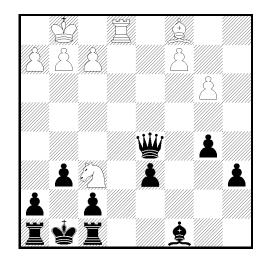
Outline

- Two formulations for learning: Inductive and Analytical
- Perfect domain theories and Prolog-EBG

1

A Positive Example



The Inductive Generalization Problem

Given:

- Instances
- Hypotheses
- Target Concept
- Training examples of target concept

Determine:

• Hypotheses consistent with the training examples

The Analytical Generalization Problem

Given:

- Instances
- Hypotheses
- Target Concept
- Training examples of target concept
- Domain theory for explaining examples

Determine:

• Hypotheses consistent with the training examples and the domain theory

An Analytical Generalization Problem

Given:

- Instances: pairs of objects
- Hypotheses: sets of horn clause rules
- Target Concept: Safe-to-stack(x,y)
- Training Example: Safe-to-stack(OBJ1,OBJ2)

```
On(OBJ1,OBJ2)
Isa(OBJ1,BOX)
Isa(OBJ2,ENDTABLE)
Color(OBJ1,RED)
Color(OBJ2,BLUE)
Volume(OBJ1,.1)
Density(OBJ1,.1)
```

- - -

Domain Theory:

```
Safe-To-Stack(x,y) :- Not(Fragile(y))
Safe-To-Stack(x,y) :- Lighter(x,y)
Lighter(x,y) :- Weight(x,wx), Weight(y,wy),
Less(wx,wy)
Weight(x,w) :- Volume(x,v), Density(x,d),
Equal(w, v*d)
Weight(x,5) :- Isa(x, ENDTABLE)
```

Determine:

 Hypotheses consistent with training examples and domain theory

Learning from Perfect Domain Theories

Assumes domain theory is *correct* (error-free)

- Prolog-EBG is algorithm that works under this assumption
- This assumption holds in chess and other search problems
- Allows us to assume explanation = proof
- Later we'll discuss methods that assume approximate domain theories

Prolog EBG

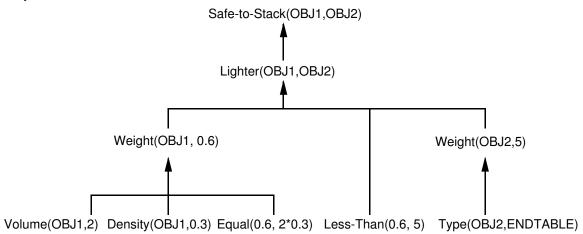
Initialize hypothesis = {}

For each positive training example not covered by hypothesis:

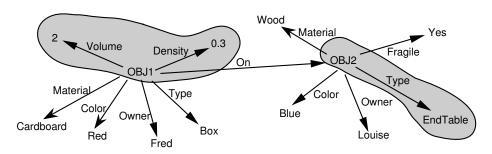
- 1. **Explain** how training example satisfies target concept, in terms of domain theory
- 2. **Analyze** the explanation to determine the most general conditions under which this explanation (proof) holds
- 3. **Refine** the hypothesis by adding a new rule, whose preconditions are the above conditions, and whose consequent asserts the target concept

Explanation of a Training Example

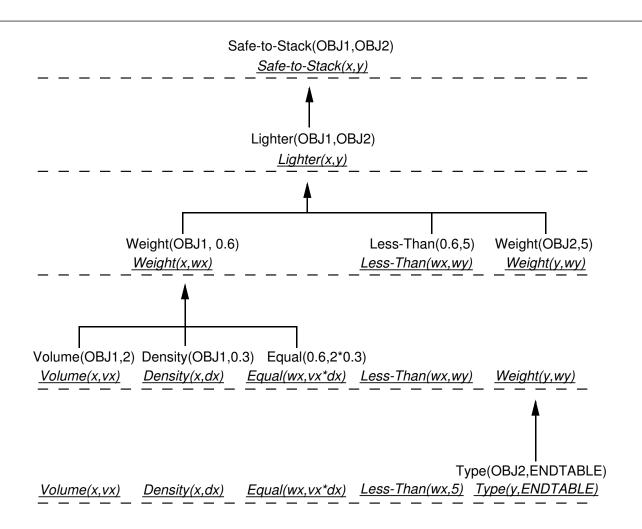
Explanation:



Training Example:



Computing the Weakest Preimage of Explanation



Regression Algorithm

```
Regress(Frontier, Rule, Expression, U<sub>I,R</sub>)
```

Frontier: the set of expressions to be regressed through Rule Rule: a horn clause.

Expression: the member of Frontier that is inferred by Rule in the explanation.

 $U_{l,R}$: the substitution that unifies Rule to the training example in the explanation

Returns the list of expressions forming the weakest preimage of Frontier with respect to Rule

```
let Consequent ← Rule consequent let Antecedents ← Rule antecedents
```

1. $U_{E,R} \leftarrow$ most general unifier of *Expression* with Consequent such that there exists a substitution S for which

$$S(U_{E,R}(Consequent)) = U_{I,R}(Consequent)$$

2. Return U_{E.R}({*Frontier* -Consequent+Antecedent})

Example:

```
\label{eq:Regress} \begin{split} &\text{Regress}(\{\text{Volume}(x,\text{vs}),\, \text{Density}(x,\text{dx}),\, \text{Equal}(\text{wx},\text{vx}^*\text{dx}),\\ &\quad \text{Less-Than}(\text{wx},\text{wy}),\, \text{Weight}(y,\text{wy})\},\\ &\quad \text{Weight}(z,5):-\text{Type}(z,\text{ENDTABLE}),\\ &\quad \text{Weight}(y,\text{wy}),\\ &\quad \{\text{OBJ2/z}\}) \end{split} &\text{Consequent} \leftarrow \text{Weight}(z,5)\\ &\quad \text{Antecedents} \leftarrow \text{Type}(z,\text{ENDTABLE})\\ &\quad \text{U}_{E,R} \leftarrow \{y/z,\, 5/\text{wy}\},\,\,\, (S=\{\text{OBJ2/y}\}) \end{split} &\text{Result} \leftarrow \{\text{Volume}(x,\text{vs}),\, \text{Density}(x,\text{dx}),\, \text{Equal}(\text{wx},\text{vx}^*\text{dx}),\\ &\quad \text{Less-Than}(\text{wx},5),\, \text{Type}(y,\text{ENDTABLE})\} \end{split}
```

Lessons from Safe-to-Stack Example

- Justified generalization from single example
- Explanation determines feature relevance
- Regression determines needed feature constraints
- Generality of result depends on domain theory
- Still require multiple examples

Perspectives on Prolog-EBG

- Theory-guided generalization from examples
- Example-guided operationalization of theories
- "Just" restating what learner already "knows"

Is it learning?

- Are you learning when you get better over time at chess?
 - Even though you already know everything in principle, once you know rules of the game...
- Are you learning when you sit in a mathematics class?
 - Even though those theorems follow deductively from the axioms you've already learned...