Probability Theory

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What is Probability Theory?

The outcome of random events like rolling a pair of dice are impossible to predict with absolute certainty. Probability theory provides a mathematical framework for quantifying uncertainty and randomness in these situations where outcome is not deterministic.

Under the umbrella of probability theory is a couple of key concepts that I will be going through, such as:

- Random Variables
- Independence
- Conditional Probability
- Bayes Theorem

Areas of Importance

Probability theory has proven to be crucial in various fields including, but not limited to:

Economics

In economics, probability models are used to forecast certain economic indicators like annual GDP growth, inflation, and unemployment rates. Through these forecasts, policymakers are able to make informed decisions about the macroeconomy [2]

Finance

Probability theory is fundamental for assessing risk in financial markets. It has a massive influence on how institutional and individual investors make decisions regarding buying and selling of securities. It also influences the way financial derivatives like options are priced by sellers [2].

Medicine

3 years ago, the whole world was shut down by COVID-19, which resulted in many people falling ill. In instances like this, epidemiologists utilize predictive algorithms to measure the probability of patients being carriers disease carriers based on symptom screening. By identifying novel cases of COVID-19, there is potential that it can be identified early, which can help to reduce long-term complications and even save lives [3]

Random Variables

In the context of probability theory, a random variable is usually designated as X and can take on different numerical values a as result of random events/experiments. The numerical values associated with the outcome of the event are determined by the underlying probability space[4].[5]. There are 2 types of random variables values:

Discrete Variable

Discrete variables are variables that can only take on certain discrete, countable values. It is restricted to integers and can not be represented as a decimal or fraction.

Examples: - Counts and integers like number of items or scores - Binary variables like pass/fail or yes/no - Rating scales like rating movies on a 1-5 star scale - Event outcomes such as a dice roll or coin flip

Continuous Variables

Continuous variables are able to take on an infinite number of real values within a range. It can take on fractional or decimal values in addition to integer values.

Examples: * Physical measurements like height, weight, temperature * Time, geographic coordinates * Natural phenomena like air pressure

Independence

In probability theory, we say that two events A and B are independent if the occurence of one of the events does not effect the probability of the other event. That is, $P(A|B) = P(A)$ and $P(B|A) = P(B)$.

For independent events, the probability of both occuring is the product of their respective probabilities. That is, $P(A \cap B) = P(A) * P(B)$

Independence can be used to determine whether the probability of one event is dependent on the outcome of another or not. Events that are not independent are dependent, meaning the occurence of one event influences the outcome of another. In statists, we want samples to be independent so we don't introduce bias.

An example of independence would be a coin flip. If we were to flip 2 coins, the outcome of one of the coins is not going to effect the other since they are independent. Another example would be rolling a pair of dice. The outcome of one dice does not affect another pair.

Conditional Probability

Conditional probability refers to the measure of probability of an event A occuring, given an another event B has already occured. That us, $P(A|B)$.

An example of conditional probability would be the probability that someone has the flu, given they are coughing.

Bayes' Theorem

Bayes' Theorem is a formula that is used to calculate conditional probabilities. Essentially, it describes the probability of an event A, given that there is some new information B. That is, $P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$.

Bayes' Theorem is really useful in Machine Learning and statistics since it is used to update probability distributions based on observed data.

Application

We start off by importing `pandas`, which will allow us to extract, aggregate, and analyze our data. We also need to import `matplotlib` since we need to visualize the data as well.

The aesthetic of the visualizations are altered as well. The size of the legend, axis, ticks, and font are all altered.

```
import pandas as pd
import matplotlib.pyplot as plt

plt.rc('font', size=12) # font size
plt.rc('axes', labelsize=14, titlesize=14) # font size of axis and label titles
plt.rc('legend', fontsize=12) # font size of legend
plt.rc('xtick', labelsize=5) # size of ticks on x-axis
plt.rc('ytick', labelsize=10) # size of ticks on y-axis
```

I decided to use a dataset from Kaggle of the first 7 generations of Pokemon. Pokemon is a really old video game franchise and essentially any time a new game was made, they would come up new Pokemon. The new Pokemon for the game would comprise of a generation and usually there's around 100 or more Pokemon per generation. This dataset shows basic information about Pokemon such as their names, weight, height, and number in the Pokedex. Additionally, it includes information that can be interesting to analyze when aggregated, such as attack, defense, hit points (HP), generation, if they are considered legendary, and much more.

