MTH 308: Numerical Analysis & Scientific Computing I
Department of Mathematics and Statistics
Indian Institute of Technology - Kanpur March 7, 2019

Lab 8

Steps you should follow :

1. Start a Matlab session by typing

   ```
   $ matlab &
   ```

   at the command prompt.

2. In the Matlab command window, type

   ```
   >> diary lab8_ID
   ```

   where ID stands for your roll number. For example, if your roll number is 12345, then the command will be `diary lab8_12345`; if your roll number is 123456, then the command will be `diary lab8_123456`. This will create a file named `lab8_ID` in the present working directory. PLEASE DO NOT EDIT THIS FILE.

3. Do your lab assignment – create a separate file to write your scripts/functions, if required; once done, at the Matlab's command prompt, type

   ```
   >> diary off
   ```

4. Attach this file (that is, `lab8_ID`) and any other Matlab code file that you may have created for the labwork in an **email with subject Lab8-ID** and send it to

   mth308.iitk@gmail.com

   before the end of the lab session, that is, by 4:30 pm. Note that **late submissions will not get any credit**. In the case that your diary file is too big to be sent as an email attachment, upload it on you Google Drive (the cloud storage space associated with your Google/Gmail account) and share the link with mth308.iitk@gmail.com.

The *condition number* of a simple (algebraic multiplicity one) eigenvalue λ of an arbitrary matrix A is defined by

$$\mathrm{Cond}(\lambda) = \frac{\|u\|_2 \|w\|_2}{|\langle u, w\rangle|}$$

in which u and w are the right and left eigenvectors, respectively, associated with λ, that is, $Au = \lambda u$ and $w^* A = \lambda w^*$. The symbol $\langle \cdot, \cdot \rangle$ denotes the inner-product.

1. What is Cond(λ) if λ is an eigenvalue of a normal matrix. Compute, using Matlab, the Cond(λ_1) for the matrix

$$A = \begin{pmatrix} 1 & 1/2 & 1/3 \\ 1/2 & 1/3 & 1/4 \\ 1/3 & 1/4 & 1/5 \end{pmatrix}$$

where λ_1 is the smallest eigenvalue of A. What is the difference between the theoretical and computed values of the condition number?

2. Consider the matrix

$$B_n = \begin{pmatrix} \lambda_1 & -1 & & & \\ & \lambda_2 & -1 & & \\ & & \cdot & \cdot & \\ & & & \cdot & -1 \\ & & & & \lambda_n \end{pmatrix}$$

with $\lambda_1 = 0$ and $\lambda_i = 1/(i-1),\ i > 1$.

   (a) Verify that a right eigenvector associated with the eigenvalue λ_1 is e_1 and a left eigenvector is w whose i-th component is equal to $(i-1)!$ for $i = 1, \ldots, n$.

   (b) Use part (a) to conclude that the condition number of λ_1 satisfies

$$(n-1)! \leq \text{Cond}(\lambda_1) \leq (n-1)!\sqrt{n}.$$

   This example shows that the condition number of an eigenvalue of a non-normal matrix can be arbitrarily high.

   (c) Now, compute Cond(λ_1) for A_n, $n = 2, 4, 8, 16$ and 32. Do they satisfy the condition obtained in part (b)?

You might find Matlab's `eig` function useful while computing the condition number of an eigenvalue in Matlab (for more details, type `help eig` at the Matlab's command prompt). Note that 2(a) and 2(b) are theoretical in nature whose solution need not be submitted as part of the lab assignment.