Q1] Discuss the history & development of Quantum Computing

→ • Quantum computing has been a subject of intrest since the mid-1980s when theoretical physicist Richard Feynman proposed that due to the physical limitation of classical computing, a quantum computer might be capable of performing calculation significantly faster.

• Since then quantum computing accelerated rapidly. It is being used nowadays for many application including cryptography, image processing etc.

• There are still many ~~a~~ challenges to takle before quantum computers are commonplace

• During 1990s there was significant progress in building small scale quantum system. After that many algorithms were devlped that are used in quantum computing like ~~g~~ Grover's Algorithm

• Quantum computing is rapidly evolving field with significant scientific, technological intrest

Q2 Diffrentiate between classical computing & quantum computing

Classical	Quantum
1] Basic unit of information is in bits (0 or 1)	1] Basic unit of information is qubits (superposition of 0 & 1)
2] Bits can be manipulated by logic gates	2] ~~Qu~~ Quantum states are manipulated by quantum gates
3] Bits are independent	3] Qubits are entangled
4] Processing is sequential	4] Processing is in parallel state.
5] Efficient but limited for complex problem	5] Efficient for complex problem
6] Low errors	6] High error, require quantum error correction
7] Hardware is silicon-based microprocessor	7] Specialized cryogenic environment
8] Classical algorithms (eg. sorting, searching)	8] Quantum algorithms (eg. Shor's algo, Grover's algo)

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Q3 Explain basics of quantum computing with qubits & quantum states

⇒ Quantum computing is a specalized technology which includes computer ~~algor~~ hardware and AI algorithms to solve complex problem.

Unlike classical computing which uses bits as ~~unit~~ of basic unit of information, quantum computing uses qubits.

1] Qubits (Quantum bits):

In classical ~~bit~~ computing, a bit can be either 0 or 1, but a qubit exist in a state of 0 & 1 ie superposition of 0 and 1. This allows quantum computers to process more information simultaneously ~~so~~.

2] Superposition:

Defn:- Ability of qubit to exist in multiple state at same time.

Mathematically, $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

where, α & β are complex numbers that represent the ~~number~~ probability amplitudes of qubit being in state $|0\rangle$ or $|1\rangle$.

The probability are given by $|\alpha|^2$ & $|\beta|^2$, and must satisfy $|\alpha|^2 + |\beta|^2 = 1$

Q4) Explain Superposition, Entanglement & Interference
⇒ 1) **Superposition**: Ability of qubit to exist in multiple state at same time ~~i.e simultaneously~~
Unlike classical bits which can either be 0 or 1 a qubit in superposition can be in a state that is combination of 0 and 1.
Mathematically,

$$\psi = \alpha|0\rangle + \beta|1\rangle$$

where, α & β are complex numbers that represent probability of qubit being in state $|0\rangle$ or $|1\rangle$
The probability is given by $|\alpha|^2$ & $|\beta|^2$ and must satisfy $|\alpha|^2 + |\beta|^2 = 1$

2) **Entanglement**: Entanglement is a quantum phenomenon where two or more ~~quantum~~ qubits become linked, such that the state of one qubit directly influences the state of the other.
Eg: If qubit A & B are entangled and A is measured to be in state $|0\rangle$ then B will also instantly become $|0\rangle$

3) **Interference**: Interference in quantum mechanics refers to phenomenon where probability amplitudes can add up or cancel out, affecting the liklihood of certain outcomes
It is ~~simalar to~~ similar to wave inteference when two waves can interfere constructively or destructively
- **Constructive Interference**: Occurs when the probability amplitude of diffrent quantum path reinforce
- **Destructive Interference**: Occurs when the probability amplitude of diffrent quantum path cancel each other

Q5) Explain the application of Quantum computing in detail
→ 1] Cryptography:
Classical Encryption methods such as RSA rely on the difficulty of factoring large number into primes. Shor's algorithm, a quantum algo. can factor these large ~~&~~ number exponentially faster than the classical algo.

