EE 208

Control Engineering Lab

Experiment-12: Transfer between Digital States.

Group Number: 20

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Objective:

- To examine a given three-variable digital system with parameters susceptible to arbitrariness of values and check out the scope and limits of possible deadbeat type performance.

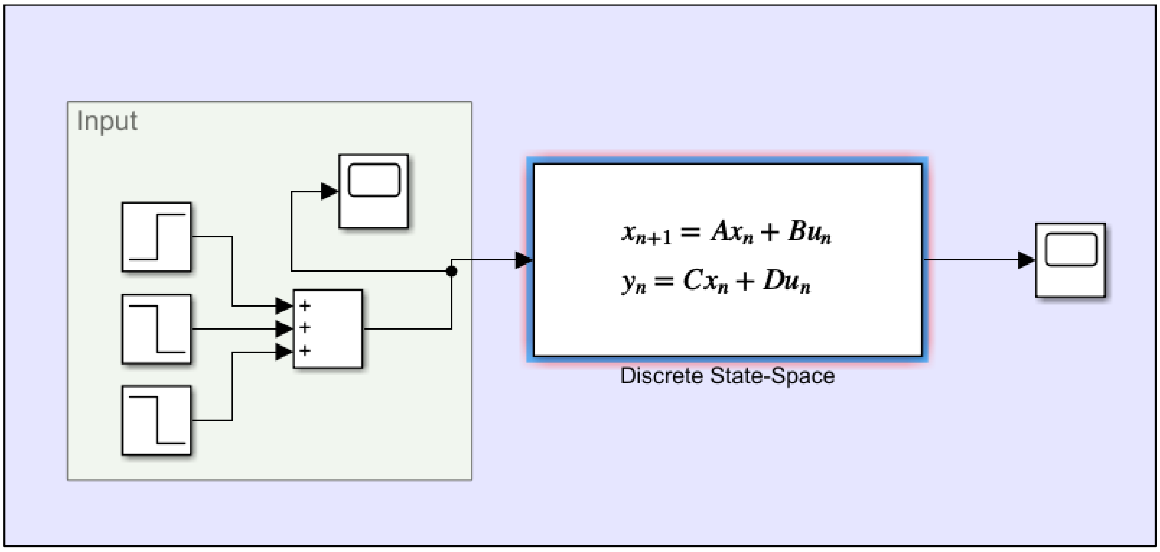
Given:

The following three variable system has arbitrary values or setting possible for parameters a and b:

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ a & b & -a/b \end{bmatrix} . \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} .u(k)$$

- With initial state x (0) = $[1 \quad 1 \quad 1]^T$, check if x (0) is achievable.
- With initial state x (0) = 0, find out if x (3) = $[1 \quad 1 \quad 1]^T$ is achievable.

Simulink-



Note- In the given system C = $I_{3\times3}$ and D = 0, this makes output (Y) = State (X)

Characteristics of Dead Beat Response:

- Steady state error is zero
- Minimum rise time and settling time
- No or very less overshoot

Case 1: X (0) = $[1\ 1\ 1]^T$ and X(3) = **0**

Finding values of a and b

On solving the given constraints, the expressions were evaluated for X (3) and were equated to **zero** to obtain the desired state output. Following equations were obtained in terms of a, b and u.

```
eqn1 = a + b - a/b + u1 == 0

eqn2 = a + b*(1+u0) -(a/b) *(a + b-a/b) +u2 ==0;

eqn3 = a*(1+u0) + b*(a+ b-a/b+u1) -(a/b) *(a + b*(1+u0) -(a/b)
*(a + b-a/b)) == 0;
```

eqn1 =

$$a + b + u_1 - \frac{a}{b} = 0$$

eqn2 =

$$a + u_2 + b\ (u_0 + 1) - \frac{a\ \left(a + b - \frac{a}{b}\right)}{b} = 0$$

eqn3 =

$$a\ (u_0 + 1) + b\ \left(a + b + u_1 - \frac{a}{b}\right) - \frac{a\ \left(a + b\ (u_0 + 1) - \frac{a\ \left(a + b - \frac{a}{b}\right)}{b}\right)}{b} = 0$$

Following solution was obtained from the above expressions:

- a =0
- b= -u (1)
- u (0) = [u (2)-u (1)] / u (1)

ans = 0

ans = $-u_1$

ans =

$$-\frac{u_1 - u_2}{u_1}$$

From the above calculations,

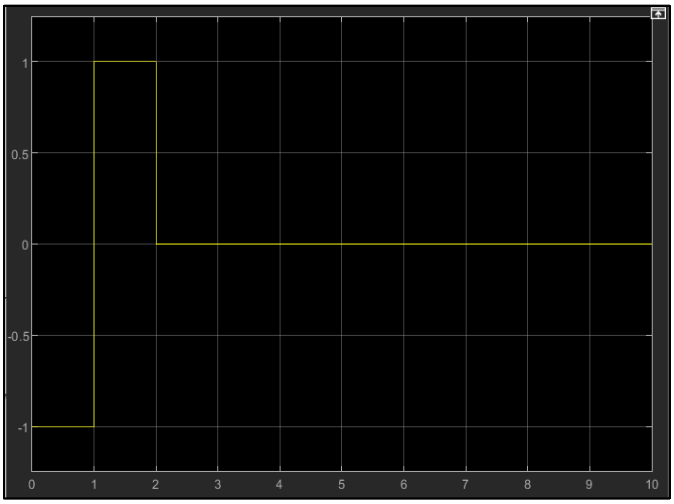
- **a has to be zero in all cases**.
- Parameter **b** and input **u** are **interrelated**, therefor they have multiple feasible solutions.

