Data Structures and Algorithms

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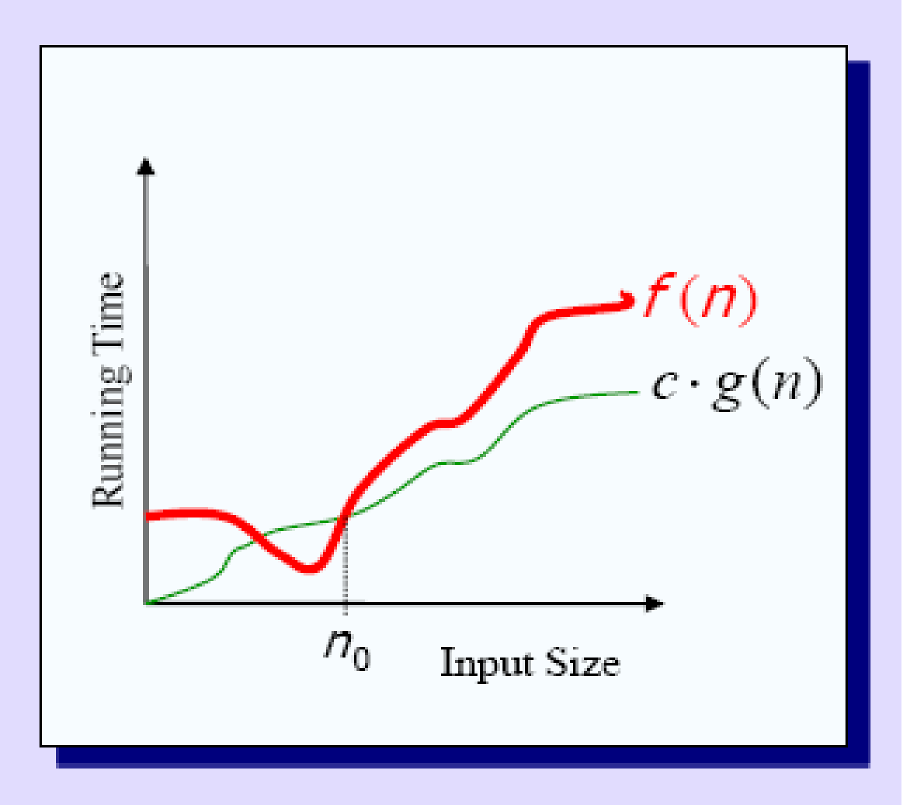
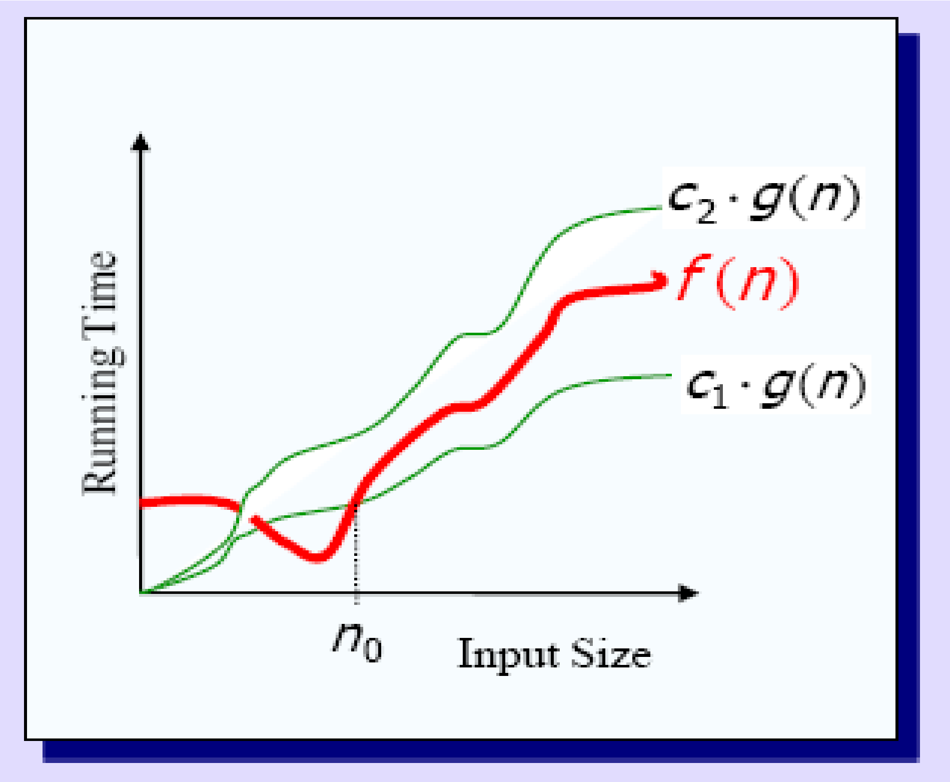
Asymptotic notation

