

Feedback — Interview Questions: Union-Find

You submitted this homework on **Sun 24 Mar 2013 2:18 PM PDT -0700**. You will be able to view your score after the deadline passes.

These interview questions are for your own enrichment and are not assessed. If you click the *Submit Answers* button, you will get a hint.

Question 1

Social network connectivity. Given a social network containing N members and a log file containing M timestamps at which times pairs of members formed friendships, design an algorithm to determine the earliest time at which all members are connected (i.e., every member is a friend of a friend of a friend ... of a friend). Assume that the log file is sorted by timestamp and that friendship is an equivalence relation. The running time of your algorithm should be $M \log N$ or better and use extra space proportional to N .

Your Answer	Score	Explanation
Total	0.00 / 0.00	

Question Explanation

Hint: union-find.

Question 2

Union-find with specific canonical element. Add a method `find()` to the union-find data type so that `find(i)` returns the largest element in the connected component containing `i`. The operations, `union()`, `connected()`, and `find()` should all take logarithmic time or better.

For example, if one of the connected components is $\{1, 2, 6, 9\}$, then the `find()` method should return 9 for each of the four elements in the connected components.

Your Answer	Score	Explanation
Total	0.00 / 0.00	

Question Explanation

Hint: maintain an extra array to the weighted quick-union data structure that stores for each root `i` the large element in the connected component containing `i`.

Question 3

Successor with delete. Given a set of N integers $S = \{0, 1, \dots, N - 1\}$ and a sequence of requests of the following form:

- Remove x from S
- Find the *successor* of x : the smallest y in S such that $y \geq x$.

design a data type so that all operations (except construction) should take logarithmic time or better.

Your Answer	Score	Explanation
Total	0.00 / 0.00	

Question Explanation

Hint: use the modification of the union-find data discussed in the previous question.

Question 4

Union-by-size. Develop a union-find implementation that uses the same basic strategy as weighted quick-union but keeps track of tree height and always links the shorter tree to the taller one. Prove a $\lg N$ upper bound on the height of the trees for N sites with your algorithm.

Your Answer	Score	Explanation
Total	0.00 / 0.00	

Question Explanation

Hint: replace the `sz[]` array with a `ht[]` array such that `ht[i]` stores the height of the subtree rooted at `i`.