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PES University, Bangalore

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UE16CS205

END SEMESTER ASSESSMENT (ESA) B.Tech. III SEMESTER - Dec. 2017

UE16CS205 - Discrete Mathematics and Logic

Time: 3 Hrs	Answer All Questions	Max Marks: 100

Instructions:

- Answer to the point.
- Make and mention reasonable assumptions wherever necessary.
- **S(n, j): Stirling numbers of the second kind.**
  $$S(n,j) = \left(\frac{1}{j!}\right)\sum_{i=0}^{j}(-1)^i\binom{j}{i}(j-i)^n$$

1.a)	Suppose the username for a system is always 5 characters long where characters are either an uppercase letter or a decimal digit. Username must contain at least one digit, but not as the first character. How many distinct usernames could be generated?	6
1.b)	Consider 4 distinct jobs to be assigned to 6 people in an organization. A job requires one person. How many ways are there to assign the jobs: (i) if a person can be assigned at most one job? (ii) where there is no restriction on the number of jobs to be assigned a person?	6
1.c)	Find the number of solutions of the equation x1 + x2 + x3 + x4 + x5 = 21, where xi are nonnegative integers, with the condition: (i) xi ≥ 2 for i = 1, 2, 3, 4, 5 (ii) 0 ≤ x1 ≤ 10 (iii) 2 ≤ x1 < 5 (iv) 0 ≤ x1 ≤ 3, 1 ≤ x2 < 4, x3 ≥ 15	8
2.a)	Prove the following logical equivalences using laws of logic (without using truth table) (i) $p \land (p \rightarrow q)) \equiv p \land q$ (ii) $\neg(p \lor (\neg p \land q)) \equiv \neg p \land \neg q$	6

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2.b) Consider a group of men and women. Let **[6]**

M(x) : x is a man
W(x) : x is a woman
L(x, y) : x likes y

Match the following assertions with their equivalent logical expressions using the above predicates.

1	Some men like all the women		**a**	$\exists x(M(x) \land \forall y(W(y) \rightarrow L(y, x)$
2	For every man, there is at least one woman who likes him		**b**	$\exists x(M(x) \land \forall y(W(y) \rightarrow L(x, y)))$
3	Some women does not like any man		**c**	$\exists x(M(x) \land \exists y(W(y) \land L(x, y)))$
4	There is a man whom all women like		**d**	$\forall x(M(x) \rightarrow \exists y(W(y) \land L(y, x)))$
5	There is a man who likes a woman		**e**	$\forall x(W(x) \rightarrow \forall y(M(y) \rightarrow L(x, y)))$
6	All women like all men		**f**	$\exists x(W(x) \land \forall y(M(y) \rightarrow \neg L(x, y)))$

2.c) Identify the rule of inference used in each of these arguments. **[8]**

(i) Universe is infinite or human stupidity is infinite. Universe is not infinite. Therefore, human stupidity is infinite.

(ii) If the traffic is bad, Ram will be late to the class. Ram was not late to the class. Therefore, the traffic was not bad.

(iii) Narendra sells chai. Therefore, Narendra either sells dreams or he sells chai.

(iv) You like Rancho or you have not watched "3 Idiots". You have watched "3 Idiots" or you are an idiot. Therefore, you are an idiot or you like Rancho.

3.a) Answer the following questions for the poset **[6]**

$(\{\{1\}, \{2\}, \{4\}, \{1, 2\}, \{1, 4\}, \{2, 4\}, \{3, 4\}, \{1, 3, 4\}, \{2, 3, 4\}\}, \subseteq)$.

(i) Write the Hasse diagram.
(ii) Find the maximal elements.
(iii) Find the least element, if there is one.
(iv) Find the least upper bound of $\{\{2\}, \{4\}\}$, if it exists.
(v) Find all lower bounds of $\{\{1, 3, 4\}, \{2, 3, 4\}\}$.

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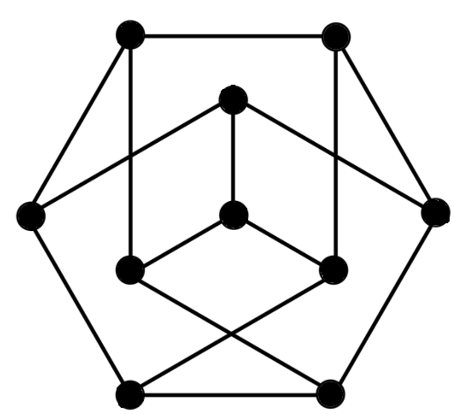
3.b)	Consider a set of 5 elements. How many equivalence relations on the set are possible consisting of 3 equivalence classes?	6
3.c)	Let ***R*** denote the relation "divides with an integer quotient" on the set of positive integers (e.g. ***5R15*** is true, but ***5R16*** is false). Let ***S*** be a relation on the set of all Web pages where ***aSb*** if and only if there are no common links found on both Web page ***a*** and Web page ***b***. Determine whether the relations R and S are reflexive, symmetric, antisymmetric, and/or transitive.	8
4.a)	Using strong induction, show that if ***n*** is an integer greater than ***1***, then ***n*** can be written as the product of prime numbers.	6
4.b)	Suppose there are a dozen identical apples and a dozen identical oranges. How many ways are there to arrange 12 fruits (apples and oranges) in a line such that no two apples are adjacent?	6
4.c)	Solve the recurrences. (i) T(n) = T(n-2) + 3, where T(2) = 1 and n is a positive even integer. (ii) H(n) = 2H(n-1) + 1, where H(1) = 1. (iii) C(n) = 6C(n-1) - 9C(n-2), where C(0) = 1, and C(1) = 6.	8
5.a)	(i) How many edges are there in a complete graph K10? (ii) How many edges are there in a complete bipartite graph K5,4? (iii) How many edges are there in the edge-complement of a simple regular graph of degree 5 having 10 vertices?	6
5.b)	Prove that there are only five regular polyhedra.	6
5.c)	Answer the following questions on the given graph with 10 vertices. (i) What is the chromatic number of the graph? (ii) Is it a regular graph? (Yes/No) (iii) Is there an Euler path in the graph? (Yes/No) (iv) Is there a Hamiltonian circuit in the graph? (Yes/No) (v) Is it a planar graph? (Yes/No) (vi) List all the cut vertices and cut edges of the graph. 	8