

Assignment-3

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Problem Statement

(NCERT Class 12, Exercise 13.5 Q9) On a multiple choice examination with three possible answers for each of the five questions, what is the probability that a candidate would get four or more correct answers just by guessing ?

Solution

Random Variables

- ① X_i : Bernoulli random variables with parameter $p, 1 \leq i \leq 5$
- ② Y : Binomial random variable given by $Y = \sum_{i=1}^{i=5} X_i$

Moment Generating Function of X_i and Y

$$M_Z(X_i) = \sum_{k=-\infty}^{k=\infty} z^{-k} P_X(k) \quad (1)$$

$$= P_X(0) + z^{-1} P_X(1) = (1 - p) + pz^{-1} \quad (2)$$

$$(3)$$

Moment Generating Function of Y

$$M_Y(Z) = E(Z^{-Y}) = E(Z^{-\sum_{i=1}^{i=5} X_i}) \quad (4)$$

$$= \prod_{i=1}^{i=5} E(Z^{-X_i}) \quad (5)$$

$$= [(1 - p) + pz^{-1}]^5 \quad (6)$$

$$= \sum_{k=0}^{k=5} z^{-k} \binom{5}{k} (1 - p)^{5-k} p^k \quad (7)$$

PMF of Y

$$\Pr(Y = k) = \begin{cases} \binom{5}{k} (1 - p)^{5-k} p^k, & 0 \leq k \leq 5 \\ 0, & \text{otherwise} \end{cases} \quad (8)$$

CDF of Y

$$F_Y(k) = \sum_{i=-\infty}^{i=k} \Pr(Y = i) = \begin{cases} 0, & k < 0 \\ \sum_{K=0}^{K=k} \binom{5}{K} (1-p)^{5-K} p^K, & 0 \leq k < 5 \\ 1, & k \geq 5 \end{cases} \quad (9)$$

Problem parameters

Given:

$$① \quad p = \frac{2}{3}$$

Solution

$$F_Y(1) = \sum_{i=0}^{i=1} \binom{5}{i} \left(1 - \frac{2}{3}\right)^{10-i} \left(\frac{2}{3}\right)^i \quad (10)$$

$$= \left(\frac{1}{3}\right)^5 + 5\left(\frac{1}{3}\right)^4 \left(\frac{2}{3}\right) \quad (11)$$

$$= \frac{11}{243} \quad (12)$$

Probability is $\frac{11}{243}$