2] AI & ML: Quantum computing could enhance machine learning algo. by speeding up processes like data classification, pattern ~~recognition~~ recognition

3] Material Science:
Quantum computers can simulate quantum system, such as molecules & material, this helps in discovery of new drug & predict its behaviour. It also can be used to discover new materials with specific property

4] Optimizing Problem:
Many real world problem ~~can to~~ like route optimization, supply chain logistics can be optimized by QAOA (Quantum Approximate Optimization Algorithm). It can be used in logistic to increase supply chain & efficiency

5] Climate ~~&~~ Modelling & Weather forecasting
Climate modelling involves simulating complex interactions between various components of Earth's system. It can also be used in ~~predic~~ predicting weather with high accuracy.

Q6] Illustrate Dirac Notation in Quantum Computing

⇒ Dirac Notation, also known as 'bra-ket' notation is a standard notation in quantum mechanics & quantum computing.

1] Ket Notation ($|\psi\rangle$)

→ A ket represents a column vector or a quantum state

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

where $|\psi\rangle$ is state of qubit &

α & β are complex probability amplitude

eg. $|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\quad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

A superposition state, such as

$|\psi\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$, can be represented

as $\begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}$

2] Bra Notation ($\langle\psi|$)

→ Bra ~~ke~~ represents the conjugate transpose of a ket

→ which is row of vector. ~~If~~

If $|\psi\rangle$ is a ket, then corresponding bra is $\langle\psi|$

$$\langle\psi| = (\alpha^*, \beta^*)$$

where α^* & β^* are complex conjugate of α & β

eg:- The bra corresponding to ket $|0\rangle$ is $\langle 0|$, which is represented by $(1, 0)$

The bra corresponding to ket $|1\rangle$ is $\langle 1|$ which is represented by $(0, 1)$

Q7] Discuss the Inner Product & Outer Product in Quantum computing

⇒ **Inner Product** $\langle \psi | \phi \rangle$

Defⁿ :- The inner product of two quantum states $|\psi\rangle$ & $|\phi\rangle$ is a complex number & is denoted by $\langle \psi | \phi \rangle$

~~For two ket the $|\psi\rangle$ & $|\phi\rangle$ the inner product~~

For two ket $|\psi\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$ $|\phi\rangle = \begin{pmatrix} \gamma \\ \delta \end{pmatrix}$

inner product is $\langle \psi | \phi \rangle = \begin{pmatrix} \alpha^* & \beta^* \end{pmatrix} \begin{pmatrix} \gamma \\ \delta \end{pmatrix}$

$= \alpha^* \gamma + \beta^* \delta$

eg. inner product of ~~$|0\rangle$ & $|1\rangle$ is~~

$|\psi\rangle = \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}$ $|\phi\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$\langle \psi | \phi \rangle = 1/\sqrt{2} + 0$

$\langle \psi | \phi \rangle = 1/\sqrt{2}$

Outer product $(|\psi\rangle\langle\phi|)$

Defⁿ :- The outer product of two quantum state $|\psi\rangle$ & $\langle\phi|$ result in an operator

The outer product is calculated by multiplying a ket & a bra

$|\psi\rangle\langle\phi| = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \begin{pmatrix} \gamma & \delta \end{pmatrix} = \begin{pmatrix} \alpha\gamma & \alpha\delta \\ \beta\gamma & \beta\delta \end{pmatrix}$

eg. outer product of $|\psi\rangle = \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}$ $|\phi\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$|\psi\rangle\langle\phi| = \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix} \begin{pmatrix} 1 & 0 \end{pmatrix} = \begin{pmatrix} 1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 0 \end{pmatrix}$

Q8 Discuss Quantum Computing & Importance of Quantum Computing

→ Quantum Computing is a type of computing that leverages principle of quantum mechanics to process information in different ways than classical computers.

