EE 208

Control Engineering Lab

Experiment-10: Dynamic studies of a nonlinear mechanical system on Simulink

Group Number: 20

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Objective:

To perform a dynamic study of the given crane trolley system using a detailed nonlinear state space system simulation in four state variables.

Given:

We have been provided with differential equations that represent a simplified model of an overhead crane. The equations are:

$$[m_L + m_C].\ddot{x}_1(t) + m_L.l.[\ddot{x}_3(t).\cos x_3(t) - \dot{x}_3^2(t).\sin x_3(t)] = u(t)$$

$$m_L.[\ddot{x}_1(t).\cos x_3(t) + l.\ \ddot{x}_3(t)] = -m_L g.\sin x_3(t)$$

Here, constants are:

m_C: Mass of trolley; 10 kg.

m_L: Mass of hook and load; the hook is again 10 kg, but the load can be zero to several hundred kg's,

but constant for a particular crane operation.

l: Rope length; 1m or higher, but constant for a particular crane operation.

g: Acceleration due to gravity, 9.8 ms^{-2}

Variables are:

- Input:
- u: Force in Newtons, applied to the trolley.
- Output:
- y: Position of load in metres, $y(t) = x_1(t) + l.\sin x_3(t)$
- States:
- x_1: Position of trolley in metres.
- x_2: Speed of trolley in m/s.
- x_3: Rope angle in rads.
- x_4: Angular speed of rope in rad/s.

Simulink Model:

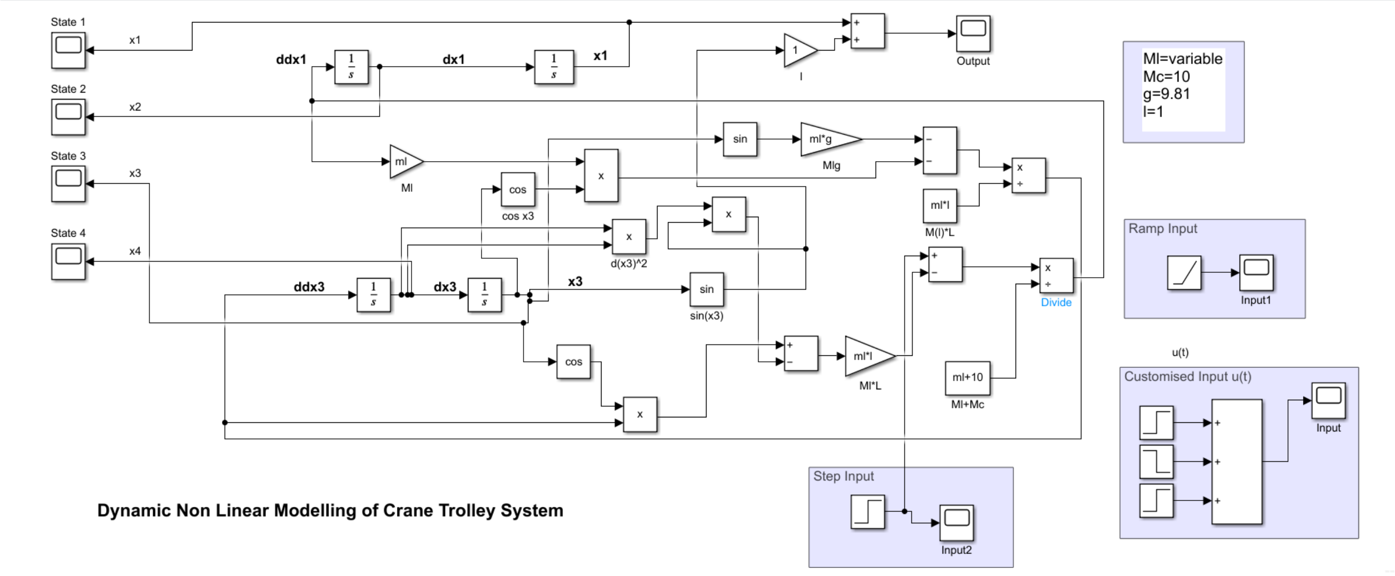


Fig 1- Simulink model for the given non-linear system

The Simulink model was rigged to simulate the given differential equations. The differential equations were rewritten to equate the highest order term to the lower order terms in a manner that they represent the highest order term.

$$\mathrm{ddx1} = \frac{\mathrm{u(t)} - (\mathrm{l} * \mathrm{ml} * (\mathrm{ddx3(t)} \cos(\mathrm{x3(t)}) - \sin(\mathrm{x3(t)}) * (\mathrm{dx3})^2)}{(\mathrm{mc} + \mathrm{ml})}$$

$$\mathrm{ddx3} = \frac{ml * (ddx1 * \cos(x3(t)) - g * \sin(x3(t))))}{(l * ml)}$$

Integral blocks (1/s) were used instead of differentiators to get the lower order terms. The specific state output (x1, x2(or dx1), x3, x4(or dx3)) were analysed via scope. The variables such as load mass was declared as workspace variable to have multiple explicit inputs [1000, 3250, 5500, 7750, 10000] in kgs.

Observations:

There are four states and the two states can be represented in form of derivative of other two states. It could be physically realised that the state x2 is the derivative of state x1 as speed of the trolley is the rate of change of the position.
It could also be observed from the graphs that state four x4 (angular speed) is a derivative of state x3 which is the rope angle.

Following are the response of states and output with respect to various input provided (mass of load (M_L)):