Now taking account of above constraints we now design an input for the system and observe the corresponding response.

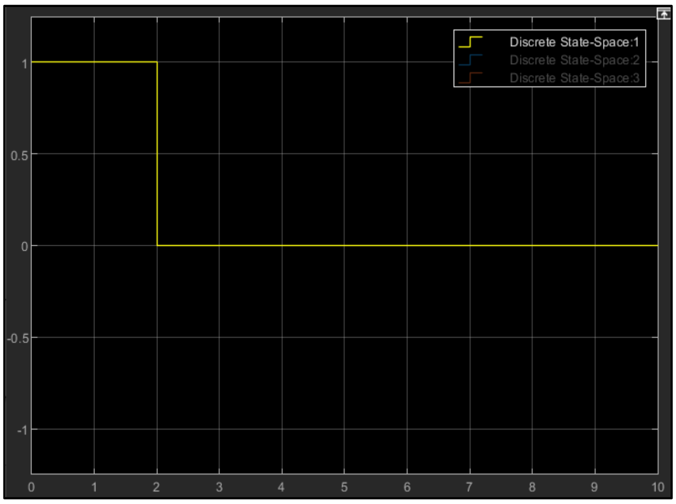
Observation:

- **a = 0, b = - 1**

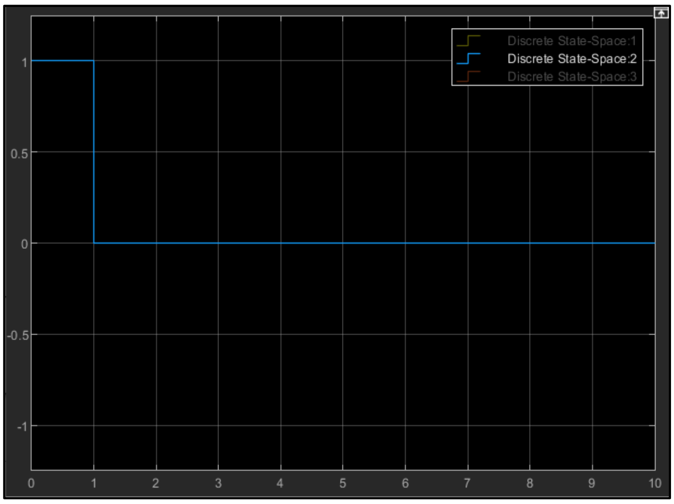
  u (0) = -1, u (1) = 1, u (2) = 0 u(k) = 0, k>2



