1. Introduction to Quantum Computing.

* History and Development in Quantum Computing :-
- The idea of quantum computer was born out of the difficult of simulating Quantum system on classical computer in 1980.

- David Deutsch is father of Quantum computing.

- Quantum computing has evolved significantly since it's inception, blending theoritical concepts with practical Advancement.

1) Origins and conceptual foundations :- (Early 1990)

2) Discovery of Quantum algorithms. (1990)

3) Experimental progress (late 1990-Earlies 2000).

4) The rise of quantum Theory

5) challenges and milestones. (2000-2010)

6) commercial & academics investments (2010-present)

* Challenges in Quantum computing :-
1. Decoherence
2. Error Rate and Quantum Gates
3. Scalability
4. Quantum Interconnects and connectivity
5. Quantum software and Algorithms
6. Hardware challanges
7. Measurement and control
8. Standardization and Protocols
9. Energy Efficiency
10. Integration with classical computing

* Introduction to imaginary and complex Numbers :-

$x^2 = 4$ but what if $x^2 = -4$
i.e. $x = \pm 2$ Number square is always positive.
which introduces imaginary numbers.

i.e. $x^2 = -4$
let $i = \sqrt{-1}$.

then $x = \pm 2\sqrt{-1}$
$x = \pm 2i$ which is imaginary Number.

complex Number:
Real Number + Imaginary Number.

std. form : $a + ib$, where $a, b \in R$ (Real Numbers)
eg. : $2+3i$, $-1-i$, $\sqrt{2} + i\sqrt{3}$

$(2+3i) + (4-8i) = 6-5i$
$(1+9i) + (-3-8i) = -2+i$

mul : $(2+3i)(4-8i) = 8 - 16i + 12i - 24i^2$
$= 8 - 4i - 24i^2$ $(i^2 = -1)$
$= 8 - 4i + 24$
$= 32 - 4i$

* complex conjugate of $a+ib = (a+ib)^* = a - ib$

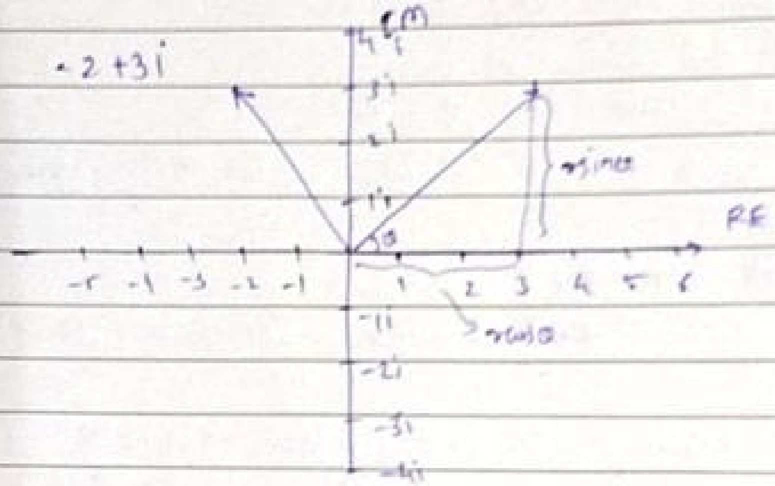
ex. $(-2+3i)^* = -2-3i$
$(3-i)^* = 3+i$

when we multiply any complex no. by its complex conjugate number it gives always real Number.

$$(a+ib)(a-ib) = a^2+b^2$$

$$(2+3i)(2-3i) = 13$$

* complex Numbers on Number plane :-



Magnitude of $-2+3i$ is $|a+ib| = \sqrt{a^2+b^2}$.

$$|-2+3i| = \sqrt{(-2)^2+(3)^2} = \sqrt{13}$$

$a+ib = r(\cos\theta + i\sin\theta)$ i.e. polar form

$a+ib = re^{i\theta}$ → Exponential form.

