EE 208

Control Engineering Lab

Experiment-11: Nonlinear system dynamics on Simulink for different Lyapunov control designs.

Group Number: 20

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Objective:

- Dynamic studies of the crane trolley system with which different Lyapunov control designs are to be incorporated.
- The systems are to be individually simulated on Simulink; each using a detailed nonlinear state space system simulation in four state variables.

Given:

We have been provided with differential equations that represent a simplified model of an overhead crane. The equations are:

$$[m_L + m_C].\ddot{x}_1(t) + m_L.l.[\ddot{x}_3(t).\cos x_3(t) - \dot{x}_3^2(t).\sin x_3(t)] = u(t)$$

$$m_L.[\ddot{x}_1(t).\cos x_3(t) + l.\ \ddot{x}_3(t)] = -m_L g.\sin x_3(t)$$

Here, constants are:
m_C: Mass of trolley; 10 kg.
m_L: Mass of hook and load; the hook is again 10 kg, but the load can be zero to several hundred kg's, but constant for a particular crane operation.
l: Rope length; 1m or higher, but constant for a particular crane operation.
g: Acceleration due to gravity, 9.8 ms^{-2}

Variables are:

- Input:
- u: Force in Newtons, applied to the trolley.
- Output:
- y: Position of load in metres, $y(t) = x_1(t) + l.\sin x_3(t)$
- States:
- x_1: Position of trolley in metres.
- x_2: Speed of trolley in m/s.
- x_3: Rope angle in rads.
- x_4: Angular speed of rope in rad/s.

Consider four different energy-based function components:
A. Proportionate to square of linear potential energy: = $K_{PE}^l . (x_{1ref} – x_1)^2$
B. Proportionate to linear kinetic energy: = $K_{KE}^l.x_2^2$
C. Proportionate to square of rotary potential energy: = $K_{PE}^r.x_3^2$
D. Proportionate to rotary kinetic energy: = $K_{KE}^r.x_4^2$

- For different control designs, Lyapunov functions can be generated by linear combinations in ones, twos, threes, or all out of "A, B, C, D".
- In Physics concepts, potential energy is proportionate to differences of distance, or angle. Function components "A" and "C" have been defined as square of these. Kinetic energy terms are proportionate to square of speeds, so these have been retained without squaring.
- "A" resembles our standard state feedback error, and can be correlated to conventional state feedback principles.
- Following are the derivatives for individual Energy profiles with respect to time:

$\dot{A} = -2K_{PE}^1(x_{1ref} - x_1)\dot{x}_1$ $\qquad$ $\dot{B} = 2K_{KE}^1 x_2\dot{x}_2$

$\dot{C} = 2K_{PE}\, x_3\dot{x}_3$ $\qquad$ $\dot{D} = 2K_{KE}\, x_4\dot{x}_4$

Lyapunov's Second Method is an extension of two fundamental physical ideas to the theory of nonlinear systems of ODEs: A conservative physical system's state is stable only if its potential energy has a local minimum at that point.

Except for the equilibrium point, the function must be positive everywhere. It should be zero at equilibrium. The fundamental idea is to create a feedback control regulation that makes the derivative of a given Lyapunov function candidate negative definite or negative semi-definite.

If a Lyapunov function with a negative definite derivative for all exists in the vicinity of an autonomous system's zero solution, the system's equilibrium point is asymptotically stable. A negative semidefinite matrix is a Hermitian matrix all of whose eigenvalues are nonpositive.

Best Lyapunov's Selection Criterion:

- In a neighbourhood of the zero solution of the system, the Lyapunov function should have a negative definite derivative for all, then the equilibrium point of the system is asymptotically stable.
- However, if the Lyapunov's derivative is negative semidefinite, then asymptotic stability cannot be concluded from that Lyapunov's method.

Step Response:

Energy Profiles

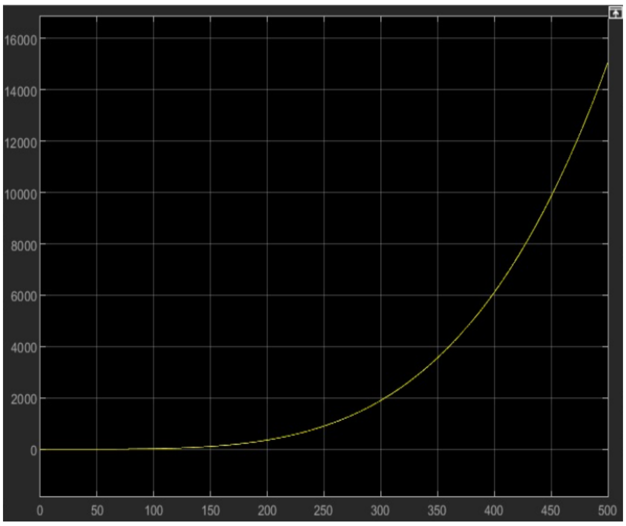


Fig-1: Variance of Linear Potential Energy

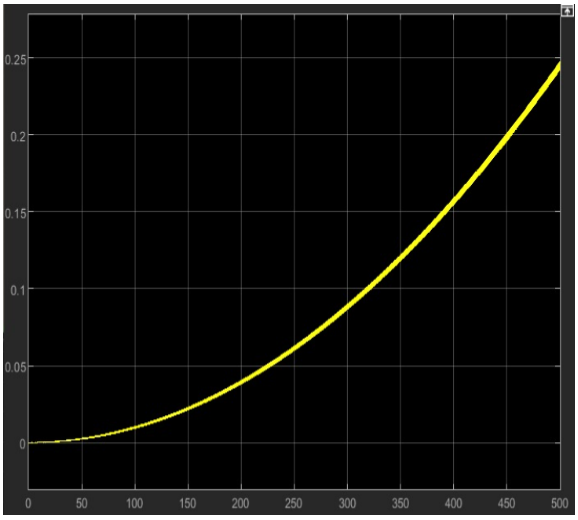


Fig-2: Variance of Linear Kinetic Energy

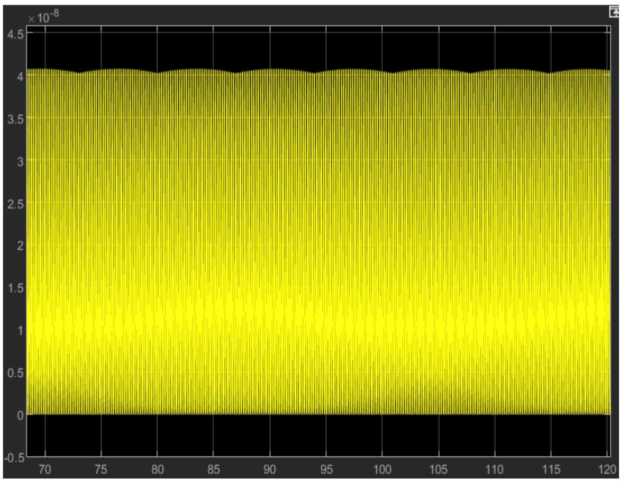


Fig-3: Variance of Rotary Potential Energy

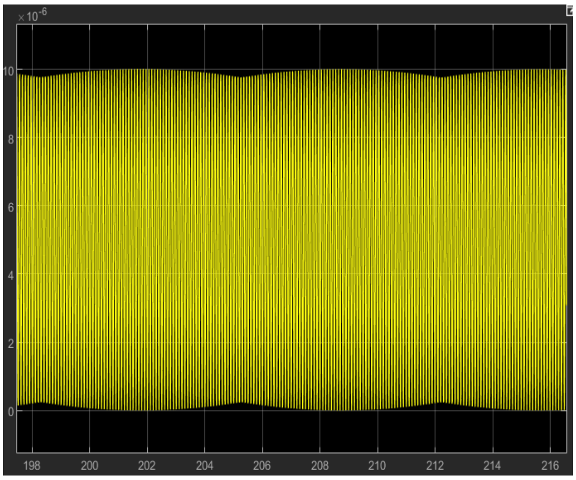


Fig-4: Variance of Rotary Kinetic Energy

Derivatives of energy profiles:

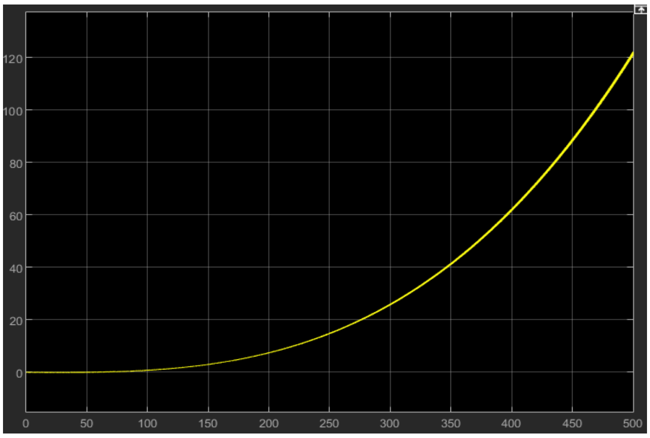


Fig-5: Variance of Derivative of Linear Potential Energy

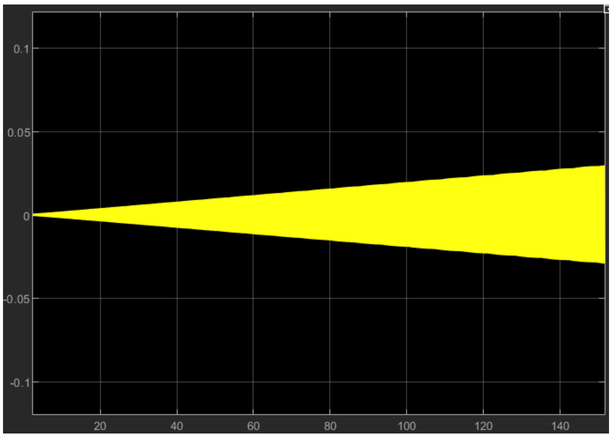


Fig-6: Variance of Derivative of Linear Kinetic Energy

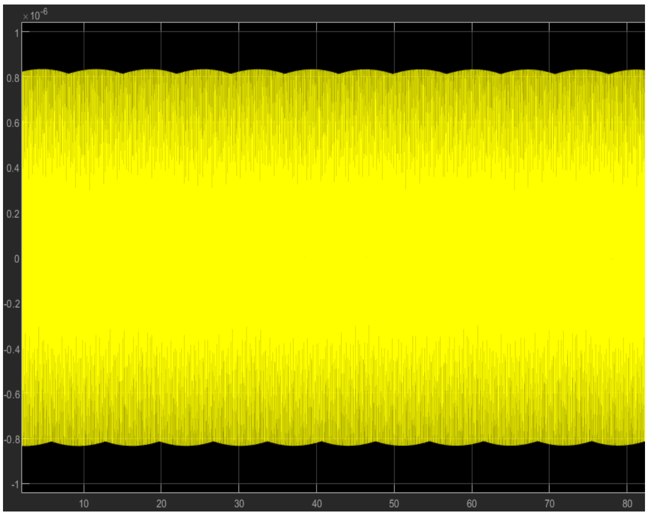


Fig-7: Variance of Derivative of Rotary Potential Energy

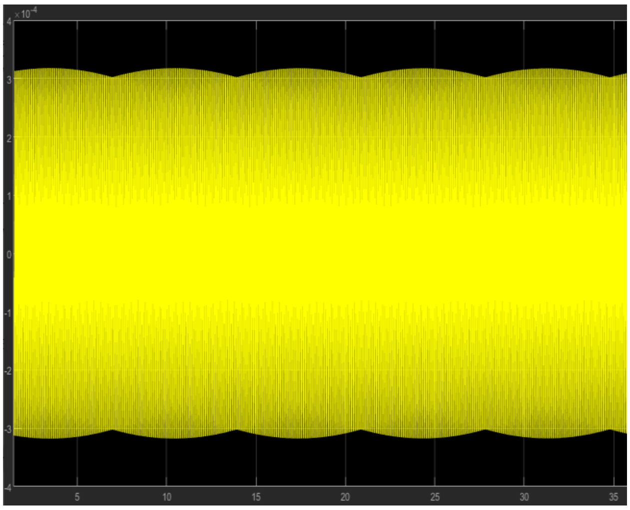


Fig-8: Variance of Derivative of Rotary Kinetic Energy

- Since the Variance of **Derivative of Linear Potential Energy is always positive** and increasing monotonically, **K_{PE1} must be negative**.
- **Magnitude of Derivative of Rotary Potential Energy and Rotary Kinetic Energy is negligible** (of order e (-6)), though they show **oscillatory nature about x axis**.
- Linear Kinetic energy shows **increasing magnitude** of the oscillations symmetrically to x axis.
- Since we require negative definite function, we can have:
  - $K_{PE1} < 0$
  - $K_{PE\,R}$, $K_{KE\,R}$, $K_{KE1} > 0$
- These proportional will render us with negative overall derivative functions and hence asymptotically stable.