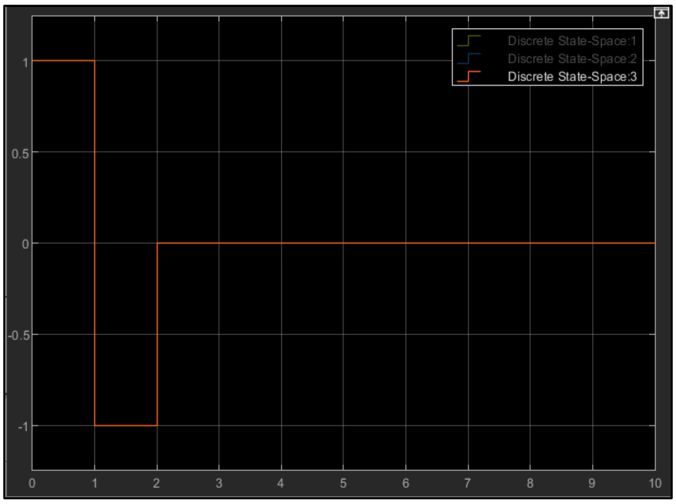
Input



X1



X2

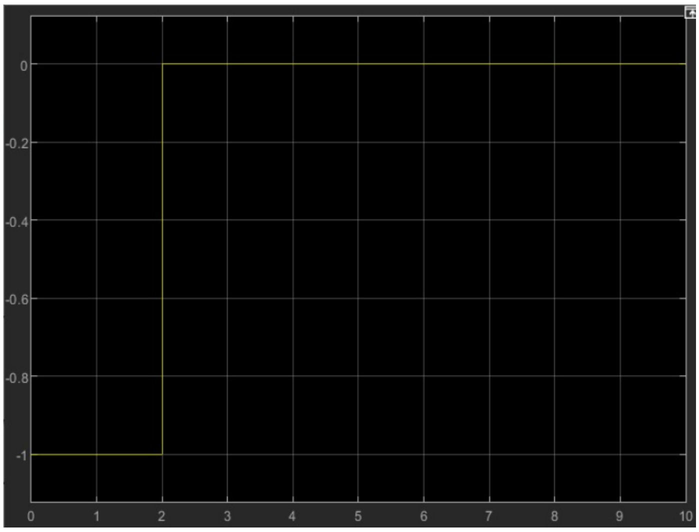


X3

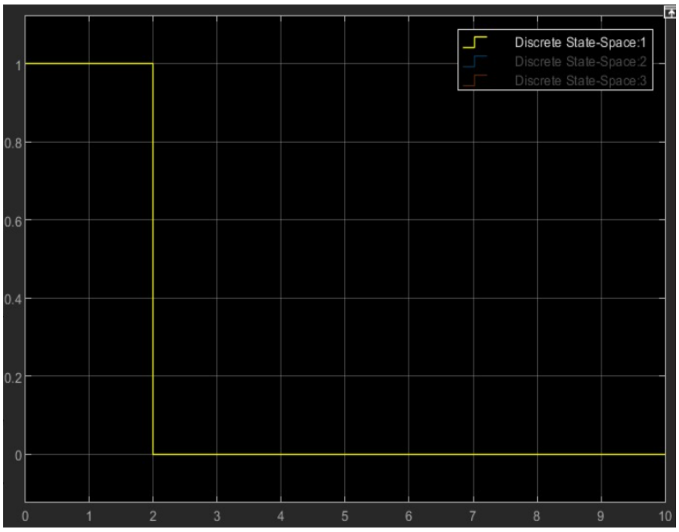
- State x1 settles after **two units** of sampling time.
- State x2 settles after **one unit** of sampling time.
- State x3 settles after **two units** of sampling time.

- **a = 0, b = 1**

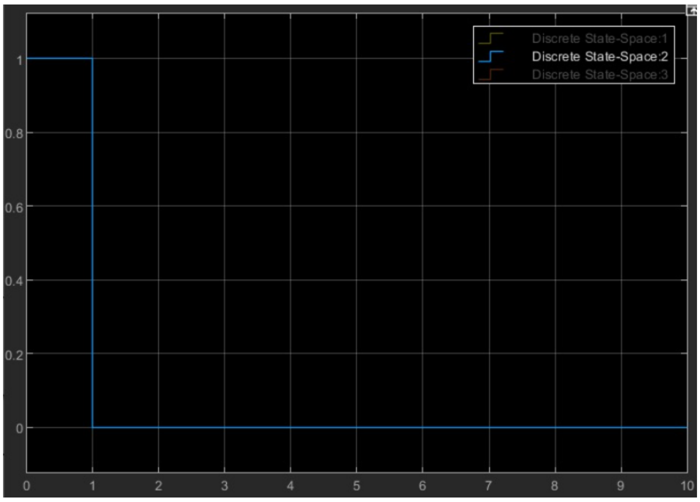
  u (0) = -1, u (1) = -1, u (2) = 0…u(k) = 0, k>2