Here, we are loading the csv file and converting it into a `pandas` Series so that it can be examined.

```
data_source_raw = "../../datasets/pokemon.csv"
data_source_result = pd.read_csv(data_source_raw)
data_source_result
```

```
/Users/aniketadhikari/anaconda3/envs/homl3/lib/python3.10/site-packages/IPython/core/formatte
  return method()
```

	attack	base_egg_steps	base_happiness	base_total	capture_rate	defense	experien
0	49	5120	70	318	45	49	
1	62	5120	70	405	45	63	
2	100	5120	70	625	45	123	
3	52	5120	70	309	45	43	
4	64	5120	70	405	45	58	
5	104	5120	70	634	45	78	
6	48	5120	70	314	45	65	
7	63	5120	70	405	45	80	
8	103	5120	70	630	45	120	
9	30	3840	70	195	255	35	
10	20	3840	70	205	120	55	
11	45	3840	70	395	45	50	
12	35	3840	70	195	255	30	
13	25	3840	70	205	120	50	
14	150	3840	70	495	45	40	
15	45	3840	70	251	255	40	
16	60	3840	70	349	120	55	
17	80	3840	70	579	45	80	
18	56	3840	70	253	255	35	
19	71	3840	70	413	127	70	
20	60	3840	70	262	255	30	
21	90	3840	70	442	90	65	
22	60	5120	70	288	255	44	
23	95	5120	70	448	90	69	
24	55	2560	70	320	190	40	
25	85	2560	70	485	75	50	
26	75	5120	70	300	255	90	
27	100	5120	70	450	90	120	
28	47	5120	70	275	235	52	
29	62	5120	70	365	120	67	
30	92	5120	70	505	45	87	
31	57	5120	70	273	235	40	
32	72	5120	70	365	120	57	
33	102	5120	70	505	45	77	
34	45	2560	140	323	150	48	
35	70	2560	140	483	25	73	
36	41	5120	70	299	190	40	
37	67	5120	70	505	75	75	
38	45	2560	70	270	170	20	
39	70	2560	70	435	50	45	
40	45	3840	70	245	255	35	
41	80	3840	70	455	90	70	
42	50	5120	70	320	255	55	
43	65	5120	70	395	120	70	
44	80	5120	70	490	45	85	
45	70	5120	70	285	190	55	
46	95	5120	70	405	75	80	
47	55	5120	70	305	190	50	
48	65	5120	70	450	75	60	
49	55	5120	70	265	255	30	
50	100	5120	70	425	50	60	
51	35	5120	70	290	255	35	

Taking a look at the data output, we can see there are 20 columns total. We can then run the `info()` command on the dataset to get a breakdown of datatypes and more.

As shown below, 16 of the 20 columns are numerical values. Of the 16, most of them are *discrete variables* since they are integer values such as:

- `attack`
- `base_egg_steps`
- `base_happiness`
- `base_total`
- `defense`
- `experience_growth`
- `hp`
- `pokedex_number`
- `sp_attack`
- `sp_defense`
- `speed`
- `generation`
- `is_legendary`

The others are *continuous variables* because they can take on an infinite number of values within a given range: - `height_m` - `percentage_male` - `weight_kg`

```
data_source_result.info()
```

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 801 entries, 0 to 800
Data columns (total 20 columns):
 #   Column             Non-Null Count  Dtype
---  ------             --------------  -----
 0   attack             801 non-null    int64
 1   base_egg_steps     801 non-null    int64
 2   base_happiness     801 non-null    int64
 3   base_total         801 non-null    int64
 4   capture_rate       801 non-null    object
 5   defense            801 non-null    int64
 6   experience_growth  801 non-null    int64
 7   height_m           781 non-null    float64
 8   hp                 801 non-null    int64
 9   name               801 non-null    object
 10  percentage_male    703 non-null    float64
 11  pokedex_number     801 non-null    int64
 12  sp_attack          801 non-null    int64
```