Key concepts:-

1] Qubits :- The fundamental unit of quantum information, which is in superposition of 0 & 1

2] Superposition :- A qubit's ability to be in multiple states at once, enabling quantum computers to process vast no. of possibilities

3] Entanglement :- A phenomenon where qubits become linked and state of one qubit affects directly another qubit

4] Quantum gates :- Operations that manipulates qubits, analogus to logic gates in classical computer

Importance :-

1] Exponential speedup for certain problems
2] Breakthrough Cryptography
3] Advance material Science
4] Optimization of Problems
5] Advancing in AI & ML
6] Driving Inovation
7] Forecasting give more accurate result

Q9 Discuss the basics of Quantum mathematics & deterministic Operation

⇒ Complex Number : Quantum states are represented using complex number. A complex number z is written as
$z = a + bi$

where a is real part & b is imaginary part
eg $z = 3 + 9i$ or $z = \sqrt{7}i$

Conjugate of complex numb $z = a + bi \Rightarrow z^* = a - bi$

Complex numb multiplication : $(2+3i)(4-8i)$
$\Rightarrow 8 - 16i + 12i \mp 24i^2$
$= 8 - 4i \pm 24$
$= \boxed{32 - 4i}$

• When we multiply ~~the~~ any complex number by its conjugate it gives a real no. $(a+ib)(a-ib) = a^2 + b^2$
eg $(2+3i)(2-3i) = 2^2 + 3^2 = 4 + 9 = 13$

Magnitude of $|a+ib| = \sqrt{a^2+b^2}$
eg $|-2+3i| = \sqrt{2^2+3^2} = \sqrt{13}$

polar form : $a+ib = r(\cos\theta + i\sin\theta)$
exponential form : $a+ib = re^{i\theta}$

Q10] Find Inner product of $|a\rangle = \begin{vmatrix} 1 \\ 0 \end{vmatrix}$ & $|b\rangle \begin{vmatrix} 0 \\ 1 \end{vmatrix}$

⇒ Inner product $|\psi\rangle = \begin{vmatrix} \alpha \\ \beta \end{vmatrix}$ & $|\phi\rangle = \begin{vmatrix} \gamma \\ \delta \end{vmatrix}$

then $\langle \psi | \phi \rangle = \alpha\gamma + \beta\delta$

$\therefore$ $|a\rangle = \begin{vmatrix} 1 \\ 0 \end{vmatrix}$ & $|b\rangle = \begin{vmatrix} 0 \\ 1 \end{vmatrix}$

$\therefore$ $\langle a | b \rangle = \begin{vmatrix} 1 \\ 0 \end{vmatrix} \begin{vmatrix} 0 \\ 1 \end{vmatrix}$

$= 1 \cdot 0 + 0 \cdot 1 = \boxed{0}$

$\therefore$ $\langle a | b \rangle = 0$

Q11] Find ~~Outer~~ Outer product of $|a\rangle = \begin{vmatrix} 1 \\ 0 \end{vmatrix}$ & $|b\rangle = \begin{vmatrix} 0 \\ 1 \end{vmatrix}$

⇒ ~~Outer~~ Outer product + $|\psi\rangle = \begin{vmatrix} \alpha \\ \beta \end{vmatrix}$ & $|\phi\rangle = \begin{vmatrix} \gamma \\ \delta \end{vmatrix}$

then $|\psi\rangle\langle\phi| = \begin{vmatrix} \alpha\gamma & \alpha\delta \\ \beta\gamma & \beta\delta \end{vmatrix}$

$\therefore$ $|a\rangle = \begin{vmatrix} 1 \\ 0 \end{vmatrix}$ $|b\rangle = \begin{vmatrix} 0 \\ 1 \end{vmatrix}$

$|a\rangle\langle b| = \begin{vmatrix} 1 \\ 0 \end{vmatrix} \begin{vmatrix} 0 & 1 \end{vmatrix}$

$= \begin{pmatrix} 1 \cdot 0 & 1 \cdot 1 \\ 0 \cdot 0 & 0 \cdot 1 \end{pmatrix}$

$|a\rangle\langle b| = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$

12] Outline the proof of 2 qubit basis states identity matrix

→ 2 Qubit states :
2 Qubit system , the basis states are :
- $|00\rangle$ • $|01\rangle$ • $|10\rangle$ • $|11\rangle$

The basis state can be represented by column vector

$$|00\rangle = \begin{pmatrix}1\\0\\0\\0\end{pmatrix} \quad |01\rangle = \begin{pmatrix}0\\1\\0\\0\end{pmatrix} \quad |10\rangle = \begin{pmatrix}0\\0\\1\\0\end{pmatrix} \quad |11\rangle = \begin{pmatrix}0\\0\\0\\1\end{pmatrix}$$