➢ ***Step input***

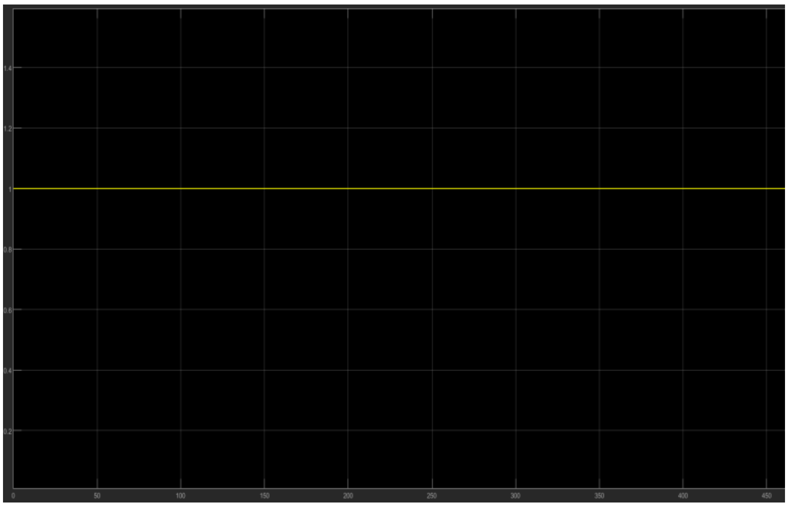


Fig 1.1 – step input

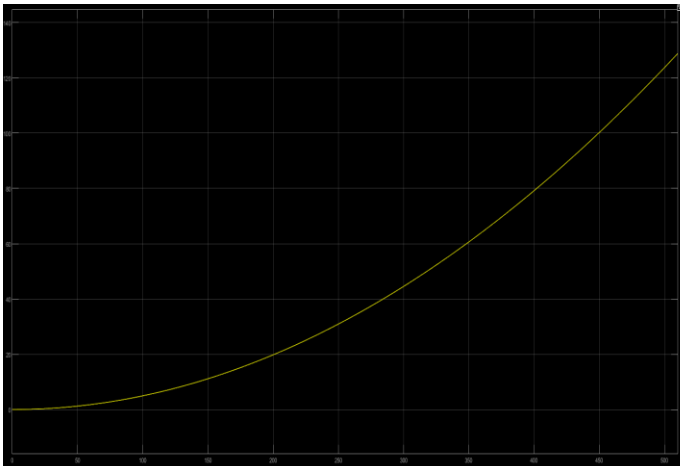


Fig 1.2 – Response of x_1

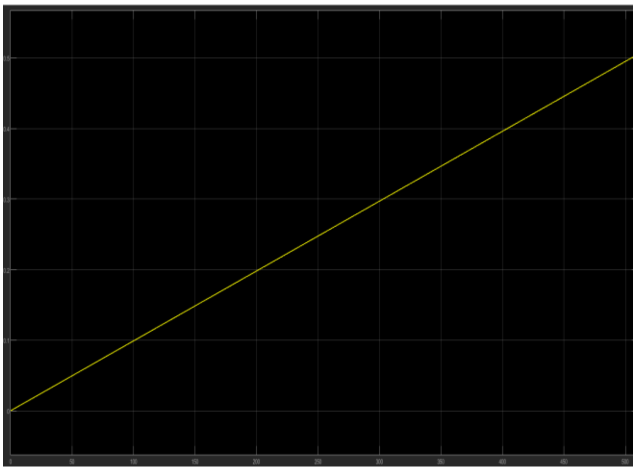


Fig 1.3 – Response of x_2

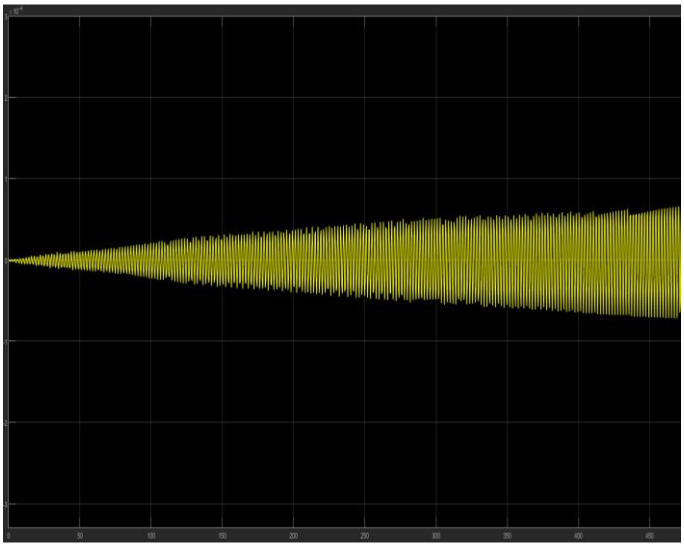


Fig 1.4 – Response of x_3

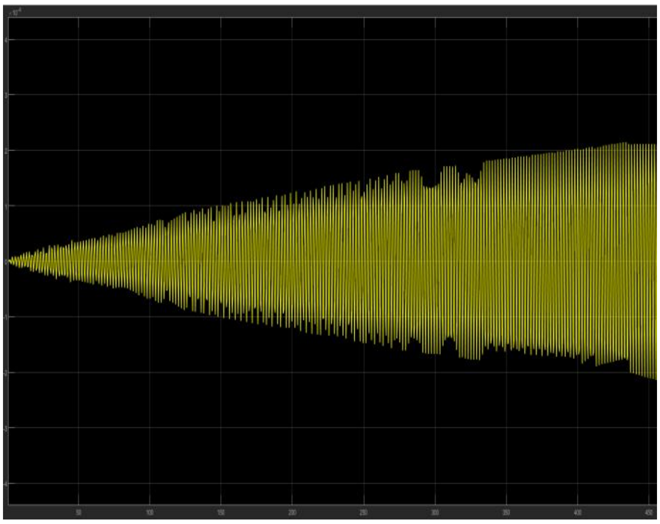


Fig 1.4 – Response of x_4

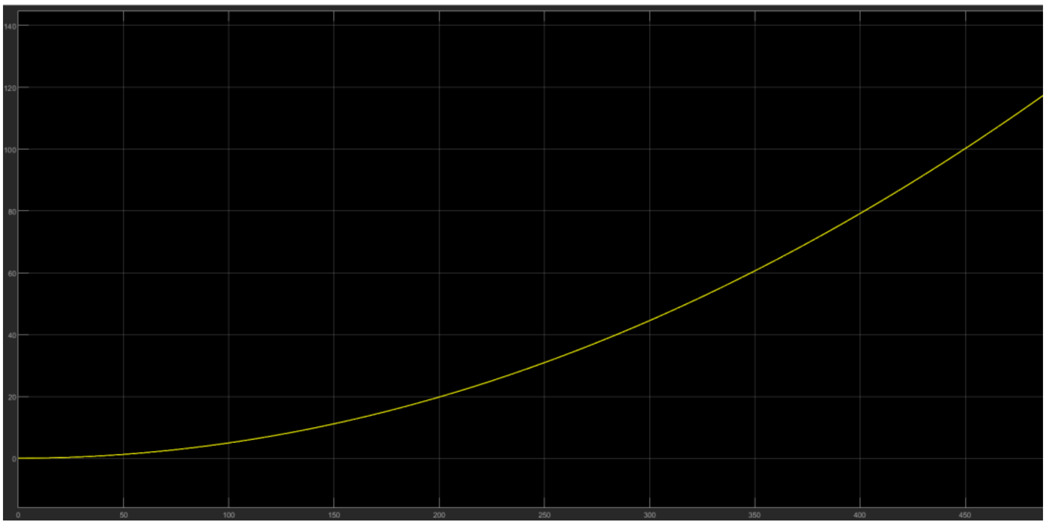


Fig 1.5 – Response of output (y)

For step input

- **x1** (positions of the trolley) **increase in biquadratic manner** with little or no oscillations.
- **x2** (speed of trolley) **increases in linear manner** with minute oscillations, this also confirms the biquadratic nature of x1 (as x2 is derivative of x1).
- **x3** (Rope angle) shows **increased oscillations** about x axis.
- **x4** (Angular speed of rope) also **increases oscillatory** at a faster rate as compared to x3.
- **y** (output) also **increases** non linearly with time.

Given non-linear system **shows unstable response** for step input.

