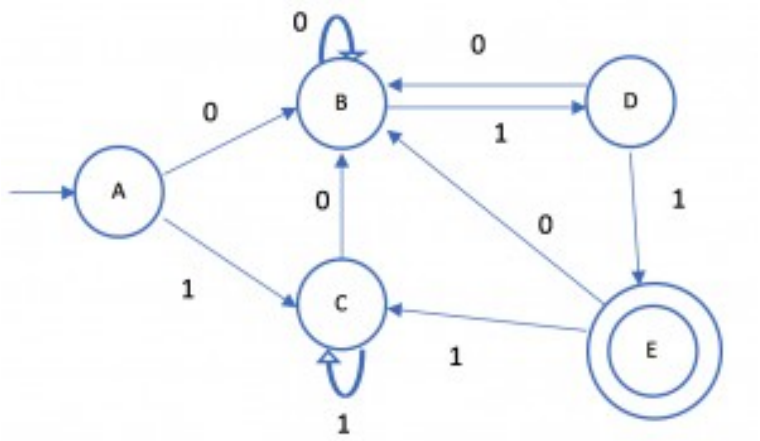
Example of Minimization of Deterministic Finite Automata (DFA)

Minimization of DFA (Table Filling Method or Myhill-Nerode Theorem)

Steps:

i. Draw a table for all pairs of states (P, Q)
ii. Mark all pairs where P∈ F and Q∉F
iii. If there are any Unmarked pairs (P, Q) such that [δ(P, x),δ(Q, x)] is marked, then mark [P, Q] where 'x' is an input symbol. Repeat this until no more marking can be made.
iv. Combine all the unmarked pairs and make them a single state in the minimized DFA.

Example: Minimize the following DFA using Table Filling Method.



Step 1: Draw a table for all pairs of states (P, Q)

	A	B	C	D	E
A					
B					
C					
D					
E					

Step 2: Mark all pairs where

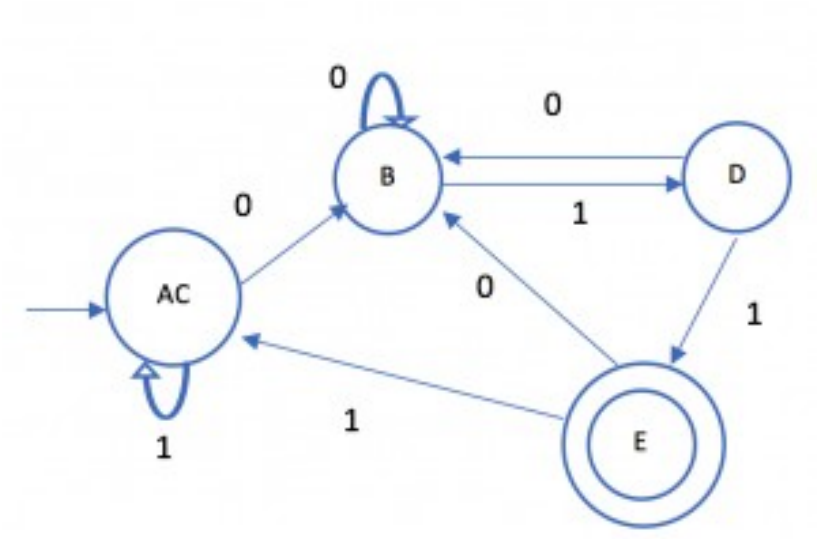
	A	B	C	D	E
A					
B					
C					
D					
E	+	+	+	+	

Step 3: If there are any Unmarked pairs (P, Q) such that [δ(P, x),δ(Q, x)] is marked, then mark [P, Q] where ‘x’ is an input symbol. Repeat this until no more marking can be made.