* Introduction to Matrices :-

$$\begin{array}{cc} & n \text{ columns} \\ m \text{ rows} & \left\{ \begin{bmatrix} 5 & \cdots & 3 \\ \vdots & \ddots & \vdots \\ 2 & \cdots & 3 \end{bmatrix} \right. \end{array} \quad m \times n \text{ matrix}$$

$$\begin{bmatrix} 4 & 1 & 2 \\ 3 & 2 & 4 \end{bmatrix} \; 2 \times 3 \text{ matrix}$$

- Matrix Addition :-

$$\begin{bmatrix} 2 & -3 \\ 7 & 0 \\ 0 & 4 \end{bmatrix} + \begin{bmatrix} 4 & 12 \\ -2 & 7 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 6 & 9 \\ 5 & 7 \\ 2 & 9 \end{bmatrix}$$

* we can only add and subtract matrices if both matrices are in the same dimensions.

* Multiplying Matrix by Scalar :-

$$k \begin{bmatrix} -4 & 3 \\ 7 & 0 \end{bmatrix} = \begin{bmatrix} -4K & 3K \\ 7K & 0 \end{bmatrix}$$

* Vectors and Matrix Multiplication to transform a vector :

① Matrix multiplication :

(1) $\begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ 11 & 7 \end{bmatrix}$

(2) $\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 11 \\ 2 \end{bmatrix}$

- To multiply two matrices together the number of columns of the left matrix must be equal to number of rows of the right matrix.

Column vector :- $\begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix}$

Column Vector.

- Multiplying a $n \times n$ matrix with a $n \times 1$ column vector gives us another $n \times 1$ column vector.

$$\begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{bmatrix} \begin{bmatrix} b_1 \\ \vdots \\ b_n \end{bmatrix} = \begin{bmatrix} c_1 \\ \vdots \\ c_n \end{bmatrix}$$

eg. $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \end{bmatrix} = \begin{bmatrix} -3 \\ 0 \end{bmatrix}$

* Identity matrix :-

2×2 Identity matrix

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

3×3 Identity matrix

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

We get,

$$A \cdot I = A.$$

eg. $\underset{I}{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}} \underset{A}{\begin{bmatrix} a \\ b \end{bmatrix}} = \underset{A}{\begin{bmatrix} a \\ b \end{bmatrix}}$

* Inverse of Matrices :-

$$A \cdot A^{-1} = I$$

* Unitary and Hermitian Matrices :-

$$A = \begin{bmatrix} 2+3i & 0 \\ 5 & 3-i \end{bmatrix} \qquad A^* = \begin{bmatrix} 2-3i & 0 \\ 5 & 3+i \end{bmatrix}$$

- to take complex conjugate A^* of matrix A we flip the signs of imaginary parts.

Transpose of matrix (T) :

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}^T = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$$

$$(A^*)^T = (A^T)^* = A^\dagger$$

unitary matrices :-

$$U^\dagger U = I \qquad ((\dagger) \text{ dagger})$$

$\Rightarrow$ Inverse of U is $U^\dagger$

given U is unitary matrix.

* Every quantum Gate Represented by unitary operator.

$$U^{-1} = U^\dagger \qquad U^{\text{Inverse}} = U^{\text{Dagger}}$$

Hermitian Matrices:

$$H = H^{\dagger}$$

where H is Hermitian matrix.

* Eigen Vectors and Eigen Values :-

$$A\vec{v} = \lambda\vec{v}$$

Eigen Vector Eigen Value

eg. $\begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 6 \end{bmatrix}$

$$\begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 3 \end{bmatrix} = 2\begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

* Introduction to the Qubit and Superposition.

Classical computers	Quantum Computers
Use Bits (0's and 1's)	Use Qubits (can be 0 and 1 at the same time)

- Qubits (Quantum Bits):

physically a qubit can be made from any quantum particle that lies 2 distinct state.

like classical computers, we still use 0's and 1's.

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Superposition :

Quantum particle is in two states simultaneously

A qubit is in superposition if it is both $|0\rangle$ and $|1\rangle$

Representing qubit Mathematically

$$|\psi\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

α represents how much the qubit is in $|0\rangle$ state.

β $|1\rangle$

* concept of quantum Mechanics :

when we measure a quantum system, it collapses into the measured state.

- if we measured $|\psi\rangle$ as 0 then $|\psi\rangle$ would collapse into the 0 state,

  so $|\psi\rangle \Rightarrow 0$

- if we measured $|\psi\rangle$ as 1 then $|\psi\rangle$ would collapse into the 1 state,

  so $|\psi\rangle \Rightarrow 1$

Measuring a qubit collapse it's superposition.

