***Abstract*—This manual provides a simple introduction to the generation of random numbers**

1 Uniform Random Numbers

Let U be a uniform random variable between 0 and 1.

1.1 Generate 10^6 samples of U using a C program and save into a file called uni.dat .
Solution: : Link to the code : C code

1.2 Load the uni.dat file into python and plot the empirical CDF of U using the samples in uni.dat. The CDF is defined as

$$F_U(x) = \Pr(U \le x) \tag{1.1}$$

Solution: The following code plots Fig: Python code

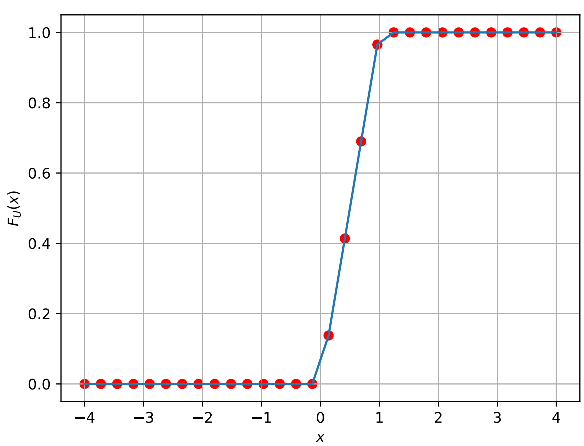


Fig. 1: The CDF of U

1.3 Find a theoretical expression for $F_U(x)$.
Solution: As U is uniformly distributed random variable in the interval (0,1) and

$$p_U(x) = \begin{cases} 1, & x \in (0,1) \\ 0, & otherwise \end{cases} \tag{1.2}$$

Hence,

$$F_U(x) = \begin{cases} 0, & x \in (-\infty, 0) \\ x, & x \in (0,1) \\ 1, & x \in (1, \infty) \end{cases} \tag{1.3}$$

1.4 The mean of U is defined as

$$E[U] = \frac{1}{N}\sum_{i=1}^{N} U_i \tag{1.4}$$

and its variance as

$$\text{var}[U] = E[U - E[U]]^2 \tag{1.5}$$

Write a C program to find the mean and variance of U.

Solution: Link to the code :C code

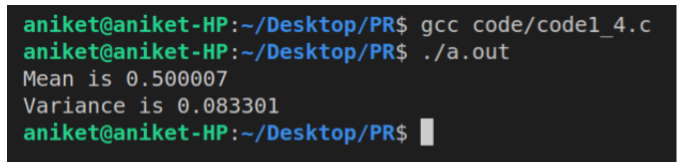


Fig. 1: Output

1.5 Verify your result theoretically given that

$$E\left[U^k\right] = \int_{-\infty}^{\infty} x^k dF_U(x) \tag{1.6}$$

Solution: we know that,

$$dF_U(x) = p_U(x)dx \tag{1.7}$$

also mean (μ) is E(U): Hence,

$$\mu = \int_{-\infty}^{\infty} x p_U(x)dx$$
$$= \int_0^1 x dx$$
$$= \left.\frac{x^2}{2}\right|_0^1$$
$$= \frac{1}{2}$$

Variance(σ^2) = $E(U^2) - E(U)^2$

$$E(U^2) = \int_{-\infty}^{\infty} x^2 p_U(x)dx$$
$$= \int_0^1 x^2 dx$$
$$= \left.\frac{x^3}{3}\right|_0^1$$
$$= \frac{1}{3}$$

$$\sigma^2 = \frac{1}{3} - \frac{1}{4}$$
$$= \frac{1}{12}$$

2 Central Limit Theorem

2.1 Generate 10^6 samples of the random variable

$$X = \sum_{i=1}^{12} U_i - 6 \tag{2.1}$$

using a C program, where $U_i, i = 1, 2, \ldots, 12$ are a set of independent uniform random variables between 0 and 1 and save in a file called gau.dat
Solution: link to the code :C code

2.2 Load gau.dat in python and plot the empirical CDF of X using the samples in gau.dat. What properties does a CDF have?
Solution: Link to Python code :Python code

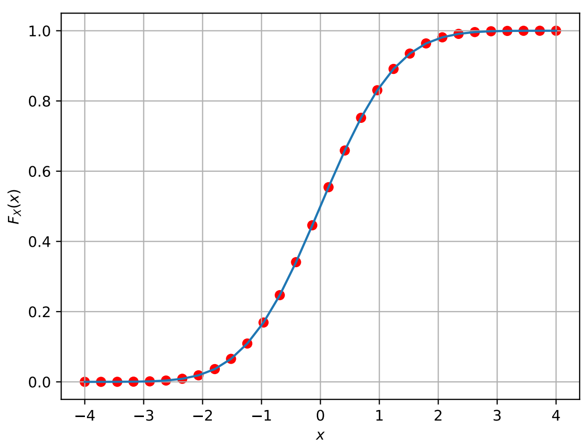


Fig. 2: The CDF of X

Properties:
(1) Graph is symmetric about a single point
(2) The $F_X(x)$ is non-decreasing function
(3) $\lim_{x\to\infty} F_X(x)$

