Lesson

2

K L E TECHNOLOGICAL UNIVERSITY

DEPARTMENT OF HUMANITIES

PROFESSIONAL APTITUDE AND LOGICAL REASONING

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Number System-1

PROFESSIONAL APTITUDE AND LOGICAL REASONING

Number System – 1

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In This Chapter You Will

☑ Learn about factor theory
☑ Greatest Common Divisor and Lowest Common Multiple
☑ Cyclicity, Remainders and Factorials

Factor Theory

Number of Factors of a Number

To find the number of factors of *a* composite number write down the number in the form of $n = a^p b^q c^r$… where a, b and c are prime numbers and p, q and r are natural numbers then the number of factors of n is given by (p+1)(q+1)(r+1)… and it includes 1 and the number itself.

Explanation:

Let us take an example of $N = 2^3 3^5 5^4$. Now, 1st observe some facts about the factors of this number:

A factor of above number

1. must have prime factor either 2, 3 or 5 or none (in case of 1)
2. is not divisible by any other prime number other than 2, 3 or 5
3. have the highest powers of 2, 3 and 5 as 3, 5 and 4 respectively
4. has power of 2 as 2^0 or 2^1 or 2^2 or 2^3 (total 3+1=4 ways)
5. has power of 3 as 3^0 or 3^1 or 3^2 or 3^3 or 3^4 or 3^5 (total 5+1=6 ways)
6. has power of 5 as 5^0 or 5^1 or 5^2 or 5^3 or 5^4 (total 4+1=5 ways)

Hence, the number of factors is given by (3+1)(4+1)(5+1)=4x5x6=120

Now, alternatively we can look at it as

A factor of a given number must have prime factors either 2, 3 or 5

So a factor must be $2^{(0 \text{ or } 1 \text{ or } 2 \text{ or } 3)}\ 3^{(0 \text{ or } 1 \text{ or } 2 \text{ or } 3 \text{ or } 4 \text{ or } 5)}\ 4^{(0 \text{ or } 1 \text{ or } 2 \text{ or } 3 \text{ or } 4)}$

3 + 1 = 4ways   5 + 1 = 6 ways   4 + 1 = 5 ways

Total number of factors = 4 x 6 x 5 = 120

Number of Even Factors of a Number $N = 2^p a^q b^r c^s$ = p(q+1)(r+1)(s+1)

Number of Odd Factors of a Number N = Total Number of Factors – Number of Even Factors

Number of Ways to Express a Number as a Product of Two Factors

To find out the number of ways to express a number of two factors 1st convert the number in the form of $n = a^p b^q c^r$… then total number of factors is (p+1)(q+1)(r+1)… and number of ways to express the number as a product of two numbers is ½ (p+1)(q+1)(r+1).

Consider a number *n* if its square root has integral part as *k* (Means √N = K.*xxxx*) then for every factor less than *k* there exists a factor more than *k* such that product of these two factors is equal to N

For example: In how many ways can you express 80 as a product of two of its factors?

80 = 2^4 x 5 so number of factors are (4+1)(1+1) = 10

Factors of 80 are 1, 2, 4, 5, 8, 10, 16, 20, 40, 80

So number of ways to express 80 as a product of two of its factors is

1 X 80 = 80   2 X 40 = 80   4 X 20 = 80   5 X 16 = 80   8 X 10 = 80

So totally 5 ways are there which is actually half of the total number of factors given by

$\frac{1}{2}$ (4+1)(1+1) = 5

Sum of All the Factors of a Number

If we need to find out the sum of all the factors of a number then initially write down the number in the form of $n = a^p b^q c^r$… here a, b and c are prime numbers

Then, sum of all the factors of the number is given by

$$n = \left\{\frac{a^{p+1}-1}{a-1} \times \frac{b^{q+1}-1}{b-1} \times \frac{c^{r+1}-1}{c-1} \dots\dots\right\}$$

Number of Ways to Express a Number as a Product of Two Co-Prime Factors

If we need to find out the sum of all the factors of a number then initially write down the number in the form of $n = a^p b^q c^r$… here a, b and c are prime numbers

If in this expression we have *k* numbers of prime factors then required number of ways = 2^{k-1}

Product of All the Factors

For a number written in the form $N = a^p b^q c^r$ the number of factors of N is given by

n(f) = (p+1)(q+1)(r+1)

