

Engineering Mathematics I-(BAS-103)

Unit 4 Multiple Integration

Tutorial 9

Que 1. Evaluate the following integrals

- (i) $\int_0^1 \int_0^{x^2} x e^y dx dy$ [2017-18] (ii) $\int_0^3 \int_0^1 (x^2 + 3y^2) dy dx$ [2019-20] (iii) $\int_0^1 \int_0^1 \frac{dx}{\sqrt{1-x^2}} \frac{dy}{\sqrt{1-y^2}}$ [2015-16]
(iv) $\int_0^1 \int_0^{x^2} e^{\frac{y}{x}} dx dy$ [2021-22] [2020-21] [2018-19]

Que2. Find by using double integration the area enclosed by the curves $y = 2 - x$ and $y^2 = 2(2 - x)$ [2021-22]

Que3. Evaluate $\iint y dx dy$ over the part of the plane bounded by the line $y = x$ and the parabola $y = 4x - x^2$. [2022-23]

Que4. Find the area between the parabola $y = 4x - x^2$ and above the line $y=x$ [2019-20]

Que5. Find the area bounded by the curves $y^2 = x$ and $x^2=y$ [2021-22]

Que 6. Evaluate the following integrals

- (i) $\int_0^1 \int_1^2 \int_2^3 xyz dx dy dz$ (ii) $\int_0^1 \int_1^x \int_0^{x+y} dx dy dz$ [2021-22] (iii) $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyz dx dy dz$. [2016-17]

Que7. Evaluate $\iiint_R (x - 2y + z) dx dy dz$, where R is the region determine by $0 \leq x \leq 1, 0 \leq y \leq x^2, 0 \leq z \leq x + y$

Que 8. Evaluate $\int \int \int_R x + y + z dx dy dz$ where R; $0 \leq x \leq 1, 1 \leq y \leq 2, 2 \leq z \leq 3$ [2017-18], [2015-16]

Que 9. Find the volume of the region bounded by the surface $y = x^2, x = y^2$ and the planes $z = 0, z = 3$ [2019-20]

Que 10. If the volume of an object expressed in spherical coordinates as following $V = \int_0^{2\pi} \int_0^\pi \int_0^1 r^2 \sin \theta dr d\theta d\phi$. Evaluate V [2016-17]

Que 11. Find the volume bounded by the cylinder $x^2 + y^2 = 4$ and the plane $y + z = 4$ and $z = 0$ [2021-22]

ANSWER

1. (i) $\frac{e}{2} - 1$ (ii) 12 (iii) $\frac{\pi^2}{4}$ (iv) $\frac{1}{2}$
2. $\frac{2}{3}$
3. $\frac{54}{5}$
4. 4.5
5. $\frac{1}{3}$
6. (i) $\frac{15}{8}$ (ii) $-\frac{1}{2}$ (iii) $\frac{a^6}{48}$
7. $\frac{8}{35}$
8. $\frac{9}{2}$
9. 1
10. $\frac{2\pi^2}{3}$
11. 16π