3 Qubit Quantum Gate

$\boxed{C^2 U}$

① TOFFOLI

$$\begin{vmatrix} I_4 & O_4 \\ O_4 & CNOT \end{vmatrix}_{8\times 8}$$

U=CNOT

$\Rightarrow$ 6 $\rightleftarrows$ 7
Column

$|110\rangle \rightarrow |111\rangle$

② FREDKIN gate   U= SWAP

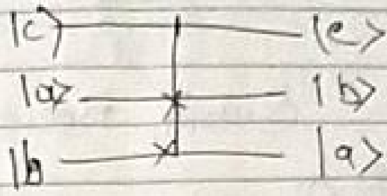
$$\begin{vmatrix} I4 & 0 \\ 0 & Swap \end{vmatrix}$$

5 $\rightleftarrows$ 6

$|101\rangle \rightleftarrows |110\rangle$

$\boxed{01 \rightarrow 10 \text{ swap}}$

Controlled bit as it is



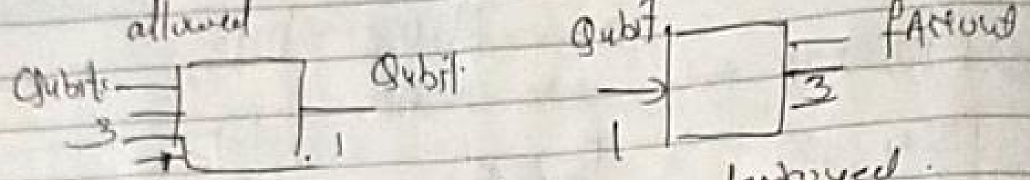
Single Qubit Gate

Quantum Circuits

C.C. if you will o/p feed back, there will loop. but in QC no loop.

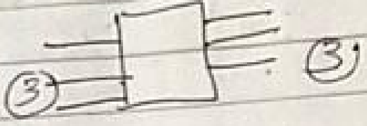
Properties 1. Quantum circuits are acyclic (No loops)
* No feedback in Q ckt.

(2) FAN-IN & FAN-OUT are not allowed



Qubit neither created nor destroyed.

(3) Q. ckts are Reversible



No of i/p = No. of o/p

* Measurement of Q. ckt

$\psi_{\text{Qubit}} = \alpha|0\rangle + \beta|1\rangle$

measurement

'0' or '1'

= Classical bit

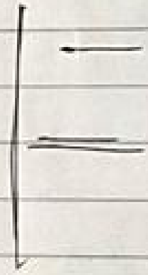
measuring thevch.

α with $|0\rangle = p = \frac{|\alpha|^2}{|\alpha|^2+|\beta|^2}$

β with $|1\rangle = p = \frac{|\beta|^2}{|\alpha|^2+|\beta|^2}$

Classical Result with '0' or '1'

Q. circuits are Reversible.
Q measurement are Irreversible.



— quantum bit

= Classical bit

I Quantum Gates in Series.

let A & B are 2 single Qubit Quantum Gates.

ψ_i —[A]—[B]— ψ_i —[BA]— $\Rightarrow BA|\psi_i\rangle$

A is operating first
so [BA] will totally ckt.

example

$|0\rangle$ —[Y]—[X]— $|0\rangle$ —[XY]— $\Rightarrow XY|0\rangle = iZ|0\rangle$

$\begin{matrix} 0 & -i \\ i & 0 \end{matrix}$ $\begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix}$

Quantum Circuits

C.C. If you will go feed back, there will loop. But in QC no loop.

Properties 1. Quantum circuits are acyclic (No loops)
No feedback in Q ckt.

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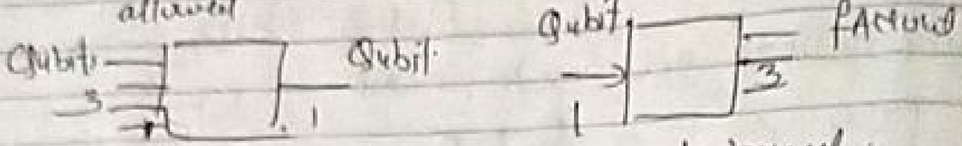

Qubit neither created nor destroyed.

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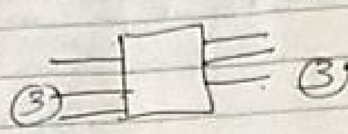
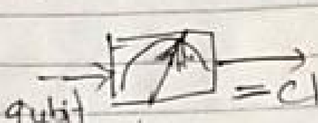
No of i/p = No. of o/p



Measurement of Q. ckt

$\psi_{Qubit} = \alpha|0\rangle + \beta|1\rangle$

measurement



measuring the value.