The **product of all the factors is given by $N^{n(f)/2}$**

Greatest Common Divisor (GCD) and Lowest Common Multiple (LCM) of Fractions

GCD of two or more fractions is given by

$$\frac{GCD\ of\ Numerators\ of\ All\ the\ Fractions}{LCM\ of\ Denominators\ of\ All\ the\ Fractions}$$

LCM of two or more fractions is given by

$$\frac{LCM\ of\ Numerators\ of\ All\ the\ Fractions}{GCD\ of\ Denominators\ of\ All\ the\ Fractions}$$

Important Points about GCD and LCM

- **Product of Two Numbers = Product of LCM and GCD** or a X b = GCD (a,b) X LCM (a,b)
- **GCD of a given set of numbers is always a factor of their LCM**
- **GCD (a,b) = LCM (a,b) then a = b**
- **For three numbers a, b and c if LCM of (a,b), (b,c) and (c,a) is L_{ab}, L_{bc} and L_{ca} and GCD of the same numbers is G_{ab}, G_{bc} and G_{ca} then**

$$\frac{\boldsymbol{LCM(a,b,c)}}{\boldsymbol{GCD)a,b,c)}} = \frac{\boldsymbol{L_{ab} \times L_{bc} \times L_{ca}}}{\boldsymbol{a \times b \times c}}$$

CYCLICITY to Find Unit's Digit

To Find Last Digit (Unit's Place Digit)

Powers of 2

Last digit of $2^1 = 2$	Last digit of $2^5 = 2$	Last digit of $2^{4k+1} = 2$ (1^{st} point of cycle)
Last digit of $2^2 = 4$	Last digit of $2^6 = 4$	Last digit of $2^{4k+2} = 4$ (2^{nd} point of cycle)
Last Digit of $2^3 = 8$	Last digit of $2^7 = 8$	Last digit of $2^{4k+3} = 8$ (3^{rd} point of cycle)
Last digit of $2^4 = 6$	Last digit of $2^8 = 6$	Last digit of $2^{4k+0} = 6$ (4^{th} point of cycle)

Therefore, unit digits of power of 2 repeats after a cycle of 4. So if we need to get unit digit of any higher power of 2 then just divide the power by 4 and find the remainder.
If remainder is 1 then unit digit is 2 (1^{st} point of cycle)
If remainder is 2 then unit digit is 4 (2^{nd} point of cycle)
If remainder is 3 then unit digit is 8 (3^{rd} point of cycle)
If remainder is 0 then unit digit is 6 (4^{th} point of cycle)

Powers of 3

Last digit of $3^1 = 3$	Last digit of $3^5 = 3$	Last digit of $3^{4k+1} = 3$ (1^{st} point of cycle)
Last digit of $3^2 = 9$	Last digit of $3^6 = 9$	Last digit of $3^{4k+2} = 9$ (2^{nd} point of cycle)
Last Digit of $3^3 = 7$	Last digit of $3^7 = 7$	Last digit of $3^{4k+3} = 7$ (3^{rd} point of cycle)
Last digit of $3^4 = 1$	Last digit of $3^8 = 1$	Last digit of $3^{4k+0} = 1$ (4^{th} point of cycle)

Example: Find unit digit of 3^{48}.

Here the power is 48. Therefore divide 48 by 4 and we get zero remainder so unit digit is same as 4^{th} point of the cycle or same as that of 3^4 which is equal to 1.

Powers of 4

Last digit of $4^1 = 4$	Last digit of $4^3 = 4$	Last digit of $4^{2k+1} = 4$ (1^{st} point of cycle)
Last digit of $4^2 = 6$	Last digit of $4^4 = 6$	Last digit of $4^{2k+0} = 6$ (2^{nd} point of cycle)

Powers of 5

Any power of 5 will give us unit digit as 5

Powers of 6

Any power of 6 will give us unit digit as 6

Powers of 7

Last digit of $7^1 = 7$	Last digit of $7^5 = 7$	→	Last digit of $7^{4k+1} = 7$ (1st point of cycle)
Last digit of $7^2 = 9$	Last digit of $7^6 = 9$		Last digit of $7^{4k+2} = 9$ (2nd point of cycle)
Last Digit of $7^3 = 3$	Last digit of $7^7 = 3$		Last digit of $7^{4k+3} = 3$ (3rd point of cycle)
Last digit of $7^4 = 1$	Last digit of $7^8 = 1$		Last digit of $7^{4k+0} = 1$ (4th point of cycle)

