EE 208

Control Engineering Lab

Experiment-2: Controller design on MATLAB platform using analog root loci.

Group Number- 20

Vedansh- 2020EEB1218

Aniket Arya- 2020EEB1157

Chirag Rathi- 2020EEB1165

Instructor- Dr. S. Roy

Date- 23/01/2022

OBJECTIVE: -

- To design of a parameter (T) for a given analog transfer function according to desired specifications.
- A sensitivity analysis for variation of key parameters is further required.

Given: -

We have been provided with an analog open loop transfer function which needs to be operated in closed loop in such a way that the complex poles have a damping ratio (ζ) in range of 0.2 to 0.25.

OLTF: $G_{OL} = \frac{30}{s(1+A\,s)(1+B\,s)(1+Ts)}$, where the parameter A = 0.2 and B= 0.1

After automation the parameters in denominator polynomial i.e. A and B are prone to variation by ±20%.

System Modelling

Open loop transfer function (OLTF), $G_{OL} = \frac{30}{s(1+0.1s)(1+0.2s)(1+Ts)}$

The OLTF is to be operated in a closed loop feedback system

Therefore, closed loop Transfer Function (CLTF), $G_{CL} = \frac{Gol}{1+Gol}$

$G_{CL} = \frac{30}{30+s(0.1s+1)(0.2s+1)(Ts+1)}$

Block Diagram:

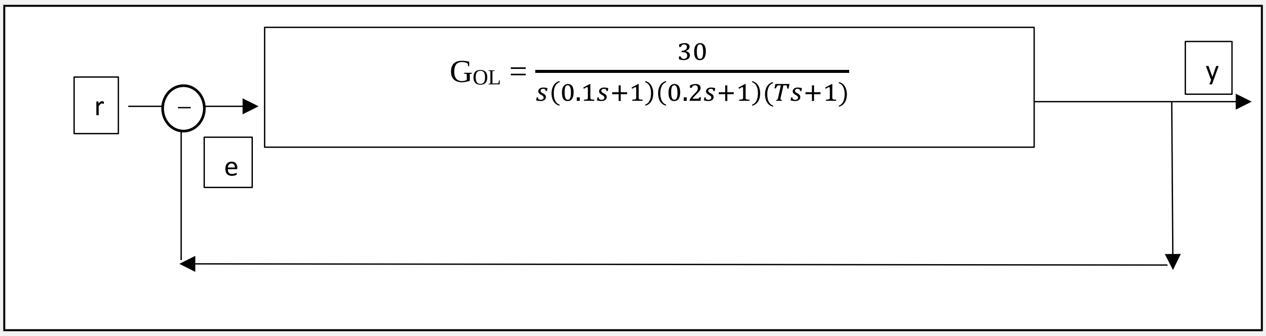


Fig.1 Given OLTF with unit negative Feedback

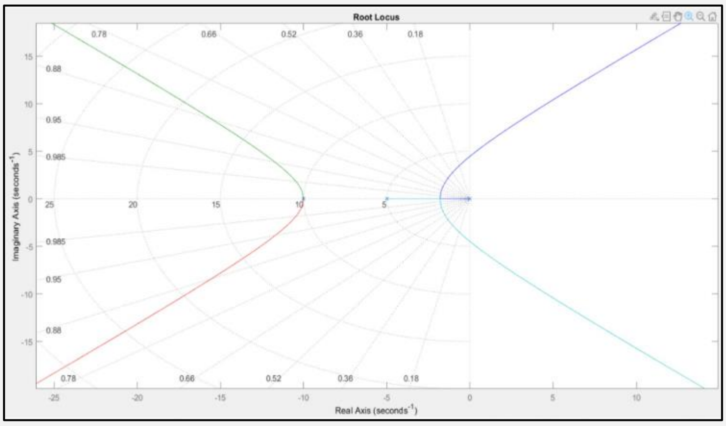
Observations and Analysis: -

Systems that have more than two poles will have a damping factor associated with each pole.
The dominant pole or poles will have the associated damping factor dominate the system behaviour.

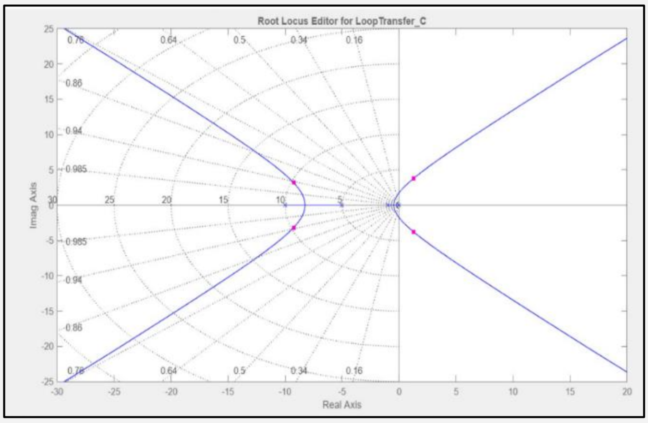
The dominant poles are the ones that are close to the imaginary axis on the root locus plane.