α with $|0\rangle = p = \frac{|\alpha|^2}{|\alpha|^2+|\beta|^2}$

β with $|1\rangle = p = \frac{|\beta|^2}{|\alpha|^2+|\beta|^2}$

Classical Result with '0' or '1'

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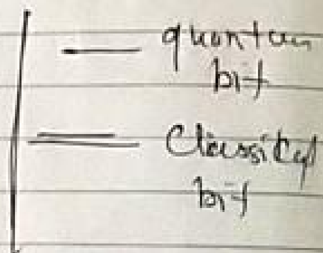


I Quantum Gates in Series.

Let A & B are 2 single Qubit Quantum Gates

ψ_i — [A] — [B] — ψ_i — [BA] — $\Rightarrow BA|\psi_i\rangle$

A is operating first
So $\boxed{BA}$ right to left ckt.

example

$|0\rangle$ — [Y] — [X] — $|0\rangle$ — [XY] — $\Rightarrow XY|0\rangle = iZ|0\rangle$

$0 = \frac{1}{0}$ $\begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix}$

$= i|0\rangle$

Right to left in series

② $|1\rangle$ —[Y]—[X]— ? ② —[XY]— $XY|1\rangle = iZ|1\rangle = -i|1\rangle$ $\begin{vmatrix}1 & 0\\ 0 & -1\end{vmatrix}$

Quantum Gate:

—[I]— NOP Gate - No operation Gate.

ψ_i —[I]— $\equiv \psi_i$ ——I—— : bare line
No gate on it → —[I]—

ψ_i

Gates in Parallel

Let A & B are 2 Single Qubits.

$|\psi_1\rangle$ ——[A]——
$|\psi_2\rangle$ ——[B]——

$$\psi_{o/p} = \left\{ \begin{matrix}\psi_1 \\ \psi_2\end{matrix} [A \otimes B] \right\} = (A\otimes B)(\psi_1 \otimes \psi_2)$$

$A\otimes B|\psi_1\psi_2\rangle$

$$\boxed{A\otimes B|\psi_1\psi_2\rangle}$$

eg:

$|0\rangle$ ——[H]——
$|1\rangle$ ——I—— Bare line.

o/p $\{(H\otimes I)(|0\rangle \otimes |1\rangle)$

$(H\otimes I)|01\rangle$

$$\frac{1}{\sqrt2}\begin{vmatrix}1 & 1\\ 1 & -1\end{vmatrix} \otimes \begin{vmatrix}1 & 0\\ 0 & 1\end{vmatrix} = \frac{1}{\sqrt2}\begin{vmatrix}1 & 0 & 1 & 0\\ 0 & 1 & 0 & 1\\ 1 & 0 & -1 & 0\\ 0 & 1 & 0 & -1\end{vmatrix}\begin{vmatrix}0\\1\\0\\0\end{vmatrix}$$

$1 \rightarrow \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$

$-1 \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$

$$= \frac{1}{\sqrt2}\begin{vmatrix}0\\1\\0\\1\end{vmatrix} = \frac{1}{\sqrt2}\left[|01\rangle + |11\rangle\right]$$

Basis
$= \{|0\rangle \,\&\, |1\rangle\}$
$\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$
$\{|000\rangle \ldots |111\rangle\}$

Identity Matrix

$\begin{matrix}0\\1\\0\\0\end{matrix} = \begin{vmatrix}0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{vmatrix}$

of order 2×2
Single bit

2 Qubit.

eg

$|0\rangle$ ——[H]——●——
$|1\rangle$ ————————⊕—— CNOT

$\Rightarrow$ Right to left

CNOT $(H\otimes I)$

$$(CNOT)\frac{1}{\sqrt2}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}\otimes\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$$

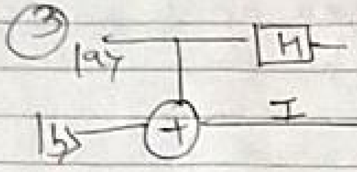
$$(CNOT)\frac{1}{\sqrt2}\begin{vmatrix}1 & 1\\ 1 & -1\end{vmatrix}\otimes\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$$

$$(CNOT)\frac{1}{\sqrt2}\begin{pmatrix}1 & 0 & 1 & 0\\ 0 & 1 & 0 & 1\\ 1 & 0 & -1 & 0\\ 0 & 1 & 0 & -1\end{pmatrix}$$

$$(CNOT)\ \frac{1}{\sqrt{2}}\begin{bmatrix}1&0&1&0\\0&1&0&1\\1&0&-1&0\\0&1&0&-1\end{bmatrix} = \begin{vmatrix} & \end{vmatrix}$$

$$\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{bmatrix} \times$$

(3)



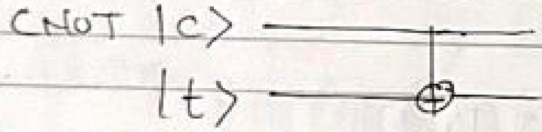
$(H \otimes I)\ CNOT$

$$\frac{1}{\sqrt{2}}\begin{vmatrix}1&0&1&0\\0&1&0&1\\1&0&-1&0\\0&1&0&-1\end{vmatrix} \times \begin{vmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{vmatrix}$$

$$= \begin{vmatrix}1&0&0&1\\0&1&1&0\\1&0&0&-1\end{vmatrix}$$

Every Q. ckt are unitary & reversible.