➢ **Ramp *input***

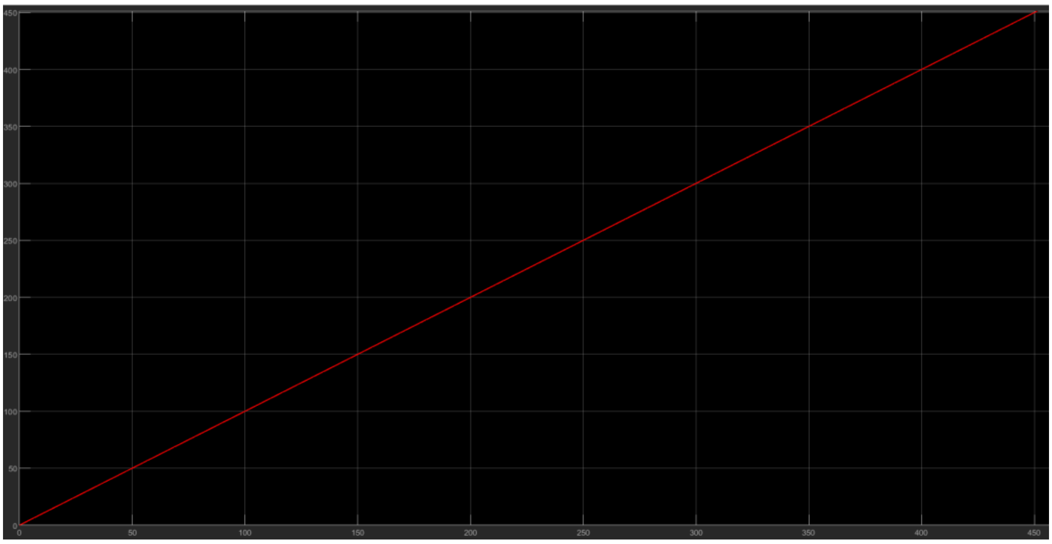


Fig 2.1 – Ramp input

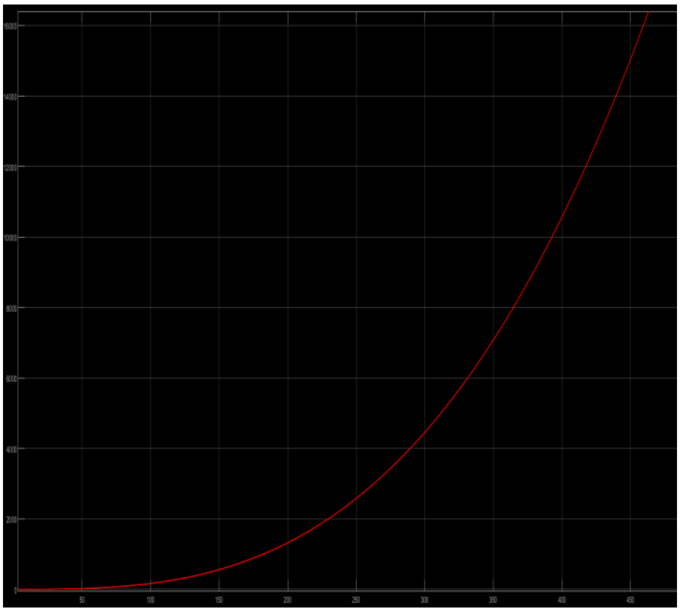


Fig 2.2 – Response of x_1

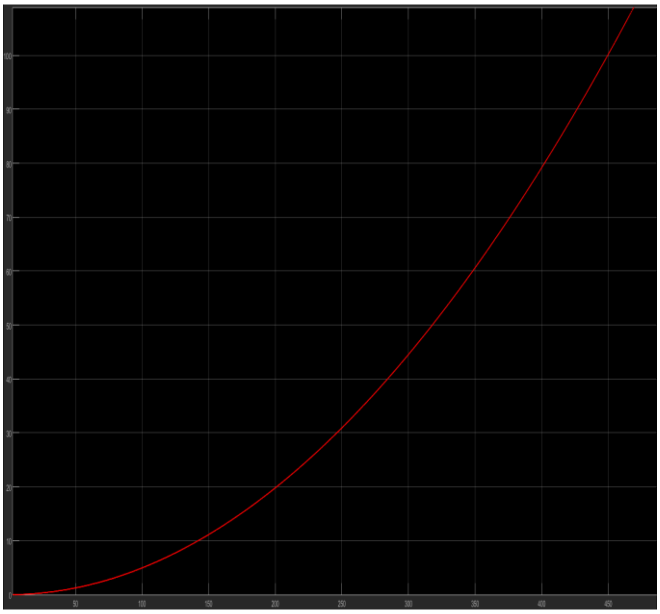


Fig 2.3 – Response of x_2

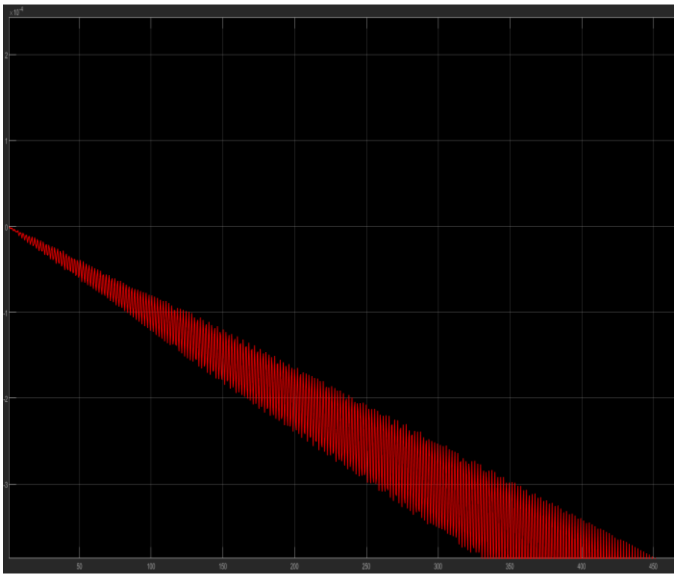


Fig 2.4 – Response of x_3

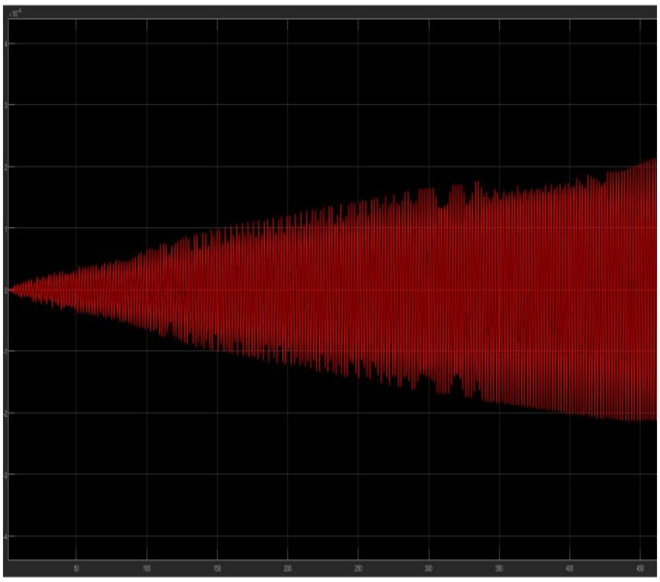


Fig 2.5 – Response of x_4

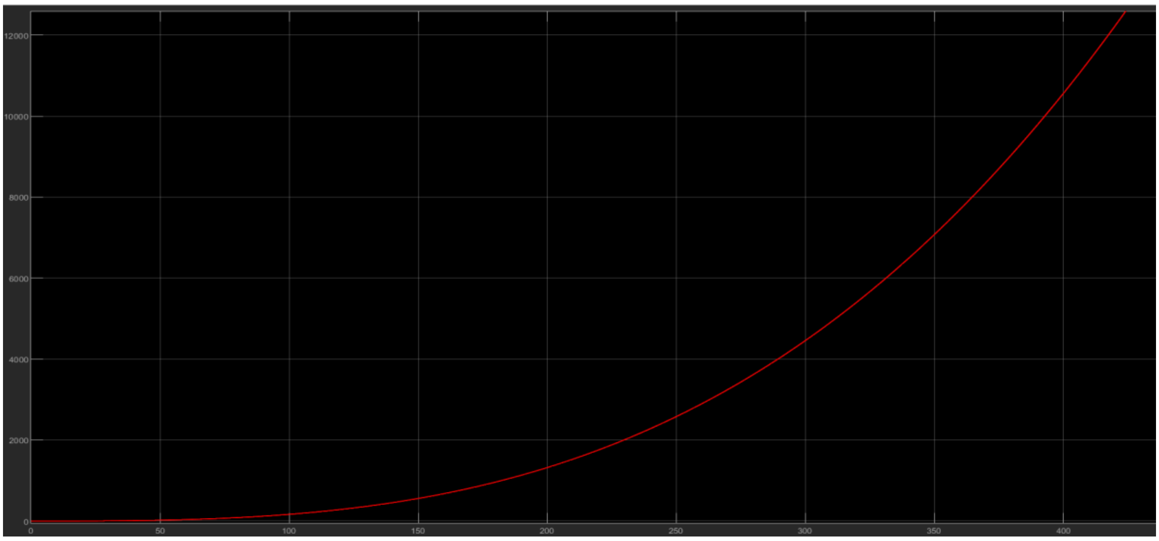


Fig 2.5 – Response of output

For ramp input:

- x1 (positions of the trolley) and x2 (speed of trolley) **increases at a fast rate as compared to step**.
- x3 (Rope angle) shows **increasing oscillatory response** but, in this case, **the mean angle, about which oscillations are taking place, increases (in negative y axis).**
- x4 (Angular speed of rope) also **increases oscillatory** but at a faster rate as compared to x3.
- y (output) also increases with **minute oscillations**.

Given non-linear system shows **unstable response for ramp input**.

