EE 208

Control Engineering Lab

Experiment-5: Controller design on MATLAB platform by discrete frequency response.

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OBJECTIVE: -

- To examine the gain and phase margins for an OLTF closed through negative feedback of different gains.
- The sensitivity to the choice of sampling time is also examined.

Given: -

The digital OLTF of a furnace model, inclusive of a first order actuator, is given by:

OLTF: $G_{OL}(z) = 10^{-5}.\ \frac{4.711z+4.644}{z^3-2.875z^2+2.753z-0.8781}$

The furnace output can be fed back to the actuator input with different positive integer gains in the feedback loop. The nominal sampling time for the system is 0.01s.

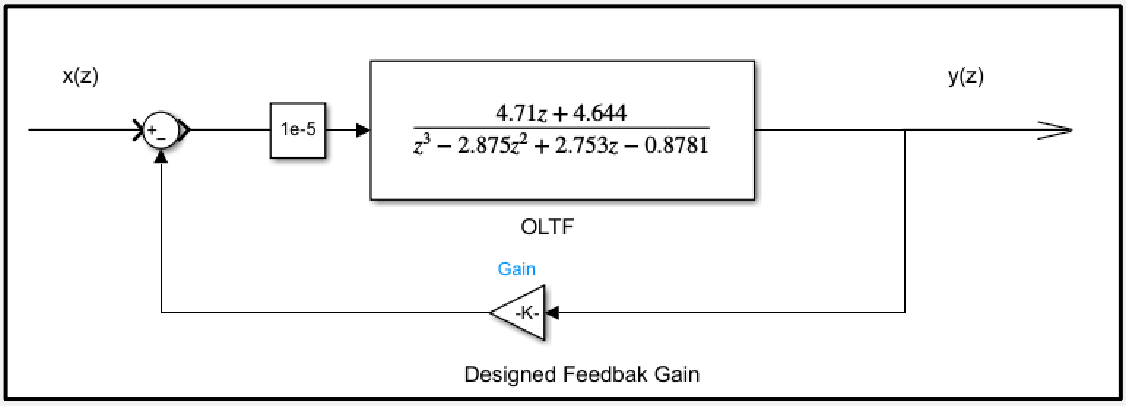
System Modelling

Transfer Function:

The closed loop Transfer function is as follows:

CLTF: $G_{CL} = \frac{10^{-5}(4.711z+4.644)}{K(10^{-5})(4.711z+4.644)+(z^3-2.875z^2+2.753z-0.8781)}$

Block Diagram:



Block diagram for final system

Observations-

$G_{CL} = \frac{G_{OL}}{1+K\ G_{OL}}$ where *K is positive integer*.

G_{CL} will become **unstable** when $K^*G_{OL} = -1$ where,

$|K\ G_{OL}| = 0$ dB Gain (at Gain Crossover frequency)

$K\ G_{OL} = 180°$ Phase (at Phase Crossover frequency)

For discrete-time systems,

$z = e^{j\omega Ts}$ and $0 \leq \omega \leq \omega_N = \frac{\pi}{Ts}$

T_S = sampling time (nominal value is 0.01 sec.)

ω_N = Nyquist frequency

The frequency components, which are **less than Nyquist frequency**, gets sampled only (which is depicted by main band of frequency plot).

Therefore, ω_{MAX} (maximum frequency in the band width) should be **less** than ω_N (Nyquist frequency) to **avoid aliasing**.

Detailed Study for various Sampling Times:

1) For Ts = 0.01s (Nominal value)

Gain(K)	Gain Margin	Phase margin	PCO	GCO	Stability
1	0.579	inf	0	NaN	**Unstable**
2	7.65	18.6	5.36	2.49	**Stable**
3	4.12	12.0	5.36	3.76	**Stable**
4	1.63	5.01	5.36	4.71	**Stable**
5	-0.312	-0.984	5.36	5.49	**Unstable**
6	-1.9	-6.02	5.36	6.15	**Unstable**

NOTE- For some values of K (where either the GCO or PCO is zero or NaN) we get ambiguous Gain and Phase margins. This generally happens when bode plots doesn't intersect 0 dB or -180^0 lines for any ω in the main band (which also depends on choice of sampling time).