Quantum ckt with CNOT.



$$CNOT \begin{bmatrix}I&0\\0&X\end{bmatrix} = \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{bmatrix}$$

$2 \rightleftarrows 3$

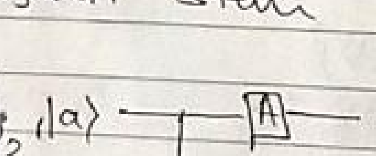
$|10\rangle \rightleftarrows |11\rangle$

****

Combination with CNOT = Bell state (Entangled state) cannot break state

Let A & B are 2 single Qubit states

$\} \psi = 1$

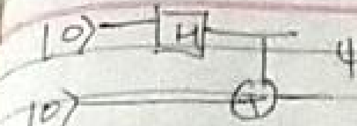


$\} \psi = 2$

$CNOT(A \otimes B)|\psi_i$

$\left(CNOT \,\middle|\, Right \,\middle|\right) \psi_i$ | $\left(\middle| left \,\middle|\, CNOT\right) \psi_1$

3 & 4th row interchange | 3rd & 4th Column interchange



$\psi_f = CNOT(H \otimes I)_{Right}$

$\psi_i = |0\rangle \otimes |0\rangle = |00\rangle$

$$= CNOT \frac{1}{\sqrt{2}} \begin{vmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{vmatrix} \begin{matrix} \\ \\ \leftarrow 3^{rd} \text{ row} \\ \leftarrow 4^{th} \text{ row} \end{matrix}$$

interchanges

$$\psi_1 = \frac{1}{\sqrt{2}} \begin{vmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 1 & 0 & -1 & 0 \end{vmatrix} \times \begin{vmatrix} 1 \\ 0 \\ 0 \\ 0 \end{vmatrix} \leftarrow |00\rangle$$

ψ_i

$$= \frac{1}{\sqrt{2}} \begin{vmatrix} 1 \\ 0 \\ 0 \\ 1 \end{vmatrix} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

$$\boxed{\psi_f = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)}$$

Bell state OR Entangled State

Bell state / Entangled state

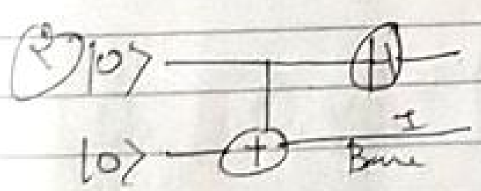
Superposⁿ state = linear combinatⁿ of all states.

Entangle state

$\psi \neq \psi_1 \otimes \psi_2$ — quantum state cannot be represented by tensor product of individual state & cannot split into individual state

Not Entangled

$$\left\{ \begin{aligned} \psi &= \frac{1}{\sqrt{2}} |01\rangle + |11\rangle \\ &= \frac{1}{2} |0\rangle + |1\rangle \otimes |1\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |11\rangle) \end{aligned} \right.$$



CNOT operating on right side rows interchange

CNOT on left hand side column interchanged

$$\psi_f = \left((H \otimes I)\ CNOT \right) \psi_1$$

$$= \left(\frac{1}{\sqrt{2}} \begin{vmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{vmatrix} CNOT \right) \psi_1$$

left side so 3rd & 4th column interch

$$= \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 0 & 1 & -1 & 0 \end{pmatrix} |00\rangle \Rightarrow \frac{1}{\sqrt{2}} \begin{vmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 0 & 1 & -1 & 0 \end{vmatrix} \begin{vmatrix} 1 \\ 0 \\ 0 \\ 0 \end{vmatrix}$$

$= \frac{1}{\sqrt{2}} \begin{vmatrix} 1 \\ 0 \\ 1 \\ 0 \end{vmatrix}$

$\frac{1}{\sqrt{2}} \left[|00\rangle + |10\rangle \right]$

NOT bell state

$\frac{1}{\sqrt{2}} \left[|0\rangle + |1\rangle \right] \otimes |0\rangle$

$\frac{1}{\sqrt{2}} \left[|00\rangle + |10\rangle \right]$ = Not bell state

CNOT Right $(A \otimes B)$ 3^{rd} & 4^{th} rows Interchange

$A \otimes B$ $(CNOT)$ 3^{rd} & 4^{th} column Interchange

History & Development in Q. C.

The idea of Q. computers was born out of the difficulty of simulating Quantum system on classical computer in 198.

David Deutsch is father of Q C.

QC has evolved significantly since its inception, blending theoritical concepts with practical advancement.

- (1) Origin & conceptual foundation 1990
- (2) Quant. Algo. Discovery 1990
- (3) Experimental progress (lt. 1990. to 2000)
- (4) Rise of Q. Theory
- (5) challanges & milestone
- (6) Commercial & ~~actoo~~ academic investment

Challanges.

