UNIT-1:LOGIC

Conditional Statement

Example:

If I am a vegetarian, I don't eat beef.

If I do eat beef, I am not a vegetarian.

If A→ B

If not B→ not A

I don't eat beef if I'm a vegetarian.

B if A = if A→ B

if I can be a vegetarian only if I don't eat beef and I tell you that I'm a vegetarian, then you can say for certain that I don't eat beef. Therefore, knowing I'm a vegetarian is sufficient for knowing I don't eat beef. **The A term is still sufficient.**

•if I tell you that I don't eat beef, you still can't be sure I'm a vegetarian because I might eat chicken or fish or pork. **The B term is not sufficient, but it is necessary for the A term to be true.**

A only if B = if A→ B

I'm a vegetarian if and only if I don't eat meat.

A if and only if B = if A→ B **and** if B→ A

If I'm a vegetarian, then you know for sure that I don't eat meat. Likewise, if I don't eat meat, then you know for sure that I'm a vegetarian.

Necessary & Sufficient Condition

Example:

"q is a necessary condition for p"

Having petrol in my car is a ***necessary condition*** for my car to start. Without petrol (q) my car (p) will not start. Of course, having petrol in the car does not guarantee that my car will start. There are many other conditions needed for my car to start.

Example:

"p is a sufficient condition for q"
Pouring a gallon of freezing water on a sleeping student is sufficient to wake him/her up. If I pour the gallon of freezing water on him/her then its guaranteed that she will wake up.

Note:

q if p means that p is a sufficient condition for q, and that q only if p means that p is a necessary condition for q.

Consider **q only if p.**

. We note that if we don't have p, then we can't have q is a logical statement in itself: $\neg p \Rightarrow \neg q$.. contrapositive of $\neg p \Rightarrow \neg q$: it is $\neg\neg q \Rightarrow \neg\neg p$, which is equivalent to $q \Rightarrow p$.

"P only if Q" means, as it says, that P will happen ONLY if Q happens. That is, P cannot happen without Q happening also, which means that if P is happening, then Q must be happening -- if P, then Q, or $P \rightarrow Q$, not $Q \rightarrow P$.

Predicates and Quantifiers:

$\forall x(P(x) \lor Q(x)) \not\equiv \forall x P(x) \lor \forall x Q(x)$

Example:

P(x) x selects Java

Q(x) x selects C++

$\forall x(P(x) \lor Q(x))$ everyone in the class selects either Java or C++

$\forall x P(x) \lor \forall x Q(x)$ everyone in the class selects Java or everyone selects C++

$\exists x(P(x) \land Q(x)) \not\equiv \exists x P(x) \land \exists x Q(x)$

Example:

Let P(x) be "x is positive" and Q(x) is "x is negative". The domain is integers.

$\exists x P(x) \land \exists x Q(x)$ means "There exist positive integers and there exist negative integers", which is obviously true.

$\exists x(P(x) \land Q(x))$ means "There exists an integer that is positive and negative" which is False.