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PES University, Bangalore

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UE17CS205

END SEMESTER ASSESSMENT (ESA) B.Tech. III SEMESTER - Dec. 2018

UE17CS205 - Discrete Mathematics and Logic

Time: 3 Hrs	Answer All Questions	Max Marks: 100

1 a	Four friends have been identified as suspects for an unauthorized access into a computer system. They have made statements to the investigating authorities. **A**mar said, "**C**haya did it." **B**haskar said, "I did not do it." **C**haya said, "**D**iya did it." **D**iya said, "**C**haya lied when she said that I did it." (i) If the authorities also know that exactly one of the four suspects is **telling the truth**, who did it? Explain your reasoning. (ii) If the authorities also know that exactly one of the four suspects is **lying**, who did it? Explain your reasoning.	6
1 b	Let **P(x)** be "**x** is a professor" **Q(x)** be "**x** is ignorant" **R(x)** be "**x** is vain" Express the following statements using quantifiers, logical connectives, and the predicates given above, where the domain consists of all people. (i) No professors are ignorant. (ii) All ignorant people are vain. (iii) No professors are vain.	6
1 c	Use the rules of inference to show that the hypotheses **"**If Superman were unable to prevent evil, then he would be powerless**"**, **"**If Superman were unwilling to prevent evil, then he would be evil-minded**"**, **"**If Superman were able and willing to prevent evil, then he would prevent evil**"**, **"**If Superman exists, then he is neither powerless nor evil-minded**"**, and **"**Superman does not prevent evil**"** imply the conclusion **"**Superman does not exist**"**.	8
2 a	Prove the following De Morgan's laws. (i) (A U B)\` = A\` ∩ B\` using membership table. (ii) (A ∩ B)\` = A\` U B\` **without** using membership table (using set builder notation and logical equivalences).	6
2 b	Find $\sum_{k=27}^{60} k^2$. (Write the final answer as a long-plain-integer without exponents and operators)	6
2 c	Let R_1 = {(a, b) | a ≡ b (mod m)} be a relation where '**a**', '**b**' and '**m**' are integers with **m** > 1. Let R_2 = {(a, b) | a divides b wholely} be a relation where '**a**' and '**b**' are positive integers. Prove if R_1 and R_2 are equivalence relation(s) and/or partial order relation(s).	8

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3 a	Suppose that an username for a computer system must have four characters, where each character in the username is a lowercase English letter, an uppercase English letter, a digit, or one of the six special characters *****, **<**, **>**, **!**, **+**, and **=**. (i) How many different usernames are available for this computer system? (ii) How many of these usernames contain at least one occurrence of at least one of the six special characters? (iii) How many of these usernames start with an English letter? (Write the final answers as a long-plain-integer without exponents, operators, and functions)	6
3 b	Suppose that there are 9 faculty members in the mathematics department and 11 in the computer science department. How many ways are there to select a committee to develop a discrete mathematics course at a school if the committee is to consist of three faculty members from the mathematics department and four from the computer science department? (Write the final answer as a long-plain-integer without exponents, operators, and functions)	6
3 c	How many solutions are there to the equation $x_1 + x_2 + x_3 + x_4 + x_5 = 21$, where x_i are nonnegative integers, with the following conditions? (i) $x_1 \geq 1$ (ii) $x_i \geq 2$ for $i = 1, 2, 3, 4, 5$ (iii) $0 \leq x_1 \leq 10$ (iv) $2 \leq x_1 < 5$ (Write the final answers as a long-plain-integer without exponents, operators, and functions)	8
4 a	Using mathematical induction, show that if **n** is a positive integer, then **$1 + 2 + \cdots + n = n (n + 1) / 2$.**	6
4 b	Which amounts of money can be formed using just 2-rupee and 5-rupee coins? Prove your answers using strong induction. Show the steps of the strong induction explicitly. Use the predicate P(**n**), which is the statement that we can form **n** rupees using just 2-rupee and 5-rupee coins.	6
4 c	Consider the problem of finding the number of bit strings of length **n** that have **three** consecutive **1**s. Write a recurrence relation with initial conditions for the problem. Use the recurrence to find the number of bit strings of length **10** that have three consecutive **1**s? (Write the final answer as a long-plain-integer without exponents, operators, and functions)	8
5 a	$\begin{array}{c\|cccc} \# & a & b & c & d \\ \hline a & a & c & d & a \\ b & a & b & c & d \\ c & a & b & a & c \\ d & a & b & b & b \end{array}$ Let {a, b, c, d} be the underlying set of an algebraic structure. The binary operation **#** of the algebraic structure is given by the above table. Find the left identity, right identity, left zero and right zero with respect to the binary operation **#**.	6
5 b	Define semigroups, monoids, and groups.	6
5 c	Define subgroup **G\`** = (**T\`**, #) of a group G = (T, #). Prove that if **T\`** is a finite subset of T, then (**T\`**, #) is a subgroup of (T, #), if **T\`** is closed under #.	8