EE 208

Control Engineering Lab

Experiment-6: State feedback controller design on MATLAB platform.

Group Number: 20
Vedansh: 2020EEB1218
Aniket Arya: 2020EEB1157
Chirag Rathi: 2020EEB1165

Professor: Dr. S. Roy
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OBJECTIVE: -

To design the state feedback gain matrix for a given analog state-space system to satisfy required performance specifications.

Given: -

The horizontal plane movement of a certain AUV has the sway speed, yaw angle, and yaw rate as state variables x1, x2, x3 respectively, and the rudder angle as the single input.

Upon linearisation about a nominal operating point, the analog system is characterised by the following matrices:

$$A=\begin{bmatrix}-0.14 & -0.69 & 0.0\\ -0.19 & -0.048 & 0.0\\ 0.0 & 1.0 & 0.0\end{bmatrix};\ b=\begin{bmatrix}0.056\\ -0.23\\ 0.0\end{bmatrix};\quad C=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\end{bmatrix}$$

The design requirements provided to us are:

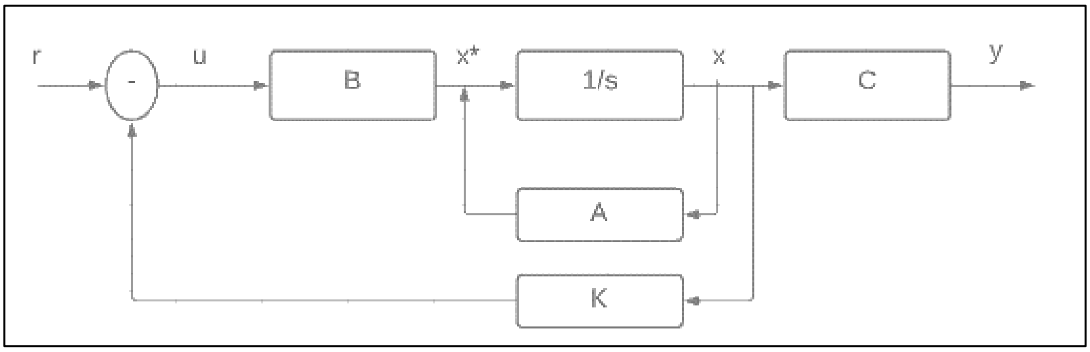
- Settling times of individual states are retained at those of the nominal eigenvalues.
- Maximum magnitude of all eigenvalues is as in the nominal set.
- The CL system is always observable and controllable.

Transfer Functions Obtained:

The system can be expressed using the following equation:

$$\dot{x}(t) = A*x(t) + b*u(t)$$
$$y(t) = C*x(t) + D*u(t)$$

'A'(3x3) is the system matrix, 'b'(3x1) is the control matrix, 'C'(2x3) is the output matrix and 'D' (0) is the feed forward matrix.



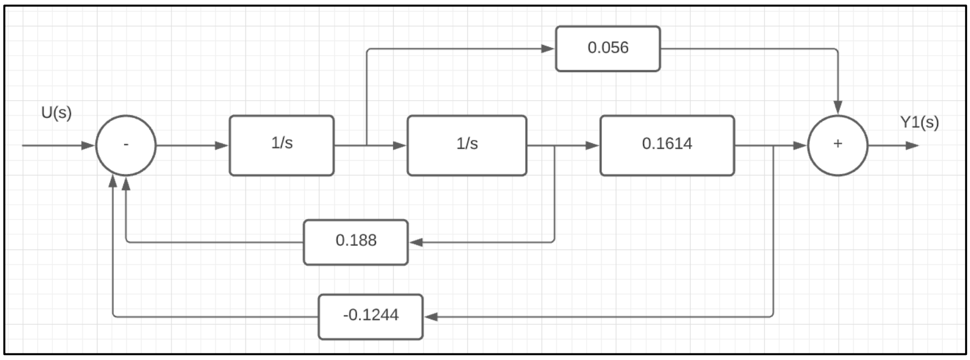
The generalised block diagram with cascaded feedback gain is depicted above. The system was investigated by considering the input, states and corresponding outputs for the given Single Input Multiple Output system. Following transfer functions were obtained for Sway speed and Yaw Angle:

```
                0.056 s + 0.1614
sway speed:  ----------------------
             s^2 + 0.188 s - 0.1244
```

