NPTEL DATA SCIENCE FOR ENGINEERS

ASSIGNMENT 5

1. The function min $f(x, y) = 3x + y$ subject to the given constraints $x^2 + y^2 < 10$ is an example of

Solution: c: Multivariate optimization with inequality constraint

In optimization <, >, ≤, ≥ are the popular notation to represent different kinds of inequalities. So, if there is given an objective function with more than one decision variable and having an inequality constraint then it is called as Multivariate optimization with inequality constraint

2. We intend to find the maxima of $\boldsymbol{f(x, y)} = 3\mathbf{x} + \mathbf{y}$ subject to the constraint $x^2 + y^2 = 10$**.**

   The Lagrangian function is:-

Solution: c

$f(x, y) = 3\mathrm{x} + \mathrm{y}$

$s.t\ x^2 + y^2 = 10$

The Lagrangian function is modified version of the objective function with the constraints incorporated.

$\boldsymbol{L}(\mathbf{x}, \mathbf{y}, \boldsymbol{\lambda}) = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{y}) + \boldsymbol{\lambda}(\mathbf{c} - \boldsymbol{g}(\boldsymbol{x}, \boldsymbol{y}))$**,** where $\boldsymbol{g}(\boldsymbol{x}, \boldsymbol{y})$ is the constraint.

$$\boldsymbol{L}(\mathbf{x}, \mathbf{y}, \boldsymbol{\lambda}) = \mathbf{3}\boldsymbol{x} + \boldsymbol{y} + \boldsymbol{\lambda}(\mathbf{10} - \boldsymbol{x}^{\mathbf{2}} - \boldsymbol{y}^{\mathbf{2}})$$

3. The values of the stationary points x^*, y^* and λ^* for the objective function $f(x, y) = 3x + y$ subject to the given constraints $x^2 + y^2 - 10 = 0$ are.

Solution: a

$\nabla L = 0$
$L(\mathrm{x}, \mathrm{y}, \lambda) = 3x + y + \lambda(10 - x^2 - y^2)$

$$\frac{\partial L}{\partial x} = 3 - 2\lambda x = 0 \ \rightarrow 1$$

$$\frac{\partial L}{\partial y} = 1 - 2\lambda y = 0 \rightarrow 2$$

From the constraint,

$$10 - x^2 - y^2 = 0 \rightarrow 3$$

By solving (1) (2) and (3) $\boldsymbol{x^* = \pm 3},\ \boldsymbol{y^* = \pm 1}$ **and** $\boldsymbol{\lambda^* = \pm 0.5}$

4. The values of the stationary points x^*, y^* and λ^* for the objective function $f(x, y) = 5x - 3y$ subject to the given constraints $x^2 + y^2 = 136$ are.

Solution: b

$$\nabla L = 0$$
$$L(\mathrm{x}, \mathrm{y}, \lambda) = 5x - 3y + \lambda(136 - x^2 - y^2)$$

$$\frac{\partial L}{\partial x} = 5 - 2\lambda x = 0 \ \rightarrow 1$$

$$\frac{\partial L}{\partial y} = -3 - 2\lambda y = 0 \rightarrow 2$$

From the constraint,

$$136 - x^2 - y^2 = 0 \rightarrow 3$$

By solving (1) (2) and (3) $\boldsymbol{x^* = \pm 10}, \boldsymbol{y^* = \pm 6}$ **and** $\boldsymbol{\lambda^* = \pm 0.25}$

5. The hessian matrix for the function $f(x, y) = -5x^2 + 4xy + 3y^2 + 2x - y$

Solution: c

$$\frac{\partial f}{\partial x} = -10x + 4y + 2 \ = 0$$

$$\frac{\partial f}{\partial y} = 4x + 6y - 1 = 0$$

$$\frac{\partial^2 f}{\partial x^2} = -10$$

$$\frac{\partial^2 f}{\partial y^2} = 6$$

$$\frac{\partial f}{\partial x\, \partial y} = 4$$

The Hessian matrix of $f(x, y)$ is $\begin{bmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial f}{\partial x\, \partial y} \\ \frac{\partial f}{\partial x\, \partial y} & \frac{\partial^2 f}{\partial y^2} \end{bmatrix} = \begin{bmatrix} -10 & 4 \\ 4 & 6 \end{bmatrix}$

6. The eigen values for the hessian matrix obtained in Q5 are: -

Solution: a

```
> m=matrix(c(-10,4,4,6),nrow=2,ncol=2,byrow=T)
> m
     [,1] [,2]
[1,]  -10    4
[2,]    4    6
> eigen(m)
eigen() decomposition
$values
[1]   6.944272 -10.944272
```

7. A predictive modeling problem where the class label is predicted for the input data is a type of

Solution: b

In machine learning, classification problem refers to a predictive modeling problem where a class label is predicted for the input data.