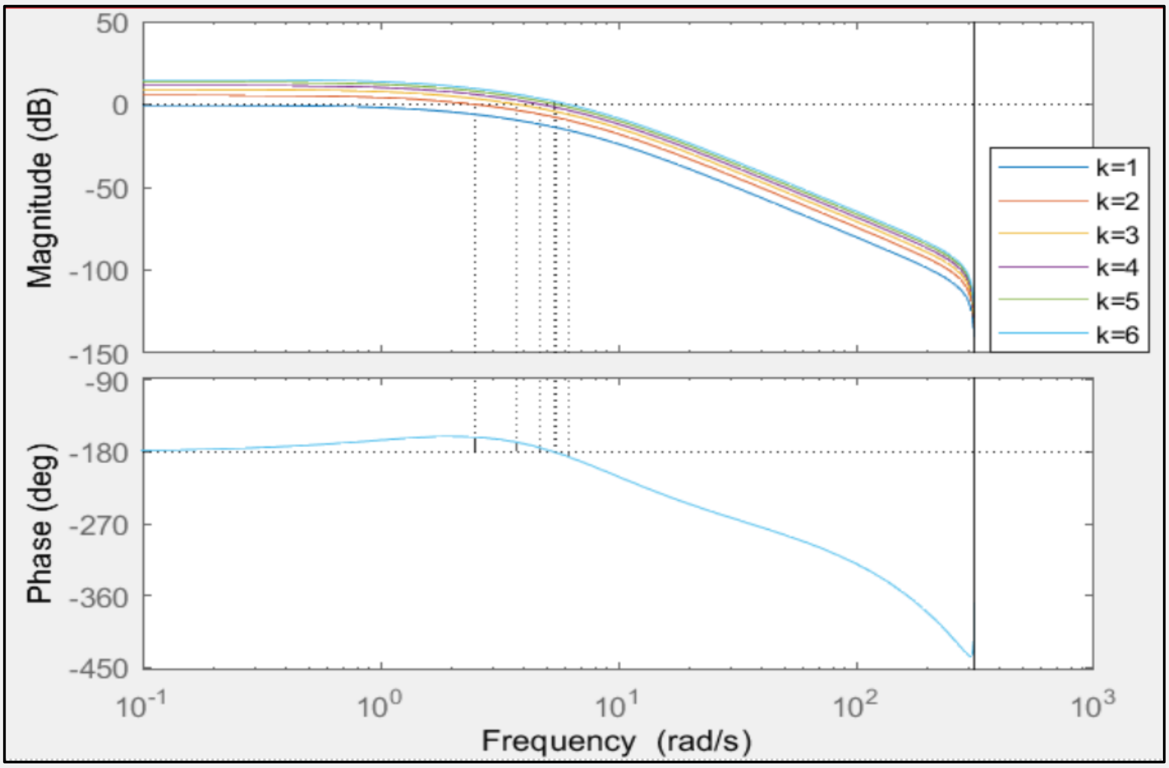


Fig-1: Bode plots for integral K

- Closed Loop system is **stable only for three integral values of K {2,3,4}** and **unstable** for rest.

As K increases (for positive integral values greater than 2):

- The value of **Phase Margin decreases** and goes negative for K = 5 and above
- The value of **Gain Margin also decreases** and goes negative for K = 5 and above.
- Since for K=1, the system is unstable, the margins drop.
- **Phase Cross Over frequency remains constant**.
- **Gain Cross Over frequency increases**.

