# Assignment-3

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# Problem Statement

**(NCERT Class 12, Exercise 13.5 Q9 )** On a multiple choice examination with three possible answers for each of the five questions, what is the probability that a candidate would get four or more correct answers just by guessing ?

# Solution

## Random Variables

1. $X_i$: Bernoulli random variables with parameter $p, 1 \leq i \leq 5$
2. $Y$: Binomial random variable given by $Y = \sum_{i=1}^{i=5} X_i$

## Moment Generating Function of $X_i$ and $Y$

$$M_Z(X_i) = \sum_{k=-\infty}^{k=\infty} z^{-k} P_X(k) \tag{1}$$

$$= P_X(0) + z^{-1} P_X(1) = (1-p) + pz^{-1} \tag{2}$$

$$\tag{3}$$

## Moment Generating Function of $Y$

$$M_Y(Z) = E(Z^{-Y}) = E(Z^{-\sum_{i=1}^{i=5} X_i}) \tag{4}$$

$$= \prod_{i=1}^{i=5} E(Z^{-X_i}) \tag{5}$$

$$= [(1-p) + pz^{-1}]^5 \tag{6}$$

$$= \sum_{k=0}^{k=5} z^{-k} \left( \binom{5}{k} (1-p)^{5-k} p^k \right) \tag{7}$$

## PMF of Y

$$\Pr(Y = k) = \begin{cases} \binom{5}{k}(1-p)^{5-k}p^k, & 0 \leq k \leq 5 \\ 0, & \text{otherwise} \end{cases} \tag{8}$$

## CDF of Y

$$F_Y(k) = \sum_{i=-\infty}^{i=k} \Pr(Y = i) = \begin{cases} 0, & k < 0 \\ \sum_{K=0}^{K=k} \binom{5}{K}(1-p)^{5-K}p^K, & 0 \leq k < 5 \\ 1, & k \geq 5 \end{cases} \tag{9}$$

## Problem parameters

Given:

1. $p = \frac{2}{3}$

# Solution

$$F_Y(1) = \sum_{i=0}^{i=1} \binom{5}{i} (1-\frac{2}{3})^{10-i} (\frac{2}{3})^i \tag{10}$$

$$= (\frac{1}{3})^5 + 5(\frac{1}{3})^4(\frac{2}{3}) \tag{11}$$

$$= \frac{11}{243} \tag{12}$$

Probability is $\frac{11}{243}$