PES UNIVERSITY, Bangalore

In-Semester Assessment

UE17CS203 - Introduction to Data Science

Assignment-1

1. Let X and Y be Bernoulli random variables. Let $Z = X + Y$.

- Show that if X and Y cannot both be equal to 1, then Z is a Bernoulli random variable.

- Show that if X and Y cannot both be equal to 1, then $pZ = pX + pY$.

- Show that if X and Y *can* both be equal to 1, then Z is not a Bernoulli random variable.

2. Two dice are rolled. Let $X = 1$ if the dice come up doubles and let $X = 0$ otherwise. Let $Y = 1$ if the sum is 6, and let $Y = 0$ otherwise. Let $Z = 1$ if the dice come up both doubles and with a sum of 6 (that is, double 3), and let $Z = 0$ otherwise.

- Let pX denote the success probability for X. Find pX.

- Let pY denote the success probability for Y. Find pY.

- Let pZ denote the success probability for Z. Find pZ.

- Are X and Y independent?

- Does $pZ = pX\,pY$?

- Does $Z = XY$? Explain

3. Find the probability mass function of the random variable X if $X \sim \text{Bin}(10, 0.4)$.
Find $P(X = 5)$.

4. A fair die is rolled eight times. Find the probability that no more than 2 sixes come up.

5. A large industrial firm allows a discount on any invoice that is paid within 30 days. Of all invoices, 10% receive the discount. In a company audit, 12 invoices are sampled at random. What is the probability that fewer than 4 of the 12 sampled invoices receive

the discount?

6. What is the probability that more than 1 of the 12 sampled invoices receives a discount?

7. Let $X \sim$ Bin(n, p), and let $Y = n - X$. Show that
$Y \sim$ Bin$(n, 1 - p)$

8. Grandma bakes chocolate chip cookies in batches of 100. She puts 300 chips into the dough. When the cookies are done, she gives you one. What is the probability that your cookie contains no chocolate chips? Find the mean and variance

9. A test of weld strength involves loading welded joints until a fracture occurs. For a certain type of weld, 80% of the fractures occur in the weld itself, while the other 20% occur in the beam. A number of welds are tested. Let X be the number of tests up to and including the first test that results in a beam fracture. What is the distribution of X? find P(X=3)

10. Of customers ordering a certain type of personal computer, 20% order an upgraded graphics card, 30% order extra memory, 15% order both the upgraded graphics card and extra memory, and 35% order neither. Fifteen orders are selected at random. Let
$X1, X2, X3, X4$ denote the respective numbers of orders in the four given categories.
a. Find $P(X1 = 3, X2 = 4, X3 = 2$, and $X4 = 6)$.
b. Find $P(X1 = 3)$.