Analysis of Shell Structures

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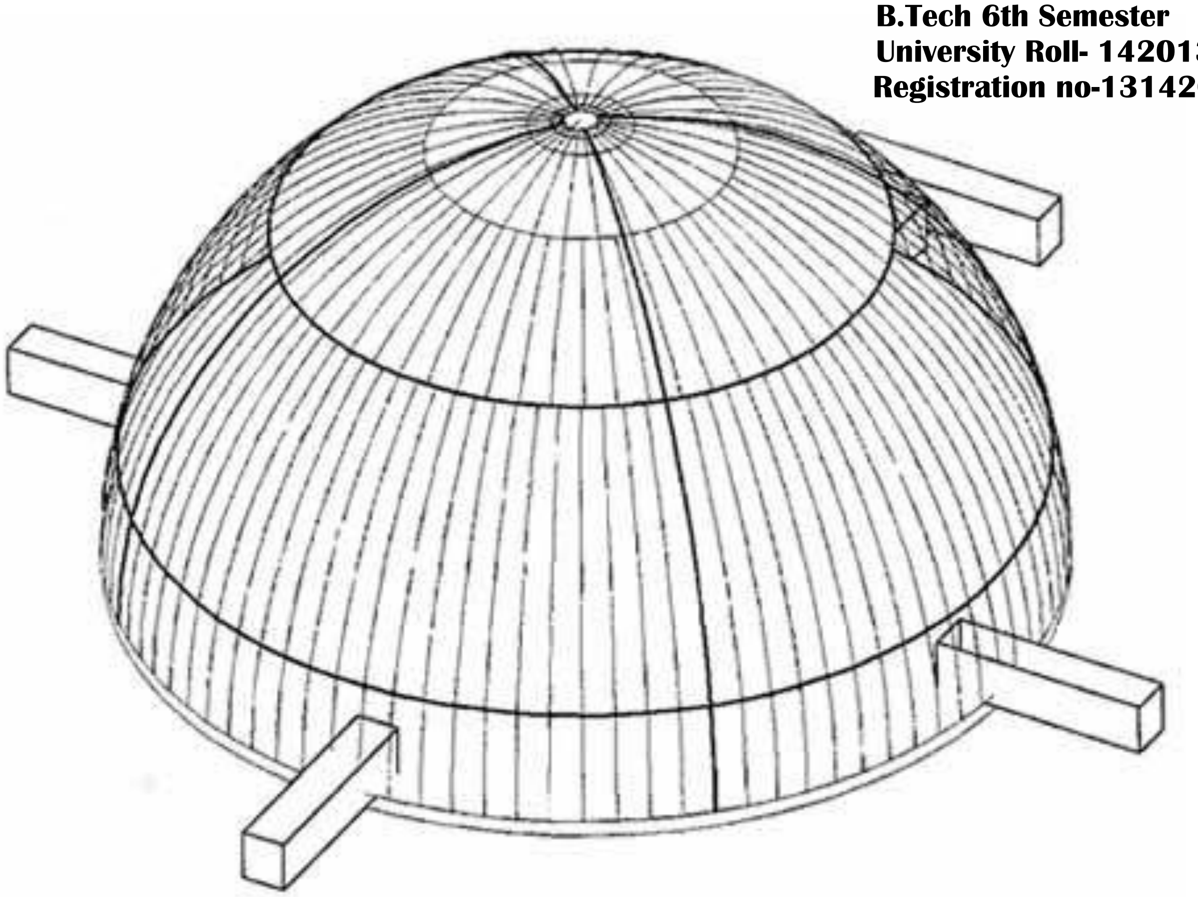

Under the guidance of

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I would also like to thank my teachers of my department of civil engineering for helping me in gaining proper knowledge in this disciple. A special thanks to my friends for helping me in completing this project .

Signature of Guide: ____________________________

Abstract: A cylindrical shell analysis are mainly done by two main theories, Membrane theory & Bending Theory. There are also certain approximation theories used for analysis of shell. In this project a 'C' program has been developed for analysis of cylindrical shell based on an approximation theory, The Beam Theory. An insight of accuracy and applicability of Beam Theory is also given in this project.

Introduction:

The shell structure are very unique due to its interesting strength to weight ratio. They are able to span over large areas, while having an exceptionally small thickness, thus reducing considerable amount of material required & dead weight on structure. This special property is mainly due to its geometry, their curved nature & structural behaviour. Due to the above reasons shell as roof have gained popularity.

A shell is considered thin when its thickness 'd' to its radius 'R' at any point i.e $d/R \leq 1/20$

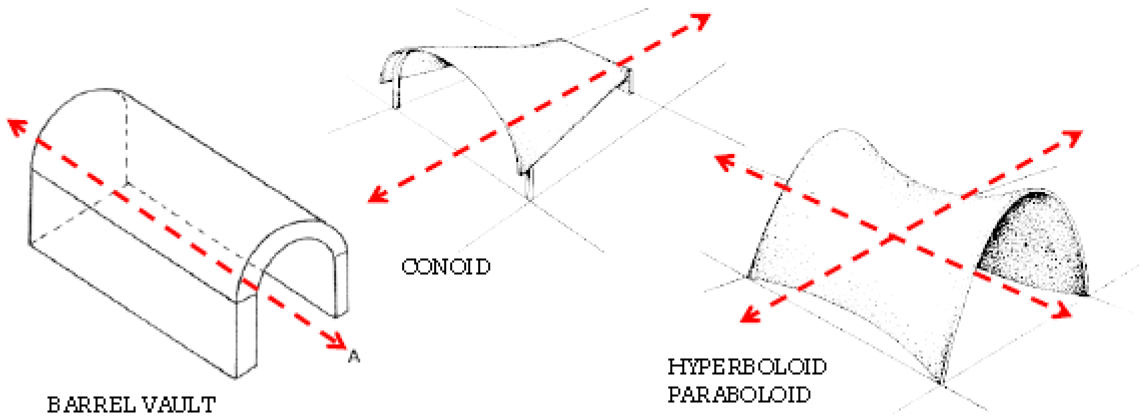
Classification Of shell:

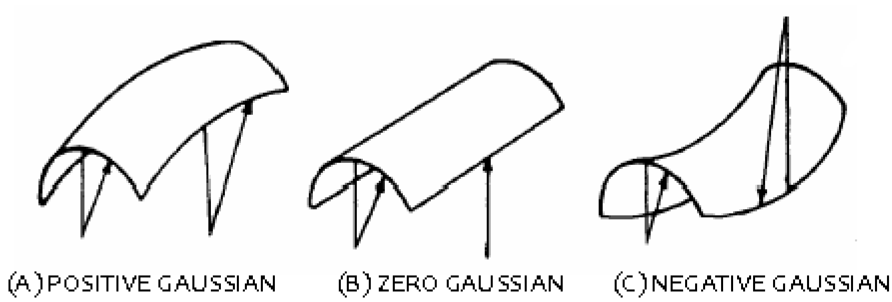
A shell can carvature along a single axis or along both axis based on that a shell can be i)single carvature- Curved along only one linear axis eg- cylinder

ii) Doule carvature- Curved along Both linear axis, eg- Hyperboloid.

