## THE MILLISECOND PULSAR MASS DISTRIBUTION: EVIDENCE FOR BIMODALITY AND CONSTRAINTS ON THE MAXIMUM NEUTRON STAR MASS

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#### ABSTRACT

The mass function of neutron stars (NSs) contains information about the late evolution of massive stars, the supernova explosion mechanism, and the equation-of-state of cold, nuclear matter beyond the nuclear saturation density. A number of recent NS mass measurements in binary millisecond pulsar (MSP) systems increase the fraction of massive NSs (with  $M>1.8\,{\rm M}_{\odot}$ ) to  $\sim20\%$  of the observed population. In light of these results, we employ a Bayesian framework to revisit the MSP mass distribution. We find that a single Gaussian model does not sufficiently describe the observed population. We test alternative empirical models and infer that the MSP mass distribution is strongly asymmetric. The diversity in spin and orbital properties of high-mass NSs suggests that this is most likely not a result of the recycling process, but rather reflects differences in the NS birth masses. The asymmetry is best accounted for by a bimodal distribution with a low mass component centred at  $1.393^{+0.031}_{-0.029}\,\mathrm{M}_{\odot}$  and dispersed by  $0.064^{+0.064}_{-0.025}\,\mathrm{M}_{\odot}$ , and a high-mass component with a mean of  $1.807^{+0.081}_{-0.132}$  and a dispersion of  $0.177^{+0.115}_{-0.072}\,\mathrm{M}_{\odot}$ . We also establish a lower limit of  $M_{\mathrm{max}} \geq 2.018\,\mathrm{M}_{\odot}$ at 98% C.L. for the maximum NS mass, from the absence of a high-mass truncation in the observed masses. Using our inferred model, we find that the measurement of 350 MSP masses, expected after the conclusion of pulsar surveys with the Square-Kilometre Array, can result in a precise localization of a maximum mass up to  $2.15 \,\mathrm{M}_{\odot}$ , with a 5% accuracy. Finally, we identify possible massive NSs within the known pulsar population and discuss birth masses of MSPs.

Keywords: Galaxy: stellar content — stellar evolution: binary — Stars: neutron stars, pulsars –X-rays: binaries – binaries: close

#### 1. INTRODUCTION

Neutron star (NS) mass measurements are motivated by central questions in physics and astrophysics, such as the final stages of stellar nucleosynthesis and mass loss, the supernova (SN) explosion mechanism, the properties of nuclear interactions, and the gravitational interaction in strong-field conditions.

At the most fundamental level the structure of NSs is determined by gravity and nuclear interactions. Below a critical threshold around  $0.1-0.3\,\mathrm{M}_\odot$ , neutron decay likely leads to rapid decompression and, ultimately, explosion of the star (Colpi et al. 1989; Haensel et al. 2002). For larger masses, the relativistic structure equations (Tolman 1939; Oppenheimer & Volkoff

1939)¹ coupled with a model for microscopic interactions (represented with the Equation of State, EoS), define a mass-radius (M-R) relation typically characterized by a canonical radius as well as a maximum mass above which NSs collapse to black holes (BHs). While the EoS and corresponding M-R relation may be directly derived from first-principle quantum chromodynamics calculations, practical limitations due to the difficulty of capturing the many-body interactions at play necessitate approximations. In the absence of experimental data, these calculations lead to a diverse range of predictions.

Owing to the properties of the M-R relation, simul-

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 $<sup>^{1}</sup>$  In what follows, we assume that General Relativity holds in the NS interior.

taneous measurements of masses and radii, as well as observations of high-mass NSs have the potential to place stringent limits on the EoS. NS radius measurements are recently becoming constraining, with  $\approx 15$  carried out to date (Ozel & Freire 2016). Systematic and modeling uncertainties have been addressed in numerous studies but some still need to be resolved (see e.g., Güver et al. 2012; Heinke et al. 2014; Ozel et al. 2015; Nättilä et al. 2015, and references therein). Similarly, the recent measurements of two extremely highmass NSs (with  $M_{\rm NS} \sim 2.0\,{\rm M}_{\odot}$  Demorest et al. 2010; Antoniadis et al. 2013) place stringent limits on the EoS (Section 5) at ultra-high densities, but still leave a wide range of possibilities (e.g., Lattimer & Prakash 2001; Ozel & Freire 2016).