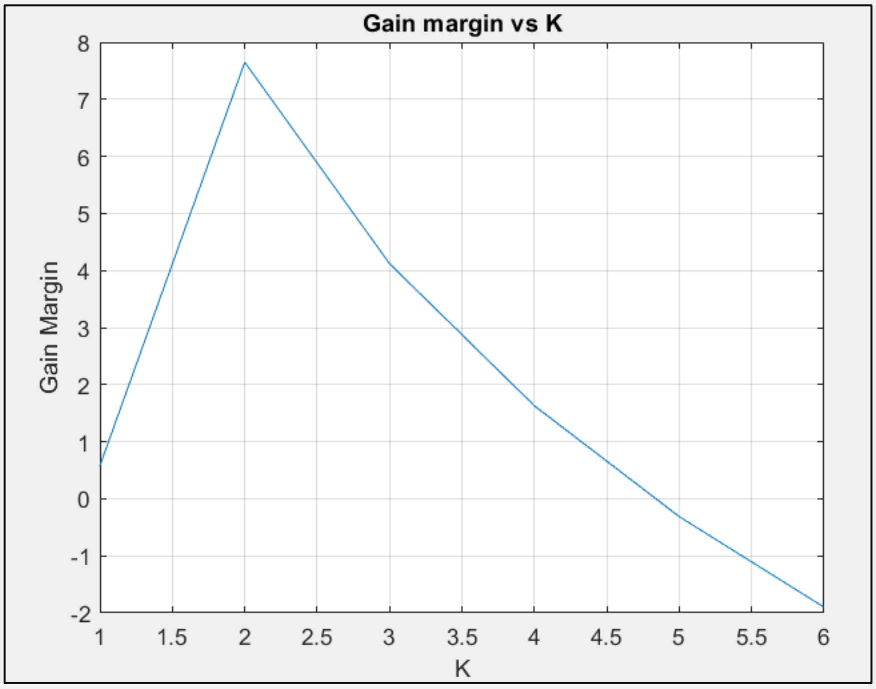


Fig.- 2: Gain Margin for integral K

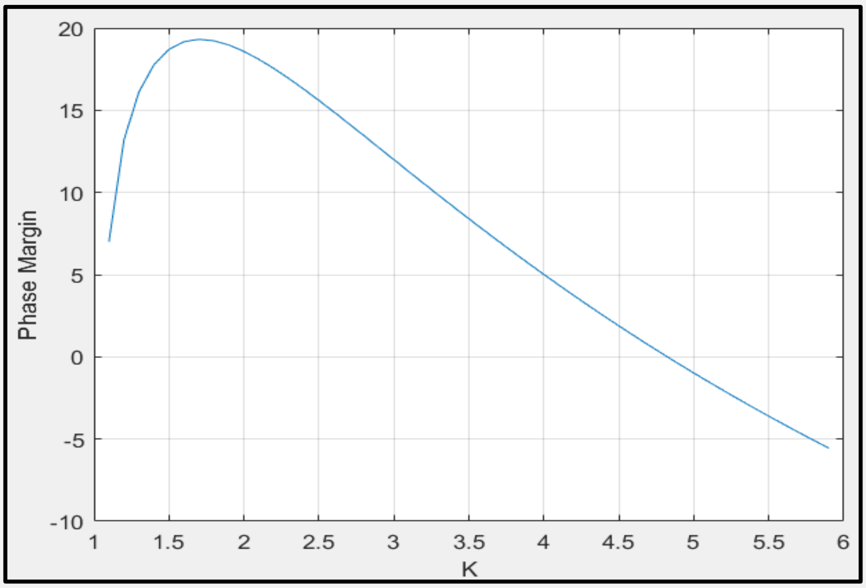


Fig.- 3: Phase Margin for integral K

- Low value of Gain Margin indicates low proximity to instability and vice-versa.
- Low value of Phase Margin indicates low proximity to instability and vice-versa.

Effect of Sampling Time

With change of sampling time **the location of poles and zero changes** for our given OLTF. Therefore, **width of the main band and frequency parameters also changes.**

- **For lower sampling time than nominal Ts:**

1) For Ts = 0.001 s

Gain(K)	Gain Margin	Phase margin	PCO	GCO	Stability
1	0.579	inf	0	NaN	**Unstable**
2	-5.44	19.2	0	2.49	**Stable**
3	4.58	13	5.55	3.76	**Stable**
4	2.08	6.22	5.55	4.71	**Stable**
5	0.14	0.428	5.55	5.49	**Stable**
6	-1.44	-4.44	5.55	6.15	**Unstable**

