Final Exam Part II

The **hard deadline** for this quiz is **Fri 31 May 2013 8:59 AM PDT (UTC -0700)**.

To specify an array or sequence of values in an answer, you must separate the values by a single space character (with no punctuation and with no leading or trailing whitespace). For example, if the question asks for the first ten powers of two (starting at 1), the only accepted answer is:

1 2 4 8 16 32 64 128 256 512

If you wish to discuss a particular question and answer in the forums, please post the entire question and answer, including the seed (which is used by the course staff to uniquely identify the question) and the explanation (which contains the correct answer).

☐ **In accordance with the Coursera Honor Code, I (Atul Gupta) certify that the answers here are my own work.**

Question 1

(seed = 637769)
Which of the following statements about connectivity in an undirected graph are true? Check all that apply.

- ☐ Removing any edge from a connected graph breaks the graph into two connected components.
- ☐ If there are two edge-disjoint paths connecting vertices u and v, then there is a simple cycle containing both u and v.
- ☐ If two vertices u and v are connected, then there is a simple cycle containing both u and v.
- ☐ If u is connected to v and v is connected to w, then u is connected to w.
- ☐ If you delete two edges from a graph, its number of connected components can increase by at most 2.

Question 2

(seed = 779272)
Which of the following statements about strong connectivity in a digraph are guaranteed to be true? Check all that apply.

- ☐ A digraph on V vertices with fewer than V-1 edges contains more than one strong component.
- ☐ Given a digraph G and a vertex s, G is strongly connected if and only if every vertex is reachable from s and s is reachable from every vertex.
- ☐ If we modify the Kosaraju-Sharir algorithm to run the first DFS in the digraph G (instead of the reverse digraph G^R) and the second depth-first search in G^R (instead of G), then it will still find the strong components.
- ☐ If two vertices u and v are strongly connected, then there is a simple cycle containing both u and v.
- ☐ If we modify the Kosaraju-Sharir algorithm to replace the second DFS with BFS, then it will still find the strong components.

Question 3

(seed = 868190)
Which of the following statements about minimum spanning trees (MSTs) are true? Check all that apply.

- [ ] Let G be a connected edge-weighted graph. Suppose that you are given a spanning tree T such that for every edge e in T, e belongs to some MST of G. Then, T is a MST of G.
- [ ] Let G be a connected edge-weighted graph with distinct edge weights. Suppose that e is the most expensive edge in some cycle C. Then e must not belong to the MST.
- [ ] Let G be a connected edge-weighted graph in which two or more edges have the same weight. Then, G has at least two different MSTs.
- [ ] Let G be a connected edge-weighted graph with distinct edge weights, no parallel edges, and at least 3 vertices. Then, the MST may contain the most expensive edge.
- [ ] Let G be a connected edge-weighted graph with distinct edge weights. Suppose that you add an edge e of positive weight to G. Then, there exists a MST of the resulting graph G' that differes from T in at most one edge.

Question 4

(seed = 667509)
Which of the following statements about shortest paths are true? Check all that apply.

- [ ] Let G be a digraph with positive edge weights. During Bellman-Ford, if you trace back the edgeTo[] entries starting from v back to s, the path has distance equal to distTo[v].
- [ ] Bellman-Ford finds the shortest simple path from s to every other vertex, even if the edge weights are arbritary positive or negative integers.
- [ ] Bellman-Ford finds the shortest simple path from s to every other vertex, even if the edge weights are positive or negative integers, provided there are no negative cycles.
- [ ] Give a DAG with positive and distinct edge weights, Dijkstra's algorithm and the topological sort algorithm relax the vertices in the same order.
- [ ] If all edges in a digraph have distinct weights, then the shortest path between two vertices is unique.

Question 5

(seed = 129015)
Which of the following statements about maxflow and mincut are guaranteeed to be true? Check all that apply.

- [ ] If the mincut is unique, then so is the maxflow.
- [ ] The value of any flow is greater than or equal to the capacity of any cut.

- [ ] The value of the maxflow equals the capacity of the mincut.
- [ ] The value of any flow is less than or equal to the capacity of any cut.
- [ ] If the capacity of some cut (A, B) equals the value of some flow f, then (A, B) is a mincut.

Question 6

(seed = 278561)
The column on the left contains the original input of 24 strings to be sorted;
the column on the right contains the strings in sorted order; the other 7 columns contain the
contents at some intermediate step during one of the 3 radix sorting algorithms listed below.

