Implied Volatility Skew: Definitions & Metrics

Aniket Dey, Dartmouth College

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1 Introduction

Implied volatility smile refers to the phenomenon in which the implied volatilities of the options with the same time to expiry vary across different strike prices. This often manifests as either a U-shaped smile or a skew, in which out-of-the-money (OTM) puts have higher implied volatilities than out-of-the-money (OTM) calls. Below we review four leading measures of implied volatility skew, each with its mathematical definition and interpretation, along with recent academic findings:

2 Calculations of Skew

2.1 Log-Moneyness Metric

One natural measure is the slope of the implied volatility curve with respect to strike (often expressed in terms of log-moneyness $x = \ln(K/F)$ where F is the forward price). Mathematically, the skew can be defined as the derivative of implied volatility σ_{imp} with respect to log-moneyness at the money (ATM):

$$S_{\text{slope}} = \left. \frac{\partial\, \sigma_{\text{imp}}(x,T)}{\partial x} \right|_{x=0}$$

This derivative, sometimes called the implied volatility smirk slope) captures how fast implied volatilities rise as strikes move downwards (OTM puts) vs. upwards (OTM calls). A **negative** slope indicates that OTM puts have higher implied vol than calls, a typical case for equity options. In practice, this continuous slope is often approximated by finite differences, for example the **"90/110"** skew defined as $\sigma_{\text{imp}}(K = 0.90F) - \sigma_{\text{imp}}(K = 1.10F)$. The slope S_{slope} is essentially the first-order measure of skew of the smile. A larger-magnitude negative slope means a more pronounced volatility skew (steeper smile tilt), which corresponds to a more left-skewed risk-neutral return distribution.

Empirically, steep negative slope has been linked to markets' perception of higher tail risk and often correlates with risk-neutral skewness. For instance, **Feng et al. (2015)** construct a put slope (IV difference between a deep OTM put and an ATM option) and a call slope similarly for OTM calls. They find that stocks with a steeper put slope (more negative skew) tend to earn lower future returns, consistent with a premium for crash risk, whereas a steeper call slope predicts higher returns, suggesting the IV slope encodes information about downside risk. Furthermore, **Borochin, Chang, & Wu (2020)** note that a steeper short-term smile slope is associated with higher crash risk and informed trading demand in the short-maturity options market. Overall, the smile slope remains a simple but powerful metric: it directly measures the volatility skew and is closely tied to risk-neutral skewness and downside risk pricing in option markets.

2.2 Delta-Based Skew

Another popular metric is the 25-delta risk reversal, a delta-based skew measure widely used in FX and equity index options. It is defined as the difference in implied volatilities between a 25-delta call and an equidistant 25-delta put with the same expiry. Using the convention of quoting put IV minus call IV, the 25 risk reversal is:

$$\text{RR}_{25\Delta} = \sigma_{\text{imp}}(25\%\Delta\,\text{put}) - \sigma_{\text{imp}}(25\%\Delta\,\text{call})$$

where a "25% delta put" is the put whose delta is –0.25 (i.e. an OTM put), and "25% delta call" has delta +0.25 (OTM call).A positive 25 RR indicates call IV ¿ put IV, but in equity and FX markets typically RR is negative, reflecting higher put volatility. Indeed, *it is observed that 25-delta risk reversals is the most frequent indicator of option skewness used in practice and across academic financial literature.*

The strength of the 25 risk reversal lies in how it measures the asymmetry between implied vols of comparable out-of-the-money calls vs. puts directly. A large negative $\mathrm{RR}_{25\Delta}$ means OTM puts are much more expensive than OTM calls, indicating a strong demand for put protection relative to call upsides. This corresponds to a left-skewed implied distribution with higher crash risk pricing. In currency options, risk reversals convey the market's bias for domestic vs. foreign currency depreciation. **Chang et al. (2020)** use the 25 RR as a primary skew proxy in examining exchange rate dynamics, noting it widened significantly post-2008 in response to hedging demand. They define the 25 RR as above and show it aligns with the pricing of tail-risk: currencies with more negative external imbalances exhibit more negative risk reversals due to hedging flows.

In equity index options, the 25 risk reversal similarly captures the skew premium. In a study of the risk-reversal premimum, **Hull and Sinclair (2022)** document that OTM puts are systematically overpriced relative to OTM calls (negative risk reversals) due to investors over-paying for downside protection. A negative risk reversal thus corresponds to the market's implicit skewness in expected returns: "a negative risk-reversal translates into a left-skewness in the option-implied asset return distribution". **Chang, Wong, and Zhang (2020)** furthermore link variations in SP 500 risk reversals to funding liquidity and crash aversion, while **Ravagli (2021)** shows harvesting the risk reversal premium as a strategy. These studies build on the notion that the 25 RR encapsulates investors' skew bias in a single number, making it a staple for comparing skew across markets and over time.

2.3 SVI Parametrization

Skew can also be quantified through parameters of option pricing models that produce a volatility smile. In popular stochastic volatility models certain parameters directly control the slope (skew) of the implied vol surface. Calibrating these models to market data yields numerical measures of skew. We highlight the Stochastic Volatility Inspired model, one such parametric model popularized by Jim Gatherall.