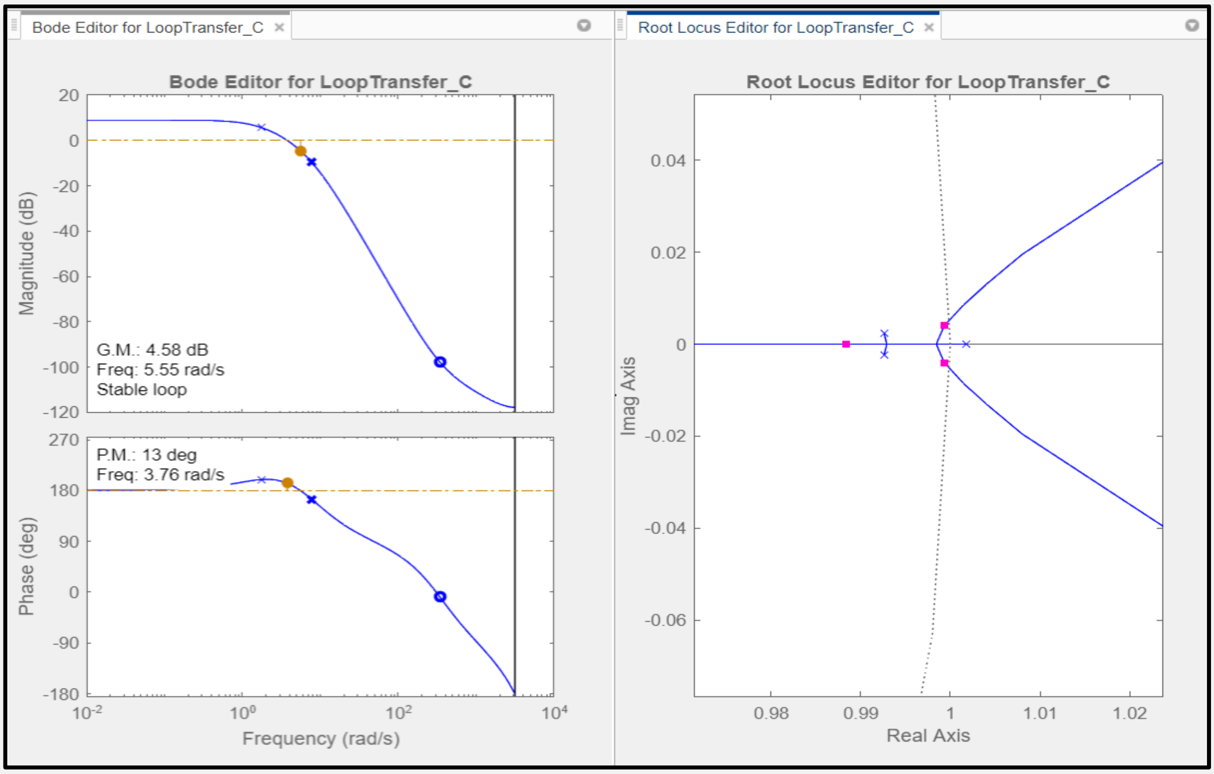


Fig.-4: Depicting the Bode plot and Root Locus for K=3, Ts =0.001s

2)For T_s = 0.005 s

Gain(K)	Gain Margin	Phase margin	PCO	GCO	Stability
1	0.579	inf	0	NaN	**Unstable**
2	-5.44	18.9	0	2.49	**Stable**
3	4.37	12.5	5.46	3.76	**Stable**
4	1.87	5.68	5.46	4.71	**Stable**
5	-0.064	-0.2	5.46	5.49	**Unstable**
6	-1.65	-5.14	5.46	6.15	**Unstable**

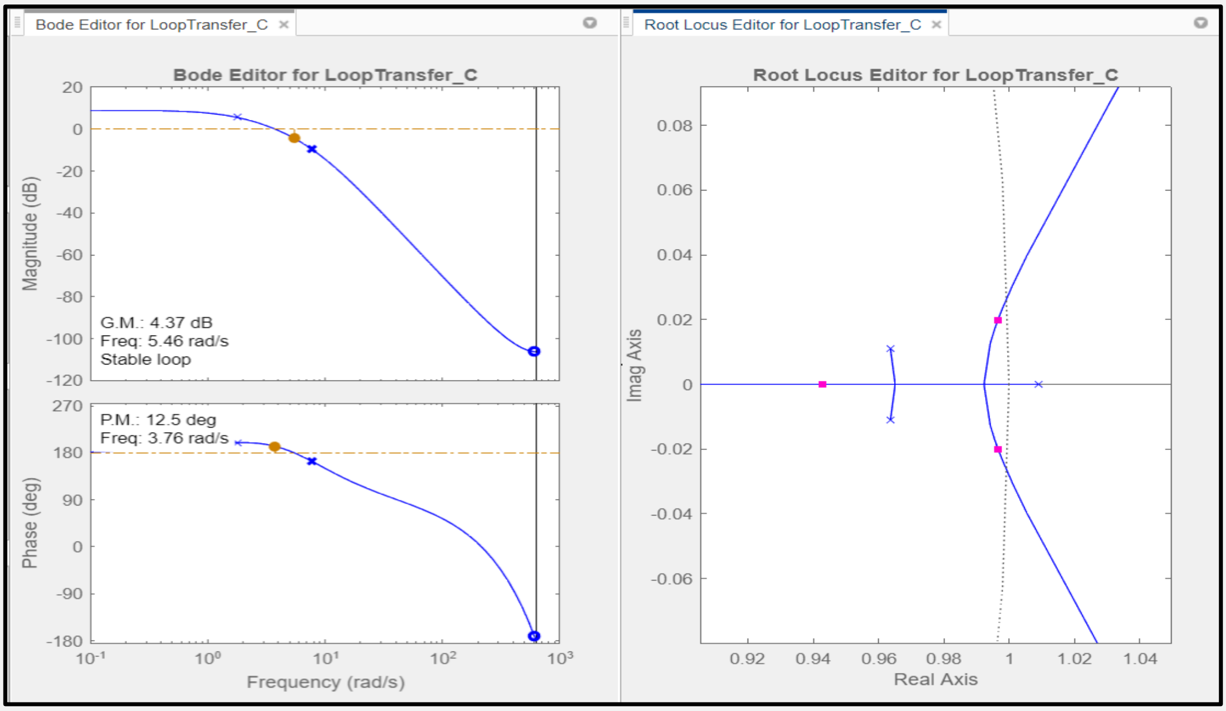


Fig.-5: Depicting the Bode plot and Root Locus for K=3, Ts =0.005s

The above two observations are for lower sampling time than nominal value,

As the value of **Sampling Time Ts decreases**,

- **Value of Gain Margin increases for a constant K.**
- **Value of Phase Margin increases for a constant K (system moves towards stability).**
- Value of **PCO increases** but remains **constant** for a **constant sampling time Ts** (regardless of K)
- **We got more integral values of K for which our Closed Loop system is stable (for T_s = 0.001s the system gets stable for k =5 also).**