- **Impulse Response**

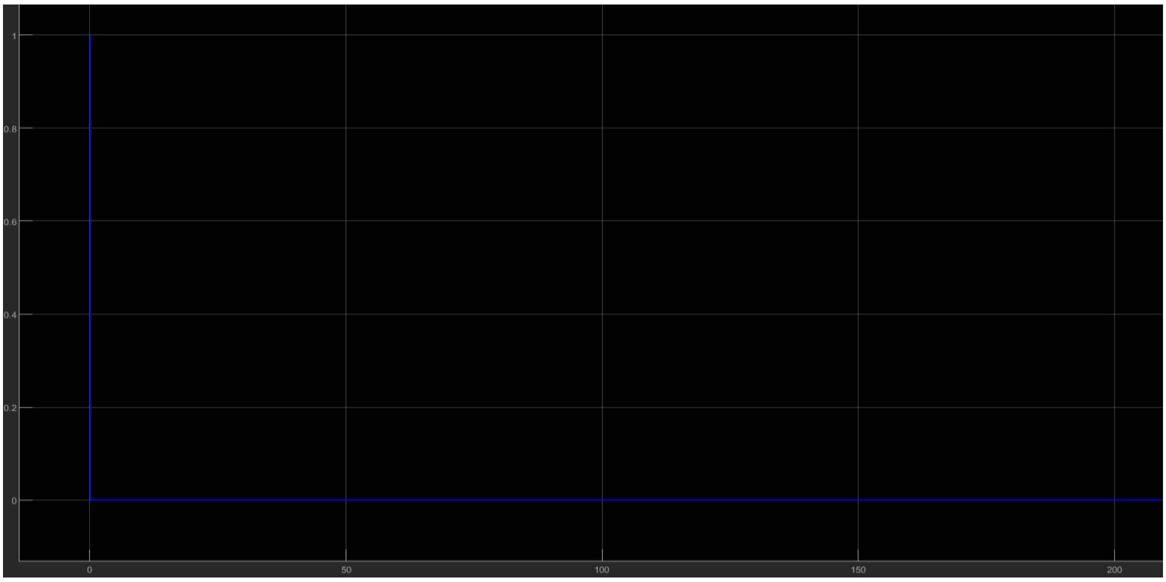


Fig 3.1 – impulse input

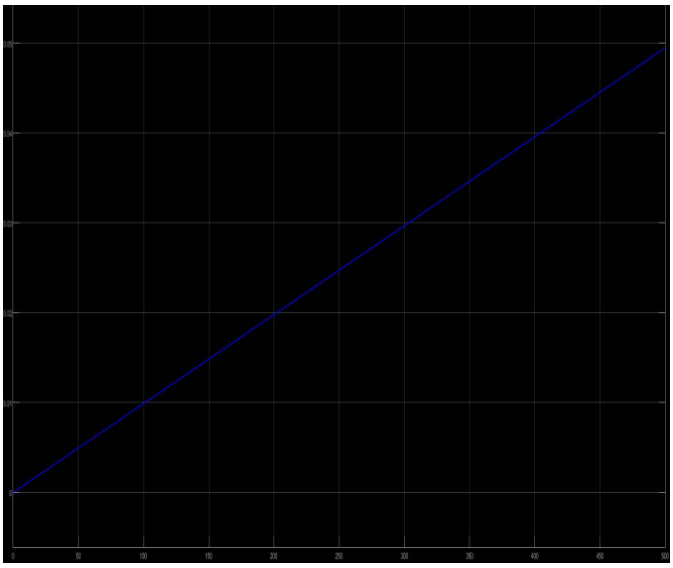


Fig 3.2 – Response of x_1

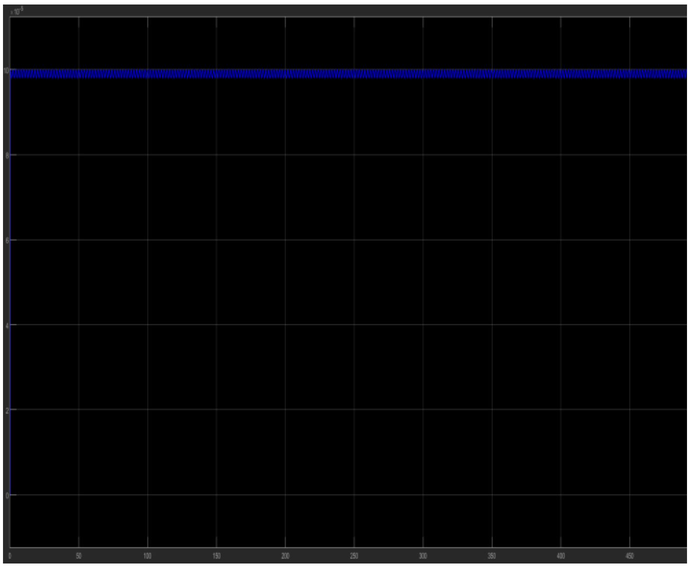


Fig 3.3 – Response of x_2

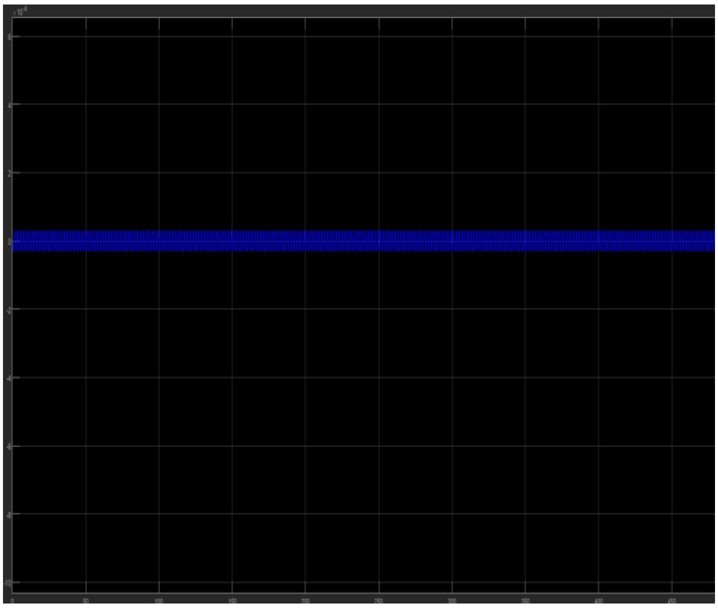


Fig 3.4 – Response of x_3

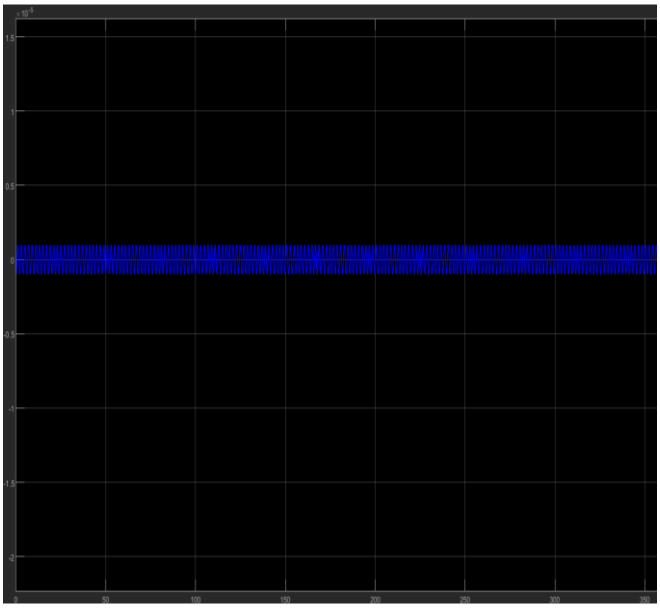


Fig 3.5 – Response of x_4

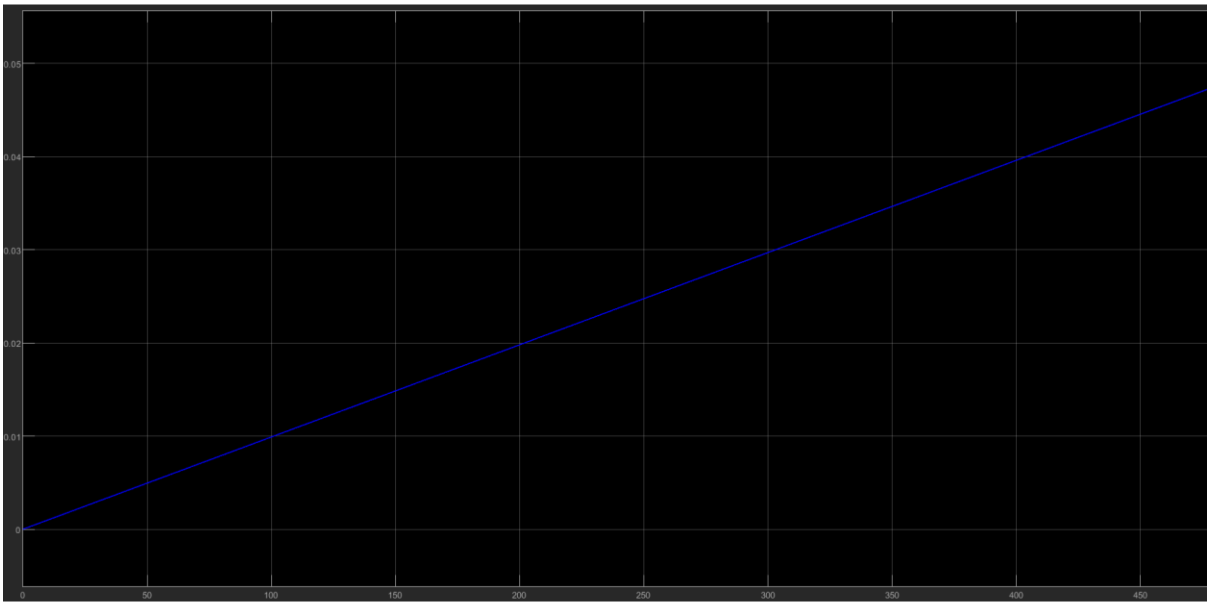


Fig 3.6 – Response of output (y)

For Impulse as input

- x1 (positions of the trolley) increase in **almost linear manner**.
- x2 (speed of trolley) remains **almost constant throughout** with very little or no oscillations (Fig 3.3 is zoomed up to 10^{-5})
- x3 (Rope angle) and x4 (Angular speed of rope) shows **non-increasing oscillatory** response, in which **magnitude of oscillations is greater in case of x4.**
- y (output) **increases linearly** with minute oscillations.

Given non-linear system shows **unstable response for ramp input.**

Analysis:

Response after Variation of load mass

Since the load mass is given of several thousand kgs, therefor we will observe the response by **varying mass from (1000 – 10,000 kg).**