SVI is a five-parameter arbitrage-free fit for implied variance $w(k)$ as a function of log-moneyness $k = \ln(K/F)$. Its parameters include a slope b and an asymmetry parameter ρ which directly encode skew. The SVI formula can be written as $w(k) = a + b\big[\rho(k-m) + \sqrt{(k-m)^2+\sigma^2}\big]$. Here $b > 0$ sets the smile's slope magnitude and $\rho \in [-1, 1]$ tilts the smile to one side. The ATM skew (derivative of implied vol at $k = m$) simplifies to:

$$\left.\frac{\partial \sigma_{\text{imp}}}{\partial k}\right|_{ATM} \approx \frac{b\rho}{2\,\sigma_{\text{imp}}(m,T)\,T}$$

which is proportional to b, ρ. A typically negative ρ in equity markets leads to a negative ATM skew. In other words, SVI's parameter ρ "drives the negative skew observed in equity markets. SVI is especially interesting because of the possibility to state explicit conditions on it parameters so that the model does not generate prices where static arbitrage opportunities can occur. As such, calibration of the SVI model to real market data requires non-linear optimization algorithms and can be quite time consuming. In this calibration, the fitted ρ serves as a quantitative measure of skew (often around -0.4 to -0.8 for index options), and b indicates the steepness.

Model-based measures like SVI extract skew in a parametric way, which can be varied across assets or time for comparison. While **Gatheral et al. (2014)** provided the necessary theoretical and methodological rigor for meaningful parametrization, **Alexander et al. (2020)** empirically validate the skew parameter's sensitivity and responsiveness to market shocks, demonstrating its real-world utility as a timely indicator of tail-risk dynamics.

2.4 Risk-Neutral Skewness & Moment-Based approaches

A more distribution-centric measure of implied volatility skew comes from the risk-neutral density (RND) implied by options. This approach defines skew in terms of the third moment (or third cumulant) of the implied risk-neutral distribution of the underlying's future price. In other words, one computes the risk-neutral skewness of returns directly from option prices – a model-free method first pioneered by **Bakshi, Kapadia & Madan**

(2003). Bakshi et al. provided formulas to extract the first three moments of the RND using portfolios of options, allowing one to calculate the skewness without assuming a specific mode.

Let S_T be the underlying price at expiry and $F = S_0 e^{rT}$ the forward price (for simplicity, assume zero dividends). The risk-neutral skewness Γ^Q is the standardized third central moment of S_T under the risk-neutral measure Q. One convenient expression for it (in terms of forward price) is:

$$\Gamma^Q(T) = \frac{e^{rT}W - 3\mu e^{rT}V + 2\mu^3}{(e^{rT}V - \mu^2)^{3/2}}$$

where $\mu = E^Q[S_T] = F$ is the first moment (mean under Q), $V = E^Q[S_T^2]$ is the second moment, and $W = E^Q[S_T^3]$ is the third moment (authors use V and W to avoid confusion with variance and "third moment" terminology). This formula gives the skewness in terms of raw moments V and W. We can also express it in terms of central moments or cumulants; essentially it is $\frac{E^Q[(S_T-\mu)^3]}{(E^Q[(S_T-\mu)^2])^{3/2}}$. Bakshi et al. derived elegant integrals to compute V and W (and hence Γ^Q) from option prices across strikes using the Breeden–Litzenberger result $f_Q(K) = e^{rT}, \partial^2 C/\partial K^2$ for the risk-neutral density:

$$V = E^Q[S_T^2] = 2e^{rT} \int_0^\infty \frac{C(K,T) - \max(F-K,0)}{K^2}\, dK$$

$$W = E^Q[S_T^3] = 3e^{rT} \int_0^\infty \frac{C(K,T)\ln(K/F) - \max(F-K,0)\ln(F/K)}{K^2}\, dK$$

where $C(K,T)$ is the call price as a function of strike. sing these, Γ^Q can be calculated. A negative Γ^Q indicates a left-skewed distribution with heavier left tai), which corresponds to a volatility skew sloping downward, with a negative risk-neutral skewness corresponding to a negative volatility skew.

Risk-neutral skewness Γ^Q is a holistic measure of skew derived from the entire smile, rather than just two points or a local slope. It effectively aggregates the information in OTM call and put prices to quantify the third moment of the implied distribution. This measure captures the volatility smile's asymmetry in probabilistic terms, detailing how lopsided the implied distribution of returns is. A strongly negative Γ^Q means the market-assigned distribution has a long left tail, and thus, large downside moves are more probable than a lognormal would suggest. This aligns with the usual notion of skew: e.g. S&P 500 options consistently show large negative risk-neutral skewness due to crash fears.

Compared to the other metrics in the study, risk-neutral skew measures have the largest growing body of modern work. **Borochin, Chang & Wu (2020)** find that short-term Γ^Q (from near-dated options) positively predicts stock returns, whereas long-term Γ^Q has the opposite sign, suggesting informed trading in short expiries and risk preferences in long expiries. Furthermore, **Bressan & Weissensteiner (2023)** compare skewness from past returns vs. option-implied skewness for financial firms, and show that the option-derived (ex ante) skewness has significantly higher explanatory power . Both of the studies compute Γ^Q following the Bakshi et al. method and integrals, underlining that this approach to skew is now standard in empirical finance. Other studies, like **Stilger, Kostakis & Poon (2016)**, demonstrate that a strategy exploiting stocks with the most negative risk-neutral skewness earns abnormal returns, consistent with an "informed trading/hedging demand" hypothesis for skew. Finally, in the commodity space, **Fuertes et al. (2022)** show Γ^Q is priced in commodity option markets as well, and can be an effective tool for illustrating skew.

Overall, the moment-based skew measure Γ^Q provides a single-number summary of the smile's asymmetry grounded in the actual risk-neutral distribution. This approach, enabled by the BKM integrals, has become a go-to in recent academic research for examining the role of skewness in asset pricing. It also complements the other measures: while slope or risk reversals are more convenient for quick intuition, the full risk-neutral skewness is a more comprehensive metric that fully utilizes all strikes' information.

3 References

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