PES University, Bangalore

(Established under Karnataka Act No. 16 of 2013)

UE18MA251

MAY 2020: END SEMESTER ASSESSMENT (ESA) B.TECH. IV SEMESTER

UE18MA251- LINEAR ALGEBRA
MODEL QUESTION PAPER

Time: 3 Hrs | Answer All Questions | Max Marks: 100

1	a)	Use the method of Gaussian Elimination to decide if the planes $6x - 3y + 3z = -2$, $2x - y + z = 1$, $3x + 2y - 4z = 4$ have a common point of intersection in R^3. What happens if the right hand side of the second equation is changed to -2/3 instead of the present number 1 ? What are all the solutions of the system in that case?	7
	b)	Find the matrices L and U for the matrix $A = \begin{bmatrix} 1 & -1 & 0 & 3 \\ 2 & -2 & 0 & 8 \\ -1 & -1 & 4 & -2 \\ -2 & -2 & 6 & -3 \end{bmatrix}$. Write down the permutation matrices, if any, used in the process of elimination.	7
	c)	If the inverse of $A = \begin{bmatrix} 1 & a & b \\ 1 & a & 2 \\ 1 & 0 & b \end{bmatrix}$ is known to be $\begin{bmatrix} 1 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix}$, use Gauss – Jordan elimination on [A I] to find the values of a and b.	6
2	a)	If the vectors (1, 1, 2) , (1, 2, 4) , (2, 4, 8) span the column space of a matrix A, determine whether or not the vector b = (2, 3 , 5) is in C (A). What value should replace the third component " 5 " in the vector b so that the system Ax = b has infinitely many solutions ? Express this new vector b as a linear combination of columns of A.	7
	b)	If the row space of a matrix A is spanned by the vectors (2, 4, 6, 4) , (2, 5, 7, 6) and (2, 3, 5, c) find the value of c for which C (A) is (i) a plane in R^3 (ii) the whole of R^3. For this value of c in case (i), identify the free variable(s) and write down the special solution(s) of Ax = 0.	7
	c)	Find the dimension and a basis for the null space and the left null space of the matrix whose columns are (1, 2, - 1), (3, 6, - 3) , (3, 9, 3) and (2, 7, 4).	6
3	a)	Find a basis for the subspace S spanned by all solutions of $x + y + z - t = 0$ and a basis for the orthogonal complement of S. Find a vector a in S and a vector b in $S^{\perp}$ such that a + b = (1, 1, 1, 1).	7
	b)	Solve Ax = b by least squares and find $p = A\hat{x}$ if $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$, $b = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$.	7
	c)	Let M denote the space of all 2 x 2 matrices and let T : M → M be a linear transformation defined by T (X) = BX , the usual product of matrices, where $B = \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$. Find the matrix that represents T.	6

Please Turn Over……

4	a)	Apply the Gram – Schmidt process to (1, 0, 1) , (1, 0, 0) and (2, 1, 0) and write the result in the form A = QR.	7
	b)	Use the power method to compute the numerically largest eigenvalue of A = $\begin{bmatrix} 1 & 3 & -1 \\ 3 & 2 & 4 \\ -1 & 4 & 10 \end{bmatrix}$ starting with an initial approximation of (0 , 0 , 1). Perform 6 iterations and correct your answer to 3 decimal places) .	7
	c)	Diagonalize A= $\begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix}$ and find one of its square roots ,a matrix R such that R^2 = A.	6
5	a)	Find the matrix A that represents the quadratic form Q (x) = $5x_1^2 - 4x_1\,x_2 + 5x_2^2$. Also check if A is positive definite by finding its eigenvalues. Show also that A is orthogonally diagonalizable.	8
	b)	Find the singular value decomposition of A = $\begin{bmatrix} 4 & 11 & 14 \\ 8 & 7 & -2 \end{bmatrix}$.	12