Lyapunov Function (A): - $1 * (x_{1ref} – x_1)^2 + x_2^2 + x_3^2 + x_4^2$

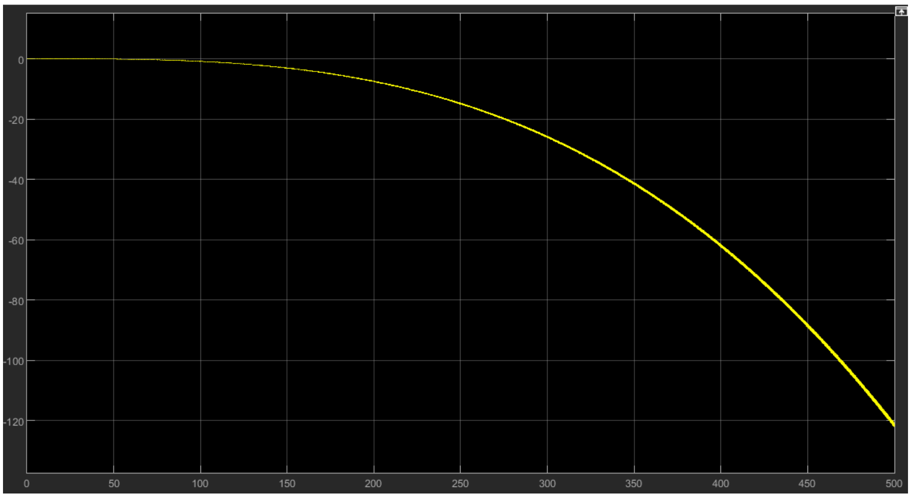


Fig-9: The derivative of the designed Lyapunov function (semidefinitely negative)

<u>Ramp Response:</u>

Derivatives of Energy Profiles:

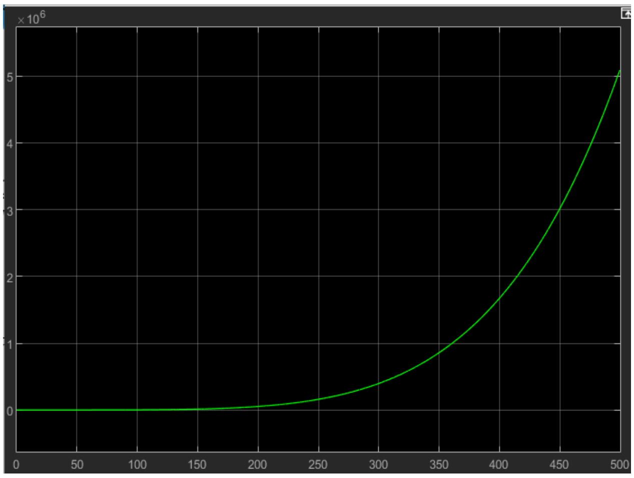


Fig-10: Variance of Derivative of Linear Potential Energy

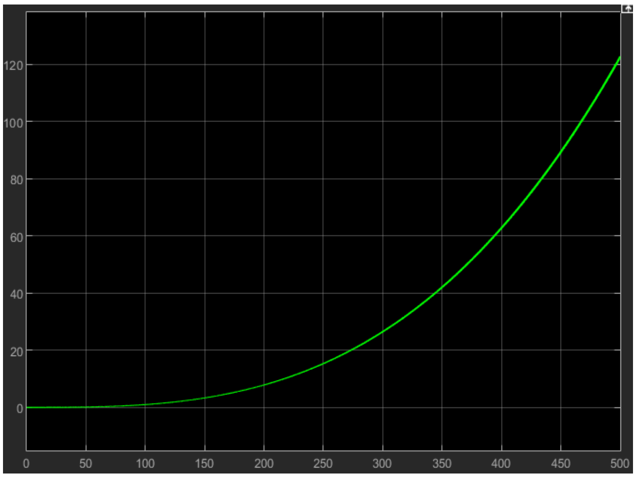


Fig-11: Variance of Derivative of Linear Kinetic Energy

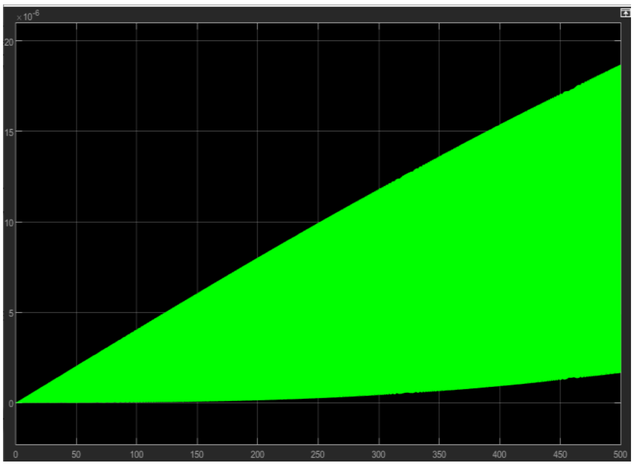


Fig-12: Variance of Derivative of Rotary Potential Energy

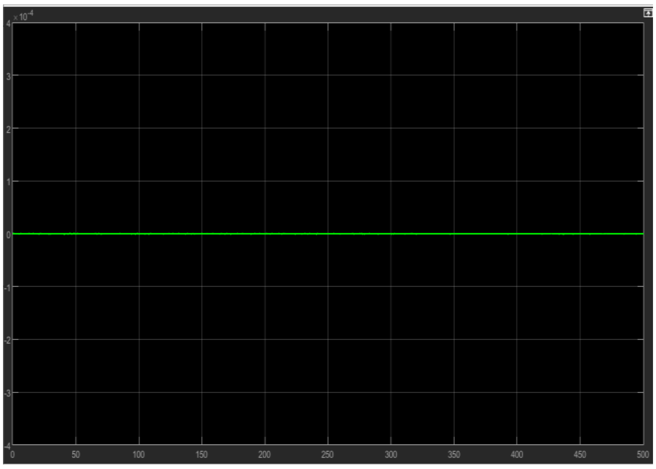


Fig-13: Variance of Derivative of Rotary Kinetic Energy

- The derivative of the Linear Potential Energy and Linear kinetic energy shows **a monotonous increasing curve**; hence they are **liable of negative proportional value** for designing the Lyapunov.
- The derivative of the rotary potential energy shows an **expected oscillatory behaviour of increasing magnitude** with a monotonous increase in the equilibrium position.

- Whereas the derivative of the rotary kinetic energy shows an oscillatory symmetric behaviour about x axis of **negligible magnitude** of the power of e (-8).
- The desired Lyapunov function should have the following proportion constant:
  - K_{PE1}, $K_{PE\ R} < 0$
  - $K_{KE\ R}$, $K_{KE1} > 0$
  - $K_{PE1} = (-0.001) * K_{PE1}$

Lyapunov Function (B): - $0.001*(x_{1ref} - x_1)^2 + x_2^2 - x_3^2 + x_4^2$

- Since rotary potential energy also shows increasing magnitude though of small magnitude has been assigned with the negative proportional.

