NFA TO DFA: SUBSET CONSTRUCTION METHOD

Steps to convert NFA → DFA:

1. Construct the transition table of given NFA machine.
2. Scan the next states column in the transition table from initial state to final state.
3. If any of the next state consists more than one state on the single input alphabet. Then merge them and make it new state. Place this new constructed state in DFA transition table as present state.
4. The next state of this new constructed state on input alphabet will be the summation of each next state which parts in the NFA transition table.
5. Repeat step 2 to step 4 until all the states in NFA transition table will be scanned completely.
6. The finial transition table must have single next state at single input alphabet.

Example 1:

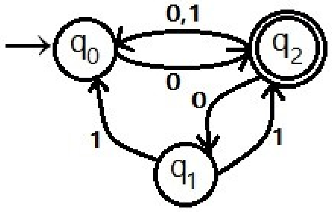


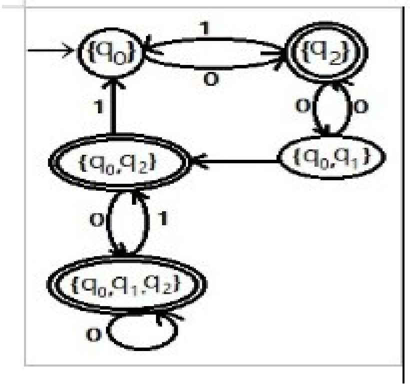
Figure (1): NFA

Construct the equivalent DFA of the NFA given in figure (1).
Step 1: Transaction Table of NFA from Figure (1):

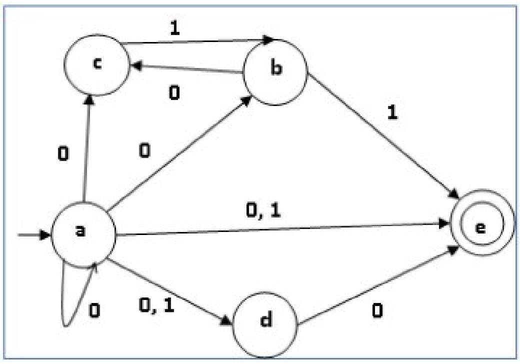
Present State	**Next State**	
	0	**1**
$\rightarrow q_0$	$\{q_2\}$	Φ
q_1	Φ	$\{q_0, q_2\}$
q_2*	$\{q_0, q_1\}$	$\{q_0\}$

Step 2: Transaction Table of DFA:

Present State	**Next State**	
	0	**1**
$\rightarrow \{q_0\}$	$\{q_2\}$	Φ
$\{q_2\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_2\}$	$\{q_0, q_2\}$
$\{q_0, q_2\}$	$\{q_0, q_1, q_2\}$	$\{q_0\}$
$\{q_0, q_1, q_2\}$	$\{q_0, q_1, q_2\}$	$\{q_0\}$



Example 2) Let us consider the NDFA shown in the figure below.

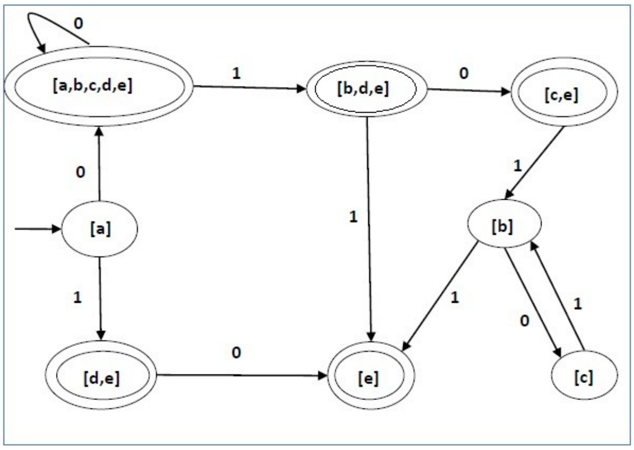


q	**δ(q,0)**	**δ(q,1)**
a	{a,b,c,d,e}	{d,e}
b	{c}	{e}
c	∅	{b}
d	{e}	∅
e	∅	∅

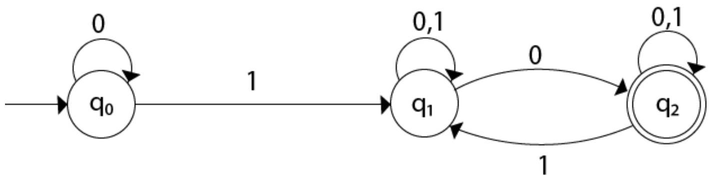
Using the above algorithm, we find its equivalent DFA. The state table of the DFA is shown in below.

q	**δ(q,0)**	**δ(q,1)**
[a]	[a,b,c,d,e]	[d,e]
[a,b,c,d,e]	[a,b,c,d,e]	[b,d,e]
[d,e]	[e]	∅
[b,d,e]	[c,e]	[e]
[e]	∅	∅
[c, e]	∅	[b]
[b]	[c]	[e]
[c]	∅	[b]

The state diagram of the DFA is as follows −



Example 3:Convert the given NFA to DFA.