Again in a double carvature structure based on curvature nature of direction can be classified into i) Synclastic Shells- The curvature are similar in both direction. Dome is a good example of this. This type of shell is also called Gauss positive surface.

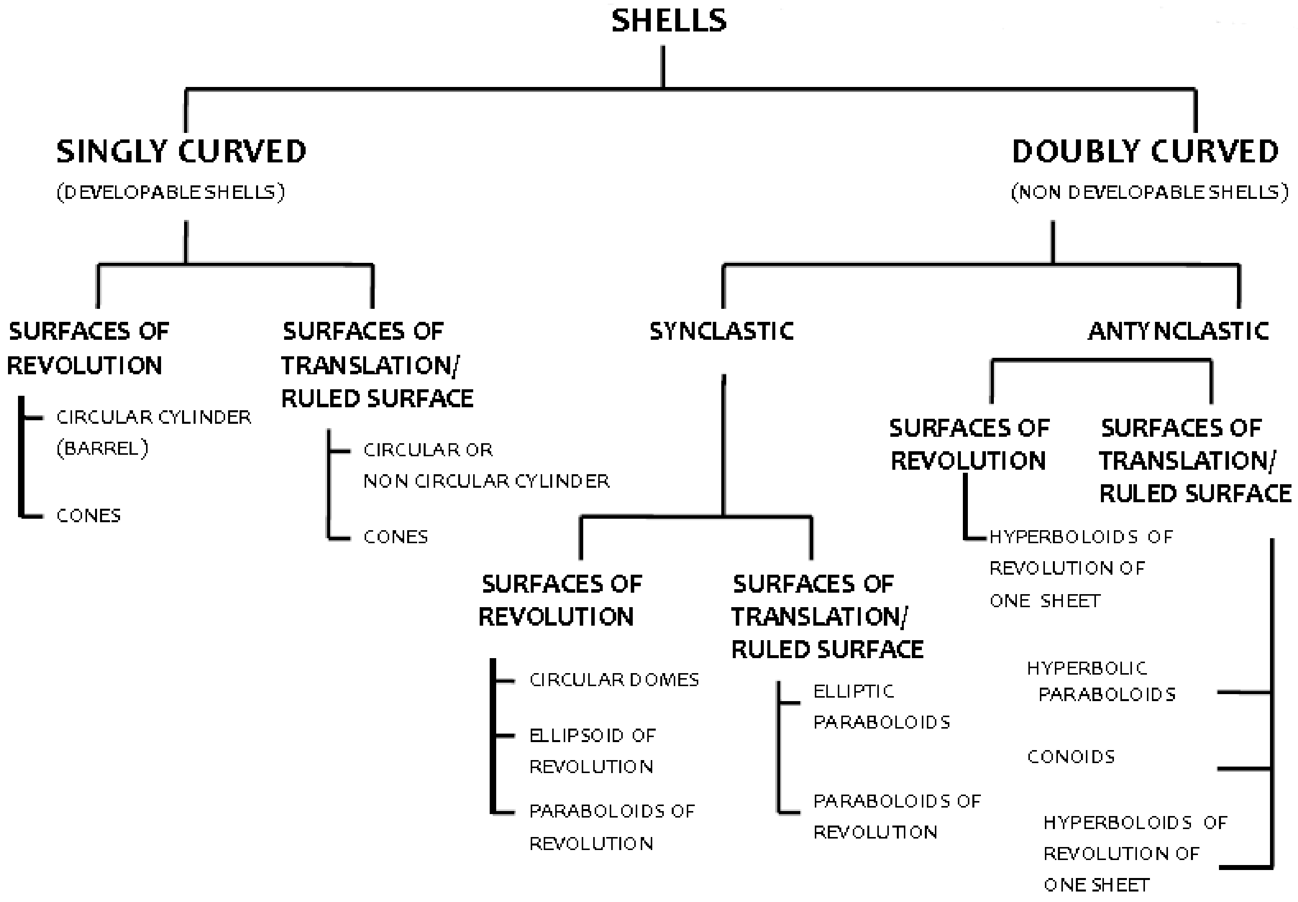
ii) Anticlastic Shell- The curvatures are in two direction are of opposite nature. Hyperberboloid is a good example of this This type of shell is also called Gauss negetive.





Among all kind of shell cylindrical shell is preferred due to its simplicity in analysis, designing and construction.