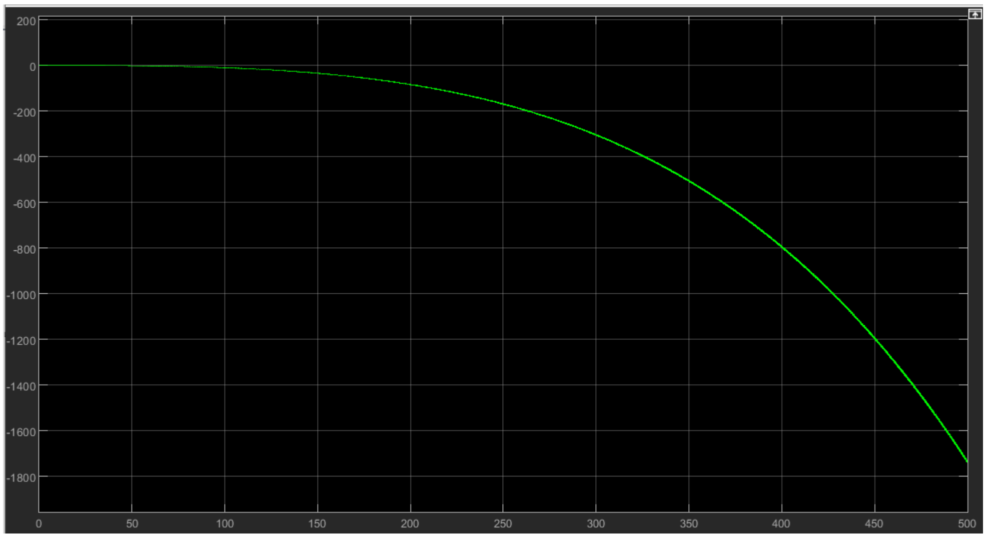


Fig-14: The derivative of the designed Lyapunov function (definitely negative)

Impulse:

Derivatives of Energy Profiles:

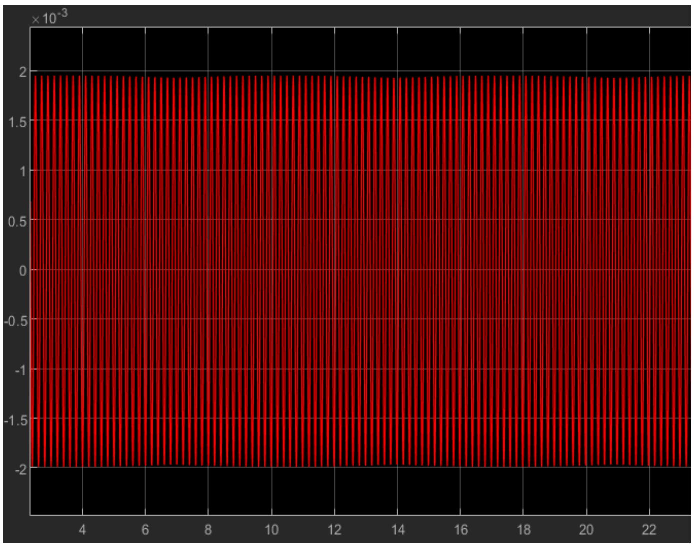


Fig-15: Variance of Derivative of Linear Potential Energy

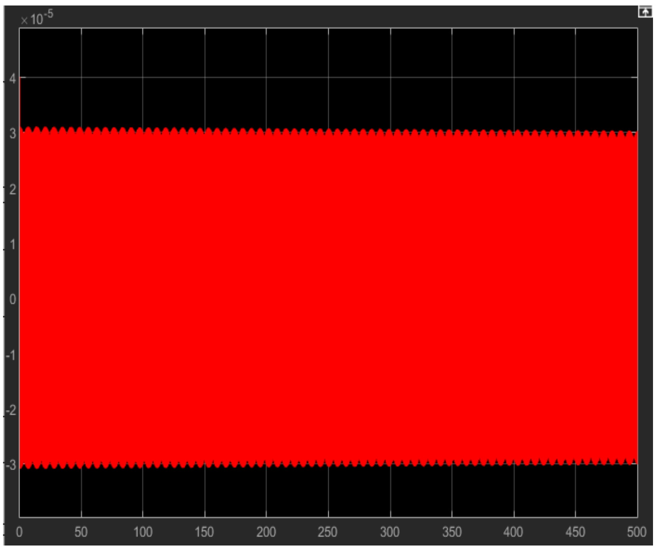


Fig-16: Variance of Derivative of Linear Kinetic Energy

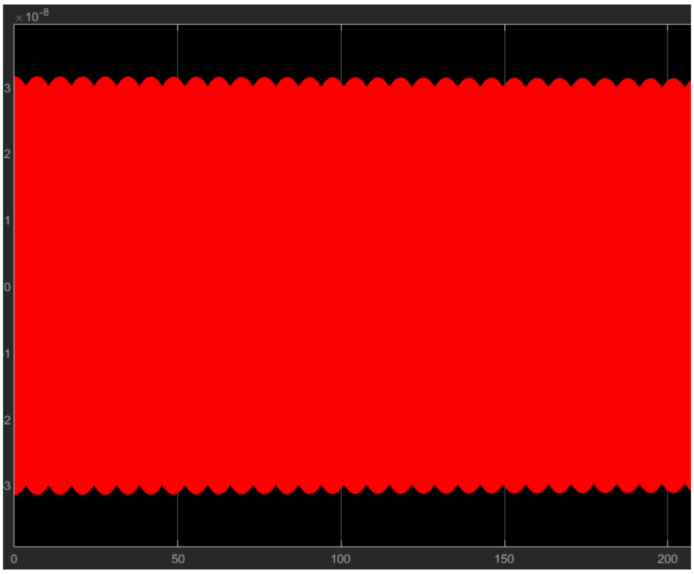


Fig-17: Variance of Derivative of Rotary Potential Energy

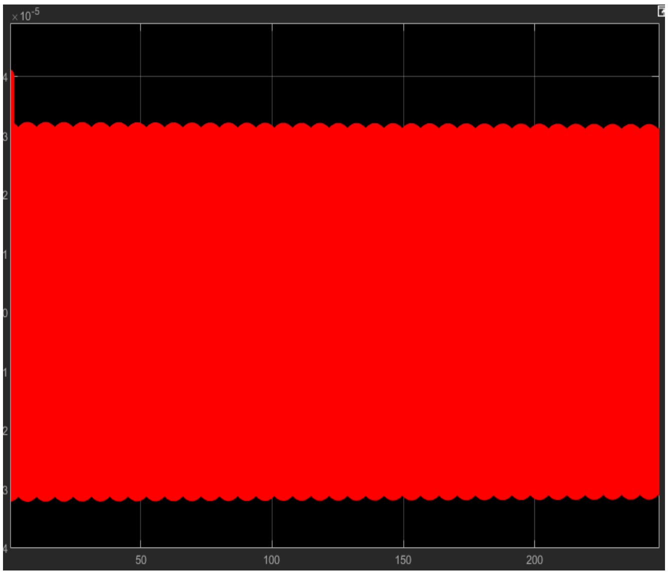


Fig-18: Variance of Derivative of Rotary Kinetic Energy

- The impulsive response shows oscillatory derivative functions, which makes the design of the suitable Lyapunov function difficult to generate through simple algebraic operations.
- The desired Lyapunov function should have the following proportion constant:
  - K_{KE1}, $K_{PE\ R} < 0$
  - $K_{KE\ R}$, $K_{PE1} > 0$
  - $K_{KE1} = (-1000) * K_{KE1}$
  - $K_{PER} = (-100) * K_{PER}$

Lyapunov Function (C): $(x_{1ref} - x_1)^2 - 1000 * x_2^2 - 100 * x_3^2 + x_4^2$

- The basis of the choice was to eliminate the oscillations and produce somewhat constant line by superimposing the scaled equal magnitudes.