While additional mass and radius measurements could help resolve those remaining uncertainties, numerous questions also remain in the evolution of massive stars and supernova (SN) explosion mechanism that can be addressed by studying the NS mass function. NS masses are not expected to be uniformly distributed between the theoretical extrema, but rather to cluster around a small number of characteristic values. In the textbook example of NS formation, the core of a massive star collapses when it surpasses the Chandrasekhar limit,

$$M_{\rm ch} \simeq 5.83 Y_{\rm e}^2 \,\mathrm{M}_{\odot}.$$
 (1)

Typical iron cores have average electron fractions of  $Y_{\rm e} \simeq 0.45$  yielding  $M_{\rm ch} \simeq 1.18\,{\rm M}_{\odot}$ . In practice, one needs to apply several corrections, e.g., taking into account the core's thermal structure, finite entropy, electrostatic interactions and surface boundary pressure, non-radial convective effects as well as neutrino radiation during the SN. All of these place the lower end of the proto-NS gravitational mass between 1.1 and 1.3  ${\rm M}_{\odot}$  (see, e.g., Timmes et al. 1996). Further uncertainties arise from the explosion energy and the location of the mass cut during the SN, as well as the final stages of nuclear shell burning (Woosley & Weaver 1995; Woosley et al. 2002; Langer 2012). In addition, the final remnant may gain significant mass due to fall-back of material from the stellar envelope (Fryer & Warren 2002).

State-of-the art numerical simulations and analytic calculations for core-collapse SNe and their progenitors predict NS initial mass functions ranging from uni-modal to highly skewed and/or multi-modal distributions (Timmes et al. 1996; Ugliano et al. 2012; Janka 2012; Pejcha & Thompson 2015; Ertl et al. 2015; Sukhbold et al. 2015; Müller et al. 2016). Akin to this work, some studies find notable differences between remnants originating from stars that burn carbon radiatively or convectively (Timmes et al. 1996; Brown et al. 2001). This bifurcation may lead to a bimodal NS mass distribution. Furthermore, recent studies of core-

collapse progenitors consistently find a highly non-linear relation between the initial (or core helium) stellar mass and the final remnant mass (e.g. Müller et al. 2016, and references therein).

Additional components may arise due to alternative formation channels, such as an electron-capture implosion, which is expected to produce a distinct peak with a small dispersion around  $1.25\,\mathrm{M}_\odot$  (Nomoto 1987; Podsiadlowski et al. 2004). Finally, two significant but still poorly understood factors for the outcome of massive star evolution, besides the initial mass, are the effects of wind mass loss, and the dynamical interaction and mass transfer in a binary system (Wellstein & Langer 1999; Brown et al. 2001; Podsiadlowski et al. 2004).

Following birth, the NS mass can further increase due to matter accretion from a binary companion (Bhattacharya & van den Heuvel 1991; Tauris & van den Heuvel 2006). Depending on the rate and duration of mass transfer, a significant amount of material may be accreted onto the NS, potentially even driving the star beyond the critical limit for collapse into a BH. On average, larger masses (typically  $\gtrsim 0.1\,\mathrm{M}_\odot$ ) are expected for "recycled" millisecond pulsars (MSPs) with low-mass companions, that have undergone a long episode of stable mass transfer (Tauris & van den Heuvel 2006; Tauris et al. 2012)

As of today, NS masses have been inferred for  $\sim 75$  NSs in X-ray binaries, double NS systems (DNS) and MSPs (Ozel & Freire 2016). If one excludes marginal measurements and strongly model-dependent or probabilistic inferences, then the sample of reliable, precision measurements reduces to 32 (Fig. 1) among the DNS and MSP populations. Notably, all of these are at least partly based on the radio timing technique, a summary of which is given in Section 2.