Classifation of shell



Hyperbolic Paraboloid shell

Group of Cylindrical shell

Analysis Cylindrical Shell

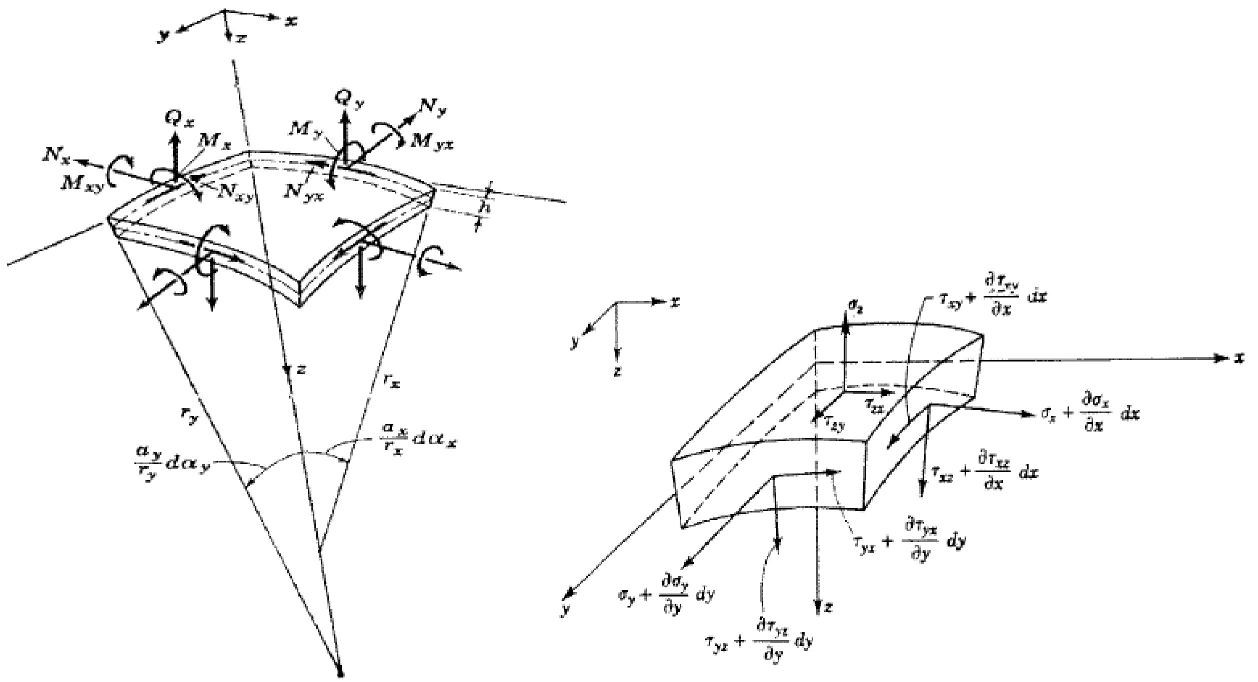
Different theories on thin shell: For the analysis of thin shell 3 different theories are available. Those are:

- Bending Theory
- Membrane Theory
- Beam Theory

Bending Theory- This is the most accurate theory as all the possible unknown forces & moments are considered in this theory. This theory is almost applicable in cases. A general shell element in equilibrium consists 6 unknowns forces and 4 unknown moments.

Assumptions:

- Bending moments, twisting moments and radial shear has non zero value in the shell structures.
- The material in which the shell is to be constructed is considered homogeneous, isotropic.



The differential shell element

The 6 unknown forces

$$N_x = \int_h \sigma_x \left(1 - \frac{z}{r_y}\right) dz \qquad N_y = \int_h \sigma_y \left(1 - \frac{z}{r_x}\right) dz$$

$$N_{xy} = \int_h \tau_{xy} \left(1 - \frac{z}{r_y}\right) dz \qquad N_{yx} = \int_h \tau_{yx} \left(1 - \frac{z}{r_x}\right) dz$$

$$Q_x = \int_h \tau_{xz} \left(1 - \frac{z}{r_y}\right) dz \qquad Q_y = \int_h \tau_{yz} \left(1 - \frac{z}{r_x}\right) dz$$

The 4 unknown moments

$$M_x = \int_h \sigma_x z \left(1 - \frac{z}{r_y}\right) dz \qquad M_y = \int_h \sigma_y z \left(1 - \frac{z}{r_x}\right) dz$$

$$M_{xy} = -\int_h \tau_{xy} z \left(1 - \frac{z}{r_y}\right) dz \qquad M_{yx} = \int_h \tau_{yx} z \left(1 - \frac{z}{r_x}\right) dz$$

Approximation Theory: A large class of cylindrical shells can be analyzed with sufficient accuracy by regarding them as a beam of curved section. The advantages of beam theory is that it brings shell analysis within a reach of those who are unfamiliar with the techniques of advanced mathematics. Unlike the analytical theory, shells with noncircular directrics can be dealt with. The theory can be applied to shells of varying depth.

Advantages of Beam Theory: The advantages of beam theory may be summed up as follows:

- It brings shell analysis within the reach of those who are unfamiliar to the techniques of advanced mathematics.
- Unlike the analytical theory,shells with noncircular directrices can be dealt with.
- It can be applied to shells with non-uniform thickness.
- It can handle shells with non-uniform thickness.
- It is also claimed that line loads carried by shells can be treated by this method.
- structural action of the shell is easily visualized.

Assumptions:

- the deformation of the cross-section in its plane are negligible.
- Mx and hence Qx and Mxθ may be ignored.
- The strain γ_{xy} caused by the shear force Nxθ and the lateral contraction are neglected.

Range of validity:

- Single shells without edge beams if l/a>5
- long single shells with not deep edge beams, if l/a>3
- Interior shells with featheredge beams of a group of multiple shells.

Beam Analysis:

There are two distict steps in the method of beam analysis.In the first, the shell is regarded as a beam of curved cross-section and the familiar Mc/I and VQ/Ib formulae are applied to determine Nx and Nxθ. This step may be called beam analysis. Refering to the fig and assuming that the shell is carrying only vertical loading, symmetrically distributed over the cross-section.

$$Nx = \frac{Myy}{Iyy} zd \qquad ....(1)$$

where Myy is the bending moment at any cross-section computed as for a simply supported beam and Iyy is the moment of inertia about axis y-y.It is easily verified that

$$I_{yy}=2d\int_{\theta=0}^{\theta=\theta c} ad\theta(a\cos\theta - \frac{a\sin\theta c}{\theta c})^2$$

$$=a^3 d[\theta c + \sin\theta c(\cos\theta c - \frac{2sin\theta c}{\theta c})] \qquad ....(2)$$

Formulae (1) and (2) are applicable only to shells of symmetrical cross-sections. As we are concerned about symmetrical structures in this report therefore these formulae will enough for analysis.