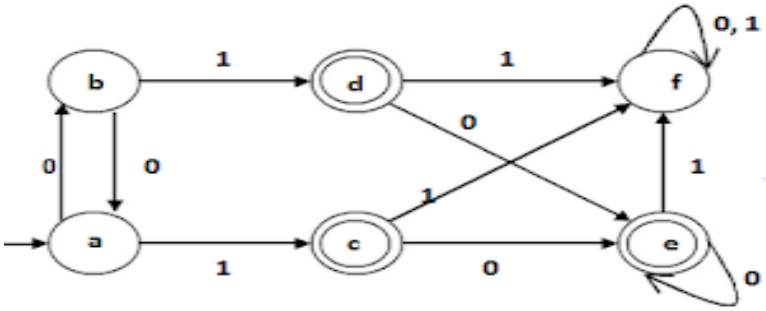
	A	B	C	D	E
A					
B	+				
C		+			
D	+	+	+		
E	+	+	+	+	

(B, A) For input 0 (B, 0) = B (A, 0) = B	(B, A) For input 1 (B,1) = D (A,1) = C	(C, A) For input 0 (C,0) = B (A,0) = B	(C, A) For input 1 (C,1) = C (A,1) = C
(C, B) For input 0 (C,0) = B (B,0) = B	(C, B) For input 1 (C,1) = C (B,1) = D	(D, A) For input 0 (D,0) = B (A,0) = B	(D, A) For input 1 (D,1) = E (A,1) = B
(D, B) For input 0 (D,0) = B (B,0) = B	(D, B) For input 1 (D,1) = E (B,1) = D	(D, C) For input 0 (D,0) = B (C,0) = B	(D, C) For input 1 (D,1) = E (C,1) = C

Step 4 :(A, C), B, D, E



Example 2: minimize the DFA shown below.



State Diagram of DFA

Step 1 : We draw a table for all pair of states.

	a	b	c	d	e	f
a						
b						
c						
d						
e						
f						

Step 2 : We mark the state pairs:

	a	b	c	d	e	f
a						
b						
c	✓	✓				
d	✓	✓				
e	✓	✓				
f			✓	✓	✓	

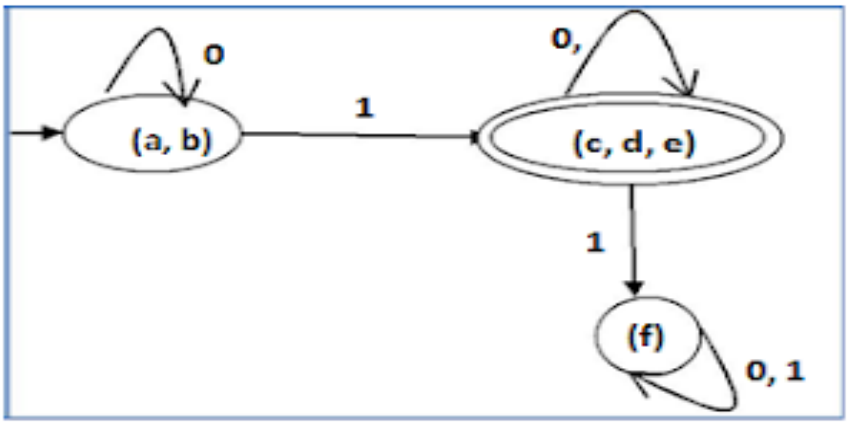
Step 3 : We will try to mark the state pairs, with green colored check mark, transitively. If we input 1 to state 'a' and 'f', it will go to state 'c' and 'f' respectively. (c, f) is already marked, hence we will mark pair (a, f). Now, we input 1 to state 'b' and 'f'; it will go to state 'd' and 'f' respectively. (d, f) is already marked, hence we will mark pair (b, f).

	a	b	c	d	e	f
a						
b						
c	✓	✓				
d	✓	✓				
e	✓	✓				
f	✓	✓	✓	✓	✓	

After step 3, we have got state combinations {a, b} {c, d} {c, e} {d, e} that are unmarked.

We can recombine {c, d} {c, e} {d, e} into {c, d, e} Hence we got two combined states as: {a, b} and {c, d, e}

So the final minimized DFA will contain three states {f}, {a, b} and {c, d, e}



State diagram of reduced DFA