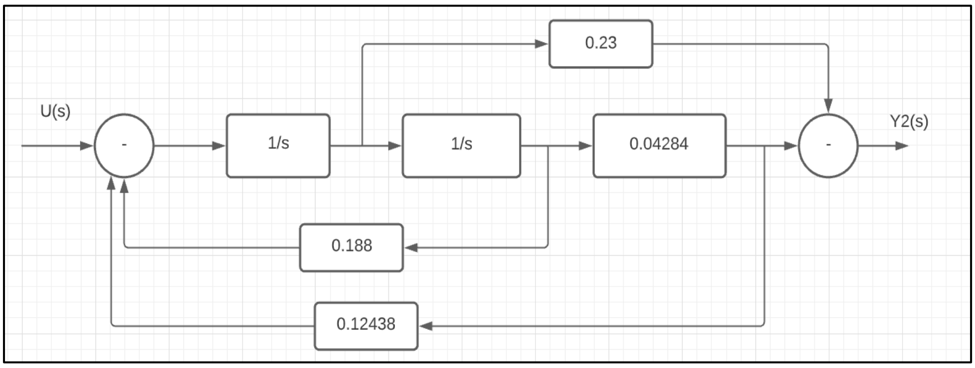
```
               -0.23 s - 0.04284
yaw angle:  ----------------------
            s^2 + 0.188 s - 0.1244
```

Block Diagram:

For Sway speed:

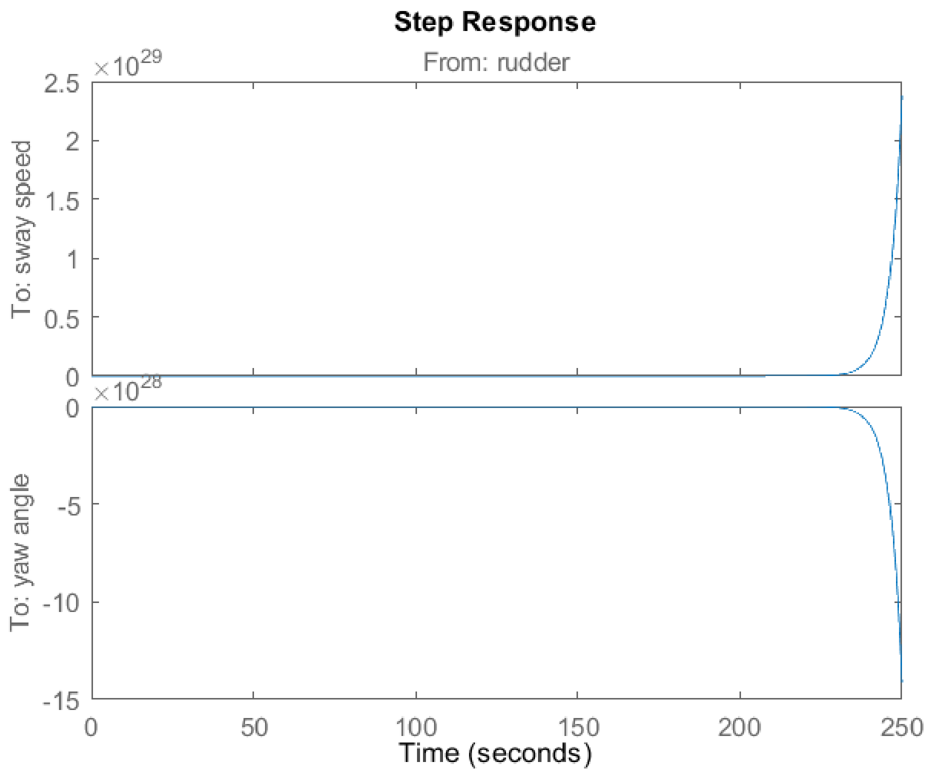


For Yaw angle:



Observations:

- The eigen values for the original system obtained are: **0, 0.2710, -0.4590**.