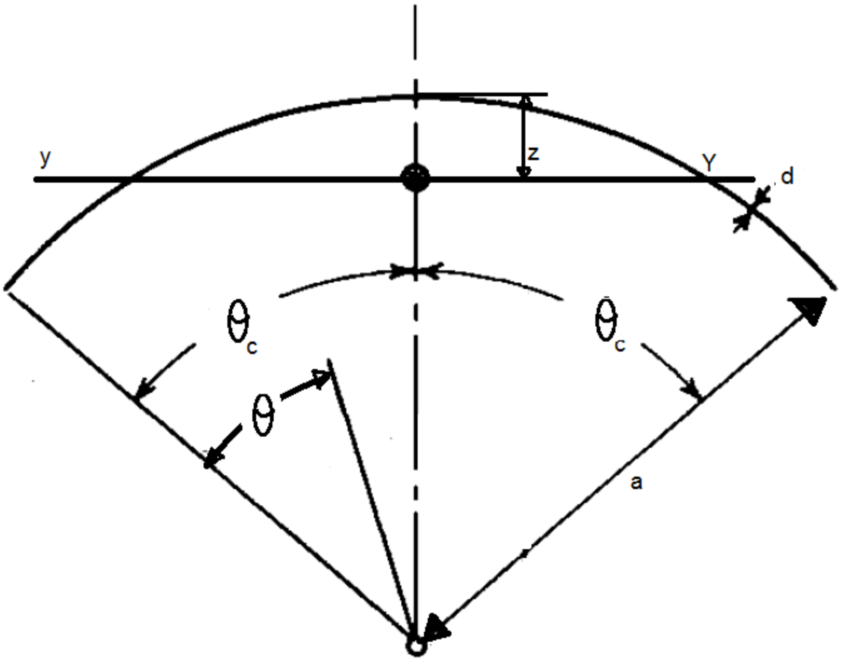
The beam analysis also enables Nxθ to be found by the use of the well-known VQ/Ib formula.For a shell subjected to vertical loading, symmetrically distributed over the cross-section, it is easily verified as:

$$Nx\theta=\frac{VQd}{2Iyyd} = VQ/2Iyy \qquad ....(3)$$

where V is the vertical shearing force at the cross-section, computed as for a simple beam and Q is the first static moment of the cross-section up to the joint under consideration about the axis

yy found from the expression:

$$Q=a\left(\frac{sin\theta}{\theta} - \frac{sin\theta c}{\theta c}\right) 2a\theta d = 2a^2 d(sin\theta - \frac{\theta}{\theta c} sin\theta c) \qquad ....(4)$$



Cross-section of a Cylindrical Arch

a= radius of curvature of shell

d= thickness of shell

θc= semi-circular angle

z= the distance of neutral axis from crown of shell

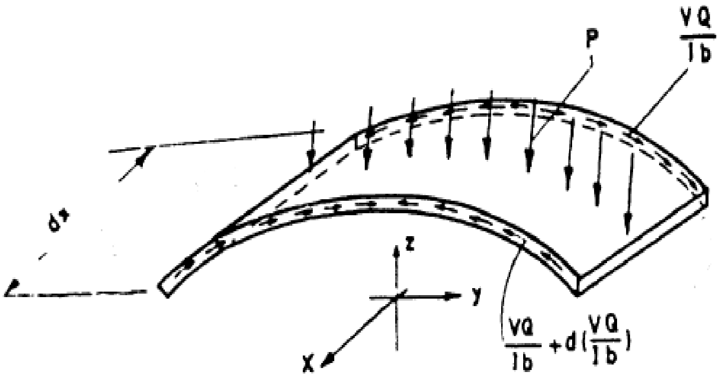
l= span of shell

Iyy= moment of inertia about Y axis

θ= Angle measured from edge

<u>Arch analysis</u>:-

In the 2nd part of analysis the long section of the cylindrical shell have been segmented to number of elementary arches. The main objective of this analysis is to determine Mθ, Nθ



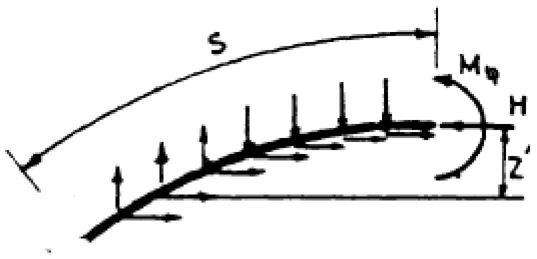
The free body diagram of a slice arch is given. The arch include a longitudinally varying shear force(Nxθ) along with vertically acted load.

The components of load and shear force causes moment in transverse direction Mθ .

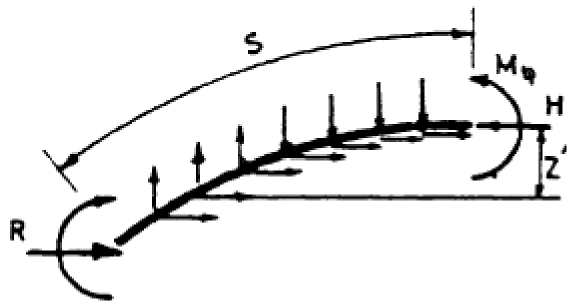
If the shell is isolated without any

edge beam then there will be one restraining moment sets up at crown. In such condition arch can be called as determinate arch.The transverse moment at any point in arch will be algebraic sum of moments due to load and component of shear force.

If the shell is interior one, belongs to a group of multiple shells, In such condition the ends of the arch are restrained so an additional restraining moments sets up at end. This type of arch slice is indeterminate in nature. Solving requires indeterminate approach. We choose column analogy for solving moments indeterminate in nature.



A determinate type slice arch of an isolated shell



A indeterminate type arch of an interior shell

In the next paragraphs code for "c" programs for beam analysis of cylindrical shell given.
The program is made in most general way to solve the shell which is applicable for shell of any span and of any radius with any semi-circular angle.
The program is valid for uniformly distributed dead and live load.
The program is applicable for any length of span
The results shown in the program are that of mid-span of shell.
Maximum number angular interval that can be ask for has a limit of 20
All the data during input must belong to same unit system (i.e. imperial or S.I.) and parameter of similar dimension must be of same unit.
In program Nxq represents Nxθ and transverse moment represents Mθ