1. For step input:

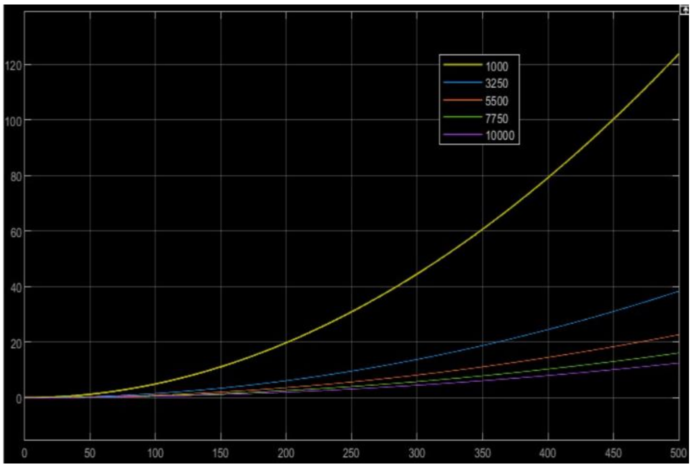


Fig 4.1 – Response of x_1

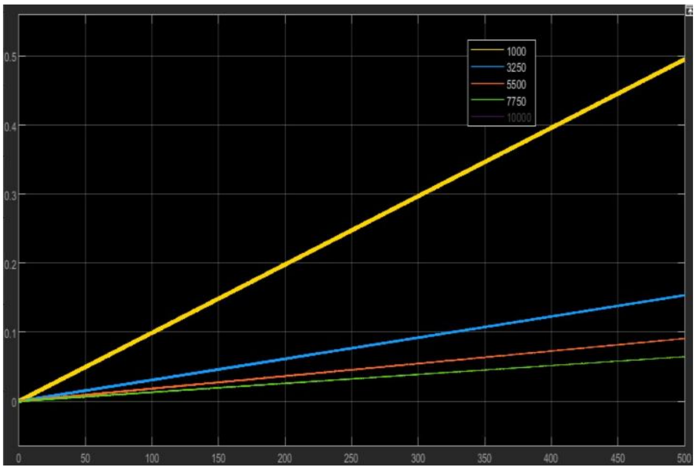


Fig 4.2 – Response of x_2

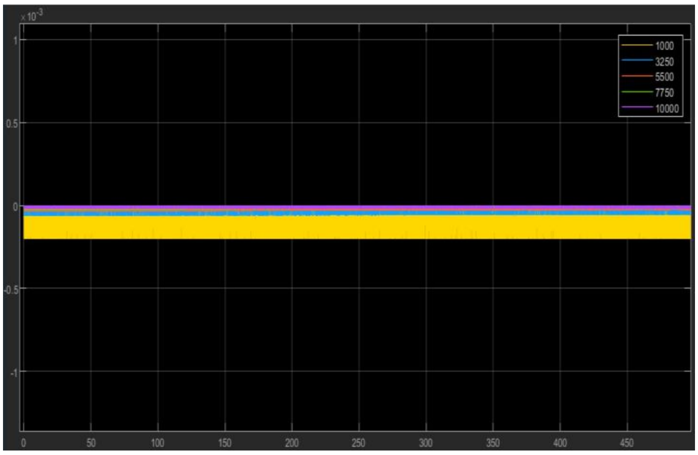


Fig 4.3 – Response of x_3

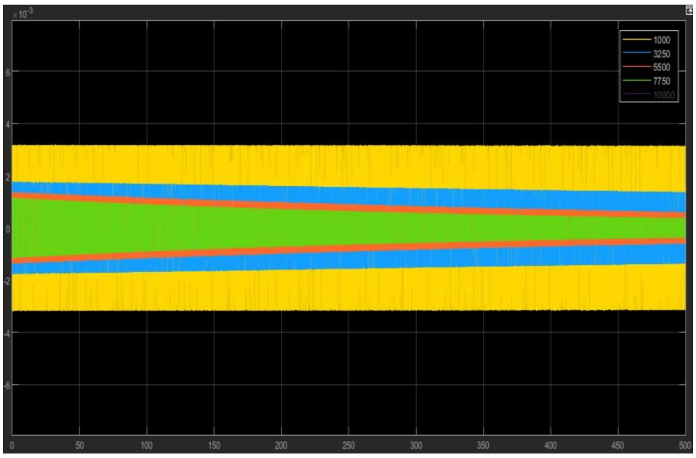


Fig 4.4 – Response of x_4

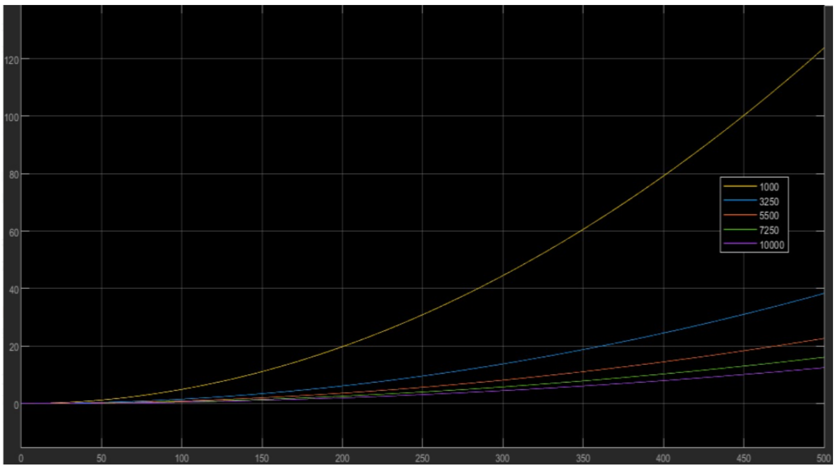


Fig 4.5 – Response of x

2. For ramp input:

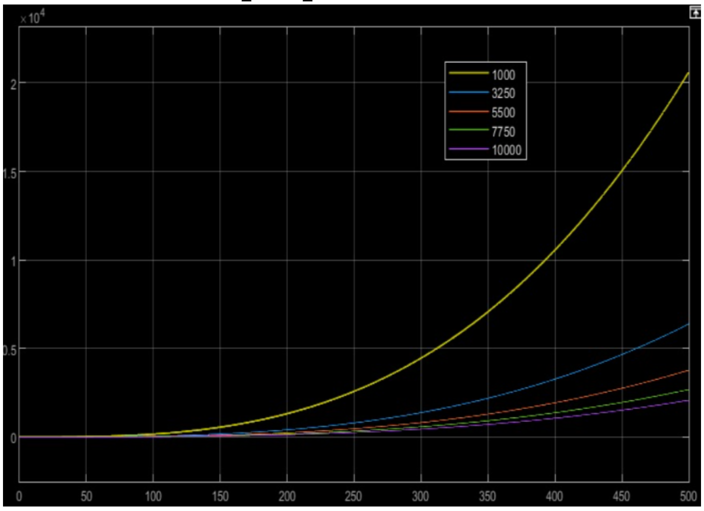


Fig 5.1 – Response of x_1

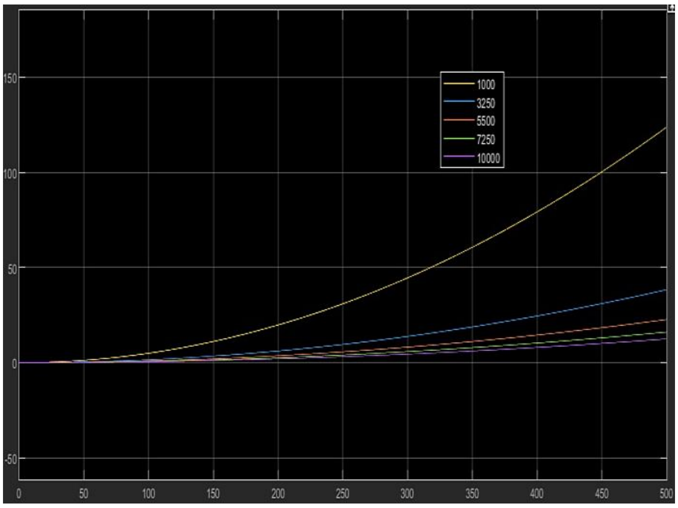


Fig 5.2 – Response of x_2

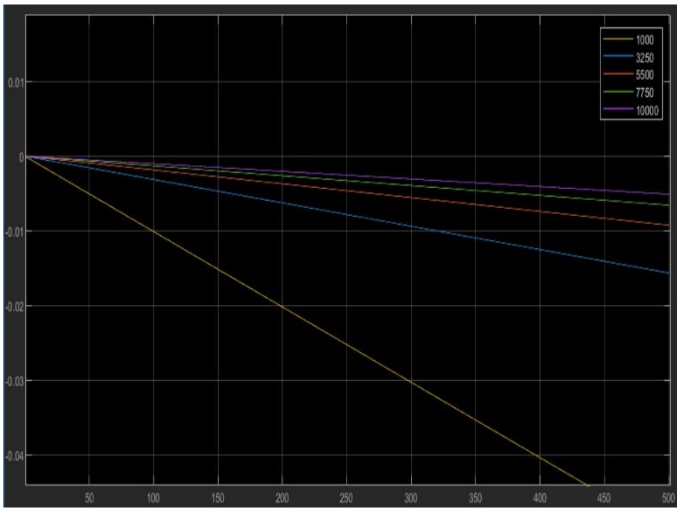


Fig 5.3 – Response of x_3

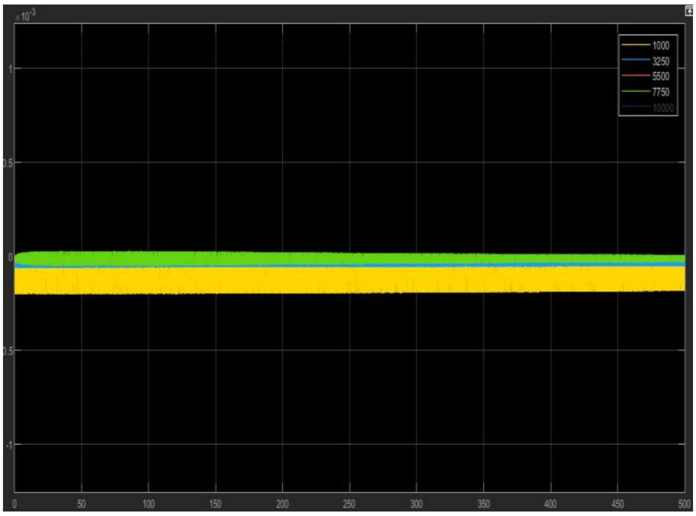


Fig 5.4 – Response of x_4

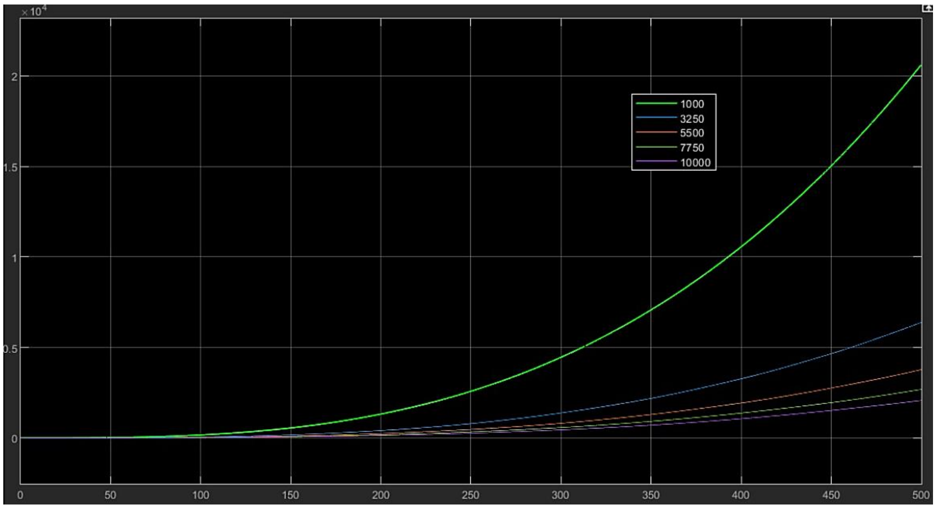


Fig 5.5 – Response of x_5

3. For impulse input:

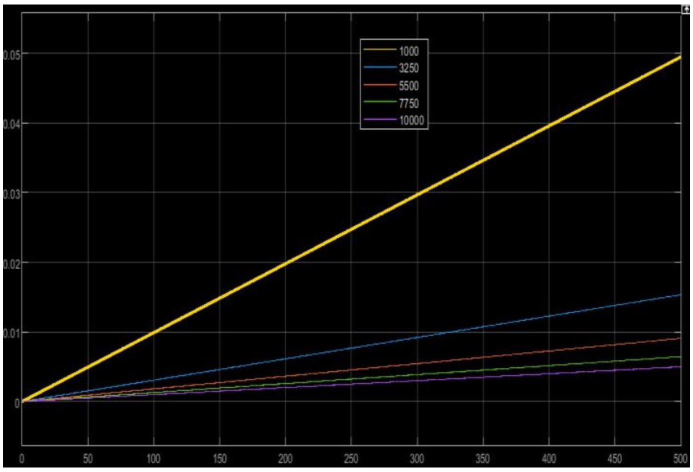


Fig 6.1 – Response of x_1

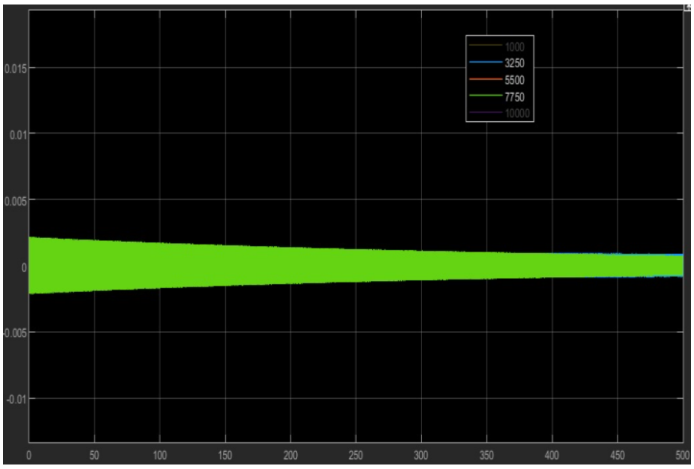


Fig 6.2 – Response of x_2

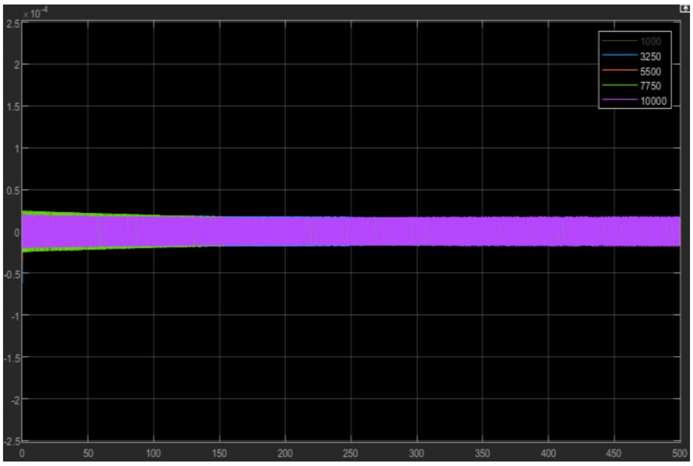


Fig 6.3 – Response of x_3

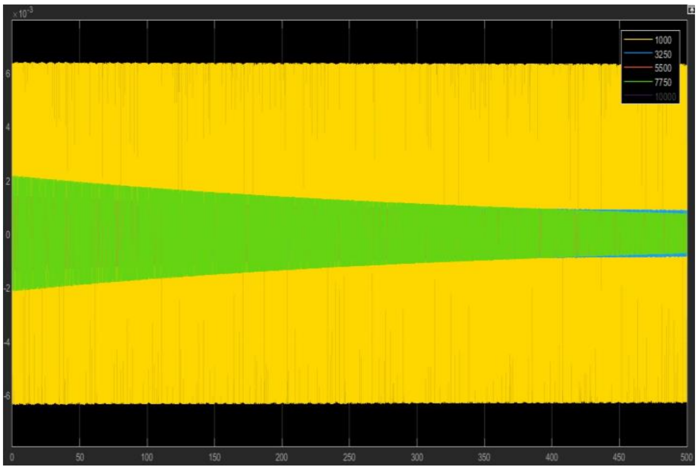


Fig 6.4 – Response of x_4

