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PES University, Bengaluru

(Established under Karnataka Act No. 16 of 2013)

JAN – MAY- 2020: END SEMESTER ASSESSMENT (ESA) B.TECH. IV SEMESTER
UE18MA251 – LINEAR ALGEBRA (Scheme and Solution)

Instructions: You may bring only Calculators.

Time: 3 Hr | Answer All Questions. | Max Marks: 100

1.		Let $$A = \begin{bmatrix} 1 & 2 & 0 & 1 \\ 2 & 4 & 1 & 4 \\ 3 & 6 & 3 & 9 \end{bmatrix}$$	
	a)	Find a lower triangular L and an upper triangular U so that A = LU. Answer: $$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$	10
	b)	Find the reduced row echelon form R = rref(A). How many independent columns in A? Answer: 2 $$R = \begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix} = U \text{ in this example.}$$	10
2.	a)	Find a basis for the nullspace of A. Answer: $$\begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 3 \\ -2 \\ 0 \\ 1 \end{bmatrix}$$	10
	b)	If the vector b is the sum of the four columns of A, write down the complete solution to Ax = b. Answer: $$x = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} + x_2 \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 3 \\ -2 \\ 0 \\ 1 \end{bmatrix}$$	10

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3.	a)	Consider a 120^0 rotation around the axis x = y = z. Show that the vector i = (1, 0, 0) is rotated to the vector j = (0, 1, 0). (Similarly j is rotated to k = (0, 0, 1) and k is rotated to i.) How is j − i related to the vector (1, 1, 1) along the axis? Answer: $j - i = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$ is orthogonal to the axis vector $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$. So are $k - j$ and $i - k$. By symmetry the rotation takes i to j, j to k, k to i.	10
	b)	This problem finds the curve $y = C + D2^t$ which gives the best least squares fit to the points (t,y) = (0, 6), (1, 4), (2, 0). Write down the 3 equations that would be satisfied if the curve went through all 3 points. Answer: $\mathrm{C} + 1\mathrm{D} = 6$ $\mathrm{C} + 2\mathrm{D} = 4$ $\mathrm{C} + 4\mathrm{D} = 0$	10
4	a)	Let $A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \\ 3 & 6 \end{bmatrix}$ Find the eigen values of A^TA and also of AA^T. For both matrices find a complete set of orthonormal eigenvectors. Answer: $A^TA = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & 4 \\ 3 & 6 \end{bmatrix} = \begin{bmatrix} 14 & 28 \\ 28 & 56 \end{bmatrix}$ has $\lambda_1 = 70$ and $\lambda_2 = 0$ with eigenvectors $x_1 = \frac{1}{\sqrt{5}} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $x_2 = \frac{1}{\sqrt{5}} \begin{bmatrix} -2 \\ 1 \end{bmatrix}$. $\mathrm{AA}^T = \begin{bmatrix} 1 & 2 \\ 2 & 4 \\ 3 & 6 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \end{bmatrix} = \begin{bmatrix} 5 & 10 & 15 \\ 10 & 20 & 30 \\ 15 & 30 & 45 \end{bmatrix}$ has $\lambda_1 = 70,\ \lambda_2 = 0,\ \lambda_3 = 0$ with $\mathrm{x}_1 = \frac{1}{\sqrt{14}} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and $x_2 = \frac{1}{\sqrt{5}} \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$ and $x_3 = \frac{1}{\sqrt{70}} \begin{bmatrix} 3 \\ 6 \\ -5 \end{bmatrix}$.	10

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	b)	If you apply the Gram-Schmidt process (orthonormalization) to the columns of this matrix A, what is the resulting output? Answer: Gram-Schmidt will find the unit vector $$q_1 = \frac{1}{\sqrt{14}} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}.$$ But the construction of q_2 fails because column $2 = 2$ (column 1).	10
5	a)	If A is any m by n matrix with m > n, tell me why AA^T cannot be positive definite. Is A^TA always positive definite? (If not, what is the test on A?) Answer: AA^T is m by m but its rank is not greater than n (all columns of AA^T are combinations of columns of A). Since $n < m$, AA^T is singular. A^TA is positive definite if A has full colum rank n. (Not always true, A can even be a zero matrix.)	10
	b)	If a 3 by3 matrix P projects every vector onto the plane x+2y+z =0, find three eigen values and three independent eigenvectors of P. No need to compute P. Answer: The plane is perpendicular to the vector $(1, 2, 1)$. This is an eigenvector of P with $\lambda = 0$. The vectors $(-2, 1, 0)$ and $(1, -1, 1)$ are eigenvectors with $\lambda = 0$.	10