Computer Program for Analysis of Interior Cylindrical Shell in Group of Shells by Beam Method

```
#include<stdio.h>
#include<conio.h>
#include<math.h>
#define rad 0.01745329
main()
{

double
ang,Rang,z,a,Iyy,d,Myy,V,dl,ll,l,Rang1,m,m1,m2,m3,m4,M,s1=0,s2=0,Marea=0,MMarea=0,Carea,Icc,Hsum=0
,Vsum=0;
    int i,j,n;
    double
zdist[20],Q[20],Nx[20],Nxq[20],ds[20],dy[20],dz[20],ss[20],ssh[20],ssv[20],dw[20],lw[20],v[20],Moment[20],
Mid[20],Mact[20],Nq[20];
    printf("span of shell \n");
    scanf("%lf",&l);
    printf("enter radius \n");
    scanf("%lf",&a);
    printf("enter thickness\n");
    scanf("%lf",&d);
    printf("enter semicircular angle\n");
```

```
    scanf("%lf",&Rang);
    printf("enter deadload \n");
    scanf("%lf",&dl);
    printf("live load\n");
    scanf("%lf",&ll);

    ang=rad*Rang;                          //ang= main angle in radian in radia Rang=main angle indegree
    z=a*(1-sin(ang)/ang);
    Iyy=a*a*a*d*(ang+sin(ang)*(cos(ang)-(2*sin(ang)/ang)));        //Iyy=moment of iertia
    printf("enter the perferred angular intervals \n");         //n= no of angular interval
    scanf("%d",&n);
    Myy=(dl*2*ang*a+(ll*2*a*sin(ang)))*l*l*0.125;             //longitudinal moment
    V=(dl*2*ang*a+ll*2*sin(ang)*a)*l*0.5;                    //longitudinal shear
    for(i=0;i<=n;i++)
    {
      Rang1=Rang-(Rang*(i)/n);
      zdist[i]=a*(1-cos(Rang1*rad))-z;                 //the angular interval.........Rang1=each interval angle
      Q[i]=2*a*a*d*(sin(Rang1*rad)-(sin(ang)*Rang1/Rang));
      Nx[i]=Myy*d*(a*(1-cos(Rang1*rad))-z)/Iyy;
      Nxq[i]=0.5*V*Q[i]/Iyy;

      if(i==0 || i==n)
      {
        ds[i]=a*Rang*rad/(n*2);
      }
      else
      {
        ds[i]=a*Rang*rad/(n);
      }
      dy[i]=ds[i]*cos(Rang1*rad);
      dz[i]=ds[i]*sin(Rang1*rad);
      ss[i]=Nxq[i]*2/l;              //ss[i]=specific shear
      ssh[i]=ss[i]*dy[i];             //ssh[i]=horizontal force component of specific shear
      ssv[i]=-ss[i]*dz[i];             //ssh[i]=vertical force component of specific shear
      dw[i]=dl*ds[i];                //dw=dead weight
      lw[i]=ll*dy[i];               //lw= live weight
      v[i]=ssv[i]+dw[i]+lw[i];
    }
   for(i=0;i<=n;i++)
  {
      printf("POINT%d Nx=%lf  Nxq=%lf\n",i,Nx[i],Nxq[i]);
  }
  m=Rang/n;    //m= angular interval
  for(i=0;i<n;i++)
  {
    for(j=0;j<=i;j++)
    {
      m1=v[j]*cos((Rang-(i+1)*m)*rad)*a*sin((m*(i+1-j))*rad);
      m2=-v[j]*sin((Rang-(i+1)*m)*rad)*a*(1-cos((m*(i+1-j))*rad));
      m3=ssh[j]*cos((Rang-(i+1)*m)*rad)*a*(1-cos((m*(i+1-j))*rad));
      m4=ssh[j]*sin((Rang-(i+1)*m)*rad)*a*sin((m*(i+1-j))*rad);
      M=m1+m2+m3+m4+M;

    }
   Moment[i+1]=M;       //determinate moment
    M=0;

  }
  Moment[0]=0;
   //column analogy
   for(i=1;i<n;i++)
```

```
  {
     if(i%2==1)
     {
        s1=s1+Moment[i];
     }
     else
     {
        s2=s2+Moment[i];
     }
  }
Marea=-2*(Moment[0]+Moment[n]+4*s1+2*s2)*a*m*rad/3;
for(i=0;i<=n;i++)
{
   if(i==0 || i==n)
   {
   MMarea=-a*m*rad*(Moment[i]*(-zdist[i]))+MMarea;     //MMarea=moment of moment area about z-axis
   }
   else
   MMarea=-2*a*m*rad*(Moment[i]*(-zdist[i]))+MMarea;
}
Carea=2*a*ang;                        //Carea=equivalent column area
Icc=Iyy/d;                            //Icc=euivalent column moment of inertia
for(i=0;i<=n;i++)
{
   Mid[i]=(Marea/Carea)+(-MMarea*zdist[i]/Icc);    //Mid=indeterminate moment from colimn analogy
   Mact[i]=-Moment[i]-Mid[i];                     //Mact=actual transverse moment
   printf("transverse moment%d %lf \n \n",i,Mact[i]);
}
//Nq calculation
for(i=0;i<=n;i++)
{
   Rang1=Rang-(Rang*(i)/n);
   if(i==0)
   {
     Hsum=-ssh[i]+Hsum+(-MMarea/Icc);          //Hsum=cumulative summation of horizontal force
   }
   else
   {
     Hsum=-ssh[i]+Hsum;
   }
   Vsum=v[i]+Vsum;                                  //Vsum=cumulative summation vertical force
   Nq[i]=Hsum*cos(Rang1*rad)+Vsum*sin(Rang1*rad);
   printf("Nq%d = %lf \n \n",i,Nq[i]);
}
getch();
}
```