```
 13   sp_defense         801 non-null    int64
 14   speed              801 non-null    int64
 15   type1              801 non-null    object
 16   type2              417 non-null    object
 17   weight_kg          781 non-null    float64
 18   generation         801 non-null    int64
 19   is_legendary       801 non-null    int64
dtypes: float64(3), int64(13), object(4)
memory usage: 125.3+ KB
```

Getting specific columns that are important.

There's a lot of information in this file, with a lot of it being unnecessary at the moment.

As a result, we're going to filter for specific columns and we're going to rename the columns. In this code, we filter for the `generation` and `is_legendary` column so we don't have to rename every single column. Instead, we're just renaming the columns that are important to us. We rename `generation` to `Generation` and `is_legendary` to `Legendary`

```
generation_legends = data_source_result[["generation", "is_legendary"]]
generation_legends = generation_legends.rename(columns={'is_legendary':"Legendary", 'gener
```

Afterwards, we determine how many legendary Pokemon are there for each generation.

In this example, `generation` represents a discrete variable since it exists as an integer within a fixed range of 1-7.

We find that the generation with the most legenedary Pokemon is generation 7. This is done by running the `groupby()` function on the `generation` column then subsequently running `agg` to find the number of legendary pokemon.

Once we sort, it becomes increasingly clear which that Generations 7, 4, 5 are the most ripe with legendary Pokemon

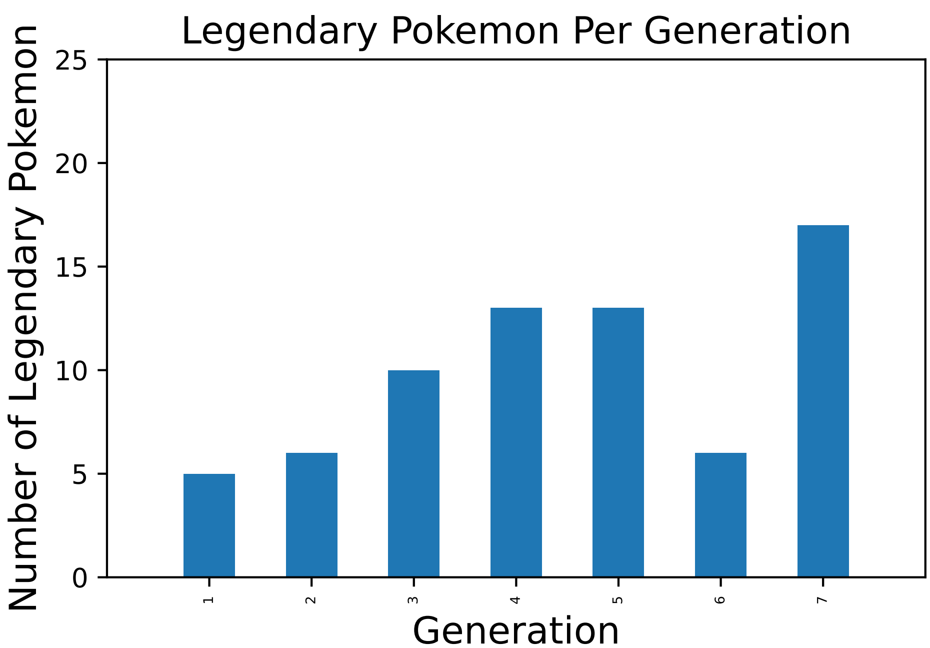
```
legendary_per_generation = generation_legends.groupby("Generation").agg({"Legendary":"sum"
legendary_per_generation[['Legendary']].sort_values(by="Legendary", ascending=False)
```

```
/Users/aniketadhikari/anaconda3/envs/homl3/lib/python3.10/site-packages/IPython/core/formatte
  return method()
```

Generation	Legendary
7	17
4	13
5	13
3	10
2	6
6	6
1	5

Getting the data, we can now visualize it on a bar graph to compare the number of legendary pokemon across generation.