⇒ probability of measuring $|\psi\rangle$ as 0 is : $|\alpha|^2$

⇒ probability of measuring $|\psi\rangle$ as 1 is : $|\beta|^2$

① $|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

probability of measuring 0 is 1

② $|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

probability of measuring 1 is 1

So, probability of measuring 0's and 1's will always 1.

$$|\alpha|^2 + |\beta|^2 = 1$$

eg. $|\psi\rangle = \begin{pmatrix} \sqrt{3}/2 \\ 1/2 \end{pmatrix}$

$$\left|\frac{\sqrt{3}}{2}\right|^2 + \left|\frac{1}{2}\right|^2 = 1$$

$$\frac{3}{4} + \frac{1}{4} = 1$$

* Dirac Notation :-

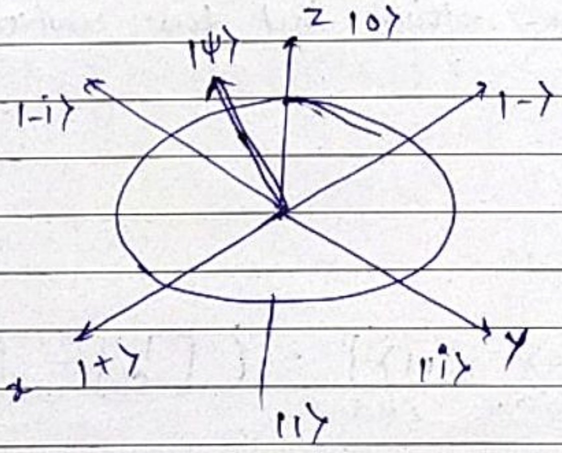
$$|\psi\rangle = \begin{pmatrix}\alpha\\ \beta\end{pmatrix} = \begin{pmatrix}\alpha\\ 0\end{pmatrix} + \begin{pmatrix}0\\ \beta\end{pmatrix}$$
$$= \alpha\begin{pmatrix}1\\ 0\end{pmatrix} + \beta\begin{pmatrix}0\\ 1\end{pmatrix}$$
$$= \underbrace{\alpha|0\rangle + \beta|1\rangle}_{\text{Dirac Notation}}$$

ex Matrix to Dirac Notation conversion :-

$$|\psi\rangle = \begin{pmatrix}1/2\\ 2\sqrt{3}/4\end{pmatrix} = \frac{1}{2}|0\rangle + \frac{2\sqrt{3}}{4}|1\rangle$$

* Bloch Sphere :-

we can represent a qubit $|\psi\rangle$ as a point on the surface of the Bloch sphere.



○ Higher vertically ⇒ Higher probability of measuring $|\psi\rangle$ as $|0\rangle$

○ ~~Higher~~ lower vertically ⇒ Higher pro. of measuring $|\psi\rangle$ as $|1\rangle$

* Quantum Gates:-

- A quantum logic gate is a basic quantum circuits operating on a small number of qubits.

- Classical gates operate on classical bits, while quantum gates operate on quantum bits (qubits)

- Quantum gates are the basic building block of quantum circuit and algorithms.

- They act on one or more qubits to change their quantum states.

- Single-qubits gates operate on a single qubit and can be used to create superposition or perform rotations.

- Two qubit gates operate on two qubits and are used to create entanglement between qubits.

* Quantum Gates are nothing but the unitary operators

⇒ Basis States:

① Single Qubit :-

$$\text{Basis} = \{ \underbrace{|0\rangle}_{2^0 \times 0 = 0}, \underbrace{|1\rangle}_{2^0 \times 1 = 1} \} = \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$$

Basis columns :-

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2\times 2} \Rightarrow \text{Identity matrix}$$

② 2 Qubit States :-

Basis States :- $\Big\{ \underset{\substack{|00\rangle \\ \hookrightarrow 2^0\times0+2^1\times0=0 \\ |0\rangle}}{|0\rangle \overset{\text{Tensor product}}{\otimes} |0\rangle}, \underset{\substack{|01\rangle \\ 2^0\times1+2^1\times0=1 \\ |1\rangle}}{|0\rangle \otimes |1\rangle}, \underset{\substack{|10\rangle \\ 2^1\times0+2^1\times1=2 \\ |2\rangle}}{|1\rangle \otimes |0\rangle}, \underset{\substack{|11\rangle \\ 2^0\times1+2^1\times1=3 \\ |3\rangle}}{|1\rangle \otimes |1\rangle} \Big\}$

Tensor product :-

$$|0\rangle \otimes |0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \otimes \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Basis State as column = $\underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{4\times4}$ Identity matrix.