Step Response of the original system

- Since **eigen values of 'A' denotes the poles** of the system (without feedback gain) and **one of the eigen value** lies on the **right-hand plane** the system is **unstable**.
- To stabilize the closed loop system, we require a feedback gain matrix to relocate the poles on left-hand plane.
- **Since output matrix (C) has order 2x3 the system output is independent of x_3 i.e., Yaw rate. Hence, we obtain only two transfer matrices corresponding to the first two states.**
- **While evolving the equation $\dot{x}$ = A*x + b*u, the derivative of x_3 (Yaw Rate) comes out to be equal to yaw angle(x_1).**
- This provides the evidence for settling time of the system to NaN, as the theta will keep growing due to proportionate acceleration.
- We derived **different values of K such that eigen values of A-b*K are negative**.

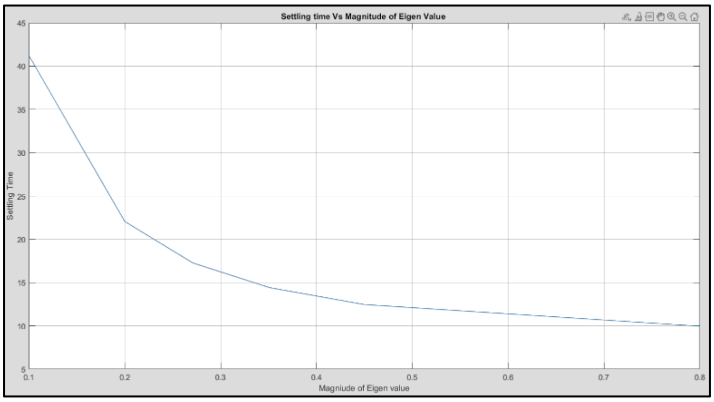
Pole Placement:

- We **replaced the positive Eigen Value with the negative one**, other two eigen values are retained. The assumed values of k were purely based on the system behaviour and the poles were placed intuitively.
- It was observed **that one pole at 0 is must to settle the step response else settling time becomes not defined.**
- The system is not observable since there is no direct connection between yaw rate and output.
- The **rank of observability matrix comes out to be 2 for the cases studied after adding feedback gain.**
- The **settling time for sway speed continuously decreases on increasing the magnitude** (though it is negative) of the only pole changed.
- The feedback matrix is obtained **by placing the poles (eigen values)** under the design requirements i.e., less than the nominal eigen values (<0.459).

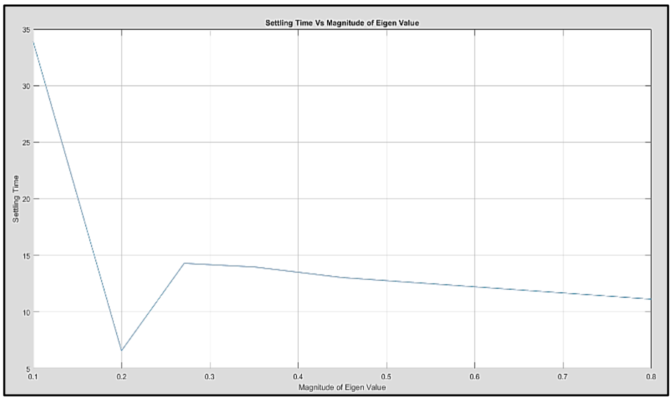
Eigen values	K	Settling time for Sway speed	Settling time for Yaw Angle	Observability	Controllability
0, 0.2710, -0.4590	Original System	NaN	NaN	No	Yes
0, -0.1, -0.4590	0.6702 -1.4499 0	41.2249	33.8801	No	Yes
0, -0.2, -0.4590	0.8509 -1.8406 0	22.0554	6.5875	No	Yes
0, -0.2710, -0.4590	0.9792, -2.1181 0	17.2864	14.3146	No	Yes
0, -0.351, -0.4590	1.1237 -2.4307 0	14.4206	13.9840	No	Yes
0, -0.45, -0.4590	1.3026 -2.8176 0	12.4663	13.0545	No	Yes