1. Decoherence
2. Error Rate & Q. Gates
3. Scalability.
4. Q. Interconnects & connectivity
5. Q. s/w & Algo
6. H/w challanges
7. Measurement & ctr.
8. standardisation & Protocol
9. Energy efficiency
10. Integration with classical Computing

= Complex & Imaginary no.

$x^2 = 4$ i.e. $x = \pm 2$ square always +ve.

imaginary no.

$\Rightarrow x^2 = -4$

let $i = \sqrt{-1}$ then $x = \pm 2 i \sqrt{-1}$

$x = \pm i$ which is imaginary

Complex No. Real + imaginary no.

std. form $a + ib$ = a, b are real nos

eg. $3+2i$, $-1-i$, $\sqrt{3} + i\sqrt{2}$

$8-4i$ $-2 4 i^2$ $(i^2 = -1)$

eg.

Complex Conjugate of $a+bi = (a+ib)^* = a-ib$

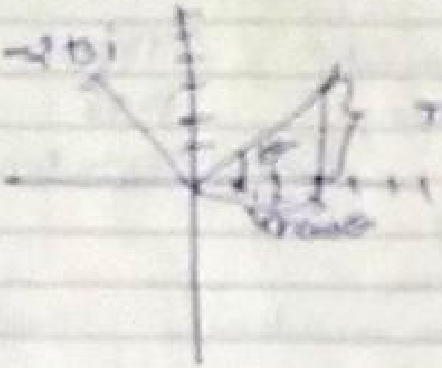
$(-2+3i)^* = -2-3i$

$(3+i)^* = 3-i$

Multiplication of complex no. with complex no. at gives real values always.

$(2+i3)(2-i3) = 2^2+9 = 13$

Complex no. of Number plane



Magnitude of $-2+3i$ is $|a+ib| = \sqrt{a^2+b^2}$

$|-2+3i| = \sqrt{(-2)^2+3^2} = \sqrt{13}$

$a+ib = r(\cos\theta + i\sin\theta)$ in Polar form

$a+ib = re^{i\theta}$ Exponential form

Matrices

$$\begin{vmatrix} 1 & \cdots & n \\ & & \vdots \\ m & \cdots & \end{vmatrix}_{m \times n}$$

m rows, n column

$$\begin{bmatrix} 4 & 1 & 2 \\ 3 & 2 & 1 \end{bmatrix}_{2\times 3}\ M$$

Matrix Addition

$$\begin{vmatrix} 3 & -7 \\ 4 & 0 \\ 5 & 4 \end{vmatrix} + \begin{vmatrix} 1 & 2 \\ -3 & 4 \\ 5 & 7 \end{vmatrix} = \begin{vmatrix} 4 & 5 \\ 1 & 4 \\ 10 & 11 \end{vmatrix}$$

both M. has same Dimension

Mat Multiplication by Scalar

$$K\begin{bmatrix} -4 & 3 \\ 7 & 0 \end{bmatrix} = \begin{bmatrix} -4K & 3K \\ 7K & 0 \end{bmatrix}$$

Coefficient

$$\begin{vmatrix} 1 & 2 \\ 1 & 3 \end{vmatrix} \times \begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix} = \begin{vmatrix} 1\times2+2\times3 & 1\times1+2\times2 \\ 11 & 7 \end{vmatrix} = \begin{vmatrix} 8 & 5 \\ 11 & 7 \end{vmatrix}$$

Column Vector

$$\begin{vmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{vmatrix}$$

Multiply $n\times n$ matrix with $n\times 1$ column vector gives $n\times 1$

$$\begin{vmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \vdots \\ a_{n1} & & a_{nn} \end{vmatrix} \begin{vmatrix} b_1 \\ \vdots \\ b_n \end{vmatrix} = \begin{vmatrix} c_1 \\ \vdots \\ \end{vmatrix}$$

Identity M.

eg:- $\begin{vmatrix} \frac{1}{2} & 0 \\ -\frac{1}{2} & 1 \end{vmatrix} \begin{vmatrix} 1 \\ -2 \end{vmatrix} = \begin{vmatrix} 3 \\ 0 \end{vmatrix}$ $= \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$

$A \times I = A$ = eg: $\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} \times \begin{vmatrix} a \\ b \end{vmatrix} = \begin{vmatrix} a \\ b \end{vmatrix}$

inverse $AA^{-1} = I$

Unitary & Hermitian Matrices

$A = \begin{vmatrix} 2+3i & 0 \\ 5 & 3-i \end{vmatrix}$ $A^* \begin{vmatrix} 2-3i & 0 \\ 5 & 3+i \end{vmatrix}$

A^* complex conjugate of A. we flip the sign of imaginary number

Transpose

$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{vmatrix}^T = \begin{vmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{vmatrix}$

$(A^*)^T = (A^T)^* = A$

Unitary M.

(+ dagger) $U^{+}U = I$

Inverse of U is U^{+}

given U is unitary Matrix.