③ 3 Qubit State :-

Basis State : $\Big\{ \overset{0}{|000\rangle}, \overset{1}{|001\rangle}, \overset{2}{|010\rangle}, \overset{3}{|011\rangle}, \overset{4}{|100\rangle}, \overset{5}{|101\rangle}, \underset{6}{|110\rangle}, \underset{7}{|111\rangle} \Big\}$

$UU^{\dagger} = I \Rightarrow U^{-1} = U^{\dagger}$, $|\det U = 1|$

and column satisfy orthonormal condn. i.e. self product = 1
2 diff. product = 0.

$U = \sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ $U = \sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$ $U = \sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

$-\boxed{X}-$ $-\boxed{Y}-$ $-\boxed{Z}-$

i.e. Pauli [X, Y, Z] Gates.

① Pauli-X Gate:-

Pauli-X Gate is Bit flip rotation around x-axis by π.

σ_x	$\|0\rangle$	$\|1\rangle$
$\langle 0\|$	0	1
$\langle 1\|$	1	0

for outer product form ignore 0
for 1 $|1\rangle$ make it as $\langle 1|$

$X = |0\rangle\langle 1| + |1\rangle\langle 0|$

$\therefore X = |0\rangle\langle 1| + |1\rangle\langle 0|$

① $X|0\rangle = (|0\rangle\langle 1| + |1\rangle\langle 0|)\,|0\rangle$

$X|0\rangle = |1\rangle$ $|0\rangle - \boxed{X} - |1\rangle$

② $X|1\rangle = (|0\rangle\langle 1| + |1\rangle\langle 0|)\,|1\rangle$

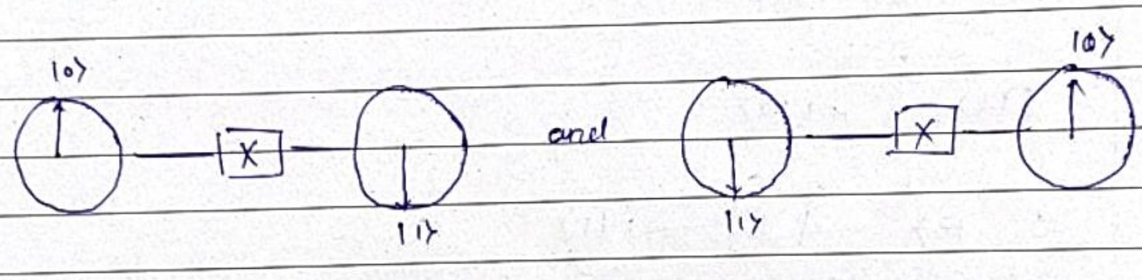
$X|1\rangle = |0\rangle$ $|1\rangle - \boxed{X} - |0\rangle$

$$\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\sigma_x |0\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} = |1\rangle$$

$$\sigma_x |1\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = |0\rangle$$

$$\sigma_x (\alpha |0\rangle + \beta |1\rangle) = \beta |0\rangle + \alpha |1\rangle$$



- Pauli-X gate is also called as quantum not gate.

③ Pauli-Y Gate :-

Pauli-Y Gate is phase flip Rotation ~~around~~ and Bit flip around Y-axis by π.

