Engineering Mathematics I-(BAS-103)
Unit 3 Differential Calculus II
Tutorial 5

Que1. Find the stationary points of $f(x,y)= x^3 + y^3 + 3axy$, a>0 **[2018-19]**

Que2. Find the extreme value of the function $f(x,y) = sinx + \ siny + \sin(x+y)$

Que3. Find the stationary points of $f(x,y)$=$x^3y^2(12-x-y)$ satisfying the condition $x > 0, y > 0$ and examine their nature.

Que4. Find the critical points of $f(x,y)= x^3 + y^3 - 3axy$, a>0 **[2021-22]**

Que5. Show that the rectangular solid of maximum volume that can be inscribed in a given sphere is a cube.

Que6. Find the extreme value of the function $f(x,y) = xy + \frac{a^3}{x}+\frac{a^3}{y}$

Que7. Divide a number 24 into three parts such that the products of first, square of second and cube of third is maximum. **[2013-14], [2020-21]**

Que8. Find the volume of largest rectangular parallelepiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ **[2019-20]**

Que9. Using Lagrange's method of maxima & minima find the dimension of rectangular box of maximum capacity whose surface area is given when (i) Box is open at top (ii) Box is closed. **[2015-16]**

Que10. Divide a number K into three parts such that the products of first, square of second and cube of third is maximum. **[2016-17]**

Que11. Find the maximum value of the function $f(xyz) = (z - 2x^2 - 2y^2)$ where $3xy - z + 7 = 0$. **[2016-17]**

Que 12. Using Lagrange's method of maxima & minima find the shortest distance from the point (1,2,-1) to sphere x^2+y^2+z^2=24. **[2014-15] [2018-19], [2017-18]**

Que13. A rectangular box open at the top is to have 32cc.By using Lagrange's method of multiplier. Find the dimensions of the box requiring least material for its construction. **[2022-23],[2021-22],[2014-15]**

Que14. The pressure P at any point(x,y,z) is P=400xy z^2 Find the highest pressure at surface of unit sphere x^2+y^2+z^2=1 by using Lagrange's method. **[2023-24]**

Que15. Use Lagrange multiplier method find the maximum value of $f(x,y,z) = x^m y^n z^p$ such that $x + y + z = a$

Answers

1. $(0,0), (-a,-a)$

2. $f(x,y)$ has max. value at $\left(\frac{\pi}{3},\frac{\pi}{3}\right)$ $f_{max}=\frac{3\sqrt{3}}{2}$

3. $f(x,y)$ has max. value at $(6,4)$

4. $(0,0), (a,a)$

6. $f(x,y)$ has min. value at (a,a) $f_{min}=3a^2$

7. First part=4, Second Part=8, Third Part=12

8. $\frac{8abc}{3\sqrt{3}}$

9. When box is open Length=$\sqrt{\frac{S}{3}}$, Breadth=$\sqrt{\frac{S}{3}}$ and height=$\frac{1}{2}\sqrt{\frac{S}{3}}$

When box is closed Length=$\sqrt{\frac{S}{6}}$, Breadth=$\sqrt{\frac{S}{6}}$ and height=$\sqrt{\frac{S}{6}}$ where S is surface area of box

10. First part=$\frac{K}{6}$, Second Part=$\frac{K}{3}$, Third Part=$\frac{K}{2}$

11. $f(x,y,z)$ has max. value at $(0,0,7)$ $f_{max}=7$

12. Shortest distance is $\sqrt{6}$
13. Dimensions of box is 4,4,2
14. Highest pressure is 50 at point$\left(\pm\frac{1}{2},\pm\frac{1}{2},\pm\frac{1}{\sqrt{2}}\right)$
15. $\frac{m^m n^n p^p a^{m+n+p}}{(m+n+p)^{m+n+p}}$