0, -0.8, -0.4590	1.9349 -4.1854 0	9.9671	11.1480	No	Yes

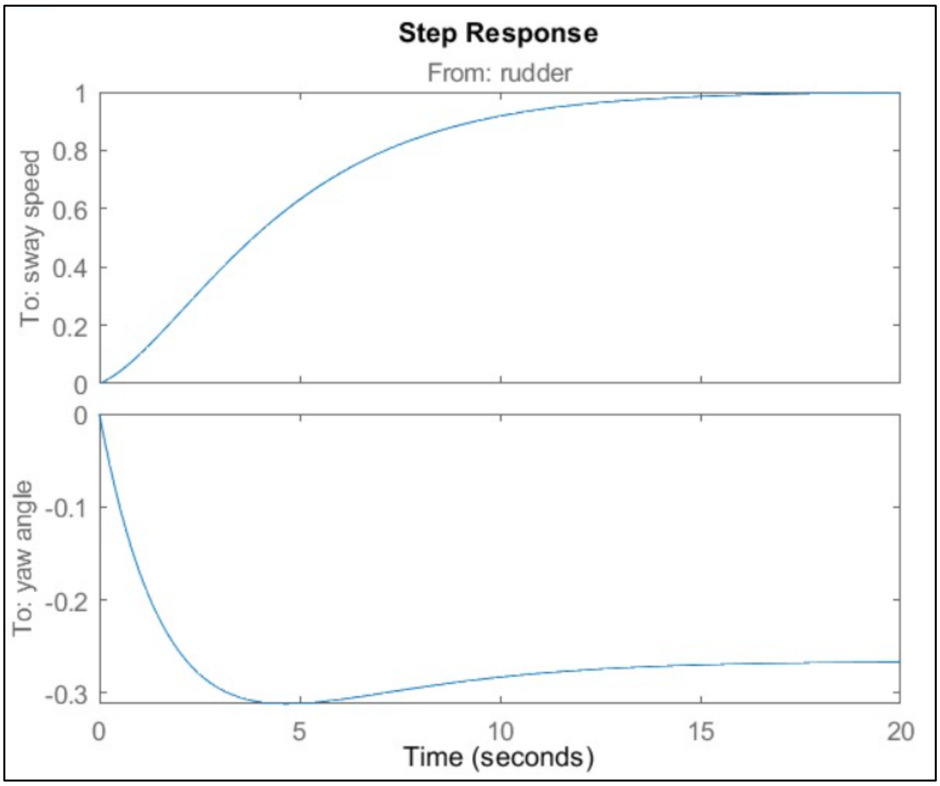
The last marked case defies the design requirements for the system in terms of magnitude of eigen value.



Variation in settling time for sway speed



Variation in settling time for Yaw Angle



Step Response by placing the poles at 0, -0.351, -0.4590

Optimal Case:

Following transfer functions were obtained for the sway speed and yaw angle by placing the **poles at 0, -0.351, -0.4590**:

Sway Speed = $\frac{0.056s+0.1614}{s^2+0.81s+0.1611}$ Yaw Angle = $\frac{-0.23s-0.04284}{s^2+0.81s+0.1611}$

- The feedback matrix for the above placed **poles** is calculated to be [**1.1237; -2.4307; 0**].
- It could be observed that both yaw angle and sway speed **settles** after a small time.
- **The settling time decreased for sway speed decreased monotonically with increase in magnitude of the eigen value** which is changed.

- The response of Yaw angle is negative.
- The steady state error is negligible for the sway speed though it is considerable for yaw angle.
- A trade off was observed between the steady state error of sway speed and Yaw angle.

Using Linear Quadratic Regulator Method:

- Since the above values of the relocated poles were completely intuitive. We designed the Linear Quadratic Regulator to find the optimal **values of K**, directly by biasing system states to meet the system requirement.