Take the outer product of each state with itself

$$|00\rangle\langle 00| = \begin{pmatrix}1&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix} \quad |01\rangle\langle 01| = \begin{pmatrix}0&0&0&0\\0&1&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix}$$

$$|10\rangle\langle 10| = \begin{pmatrix}0&0&0&0\\0&0&0&0\\0&0&1&0\\0&0&0&0\end{pmatrix} \quad |11\rangle\langle 11| = \begin{pmatrix}0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&1\end{pmatrix}$$

Now, sum the outer product

$$|00\rangle\langle 00| + |01\rangle\langle 01| + |10\rangle\langle 10| + |11\rangle\langle 11| = \begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix} = I$$

This proves that the sum of the outer product of 2 qubit basis state forms identity matrix.

Q3 List out diffent types of quantum gates. Explain 1 in detail

$\Rightarrow$ There are diffent types of quantum gates like:-

I] Pauli's gates (x, y, z)

- Pauli X gate -[X]- : $\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$; -[X]-
- Pauli Y gate -[Y]- : $\sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$; -[Y]-
- Pauli Z gate -[Z]- : $\sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$; -[Z]-

2) Phase shift gate:

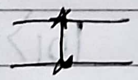
It maps $|0\rangle$ to $|0\rangle$ & $|1\rangle$ to $e^{i\varphi}|1\rangle$

$$P(\varphi) = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\varphi} \end{bmatrix}$$

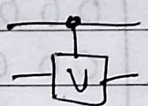
3) Handemard gate : -[H]-

$$|0\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}} \qquad |1\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

4) Swap gates -



5) Controled Not gate (CNOT):



Now,

Haadamard gate: -[H]-

It is used to create surperposition of $|0\rangle$ & $|1\rangle$

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

Handmard gate affects qubits if we intialize it to $|0\rangle$ or $|1\rangle$

a) intialized by $|0\rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$$= \frac{1}{\sqrt{2}} \begin{bmatrix} (1 \cdot 1) + (1 \cdot 0) \\ (1 \cdot 1) + (-1 \cdot 0) \end{bmatrix}$$

$$= \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$= \boxed{\frac{|0\rangle + |1\rangle}{\sqrt{2}}}$$

b) intialized by $|1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$= \frac{1}{\sqrt{2}}\begin{pmatrix} (1\cdot 0) + (1)(1) \\ (1\cdot 0) + (1)(-1) \end{pmatrix}$

$= \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$= \boxed{\frac{|0\rangle - |1\rangle}{\sqrt{2}}}$

Q14. Describe the below quantum gates
① Pauli - X, Y, Z ② Hadamard gate

→ ① Pauli - ~~X, Y, Z~~ gate :

- Pauli X gate : $-\boxed{X}-$
  It is equivalent to classical NOT gate.
  It maps $|0\rangle$ to $|1\rangle$ & vice versa

  $X = \sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

- Pauli Y gate ; $-\boxed{Y}-$
  It maps $|0\rangle$ to $i|1\rangle$ & $|1\rangle$ to $-i|0\rangle$

  $Y = \sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$

- Pauli Z gate : $-\boxed{Z}-$
  It ~~maps~~ keep $|0\rangle$ unchanged & maps $|1\rangle$ to $-|1\rangle$

  $Z = \sigma_z = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

(2) Hadmarid gate : $-\boxed{H}-$

It creates superposition state of $|0\rangle$ & $|1\rangle$

$$H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

~~Hand~~ Hadmard gate affects the qubits based on which it is intialized

• If intialized to $|0\rangle$ =

$$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

If intialized to $|1\rangle$ =

$$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$= \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

Q15] Differential NOT gate & CNOT gate

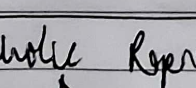
Classical NOT gate	Quantum CNOT gate
1) The classical NOT gate is a unary operation that flips the input bit	1) The controlled-NOT gate is a two qubit quantum gate that flips the second qubit if first qubit is 1

