Engineering Mathematics I-(BAS-103)
Unit 1 Matrices
Tutorial 8

Que 1. Verify Cayley - Hamilton theorem for A = $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$. Hence find inverse of A.

Also express the polynomial $B = A^6 - 6A^5 + 9A^4 - 2A^3 + 12A^2 + 23A - 9I$ **[2022-23], [2017-18] ,[2015-16]**

Que 2. Verify Cayley Hamilton theorem for matrix A=$\begin{bmatrix} 4 & 0 & 1 \\ 0 & 1 & 2 \\ 1 & 0 & 1 \end{bmatrix}$and hence find A^{-1}. **[2019-20]**

Que 3. Determine A^{-1} , A^{-2} and A^{-3} if A = $\begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{bmatrix}$by using Cayley Hamilton theorem **[2023-24]**

Que 4. Check whether the given vectors are linearly dependent or linearly independent.
$(\boldsymbol{i})(1,6,4), (0,2,3)\ and (0,1,2)$ **[2019-20]** $(\boldsymbol{ii})(1,-1,1), (2,1,1)\ and (3,0,2)$ **[2021-22]**

Que 5. Find the product and sum of Eigen values of matrix A= $\begin{bmatrix} 8 & -4 \\ 2 & 2 \end{bmatrix}$ **[2023-24]**

Que 6. Find the eigenvalue of the matrix $\begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix}$ corresponding to the eigen vector $\begin{bmatrix} \mathbf{101} \\ \mathbf{101} \end{bmatrix}$. **[2016-17]**

Que 7. If the matrix A = $\begin{bmatrix} -2 & 1 & 3 \\ 0 & 3 & 5 \\ 0 & 0 & -2 \end{bmatrix}$ then find the eigenvalues of $A^3 + 5A + 8I$ **[2021-22]**

Que 8. Find the Eigen values and corresponding eigen vectors of matrix

$(\boldsymbol{i})$ $\begin{bmatrix} 2 & 1 & 1 \\ 2 & 3 & 4 \\ -1 & -1 & -2 \end{bmatrix}$ **[2022-23]** $(\boldsymbol{ii})$ $\begin{bmatrix} 2 & 1 & 1 \\ 2 & 3 & 2 \\ 3 & 3 & 4 \end{bmatrix}$ **[2021-22]** $(\boldsymbol{iii})$ $\begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{bmatrix}$ **[2023-24]**

Que 9. For what value of x the Eigenvalues of given matrix A are real A =$\begin{bmatrix} 10 & 5+i & 4 \\ x & 20 & 2 \\ 4 & 2 & -10 \end{bmatrix}$ **[2016-17]**

Que 10. Show that following matrices are unitary.

$(\boldsymbol{i})$ A=$\frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{bmatrix}$ where ω is complex cube root of unity. **[2016-17]**

$(\boldsymbol{ii})$ A=$\begin{bmatrix} \alpha + i\gamma & -\beta + i\delta \\ \beta + i\delta & \alpha - i\gamma \end{bmatrix}$ if $\alpha^2 + \beta^2 + \gamma^2 + \delta^2 = 1$ **[2017-18]**

$(\boldsymbol{iii})$ A = $\frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$ unitary**.** **[2020-21]**

Que 11. If A is Hermitian matrix then show that iA skew Hermitian matrix . **[2022-23]**

Answers

1. $A^{-1}=\frac{1}{4}\begin{bmatrix}3 & 1 & -1\\ 1 & 3 & 1\\ -1 & 1 & 3\end{bmatrix}$, $B=5A-I=\begin{bmatrix}9 & -5 & 5\\ -5 & 9 & -5\\ 5 & -5 & 9\end{bmatrix}$

2. $A^{-1}=\frac{1}{3}\begin{bmatrix}1 & 0 & -1\\ 2 & 3 & -8\\ -1 & 0 & 4\end{bmatrix}$

3. $A^{-1}=\begin{bmatrix}\frac{1}{4} & \frac{3}{2} & \frac{3}{2}\\ \frac{-1}{4} & \frac{3}{2} & \frac{1}{2}\\ \frac{1}{4} & \frac{-5}{2} & \frac{-3}{2}\end{bmatrix}$, $A^{-2}=\begin{bmatrix}\frac{1}{16} & \frac{-9}{8} & \frac{-9}{8}\\ \frac{-5}{16} & \frac{5}{8} & \frac{-3}{8}\\ \frac{5}{16} & \frac{3}{8} & \frac{11}{8}\end{bmatrix}$ $A^{-3}=\begin{bmatrix}\frac{1}{64} & \frac{39}{32} & \frac{39}{32}\\ \frac{-21}{64} & \frac{45}{32} & \frac{13}{32}\\ \frac{21}{64} & \frac{-77}{32} & \frac{-45}{32}\end{bmatrix}$

4. (i) Linearly independent. (ii) Linearly dependent.

5. Products of Eigenvalues=24, Sum of Eigenvalue=10

6. 6

7. $-10, 50, -10$

8. (i) 3,-1,1: $k_1\begin{bmatrix}2\\ 3\\ -1\end{bmatrix}$, $k_2\begin{bmatrix}0\\ -1\\ 1\end{bmatrix}+k_3\begin{bmatrix}1\\ -1\\ 0\end{bmatrix}$ (ii) 1,1,7: $k_1\begin{bmatrix}1\\ 2\\ 3\end{bmatrix}$, $k_2\begin{bmatrix}-1\\ 1\\ 0\end{bmatrix}+k_3\begin{bmatrix}-1\\ 0\\ 1\end{bmatrix}$

(iii) 1,2,2: $k_1\begin{bmatrix}1\\ 1\\ -1\end{bmatrix}$, $k_2\begin{bmatrix}2\\ 1\\ 0\end{bmatrix}$

9. $x=5+i \; or \; x=5-i$