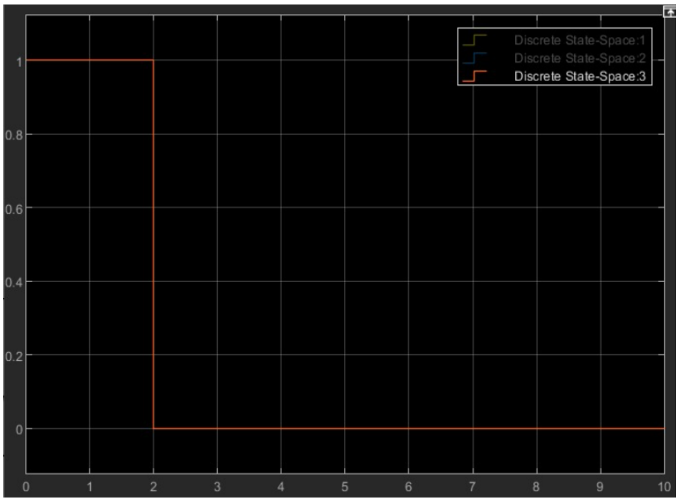
Input



X1



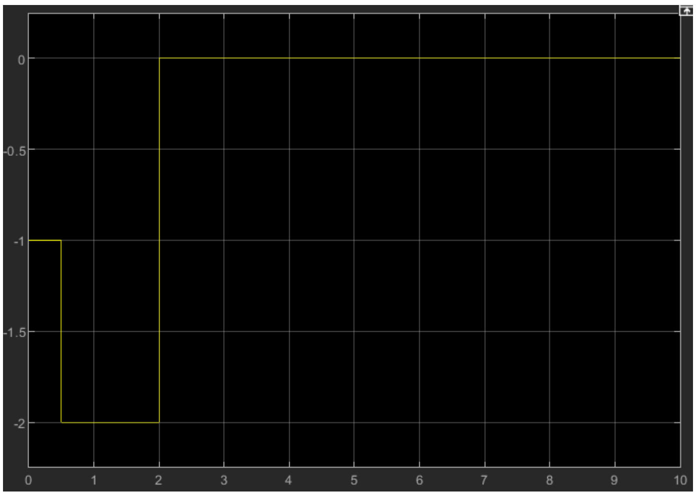
X2



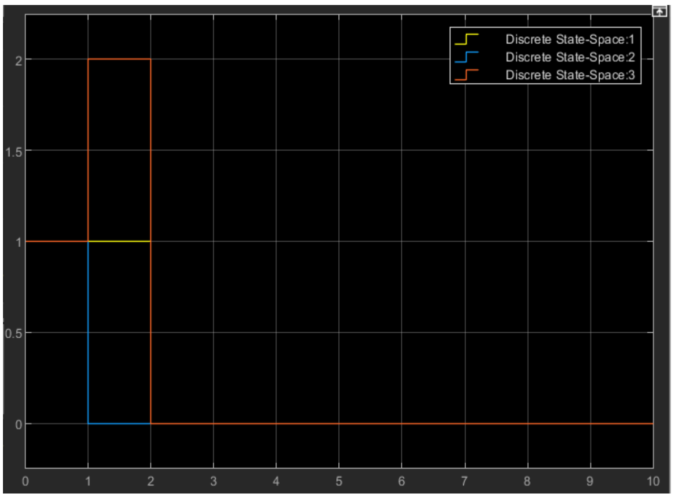
X3

- State x1 settles after **two units** of sampling time.
- State x2 settles after **one unit** of sampling time.
- State x3 settles after **two units** of sampling time.

➢ **a = 0, b =2**

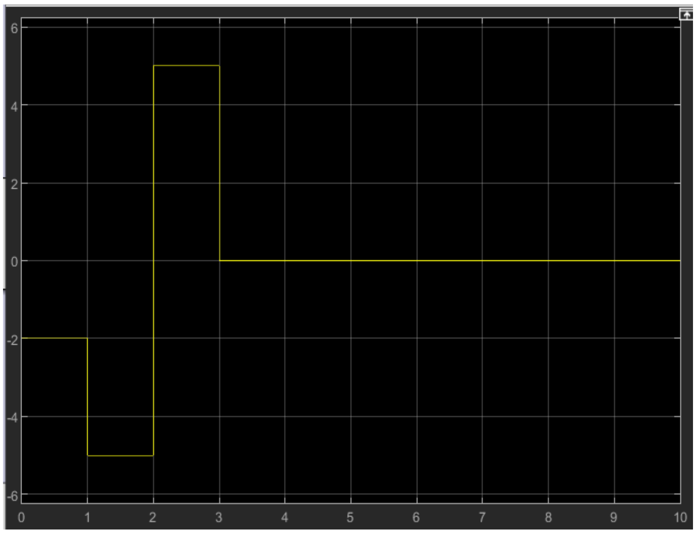


Input

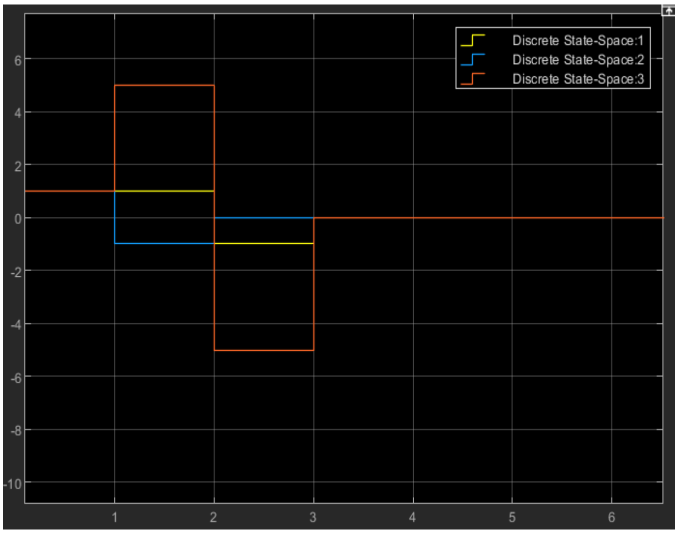


State Response

➢ **a = 0, b =5**



Input



State Response

<u>Case 2:</u> X (0) = **0** and X (3) = $[1\ 1\ 1]^T$

Finding values of a and b:

After evaluating the expression of X (3) and evaluating it to $[1\ 1\ 1]^T$ we get the following equations:

eqn1 = u (1) == 1

eqn2 = u (2) + b u (0) == 1

eqn3 = b u (1) == 1

$$\text{eqn1} = u_1 = 1$$
$$\text{eqn2} = u_2 + b\,u_0 = 1$$
$$\text{eqn3} = b\,u_1 = 1$$

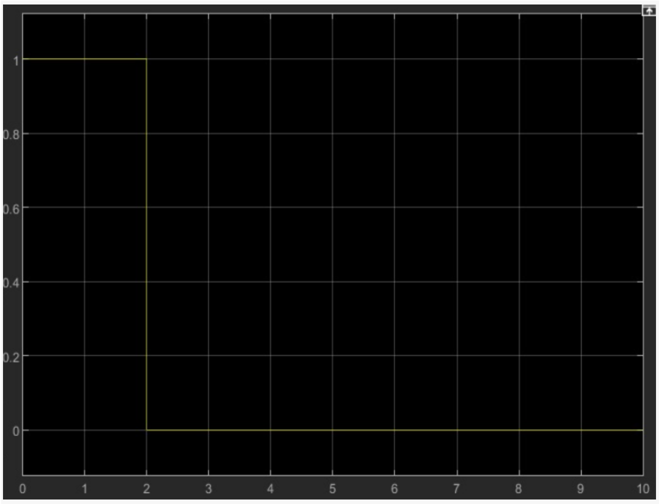
From the above equations:

- **u (1) =1 in every case** of input.
- **b = 1**
- solution is independent of **parameter a.**
- **u (2) + u (0) = 1**

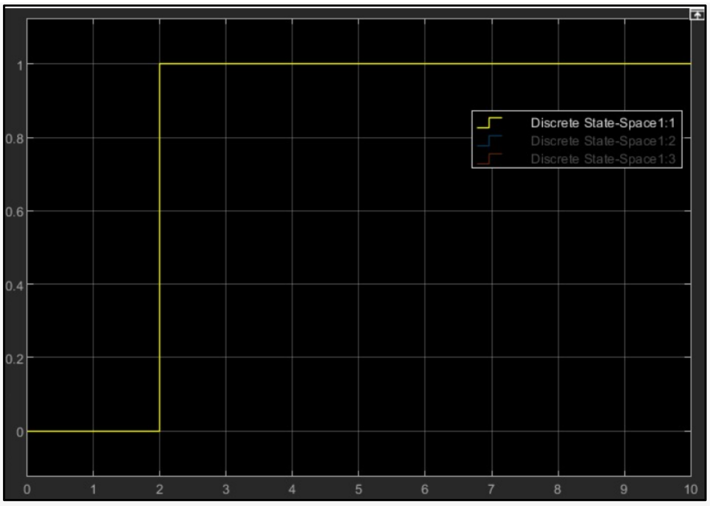
Observation:

➢ **a=0, b=1**

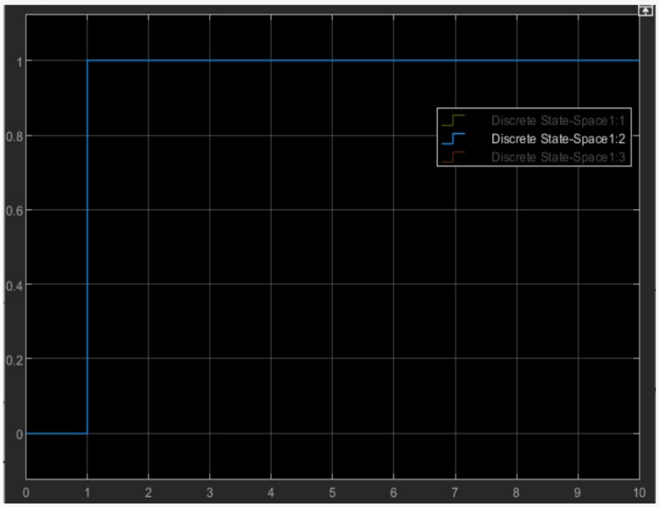
```
u (0) = 1, u (1) = 1, u (2) = 0. . . u(k)=0, k>3
```



