Programming and Problem Solving CSIS 2610

Fall Semester 2017 — CRN 41507

Project 2 — Stern-Brocot Approximations

Due date: Friday, November 4, 2017

Goal

Write a program that computes a Farey/Stern-Brocot approximation to a real number.

Details

Farey sequences (named after John Farey, a 19^{th} century geologist) are an interesting means of approximating real numbers between 0 and 1; the Stern-Brocot system extends the idea to all real numbers. At one point in the early history of computers, engineers would use the Farey approximation 16/113 for the fractional part of π . This fraction is accurate to six decimal places, while the more commonly known schoolbook approximation 1/7 is only good to two places.

To generate a Farey sequence to approximate some real number x, start with two endpoints — 0/1 and 1/0. Next, perform the following steps in a loop:

- Compute the *mediant* of the endpoints. If the endpoints are a/b and c/d, compute (a + c)/(b+d).
- Determine whether x is larger than, smaller than, or equal to the mediant.
- If it is smaller, replace the larger endpoint with the mediant.
- If it is larger, replace the smaller endpoint with the mediant.
- If the mediant is sufficiently close to x, or the mediant's numerator or denominator are larger than some threshold, stop the loop.

When the loop stops, output the last mediant, as a fraction. For our purposes, stop when either the mediant denominator is larger than 1,000,000 or when the mediant is within 10^{-6} of x.

Your program should read the value of x from the command line, rather than prompting for it. We will discuss how this is done in class.

▶Important Note

You may not use division anywhere in this program!

What to turn in

Turn in your source code and the output from two test runs.

Examples

▶Example 1

Input

./a.out 3.1415926535897932384

Output

3.14159265358979 is approximately 355 / 113

▶Example 2

Input

./a.out 2.718281828459

Output

2.718281828459 is approximately 2721 / 1001

▶Example 3

Input

./a.out 1.414213562373

Output

1.414213562373 is approximately 1393 / 985