{a}_4\}$.

A. Is $\mathbf{b}$ is W?

B. If $\mathbf{b}$ is in W, then express $\mathbf{b}$ as a linear combination of the vectors $\mathbf{a}_1$, $\mathbf{a}_2$, $\mathbf{a}_3$ and $\mathbf{a}_4$.

Question 4. THE MATRIX EQUATION A$\mathbf{x}$=$\mathbf{b}$

A.) Write the given matrix equation given below as a system of linear equations:

$$\begin{bmatrix} 1 & 2 & 13 \\ 1 & -1 & -2 \\ 2 & 4 & 26 \\ 2 & 1 & 11 \\ 3 & 3 & 24 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -13 \\ 2 \\ -26 \\ -11 \\ -24 \end{bmatrix}.$$

B.) Solve the system and write the general solution in the parametric-vector form.

Question 5. SOLUTION SETS OF LINEAR SYSTEMS

A.) Solve the nonhomogeneous system $A\mathbf{x}=\mathbf{b}$ and write the solution in parametric vector form where

$$A = \begin{bmatrix} 2 & 4 & 6 \\ 1 & 2 & 3 \\ -1 & -2 & -3 \end{bmatrix} \text{ and } \mathbf{b} = \begin{bmatrix} -4 \\ -2 \\ 2 \end{bmatrix}.$$

B.) Using the parametric vector form of the solution set in part **A.**, determine a particular solution $\mathbf{p}$.

C.) Write the general solution for the system $A\mathbf{x}=\mathbf{0}$ in parametric vector form.

QUESTION 6. LINEAR INDEPENDENCE

Suppose $\{\mathbf{v}_1, \mathbf{v}_2\}$ is a linearly independent set in $\mathbb{R}^n$. Show that $\{\mathbf{v}_1, \mathbf{v}_1 + \mathbf{v}_2\}$ is also linearly independent.