1) For T greater than zero

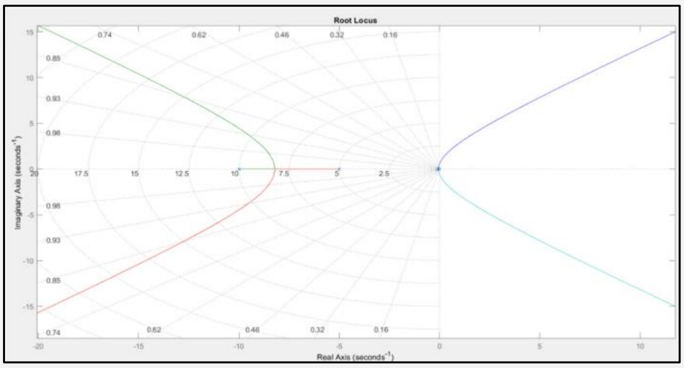
T	Pole 1	Pole 2	Pole 3	Pole 4	Damping Ratio (ζ) for dominant poles	Stability
0.1	2.12 + 7.17i	2.12 - 7.17i	-14.6 + 7.37i	-14.6 - 7.37i	0.893	Unstable
1	1.25 + 3.75i	1.25 - 3.75i	-9.25 + 3.18i	-9.25- 3.18i	0.946	Unstable
10	0.302 + 1.57i	0.302 - 1.57i	-6.05	-9.65	1	Unstable
100	0.0386 + 0.541i	0.0386 – 0.541i	-5.12	-9.97	1	Unstable
1000	0.00398 + 0.173i	0.00398 – 0.173i	-5.01	-10.00	1	Unstable



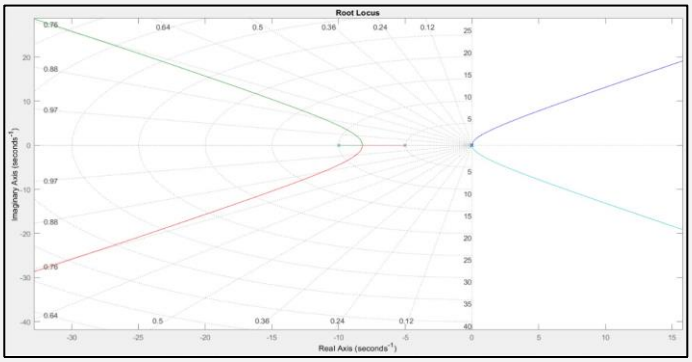
T = 0.1



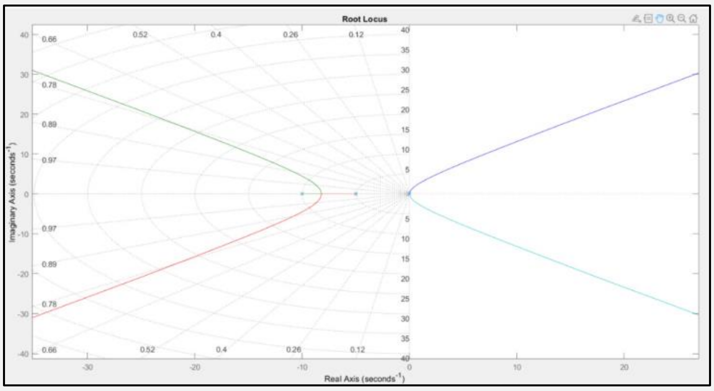
T = 1



T = 10



T = 100



T = 1000

Fig.2 Variation off closed loop pole on root locus

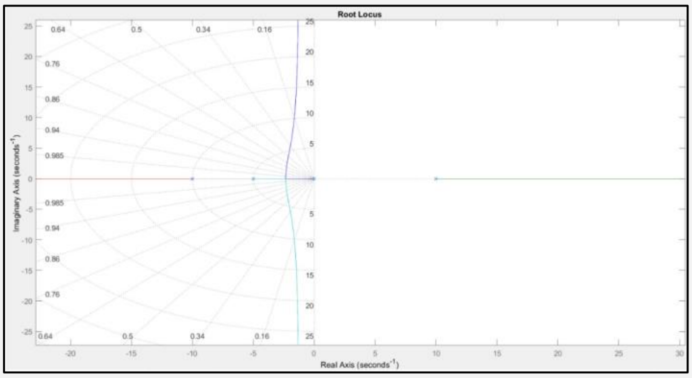
For positive values of T:

- **Two** of the closed loop poles (dominant ones) lie in **positive half of the plane**.
- With increase in T, the above two poles **approach the imaginary axis**, but still **remains in the right half of the plane.**
- **We can't achieve ζ between 0.2 – 0.25 for any T.**