Solution: For the given transition diagram we will first construct the transition table.

State	0	1
→q0	q0	q1
q1	{q1, q2}	q1
*q2	q2	{q1, q2}

Now we will obtain δ' transition for state q0.

δ'([q0], 0) = [q0]

δ'([q0], 1) = [q1]

The δ' transition for state q1 is obtained as:

δ'([q1], 0) = [q1, q2] (**new** state generated)

δ'([q1], 1) = [q1]

The δ' transition for state q2 is obtained as:

δ'([q2], 0) = [q2]

δ'([q2], 1) = [q1, q2]

Now we will obtain δ' transition on [q1, q2].

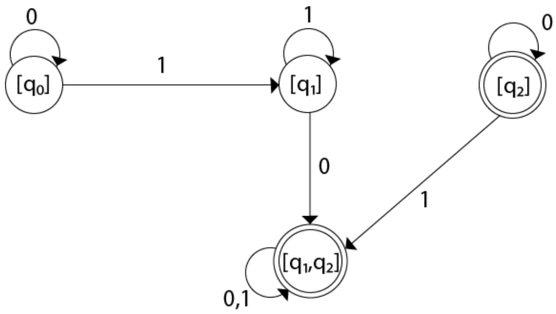
δ'([q1, q2], 0) = δ(q1, 0) ∪ δ(q2, 0)
= {q1, q2} ∪ {q2}
= [q1, q2]

δ'([q1, q2], 1) = δ(q1, 1) ∪ δ(q2, 1)
= {q1} ∪ {q1, q2}
= {q1, q2}
= [q1, q2]

The state [q1, q2] is the final state as well because it contains a final state q2. The transition table for the constructed DFA will be:

State	0	1
→[q0]	[q0]	[q1]
[q1]	[q1, q2]	[q1]
*[q2]	[q2]	[q1, q2]
*[q1, q2]	[q1, q2]	[q1, q2]

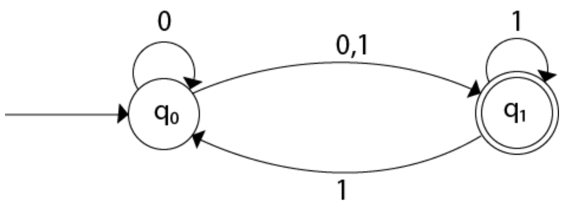
The Transition diagram will be:



The state q2 can be eliminated because q2 is an unreachable state.

Example 4:

Convert the given NFA to DFA.



Solution: For the given transition diagram we will first construct the transition table.

State	**0**	**1**
→q0	{q0, q1}	{q1}
*q1	ϕ	{q0, q1}

Now we will obtain δ' transition for state q0.

δ'([q0], 0) = {q0, q1}
= [q0, q1] (**new** state generated)
δ'([q0], 1) = {q1} = [q1]

The δ' transition for state q1 is obtained as:

δ'([q1], 0) = ϕ
δ'([q1], 1) = [q0, q1]

Now we will obtain δ' transition on [q0, q1].

δ'([q0, q1], 0) = δ(q0, 0) ∪ δ(q1, 0)
= {q0, q1} ∪ ϕ
= {q0, q1}
= [q0, q1]

Similarly,

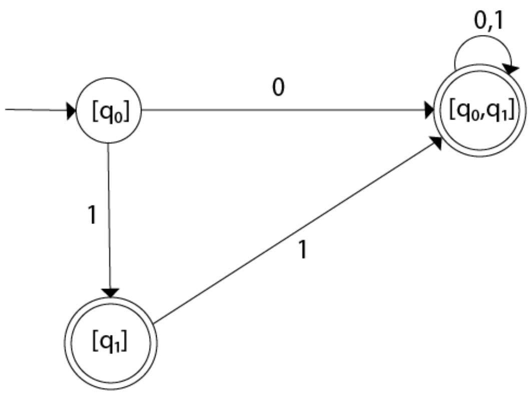
δ'([q0, q1], 1) = δ(q0, 1) ∪ δ(q1, 1)
= {q1} ∪ {q0, q1}
= {q0, q1}
= [q0, q1]

As in the given NFA, q1 is a final state, then in DFA wherever, q1 exists that state becomes a final state. Hence in the DFA, final states are [q1] and [q0, q1]. Therefore set of final states F = {[q1], [q0, q1]}.

The transition table for the constructed DFA will be:

State	0	1
→[q0]	[q0, q1]	[q1]
*[q1]	ϕ	[q0, q1]
*[q0, q1]	[q0, q1]	[q0, q1]

The Transition diagram will be:



Even we can change the name of the states of DFA.

Suppose

1. A=[q0]
2. B=[q1]
3. C=[q0,q1]

With these new names the DFA will be as follows: