MT210 Midterm 1 Sample 1

Ilker S. Yuce

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Question 1. SYSTEMS OF LINEAR EQUATIONS

Determine the values of k such that the linear system

$$\begin{array}{rcrcr} 9x_1 & + & kx_2 & = & 9 \\ kx_1 & + & x_2 & = & -3 \end{array}$$

is consistent.

QUESTION 2. ROW REDUCTION AND ECHELON FORMS

Determine when the augmented matrix below represents a consistent linear system.

$$\begin{bmatrix} 1 & 0 & 2 & a \\ 2 & 1 & 5 & b \\ 1 & -1 & 1 & c \end{bmatrix}$$

Question 3. VECTOR EQUATIONS

Determine if $\mathbf{b}$ is a linear combination of the vectors $\mathbf{a}_1$, $\mathbf{a}_2$, and $\mathbf{a}_3$ where

$$\mathbf{a}_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \mathbf{a}_2 = \begin{bmatrix} -2 \\ -1 \\ -1 \end{bmatrix}, \mathbf{a}_3 = \begin{bmatrix} 3 \\ -1 \\ -3 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 5 \\ -4 \\ -7 \end{bmatrix}.$$

If $\mathbf{b}$ is a linear combination of the vectors $\mathbf{a}_1$, $\mathbf{a}_2$, and $\mathbf{a}_3$, express $\mathbf{b}$ as a linear combination of the vectors $\mathbf{a}_1$, $\mathbf{a}_2$, and $\mathbf{a}_3$.

Question 4. THE MATRIX EQUATION A$\mathbf{x}$=$\mathbf{b}$

A. Solve the matrix equation $A\mathbf{x} = \mathbf{b}$ where

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 3 & 0 \\ 1 & 1 & 2 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

B. Is it possible to solve $A\mathbf{x} = \mathbf{b}$ for **any given** $\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ where A is the matrix given in part **A**? Explain.

C. Describe the set of all $\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ for which $A\mathbf{x} = \mathbf{b}$ does have a solution.

Question 5. SOLUTION SETS OF LINEAR SYSTEMS

Consider the linear system $A\mathbf{x} = \mathbf{b}$ where

$$A = \begin{bmatrix} 1 & -1 & -2 & -2 & -2 \\ 3 & -2 & -2 & -2 & -2 \\ -3 & 2 & 1 & 1 & -1 \end{bmatrix}, \ \mathbf{b} = \begin{bmatrix} 3 \\ -1 \\ -1 \end{bmatrix}$$

A. Solve the linear system.
B. Write the general solution in parametric-vector form.
C. Give a particular solution $\mathbf{p}$.
D. Write the solution set for the homogeneous equation $A\mathbf{x} = \mathbf{0}$.

Question 6. LINEAR INDEPENDENCE

Determine if the following sets of vector are linearly independent. If **not**, write one vector as a linear combination of other vectors in the set.

A.) $\left\{ \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}.$

B.) $\left\{ \begin{bmatrix} -5 \\ 10 \end{bmatrix}, \begin{bmatrix} -4 \\ -2 \end{bmatrix}, \begin{bmatrix} 36 \\ 12 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \end{bmatrix} \right\}$

C.) $\left\{ \begin{bmatrix} -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \end{bmatrix} \right\}$

D.) $\left\{ \begin{bmatrix} 1 \\ 2 \\ -4 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ -2 \end{bmatrix}, \begin{bmatrix} 4 \\ 5 \\ -6 \end{bmatrix} \right\}$

E.) $\left\{ \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \\ -4 \end{bmatrix}, \begin{bmatrix} -4 \\ 2 \\ -1 \end{bmatrix} \right\}$