Classical NOT gate

2] Truth table

Input	Output
0	1
1	0

3] Symbolic Representation

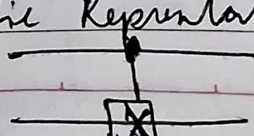


Quantum CNOT gate

2] Truth Table:

control(C)	Target(T)	Output(C)	output(T)
0	0	0	0
0	1	0	1
1	0	1	1
1	1	1	0

3] Symbolic Representation



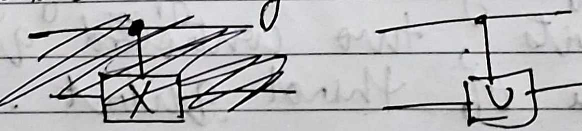
4] Mathematical representation, NOT gate can be represented in 2x2 matrix by $NOT = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	4] Mathematical representation of CNOT gate can be represented by ~~3~~ 4x4 matrix by, $CNOT = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$
5] Flips single bit	5] Flips the target qubit based on the control qubit
6] eg. If input is 0 then output will be 1 & vice versa	6] eg. If the control qubit is $\|0\rangle$ & the target qubit is $\|1\rangle$ then the output will be $\|01\rangle$.

16] Describe the ① CNOT & ② SWAP quantum gates

→ 1] CNOT:

CNOT is controlled NOT gate, it acts on two qubit. It flips the target qubit if control qubit is 1. If controled qubit is 0 the result remains unchanged

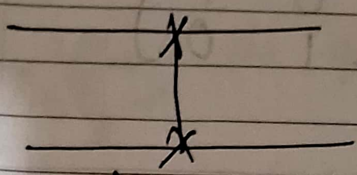
Symbol :



Control (C)	Target (T)	Output (C)	Output (T)
0	0	0	0
0	1	0	1
1	0	1	1
1	1	1	0

Matrix repentation =
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{matrix} \left.\vphantom{\begin{matrix}1\\0\end{matrix}}\right\} Q_1 \\ \left.\vphantom{\begin{matrix}1\\0\end{matrix}}\right\} Q_2 \end{matrix}$$

SWAP gate :
The SWAP gate is a gate in quantum computing that swaps the states of two qubits

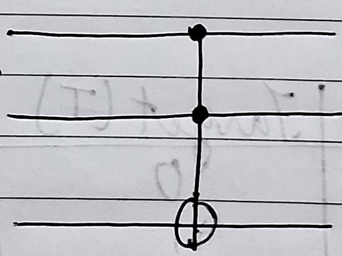
Represted by 

Here $|00\rangle$ remain $|00\rangle$
$|01\rangle$ becomes $|10\rangle$
$|10\rangle$ becomes $|01\rangle$
$|11\rangle$ Remains $|11\rangle$

Matrix representation =

$$SWAP = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

7) Describe ① Toffoli & ② Fredkin quantum gates
→ Toffoli gate :
A Toffoli gate is a type of gate that acts on three qubits, two controlled qubits which affect state of third qubit. It is also called CCNOT

Symbol : 

Input	Output
000	000
001	001
010	010
011	011
100	100
101	101
110	111
111	110

} first two qubits are $|11\rangle$

Matrix form:

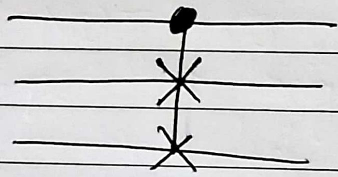
$$\begin{bmatrix} 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&1&0 \end{bmatrix}$$

Toffoli gate

<u>Fredkin gate</u>:

It is also known as controlled SWAP (CSWAP) gate.
It is three qubit gate that swaps the state of two qubit based on state of third qubit.

