MTH 308: Numerical Analysis & Scientific Computing I
Department of Mathematics and Statistics
Indian Institute of Technology - Kanpur April 11, 2019

Lab 12

Steps you should follow :

1. Start a Matlab session by typing

   ```
   $ matlab &
   ```

   at the command prompt.

2. In the Matlab command window, type

   ```
   >> diary lab12_ID
   ```

   where ID stands for your roll number. For example, if your roll number is 12345, then the command will be `diary lab12_12345`; if your roll number is 123456, then the command will be `diary lab12_123456`. This will create a file named `lab12_ID` in the present working directory. PLEASE DO NOT EDIT THIS FILE.

3. Do your lab assignment – create a separate file to write your scripts/functions, if required; once done, at the Matlab's command prompt, type

   ```
   >> diary off
   ```

4. Attach this file (that is, `lab12_ID`) and any other Matlab code file that you may have created for the labwork in an **email with subject Lab12-ID** and send it to

   mth308.iitk@gmail.com

   before the end of the lab session, that is, by 4:30 pm. Note that **late submissions will not get any credit**. In the case that your diary file is too big to be sent as an email attachment, upload it on you Google Drive (the cloud storage space associated with your Google/Gmail account) and share the link with mth308.iitk@gmail.com.

Use the Matlab code developed in Lab 9 to find the roots $\{x_j^{g,n}\}_{j=1}^n$ of the sequence of functions $(P_n(x))_{n\geq 0}$ defined recursively by

$$(n+1)P_{n+1}(x) = (2n+1)xP_n(x) - nP_{n-1}(x)$$

with $P_0(x) = 1$ and $P_1(x) = x$. Recall that these are the quadrature points of the corresponding n-point *Gauss quadrature*

$$Q_n^g(f) = \sum_{j=1}^{n} w_j^{g,n} f(x_j^{g,n}).$$

In case you can't access your Lab 9 code, you can use the following lines to obtain $x_j^{g,n}$:

```
J = zeros(n);
for j = 1:n-1
    J(j,j+1) = j/sqrt(4*j^2-1);
    J(j+1,j) = j/sqrt(4*j^2-1);
end
xg = eig(J);
```

As the degree of precision the n-point Gauss quadrature is $2n-1$, we observe that the weights $w_j^{g,n}$ can be obtained as the solution of the linear system given by

$$\sum_{j=1}^{n} w_j^{g,n} (x_j^{g,n})^i = \begin{cases} 0, & i \text{ odd}, \\ 2/(i+1), & i \text{ even}, \end{cases} \quad \text{for } i = 0, 1, 2, \ldots n-1.$$

One can similarly obtain the quadrature weights $w_j^{c,n}$ for the n-point *Clenshaw-Curtis* rule

$$Q_n^c(f) = \sum_{j=1}^{n} w_j^{c,n} f(x_j^{c,n}),$$

with quadrature points $x_j^{c,n} = \cos((2j-1)\pi/(2n)), j = 1, \ldots, n$, utilizing the fact that it has a degree of precision at least $n-1$.
Use the quadratures to approximate the integrals

$$I_j = \int_{-1}^{1} f_j(x)\, dx, \quad j = 1, 2,$$

where

$$f_1(x) = \frac{1}{1+x^2} \quad \text{and} \quad f_2(x) = \sin(\pi|x|).$$

If the corresponding errors are denoted as

$$e_{j,n}^g = |I_j - Q_n^g(f_j)|, \quad e_{j,n}^c = |I_j - Q_n^c(f_j)|,$$

then complete the following table for $j = 1$ and $j = 2$:

n	$e^g_{j,n}$	$e^g_{j,n}/e^g_{j,n/2}$	$e^c_{j,n}$	$e^c_{j,n}/e^c_{j,n/2}$
2		—		—
4				
8				
16				
32				
64				
128				

Comment on the results.