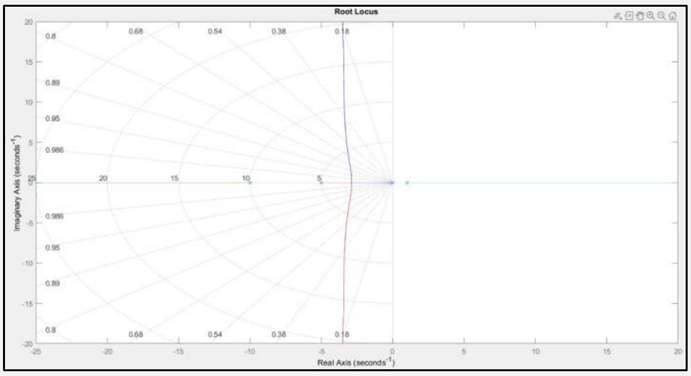
- **System is unstable for all T.**

2) For T less than zero

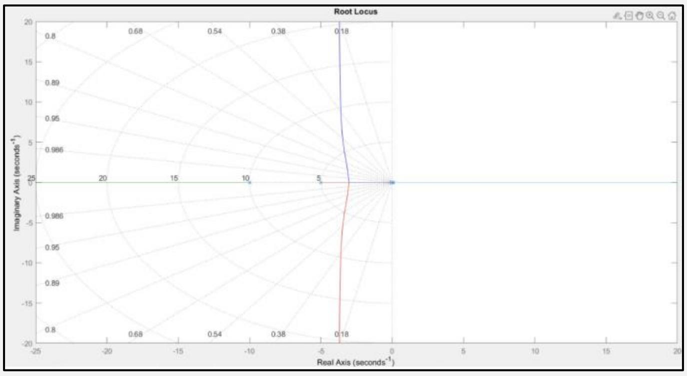
T	Pole 1	Pole 2	Pole 3	Pole 4	Damping Ratio (ζ) for dominant poles	Stability
-0.1	-1.69+ 8.82i	-1.69 - 8.82i	1.29e+01	-1.45e+01	0.189	Unstable
-1	3.99	-3.21+ 4.71i	-3.21 - 4.71i	-11.6	0.563	Unstable
-10	1.47	-3.05 + 0.779i	-3.05 – 0.779i	-10.3	0.969	Unstable
-100	0.514	-0.597	-4.88	-10.0	1	Unstable
-1000	0.169	-0.177	-4.99	-10.0	1	Unstable



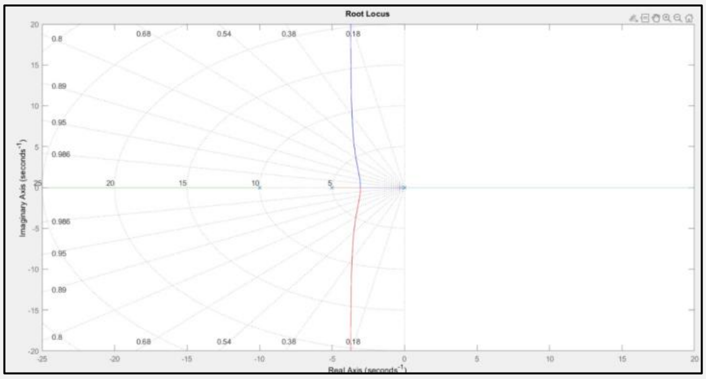
T = -0.1



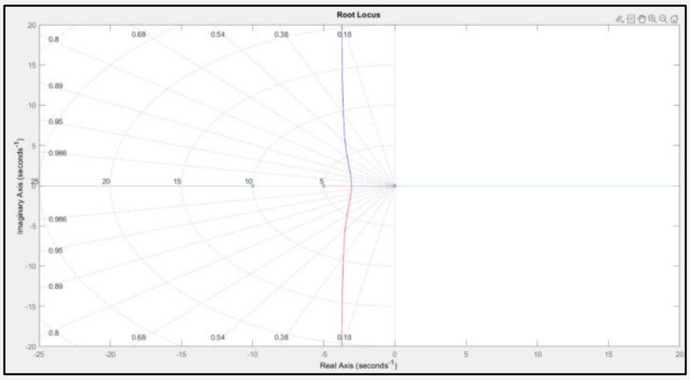
T = -1



T =- 10



T = -100



T = -1000

For negative values of T

- The closed loop poles **intersect the region** having ζ between (0.2 - 0.25) for T (-0.11 to -0.14).
- But one of the closed loop poles **always** lie in the **Right Half of the plane** for all values of T.
- **The system remains unstable**.

From above discussions, required ζ and stability **can't be achieved simultaneously** for the above CLTF.
In order to achieve the performance we have to **add** an extra **proportional controller** (parameter K) in the system.

For our new system,

$$G_{OL} = \frac{30\,K}{s(1+0.1s)(1+0.2s)(1+Ts)}$$

$$G_{CL} = \frac{30\,K}{s(1+0.1s)(1+0.2s)(1+Ts)+30K}$$

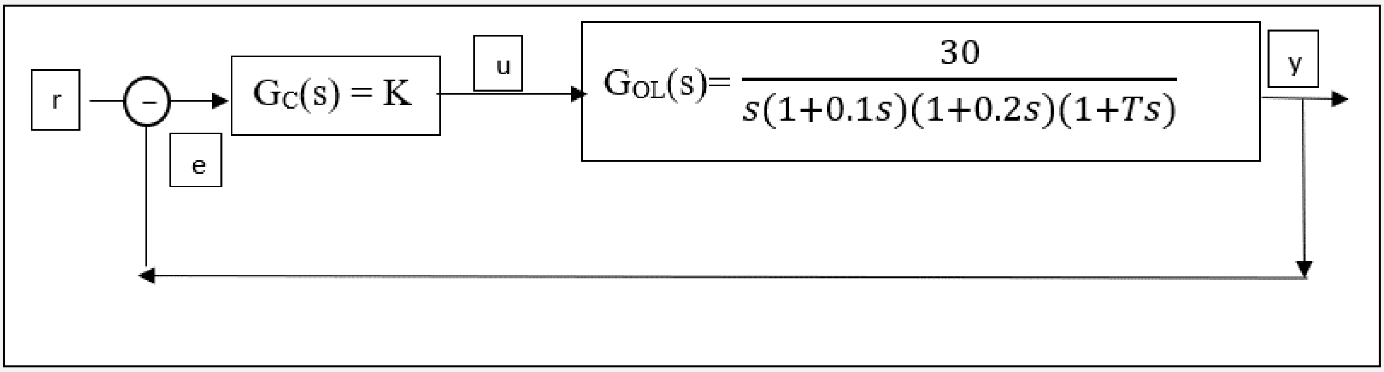


Fig. -3: New designed cascaded system

Now we will analyse our new system to achieve the desired damping ratio ζ, let's say 0.225.

