Model Question Paper

B.Tech, IV SEMESTER,

LINEAR ALGEBRA(Scheme and Solution)

<table>
<tr><td>1.</td><td>a)</td><td>

$$A \approx \left[\begin{array}{ccc|c}1&2&2&10\\2&3&-4&3\\1&1&1&7\end{array}\right] \approx \left[\begin{array}{ccc|c}1&2&2&10\\0&-1&-8&-17\\0&-1&-1&-3\end{array}\right] \approx \left[\begin{array}{ccc|c}1&2&2&10\\0&-1&-8&-17\\0&0&7&13\end{array}\right]$$

----1+1+1 Marks

By back substituition (u,v,w)=(4,1,4)----**3 marks**
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<tr><td></td><td>b)</td><td>

We can factorize PA=LDU where

$$P = P_{23} = \begin{bmatrix}1&0&0\\0&0&1\\0&1&0\end{bmatrix}, \quad L = \begin{bmatrix}1&0&0\\-2&1&0\\2&0&1\end{bmatrix}, \quad D = \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix} \quad \text{and} \quad U = \begin{bmatrix}1&-2&2\\0&1&0\\0&0&1\end{bmatrix}$$

--------1+2+1+3 marks
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<tr><td></td><td>c)</td><td>

$$[A:I] \approx \left[\begin{array}{ccc|ccc}1&2&0&1&0&0\\0&-1&1&-1&1&0\\0&-2&1&-1&0&1\end{array}\right] \approx \left[\begin{array}{ccc|ccc}1&2&0&1&0&0\\0&-1&1&-1&1&0\\0&0&-1&1&-2&1\end{array}\right] \approx \left[\begin{array}{ccc|ccc}1&2&0&1&0&0\\0&-1&0&0&-1&1\\0&0&-1&1&-2&1\end{array}\right] \approx \left[\begin{array}{ccc|ccc}1&0&0&1&-2&2\\0&-1&0&0&-1&1\\0&0&-1&1&-2&1\end{array}\right]$$

$$\approx \left[\begin{array}{ccc|ccc}1&0&0&1&-2&2\\0&1&0&0&1&-1\\0&0&1&-1&2&-1\end{array}\right] = [I : A^{-1}]$$

.--------**1 mark each**.
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<tr><td>2.</td><td>a)</td><td>

$Ux = 0 \Rightarrow \begin{bmatrix}0&2&4&6\\0&0&1&-1\\0&0&0&0\end{bmatrix} --2marks,$ Special solution is $x = x_1\begin{bmatrix}1\\0\\0\\0\end{bmatrix} + x_4\begin{bmatrix}0\\-5\\1\\1\end{bmatrix}$.----**1+2marks**

Pivots are 2 and 1.---**1 mark**.

Rows of A are perpendicular to the solution of Ax=0-----**1mark**
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<tr><td></td><td>b)</td><td>

Let $A = \begin{bmatrix}1&0&-2\\3&5&-1\\-5&\lambda&0\end{bmatrix}$. When $\lambda = -10$ the given vectors span 2 dimensional space.-----

Basis for $C(A) = \left\{\begin{pmatrix}1\\3\\5\end{pmatrix}, \begin{pmatrix}0\\5\\-10\end{pmatrix}\right\}$ and basis for $N(A^T) = \left\{\begin{pmatrix}-1\\2\\1\end{pmatrix}\right\}$
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<tr><td></td><td>c)</td><td>

Let $c_1(u+v) + c_2(u+2v+3w) + c_3(u+v-2w) = 0$ combine the coefficients of u, v and w. then $c_1 + c_2 + c_3 = 0, c_1 + 2c_2 + c_3 = 0 \; and \; 3c_2 - 2c_3 = 0$ since u,v and w are Linearly independent and $c_1 = c_2 = c_3 = 0$.Therefore given set is Linearly independent.
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<tr><td>3.</td><td>a)</td><td>

$A_T = \begin{bmatrix}1&2&-1\\0&1&1\\1&1&-2\end{bmatrix}$ Range is plane $\left\{c_1\begin{pmatrix}1\\0\\1\end{pmatrix} + c_2\begin{pmatrix}2\\1\\1\end{pmatrix}\right\}$ and Kernel is a line $\left\{c_3\begin{pmatrix}3\\-1\\1\end{pmatrix}\right\}$

Dimensions are null space and column space are 1 and 2.
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	b)	$A^T A\hat{x} = A^T b \Rightarrow \begin{bmatrix} 6 & -8 \\ -8 & 18 \end{bmatrix}\hat{x} = \begin{bmatrix} -11 \\ 27 \end{bmatrix} \Rightarrow \hat{x} = \begin{bmatrix} 9/22 \\ 37/22 \end{bmatrix}$ $A\hat{x} = p \Rightarrow p = \begin{bmatrix} 23/11 \\ -14/11 \\ 93/22 \end{bmatrix} \therefore q = b - p = \begin{bmatrix} -12/11 \\ 36/11 \\ -16/11 \end{bmatrix}$
	c)	$b = c + mt \Rightarrow \begin{pmatrix} 4 \\ 3 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 & -2 \\ 1 & -1 \\ 1 & 0 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} \hat{c} \\ \hat{d} \end{pmatrix}$ on solving this we get best line $= \frac{61}{35} - \frac{36}{35}t$
4.	a)	Eigen values are 1,2 and 3. Eigen vectors are $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix}$ *and* $\begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$
	b)	$q_1 = \begin{pmatrix} 1/3 \\ 2/3 \\ -2/3 \end{pmatrix}, q_2 = \begin{pmatrix} 2/3 \\ 1/3 \\ 2/3 \end{pmatrix} and\, q_3 = \begin{pmatrix} -2/3 \\ 2/3 \\ 1/3 \end{pmatrix} \quad A = QR = \begin{pmatrix} 1/3 & 2/3 \\ 2/3 & 1/3 \\ -2/3 & 2/3 \end{pmatrix}\begin{pmatrix} 3 & -3 \\ 0 & 3 \end{pmatrix}$
	c)	$\Lambda = \begin{pmatrix} 2 & 0 \\ 0 & 5 \end{pmatrix},\ S = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}\ and\ S^{-1} = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} \therefore A^k = S\Lambda^k S^{-1} \quad A^k = \begin{pmatrix} 2^k & 5^k - 2^k \\ 0 & 5^k \end{pmatrix}$
5.	a)	$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix} \approx \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix}$ A is not positive definite. $B = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 2 & -2 \\ -2 & -2 & 7 \end{bmatrix} \approx \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ B is positive definite.
	b)	$AA^T = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}, and \quad A^T A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 1 \end{bmatrix}\ , A = U\Sigma V^T,\ U = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix},\ \Sigma = \begin{bmatrix} \sqrt{3} & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ And $V = \begin{bmatrix} \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} \\ \frac{2}{\sqrt{6}} & 0 & -\frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} \end{bmatrix}$

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