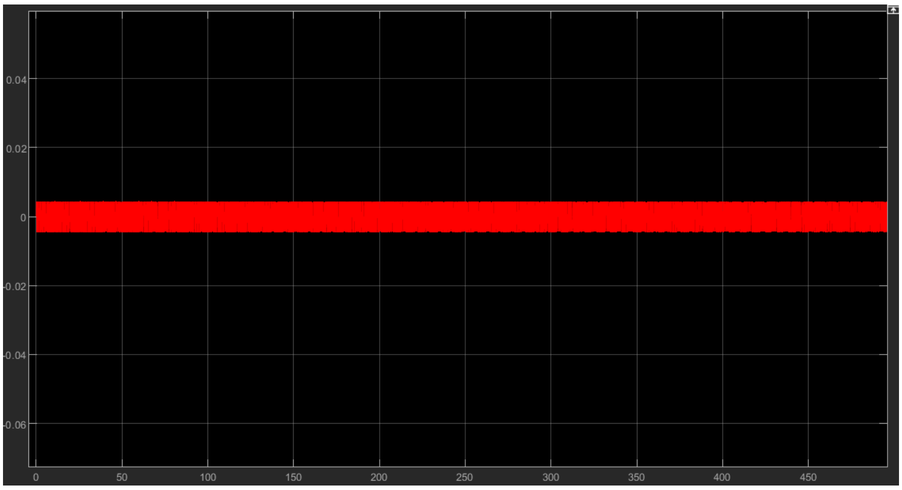


Fig-19: Lyapunov Response

- The obtained response doesn't satisfy the criteria of negative definite and does not meet the expectation, though the magnitude is decreased significantly.'

<u>Customised Input:</u> u(t)-2*u(t-5) + u(t-10)

Derivatives of Energy Profiles:

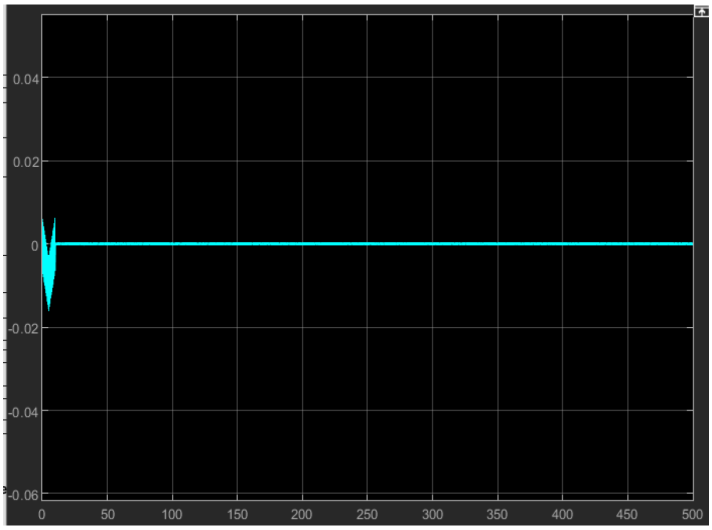


Fig-20: Variance of Derivative of Linear Potential Energy

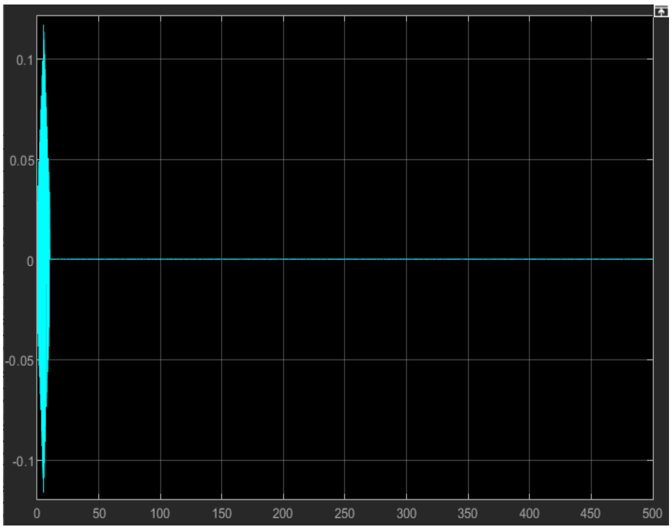


Fig-21: Variance of Derivative of Linear Kinetic Energy

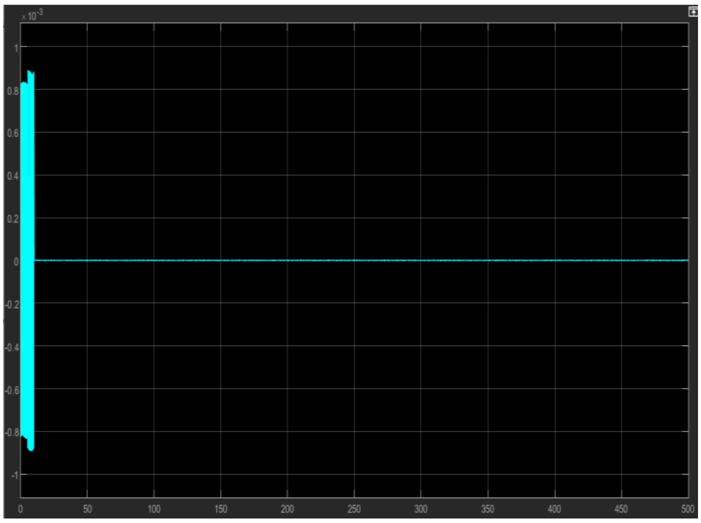


Fig-22: Variance of Derivative of Rotary Potential Energy

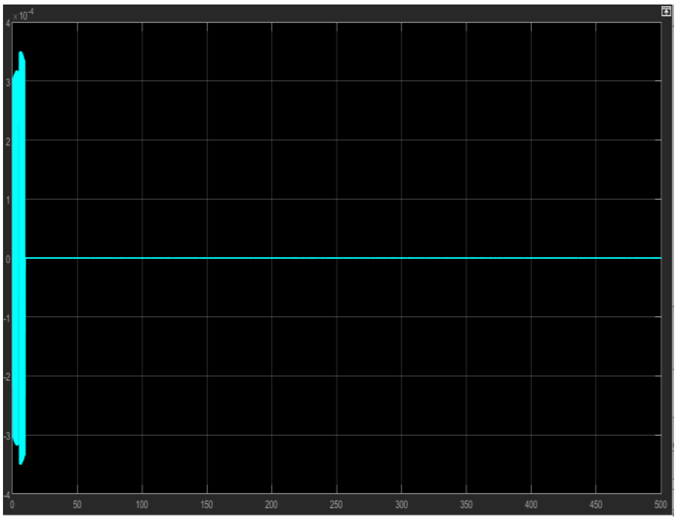


Fig-23: Variance of Derivative of Rotary Kinetic Energy

- It could be observed that the graphs **except for derivative of linear energy** are **symmetrical about x axis.**
- The linear energy provides us with more **negative nature** of the derivative since the direction of the force is reversed in the input after a certain interval of time.
- Since we require the **negative definite derivative** for the **best** representation, the inclusion of linear energy derivative proves to be must.
- It could also be observed that the magnitude of **the rotatory energy** is less than that of linear energy by a **factor of more than ten** times.
  - K_{KE1}, $K_{KE\,R} = 0$
  - $K_{KE1}= 0.001* K_{KE1}$
  - $K_{PE1} < 0$
  - $K_{PER}= (-2.67) *K_{PE1}$