* Every quantum gate represented by unitary operator.

$U^{-1} = U^{+}$ $U^{inverse} = U^{dagger}$

Hermitian Matrices

$H = H^{+}$

Eigen Value & Vector.

$A\vec{v} = \lambda \vec{v}$

Eigen Vector Eigen Value.

eg $\begin{vmatrix} 2 & 0 \\ 1 & 2 \end{vmatrix} \begin{bmatrix} 0 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 6 \end{bmatrix} = 2 \begin{bmatrix} 0 \\ 3 \end{bmatrix}$

Classical Computer — uses bits '0' & '1'

Quantum Com... — uses Qubits (combo 0 & 1 at the same time)

physically qubit can be made from any quantum partical that lies 2 distinct state

like classical computer, we use '0' & '1'

in Q.C. $|0\rangle = \begin{vmatrix} 1 \\ 0 \end{vmatrix}$, $|1\rangle = \begin{vmatrix} 0 \\ 1 \end{vmatrix}$

$\langle 0| = (1\ 0)$ $\langle 1| = (0\ 1)$

Superposition:

Quantum partical in two states simultaneously

A qubit is in superposition if it is both $|0\rangle$ & $|1\rangle$

Representing Qubit Mathematically.

$|\psi\rangle = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$ α represent qubit in $|0\rangle$ st.

β " qubit in $|1\rangle$ st.

Concept of Q. Matrices.

When we measure a quantum state, system, it collapses into the measured state.

- If we measured $|\psi\rangle$ as 0 then $|\psi\rangle$ ~~would~~ could collapse into 0 state.

  so $|\psi\rangle \Rightarrow 0$

- If we measured $|\psi\rangle$ as 1 then $|\psi\rangle$ could collapse into the state '1'.

  So $|\psi\rangle \Rightarrow 1$

Measuring a qubit collapse its superposition

* probability of measuring $|\psi\rangle$ is 0 is $= |\alpha|^2$

" " measuring $|\psi\rangle$ is 1 is $= |\beta|^2$

① $|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, Probability of measuring '0' is

② $|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$, probability of measuring '1' is 1

So probability of measuring 0's & 1's will be always '1'.

$|\alpha|^2 + |\beta|^2 = 1$

eg. $|\psi| = \begin{pmatrix} \sqrt{3}/2 \\ 1/2 \end{pmatrix}$

$\left|\frac{\sqrt{3}}{2}\right|^2 + \left|\frac{1}{2}\right|^2 = 1$

$\frac{3}{4} + \frac{1}{4} = \frac{4}{4} = 1$

Dirac Notation

$|\psi\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} \alpha \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ \beta \end{pmatrix}$

$= \alpha\begin{pmatrix} 1 \\ 0 \end{pmatrix} + \beta\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$= \underbrace{\alpha|0\rangle + \beta|1\rangle}$ Dirac Notaⁿ

$\begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

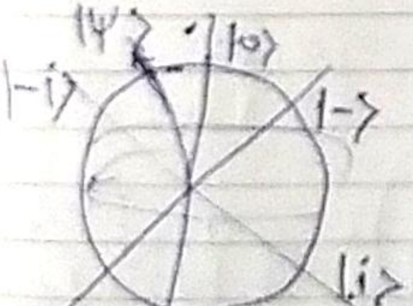
$\begin{pmatrix} 1 \\ 1 \end{pmatrix} I$

eg: Matrix to Dirac Notatⁿ conversion

$|\psi\rangle = \begin{pmatrix} 1/2 \\ \frac{2\sqrt{3}}{4} \end{pmatrix} = \frac{1}{2}|0\rangle + \frac{2\sqrt{3}}{4}|1\rangle$

Bloch sphere:

we can represent a qubit $|\psi\rangle$ as a point on the surface of the Bloch sphere.



Higher vertically ⇒
~~Higher propn~~
Higher prob. of measuring $|\psi|$ as $|0\rangle$
lower prob. of measuring $|\psi|$ as $|1\rangle$

$|0\rangle$ $|\psi\rangle$

$|1\rangle$ $|\psi\rangle$

Quantum Gates

$$\psi = \alpha|0\rangle + \beta|1\rangle \qquad |\uparrow\rangle \quad |\downarrow\rangle$$

$$\therefore \psi = \alpha|\uparrow\rangle + \beta|\downarrow\rangle$$

$$\langle\psi|\psi\rangle = (\alpha^*\langle 0| + \beta^*\langle 1|)(\alpha|0\rangle + \beta|1\rangle)$$

$$= |\alpha|^2 + |\beta|^2 = 1$$

$\left.\begin{array}{l} \underline{\langle 1|\,|1\rangle} = (0\ 1)\begin{pmatrix}0\\1\end{pmatrix} = (0+1) = \underline{1} \\ \langle 1|1\rangle \\ \underline{\langle 0|\,|0\rangle} = (1\ 0)\begin{pmatrix}1\\0\end{pmatrix} = (1+0) = \underline{1} \\ \text{OR } \langle 0|0\rangle \end{array}\right\}$ Normal and State