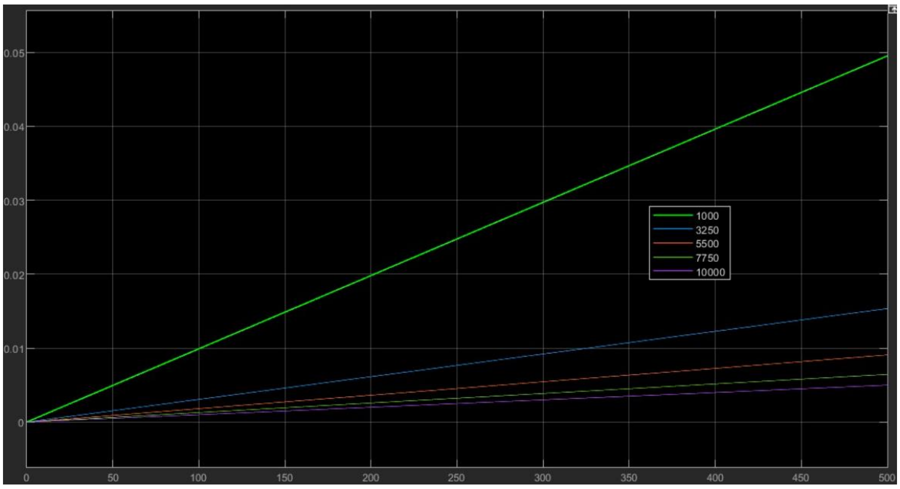


Fig 6.5 – Response of x_5

For all three of the above inputs, we conclude that with **increase in M_L:**

- **State x1 and x2 rises sharply for lower m_L** as compared to **higher values**.

- **Oscillations in x2 and x3 are higher for lower masses**, therefor have an **inverse relation**.
- Response of **output (y) is much similar to that of x1** as y(t) = x1(t) + l*sin x3(t) and l is one, **with more increasing nature for lower value of load mass**.

From all the observations, we conclude that

- The **output (y) goes on increasing, without settling to any steady input.**
- This is **probably due to absence of any damping factor in the system**, for any **positive input the system dynamics keeps increasing**.
- In order to stabilize the system, **we need to provide an input that must also work to break off the crane's motion after reaching the final position.**
- We chose a force **which has net zero area enclose with x- axis**. (means equal amount of work is done to counter the motion, as it was required to start it)

