Engineering Mathematics I-(BAS-103)
Unit 3 Differential Calculus II
Tutorial 4

Que1 *If* $f(x) = x^3 - 2x + 5$, Find the value of $f(2.001)$ with the help of Taylor's Theorem. Find the approximate change in the value of $f(x)$ when x changes from 2 to 2.001

Que2. Apply Maclaurin's theorem to show that $e^x cosx = 1 + x - \frac{2x^3}{3!} - \frac{2^2x^4}{4!} - \frac{2^2x^5}{5!}$ ...

Que3. Expand by Maclaurin's theorem $\frac{e^x}{1+e^x}$ as far as x^3

Que4. Expand $f(x, y) = y^x$ about (1,1) up to second degree term and hence evaluate $(1.02)^{1.03}$ **[2022-23]**

Que5. Expand e^xlog(1+y) in powers of x and y up to terms of third degree. **[2014-15]**

Que6. Find the Taylor series expansion of $f(x, y) = e^x \cos y$ about point $\left(1, \frac{\pi}{4}\right)$ by Taylor's series. **[2023-24]**

Que7. Expand $(x^2y + siny + e^x)$ in powers of $(x - 1)$ and $(y - \pi)$ **[2014-15]**

Que8. Expand $\tan^{-1}\frac{y}{x}$ in the neighbourhood of (1,1) up to and inclusive of second degree terms. Hence compute $f(1.1, 0.9)$ approximately. **[2013-14]**

Que9. Expand x^y in powers of $(x - 1)$ and $(y - 1)$ upto third degree term and hence evaluate $(1.1)^{1.02}$ **[2021-22]**

Que10. Express the function $f(x, y) = x^2 + 3y^2 - 9x - 9y + 26$ as Taylor's series expansion about the point (1,2) **[2017-18],[2016-17]**

Answers

1. $f(2.001) = 9 + (.001)10 + \frac{1}{2!}(.001)^2\ 12 + \frac{1}{3!}(.001)^3\ 6$, the approximate change is 0.01

3. $\frac{e^x}{1+e^x} = \frac{1}{2} + \frac{x}{4} - \frac{x^3}{48} + \cdots$

4. $y^x = 1 + (y-1) + (x-1)(y-1) + \frac{1}{2}(x-1)(y-1)^2 + \cdots$. $(1.02)^{1.03} = 1.020606$

5. $e^x log(1+y) = y + xy - \frac{1}{2}y^2 + \frac{1}{2}x^2y - \frac{1}{2}xy^2 + \frac{1}{3}y^3 + \cdots$ ...

6. $e^x \cos y = \frac{e}{\sqrt{2}}\left[1 + (x-1) - \left(y - \frac{\pi}{4}\right) + \frac{1}{2}(x-1)^2 - (x-1)\left(y - \frac{\pi}{4}\right) - \frac{1}{2}\left(y - \frac{\pi}{4}\right)^2 + \cdots\right]$

7. $x^2y + siny + e^x = \pi + e + (x-1)(2\pi + e) + \frac{1}{2}(x-1)^2(2\pi + e) + 2(x-1)(y-\pi) + \cdots$.

8. $\tan^{-1}\frac{y}{x} = \frac{\pi}{4} + \frac{1}{2}[(y-1) - (x-1)] + \frac{1}{4}[(x-1)^2 - (y-1)^2] - \frac{1}{12}[(x-1)^3 + 3(x-1)^2(y-1) - 3(x-1)(y-1)^2 - (y-1)^3]$
$f(1.1, 0.9) = 0.6857$

9. $x^y = 1 + (x-1) + (x-1)(y-1) + \frac{1}{2}(y-1)(x-1)^2 + \cdots$. $(1.1)^{1.02} = 1.1021$

10. $x^2 + 3y^2 - 9x - 9y + 26 = 12 - 7(x-1) + 3(y-2) + (x-1)^2 + 3(y-2)^2$