Sr. No.	T	K (For ζ = 0.225)	Value of ζ for selected (T, K) and variation in A (20%)			Value of ζ for selected (T, K) and variation in B (20%)		
			A=0.2	A=0.16	A=0.24	B=0.1	B=0.08	B=0.12
1.	0.2	0.0911	0.225	0.256	0.202	0.225	0.247	0.207
2.	0.3	0.0788	0.225	0.254	0.2	0.225	0.244	0.208
3.	0.4	0.0709	0.225	0.254	0.200	0.225	0.242	0.209
4.	0.5	0.0653	0.225	0.253	0.201	0.225	0.241	0.210
5.	0.6	0.0611	0.225	0.252	0.201	0.225	0.240	0.211
6.	0.78	0.0552	0.225	0.250	0.203	0.225	0.238	0.213
7.	0.8	0.0547	0.225	0.249	0.203	0.225	0.238	0.213
8.	0.9	0.0522	0.225	0.248	0.204	0.225	0.237	0.214
9.	1.2	0.0464	0.225	0.245	0.206	0.225	0.235	0.215

Form the above table we can conclude that

- For any positive value of T, **we can find a K to achieve ζ = 0.225** while preserving **the stability of system.**
- For **low values** of T, we observe **high variation in** ζ while varying the parameters A and B.
- The variation in ζ with respect to A and B **decreases** as the parameter T **increases** for given (T, K) pair.

- For **T > 0.78** and respective K, with 20% variation in parameters A and B (after system gets automated), **the damping ratio of dominant poles remain between 0.2 – 0.25.**

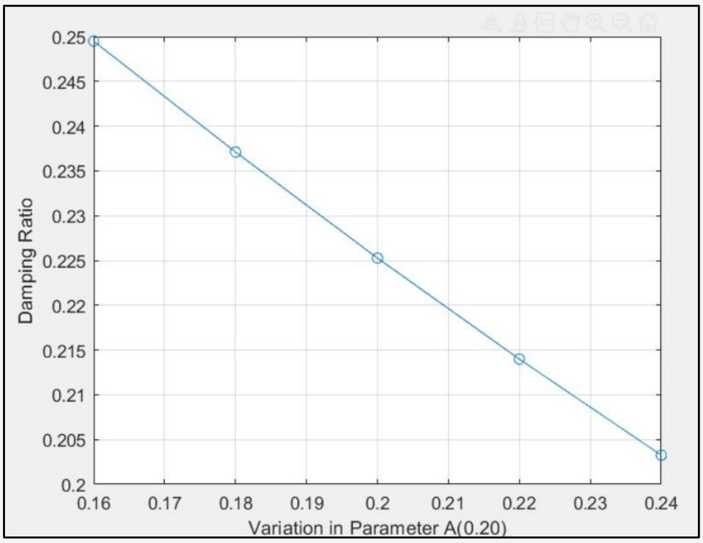
Performance analysis, with respect to variation in A (± 20%):

For T = 0.8, k = 0.0547

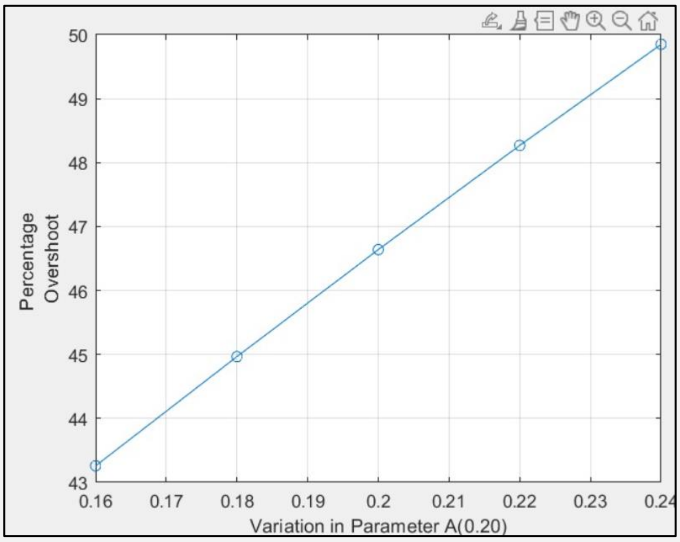
A	Damping ratio	Overshoot (%)	Rise time	Settling time
0.16	0.249493863901911	43.25821	0.970034016384244	10.636137276114319
0.18	0.237106840027973	44.966844064029000	0.972207829553827	12.503263918771646
0.20	0.225265476024734	46.640490841821070	0.975218853154736	12.887841466224970
0.22	0.213980298507656	48.269255534746925	0.978985002244619	13.146176436165852
0.24	0.203251129129841	49.849265023076825	0.983283122576928	13.362387253824080