Lyapunov Function (D): - $0.001* [\ (x_{1ref} – x_1)^2\ \ - 2.67.x_3^2]$

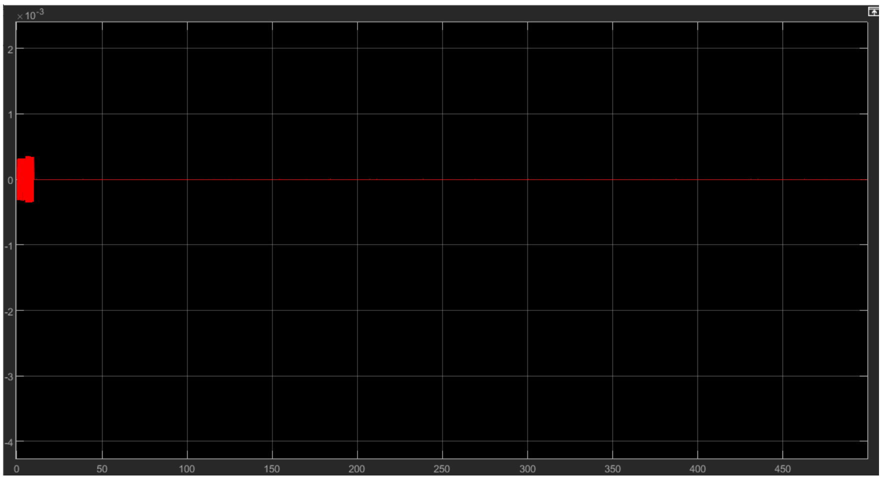
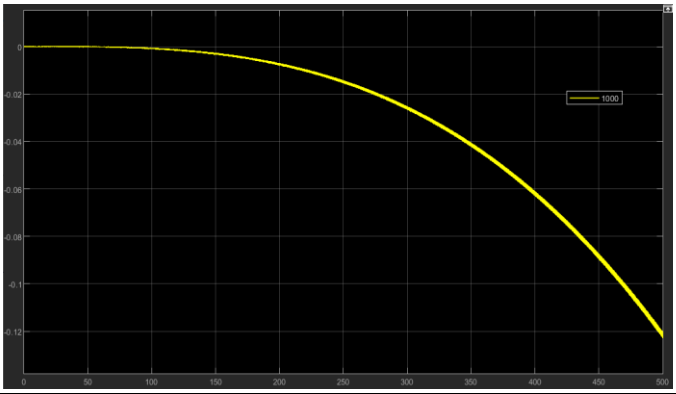


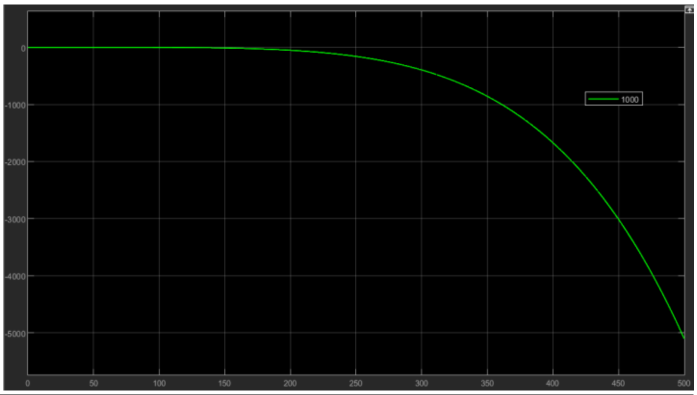
Fig-24: Lyapunov Response

Best Designed Lyapunov

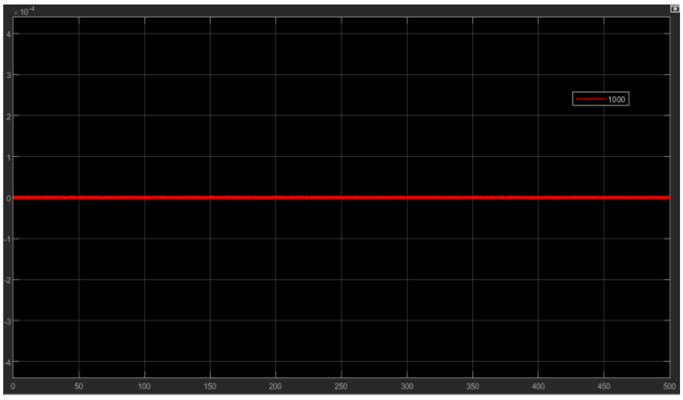
Derivative of the Lyapunov's Function for various inputs:



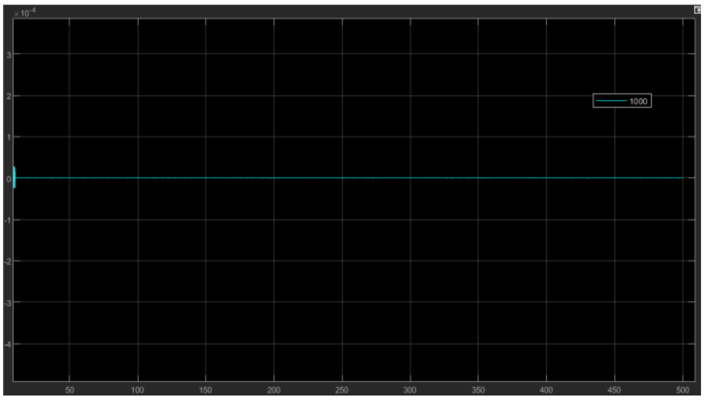
For step input



For Ramp Input



For Impulse Input



For Customised Input

- The designed Lyapunov function shows desired results especially for step and ramp input, also for impulse and the customised input, the system is stable.
- The desired Lyapunov function should have the following proportion constant:
  - K_{KE1}, $K_{KE\ R}$, $K_{PE1} < 0$
  - $K_{PER} > 0$
  - $K_{PE1}= -1$; $K_{KE1}= -0.01$
  - $K_{PER}= 0.02$; $K_{PE1}=-0.05$

Lyapunov Function (E): - $(x_{1ref} - x_1)^2 -0.01*x_2^2 + 0.02*.x_3^2 - 0.05.x_4^2$