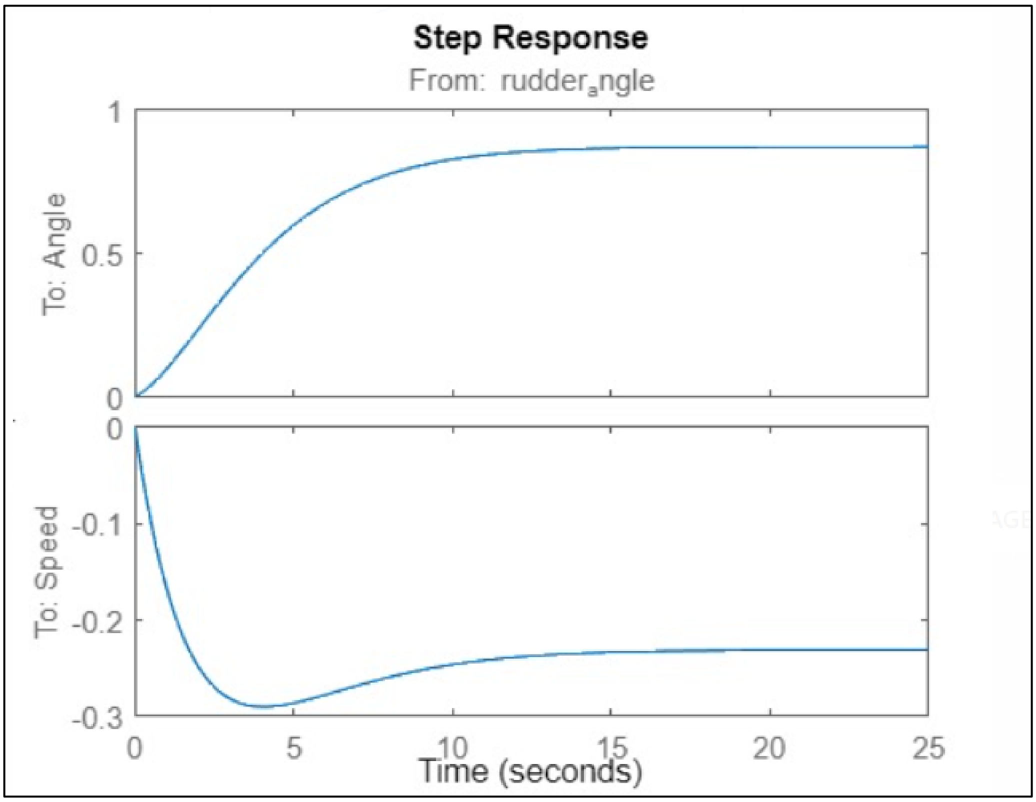
$$\text{Cost Function: J} = \int_0^{\infty} (x^T Q x + u^T R u) dt$$

- We have to minimise this cost function. Here **Q depicts the penalisation of different states and R penalises the input function**. Q is a diagonal matrix and each diagonal matrix correspond a state.
- We found the poles using the LQR function by penalising the states and input by different values.

Q	R	Eigen Values	Gain Feedback Matrix	Settling Time for Sway speed	Settling time for Yaw Angle	Controllability	Observability
$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	5	-0.1039 -0.4076+0.1352i -0.4076-0.1352i	$\begin{bmatrix} 1.0232 \\ -2.9296 \\ -0.4472 \end{bmatrix}$	NaN	NaN	Yes	Yes
$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	5	0 -0.4076+0.1352i -0.4076-0.1352i	$\begin{bmatrix} 1.2717 \\ -2.4173 \\ 0 \end{bmatrix}$	11.8541	13.2458	Yes	No
$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	5	-0.1267 -0.4168+0.1468i -0.4168-0.1468i	$\begin{bmatrix} 0.9669 \\ -2.8905 \\ -0.4472 \end{bmatrix}$	NaN	NaN	Yes	Yes
$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	5	0 -0.4168+0.1468i -0.4168-0.1468i	$\begin{bmatrix} 1.3209 \\ -2.4853 \\ 0 \end{bmatrix}$	11.3542	12.9380	Yes	No
$\begin{bmatrix} 4 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	8	-0.0824 -0.4081-0.1311i -0.4081+0.1311i	$\begin{bmatrix} 1.0713 \\ -2.8289 \\ -0.3536 \end{bmatrix}$	NaN	NaN	Yes	Yes
$\begin{bmatrix} 4 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	8	0 -0.4081-0.1311i -0.4081+0.1311i	$\begin{bmatrix} 1.2659 \\ -2.4231 \\ 0 \end{bmatrix}$	11.9556	13.2812	Yes	No
$\begin{bmatrix} 4 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	8	-0.0817 -0.4151+0.1140i -0.4151-0.1140i	$\begin{bmatrix} 1.0692 \\ -2.8875 \\ -0.3536 \end{bmatrix}$	NaN	NaN	Yes	Yes
$\begin{bmatrix} 4 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	8	0 -0.4151+0.1140i -0.4151-0.1140i	$\begin{bmatrix} 1.2591 \\ -2.4856 \\ 0 \end{bmatrix}$	12.2306	13.3138	Yes	No

- While using the LQR method, we could always **make the system observable** but at the **cost of settling time**.