Past attempts to infer the underlying mass distribution based on growing subsets of these data suggest a strong clustering of masses between  $\sim 1.3$  and  $1.5 \, \mathrm{M}_{\odot}$ (Finn 1994; Thorsett & Chakrabarty 1999; Schwab et al. 2010; Zhang et al. 2011; Valentim et al. 2011; Ozel et al. 2012; Kiziltan et al. 2013). Recent studies by Özel et al. (2012) and Kiziltan et al. (2013) distinguish between different NS types and find statistically significant differences between those believed to be close to their birth masses and the ones that have undergone at least one long-term accretion episode. A distinct property of the former NS type, as manifested in the DNS mass distribution, is a relatively small dispersion of only  $\Delta M \simeq 0.05 \,\mathrm{M}_{\odot}$  around the mean mass of  $M \simeq 1.35 \,\mathrm{M}_{\odot}$ (Özel et al. 2012). As argued by Özel et al. (2012), it is possible that the small dispersion reflects a highly tuned formation channel for DNSs. For example, this likely implies inefficient but precise amount of accretion of fall-back material during the SN, which may be dif-

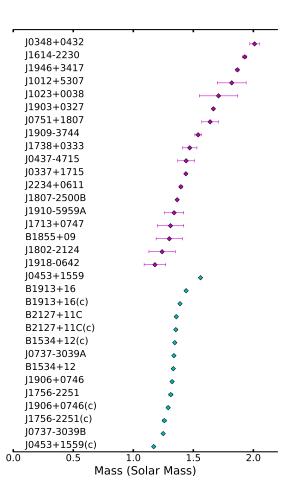


Figure 1. Mass measurements and 68% uncertainty intervals for NSs in DNS (blue) or MSP (purple) systems. See Ozel & Freire (2016) and references therein for the masses of DNS systems.

ficult to understand in the context of stellar evolution (Janka 2012; Wongwathanarat et al. 2013).

Recent developments in pulsar searches and timing have led to a nearly exponential increase in the mass measurements of MSPs and, in particular, in the discovery of some fairly massive pulsars (Ozel & Freire 2016). Mass distributions inferred based on earlier data do not predict many massive ones. For example, Özel et al. (2012) expect about 5-7% of MSPs with masses above  $1.8\,\mathrm{M}_\odot$ , whereas the new discoveries of J0348+0432 ( $M=2.01(4)\,\mathrm{M}_\odot$ ; Antoniadis et al. 2013) and J1946+3417 ( $M=1.867(13)\,\mathrm{M}_\odot$  Barr et al. 2016), as well as a number of other mass refinements (see Fig. 1) suggest the actual fraction to be larger than 20%. These systems have very distinct orbital properties, and their masses have been measured with different methods. It is therefore unlikely that the new masses result from

selection effects caused by observational bias.

In this paper, we model the MSP mass distribution using the most up-to-date ensemble of mass measurements. We compare uni-modal and bi-modal approximations using Bayesian inference techniques and find that the bimodal distribution in the MSP masses is preferred by the current data. We examine the implications of these different intrinsic distributions for stellar evolution and the EoS. Furthermore, we use our findings to make zero-order estimates for future large-scale pulsar surveys, such as those planned for the Square Kilometre Array (SKA). The lay-out of the paper is as follows: In Section 2 we provide a brief overview of mass measurement methods and discuss our dataset. In Section 3 we outline our statistical method and then present our main results in Section 4. We examine the ramifications for the EoS in Section 5. Finally, we conclude with a broader discussion in Section 6.

#### 2. MILLISECOND PULSAR MASSES

Pulsar mass measurements can be obtained using a broad range of techniques at different wavelengths. For MSPs, most constraints come from precision radio timing, sometimes supplemented by optical observations of their binary companions. In what follows, we shall briefly review these methods and discuss their strengths and weaknesses.

### 2.1. Radio Timing

Radio timing observations of binary pulsars yield precise measurements of the orbital period  $P_{\rm b}$  and projected semi-major axis,  $x \equiv a_p \sin i$ . These quantities allow to determine the mass function,

$$f(m_{\rm p}, m_{\rm c}, i) = \frac{(m_{\rm c} \sin i)^3}{(m_{\rm p} + m_{\rm c})^2} = \left(\frac{2\pi}{P_{\rm b}}\right)^2 \frac{x^3}{G},$$
 (2)

which relates the unknown stellar masses,  $m_{\rm p}$  and  $m_{\rm c}$ , and inclination, i.