$\left.\begin{array}{l} \langle 1|\,|0\rangle = (0\ 1)\begin{pmatrix}1\\0\end{pmatrix} = (0+0) = 0 \\ \text{OR } \langle 1|0\rangle \\ \langle 0|\,|1\rangle = (1\ 0)\begin{pmatrix}0\\1\end{pmatrix} = (0+0) = 0 \\ \text{OR } \langle 0|1\rangle \end{array}\right\}$ Orthogonal State

$$\underline{|1\rangle\ |0\rangle} = \begin{pmatrix}0\\1\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}0\\1\end{pmatrix}$$

$$|1\rangle\langle 0| = \begin{pmatrix}0\\1\end{pmatrix}(1\ 0) = \begin{pmatrix}0\times1 & 0\times0\\ 1\times1 & 1\times0\end{pmatrix} = \begin{pmatrix}0 & 0\\ 1 & 0\end{pmatrix}$$

$$|0\rangle\langle 1| = \begin{pmatrix}1\\0\end{pmatrix}(0\ 1) = \begin{pmatrix}1\times0 & 1\times1\\ 0\times0 & 0\times1\end{pmatrix} = \begin{pmatrix}0 & 1\\ 0 & 0\end{pmatrix}$$

$$|0\rangle\langle 0| = \begin{pmatrix}1\\0\end{pmatrix}(1\ 0) = \begin{pmatrix}1\times1 & 1\times0\\ 0\times1 & 0\times0\end{pmatrix} = \begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix}$$

$$|1\rangle\langle 1| = \begin{pmatrix}0\\1\end{pmatrix}(0\ 1) = \begin{pmatrix}0\times0 & 0\times1\\ 1\times0 & 1\times1\end{pmatrix} = \begin{pmatrix}0 & 0\\ 0 & 1\end{pmatrix}$$

Qubit on {0, 1} basis

$$\langle\psi|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\end{pmatrix} + \frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}1/\sqrt{2}\\ 1/\sqrt{2}\end{pmatrix}$$

z-axis

$$|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\end{pmatrix} - \frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}1/\sqrt{2}\\ -1/\sqrt{2}\end{pmatrix}$$

(1)

Y axis

$|i\rangle = \frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle) = \begin{bmatrix} 1/\sqrt{2} \\ i/\sqrt{2} \end{bmatrix}$

$|-i\rangle = \frac{1}{\sqrt{2}}(|0\rangle - i|1\rangle) = \begin{pmatrix} 1/\sqrt{2} \\ -i/\sqrt{2} \end{pmatrix}$

Measuring state $|+\rangle$ on basis $\{0,1\}$

$\langle 0|+\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$

$= \frac{1}{\sqrt{2}}\underbrace{\langle 0|0\rangle}_{1} + \frac{1}{\sqrt{2}}\langle 0|1\rangle$

$= 1/\sqrt{2}$

$\langle 1|+\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$

$= \frac{1}{\sqrt{2}}\langle 1|0\rangle + \frac{1}{\sqrt{2}}\underbrace{\langle 1|1\rangle}_{1}$

$= 1/\sqrt{2}$

$\therefore$ prob $= \alpha^2 + \beta^2$

$= \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2} + \frac{1}{2} = \frac{2}{2} = 1$

Quantum Gate

Any Single qubit Quantum

$\langle 0|0\rangle$ = inner prod

$|0\rangle\langle 1|$ = outer prod

$\begin{vmatrix} 0 \\ 1 \end{vmatrix}$ $|1\rangle$ $\begin{pmatrix} 1 & 0 \end{pmatrix} \cdot \langle 0|$

$|0\rangle \langle 1|$ (a) $\begin{vmatrix} 0 \\ 1 \end{vmatrix}$ (b) $\begin{vmatrix} 1 \\ 0 \end{vmatrix}$