Important Points to Remember About Cyclicity

- If you get 1 then that is the terminating point and after that the cycle will start again as we have seen in the case of 7
- If X is the digit then unit digit of X^5 is same as that of X
- Cyclicity of different digits is as follows

Digit	Cyclicity
1	1
2	4
3	4
4	2
5	1
6	1
7	4
8	4
9	2

1. Find the difference between largest and smallest 2 digit number that leaves 18 when subtracted by the product of its digits.
   a) 35 b) 77 c) 79 d) 88 e) None of these
2. A teacher gave a student to do multiplication of 6 consecutive natural numbers. When the student reported his result the teacher found that the result is divisible by 540 but not by 360. Then, he told the student, "You missed one number and instead you multiplied one nearby number of the missed number twice." Which of the following could be a possible missed number?
   a) 35 b) 122 c) 84 d) 71 e) 99
3. Which of the following statement is correct?
   a) $(+) \times (-) + (-) \times (-) + (-) \times (-) = (+)$ b) $(-)^{(-)} + (-)^{(+)} = (-)$
   c) $(-)^{(odd)} \times (+)^{(even)} = (+)$ d) $(-)^{(even)} \times (+)^{(odd)} = (+)$
   e) None of these
4. The total number of numbers that can be formed by using digits 1, 2, 3, 4, 5 and 6 exactly once is 6! How many of them is/are perfect squares?
   a) 1 b) 3 c) 0 d) None of these e) all of these
5. How many 5 digit numbers exist that have unit digit 6 and is/are perfect square?
   a) 44 b) 42 c) 40 d) 46 e) None of these
6. Find the number of factors of 1200.

a) 30 b) 24 c) 32 d) 36 e) None of these

7. If N has 16 factors then N^2 may have how many factors?

a) 32 b) 40 c) 45 d) 48 e) 56

8. Consider the number $n = 2^6 \times 3^5 \times 5^4 \times 7^3 \times 11^2$. How many factors of n have 5 as unit digit?

a) 288 b) 360 c) 720 d) 320 e) None of these

9. Find the number of odd factors of 300300

a) 128 b) 256 c) 192 d) 240 e) None of these

10. Find the smallest number that has 5 composite factors

a) 48 b) 36 c) 24 d) 30 e) 60

11. How many factors of 210^{50} end with at most 5 zeroes?

a) 65025 b) 90000 c) 93636 d) 75820 e) None of these

12. If the HCF of two numbers is 17 and their LCM is 765 then one of the two numbers may be

a) 153 b) 34 c) 37 d) 39 e) 36

13. Find the LCM of $2^3 \times 3^5 \times 5^2$, $3^3 \times 5^5 \times 7^7$ and $2^6 \times 3^6 \times 7^4$

a) $2^5 \times 3^6 \times 5^5 \times 7^7$ b) $2^6 \times 3^6 \times 5^5 \times 7^3$ c) $2^6 \times 3^6 \times 5^5 \times 7^7$

d) $2^5 \times 3^6 \times 5^4 \times 7^7$ e) None of these

14. If LCM of two numbers is 420 and their HCF is 7, then how many such pairs of numbers exist?

a) 2 b) 4 c) 6 d) 8 e) None of these

15. In a class there are 2 sections A and B. 1/3 of section A play cricket, 1/7 like chocolate and 1/5 like Amitabh. While in section B 1/11 of the section like Aamir, 1/4 like ice-cream and 1/5 play football. If strength of section A is more than that of section B then find the minimum possible total number of students in section A and B.

a) 325 b) 535 c) 330 d) 220 e) 450

16. Find the unit digit of 9^8

a) 9 b) 1 c) 3 d) 7 e) None of these

17. Find the unit digit of $9^9 + 99^{99} + 999^{999}$

a) 9 b) 1 c) 3 d) 7 e) None of these

18. Find the unit digit of $4^{6^{8^{10^{\cdots^{100}}}}}$

a) 2 b) 4 c) 6 d) 8 e) None of these

19. Find the last two digits of 3^{171}

a) 41 b) 23 c) 47 d) 69 e) None of these

20. Find the last two digits of $17^{16^{15^{14^{\cdots^{2^1}}}}}$

a) 17 b) 21 c) 81 d) 31 e) None of these