Sl. No	Questions and Answers
1	Prediction of an AR(p) process Applying to the ARMA(p, 1) process with $\theta 1$ _ 0, we easily find that $\hat{} Xn+1$ _ $\varphi 1Xn + \cdot\cdot\cdot + \varphi pXn+1-p, n \geq p$.
2	Estimation of a missing value Consider again the stationary series defined in Example 2.2.1 by the equations Xt _ φXt−1 + Zt, t_ 0,±1, . . . , Where |φ| < 1 and {Zt} ∼ WN (0, σ2). Suppose that we observe the series at times 1 and 3 and wish to use these observations to find the linear combination of 1,X1, and X3 that estimates X2 with minimum mean squared error. The solution to this problem can be obtained directly from (2.5.12) and (2.5.13) by setting Y _ X2 and **W** _ (X1,X3) _. This gives the equations $$\begin{bmatrix} 1 & \phi^2 \\ \phi^2 & 1 \end{bmatrix} \mathbf{a} = \begin{bmatrix} \phi \\ \phi \end{bmatrix},$$ with solution $$\mathbf{a} = \frac{1}{1+\phi^2} \begin{bmatrix} \phi \\ \phi \end{bmatrix}.$$ The best estimator of X2 is thus $$P(X_2 \mid \mathbf{W}) = \frac{\phi}{1+\phi^2}(X_1 + X_3),$$ with mean squared error $$E[(X_2 - P(X_2 \mid \mathbf{W}))^2] = \frac{\sigma^2}{1-\phi^2} - \mathbf{a}' \begin{bmatrix} \frac{\phi\sigma^2}{1-\phi^2} \\ \frac{\phi\sigma^2}{1-\phi^2} \end{bmatrix} = \frac{\sigma^2}{1+\phi^2}.$$

	The Prediction Operator P(·|**W**) For any given **W** _ (Wn, . . . , W1)' and Y with finite second moments, we have seen' how to compute the best linear predictor P(Y|**W**) of Y in terms of 1, Wn, . . . , W1 . The function P(·|**W**), which converts Y into P(Y|**W**), is called a **prediction operator**. (The operator Pn defined by equations (2.5.7) and (2.5.8) is an example with **W** _ (Xn,Xn−1, . . . , X1) Prediction operators have a number of useful properties that can sometimes be used to simplify the calculation of best linear predictors. We list some of these below. **Properties of the Prediction Operator** P(·|**W**)**:** Suppose that EU2 < ∞, EV 2 < ∞, _ cov(**W**,**W**), and β, α1, . . . , αn are constants. **1.** P(U|**W**) _ EU + **a'** (**W**− E**W**), where Γ **a** cov(U,**W**). **2.** E[(U − P(U|**W**))**W**] = **0** and E[U − P(U|**W**)]= 0. **3.** E[(U − P(U|**W**))^2] = var(U) – **a**'cov(U,**W**). **4.** P(α_1U + α_2V + β|**W**) = α_1P(U|**W**) + α_2P(V|**W**) + β. **5.** P$\sum^n_{i_1}$ αiWi + β|**W =** $\sum^n_{i_1}$Wi + β **6.** P(U|**W**) = EU if cov(U,**W**) _=**0**. **7.** P(U|**W**) =P(P(U|**W**,**V**)|**W**) if **V** is a random vector such that the components of E(**VV**') are all finite.
3	Numerical prediction of an ARMA (2,3) process In this example we illustrate the steps involved in numerical prediction of an ARMA (2,3) process. Of course, these steps are shown for illustration only. The calculations are all carried out automatically by ITSM in the course of computing predictors for any specified data set and ARMA model. The process we shall consider is the ARMA process defined by the equations *Xt* − *Xt*−1 + 0.24*Xt*−2 _ *Zt* + 0.4*Zt*−1 + 0.2*Zt*−2 + 0.1*Zt*−3, where {*Zt*} ∼ WN *(*0, 1*)*. Ten values of *X*1, . . . , *X*10 simulated by the program ITSM are shown in Table 3.1. (These were produced using the option Model>Specify to specify the order and parameters of the model and then Model>Simulate to generate the series from the specified model.) The first step is to compute the covariances *γX(h)*, *h* _ 0, 1, 2, which are easily found from equations with *k* _ 0, 1, 2 to be