For a constant value of T and K, as the value of Parameter A increases:

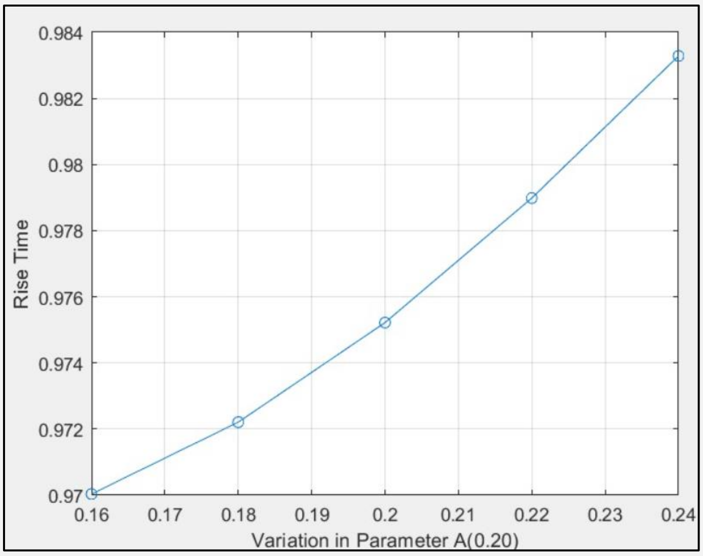
- Damping ratio decreases.
- Percentage Overshoot increases.
- Rise Time increases.
- Settling Time increases



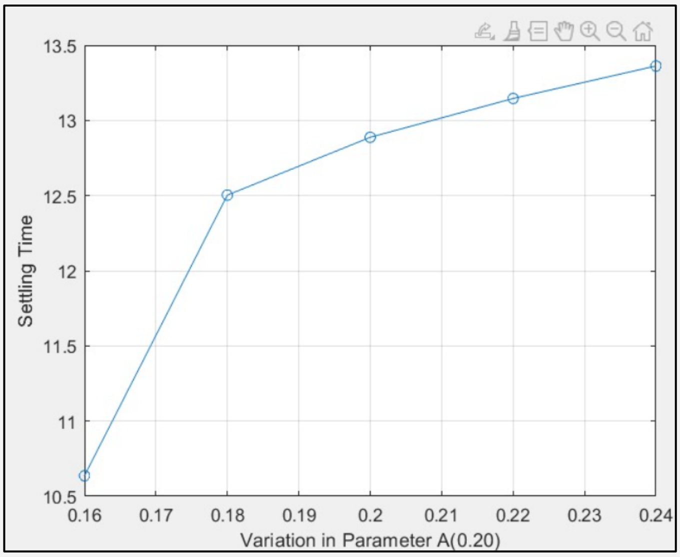
Graph: Variation of damping ratio against A



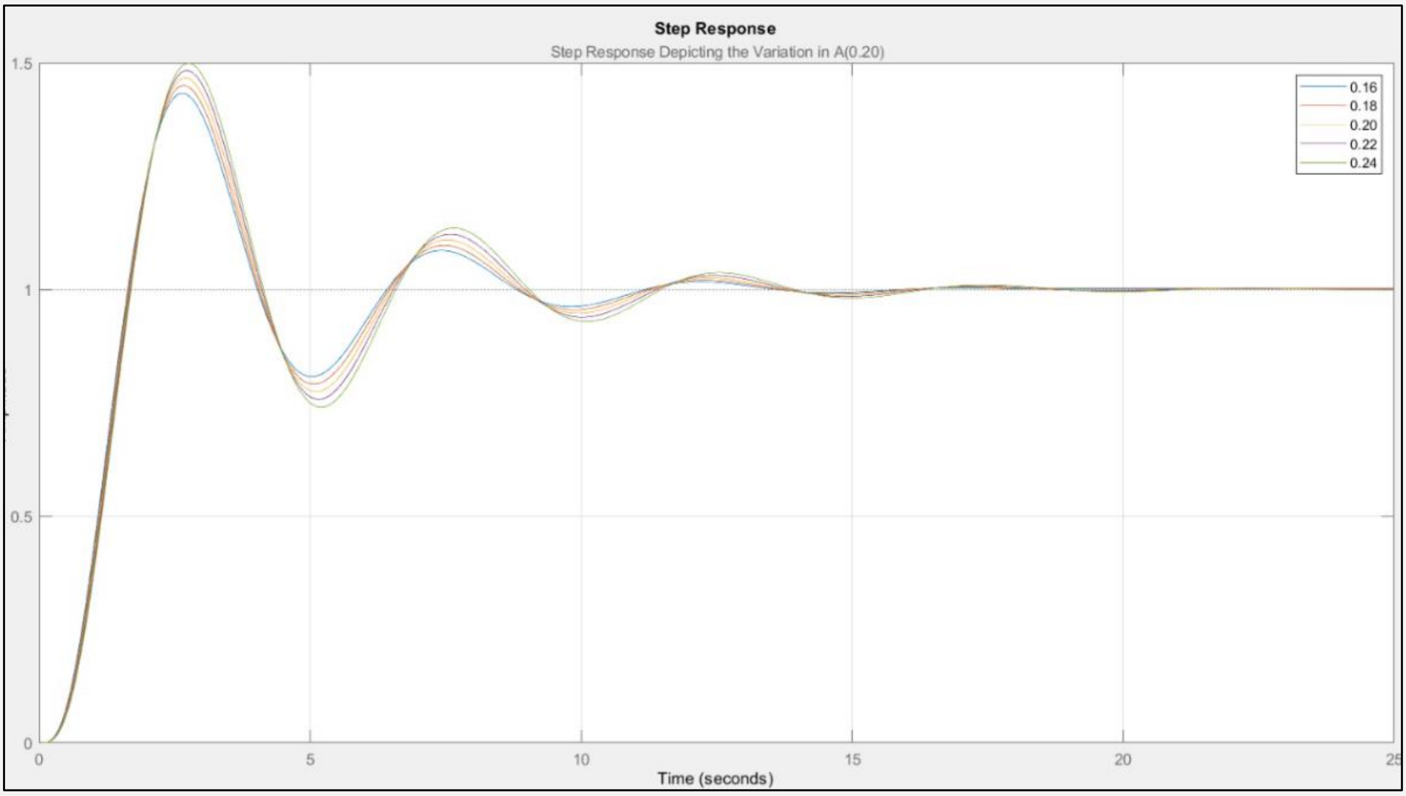
Graph: Variation of Percentage overshoot against A