- The **rank of the observability matrix using the poles derived by LQR is 3 i.e., it is observable.**
- The **rank of the controllability matrix is always 3** (equal to that of A*) which implies system is controllable.
- In order to achieve the settling time, **we relocated one pole to zero**, and **took other two poles, the same we obtained from LQR**.
- When we **could achieve the desired settling time, we lost the observability** of the system.
- **Clearly, there was a trade-off between observability and settling time and was due to pole at zero.**
- To achieve the eigen values in the nominal range we required **comparatively larger values of R to compensate increase in Q.**



Step Response for the poles: 0, -0.4151+0.1140i, -0.4151-0.1140i

Optimal Case:
Following transfer functions were obtained for the sway speed and yaw angle using LQR method, and **placing k at 1.2591, -2.4856, 0.**

Sway Speed = $\frac{0.056s+0.1614}{s^2+0.8302s+0.1853}$ Yaw Angle = $\frac{-0.23s-0.04284}{s^2+0.8302s+0.1853}$

- The following case have **eigen values: 0, -0.4151+0.1140i, -0.4151-0.1140i** which are in nominal range.
- The settling time for both states is about 13 seconds which is also quite nominal.
- The system is not observable.

Conclusion:

- We analysed the given system, and it was not stable as one of the eigen value was positive.
- The **derivative of yaw rate is equal to yaw angle** which implies that yaw acceleration is proportional to yaw angle, which justifies the settling time of the original system as NaN.
- We designed a Feedback Gain Matrix to achieve stability by:
  - ➢ Relocating the poles by pole placement method.
  - ➢ Designing Linear Quadratic Regulator.
- **It was observed that system could not fulfil the design requirement of achieving nominal settling time and be observable simultaneously.**
- **Though other requirement on eigen value and controllability were simultaneously achieved with either settling time or observability.**

MATLAB Script:

```
A=[-0.14 -0.69 0.0;-0.19 -0.048 0.0; 0.0 1.0 0.0 ];
B=[0.056;-0.23;0.0];
C=[1 0 0; 0 1 0];
D=0;
states = {'swayspeed' 'yawangle' 'yawrate'};
inputs = {'rudder'};
outputs = {'sway speed' 'yaw angle'};
eig(A)
sys_mimo = ss(A,B,C,D,'statename',states,'inputname',inputs,'outputname',outputs);
%eq=tf(sys_mimo);
%stepinfo(eq)
p=[-1 + 0.059i, -1 - 0.059i, 0];
k=place(A,B,p);
Anew=A-B*k;
eig(Anew)
sys_new = ss(Anew,B,C,D,'statename',states,'inputname',inputs,'outputname',outputs);
z=stepinfo(sys_new)
stepplot(sys_new)
damp(sys_new)
rank(obsv(sys_new))
rank(ctrb(sys_new))
```

Script-1: Script describing Pole Placement method

```
clc
clear
%Cascaded
settling_time=[33.8801 6.5875 14.3146 13.9840 13.0545 11.1480];
eigen_values=[0.1 0.2 0.2710 0.351 0.45 0.8];
plot(eigen_values,settling_time);
grid on
```

Script-2: Script for plotting variations

```
A=[-0.14 -0.69 0.0;-0.19 -0.048 0.0; 0.0 1.0 0.0 ];
B=[0.056;-0.23;0.0];
C=[1 0 0; 0 1 0];
D=0;
states = {'swayspeed' 'yawangle' 'yawrate'};
inputs = {'rudder'};
outputs = {'sway speed' 'yaw angle'};

sys_mimo = ss(A,B,C,D,'statename',states,'inputname',inputs,'outputname',outputs);
tf(sys_mimo);
Q=[4 0 0;0 6 0; 0 0 1];
R=8;
k=lqr(A,B,Q,R);
Anew=A-B*k;
eig(Anew)
p=[ 0, -0.7222 + 0.5081i, -0.7222 - 0.5081i];
A_n=A-B*k_n;
k_n=place(A_n,B,p);

sys_new = ss(A_n,B,C,D,'statename',states,'inputname',inputs,'outputname',outputs);
z=stepinfo(sys_new)
stepplot(sys_new)
damp(sys_new)
rank(obsv(sys_new))
rank(ctrb(sys_new))
```

Script-3: Script describing Linear Quadratic Regulated Method

Thank You!