➢ **<u>For higher sampling time than Nominal Ts:</u>**

3) For Ts = 0.05 s

Gain(K)	**Gain Margin**	**Phase margin**	**PCO**	**GCO**	**Stability**
1	0.579	inf	0	NaN	**Unstable**
2	-5.44	15.7	0	2.49	**Stable**
3	2.4	7.72	4.67	3.75	**Stable**
4	-0.094	-0.325	4.67	4.71	**Unstable**
5	-2.03	-7.18	4.67	5.48	**Unstable**
6	-3.62	-12.9	4.67	6.14	**Unstable**

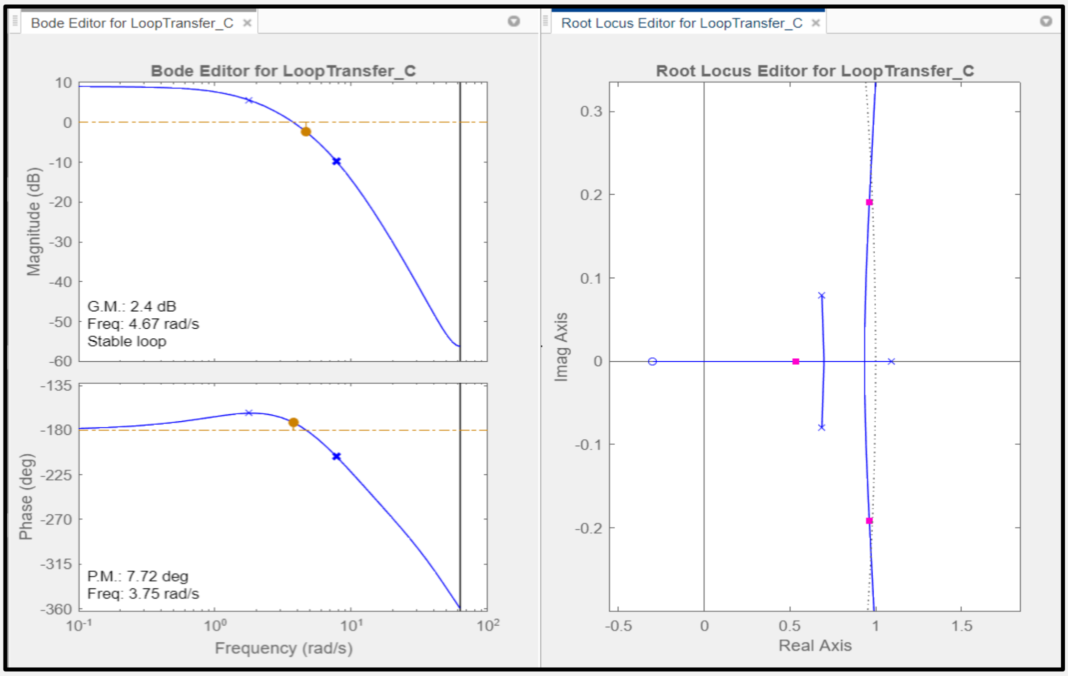


Fig.-6: Depicting the Bode plot and Root Locus for K=3, Ts =0.05s

4) For Ts = 0.1 s

Gain(K)	Gain Margin	Phase margin	PCO	GCO	Stability
1	0.579	inf	0	NaN	**Unstable**
2	4.22	12.2	4	2.49	**Stable**
3	0.7	2.47	4	3.74	**Stable**
4	-1.8	-6.84	4	4.68	**Unstable**
5	-3.74	-14.7	4	5.45	**Unstable**
6	-5.32	-21.3	4	6.09	**Unstable**

