EE 208

Control Engineering Lab

Experiment-4: Controller design on MATLAB platform using discrete root loci.

Group Number- 20

Vedansh- 2020EEB1218

Aniket Arya- 2020EEB1157

Chirag Rathi- 2020EEB1165

Instructor- Dr. S. Roy

Date- 06/02/2022

OBJECTIVE: -

- To design a cascade feedback controller for a given digital transfer function according to desired specifications.
- To perform sensitivity analysis for variation of parameters.

Given: -

An OLTF of a digital system which has three marginally stable poles.

OLTF: $G_{OL} = \frac{(z-a)(z-b)}{(z-1)^3}$, where the parameter a= -2 and b= 0.5.

A proportional gain within the range $-\infty<K<\infty$ may be assumed for closing the loop.

System Modelling

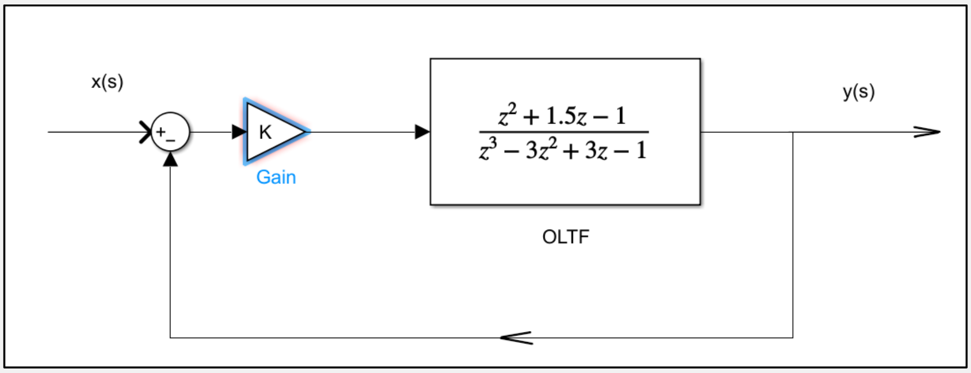
Transfer Function:

The closed loop Transfer function is as follows:

CLTF: $G_{CL}= \frac{K(z^2+1.5z-1)}{(z-1)^3+K(z^2+1.5z-1)}$

Which can further be written as $G_{CL}= \frac{K(z+2)(z-0.5)}{(z-1)^3+K(z+2)(z-0.5)}$

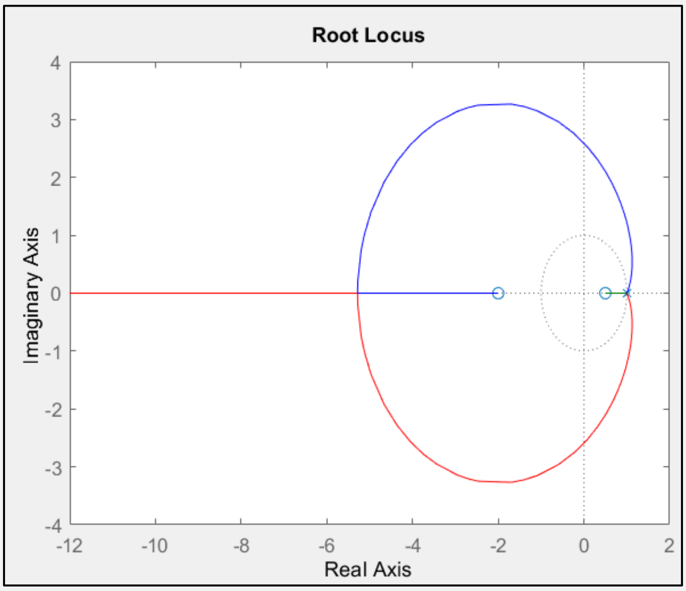
Block Diagram:

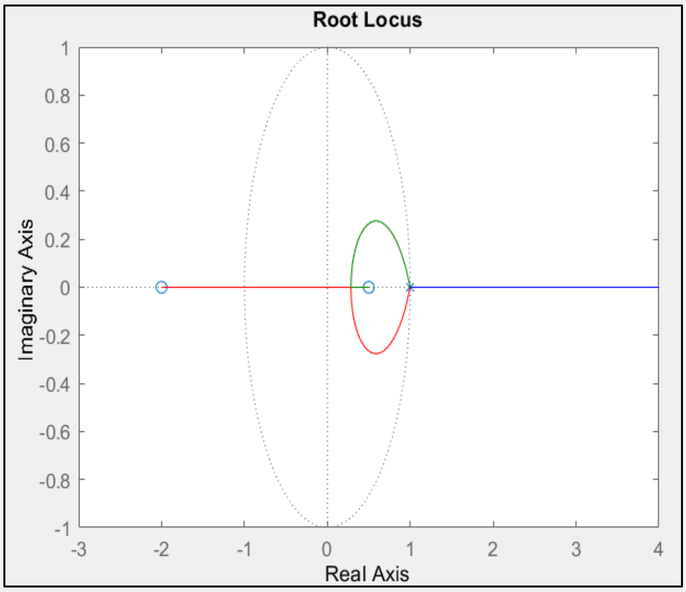


Block diagram for final system

Observations and Analysis: -

For the CLTF derived from the given OLTF without any parameter's variation,





$0<K<\infty$

$-\infty<K<0$

For the entire range of K,

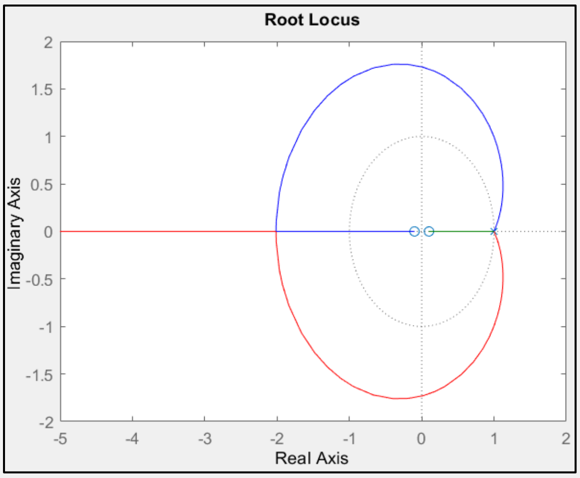
- At least one of the closed loop poles, **always lie outside** the root locus.
- The closed loop system is **always unstable** (for non-zero K).

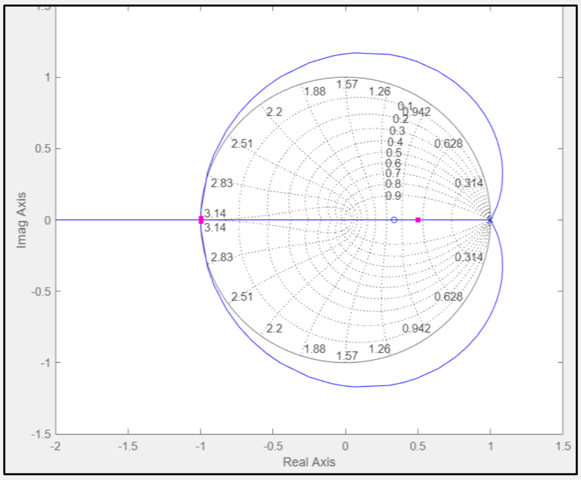
Sustained oscillation frequencies are obtained in z-domain by taking the **intersection of root loci with unit circle** (which represents damping factor=0). Therefore, the root loci have to **intersect the unity circle**, at point other than z=1 (because K=0 at z = 1).

At K = 0, the CLTF will **reduced** to the OLTF (won't get ω for a closed loop system), and our closed loop system isn't realizable. Therefore, we will aim for those cases in which rlocus **intersect** the unity circle other than z =1.

As **zeroes of OLTF vary**, the **corresponding CLTF root loci also changes** and hence different frequencies of sustained oscillations are obtained.