σ_y	$\|0\rangle$	$\|1\rangle$
$\langle 0\|$	0	$-i$
$\langle 1\|$	i	0

$$Y = -i|0\rangle\langle 1| + i|1\rangle\langle 0|$$

$$Y|0\rangle = i|1\rangle$$

$$Y|1\rangle = -i|0\rangle$$

$$\therefore \quad |0\rangle \text{ —} \boxed{Y} \text{— } i|1\rangle$$

$$|1\rangle \text{ —} \boxed{Y} \text{— } -i|0\rangle$$

i.e. $\sigma_y|0\rangle = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} = i\begin{bmatrix} 0 \\ 1 \end{bmatrix} = i|1\rangle$

$\sigma_y|1\rangle = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -i \\ 0 \end{bmatrix} = -i\begin{bmatrix} 1 \\ 0 \end{bmatrix} = -i|0\rangle$

③ Pauli - Z Gate :-

- Pauli - Z Gate is phase flip Rotation around Z - axis by π.

σ_z	$\|0\rangle$	$\|1\rangle$
$\langle 0\|$	1	0
$\langle 1\|$	0	-1

$$Z = |0\rangle\langle 0| - |1\rangle\langle 1|$$

$$Z|0\rangle = |0\rangle$$

$$Z|1\rangle = -|1\rangle$$

e $\quad |0\rangle$ —$\boxed{Z}$— $|0\rangle$

$|1\rangle$ —$\boxed{Z}$— $-|1\rangle$ (phase change)

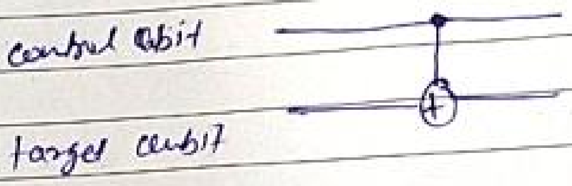
$$\sigma_z|0\rangle = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = |0\rangle$$

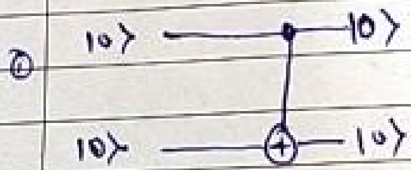
$$\sigma_z|1\rangle = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix} = -\begin{bmatrix} 0 \\ 1 \end{bmatrix} = -|1\rangle$$

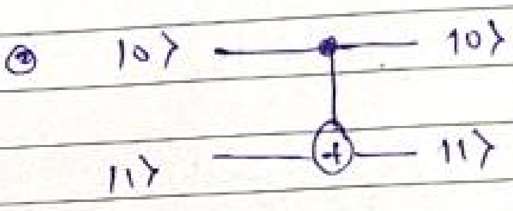
* 2 Qubit Gates:

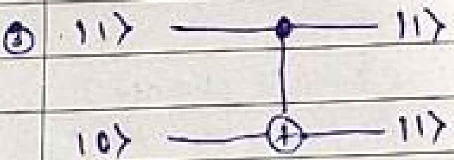
① CNOT Gate :- Controlled NOT

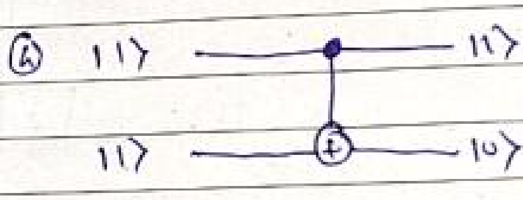
- CNOT Gate operates on 2 qubits out of which one is control bit and second is target bit.











if control bit is $|1\rangle$ then target qubit flips.

$$|00\rangle \rightarrow |00\rangle$$
$$|01\rangle \rightarrow |01\rangle$$
$$|10\rangle \rightarrow |11\rangle$$
$$|11\rangle \rightarrow |10\rangle$$

matrix Representation of CNOT Gate.

$$\begin{bmatrix} I_2 & O_2 \\ O_2 & \underbrace{U_2}_{X\ \text{Gate}} \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{4\times4}$$

* C Phase Gate :-

C phase Gate can be represented as —[S]—

the matrix representation for —[S]— is $\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$

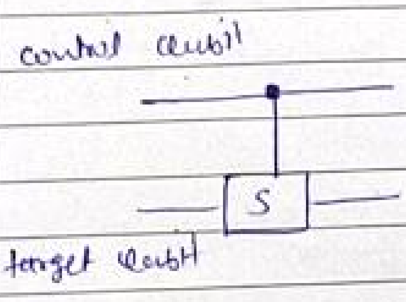
∴ two represent C phase gate using matrix form is.