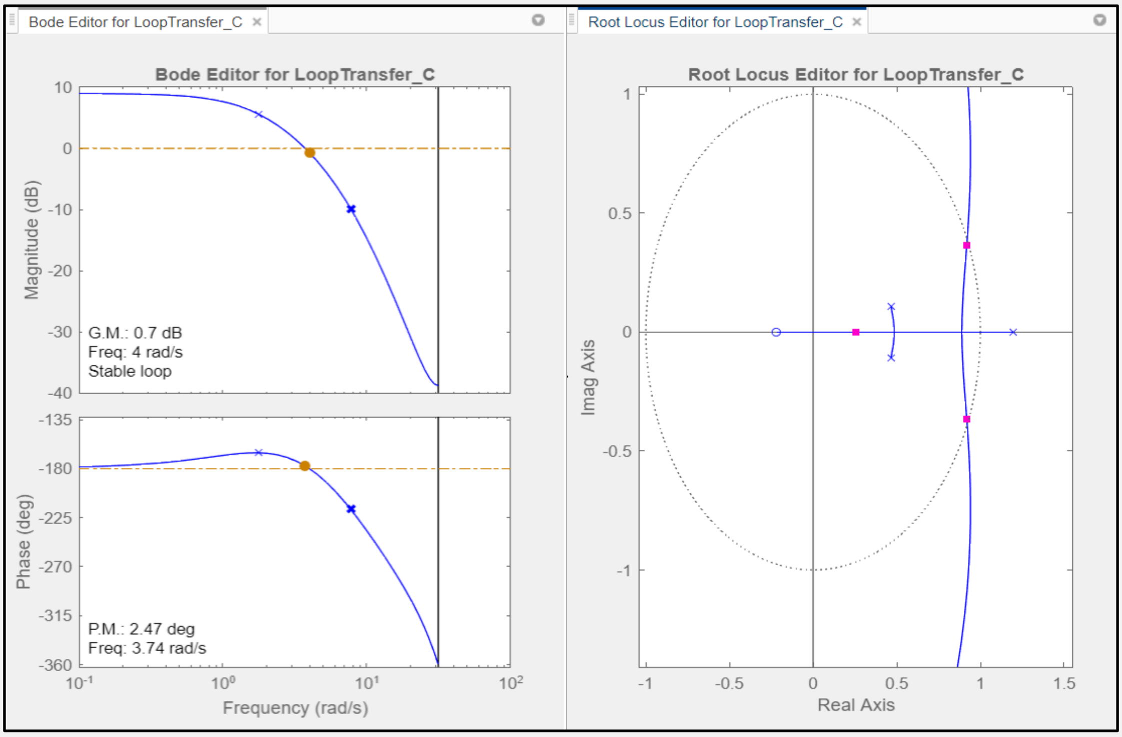


Fig.-7: Depicting the Bode plot and Root Locus for K=3, Ts =0.1s

5) For Ts = 0.5 s

Gain(K)	Gain Margin	Phase margin	PCO	GCO	Stability
1	0.579	inf	0	NaN	**Unstable**
2	-3.82	-13.4	1.07	2.26	**Unstable**
3	-7.34	-31.1	1.07	3.21	**Unstable**
4	-9.84	-44.2	1.07	3.81	**Unstable**
5	-11.8	-54.0	1.07	4.23	**Unstable**
6	-13.4	-61.7	1.07	4.53	**Unstable**

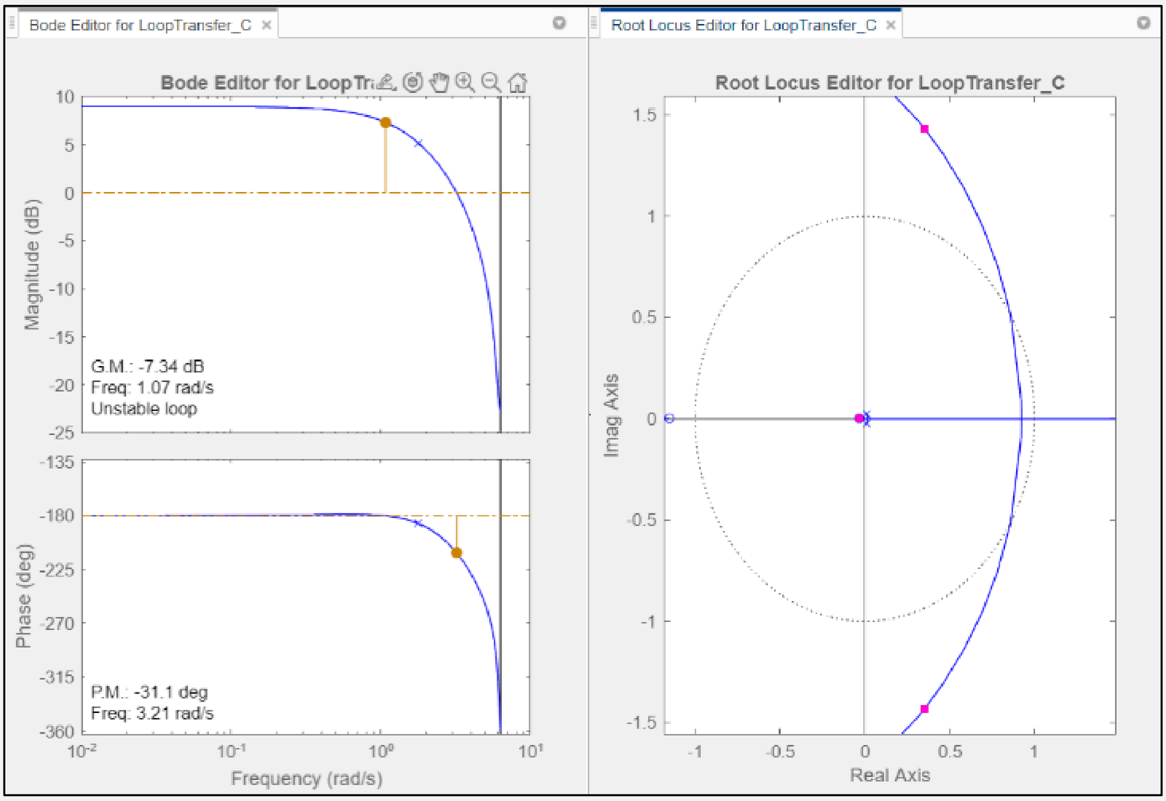


Fig.-8: Depicting the Bode plot and Root Locus for K=3, Ts =0.5s

The above observations are for **higher sampling time than nominal value**,

As the value of **Sampling Time Ts increases**,

- Value of **PCO decreases** but remains **constant for a constant Ts (regardless of K)**
- **The range of K, for which the system is stable, decreases (for Ts = 0.5s the system gets unstable for all positive integral value of K).**
- Value of **Gain Margin decreases** for a constant K
- Value of **Phase Margin** also **decreases** for a constant K (system moves towards instability)