Note- For analysis purpose, we always maintain the value of b less than that of a. (Therefor in all the plots b is the left root).
Case I: - When both a and b are less than 0.333.
Root Locus Plots:





a=0.1 b= -0.1

a=0.3 and b=0.3

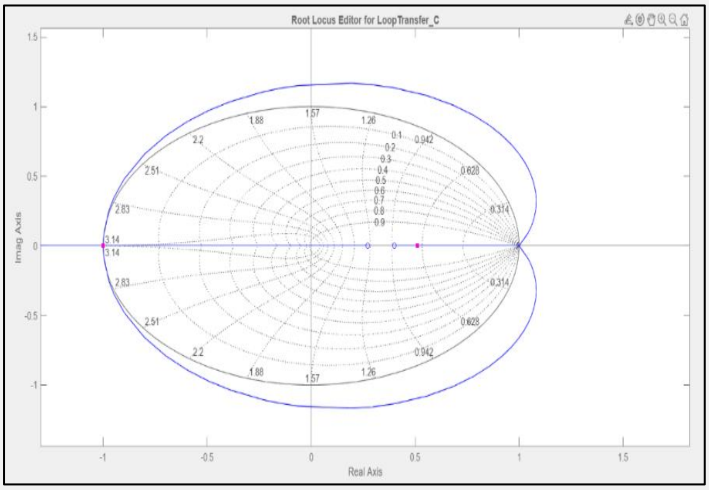
- For any pair (a, b), in which a < 0.333 & b < 0.333, we **won't get a favourable** root locus, which **intersect the unity circle** other than z=1.
- For a = b = 0.333, we got a **closed loop pole on unity circle** (and other two inside the circle), for **K=4.5 and ω = 3.14 (normalised freq.)**

Case II: - When (0.333 < a < 1)

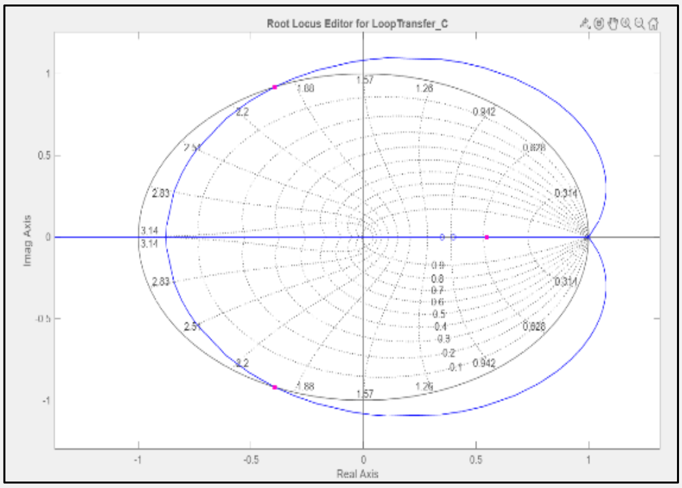
When b is varied:

a	b	ω	K
0.333	0.333	3.14	4.5
0.4	0.274 (b_{min})	3.14	4.49
0.4	0.35	1.97	3.23
0.4	0.4	1.69	2.67

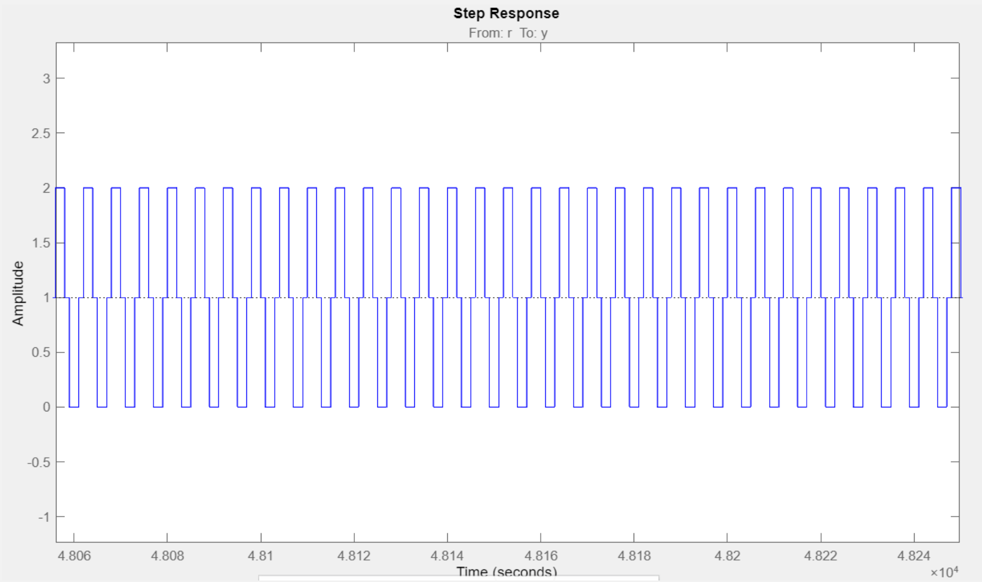
Root Locus Plots:



a=0.4 and b=0.274



a=0.4 and b=0.35

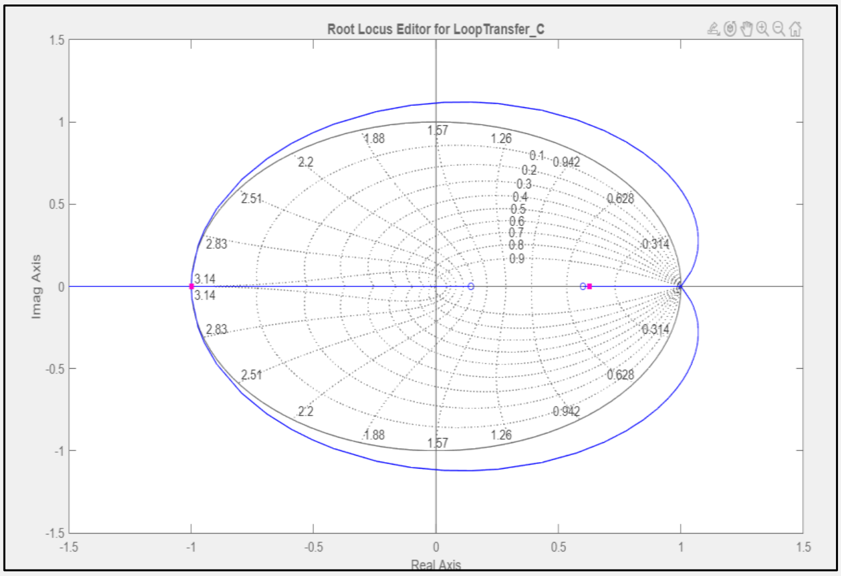


Step Response depicting the Sustained Oscillation for the designed system

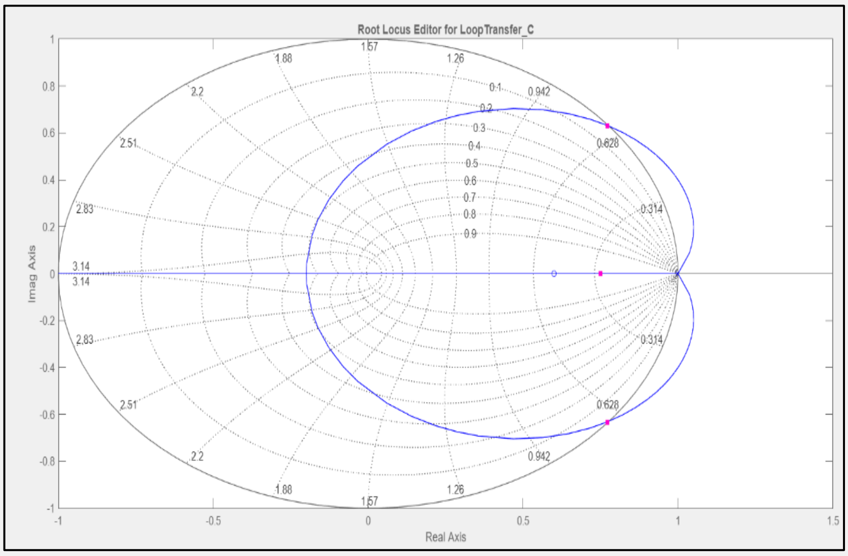
When a is varied:

a	b	ω	K
0.6	0.143 (b_{min})	3.14	4.37
0.6	0.4	1.04	1.31
0.6	0.6	0.679	0.694

Root locus Plots:



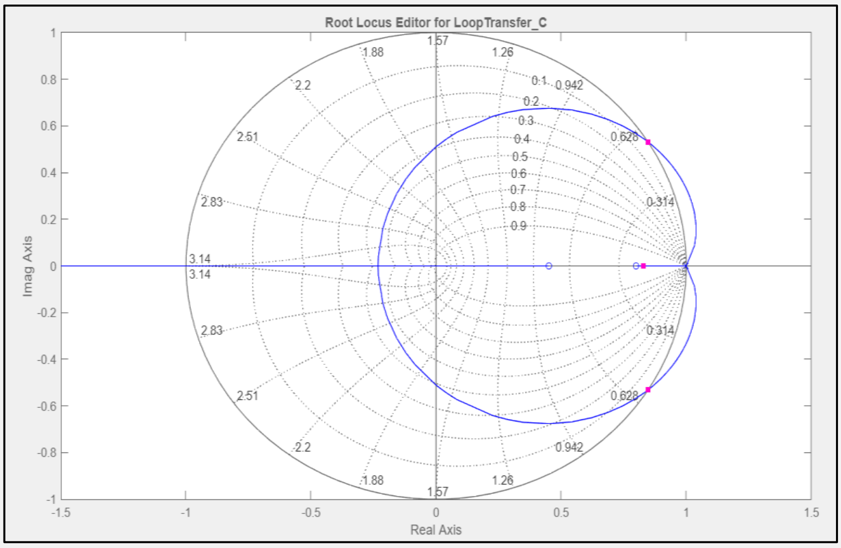
a=0.6 and b=0.143



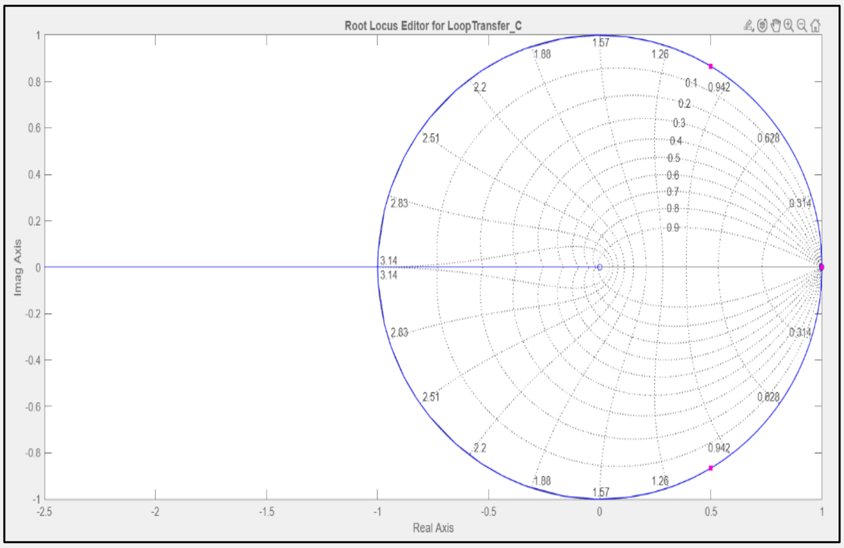
a= 0.6 and b=0.6

a	b	ω	K
0.8	0.06 (b_{min})	3.14	4.19
0.8	0.45	0.559	0.476
0.8	0.8	0.251	0.174
1.0	0	ALL	Multiple K

Root Locus Plot:



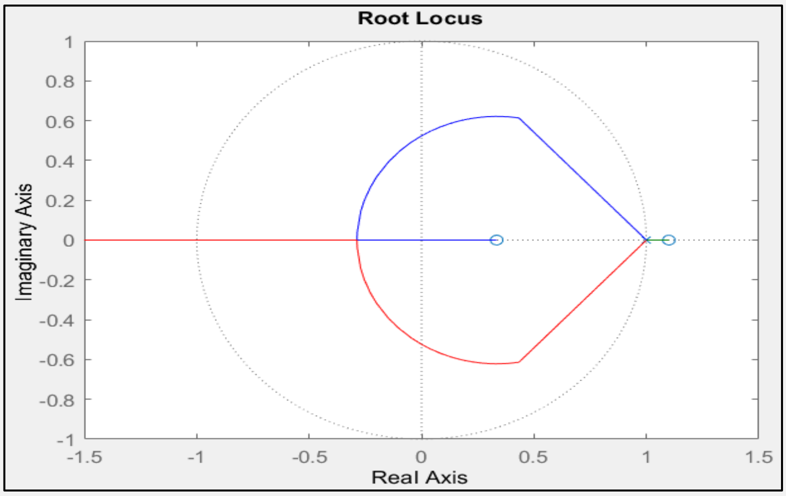
a=0.8 and b=0.45



a= 1.0 and b= 0

- For every 0.333 < a < 1, we always have a **minimum value** of b, such that for (b_{min}< b < a), we get a **valid frequency** for sustained oscillation while **maintaining the marginal stability** (for non-zero K).
- **b_{min} always** remains **greater than zero**.
- For a **constant** a, as b **increases** (up to a), both and K **decreases**.
- For a **greater** (a, b) pair, maintaining (a > b), we get **lower** ω and K.
- For zeros at 1 and 0, the root loci coincide with the unity circle providing sustained oscillations for multiple pair of k, ω.

Case III: - For (a > 1), when at least one zero is greater than z=1



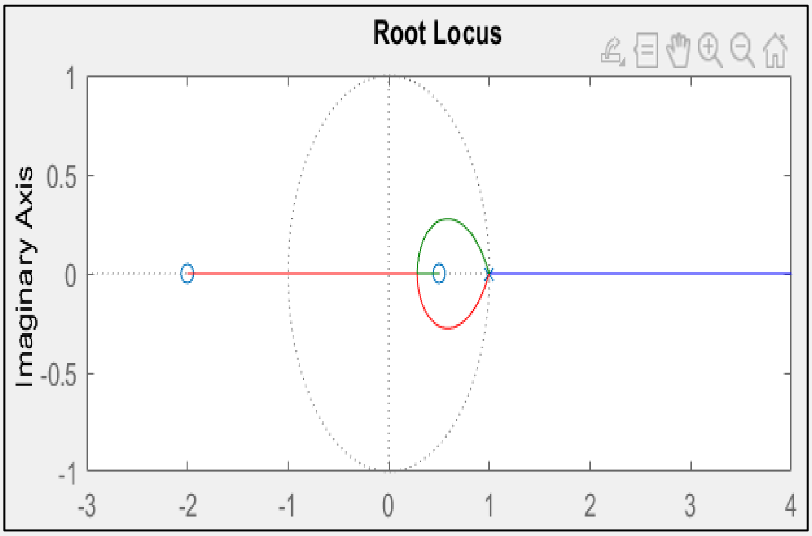
a= 1.1 and b= 0.334

- For every non zero K, one of the **Closed loop poles always** lie in the **unstable region**.
- This is the same case as Case I.

Therefore, we **won't be able to obtain** any frequency of sustained oscillation (for non-zero K) **for the above case.**

In the above cases, **we get a positive K for all positioning of zeroes**.

For negative K,



For negative K

- **One pole always lies outside the unit circle.**
- **No marginal stability**, for non-zero negative K, can be obtained.

This case is also same as that of Case I (discussed above).

Sensitivity Analysis:

Sensitivity of G_{CL} (s) with respect to a can be found using:

$$S_a{}^G(s) = \frac{dG}{G} \times \frac{a}{da} = -\frac{a}{(z-a)} + \frac{a*k*(z-b)}{((z-1)*(z-1)*(z-1))+k*(z-a)*(z-b)}$$

Sensitivity is defined as max| Sa ^{G} (s) |.