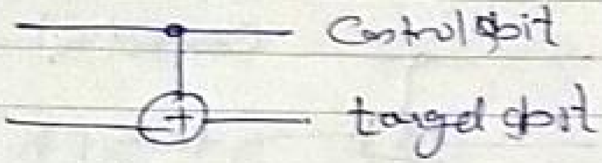
$\langle 1|$ $\langle 0|$

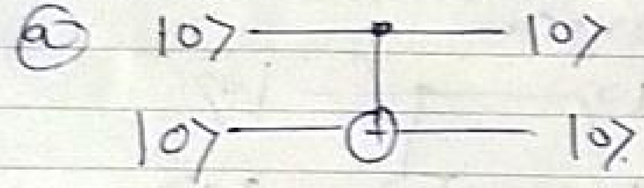
outer

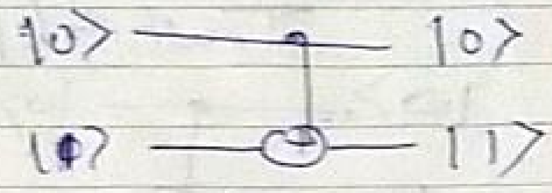
2 Qubit Gates

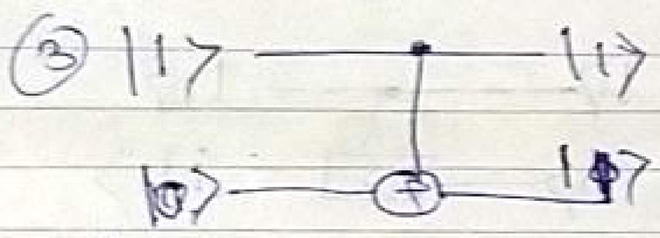
CNOT gate : Controlled NOT.

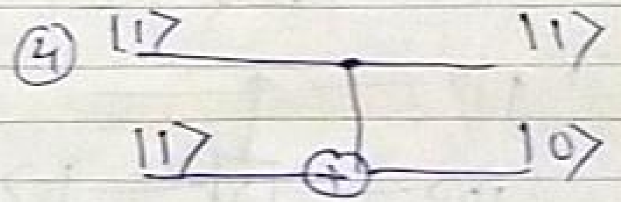
CNOT gate operates on 2 qubit out of which one is control bit & other is target bit.











if control bit is $|1\rangle$, then target Qubit flips

$|00\rangle \Rightarrow |00\rangle$
$|01\rangle \Rightarrow |01\rangle$
$|10\rangle \rightarrow |11\rangle$
$|11\rangle \rightarrow |10\rangle$

Matrix Representⁿ of CNOT Gate

$$\begin{vmatrix} I_2 & O_2 \\ O_2 & \underset{\uparrow \; X \text{ Gate}}{U_2} \end{vmatrix} \Rightarrow \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{vmatrix}_{4\times4}$$

C phase Gate:

It is represented by $-\boxed{S}-$

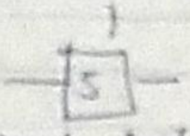
Matrix representaⁿ for $-\boxed{S}-$ is $\begin{vmatrix} 1 & 0 \\ 0 & i \end{vmatrix}$

C phase gate using Matrix is represented as

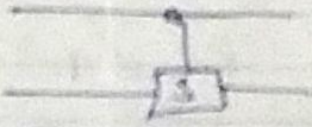
$$\begin{vmatrix} I_2 & O_2 \\ O_2 & U_2 \end{vmatrix} \Rightarrow \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & i \end{vmatrix}$$

control bit

target bit



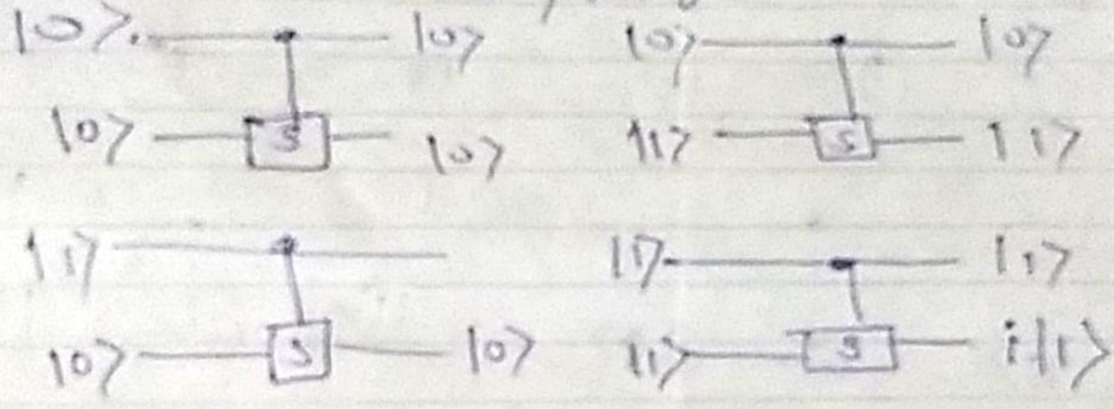
Control bit



Control bit

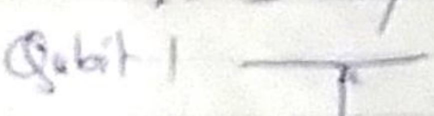
target bit

Truth table / Diagram



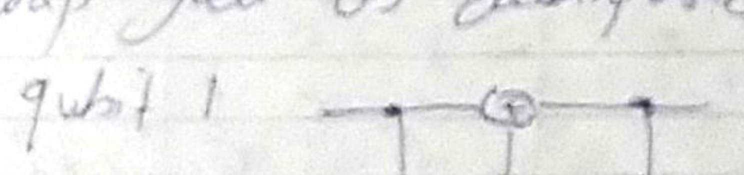
SWAP Gate

It swaps the states of qubits

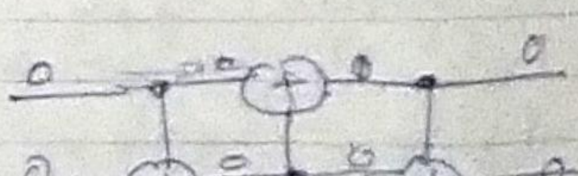


} Diagramatical Representation of Swap Gate

Swap gate is decomposed into 3 CNOT Gates.