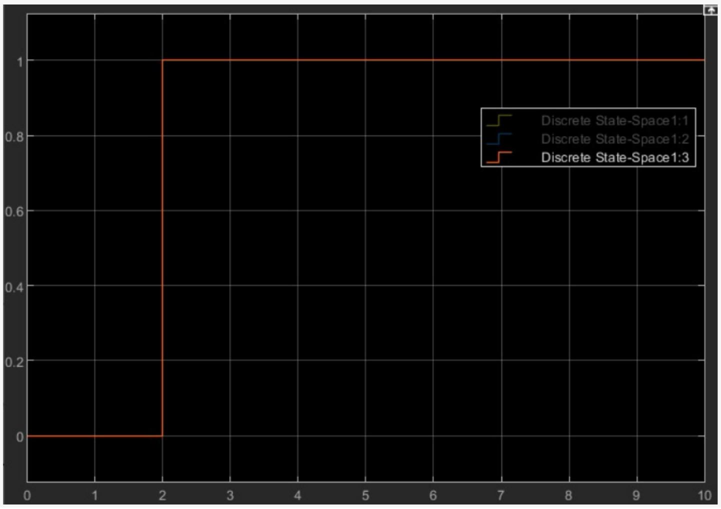
Input



X1



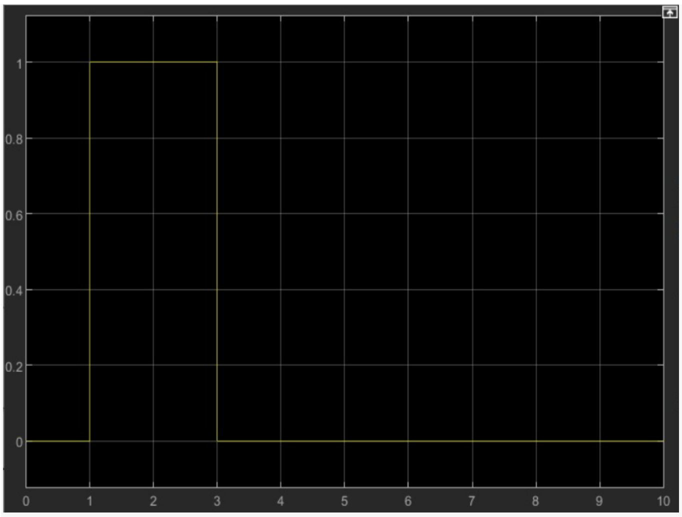
X2



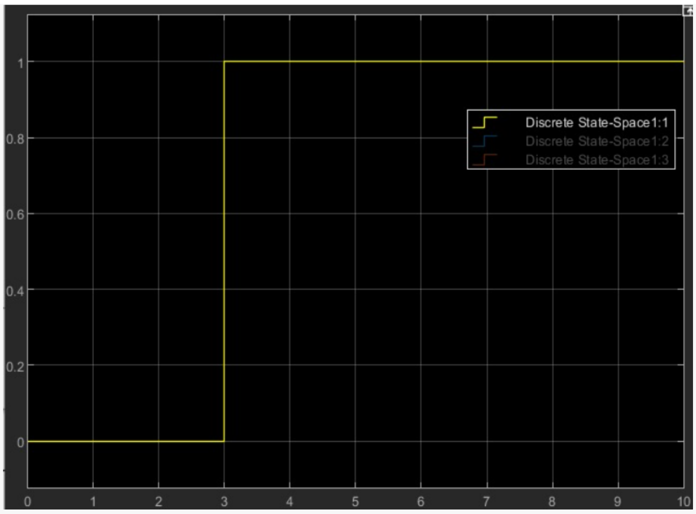
X3

- State x1 settles after **two units** of sampling time.
- State x2 settles after **one unit** of sampling time.
- State x3 settles after **two units** of sampling time.

