Engineering Mathematics I-(BAS-103)
Unit 2 Differential Calculus I
Tutorial 1

Que 1. Find the n^{th} derivative of

$(\boldsymbol{i})\ y = \frac{x}{x^2+a^2}$ $(\boldsymbol{ii}) y = e^x sinx\ cosx$ $(\boldsymbol{iii})\ y = x^2 e^x$ $(\boldsymbol{iv}) y = \frac{x^2}{(x+2)(2x+3)}$. $(\boldsymbol{v}) y = \frac{ax+b}{cx+d}$

$(\boldsymbol{vi})\ y = \frac{x^n-1}{x-1}$ $(\boldsymbol{vii})$ y = $\log\sqrt{\frac{(2x+1)}{(x-3)}}$ $(\boldsymbol{viii}) y = x^{n-1} logx.$ **[2017-18]**

Que 2. If $y = \tan^{-1}\left(\frac{1+x}{1-x}\right)$ then show that $y_n = (-1)^{n-1}\ n-1!\ sin^n\theta \sin n\theta$ where $\theta = \tan^{-1}\frac{1}{x}$

Que 3. If $u = \sin nx + \cos nx$ then show that $u_r = n^r\{1+(-1)^r\ sin2nx\}^{\frac{1}{2}}$ where u_r is the r^{th} differential coefficient of u w.r.t. x **.**Hence show that $u_8(\pi) = \left(\frac{1}{2}\right)^{\frac{31}{2}}$ when $n = \frac{1}{4}$ **[2017-18]**

Que 4. If $I_n = \frac{d^n}{dx^n}(x^n logx)$, then show that $I_n = nI_{n-1} + n - 1!$ **[2016-17]**

Que 5. If $y = \cos^{-1} x$, prove that $(1-x^2)y_2 - xy_1 = 0$ **.** **[2022-23]**

Que 6. If $y = e^{tan^{-1}x}$ then show that $(1+x^2)y_{n+2} + [2(n+1)x-1]y_{n+1} + n(n+1)y_n = 0$ **[2020-21], [2017-18]**

Que 7. If $y = e^{mcos^{-1}x}$ *then show that* $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+m^2)y_n = 0$. Hence find y_n at x=0 **[2019-20]**

Que 8. If $y = \left[x+\sqrt{x^2+1}\right]^m$ *then* find $y_n(0)$ **[2017-18], [2021-22]**

Que 9. If $y = \sin(msin^{-1}x)$ *then* find $y_n(0)$ **[2020-21], [2018-19]**

Que 10. If $y\sqrt{x^2-1} = \log\left[x+\sqrt{x^2-1}\right]$ *then* prove that $(x^2-1)y_{n+1} + (2n+1)xy_n + n^2 y_{n-1} = 0$ **[2022-23]**

Que 11. If $sin^{-1}y = 2\log(x+1)$ *then show that* $(x+1)^2\ y_{n+2} + (2n+1)(x+1)y_{n+1} + (n^2+4)y_n = 0$ **[2018-19]**

Que 12. If $y^{1/m} + y^{-1/m} = 2x$, Prove that $(x^2-1)y_{n+2} + (2n+1)xy_{n+1} + (n^2-m^2)y_n = 0$ [**2014-15]**

Que13. If $y = \left(\frac{1+x}{1-x}\right)^{1/2}$ *prove that* $(1-x^2)y_n - [2(n-1)x+1]y_{n-1} - (n-1)(n-2)y_{n-2} = 0$

Que14. If $y = \cos(msin^{-1}x)$ then show that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} + (m^2-n^2)y_n = 0$. Also find $y_n(0)$ **[2023-24]**

Answers

1. $(\boldsymbol{i})\ \ y_n = \frac{(-1)^n n!}{a^{n+1}} sin^{n+1}\theta \cos(n+1)\theta$ where $\theta = \tan^{-1}\frac{a}{x}$

1. $(\boldsymbol{ii})\ y_n = \frac{1}{2}\ 5^{\frac{n}{2}} e^x \sin(2x + n\tan^{-1} 2)$

1. $(\boldsymbol{iii})\ y_n = e^x[x^2 + 2nx + n(n-1)]$

1. $(\boldsymbol{iv}) y_n = \frac{(-1)^n n!}{2}\left[\frac{9.2^n}{(2x+3)^{n+1}} - \frac{8}{(x+2)^{n+1}}\right]$

1. $(\boldsymbol{v}) y_n = \frac{(-1)^n n!(bc-ad)\ \ c^{n-1}}{(cx+d)^{n+1}}$

1. $(\boldsymbol{vi})\ y_n = \mathbf{0}$

1. $(\boldsymbol{vii})\ y_n = \frac{(-1)^{n-1}\ \ n-1!}{2}\left[\frac{2^n}{(2x+1)^n} - \frac{1}{(x-2)^n}\right]$

1 . $(\boldsymbol{viii})\ y_n = \frac{(\boldsymbol{n-1})!}{\boldsymbol{x}}$

7. $y_n(0) = \begin{cases} -m\ \ e^{\frac{m\pi}{2}}(1^2 + m^2)(3^2 + m^2) \ldots\ldots((n-2)^2 + m^2)\ , when\ n\ is\ odd \\ m^2\ e^{\frac{m\pi}{2}}(2^2 + m^2)(4^2 + m^2) \ldots\ldots((n-2)^2 + m^2)\ , when\ n\ is\ even \end{cases}$

8. $y_n(0) = \begin{cases} m\ \ (m^2 - 1^2)(m^2 - 3^2) \ldots\ldots(m^2 - (n-2)^2)\ , when\ n\ is\ odd \\ m^2\ (m^2 - 2^2)(m^2 - 4^2) \ldots\ldots(m^2 - (n-2)^2), when\ n\ is\ even \end{cases}$

9. $y_n(0) = \begin{cases} m\ \ (1^2 - m^2)(3^2 - m^2) \ldots\ldots((n-2)^2 - m^2)\ , when\ n\ is\ odd \\ 0 \qquad , when\ n\ is\ even \end{cases}$

14. $y_n(0) = \begin{cases} 0 \qquad , when\ n\ is\ odd \\ (-1)^{\frac{n}{2}}\ \ m^2\ (m^2 - 2^2)(m^2 - 4^2) \ldots\ldots(m^2 - (n-2)^2), when\ n\ is\ even \end{cases}$