$\gamma X(0)$ _ 7.17133, $\gamma X(1)$ _ 6.44139, and $\gamma X(2)$ _ 5.0603.
From we find that the symmetric matrix K _ [$\kappa(i, j)$]i,j_1,2,... is given by

$$K = \begin{bmatrix} 7.1713 &&&&&&&& \\ 6.4414 & 7.1713 &&&&&&& \\ 5.0603 & 6.4414 & 7.1713 &&&&&& \\ 0.10 & 0.34 & 0.816 & 1.21 &&&&& \\ 0 & 0.10 & 0.34 & 0.50 & 1.21 &&&& \\ 0 & 0 & 0.10 & 0.24 & 0.50 & 1.21 &&& \\ \cdot & 0 & 0 & 0.10 & 0.24 & 0.50 & 1.21 && \\ \cdot & \cdot & 0 & 0 & 0.10 & 0.24 & 0.50 & 1.21 & \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{bmatrix}.$$

The next step is to solve the recursions of the innovations algorithm for θnj and rn using these values for $\kappa(i, j)$. Then

$$\hat{X}_{n+1} = \begin{cases} \sum_{j=1}^{n} \theta_{nj} \left(X_{n+1-j} - \hat{X}_{n+1-j} \right), & n = 1, 2, \\ X_n - 0.24 X_{n-1} + \sum_{j=1}^{3} \theta_{nj} \left(X_{n+1-j} - \hat{X}_{n+1-j} \right), & n = 3, 4, \ldots, \end{cases}$$

And

$$E\left(X_{n+1} - \hat{X}_{n+1} \right)^2 = \sigma^2 r_n = r_n.$$

4 Prediction of an MA(q) process
Applying to the ARMA(1, q) process with $\varphi 1$ =0 gives

$$\hat{X}_{n+1} = \sum_{j=1}^{\min(n,q)} \theta_{nj} \left(X_{n+1-j} - \hat{X}_{n+1-j} \right), \quad n \geq 1,$$

where the coefficients θ_{nj} are found by applying the innovations algorithm to the covariances $\kappa(i, j)$ defined. Since in this case the processes $\{Xt\}$ and $\{\sigma^{-1}W_t\}$ are identical, these covariances are simply

$$\kappa(i, j) = \sigma^{-2}\gamma_X(i-j) = \sum_{r=0}^{q-|i-j|} \theta_r \theta_{r+|i-j|}.$$

5 Prediction of an ARMA(1,1) process If
$Xt - \varphi Xt{-}1$ _ $Zt + \theta Zt{-}1$, $\{Zt\} \sim$ WN $(0, \sigma 2)$,
and $|\varphi| < 1$, then equations reduce to the single equation
$\hat{}Xn{+}1$ _ $\varphi Xn + \theta n1(Xn - \hat{}Xn), n \geq 1$.
To compute $\theta n1$ wefirst use Example 3.2.1 to find that $\gamma X(0) = \sigma 2(1 + 2\theta\varphi + \theta 2)/(1-\varphi 2)$. Substituting this, then gives, for $i, j \geq 1$,

$$\kappa(i, j) = \begin{cases} \left(1 + 2\theta\phi + \theta^2\right) / \left(1 - \phi^2\right), & i = j = 1, \\ 1 + \theta^2, & i = j \geq 2, \\ \theta, & |i - j| = 1, i \geq 1, \\ 0, & \text{otherwise.} \end{cases}$$

With these values of $\kappa(i, j)$, the recursions of the innovations algorithm reduce to
$r_0 = 1 + 2\theta\varphi + \theta^2)/(1 - \varphi 2)$,
$\theta_{n1} = \theta/rn{-}1$,
$r_n = 1 + \theta^2 - \theta^2/r_{n-1}$,
which can be solved quite explicitly