PESU Center for
Information Security,
Forensics and
Cyber Resilience

Welcome to

PES University

Ring Road Campus, Bengaluru

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PESU Center for
Information Security,
Forensics and
Cyber Resilience

APPLIED CRYPTOGRAPHY

Private key systems

Lecture 2

Pseudo Random Numbers

generating a sequence of numbers

Pseudorandom generators

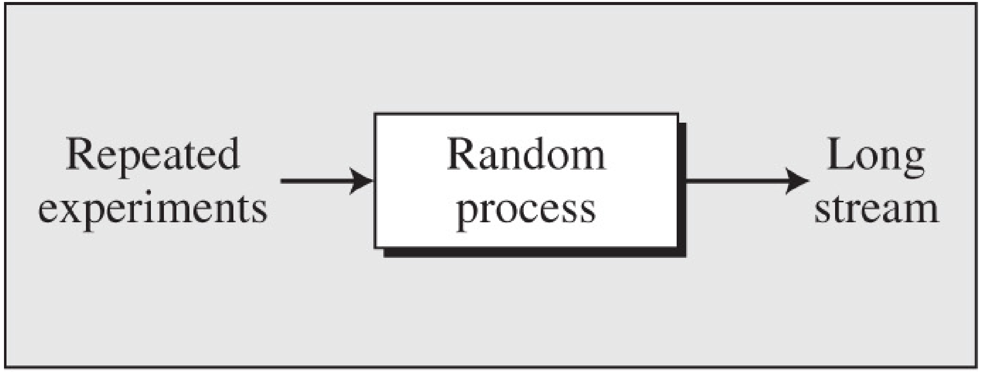
- A pseudorandom generator G is an efficient deterministic algorithm for transforming a short, uniform string called seed into a longer "uniform-looking" output string
- Let G: $\{0,1\}^n \rightarrow \{0,1\}^l$ be a function and define Dist to be the distribution on l-bit string obtained by choosing a uniform s$\in \{0,1\}^n$ and outputting G(s)

Randomness

- Truly random
- Pseudo random

Truly random

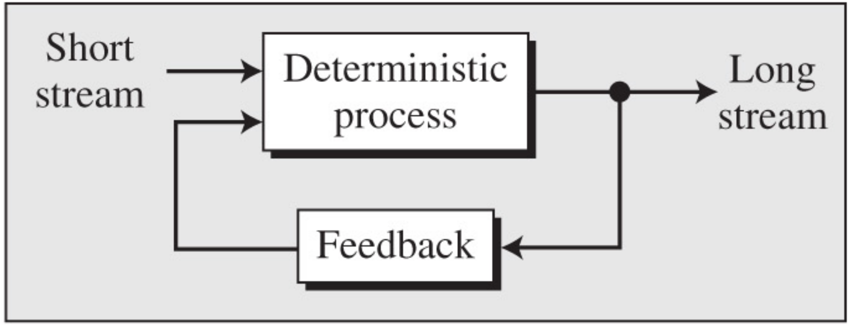
- TRG G’ is a randomised algorithm
- Output $R \in \{0,1\}^L$
- Uniformly randomly output a bit string of length L bits



a. TRGN

Pseudorandom generators

- G should be an ***efficient algorithm***
- ***Expansion:*** $L > l$
- ***Pseudo randomness:*** No efficient statistical test should significantly separate an output of G from L bit truly random generator



b. PRNG

Generator Algorithm

- Function G is called a **pseudorandom generator**.
- G is a deterministic algorithm

Input: s$\in \{0,1\}^{l}$

Output: G(s) $\in \{0,1\}^{L}$

Note: G should be a polynomial function of a security parameter (efficient).

Seed

- Kept secret
- Chosen uniformly

ALGORITHM 3.16
Constructing G_ℓ from $(\mathsf{Init}, \mathsf{GetBits})$

Input: Seed s and optional initialization vector IV
Output: $y_1, \ldots, y_\ell$

$\mathsf{st}_0 := \mathsf{Init}(s, IV)$
for $i = 1$ to ℓ:
  $(y_i, \mathsf{st}_i) := \mathsf{GetBits}(\mathsf{st}_{i-1})$
return $y_1, \ldots, y_\ell$

Encrypting long messages using short keys

$$M = K = C = \{0,1\}^L \quad G\{0,1\}^l \rightarrow \{0,1\}^L \qquad l < L$$

Consider mϵM$= \{0,1\}^L$

Encryption done by computing m$\oplus$G(s).

s is the seed = $\{0,1\}^l$

- A computationally bounded adversary will not be able to distinguish between G(s) and uniformly random string from $\{0,1\}^L$
- Both l and L are polynomial functions of security parameter n

PRG indistinguishability game

1. Hypothetical verifier:

   Challenges the distinguisher by a string or a sample of length L bits.