Computer Program for Analysis of Isolated Simply Supported Cylindrical Shell without Edge Beam:-

```
#include<stdio.h>
#include<conio.h>
#include<math.h>
#define rad 0.01745329
main()
{

double ang,Rang,z,a,Iyy,d,Myy,V,dl,ll,l,Rang1,m,m1,m2,m3,m4,M,Hsum=0,Vsum=0;
   int i,j,n;
```

```
    double
zdist[20],Q[20],Nx[20],Nxq[20],ds[20],dy[20],dz[20],ss[20],ssh[20],ssv[20],dw[20],lw[20],v[20],Moment[20],
Nq[20];
    printf("span of shell \n");
    scanf("%lf",&l);
    printf("enter radius \n");
    scanf("%lf",&a);
    printf("enter thickness\n");
    scanf("%lf",&d);
    printf("enter semicircular angle\n");
    scanf("%lf",&Rang);
    printf("enter deadload \n");
    scanf("%lf",&dl);
    printf("live load\n");
    scanf("%lf",&ll);

    ang=rad*Rang;                                   //ang= main angle in radian in radian......Rang=main angle in
degree
    z=a*(1-sin(ang)/ang);
    printf("depth of nutral axis %lf\n",z);
    Iyy=a*a*a*d*(ang+sin(ang)*(cos(ang)-(2*sin(ang)/ang)));
    printf("moment of inertia %lf \n",Iyy);
    printf("enter the perferred angular intervals \n");          //n= no of angular interval
    scanf("%d",&n);
    Myy=(dl*2*ang*a+(ll*2*a*sin(ang)))*l*l*0.125;
    printf(" total load %lf \n",dl*ang*a+(ll*2*sin(ang)*a));
    printf("Myy=%lf \n",Myy);
    V=(dl*2*ang*a+ll*2*sin(ang)*a)*l*0.5;
    printf("shear %lf\n",V);
    for(i=0;i<=n;i++)
    {
        Rang1=Rang-(Rang*(i)/n);
        zdist[i]=a*(1-cos(Rang1*rad))-z;                //the angular interval.........Rang1=each interval
angle
        Q[i]=2*a*a*d*(sin(Rang1*rad)-(sin(ang)*Rang1/Rang));
        Nx[i]=Myy*d*(a*(1-cos(Rang1*rad))-z)/Iyy;
        Nxq[i]=0.5*V*Q[i]/Iyy;

        if(i==0 || i==n)
        {
            ds[i]=a*Rang*rad/(n*2);
        }
        else
        {
            ds[i]=a*Rang*rad/(n);
        }
        dy[i]=ds[i]*cos(Rang1*rad);
        dz[i]=ds[i]*sin(Rang1*rad);
        ss[i]=Nxq[i]*2/l;                   //ss[i]=specific shear
        ssh[i]=ss[i]*dy[i];                  //ssh[i]=horizontal force component of specific shear
        ssv[i]=-ss[i]*dz[i];                 //ssh[i]=vertical force component of specific shear
        dw[i]=dl*ds[i];                      //dw=dead weight
        lw[i]=ll*dy[i];                     //lw= live weight
        v[i]=ssv[i]+dw[i]+lw[i];
    }
for(i=0;i<=n;i++)
{
        printf("POINT%d Nx=%lf  Nxq=%lf\n",i,Nx[i],Nxq[i]);
}
m=Rang/n;    //m= angular interval
for(i=0;i<n;i++)
```

```
{
  for(j=0;j<=i;j++)
  {
     m1=v[j]*cos((Rang-(i+1)*m)*rad)*a*sin((m*(i+1-j))*rad);
     m2=-v[j]*sin((Rang-(i+1)*m)*rad)*a*(1-cos((m*(i+1-j))*rad));
     m3=ssh[j]*cos((Rang-(i+1)*m)*rad)*a*(1-cos((m*(i+1-j))*rad));
     m4=ssh[j]*sin((Rang-(i+1)*m)*rad)*a*sin((m*(i+1-j))*rad);
     M=m1+m2+m3+m4+M;

  }
 Moment[i+1]=M;         //determinate moment
  M=0;
}
Moment[0]=0;
for(i=0;i<=n;i++)
{

   printf("transverse moment%d %lf\n",i,Moment[i]);
}
//Nq calculation
for(i=0;i<=n;i++)
{
   Rang1=Rang-(Rang*(i)/n);
   Hsum=-ssh[i]+Hsum;          //Hsum=cumulative summation of horizontal force
   Vsum=v[i]+Vsum;                              //Vsum=cumulative summation vertical force
   Nq[i]=Hsum*cos(Rang1*rad)+Vsum*sin(Rang1*rad);
   printf("Nq%d = %lf \n \n",i,Nq[i]);
}
getch();
}
```

Comparison Of Program Results

An analysis of interior shell in a multiple group of shell is done by Beam Theory in book G. Ramaswami. The same problem have been analysed with the program for "interior cylindrical shell analysis in a group of multiple shell"

PROBLEM-
Geometry
span l =62ft
Radius a =31ft
Thickness d =0.3125ft
semicircular angle ϕ =40°
The shell has no edge beam and uniformly loaded.
Dead load =47lb/ft^2
live load =25lb/ft^2(Horizontal projection)
The following table shows the comparisons of results
Angle 0° signifies the edge of shell and angle 40° the crown of shell

Comparison of Nx										
degree	Angle	0	5	10	15	20	25	30	35	40
lb/ft	NX(program)	34099	22392	12058	3178	-4181	-9964	-14127	-16637	-17476
	Nx(book)	34145	22421	12074	3183	-4185	-9977	-14144	-16657	-17497

Comparison of Nxθ										
degree	Angle	0	5	10	15	20	25	30	35	40
lb-ft/ft	Nxθ(program)	0	4910	7896	9204	9094	7836	5710	3001	0
	Nxθ(book)	0	4911	7906	9216	9185	7847	5718	3007	0

Comparison of Mθ										
degree	Angle	0	5	10	15	20	25	30	35	40
lb/ft	Mθ(program)	-1378	-179.74	477.73	667.036	515.52	172.904	-215.641	-530.76	-694.3
	Mθ(book)	-1381	-182	475	665	514	173	-215	-526	-687

Comparison of Nθ										
degree	Angle	0	5	10	15	20	25	30	35	40
lb/ft	Nθ(program)	704.57	419.43	-155.503	-873.062	-1606.34	-2251.44	-2728.65	-2983.78	-2989.06
	Nθ(book)	706	420	-156	-875	-1709	-2255	-2733	-2988	-2993

The data from above tables suggests the results obtained from program are remarkably close with that of the value given in book with little or no error.

Comparison of Beam theory with Bending theory

Bending theory is considered to be the most accurate theory for analysis of shell. The results obtained from bending theory analysis are very reliable. Here an attempt have been made to check the accuracy of the approximate theory, Beam Theory by comparing its result with bending theory.
A problem of simply supported cylindrical shell analysis is done based on bending theory by following Schorer's approach in book of J. E. Gibson & B. G. Neil. The same problem is solved by the above given program for isolated simply supported cylindrical shell without edge beam.