$$\begin{bmatrix} I_2 & O_2 \\ O_2 & U_2 \end{bmatrix} \quad \text{i.e.} \quad \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & i \end{bmatrix}$$

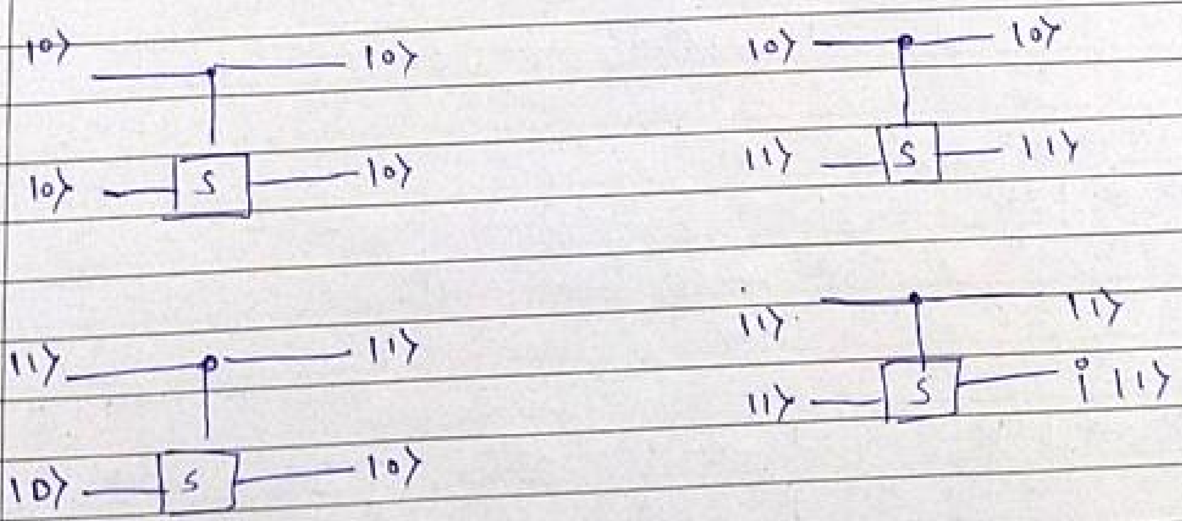
U_2 ↓ —[S]—

control bit

target bit

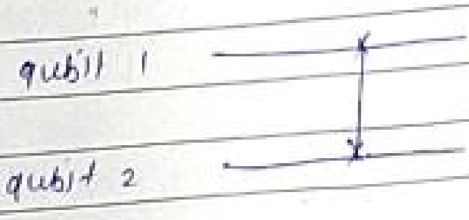


Truth table / diagram :-



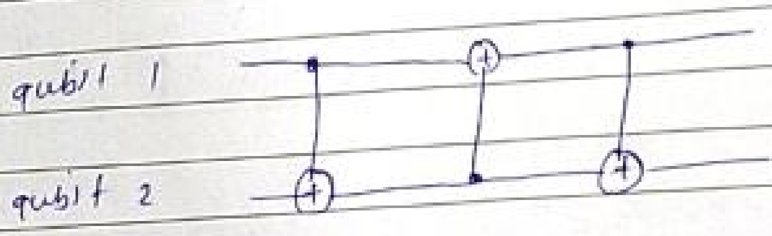
* Swap Gate:-

- SWAP gate is a gate in quantum computing that swaps the states of two qubits.



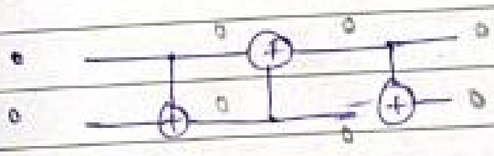
} diagramatical representation of swap gate

- The SWAP Gate can be decomposed into 3 CNOT Gates.