```
legendary_per_generation.plot(kind="bar", x="Generation", y="Legendary", title="Legendary
plt.axis([-1,7 , 0, 25])
plt.show()
```



So we've simply pointed out discrete variables and continuous variables here. We've managed to create a discrete variable by counting the number of legendary Pokemon. But what does this have to do with probability? Well we can take a closer look at Pokemon from a specific generation and break it down further. Here we can figure out what are the odds that a Pokemon from Generation 7 is classified as Legendary?

We've already imported everything we need, so we need to start by filtering by the generation. Here we have filtered so that only Pokemon from generation 7 will appear.

```
generation_num = 7
gen7_pokemon = data_source_result[data_source_result['generation']==generation_num]
```

Now that we've gotten the specific Pokemon that we want, we also need to get the column we want. Here we are using the `is_legendary` column because we are evaluating which Pokemon are and aren't legendary. While we could get the count of each, it is probably more meaningful to get percentage breakdowns of them. As a result, we use `value_counts` to get the probability of 0 or 1, but we also normalize the results so as to get percentage values rather than just a count.

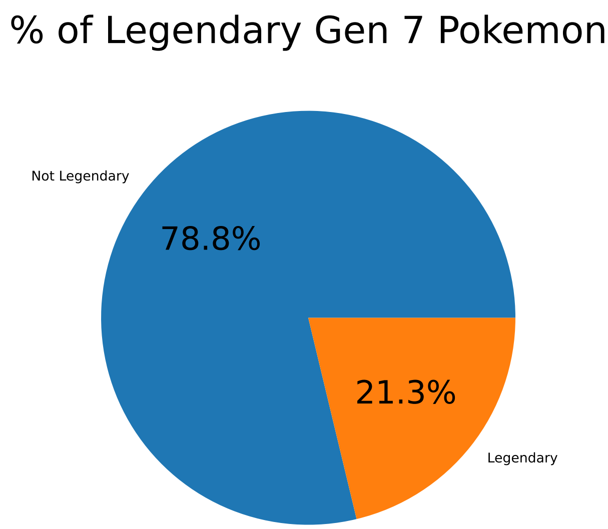
```
legendary_pokemon = gen7_pokemon['is_legendary']
legendary_percentages = legendary_pokemon.value_counts(normalize=True)
```

From there, we use 0 and 1 as index values and the percentages that were calculated earlier are used as the values.

```
legendary_percentages = pd.Series(legendary_percentages.values, index=legendary_pokemon.un
```

From there, I created a pie graph since we're only comparing 2 values. As we can see, there is 78.75% non-legendary Pokemon and 21.25% legendary Pokemon in Generation 7.

```
legendary_percentages.plot(kind="pie", ylabel="", title="% of Legendary Gen 7 Pokemon", la
plt.show()
```



We could also factor in conditional probablity and Bayes' Theorem. If we were to play a guessing game for every single Pokemon in this dataset, guessing the right Pokemon would be really challenging because there is over 800 Pokemon. We would have only a 0.12% chance of guessing right!

```
total_num_pokemon = data_source_result['pokedex_number'].count()
1/total_num_pokemon * 100
```

```
0.12484394506866417
```

But what if we found out that the Pokemon is in Generation 1 and is also a legendary pokemon? That would certainly increase our odds!

Here we can filter for Pokemon that appear in Generation 1, filter for the legendary Pokemon, and then get the count which is 5.

```
gen1_pokemon = data_source_result[data_source_result['generation']==1]
legendary_gen1 = gen1_pokemon[gen1_pokemon['is_legendary']==1]
legendary_gen1_count = legendary_gen1['name'].count()
```

We then do 1 divided by the count to get our new odds. We have improved our guessing odds from 0.1% to 20%!

```
1/legendary_gen1_count * 100
```

```
20.0
```

This showcasing *Bayes' Theorem*, which suggests bringing in new evidence will effect the outcome of the event. In our case, we found out that the Pokemon we were looking for is in Generation 1 and Legendary. This narrowed down the choices Pokemon we can guess from