PROBLEM

Span = 120ft
Radius =30ft
Semicircular angle =40°
load =50lb/ft^2 (horizontal projection)

The following tables shows the comparisons of results between two theories and graphs also given based on that.
Angle 0° signifies the edge of shell and angle 40° the crown of shell

Comparison of Nx						
degree	Angle(θ)	0	10	20	30	40
lb/ft	NX(beam theory)	86788	30690	-10642	-35955	-44479
	Nx(bending theory)	101474	20276	-16684	-29275.8	-31738.3

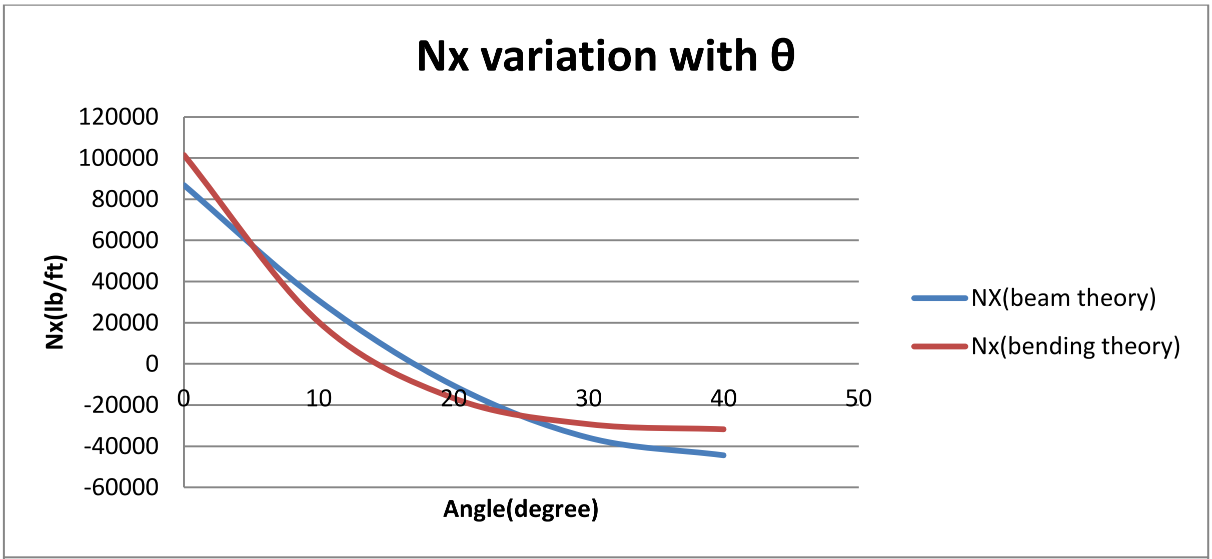
Comparison of Nxθ						
degree	Angle(θ)	0	10	20	30	40
lb/ft	Nxθ(beam theory)	0	10048	11572	7266	0
	Nxθ(bending theory)	0	7715.6	7577.6	4256.3	0

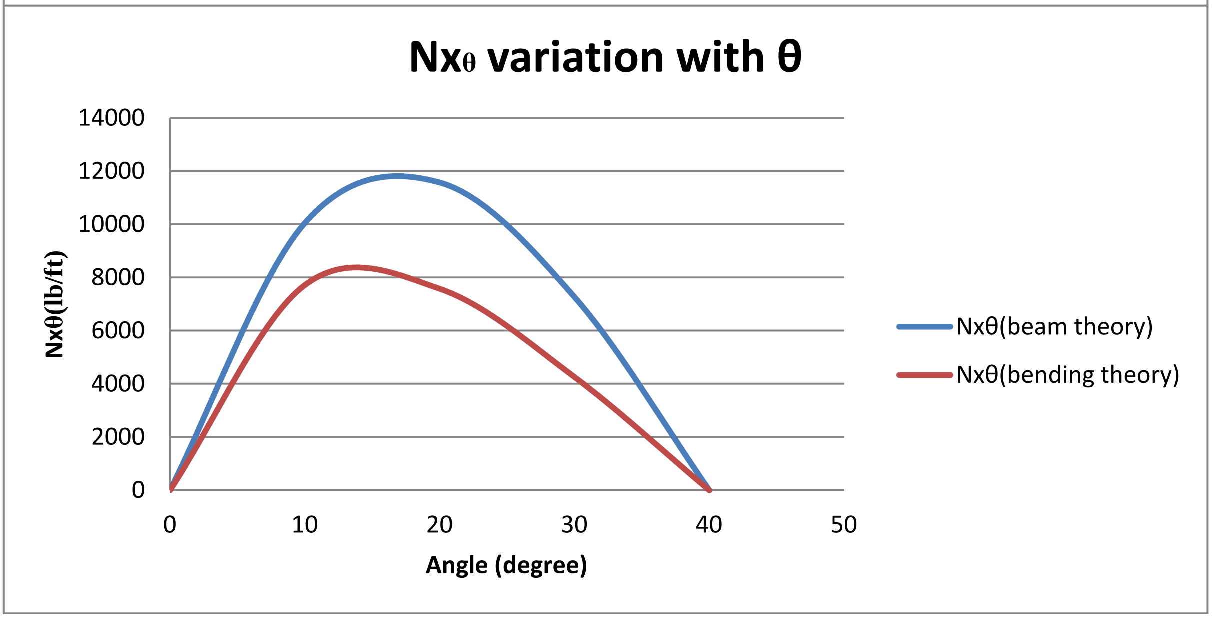
Comparison of Mθ						
degree	Angle(θ)	0	10	20	30	40
lb-ft/ft	Mθ(beam theory)	0	429.5	1579.6	2826.8	3561.19
	Mθ(bending theory)	0	515.2	1718.6	2855.2	3308.7