Because Eq. 2 connects three unknowns, inference of the pulsar mass requires the measurement of at least two additional quantities that depend on those parameters. For sufficiently compact binaries, this can be achieved with the measurement of post-Keplerian (pK) parameters induced by relativistic effects. These include the precession of the orbital periastron  $\dot{\omega}$ , the Einstein-delay  $\gamma$  (which accounts for time-dilation effects and the varying gravitational redshift along the orbit), the Shapirodelay  $\Delta t_{\rm s}$ , as modelled by the parameters r and s (describing the extra travel-time due to the companion's gravitational potential), and the orbital decay  $\dot{P}_{\rm b}^{\rm GW}$  due to emission of gravitational waves. In General Relativity (GR) the pK parameters become functions of the stellar masses and Keplerian parameters (see Lorimer

& Kramer 2012, for details):

$$\dot{\omega} = 3 \left(\frac{P_{\rm b}}{2\pi}\right)^{-5/3} \left(T_{\odot} M_{\rm T}\right)^{2/3} \left(1 - e^2\right)^{-1},$$
 (3)

$$\gamma = e \left(\frac{P_{\rm b}}{2\pi}\right)^{1/3} T_{\odot}^{2/3} M_{\rm T}^{-4/3} m_{\rm c} \left(m_{\rm p} + 2m_{\rm c}\right), \quad (4)$$

$$r = T_{\odot} m_{\rm c},\tag{5}$$

$$s = \sin i = x \left(\frac{P_{\rm b}}{2\pi}\right)^{-2/3} T_{\odot}^{-1/3} M_{\rm T}^{2/3} m_{\rm c}^{-1}, \qquad (6)$$

$$\dot{P}_{\rm b}^{\rm GW} = -\frac{192\pi}{5} \left( 1 + \frac{73}{24} e^2 + \frac{37}{96} e^4 \right) \times \left( 1 - e^2 \right)^{-7/2} \left( \frac{2\pi \mathcal{M} T_{\odot}}{P_{\rm b}} \right)^{5/3}, \quad (7)$$

where  $T_{\odot} \equiv G M_{\odot}/c^3 = 4.925490947 \,\mu s$  is the solar mass in time units,  $M_{\rm T} = m_{\rm c} + m_{\rm p}$  is the total mass of the binary, and  $\mathcal{M} = (m_{\rm p} m_{\rm c})^{3/5} (m_{\rm p} + m_{\rm c})^{-1/5}$  is the chirp mass of the system.

Due to their formation history, most binary MSPs in the Galactic disk have eccentricities of order  $10^{-7}-10^{-3}$ , rendering the measurement of  $\dot{\omega}$  and  $\gamma$  extremely challenging. Similarly, the Shapiro delay magnitude depends sensitively on the inclination, and is typically relevant only for systems viewed nearly edge-on. Finally, the measurement of  $\dot{P}_{\rm b}^{\rm GW}$  is only possible in extremely compact binary MSPs ( $P_{\rm b} \lesssim 1\,\rm d$ ) with point-mass like companions (i.e., in double NS and NS–white dwarf binaries).

On the other hand, a substantial number of MSPs in globular clusters, as well as a handful of systems in the Galactic field have sufficiently high eccentricities to allow for constraints on  $\dot{\omega}$  and consequently the total mass  $M_{\rm T}$  (see Antoniadis 2014; Verbunt & Freire 2014, and references therein).

## 2.2. Optical Spectroscopy

Additional information on the masses can also be obtained when the pulsar companion has an optically bright counterpart. Phase-resolved spectroscopy yields the orbital radial velocity amplitude  $K_c$ , which together with x and  $P_b$  for the pulsar, yields the mass ratio of the binary,  $q \equiv m_{\rm p}/m_{\rm c} = K_{\rm c}/K_{\rm p}$ . Furthermore, the spectrum of the companion contains information about its composition and atmospheric properties, which in turn depend on the stellar mass and radius.

Most known MSPs have He-core white-dwarf companions with a pure hydrogen atmosphere. Despite the model dependences implicit in the spectroscopic method, the mapping between atmospheric parameters

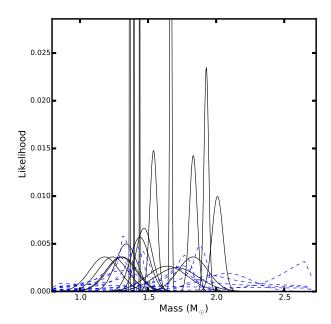


Figure 2. Mass likelihoods for the systems in Tables 1–3, based on Eq. 1 (Table 1, solid lines) and Eqs. 9 & 10 (dashed lines). All likelihoods are normalized so that the enclosed area is the same. The three precise MSP measurements in the  $1.3-1.4\,\mathrm{M}_\odot$  range, have peak likelihoods around  $\sim 0.35$ .

and white-dwarf masses has reached a sufficient level of precision to allow for accurate mass determinations (see Antoniadis 2013; Istrate et al. 2014b; Tremblay et al. 2013; Althaus et al. 2013; Tremblay et al. 2015, and references therein).