Graph: Variation of Rise Time against A



Graph: Variation of settling time against A



Graph: Step response for varying A

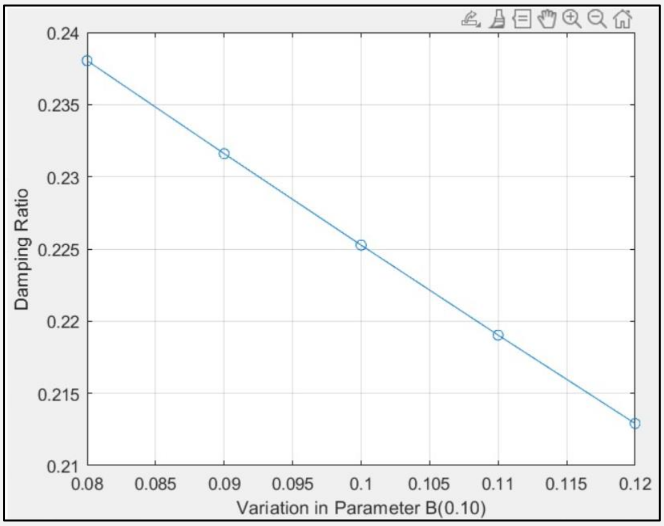
Performance analysis, with respect to variation in B (± 20%)

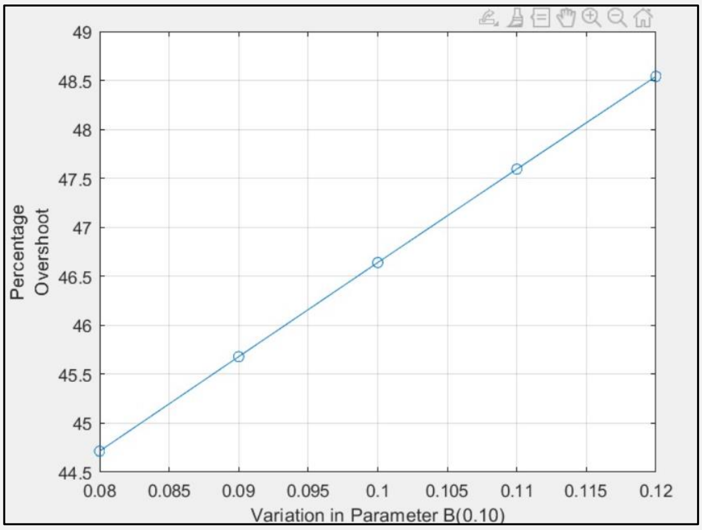
For T = 0.8, K = 0.0547

B	Damping ratio	Overshoot (%)	Rise time	Settling time
0.08	0.238040413704767	44.715314073274470	0.977089101058212	12.474315467160789
0.09	0.231603431337153	45.679791331844300	0.975971226471345	12.724298395207438
0.1	0.225265476024734	46.640490841821070	0.975218853154736	12.887841466224970
0.11	0.219032653410678	47.594584568536710	0.974838810551984	13.021705049547696
0.12	0.212910280536538	48.541125586366520	0.974768491624453	13.138984129198429

For a constant value of T and K, as the value of Parameter B increases

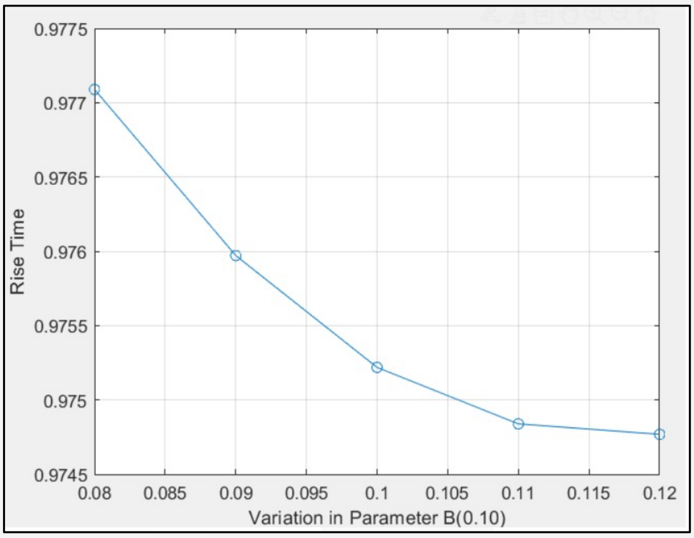
- Damping ratio decreases.
- Percentage overshoot increases.
- Rise time decreases.
- Settling Time increases.