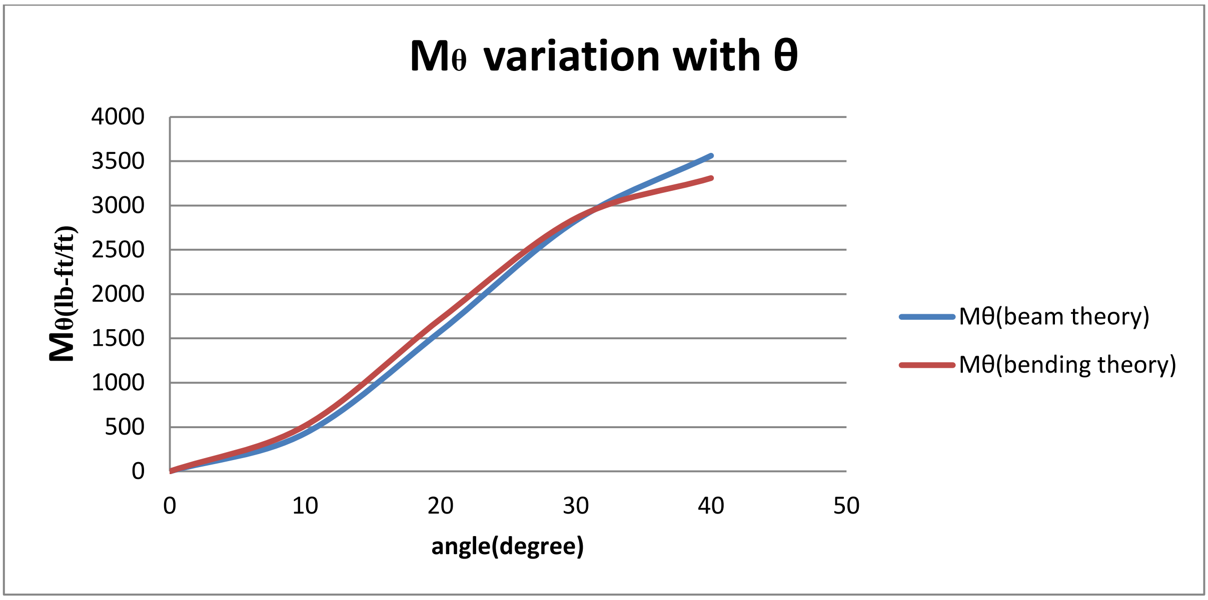
Comparison of Nθ						
degree	Angle(θ)	0	10	20	30	40
lb/ft	Nθ(beam theory)	64	-713.4	-1677.52	-2308.47	-2332.94
	Nθ(bending theory)	0	-505.4	-1500.4	-2262.4	-2534.4

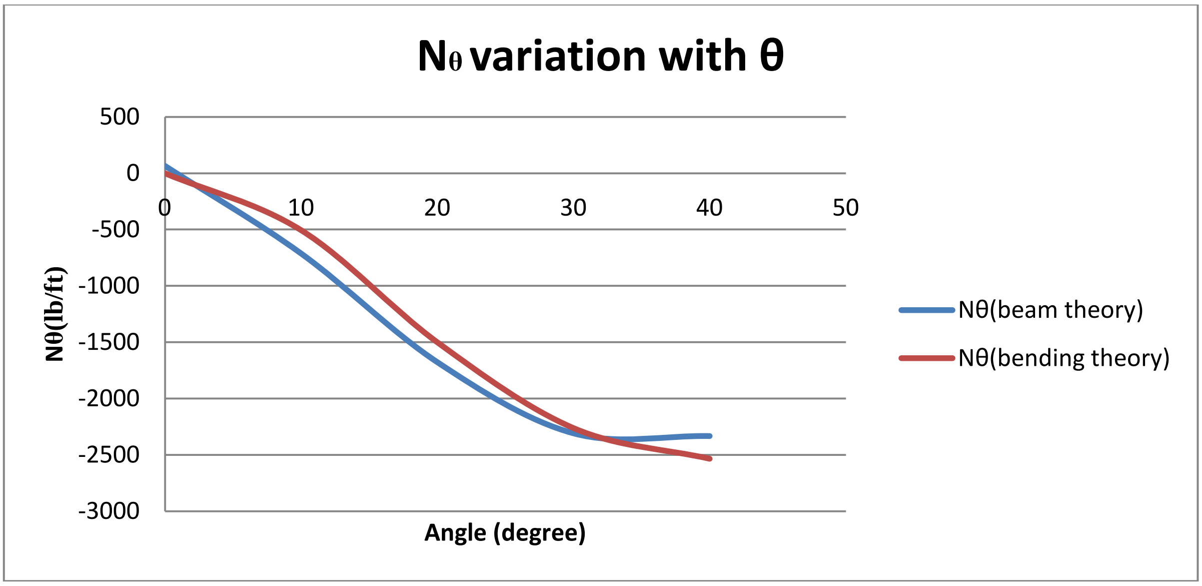
Comparison of Q						
degree	Angle(θ)	0	10	20	30	40
lb/ft	Q(beam theory)	0	0	0	0	0
	Q(bending theory)	0	2.4	3.6	2.7	0

The graph are plotted based on above results. Variation of the forces and moments with angles are plotted. X-axis representing the angle and Y-axis the forces and moments.









Discussions

- The above graphs shows the results obtained from both the theories are quite close to each other specially for Nx. Mθ, Nθ.
- Value of Nxθ obtained from two theories differed by a considerable amounts in between angles 0° to 40°. Value obtained from beam theory is quite larger than the value obtained from bending theory.
- The value of Nx obtained from beam theory is lesser at edge of shell and larger at crown when compared with bending theory.
- The value of Mθ obtained from beam theory is larger at crown compared to bending theory.
- Nθ value obtained from beam theory is found to be more throughout the shell except near the crown of shell.
- The value of Q obtained from bending theory is very small compared to other values.

Conclusion

- Beam theory being an approximate theory is quite accurate in determining the forces and moments in this case. This is due to the fact of long nature of the shell where span to radius ratio was 4. As shell is long its behaviour as beam is predominant. The theory is expected to give more accurate result with increase in span to radius ratio as beam like behaviour of shell will become more predominant.

- In this particular case the value of force Q obtained from bending theory is sufficiently less and can be ignored. Thus the assumption of beam theory to neglects Q holds good for this long shell.

- It is also found beam theory overestimate the value of N_x, M_θ at crown and $N_{x\theta}$ throughout the shell. At the same time the value of N_x & N_θ is underestimated at around the edge of shell and crown respectively.

- Overall, Beam theory can be well used of predetermining forces and moment of cylindrical shell and can also be used for full analysis of a cylindrical shell which is sufficiently long with span to radius ratio is 5 or more.

- It has been already shown before that program is very accurate in analysing shell by beam theory. Thus the program can be extensively used in analysing shell very quickly and accurately.

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