Time Frame Analyses:

Comparison Table (at 450 seconds):

State comparisons:

Input	x1	x2	x3	x4	y
Step u(t)	100	0.50	0.00005	0.00002	100
Ramp	16000	105	0.0004	0.0002	16000
Impulse	0.035	4*e (-3)	6*e (-4)	2*e (-3)	0.05
Customised Input: **u(t)-2*u(t-5) + u(t-10)**	0.045	5*e (-4)	7*e (-5)	2*e (-4)	0.045

Comparison for various states

For ml=1000kgs

Input	Lyapunov Function	Derivative of Lyapunov
Step u(t)	1*e (-4)	-88
Ramp	2.43*e (-4)	-204
Impulse	0.991	-0.0004
Customised Input: **u(t)-2*u(t-5) + u(t-10)**	9.56*e (-4)	-0.06

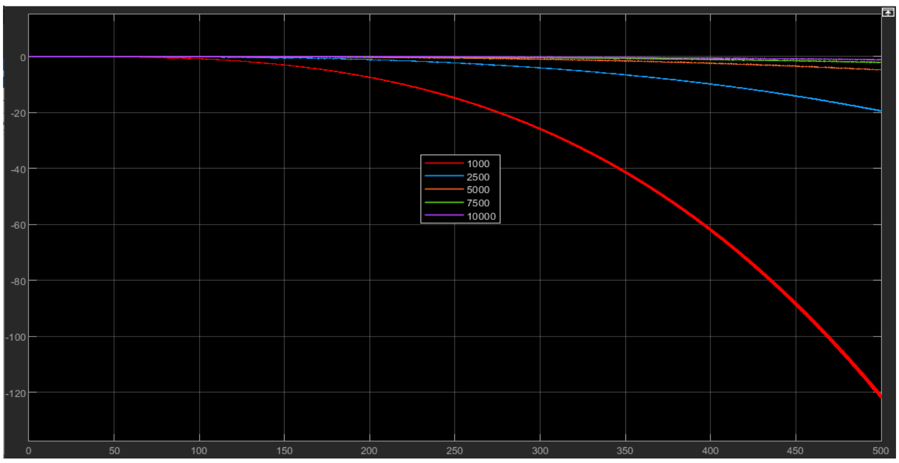
Above: All state Designed Lyapunov's except (E)

Comparison of Lyapunov's Functions

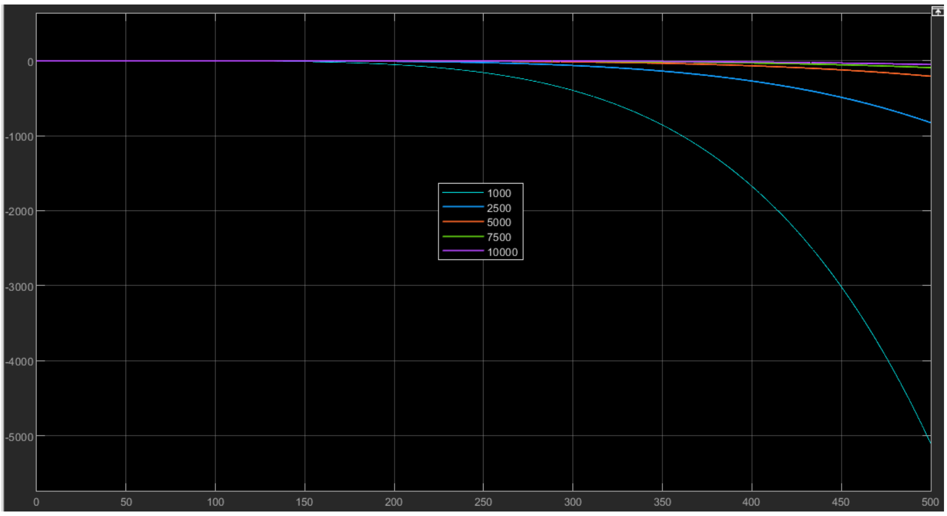
Input	K_{PE1}	K_{KE1}	K_{PER}	K_{KER}
Lyapunov (A)	-1	+1	+1	+1
Lyapunov (B)	-0.001	+1	-1	+1
Lyapunov (C)	+1	-1000	-100	+1
Lyapunov (D)	-0.001	0	+0.00267	0
Best Designed Lyapunov (E)	-1	-0.01	0.02	-0.05

Variations with Variable Load:

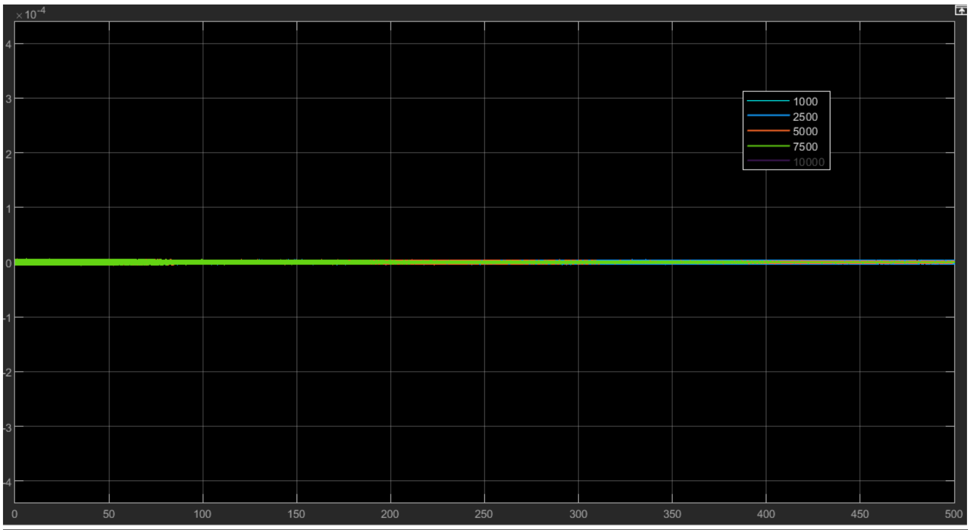
Step Response using Lyapunov (E):