- "Big-Omega" notation (Ω-notation)
  - Asymptotic lower bound on running time
  - f(n) is $\Omega(g(n))$ if there exists constants "$c > 0$" and "$n_0 \geq 1$", s. t. $c\, g(n) \leq f(n)$, for all $n \geq n_0$.



Asymptotic notation

- "Big-Theta" notation (θ-notation)
  - Asymptotically tight bound
  - f(n) is θ(g(n)) if there exists constant "$c_1 > 0$", "$c_2 > 0$" and "$n_0 \geq 1$" s.t. c_1 g(n) ≤ f(n) ≤ c_2 g(n), for n ≥ n_0



Asymptotic Notation

- Two more relatives of Big-Oh notation are:
  - “Little-Oh” notation: If $f(n)$ is $o(g(n))$, then, for every $c > 0$, there should exist $n_0 > 0$, s.t. $f(n) \le c\, g(n)$ for $n \ge n_0$
  - “Little-Omega” notation: If $f(n)$ is $\omega(g(n))$, then, for every $c > 0$, there should exist $n_0 > 0$, s.t. $c\, g(n) \le f(n)$ for $n \ge n_0$

Importance of Asymptotics

Running Time	Maximum problem size (n)		
	1 second	1 minute	1 hour
$400n$	2500	150000	9000000
$20n \log n$	4096	166666	7826087
$2n^2$	707	5477	42426
n^4	31	88	244
2^n	19	25	31

Stack operations: Running time

```
Algorithm push(o)
        if size()  = N then
                indicate a stack-full error has occurred
        t ←t+1
        S[t] = o
Running time: O(1)
Algorithm pop()
        if(isEmpty()) then
                indicate a stack-empty error has occurred
        e ← S[t]  {e is a temporary variable}
        S[t] ← null
        t ← t-1
        return e

Running time: O(1)
```

Queue Operations: Running time

```
Algorithm enqueue(e)
        if (size() == N-1)
                raise QueueFull exception
        Q[r] = e
        r ← (r+1) mod N
Running time: O(1)
Algorithm dequeue()
        if isEmpty() then
                raise QueueEmpty Exception
        temp ← Q[f]
        Q[f] = null
        f ← (f+1) mod N
        return temp
Running time: O(1)
```

List Operations: Running time

Algorithm insert(e, p)

```
    for i=n-1, n-2, . . ., p do
        S[i+1] ← S[i]  {Make room for "e" to be inserted}
    S[p] ← e
    n ← n+1
```

Running time: O(n)

Algorithm removeAtPosition(p)

```
    e ← S[p]  {e is a temporary variable}
    for i=p, p+1, . . ., n-2 do
        S[i] = S[i+1]  {fill in for the removed element}
    n ←n-1
```

Running time: O(n)

Linked list: Running time

- Assume that a list is implemented using a singly linked list
- The list is maintained in sorted order
- We have reference to the head of the list
- The running time of insert(e) operation?
- The running time of delete(e) operation?