2.3 Load gau.dat in python and plot the empirical PDF of X using the samples in gau.dat. The PDF of X is defined as

$$p_X(x) = \frac{d}{dx} F_X(x) \tag{2.2}$$

What properties does the PDF have?
Solution: Link to the code :Python code

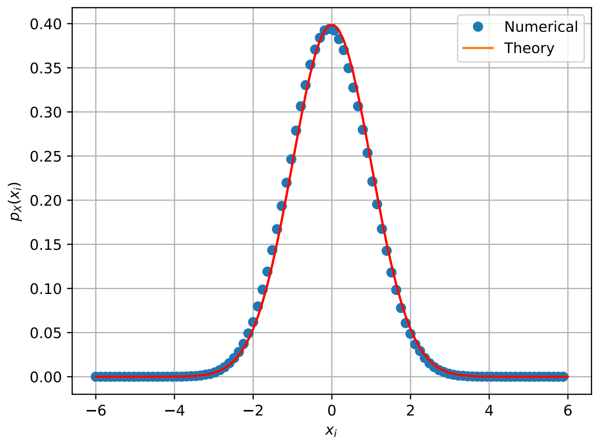
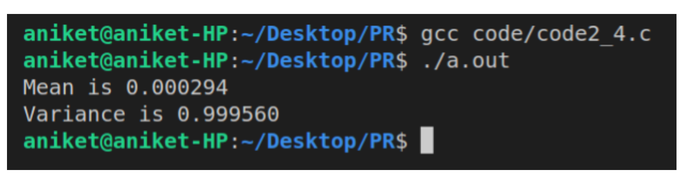


Fig. 2: The PDF of X

Properties :
(1) Area under the curve is One.
(2) Symmetric about line $x = \mu$.
(3) Increasing in first half and decreasing in other half.

2.4 Find the mean and variance of X by writing a C program.
Solution: Link to the code :C code



```
aniket@aniket-HP:~/Desktop/PR$ gcc code/code2_4.c
aniket@aniket-HP:~/Desktop/PR$ ./a.out
Mean is 0.000294
Variance is 0.999560
aniket@aniket-HP:~/Desktop/PR$
```

Fig. 2: Output

2.5 Given that

$$p_X(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right), -\infty < x < \infty, \tag{2.3}$$

repeat the above exercise theoretically. **Solution:**
(1) CDF is

$$F_X(x) = \int_{-\infty}^{\infty} p_X(x) dx$$
$$= 1$$

(2) Mean is

$$\mu = E(x) = \int_{-\infty}^{\infty} x p_X(x) dx$$

Due to symmetry clearly, $\mu = 0$

(3) Variance is

$$\begin{aligned} var[X] &= E(X^2) - E(X)^2 \\ &= \int_{-\infty}^{\infty} x^2 p_X(x) dx - 0 \\ &= 1 \end{aligned}$$

3 From Uniform to Other

3.1 Generate samples of

$$V = -2\ln(1 - U) \tag{3.1}$$

and plot its CDF.
Solution: Link to the code :Python code

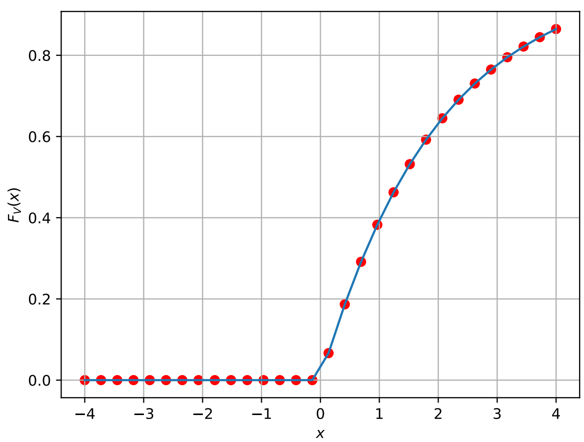


Fig. 3: CDF of V

3.2 Find a theoretical expression for $F_V(x)$.
Solution:

$$\begin{aligned} F_V(X) &= Pr(V \le x) \\ &= Pr(-2ln(1 - U) \le x) \\ &= Pr((1 - U) \ge exp(\frac{-x}{2}) \\ &= Pr(U \le (1 - exp(\frac{-x}{2})) \\ &= F_U(1 - exp\frac{-x}{2}) \end{aligned}$$

from equation (1.3),

$$F_U(x) = \begin{cases} 0, & x \in (-\infty, 0) \\ (1 - exp\frac{-x}{2}), & x \in (0, \infty) \end{cases}$$