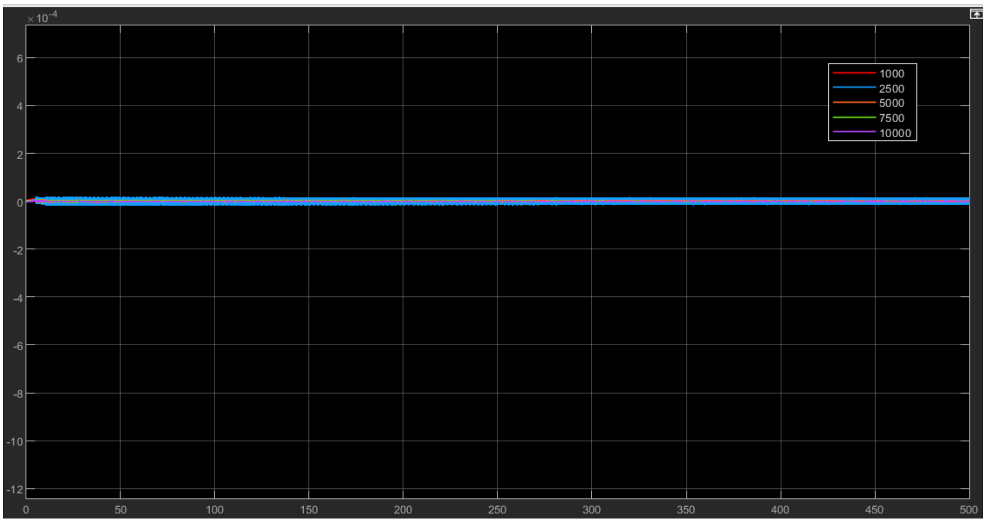
Ramp Response using Lyapunov (E):



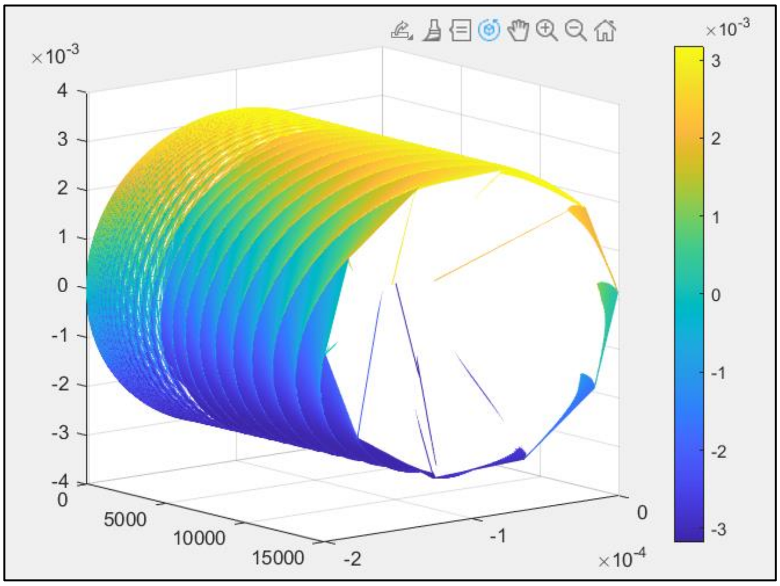
Impulse Response using Lyapunov (E):



Customised Input using Lyapunov (E):



- It could be well observed that **with increase in load mass** the Lyapunov function **tends to destabilise** in each case.
- Though **none of the derivative crossed the x axis** until **10,000 kgs** of load mass, but gradual upward increase is observed.



Multi-Dimensional Plot for Lyapunov Function (E)
with respect to states

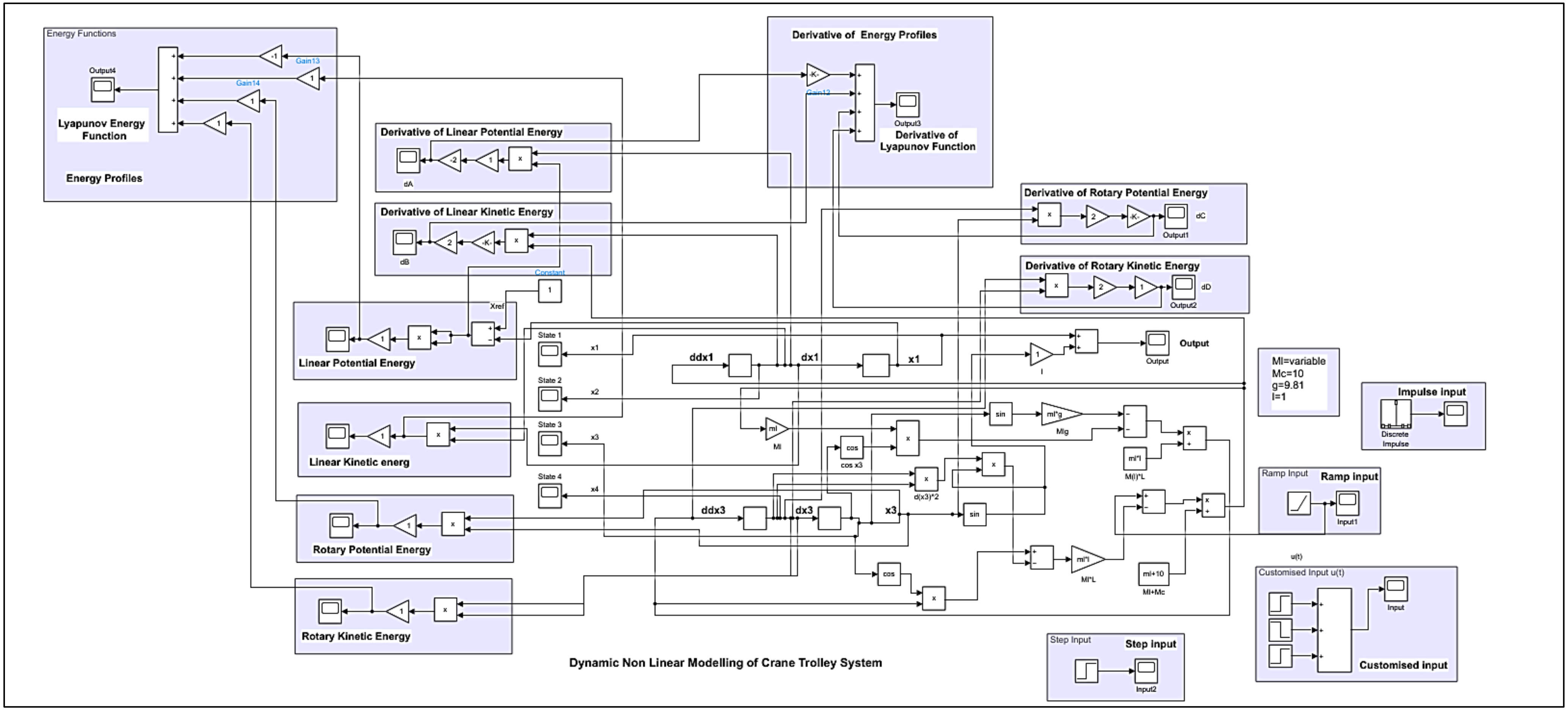
The plot of the designed Lyapunov (E) function describes the convergence of the system to zero, as desired.

Conclusion:

The system was analysed for various inputs and various Lyapunov functions (five) were obtained. The Lyapunov model was build in Simulink and its variations were observed and noted, the load mass was also varied to observe its impact.

- The Lyapunov function was first designed for the specific Inputs, thus 4 Lyapunov's functions were generated to show best response to the specific applied input.
- An overall Lyapunov Function was designed, capable of providing stability at all the applied inputs.

Simulink Model:



Thankyou!