Sensitivity of G_{CL} (s) with respect to b can be found using:

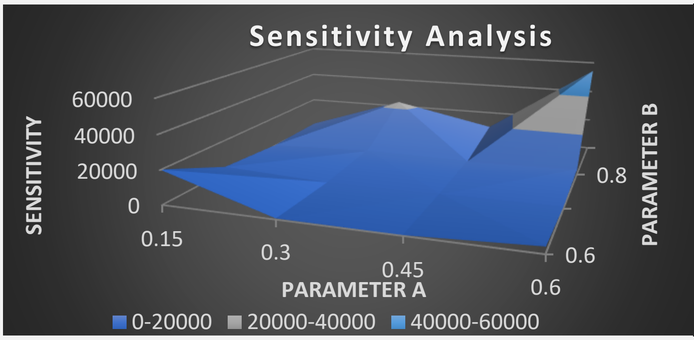
$$S_b{}^G(s) = \frac{dG}{G} \times \frac{b}{db} = -\frac{b}{(z-b)} + \frac{b*k*(z-b)}{((z-1)*(z-1)*(z-1))+k*(z-b)*(z-a)}$$

The table shown below depicts the sensitivity of the transfer function with respect to deigned zeros in the frequency band of [0:3].

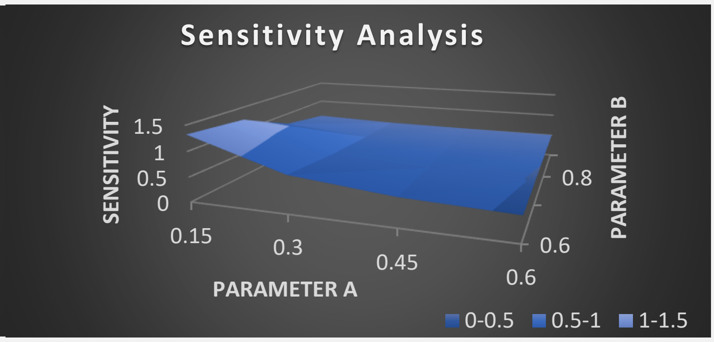
Sensitivity in Frequency band [0:3]

b	a			
	0.6	**0.7**	**0.8**	**0.9**
0.15	20397.7153697138	3069.82362017413	207.27	919.4823
0.3	619.7157	244.9393	2342.93297447746	23041.9950084778
0.45	405.6817	645.7481	1610.76851789211	9426.90499259911
0.6	3830.65142918663	1034.52402427028	3406.74708520233	54454.3941912450

The values of the sensitivity are quite high for this frequency band and hence limiting the robustness of the system in this band.



Graph-1: For frequencies less than 3 rad/sec



Graph-2: For frequencies more than 3 rad/sec

Sensitivity in Frequency band [3:1000]

b	a			
	0.6	**0.7**	**0.8**	**0.9**
0.15	1.432	1.2316	0.7523	0.5917
0.3	0.7413	0.6315	0.5747	0.5335
0.45	0.5550	0.5396	0.5277	0.5154
0.6	0.4832	0.4957	0.5039	0.5059

Further the frequency band of **above 3 rad/s**, shows **low sensitivity for the designed zeros** and hence making the system **dynamically robust for higher frequency ranges**.

It could also be observed that **sensitivity increases** as we place zero at **larger value** on the right plane.