LIVE	INXS	UB40	AQUA	FUEL	UB40	VAIN	ABBA	ABBA
INXS	FUEL	SOAD	ABBA	ENYA	EVE6	CARS	ACDC	ACDC
FUEL	ENYA	WHAM	ACDC	AQUA	ENYA	UB40	AQUA	AQUA
ENYA	AQUA	ABBA	BUSH	ACDC	AQUA	ABBA	BUSH	BUSH
AQUA	ACDC	ACDC	CARS	EVE6	ABBA	ACDC	CARS	CARS
VAIN	EVE6	EVE6	DEVO	CARS	ACDC	WEEN	DEVO	DEVO
PINK	CARS	FUEL	DOOM	DEVO	SOAD	DEVO	DOOM	DOOM
CARS	DEVO	WEEN	ENYA	ABBA	LIVE	WHAM	ENYA	ENYA
DEVO	ABBA	TUFF	EVE6	FIXX	HOLE	PINK	EVE6	EVE6
ABBA	FIXX	VAIN	FUEL	BUSH	TUFF	LIVE	FUEL	FIXX
FIXX	BUSH	HOLE	FIXX	DOOM	BUSH	FIXX	FIXX	FUEL
UB40	DOOM	PINK	HOLE	HOLE	PINK	INXS	HOLE	HOLE
TOTO	HOLE	TOOL	INXS	INXS	FUEL	ENYA	INXS	INXS
HOLE	LIVE	DOOM	LIVE	LIVE	TOOL	SOAD	LIVE	LIVE
STYX	WEEN	CARS	PINK	WEEN	DOOM	HOLE	PINK	PINK
WEEN	TUFF	BUSH	STYX	TUFF	WHAM	TOOL	STYX	SOAD
TUFF	STYX	TOTO	SOAD	STYX	VAIN	DOOM	SOAD	STYX
DOOM	TOTO	AQUA	TOTO	TOTO	WEEN	TOTO	TOTO	TOOL
BUSH	SOAD	LIVE	TUFF	SOAD	DEVO	AQUA	TUFF	TOTO
SOAD	UB40	DEVO	TOOL	UB40	TOTO	STYX	TOOL	TUFF
EVE6	WHAM	INXS	UB40	WHAM	INXS	FUEL	UB40	UB40
WHAM	PINK	FIXX	VAIN	PINK	CARS	TUFF	VAIN	VAIN
ACDC	TOOL	ENYA	WEEN	TOOL	FIXX	BUSH	WEEN	WEEN
TOOL	VAIN	STYX	WHAM	VAIN	STYX	EVE6	WHAM	WHAM
----	----	----	----	----	----	----	----	----
0	?	?	?	?	?	?	?	4

Match up each column with the corresponding sorting algorithm from the given list:

0. Original input
1. LSD radix sort
2. MSD radix sort
3. 3-way radix quicksort (no shuffle)
4. Sorted

You may use an algorithm more than once. Your answer should be a sequence of 9 integers between
0 and 4 (starting with 0 and ending with 4) and with each integer separated by a single space.

Hint: think about algorithm invariants. Do not trace code.

Question 7

(seed = 970127)
What is the Burrows-Wheeler transform of the following string of length 8?

```
    C B B A B D B B
```

Your answer should be an integer (between 0 and 7) followed by a sequence of 8 characters,
with a single space between each of the integer and character entries.

Question 8

(seed = 258976)
Suppose that 3-SUM has a N^(3/2) lower bound and that 3-SUM linear-time reduces to 3-COLLINEAR. Which of the following can you infer?
Check all that apply.

- [ ] 3-COLLINEAR cannot be solved in N^(5/4) time.
- [ ] 3-COLLINEAR can be solved in N^(3/2) time.
- [ ] If 3-SUM can be solved in N^(3/2) time, then so can 3-COLLINEAR.
- [ ] If 3-SUM cannot be solved in N^(5/3) time, then neither can 3-COLLINEAR.
- [ ] If 3-COLLINEAR can be solved in N^(3/2) time, then so can 3-SUM.

Question 9

(seed = 801637)
Consider the following linear programming simplex tableaux with 6 equations and 8 variables:

```
maximize Z
            +    3 x1                                                        -    1 x7    -  Z    = -138
-----------------------------------------------------------------------------------------------------------
            +  1/2 x1  +    1 x2                                             -    1 x7            =   54
 +    1 x0  -  9/5 x1                                                        -    3 x7            =   42
            +    5 x1                                           +    1 x6  +    4 x7            =   18
            +  1/2 x1                    +    1 x4                                            =   36
            +  2/3 x1          +    1 x3                                     +  2/5 x7            =   48
            -    2 x1                                 +    1 x5              +  7/4 x7            =   12
        x0  ,      x1  ,      x2  ,      x3  ,      x4  ,      x5  ,      x6  ,      x7           >=    0
```

What is the value of variable x3 in the current basic feasible solution?
Specify your answer with two digits after the decimal place.

Question 10

(seed = 797827)
What is the definition of the complexity class P?

- [ ] All NP problems solvable in polynomial time.
- [ ] All search problems solvable in exponential time.
- [ ] All search problems solvable in polynomial space.
- [ ] All problems solvable in exponential time.
- [ ] All problems solvable in polynomial space.

Question 11

(seed = 631772)
You are applying for a job at a new software technology company. Your interviewer asks you to identify which of the following tasks are known to be possible. Check all that apply.

- [ ] Given an undirected graph and two vertices s and t, find the longest simple path from s to t in E + V time.
- [ ] Given an undirected graph, find a simple cycle in V + E time.
- [ ] Given an undirected graph, determine if it is bipartite in E + V time.
- [ ] Determine whether two undirected graphs are the same except for vertex names in V^3 time.
- [ ] Given a digraph and two subsets of vertices S and T, find a path from any vertex in S to any vertex in T that uses the fewest number of edges in E + V time.

Question 12

(seed = 907200)
You are applying for a job at a new software technology company. Your interviewer asks you to identify which of the following tasks are known to be possible. Check all that apply.

- [ ] Given an array of N strings, sort them in time linear in the total number of characters in the input.
- [ ] Given an array of N items whose keys are integers between 0 and R, stably sort them in time linear in N + R.
- [ ] Determine whether a pattern string of length M is a substring of a text string of length N in time proportional to M + N in the worst case.
- [ ] Determine whether one string s of length N is a circular shift of another string t of length N in time proportional to N.
- [ ] Design a data compression algorithm that achieve a 50% compression ratio for random inputs.

- [ ] **In accordance with the Coursera Honor Code, I (Atul Gupta) certify that the answers here are my own work.**

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