#### 2.3. MSP mass measurements and uncertainties

The sample of MSPs collected here consists of systems with constraints on at least the total mass  $M_{\rm T}$ , or mass ratio q. Compared to previous work, our definition of MSPs slightly differs. Instead of selecting our sample solely based on the pulsar spin period, we make choices on a case-by-case basis, taking into consideration other observed properties such as the orbital period and companion type. For example, the massive pulsar PSR J0348+0432 with  $P_{\rm s}=39\,{\rm ms}$ , would not normally qualify as an MSP ( $P_{\rm s}\lesssim30\,{\rm ms}$ ). Nevertheless, the system most likely evolved from a low-mass X-ray binary (LMXB) and therefore might have experienced significant accretion (recycling) from its binary companion (Antoniadis 2013; Istrate et al. 2014a,b).

Overall, our sample consists of 19 MSPs with precisely determined masses, 10 MSPs with constraints only on  $M_{\rm T}$ , and 4 systems with constraints on q. We show the likelihoods over mass for each of these pulsars in Fig. 2 and describe them in more detail below.

#### 2.3.1. Precision mass measurements

These systems are shown in Table 1. The subsample primarily consists of pulsars with constraints on either two pK parameters, or spectroscopically resolved He white dwarf companions. We also include PSR J0337+1715 (Ransom et al. 2014), a pulsar in a hierarchical triple system, where the masses are obtained from the timing signature of the 3-body interactions, and PSR J1023+0038 (Archibald et al. 2009), a transitional MSP with a measured parallax and an optically-bright companion. For these binaries, we write the likelihood that pulsar j has a mass  $m_{\rm p}$  as:

$$E_j(\text{data}|m_p) = C_j \exp\left[-\frac{(m_p - m_0^j)^2}{2\sigma_{m_0^j}^2}\right],$$
 (8)

where  $\sigma_{m_0^j}$  is the inferred measurement uncertainty on  $m_{\rm p}$  and  $C_j$  a proper normalization factor to ensure that  $\int E_j dm_{\rm p} = 1$ . The mass likelihoods for the systems in Table 1 based on Eq. 8 are shown in Fig. 2 as solid lines.

It is worth noting that for systems studied with optical spectroscopy, the actual estimate is slightly asymmetric around the mean, with a skewness towards larger masses. However, for the systems considered here, this asymmetry is small and therefore can be safely accounted for with an appropriate increase in  $\sigma_{m_p^i}$ .

## 2.3.2. Pulsars with constraints on the total mass

This group includes systems with a constraint on the total mass  $M_{\rm T}$  (Table 2). Assuming an inclination with a probability distribution that is uniform in  $\cos i$ , the likelihood for the mass of the  $j^{\rm th}$  pulsar can be written as

$$E_j(\text{data}|m_p) = C_j \int dM_T \int d(\cos i) \times \times \delta \left[ f_0 - f(M_T, m_p, i) \right] \times \exp \left[ -\frac{(M_T - M_0^j)^2}{2\sigma_{M_0^j}^2} \right], \quad (9)$$

where again  $C_j$  is a normalization coefficient. For each i, the Dirac delta function involving the mass function can be evaluated from  $\delta(i-i_0)$ , where  $i_0$  is the solution to the mass function equation for a given set of stellar masses (see Özel et al. 2012 for details).

#### 2.4. Systems with constraints on the mass ratio

The final category considered here consists of three MSPs with optically bright low-mass companions (Table 3). PSRs B1957+20 and J1311-3430 (van Kerkwijk et al. 2011; Romani et al. 2012, 2015) belong to a class of  $\gamma$ -ray bright eclipsing MSPs with extremely low-mass irradiated companions.

For PSR B1957+20, van Kerkwijk et al. (2011) derived the mass ratio shown in Table 3 after accounting

for the fact that due to the strong irradiation of the companion's surface, radial velocities track the area facing the pulsar (center of light) rather than the center of mass. Using extra constraints on the inclination from the companion's lightcurve (Callanan et al. 1995), the pulsar mass at face value is  $2.39(36)\,\mathrm{M}_{\odot}$ . However, van Kerkwijk et al. (2011) find that the impact of modeling uncertainties is large and the pulsar mass could be as low as  $1.66\,\mathrm{M}_{\odot}$ .

