

# Engineering Mathematics I-(BAS-103)

## Unit 2 Differential Calculus I

### Tutorial 3

**Que1.** If  $e^{-z/x^2-y^2} = x - y$  then show that  $y \frac{\partial z}{\partial x} + x \frac{\partial z}{\partial y} = x^2 - y^2$  [2016-17]

**Que2.** If  $u = f(r)$  and  $r^2 = x^2 + y^2$  show that  $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r} f'(r)$  [2015-16]

**Que3.** If  $x^2 = au + bv$ ,  $y^2 = au - bv$  Find  $\left(\frac{\partial u}{\partial x}\right)_y \left(\frac{\partial x}{\partial u}\right)_v$  [2017-18]

**Que4.** If  $z = x^2 \tan^{-1} \frac{y}{x} - y^2 \tan^{-1} \frac{x}{y}$ , Find the value of  $\frac{\partial^2 u}{\partial x \partial y}$  [2018-19]

**Que5.** If  $u = x^3 y^2 \sin^{-1} \left(\frac{y}{x}\right)$  then find  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$  [2018-19]

**Que6.** If  $v = (x^2 + y^2 + z^2)^{-1/2}$  then find  $xv_x + yv_y + zv_z$  [2015-16]

**Que 7.** If  $u = \sin^{-1} \left(\frac{x^3+y^3}{\sqrt{x}+\sqrt{y}}\right)$  prove that  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \frac{5}{2} \tan u$  [2022-23]

**Que8.** If  $u = \sec^{-1} \left(\frac{x^3-y^3}{x+y}\right)$  then prove that  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 2 \cot u$ . Also evaluate  $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2}$  [2020-21]

**Que9.** If  $u = y^2 e^{\frac{y}{x}} + x^2 \tan^{-1} \frac{x}{y}$  then prove that [2023-24]

$$(i) x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 2u \quad (ii) x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = 2u$$

**Que10.** If  $u = \frac{x^2 y^2}{x^2 + y^2} + \cos \left(\frac{xy}{x^2 + y^2}\right)$  then prove that  $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = 2 \frac{x^2 y^2}{x^2 + y^2}$  [2022-23]

**Que11.** If  $u = \sin^{-1} \left(\frac{x^3+y^3+z^3}{ax+by+cz}\right)$  prove that  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} = 2 \tan u$  [2017-18]

**Que12.** If  $u = \cos^{-1} \left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$  then show that  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + \frac{1}{2} \cot u = 0$  [2018-19]

**Que13.** If  $w = \sqrt{x^2 + y^2 + z^2}$  and  $x = u \cos v$ ,  $y = u \sin v$ ,  $z = uv$  Show that  $u \frac{\partial w}{\partial u} - v \frac{\partial w}{\partial v} = \frac{u}{\sqrt{1+u^2}}$  [2016-17]

**Que14.** Prove the following

(i) If  $v = f(2x - 3y, 3y - 4z, 4z - 2x)$  then  $\frac{1}{2} \frac{\partial v}{\partial x} + \frac{1}{3} \frac{\partial v}{\partial y} + \frac{1}{4} \frac{\partial v}{\partial z} = 0$  [2019-20]

(ii) If  $u = f(r, s, t)$   $r = \frac{x}{y}$ ,  $s = \frac{y}{z}$ ,  $t = \frac{z}{x}$  then  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} = 0$  [2017-18]

(iii) If  $u = f(y - z, z - x, x - y)$  then  $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$  [2020-21]

(iv) If  $Z = f(x, y)$  and  $x = e^u + e^{-v}$ ,  $y = e^{-u} - e^v$  then  $\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = x \frac{\partial z}{\partial x} - y \frac{\partial z}{\partial y}$  [2023-24]

**Que15.** Find  $du/dt$  as a total derivative

(i)  $x = e^{2t}$ ,  $y = e^{2t} \cos 3t$ ,  $z = e^{2t} \sin 3t$  [2014-15] (ii)  $x = a \cos t$ ,  $y = b \sin t$  [2019-20]

### Answers

**Ans 3.**  $\frac{1}{2}$

**Ans 4.**  $\frac{x^2 - y^2}{x^2 + y^2}$

**Ans 5.**  $5x^3 y^2 \sin^{-1} \left(\frac{y}{x}\right)$

**Ans 6.**  $-(x^2 + y^2 + z^2)^{-1/2}$  **Ans 8.**  $-2 \cot u (2 \operatorname{cosec}^2 u + 1)$

**Ans 15** (i)  $2e^{4t}$  (ii)  $-3a^3 \cos^2 t \sin t + 3b^3 \sin^2 t \cos t$