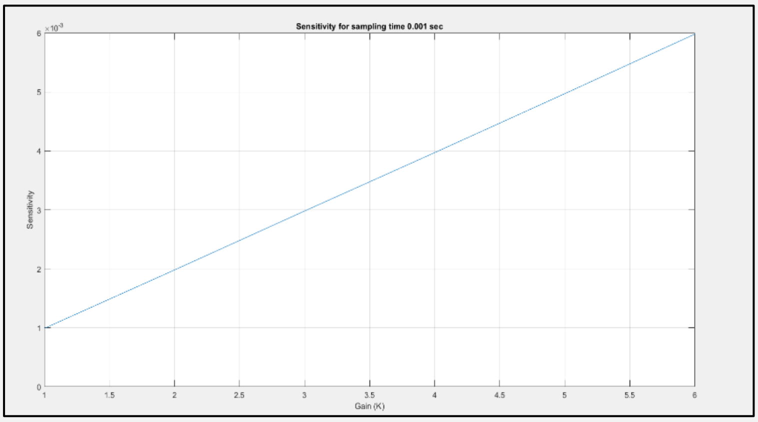
Sensitivity Analysis:

We calculated the Gol Transfer function for the corresponding sampling time using the sisotool in z domain. The corresponding sensitivity expression was derived. For a specific sampling time the sensitivity of the system was analysed by varying the Feedback Gain K.

For sampling time of 0.001s:

$$S_K{}^G = \frac{dG}{G} \text{ X } \frac{K}{dK} = \frac{-K(10^{-5})(0.06273z^2-0.1658z+0.113)}{K(10^{-5})(0.06273z^2-0.1658z+0.113)+(z^3-2.987z^2+2.974z-0.9871)}$$

```
Fixed Block
Name: G
Sample Time: 0.001
Value:
  6.273e-07 z^2 - 1.658e-06 z + 1.13e-06
  --------------------------------------
    z^3 - 2.987 z^2 + 2.974 z - 0.9871
```



Gol obtained for Ts=0.001s

Sensitivity vs Gain for Ts=0.001s

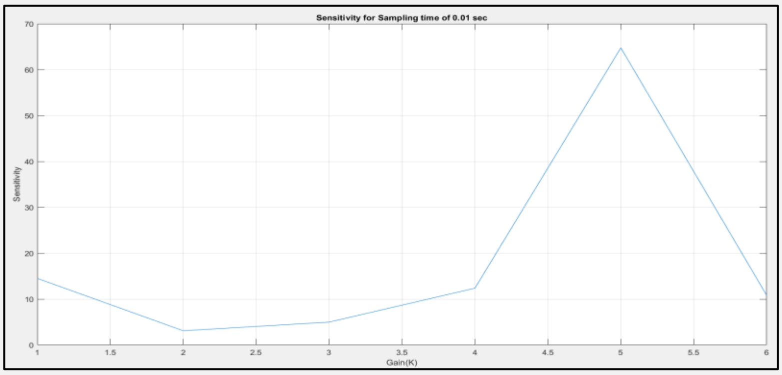
Gain (K)					
1	2	3	4	5	6
0.00099	0.00198	0.00298	0.00397	0.00497	0.00598

- The sensitivity was calculated over the main band of frequency range (calculated Nyquist criteria) i.e., **[0: pi/Ts], (pi/Ts=3141.5).**
- For sampling time of 0.001s, the system is quite robust as the values of sensitivity are quite low.
- Though the sensitivity increases with increase in feedback gain.

For sampling time 0.01s (nominal sampling time):

$$S_K{}^G = \frac{dG}{G} \text{ X } \frac{K}{dK} = \frac{-K(10^{-5})(4.711z+4.644)}{K(10^{-5})(4.711z+4.644)+(z^3-2.875z^2+2.753z-0.8781)}$$

```
Fixed Block
Name: G
Sample Time: 0.01
Value:
        4.711e-05 z + 4.644e-05
  ----------------------------------
  z^3 - 2.875 z^2 + 2.753 z - 0.8781
```