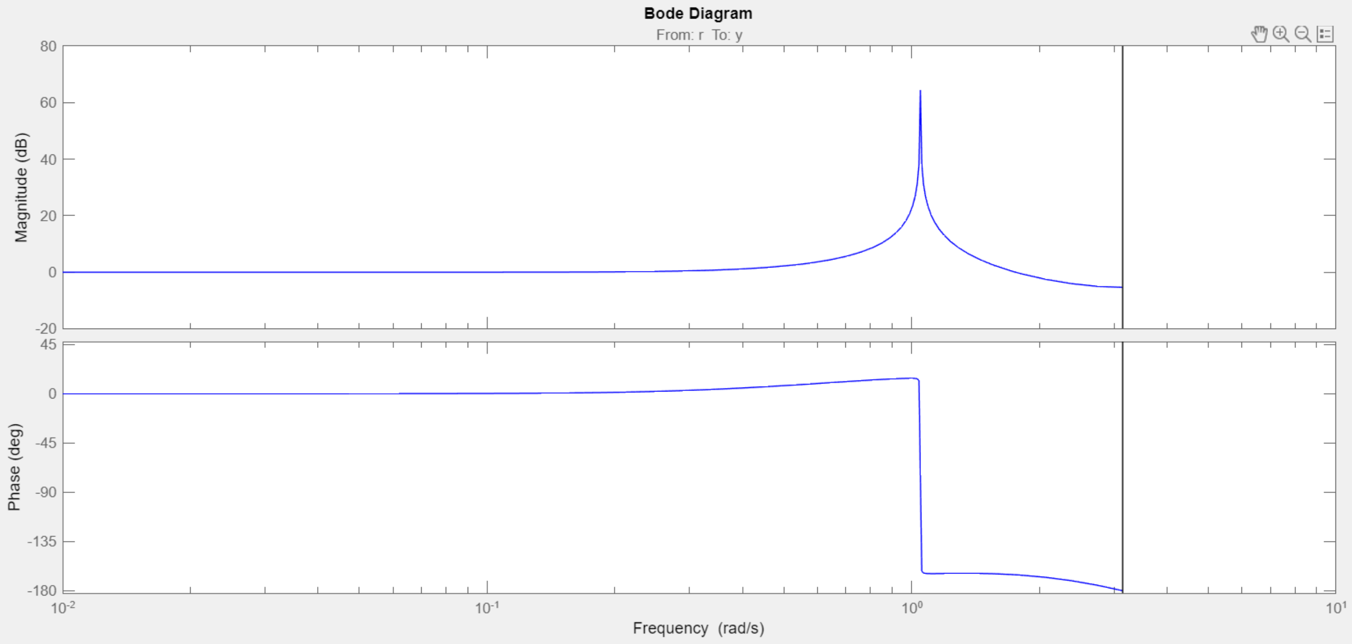
Hence **lower values (**on the right-hand plane) of the found zeros are **more suitable**.

Since **both zeros** are have **identical sensitivity function**, both depend on the choice of first and the second zero for a specific loop gain.

To assure the above mentioned points **the frequency of the peaks for sensitivity** were calculated and formulated in the table. It was found that **maximum value to attain peak was 2.6640 rad/sec.**

b	**a**			
	0.6	**0.7**	**0.8**	**0.9**
0.15	2.6640	1.7849	1.2550	0.811
0.3	1.3434	1.0394	0.7828	0.5147
0.45	0.9336	0.7387	0.5592	0.3704
0.6	0.6795	0.5402	0.4114	0.2730

Frequencies for Peaks of the corresponding sensitivities



Bode plot for zeros at 0.7 and 0.3 depicting the peak at 1.0394

Conclusion:

- We analysed the given system and optimised it for sustained oscillations by adjusting the open loop zeros and a positive gain cascaded to it.
- Though for negative gain in cascade, the sustained oscillations were not be achieved.
- We then analysed the sensitivity for the designed system and found out that it works robustly in band of frequency greater than 3 rad/sec.

Script:

```
numerator = [1, 1.5, -1];
denominator = [1, -3, 3,-1];
sys = tf(numerator,denominator,-1);
k=1;
%rlocus(sys);
sisotool(sys);
```

Script-1: Script for defining the original system

```
clc
clear
a=0.7;  %right
b=0.3;  %left
sys=zpk([a,b],[1,1,1],1,1);
%rlocus(sys);
sisotool(sys);
%damp(sys);
```

Script-2: Script for analysing the various designs

```
clear;
clc;
z=tf('z',1);
k=0.161;
a=0.9;
b=0.60;
fband=[3,1000];
Sa=-(a/(z-a))+((a*k*(z-b))/((z-1)*(z-1)*(z-1)+k*(z-a)*(z-b)));
[gpeak, fpeak]=getPeakGain(Sa,0.1, fband);
%gpeak
%fpeak
```

Script-3: Script for analysing the sensitivity of the function

Thankyou!