Assignment-3 (NCERT Class 12)

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Abstract—**This document contains the solution to Question 9 of Exercise 13.5 in Chapter 13 of the NCERT Class 12 Mathematics Textbook.**

Exercise 13.5, Q9. On a multiple choice examination with three possible answers for each of the five questions, what is the probability that a candidate would get four or more correct answers just by guessing ?

Solution: Let $X_i, 1 \leq i \leq 5$ represent 5 Bernoulli random variables with parameter p. Then,

$$\Pr(X_i = k) = \begin{cases} 1-p, & k = 0 \\ p, & k = 1 \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

Let Y be a random variable given by

$$Y = \sum_{i=1}^{i=5} X_i \tag{2}$$

Using Equation 1 the moment generating function of X_i is given by

$$M_Z(X_i) = \sum_{k=-\infty}^{k=\infty} z^{-k} P_X(k) \tag{3}$$

$$= P_X(0) + z^{-1}P_X(1) = (1-p) + pz^{-1} \tag{4}$$

Here all the X_i are independent and identically distributed hence , the moment generating function of Y is

$$M_Y(Z) = E(Z^{-Y}) = E(Z^{-\sum_{i=1}^{i=5} X_i}) \tag{5}$$

$$= \prod_{i=1}^{i=5} E(Z^{-X_i}) \tag{6}$$

$$= [(1-p) + pz^{-1}]^5 \tag{7}$$

$$= \sum_{k=0}^{k=5} z^{-k} (\binom{5}{k}(1-p)^{5-k} p^k) \tag{8}$$

The PMF of the Binomial random variable Y is

$$\Pr(Y = k) = \begin{cases} \binom{5}{k}(1-p)^{5-k}p^k, & 0 \leq k \leq 5 \\ 0, & \text{otherwise} \end{cases} \tag{9}$$

Therefore, the CDF of Y is given by

$$\mathrm{F}_Y(k) = \sum_{i=-\infty}^{i=k} \Pr(Y = i)$$

$$= \begin{cases} 0, & k < 0 \\ \sum_{K=0}^{K=k} \binom{5}{K}(1-p)^{5-K}p^K, & 0 \leq k < 5 \\ 1, & k \geq 5 \end{cases} \tag{10}$$

Probability of 4 or more correct is same as 1 or less incorrect

For this problem we have $p = \frac{2}{3}$. Hence,

$$\mathrm{F}_Y(1) = \sum_{i=0}^{i=1} \binom{5}{i}(1 - \frac{2}{3})^{5-i}(\frac{2}{3})^i \tag{11}$$

$$= (\frac{1}{3})^5 + 5(\frac{1}{3})^4(\frac{2}{3}) \tag{12}$$

$$= \frac{11}{243} \tag{13}$$

Probability is $\frac{11}{243}$