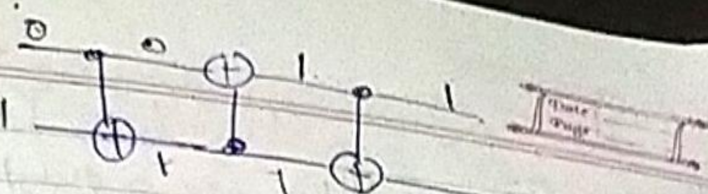


① $|00\rangle \rightarrow |00\rangle$



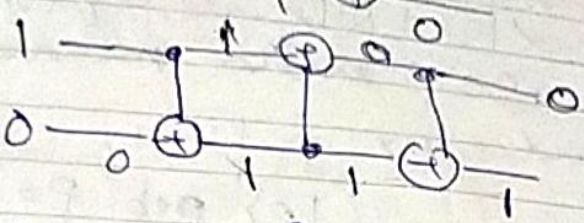
② $|01\rangle \to |10\rangle$

$1 \to 2$

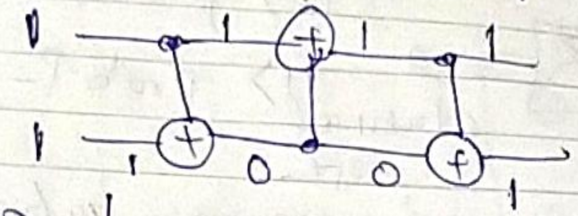


③ $|10\rangle \to |01\rangle$

$2 \to 1$



④ $|11\rangle \to |11\rangle$



$$\begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} \quad \text{SWAP}$$

3 Qubit Gate

Toffoli gate: Controlled CNOT gate.
in toffoli gate out of three qubit 2 qubits are controlled bit & 1 qubit is target bit.
The target qubit (third qubit) will be inverted if first & second quibits both are '1'.

$|000\rangle \to$	$|000\rangle$	$|100\rangle \to$	$|100\rangle$
$|001\rangle$	$|001\rangle$	$|101\rangle$	$|101\rangle$
$|010\rangle$	$|010\rangle$	$|110\rangle$	$|111\rangle$
$|011\rangle$	$|011\rangle$	$|111\rangle$	$|110\rangle$

$$\begin{vmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & 1 & & & & & & \\ & & 1 & & & & & \\ & & & 1 & & & & \\ & & & & 1 & & & \\ & & & & & 1 & & \\ & & & & & & & 1 \\ & & & & & & 1 & \end{vmatrix}$$

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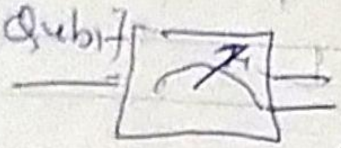
Measurement in Quantum Computing

$\psi = \alpha|0\rangle + \beta|1\rangle$ is a qubit

In QC, meas measurement is represented by

Qubit



$\Rightarrow$ $|0\rangle$ prob. $P = |\alpha|^2 / |\alpha|^2 + |\beta|^2$

$|1\rangle$ prob. $P = |\beta|^2 / |\alpha|^2 + |\beta|^2$

classical bit

When we measure qubit, it collapses to $|0\rangle$ & $|1\rangle$ which are classical state

$\therefore$ In QC all states of measurement are classical & irreversible

Measurement in QC is critical operation that extracts classical information from q. information.

purpose of measuring is to extract classical information from Quantum State

Parallism in Quantum

let A & B are 2 single qubits g

$|\phi_1\rangle$ —$\boxed{A}$—
$|\phi_2\rangle$ —$\boxed{B}$—

$\psi_{0,1}$

$(A \otimes B)\, |\psi\rangle \otimes |\psi\rangle$

$(A \otimes B)\, |\psi_1 \psi_2\rangle$

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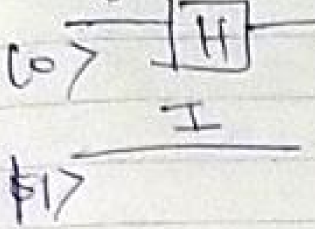
eg :-



$$(H \otimes I)(|0\rangle \otimes |1\rangle)$$

$$(H \otimes I)(|01\rangle)$$

$$= \frac{1}{\sqrt{2}} \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} \otimes \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$$

$$= \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$= \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} \Rightarrow \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$$= \frac{1}{\sqrt{2}} \left[|01\rangle + |11\rangle \right]$$

Every Q. ckt are unitary & Reversible