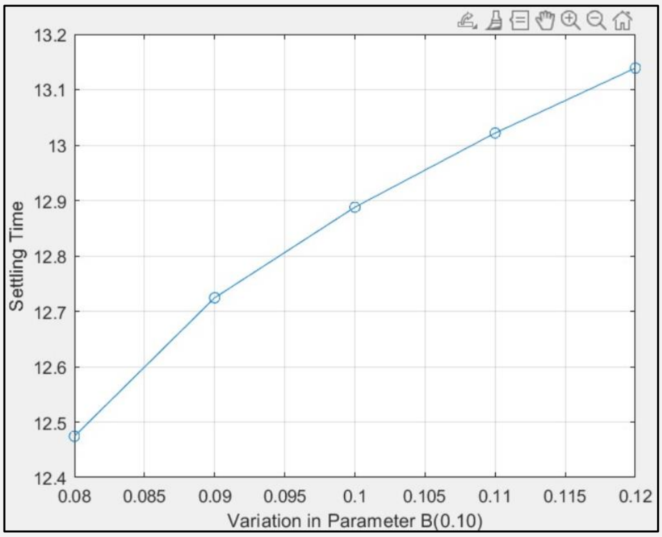




Graph: Variation of Damping ratio against B

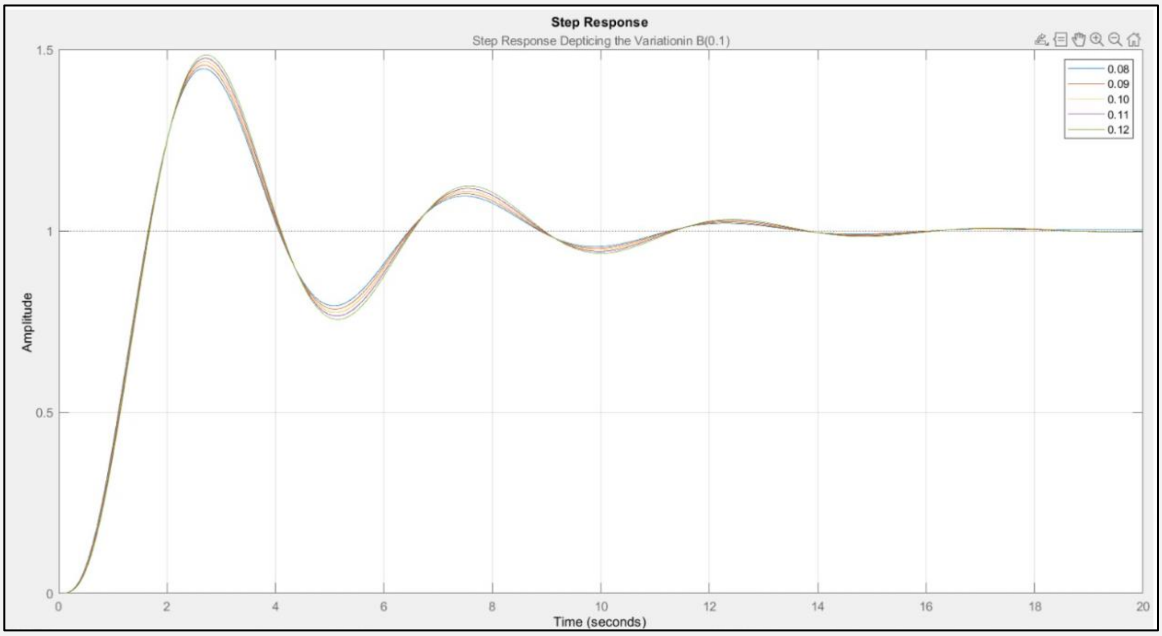
Graph: Variation of Percentage overshoot against B





Graph: Variation of Rise Time against B

Graph: Variation of settling time against B



Graph: Step response for varying B

Sensitivity Analysis: -

Sensitivity of closed loop with respect to –

1) Parameter A

Sensitivity of G_{CL} (s) with respect to A can be found using:

$$S_A^G(s) = \frac{dG}{G} \times \frac{A}{dA} = \frac{-A\,s^2\,(1+Bs)(1+Ts)}{s(1+As)(1+Bs)(1+Ts)+30K}$$

Sensitivity is defined as max| $S_A^G(s)$ |.

Note: While analysing the sensitivity, we assumed that the system is operated at a **large range of frequencies(bandwidth)**, so considered the **maximum** of the sensitivity over a range of frequencies.

- For a constant (T, K) pair, sensitivity of G_{CL} **increases** with **increase** in parameter A.
- For **constant A**, sensitivity of G_{CL} initially **increases** with **increase** in T, but eventually became **constant**.