Sorting

- Storing data in an ordered (increasing or decreasing or alphabetical, etc.) manner
- Searching becomes efficient when data is maintained sorted order
- Computer graphics and computational geometry
- A large number of sorting algorithms representing different design techniques have been developed

Sorting Problem

Input: a sequence of n numbers $(a_1, a_2, . . ., a_n)$

Output: A permutation or reordeing $(a_1', a_2', . . ., a_n')$ of the input sequence such that $a_1' \leq a_2' \leq . . . \leq a_n'$

Requirements for the output: output should be a permutation of the input

Factors affecting the running time: The input size, how sorted the input sequence is, and the algorithm used

Insertion Sort

- At any point of time, the given sequence can be divided into two parts:
  - Sorted part
  - Unsorted part
- Initially, the sorted part is empty
- Pick the first element from the unsorted part
- Place the picked element in the correct position in the sorted part
- Repeat the same process with the remaining elements

Insertion Sort

- Strategy

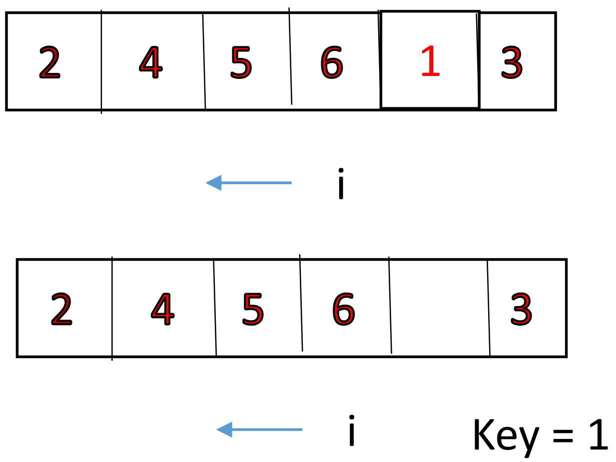
5	2	4	6	1	3
5	**2**	4	6	1	3
2	5	**4**	6	1	3
2	4	5	**6**	1	3
2	4	5	6	**1**	3
1	2	4	5	6	**3**

5	2	4	6	1	3
2	5	4	6	1	3
2	**4**	5	6	1	3
2	4	5	**6**	1	3
1	2	4	5	6	3
1	2	**3**	4	5	6

Insertion sort algorithm

Algorithm Insertion_Sort(A)
Input: A[0. .n-1] – an array of integers
Output: a permutation of A such that A[0] ≤ A[1] ≤ . . . ≤ A[n-1]

```
for j ← 1 to n-1 do
        key ← A[j]
        {insert A[j] into the sorted
                sequence A[0. .j-1]}
        i ← j-1
        while i ≥ 0 and A[i] > key do
                A[i+1] ← A[i]
                i - -
        A[i+1] ← key
```



Insertion Sort: Example

3	**4**	6	8	9	7	2	5	1
3	4	**6**	8	9	7	2	5	1
3	4	6	**8**	9	7	2	5	1
3	4	6	8	**9**	7	2	5	1
3	4	6	8	9	**7**	2	5	1
3	4	6	7	8	9	**2**	5	1
2	3	4	6	7	8	9	**5**	1
2	3	4	5	6	7	8	9	**1**

3	**4**	6	8	9	7	2	5	1
3	4	**6**	8	9	7	2	5	1
3	4	6	**8**	9	7	2	5	1
3	4	6	8	**9**	7	2	5	1
3	4	6	**7**	8	9	2	5	1
2	3	4	6	7	8	9	5	1
2	3	4	**5**	6	7	8	9	1
1	2	3	4	5	6	7	8	9

Analysis of insertion sort

for j ← 1 to n-1 do	n
key ← A[j]	n-1
{insert A[j] into the sorted sequence A[0. .j-1]}	
i ← j-1	n-1
while i ≥ 0 and A[i] > key do	$\sum_{j=1}^{n-1} t_j$
A[i+1] ← A[i]	$\sum_{j=1}^{n-1} (t_j - 1)$
i - -	$\sum_{j=1}^{n-1} (tj - 1)$
A[i+1] ← key	n-1

Analysis of insertion sort

- Best-case: O(n)
- Worst-case: $O(n^2)$
- Average case: $O(n^2)$