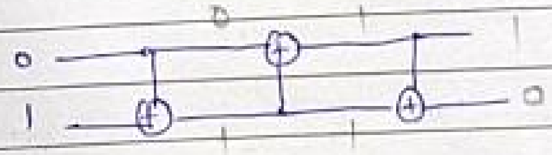
① $|00\rangle \rightarrow |00\rangle$

$\quad 0 \qquad\quad 0$



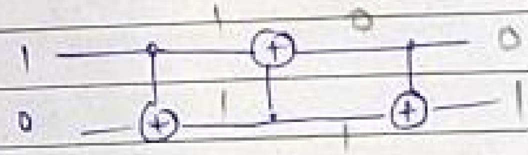
② $|01\rangle \rightarrow |10\rangle$

$\quad 1 \qquad\quad 2$



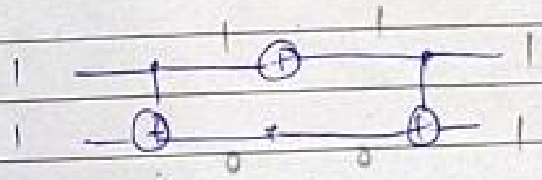
③ $|10\rangle \rightarrow |01\rangle$

$\quad 2 \qquad\quad 1$



④ $|11\rangle \rightarrow |11\rangle$

$\quad 3 \qquad\quad 3$



$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

SWAP.

* 3 Qubit Gates:-

① Toffoli Gate :- controlled CNOT gate.

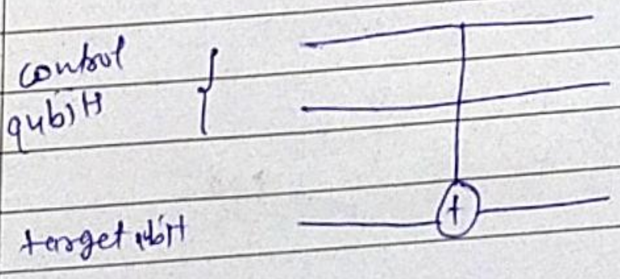
in Toffoli Gate out of ~~two~~ three qubits
2 - qubits are controlled qubit
1 - qubit is target qubit

- The target qubit (third qubit) will be inverted if the First and Second qubits both are 1.

$|000\rangle \rightarrow |000\rangle$ $\quad$ $|100\rangle \rightarrow |100\rangle$
$|001\rangle \rightarrow |001\rangle$ $\quad$ $|101\rangle \rightarrow |101\rangle$
$|010\rangle \rightarrow |010\rangle$ $\quad$ $|110\rangle \rightarrow |111\rangle$ $\quad$ $7 \rightarrow 8$
$|011\rangle \rightarrow |011\rangle$ $\quad$ $|111\rangle \rightarrow |110\rangle$ $\quad$ $8 \rightarrow 7$

∴ the matrix representation of Toffoli gate U.

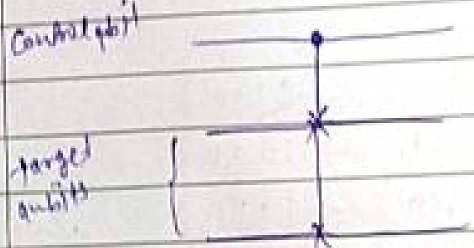
$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}_{8 \times 8}$$



① Fredkin Gate :-

The Fredkin gate is circuit or device with three inputs and three output qubits that transmits.
- the first bit unchanged
- swap last 2 bits if and only if first bit is 1.

representation of fredkin gate is:



$|000\rangle \rightarrow |000\rangle$ $|100\rangle \rightarrow |100\rangle$

$|001\rangle \rightarrow |001\rangle$ $|101\rangle \rightarrow |110\rangle$ 5 → 6

$|010\rangle \rightarrow |010\rangle$ $|110\rangle \rightarrow |101\rangle$ 6 → 5

$|011\rangle \rightarrow |100\rangle$ $|111\rangle \rightarrow |111\rangle$

matrix representation is

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}_{8\times 8}$$

* Measurement in quantum computing :-

- $\psi = \alpha|0\rangle + \beta|1\rangle$ is a qubit.

- In quantum circuit measurement can be represented as.

$|0\rangle$ probability $P = |\alpha|^2 / |\alpha|^2 + |\beta|^2$

$|1\rangle$ probability $P = |\beta|^2 / |\alpha|^2 + |\beta|^2$.

- when we measure a qubit it will collapse either into $|0\rangle$ or $|1\rangle$ which is a classical state.