<u>Best Designed Input:</u>

Following observations are for **mL = 1000kg**

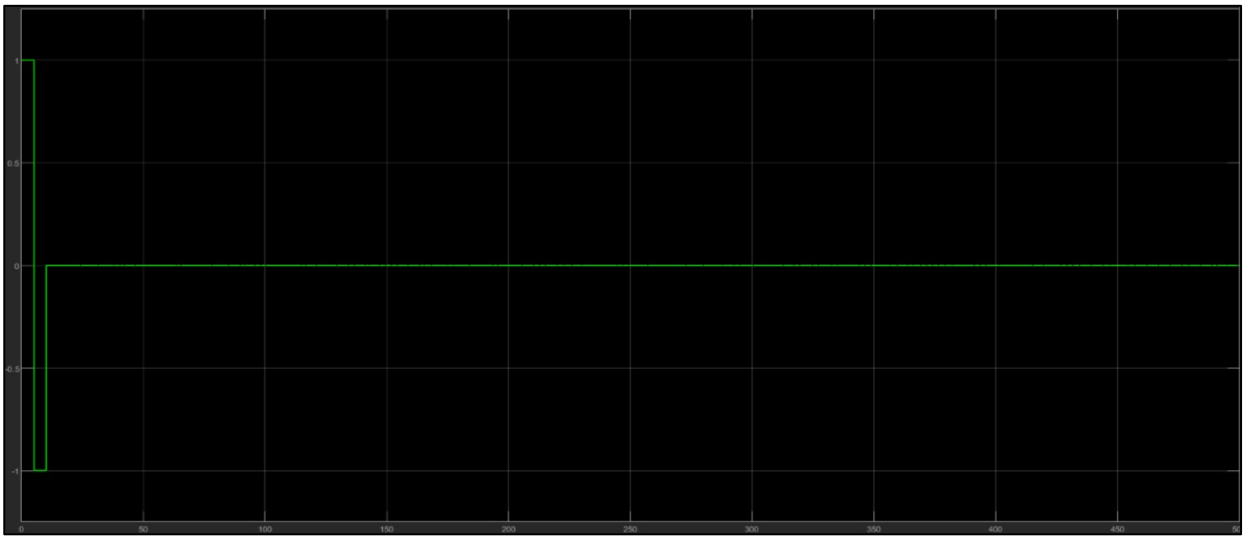


Fig 4.1 – Custom input: **u(t)-2*u(t-5) + u(t-10)**

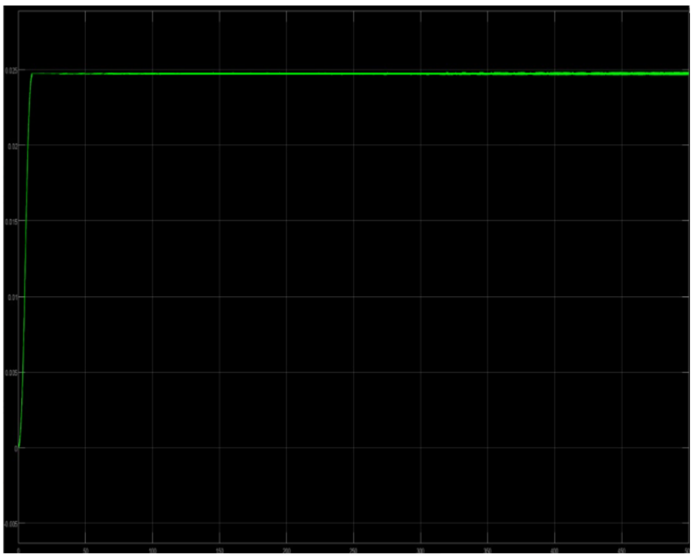


Fig 4.2 – Response of x_1

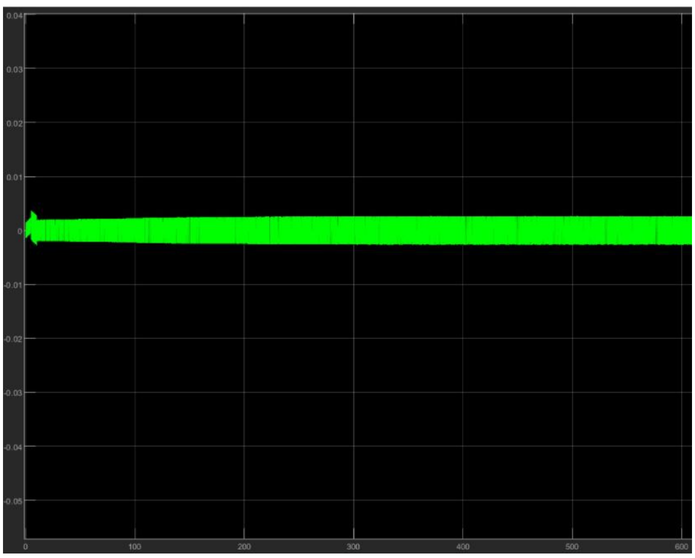


Fig 4.3 – Response of x_2

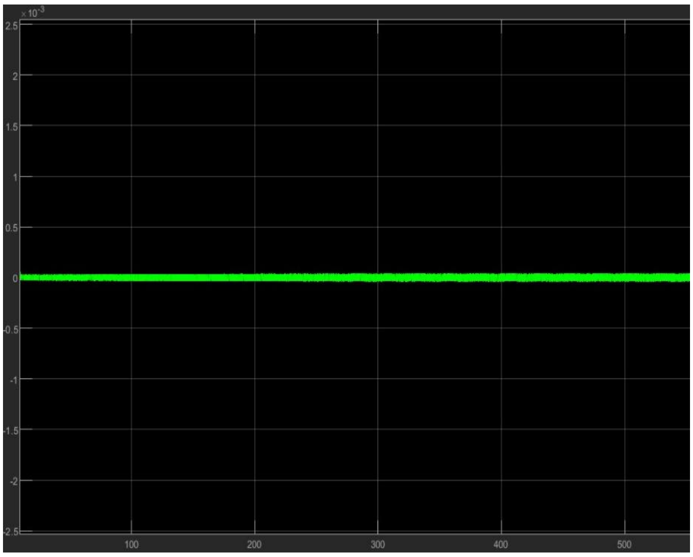


Fig 4.4 – Response of x_3

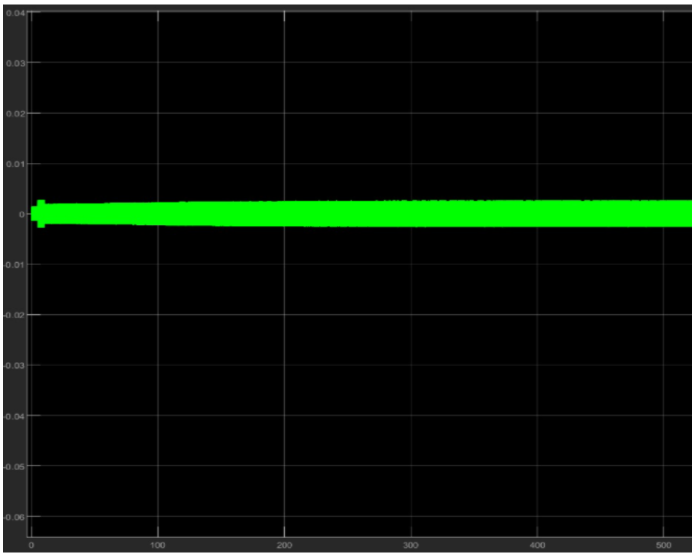


Fig 4.5 – Response of x_4

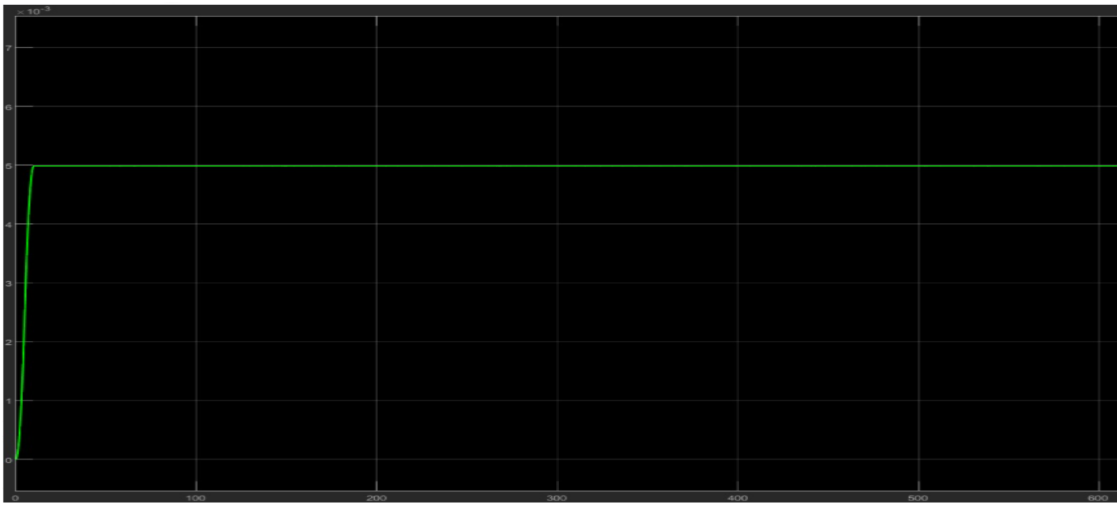


Fig 4.6 – Response of y (Output)

For the customized input,

- The designed input is of form **u(t) - 2*u(t-5) + u(t-10).**
- All the states x1, x2, x3, x4, now **settles to a steady value with sustained oscillations of microscopic magnitude.**
- **The equilibrium is observed since x2 and x4 converges to zero for an arbitrary fixed value of x1.**
- **Output y also achieves a steady value**, with sustained oscillations of **negligible magnitude**.

Comparison Table (at 450 seconds):

Input	x1	x2	x3	x4	y
Step u(t)	100	0.50	0.00005	0.00002	100
Ramp	16000	105	0.0004	0.0002	16000
Impulse	0.035	$4*10^{(-3)}$	$6*10^{(-4)}$	$2*10^{(-3)}$	0.05
Customised Input: **u(t)-2*u(t-5) + u(t-10)**	0.045	$5*10^{(-4)}$	$7*10^{(-5)}$	$2*10^{(-4)}$	0.045