Gol obtained for Ts=0.01s

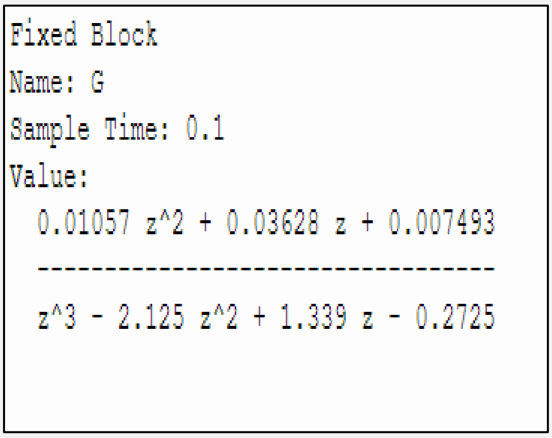
Sensitivity vs Gain for Ts=0.01s

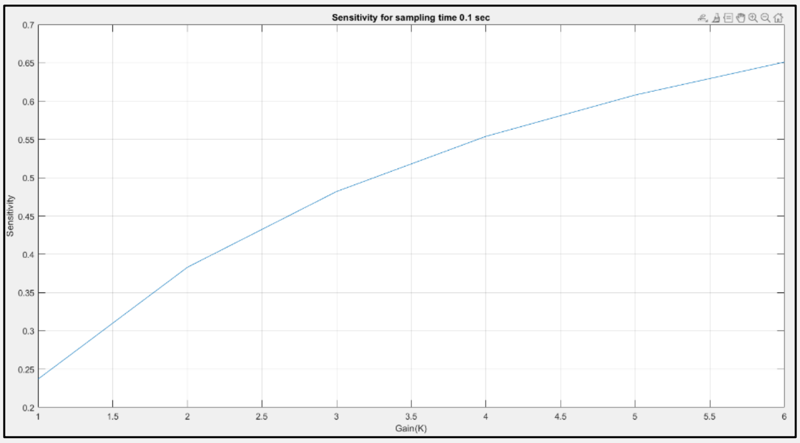
- The sensitivity analysis for the nominal system show that the system is frail for the values of gain which otherwise shows instability, though for the considered gain (stable system) the system shows nominal sensitivity.
- The sensitivity was calculated over the different **frequency band [0: pi/Ts], (pi/Ts=314.1).**

Gain (K)					
1	2	3	4	5	6
14.50	3.09	4.97	12.37	64.76	10.80

For sampling time of 0.1s:

$$S_K{}^G = \frac{dG}{G} \text{ x } \frac{K}{dK} = \frac{-K(0.01057z^2+0.03628z+0.007493)}{K(0.01057z^2+0.03628z+0.007493)+(z^3-2.125z^2+1.339z-0.2725)}$$


Fixed Block
Name: G
Sample Time: 0.1
Value:
0.01057 z^2 + 0.03628 z + 0.007493
----
z^3 - 2.125 z^2 + 1.339 z - 0.2725



Gol obtained for Ts=0.1s

Sensitivity vs Gain for Ts=0.1s

- The system for sampling time of 0.1s, shows less sensitivity than that corresponding to nominal sampling time.
- Here again the sensitivity increased with increase in feedback gain k.
- The band of frequency for sampling time of 0.1 here is: **[0: 31.41], (pi/Ts=31.41)**.

Gain (K)					
1	2	3	4	5	6
0.237	0.383	0.482	0.554	0.608	0.651

Conclusions:

- We analysed the system and figured out the integral values of feedback gain for various sampling times which provide stability to the system.
- We studied the variations in Phase Margins and Gain Margins for various feedback gains and various sampling times to conclude that the sampling time does affect the stability/instability of the system.
- Sensitivity of the system was analysed with respect to sampling time and gain factor K.

Scripts:

```
clear;
clc;
z=tf('z',1);
num=[4.711 4.644];
den=[1 -2.875 2.753 -0.8781];
Gol=(1e-5)*tf(num, den, 0.01);
k=1;
Gcl=feedback(Gol, k);
%step(Gcl);
%damp(Gcl);
%rlocus(Gol);
sisotool(Gol);
```

Script 1: For analysing Phase Margins and Gain Margins

```
clear;
clc;
z=tf('z', 0.01);
k=6;
Sa=((-k)*(1e-5)*(4.711*z + 4.644))/((k*(1e-5)*(4.711*z + 4.644)) ...
    +(z*z*z-2.875*z*z+2.753*z-0.8781));
%Sa=((-k)*(0.010572*z*z-0.03628*z+0.007493))/((k*(0.010572*z*z-0.03628*z+
% 0.007493))+(z*z*z-2.125*z*z+1.339*z-0.2725));
fband=[0,pi*100];
[gpeak, fpeak]=getPeakGain(Sa,0.1);
```

Script 2: For analysing the sensitivity of system for different sampling times

```
clc
clear
%feedback
gm=[0.579 7.65 4.12 1.63 -0.312 -1.9];
k=[1 2 3 4 5 6];
plot(k, gm);
grid on;
```

Script 3: For plotting the variations

Thankyou!