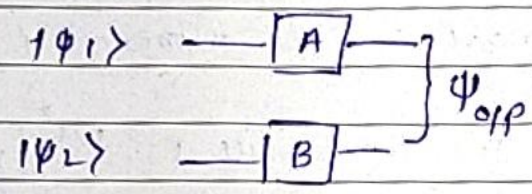
- all quantum measurements are classical and irreversible.

def^n - Measurement in quantum computing is a critical operation that extracts classical information from quantum information

- the purpose of measuring is to extract classical information from quantum state.

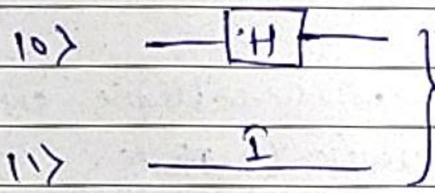
* Parallelism in Quantum :-

- let A and B are 2 single quantum qubit states.



$$(A \otimes B)\, |\psi_1\rangle \otimes |\psi_2\rangle$$
$$\Rightarrow (A \otimes B)\, |\varphi_1 \psi_2\rangle$$

example: ①



$$(H \otimes I)\,(|0\rangle \otimes |1\rangle)$$
$$= (H \otimes I)\,(|01\rangle)$$

$$= \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix} \otimes \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 0 & 1 & 0\\ 0 & 1 & 0 & 1\\ 1 & 0 & -1 & 0\\ 0 & 1 & 0 & -1\end{bmatrix}\begin{bmatrix}0\\ 1\\ 0\\ 0\end{bmatrix}$$

$$= \frac{1}{\sqrt{2}}\begin{bmatrix}0\\ 1\\ 0\\ 1\end{bmatrix} \Rightarrow \frac{1}{\sqrt{2}}\begin{bmatrix}0\\ 1\\ 0\\ 0\end{bmatrix} + \begin{bmatrix}0\\ 0\\ 0\\ 1\end{bmatrix}$$

$$= \frac{1}{\sqrt{2}}\left[|01\rangle + |11\rangle\right]$$

● Every quantum circuit are unitary and Reversible.

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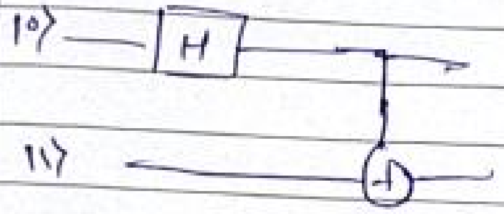

$\Rightarrow CNOT\ (H \otimes I)$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \times \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix}$$

Hadamard Gate :-

$U = \vec{\sigma} \cdot \hat{n}$

$\hat{n} = \left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}} \right)$

$$U = \frac{1}{\sqrt{2}} \sigma_x + 0 + \frac{1}{\sqrt{2}} \sigma_z = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = H$$

i.e. Hadamard Gate.

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$H^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

i.e. when we apply Hadamard gate twice we will get same state.

H	$\|0\rangle$	$\|1\rangle$
$\langle 0\|$	$1 \cdot 1/\sqrt{2}$	$1 \cdot 1/\sqrt{2}$
$\langle 1\|$	$1 \cdot 1/\sqrt{2}$	$-1 \cdot 1/\sqrt{2}$

=

H	$\|0\rangle$	$\|1\rangle$
$\langle 0\|$	$1/\sqrt{2}$	$1/\sqrt{2}$
$\langle 1\|$	$1/\sqrt{2}$	$-1/\sqrt{2}$

$$H = \frac{1}{\sqrt{2}} \left[|0\rangle\langle 0| + |0\rangle\langle 1| + |1\rangle\langle 0| - |1\rangle\langle 1| \right]$$

$$H|0\rangle = \frac{1}{\sqrt{2}} \left[|0\rangle + |1\rangle \right]$$

$$H|1\rangle = \frac{1}{\sqrt{2}} \left[|0\rangle - |1\rangle \right]$$

- Hadamard Gate transforms the basis States into equal superpositions, which is crucial for creating and manipulating superposition states in quantum algorithm.

- If you apply Hadamard gate twice you will gate the same state again.

- Hadamard gate is used to convert the qubit from clustering state to uniform superposition state.

- Hadamard gate is self-reversible. Its unitary nature allows for the transformation of qubit states into superposition state and back to the original states.