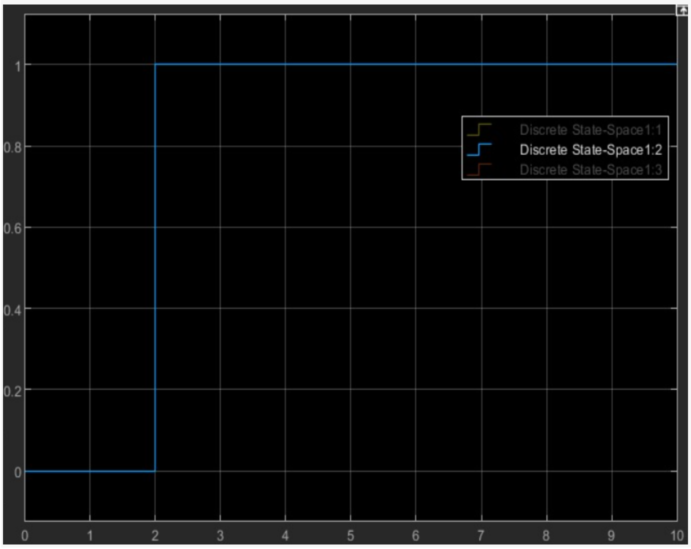
➢ ***a=0, b=1***



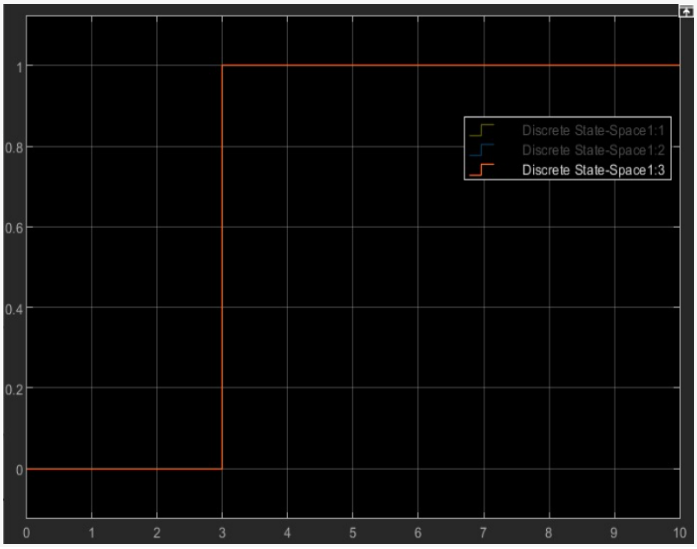
Input



X1



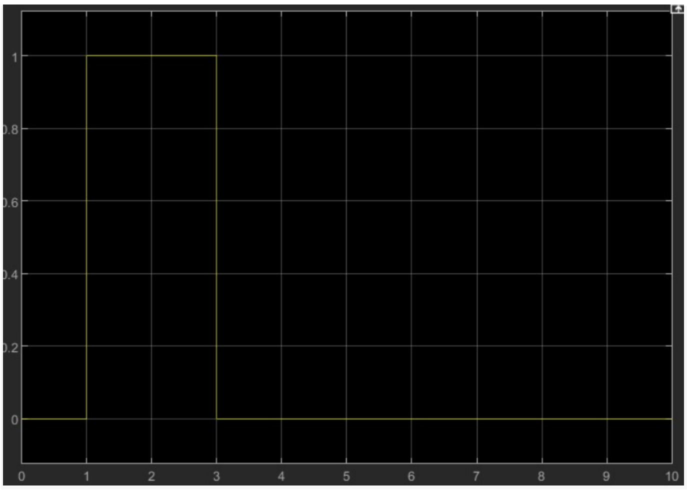
X2



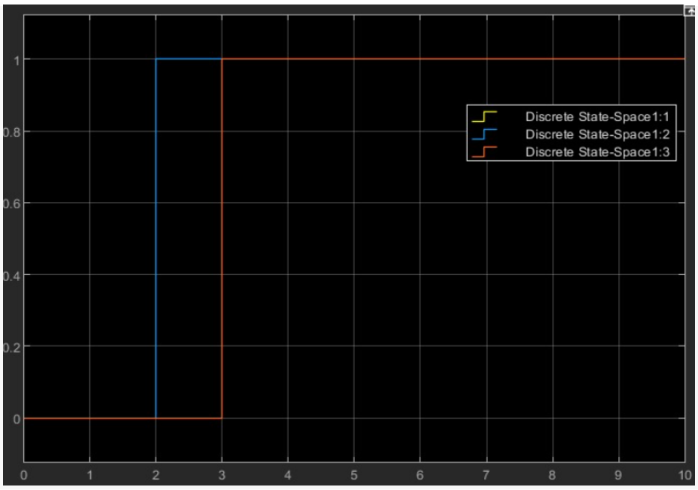
X3

- State x1 settles after **three units** of sampling time.
- State x2 settles after **two unit** of sampling time.
- State x3 settles after **three units** of sampling time.

➢ ***a = 10, b = 1***

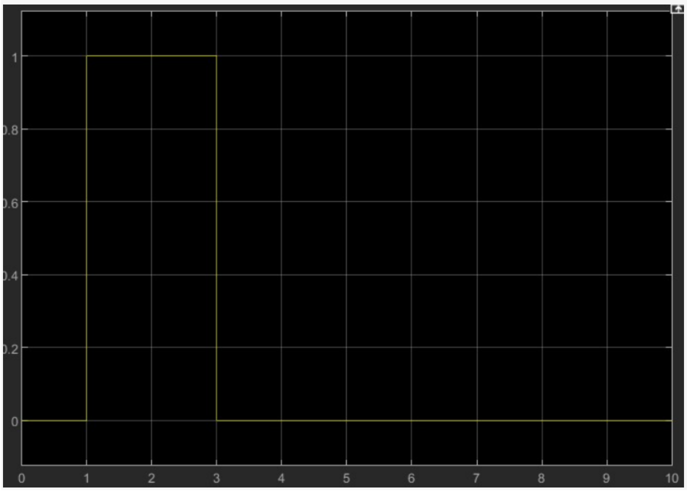


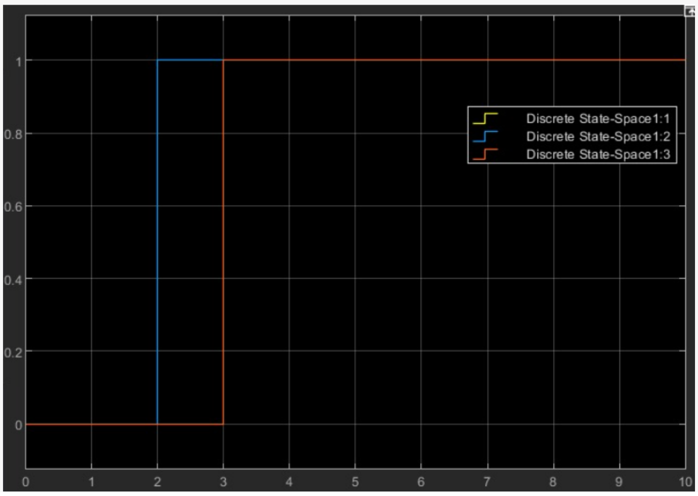
Input



State Response

a = -10, b = 1





Input State Response

Analysis:

Case 1:

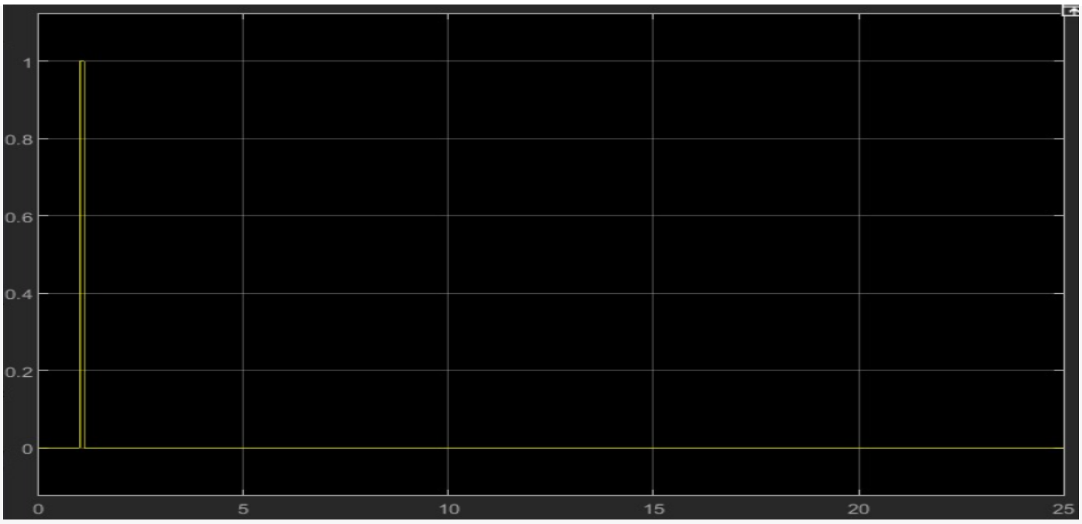
- value of **parameter a** is taken zero always, this drives **one of the pole to *origin* of *z-plane.***
- The **other two poles can’t be dragged to origin**, for any combination of a and b (as b can’t be zero).

```
A =

[0, 1,     0]
[0, 0,     1]
[a, b, -a/b]
```

