How to convert finite automata to regular expression by using Arden's theorem:

Arden's theorem state that:

"If P and Q are two regular expressions over $\sum$, and if P does not contain ϵ, then the following equation in R given by R = Q + RP has a unique solution i.e., R = QP*."

That means, whenever we get any equation in the form of R = Q + RP, then we can directly replaced by R = QP*.

The following method is the utilization of the Arden's theorem. This is used to find the regular expression recognized by a transition system.

1. To get the regular expression from the automata we first create the equations for each state in presenting in the finite automata transition diagram. Note: Consider the incoming edges only to a state in transaction diagram to construct the equation of each state.The state equation in the form of

$q_1 = q_1w_{11} + q_2w_{21} + \ldots + q_nw_{n1} + \epsilon$ (q_1 is the initial state)

$q_2 = q_1w_{12} + q_2w_{22} + \ldots + q_nw_{n2}$

$\vdots \qquad \vdots$

$q_n = q_1w_{1n} + q_2w_{2n} + \ldots + q_nw_{nn}$

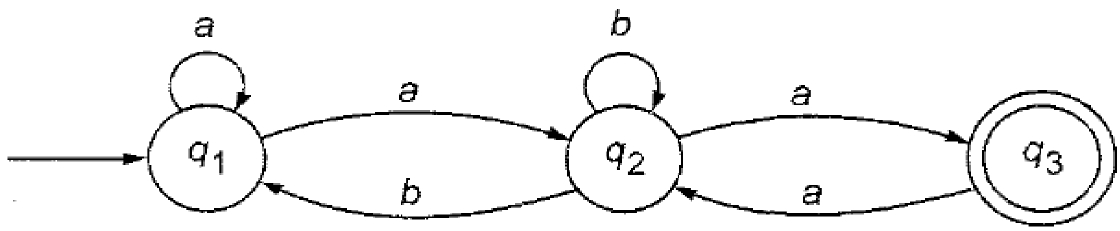
Where, w_{ij} is the regular expression representing the set of labels of edges from q_i to q_j.

Note: For parallel edges there will be that many expressions for that state in the expression.

2. Then we solve these equations to get the equation for q_i in terms of w_{ij} and that expression is the required solution, where q_i is a final state.
3. Repeatedly applying substitutions method and Arden's theorem over the state equations and we can express q_i in terms of w_{ij}'s.
4. For getting the set of string recognized by the transition system, we have to take the 'union' of all V_i's corresponding to final states.

Assumptions are made regarding the transition system:

- The transition graph does not have ε-moves.
- It has only one initial state q_0.
- Its vertices are represented as $q_1, q_1, \ldots \ldots, q_n$.
- Q_i the regular expression represents the set of string accepted by the system even though q_i is a final state.



Arden's Theorem Example Transaction Diagram – 1

Figure (1): Transaction Diagram

Consider the Transaction diagram (1), covert it to an equivalent regular expression using Arden's Theorem.

We can directly apply the Arden's procedure directly since the graph does not contain any ε-moves and there is only one initial state.

The three equations for q_1, q_2 and q_3 can be written as;

$q_1 = q_1a + q_2b + \varepsilon$ —- (1)
$q_2 = q_1a + q_2b + q_3a$ —- (2)
$q_3 = q_2a$ —- (3)

It is necessary to reduce the number of unknowns by repeated substitution. By substituting q3 in the q2-equation, we get by applying Arden's Theorem:

$q_2 = q_1a + q_2b + q_2aa$
$= q_1a + q_2 (b + aa)$
$= q_1a (b + aa)^*$

Substituting q2 in q1, we get

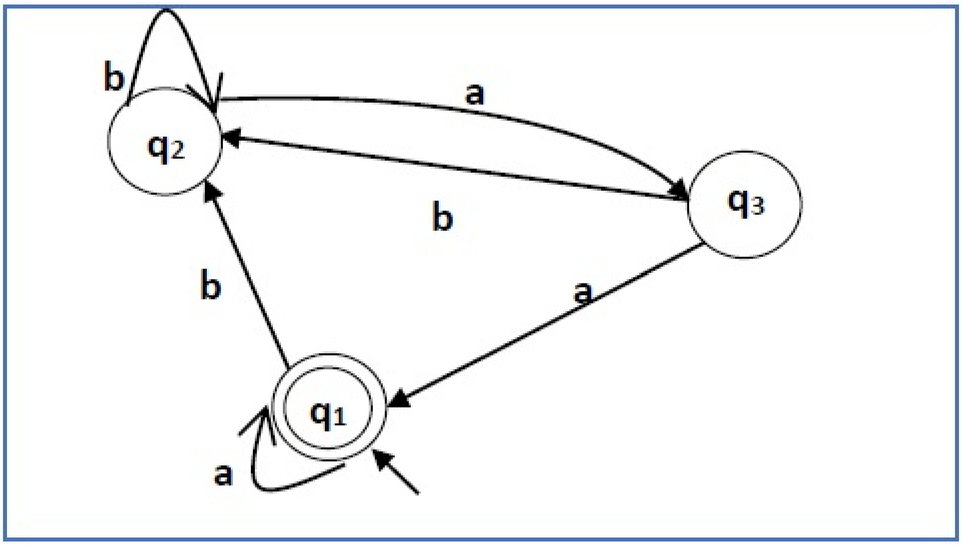
$q_1 = q_1a + q_1a(b + aa)^*b + \varepsilon$
$= q_1 (a + a(b + aa)^*b) + \varepsilon$