2. Distinguisher D

   Distinguish apart a sample generated by the pseudorandom generator from a sample generated by a truly random generator

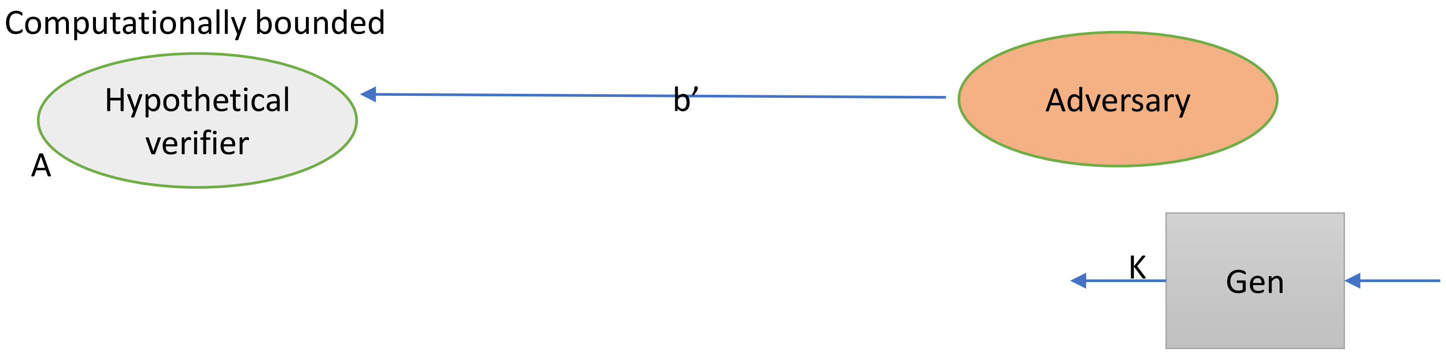
Hypothetical verifier

Uses two method to generate string of L bits

1. Truly random number generator (b=0) $y_R \in \{0,1\}^L$
2. Pseudorandom number generator (b=1) $s\{0,1\}^l \rightarrow y_p \in \{0,1\}^L$

And sends the y bits to the distinguisher

The indistinguishability experiment



Distinguisher D

Should find how y bits are generated

If for every distinguisher D participating in this experiment

$$\boldsymbol{pr(D\ outputs\ b' = b) \le \frac{1}{2} + negl(n)}$$

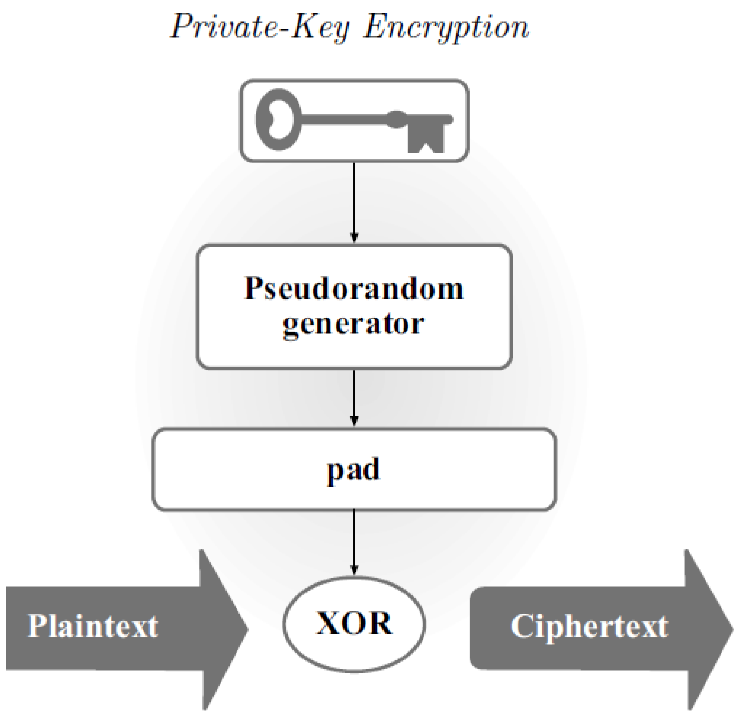
$$\boldsymbol{pr(D\ outputs\ b' = 1|b = 0)}$$

-

$$\boldsymbol{pr(D\ outputs\ b' = 1|b = 1) \le negl(n)}$$

Probability of D labelling y as outcome of PRG given that y is generated by TRG - Probability of D labelling y as outcome of PRG given that y is generated by PRG $\le negl(n)$

Encryption with a Pseudorandom generator



Pseudorandom Functions

- PRF does not require a one-to-one mapping between the input space and output space.
- A Pseudo Random Permutation is a PRF that happens to have the property that every element in the input domain has a single associated member in the output co-domain
- PRP is a bijection function (one-to-one mapping)

PRP (pseudorandom permutation)

- PRP is invertible.

*Let $F, F^{-1} : \{0,1\}^{\lambda} \times \{0,1\}^{blen} \rightarrow \{0,1\}^{blen}$ be deterministic and efficiently computable functions. Then F is a **pseudorandom permutation (PRP)** if for all keys $k \in \{0,1\}^{\lambda}$, and for all $x \in \{0,1\}^{blen}$, we have:*

- $F^{-1}(k, F(k,x)) = x.$

For example initial permutation and final permutation in DES

Thank you

Next Class

- ☞ Mandatory reading for the next class
  - ☞ https://www.coursera.org/lecture/symmetric-crypto/feistel-cipher-YgMcO

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