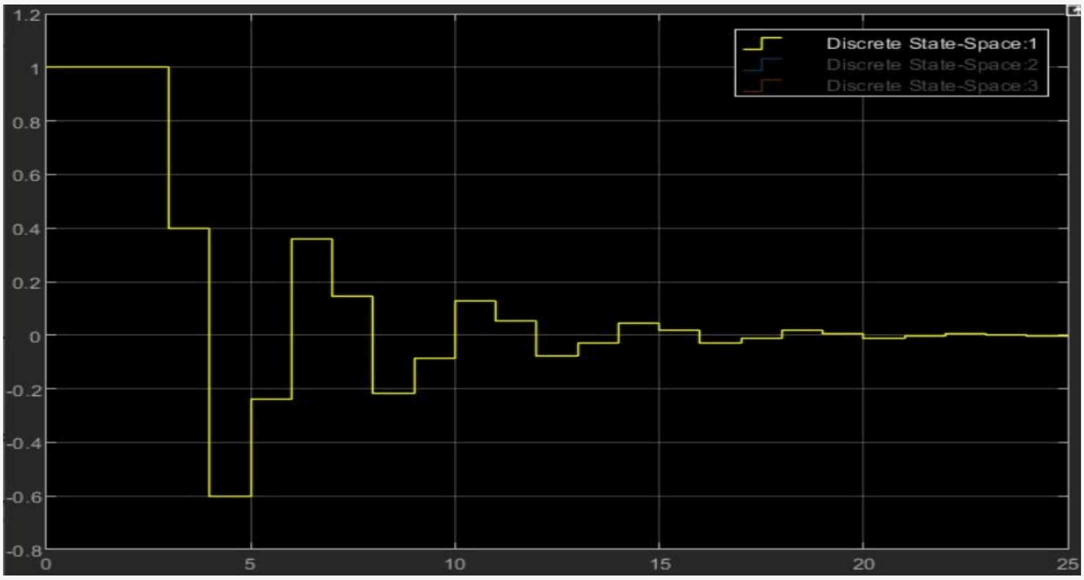
```
>> eig(A)

ans =

 1/(1/b)^(1/2)
-1/(1/b)^(1/2)
          -a/b
```

First two poles can’t be zero for any non-zero b

- The response can’t be ideal dead beat, without placing all the poles to the origin.



Input Impulse excited at 1s



Response of X1

- Without placing all the digital poles to origin we will get ripples in the state response, although our input settles to 0, but there is delay in settling of response of the states. It is not an ideal dead beat response.

- This lead us to **design a custom input (*based on the value for initial and steady state*), for early settlement of the response.**
- We have designed the corresponding **input u(t),** based on the value of **parameter b (always non-zero).**
- In each of the cases, our response **settles under at max 3 unit of sampling time** (taken normalized value).

Case 2:

- We proceed similarly as in case 1.
- Unlike of the previous case, the selection of **input (u)** comes out to be **independent** of **parameter a**, this is also proved by taking observation for multiple values of **a** in the previous section,
- **Parameter b** comes out be constant with **value 1** for all cases.
- **Input (u)** for the system is designed accordingly.

Conclusion:

We analysed the given system by converting it into discrete state space using Simulink and calculated the values of parameter a and b for multiple inputs provided to the system to achieve dead beat response. Exhaustive set of parameters was determined under the given liabilities and constraints.

MATLAB Script

```
clc;
clear;
syms a b u0 u1 u2 ;
eqn1 = a + b - a/b + u1 == 0
eqn2 = a + b*(1+u0)-(a/b)*(a+b-a/b)+u2 ==0;
eqn3 = a*(1+u0) + b*(a+b-a/b+u1)-(a/b)*(a+b*(1+u0)-(a/b)*(a+b-a/b)) == 0;
sol = solve([eqn1,eqn2,eqn3], [a,b,u0]);
sol.a
sol.b
sol.u0
```

```
clc;
clear;
close all;
syms a b u z k1 k2 k3
%z = tf('z',-1)
k = [k1 k2 k3]
F = [0 1 0; 0 0 1;a b -a/b]
G = [0; 1; 0]
[V,D] = eig(Ff)
I = eye(3);
D1 = D(1,1)
D2 = D(2,2)
D3 = D(3,3)
eq1 = D1 == 0
eq2 = D2 == 0
eq3 = D3 == 0


sol = solve([eq1 eq2 eq3],[a b k2])
sol.a
sol.b
sol.k1
```

MATLAB Script for solving simultaneous equations

THANKYOU!