Hence,
$q_1 = \varepsilon (a + a(b + aa)^*b)^*$
$q_2 = (a + a(b + aa)^*b)^* a(b + aa)^*$
$q_3 = (a + a(b + aa)^*b)^* a(b + aa)^*a$

Since q3 is a final state, the set of strings recognized by the transaction graph is given by

(a + a(b + aa)*b)* a(b + aa)*a

2) **Problem :** Construct a regular expression corresponding to the automata given below −



Solution −

Here the initial state and final state is $\mathbf{q_1}$.

The equations for the three states q1, q2, and q3 are as follows −

$q_1 = q_1a + q_3a + \varepsilon$ (ε move is because q1 is the initial state0

$q_2 = q_1b + q_2b + q_3b$

$q_3 = q_2a$

Now, we will solve these three equations −

$q_2 = q_1b + q_2b + q_3b$

$= q_1b + q_2b + (q_2a)b$ (Substituting value of q_3)

$= q_1b + q_2(b + ab)$

$= q_1b (b + ab)^*$ (Applying Arden's Theorem)

$q_1 = q_1a + q_3a + \varepsilon$

$= q_1a + q_2aa + \varepsilon$ (Substituting value of q_3)

$= q_1a + q_1b(b + ab^*)aa + \varepsilon$ (Substituting value of q_2)

$= q_1(a + b(b + ab)^*aa) + \varepsilon$

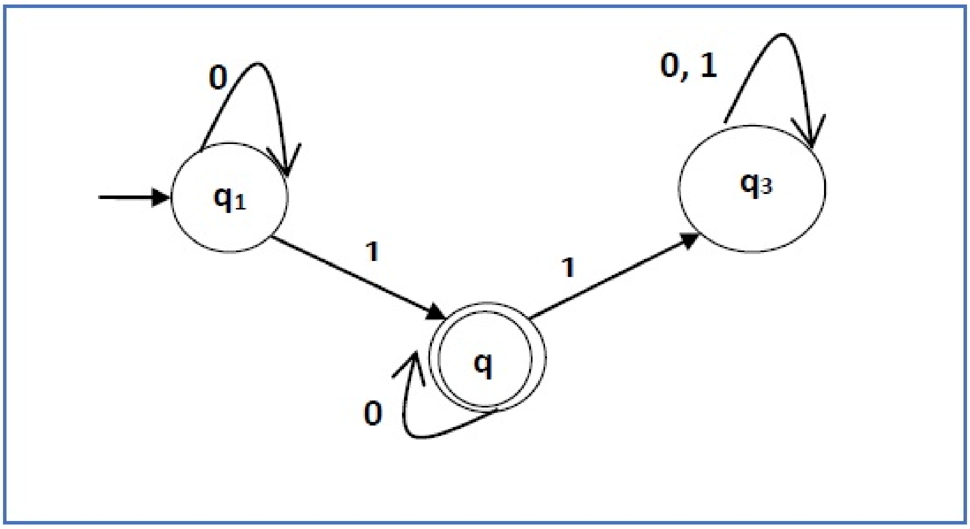
$= \varepsilon (a+ b(b + ab)^*aa)^*$

$= (a + b(b + ab)^*aa)^*$

Hence, the regular expression is (a + b(b + ab)*aa)*.

3) Problem

Construct a regular expression corresponding to the automata given below −



Solution −

Here the initial state is q_1 and the final state is q_2

Now we write down the equations −

$q_1 = q_1 0 + \varepsilon$

$q_2 = q_1 1 + q_2 0$

$q_3 = q_2 1 + q_3 0 + q_3 1$

Now, we will solve these three equations −

$q_1 = \varepsilon 0^*$ [As, $\varepsilon R = R$]

So, $q_1 = 0^*$

$q_2 = 0^*1 + q_2 0$

So, $q_2 = 0^*1(0)^*$ [By Arden's theorem]

Hence, the regular expression is 0^*10^*.