2) Parameter B

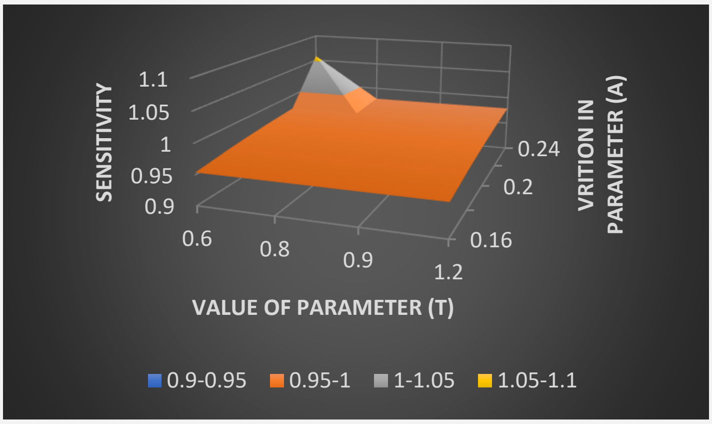
Sensitivity of G_{CL} (s) with respect to B can be found using:

$$S_B^G(s) = \frac{dG}{G} \times \frac{B}{dB} = \frac{-B\,s^2\,(1+As)(1+Ts)}{s(1+As)(1+Bs)(1+Ts)+30K}$$

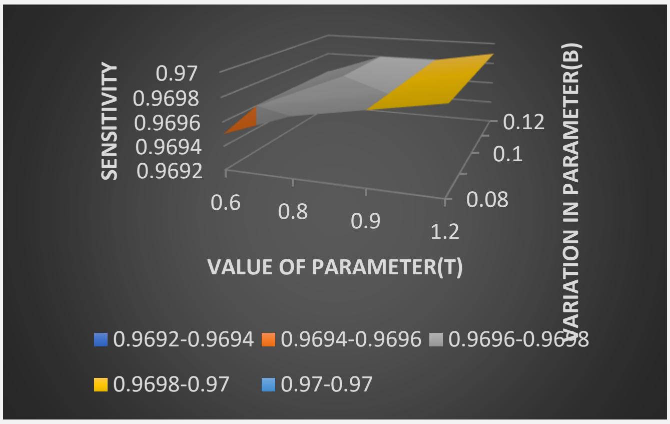
- Similarly, for constant (T, K) pair, sensitivity of G_{CL} **increases** with **increase** in parameter B.
- Though the variation in sensitivity with respect to B is **not as significant** as of A.

T	**A**				
	0.16	**0.18**	**0.20**	**0.22**	**0.24**
0.6	0.9538	0.9629	0.9696	0.9746	1.0585
0.8	0.9540	0.9631	0.9698	0.9748	0.9786
0.9	0.9541	0.9632	0.9698	0.9748	0.9787
1.2	0.9542	0.9633	0.9699	0.9749	0.9788

T	**B**				
	0.08	**0.09**	**0.10**	**0.11**	**0.12**
0.6	0.9695	0.9696	0.9696	0.9696	0.9696
0.8	0.9697	0.9697	0.9698	0.9698	0.9698
0.9	0.9698	0.9698	0.9698	0.9698	0.9698
1.2	0.9699	0.9699	0.9699	0.9699	0.9699



Graph: Sensitivity Variations w.r.t to A



Graph: Sensitivity Variations w.r.t to B

Conclusions: -

- For the given system there was no permissible T to achieve the desired performance and stability simultaneously.
- Therefore, we have to cascade an extra parameter K in the system.
- Then analysed the new system for various ranges of T and K to achieve the damping ratio such that it maintains its range after the system gets automated.
- Sensitivity analysis with respect to different parameters (A and B) are also obtained.

Script: -

```
s = tf('s');
T=-1000;
k=1;
Gol=30/(s*(1+0.10*s)*(1+0.2*s)*(1+T*s));
Gcl=feedback(Gol,1);
rlocus(Gol)
%sisotool(Gol)
damp(Gcl)
[wn,zeta]= damp(Gcl);
%data = stepinfo(Gcl);
```

Script 1: To analyse the given OLTF

```
s=tf('s');
T=-0.1;
a=0.20;
b=0.10;        %0.1
%k=0.0547;
Gol=30/(s*(1+0.1*s)*(1+0.24*s)*(1+T*s));
%Gol=zpk([],[0 -(1/b) -(1/a) -(1/T)] ,[(1500*a*b)/(T)]);
%GF=(30*k)/(s*(1+(a*s))*(1+(b*s))*(1+(T*s))+(30*k));
rlocus(Gol)
%p=pole(Gcl)
sisotool(GF)
%G=feedback(GC,1);
damp(Gol)
isstable(Gol)
stepinfo(GF)
%step(GF)
%hold on;
```

Script 2:To analyse the designed cascaded system

```
clear;
s = tf('s');
T = 1.2;
k=0.0464;
A=0.20  ;
B=0.12;
fband = [1,20];
Sa = -( A* s^2 * (1+B*s) * (1+T*s) )/ (s*(1+A*s)*(1+B*s)*(1+T*s)+30*k);
gpeak = getPeakGain(Sa,0.01,fband);
gpeak;
```

Script 3: To analyse the sensitivity

```
AZ=[B; ST];
plot(B, ST, '-o');
xlabel( 'Variation in Parameter B(0.10)')
ylabel('Settling Time')
grid on;
```

Script 4: To plot variations

THANK YOU!