Symbol :



One controlled qubit $|c\rangle$ two target qubit $|a\rangle$ & $|b\rangle$
If $|c\rangle$ is $|0\rangle$ then target remain unchanged
& if $|c\rangle$ is $|1\rangle$ the gate swaps state of $|a\rangle$ & $|b\rangle$

Input (c a b)	Output (c a b)
$\|0\ 0\ 0\rangle$	$\|0\ 0\ 0\rangle$
$\|0\ 0\ 1\rangle$	$\|0\ 0\ 1\rangle$
$\|0\ 1\ 0\rangle$	$\|0\ 1\ 0\rangle$
$\|0\ 1\ 1\rangle$	$\|0\ 1\ 1\rangle$
$\|1\ 0\ 0\rangle$	$\|1\ 0\ 0\rangle$
$\|1\ 0\ 1\rangle$	$\|1\ 1\ 0\rangle$
$\|1\ 1\ 0\rangle$	$\|1\ 0\ 1\rangle$
$\|1\ 1\ 1\rangle$	$\|1\ 1\ 1\rangle$

Matrix form

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Fredkin gate

Q18) Describe below quantum gate ① Phase ② $\pi/8$

⇒ **Phase gate**: The Phase gate is a single qubit operational gate. It is used for rotation of qubit about the Z-axis

Phase gate maps $|0\rangle$ to $|0\rangle$ & $|1\rangle$ to $e^{i\varphi}|1\rangle$

$$P(\varphi) = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\varphi} \end{bmatrix}$$ where φ is the phase shift with period 2π

It is represented by $-\boxed{P}-$

$\pi/8$ gate: It is also called as T gate. It is a single qubit quantum gate. It induces a $\pi/4$ phase

Symbol : $-\boxed{T}-$

$$T = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix}$$

T gate maps $|0\rangle$ to $|0\rangle$ & $|1\rangle$ to $e^{i\pi/4}|1\rangle$

Q19 Show that two orthogonal state are linearly independent.

$\Rightarrow$ Let us assume tha $|\psi\rangle$ & $|\phi\rangle$ are orthogonal state basis states.

Consider the linear combination,

$$a|\psi\rangle + b|\phi\rangle = 0 \quad \text{--- (1)}$$

Take inner product of both side with $\langle\psi|$

$$\langle\psi|\left(a|\psi\rangle + b|\phi\rangle\right) = \langle\psi|0\rangle$$
$$a\langle\psi|\psi\rangle + b\langle\psi|\phi\rangle = 0$$

since $|\psi\rangle$ is normalized $\langle\psi|\psi\rangle = 1$
& $|\psi\rangle$ and $|\phi\rangle$ are orthogonal, $\therefore \langle\psi|\phi\rangle = 0$

$$a(1) + b(0) = 0$$
$$a = 0 \quad \text{--- (11)}$$

Again taking inner product of eq (1) with $\langle\phi|$

$$\therefore \langle\phi|\left(a|\psi\rangle + b|\phi\rangle\right) = \langle\phi|0\rangle$$
$$a\langle\phi|\psi\rangle + b\langle\phi|\phi\rangle = 0$$

again $\langle\phi|\psi\rangle = 0$ & $\langle\phi|\phi\rangle = 1$

$$\therefore a(0) + b(1) = 0$$
$$b = 0 \quad \text{--- (111)}$$

$\therefore$ the sol^n to
$a|\psi\rangle + b|\phi\rangle = 0$ is $a = 0$ & $b = 0$
& $|\psi\rangle$ & $|\phi\rangle$ are linearly independent

Q20 Discuss at least 5 differnce of any 3 quantum gates

⇒

Handmard gate	Pauli-X Gate	CNOT gate
1] It is an single qubit gate	1] It is an single qubit gate	1] It is two-qubit gate
2] Creates superposition from a basis state	2] Flips ~~the~~ the qubit state (like classical NOT gate)	2] Flips target qubit if control qubit is 1
3] Matrix Representation $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$	3] Matrix Representation $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	3] Matrix Representation $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$
4] It transforms $\|0\rangle$ to $\frac{\|0\rangle + \|1\rangle}{\sqrt{2}}$ & $\|1\rangle$ to $\frac{\|0\rangle - \|1\rangle}{\sqrt{2}}$	4] It flips $\|0\rangle$ to $\|1\rangle$	4] CNOT affects the target qubit based on the control qubit state
5] Does not create entanglement	5] Does not create entanglement	5] Can create entanglement between qubits
6] Simple operation that interacts with both amplitude & phase	6] Simple operation that flips the qubit state	6] More complex due to its conditional operation on two qubits