ml=1000kgs

- ***Variation of our load mass for custom input***

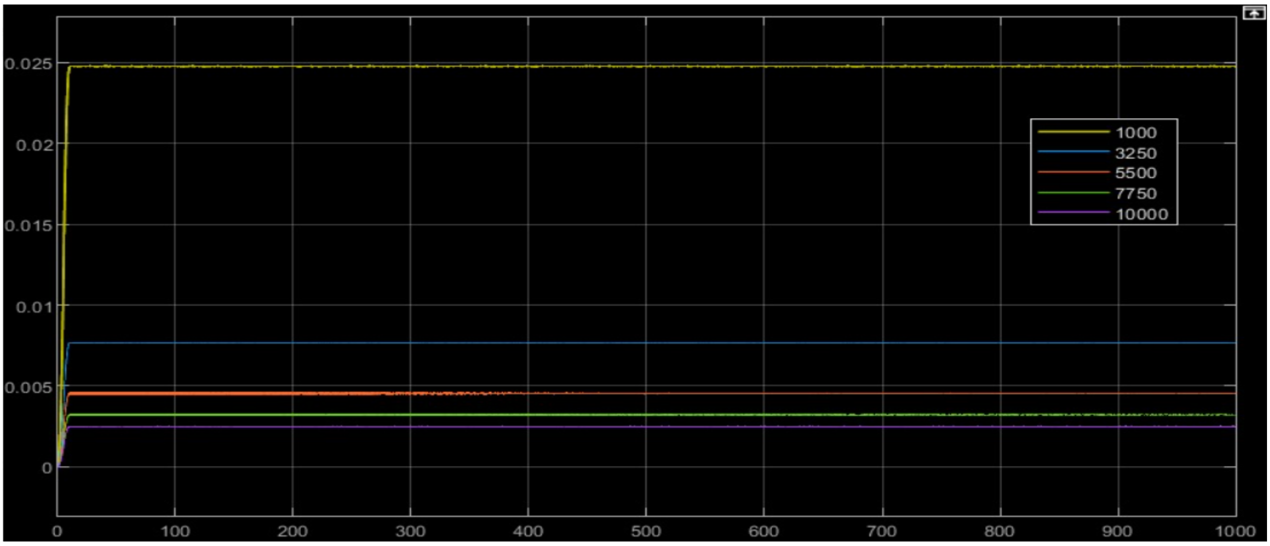


Fig 4.2 – Response of x_1

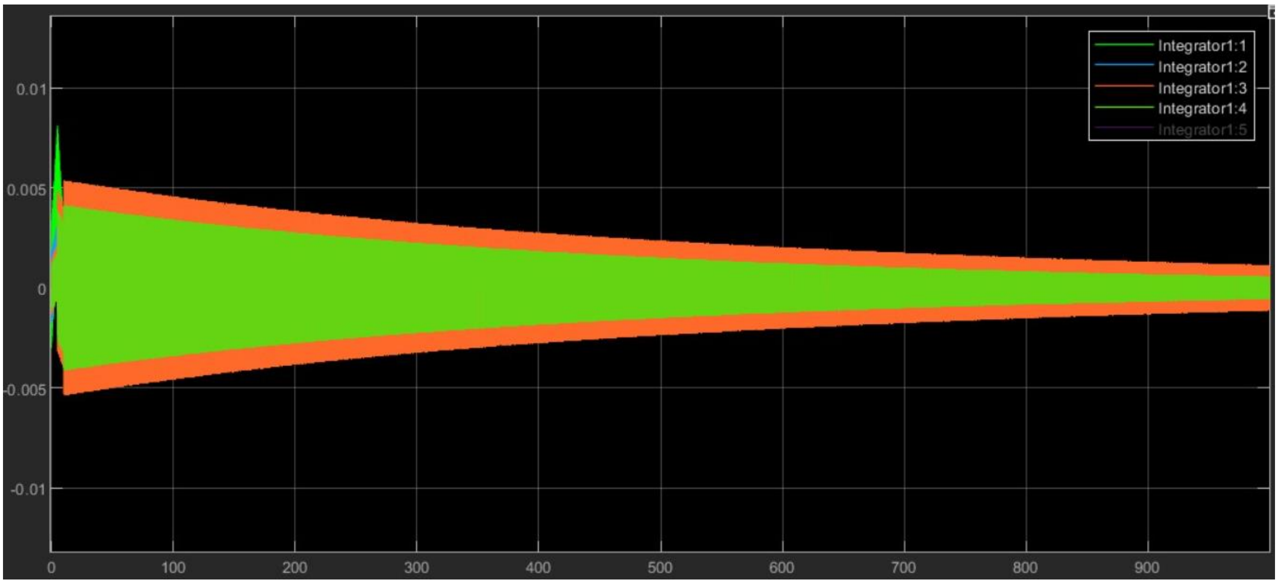


Fig 4.2 – Response of x_2

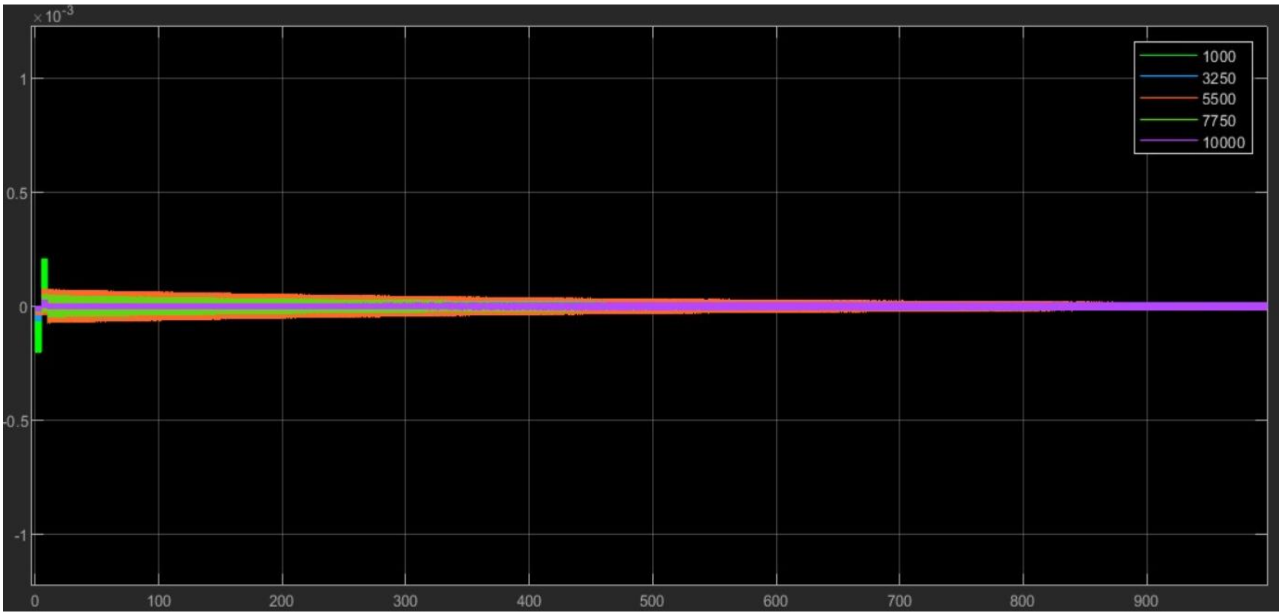


Fig 4.2 – Response of x_3

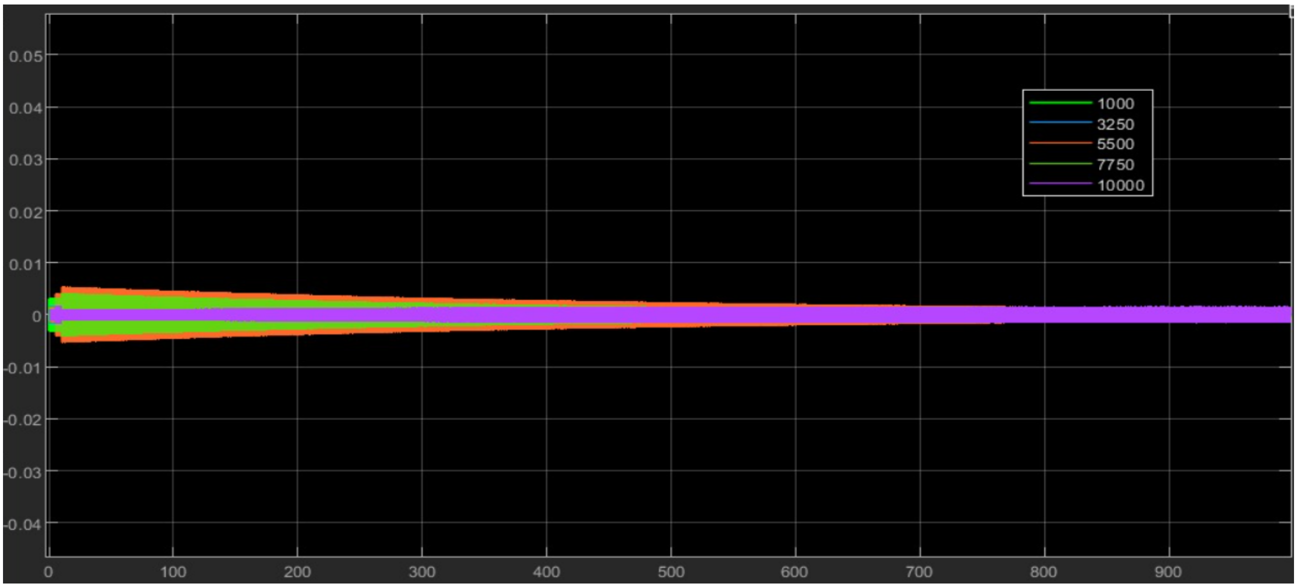


Fig 4.2 – Response of x_4

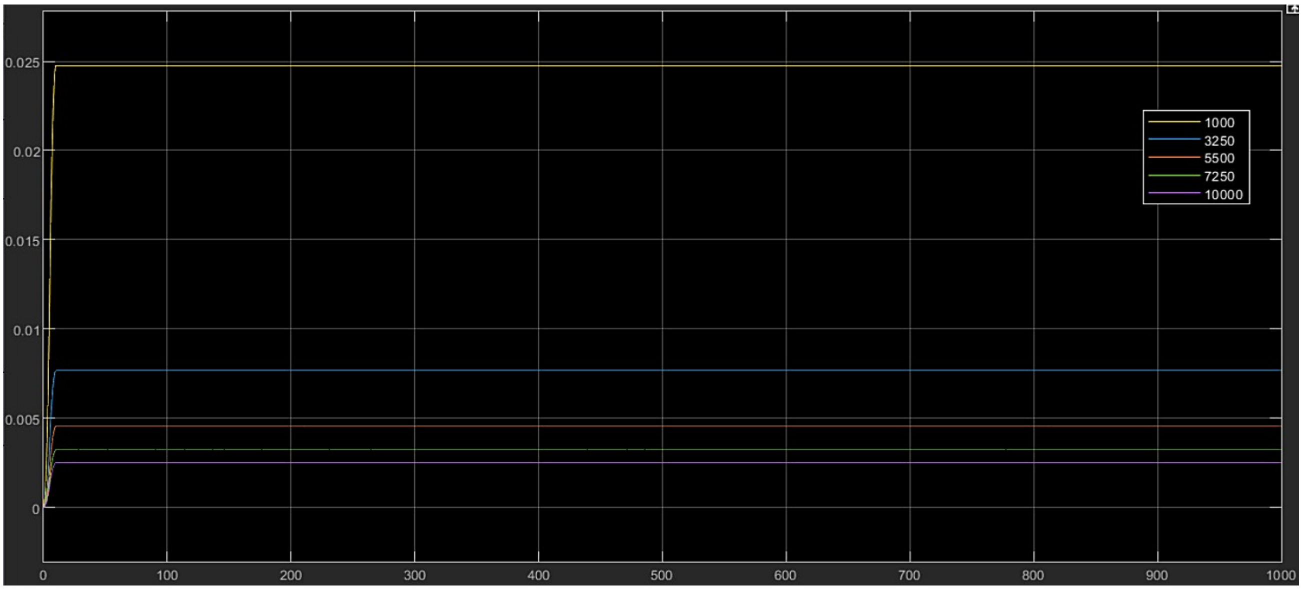


Fig 4.2 – Response of y

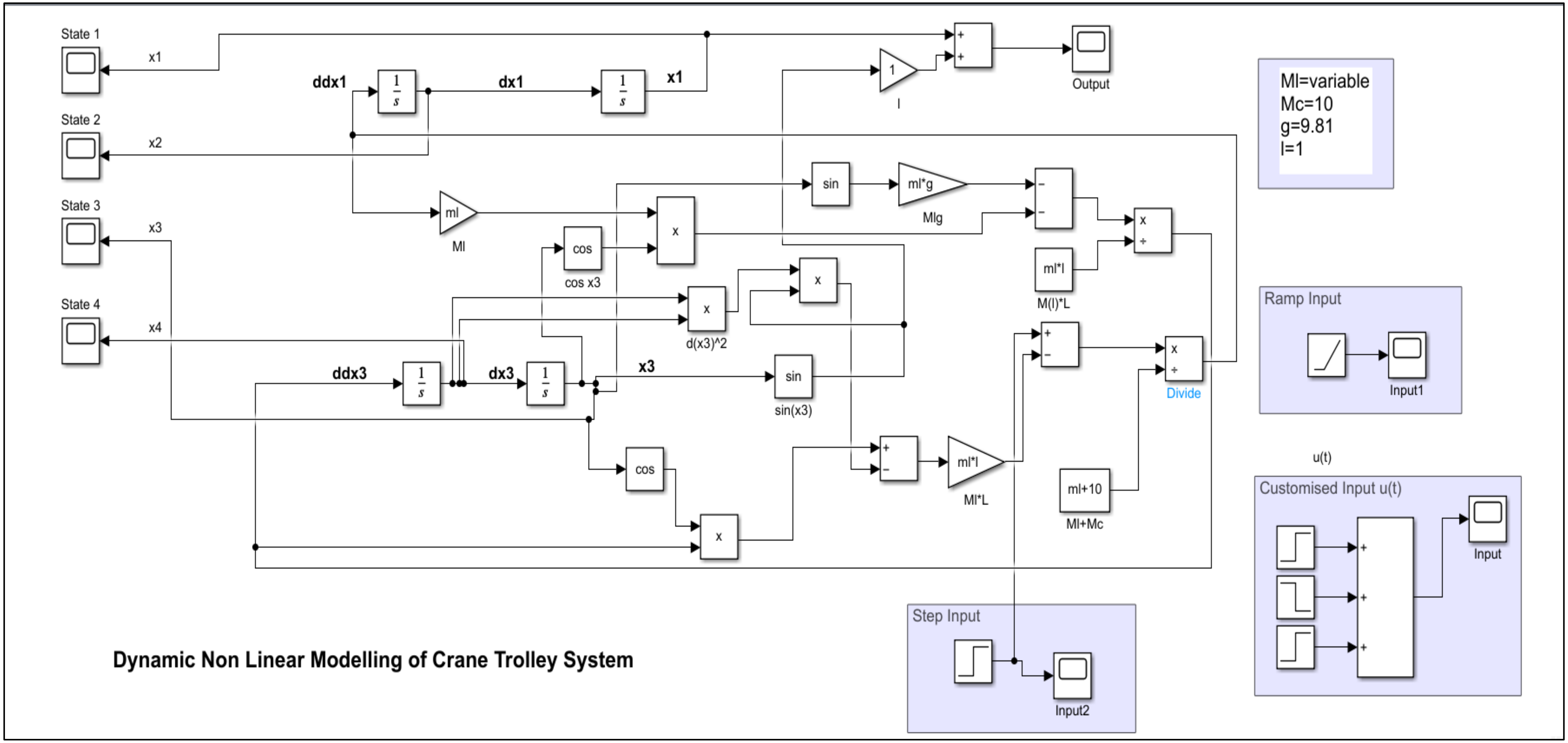
With **increase in Load Mass**:

- State x_1 and y **settles at a higher** value than before and **settling time also increases**.
- The response and x2, x3 and x4 **remains oscillatory** but settling time increase with increase in load mass
- Overall, the system shows **a stable response** for custom **input up to a certain value of load mass**.
- Values are in approximate correspondence to what derived from lab 8 x3=sin^{-1} $[-2u(t)mlg]$, u = ± $g/2\sqrt{(4mc2-3ml2+8ml\, mc)}$ at equilibrium.

Conclusion:

The given non-linear system was modelled in the Simulink. The system was analysed for all four states and the single output by applying various designed inputs (u(t)) under a defined sets of load mass. The best designed input was proposed and its behaviour was thoroughly studied with all parameters.

Simulink Model:



Thank You!