For PSR J1311–3430 (Romani et al. 2012), the initial reported value based on the same technique suggested a pulsar mass with  $M > 2.5 \,\mathrm{M}_{\odot}$ , but a more recent analysis by Romani et al. (2015) shows that a mass as low as  $\sim 1.6 \,\mathrm{M}_{\odot}$  is still possible.

PSR J1816+4510 is a binary MSP with an orbital period of 8.7 h and a metal-rich, low mass ( $\gtrsim 0.16\,\mathrm{M}_\odot$ ) companion, the radial velocity of which implies a high-mass of  $m_\mathrm{p} \sin^3 i = 1.84(11)\,\mathrm{M}_\odot$ .

Finally, PSR J1740-5340 is an eclipsing MSP with a  $\sim 0.2\,\mathrm{M}_\odot$  red-straggler companion in the globular cluster NGC 6397. This system resembles closely PSR J1023+0038 which has been observed to switch between a rotation- and an accretion-powered phase.

Given the unresolved discrepancies in the modeling of these systems, we conservatively assume a randomly oriented orbit and only use the mass ratio q for our analysis. We evaluate the mass of the  $j^{\rm th}$  pulsar as:

$$E_{j}(\text{data}|m_{p}) = C_{j} \int dq \int d(\cos i) \times \delta \left[f_{0} - f(q, m_{p}, i)\right] \times \exp \left[-\frac{(q - q_{0}^{j})^{2}}{2\sigma_{q_{0}^{j}}^{2}}\right]$$
(10)

where  $q_0$  and  $\sigma_{q_0}$  correspond to the inferred value of q and its formal uncertainty. The resulting mass likelihoods are broad and therefore have a small impact on the analysis following below. In fact, we reach the same main conclusions even if we neglect these systems entirely.

#### 3. STATISTICAL METHOD

Our main goal is to select the empirical model that best describes the intrinsic MSP mass distribution. For each model with a parameter vector  $\boldsymbol{\theta}$ , we compute the likelihood as

$$\mathcal{L}(\text{data}|\boldsymbol{\theta}) = \prod_{j}^{n} \int dm_{p} E_{j}(\text{data}|m_{p}) \times P(m_{p}|\boldsymbol{\theta}), \quad (11)$$

and then calculate the posterior probability using Bayes' theorem:

$$P(\boldsymbol{\theta}|\text{data}) = \frac{P(\boldsymbol{\theta}) \times \mathcal{L}(\text{data}|\boldsymbol{\theta})}{P(\text{data})},$$
 (12)

Table 1. Radio Millisecond pulsars with precise mass measurements

#	PSR Name	Mass $[M_{\odot}]$	Reference
1	J0337+1715	1.4378(13)	Ransom et al. (2014)
2	J0348 + 0432	2.01(4)	Antoniadis et al. (2013)
3	J0437 - 4715	1.44(7)	Reardon et al. (2016)
4	J0751 + 1807	1.64(15)	Desvignes et al. (2016)
5	J1012+0507	1.83(11)	this work (Appendix)
6	J1023+0038	1.71(16)	Deller et al. (2012)
7	J1614 - 2230	1.928(17)	Fonseca et al. (2016)
8	J1713+0747	1.31(11)	Zhu et al. (2015)
9	J1738+0333	1.47(7)	Antoniadis et al. (2012)
10	J1802 - 2124	1.24(11)	Ferdman et al. (2010)
11	J1807 - 2500B	1.3655(21)	Lynch et al. (2012)
12	B1855+09	1.30(11)	Fonseca et al. (2016)
13	J1903+0327	1.667(7)	Freire et al. (2011)
14	J1909 - 3744	1.540(27)	Desvignes et al. (2016)
15	J1910 - 5959A	1.34(8)	Corongiu et al. (2012)
16	J1918 - 0642	1.18(11)	Fonseca et al. (2016)
17	J1946 + 3417	1.832(13)	Barr et al. (2016)
18	J2234+0611	1.396(11)	Stovall et al. (2016)

Table 2. Millisecond pulsar binaries with constraints on the total mass

#	PSR Name	$f(m)~[{ m M}_{\odot}]$	$M_{\rm T}~[M_{\odot}]$	Reference
1	J0024-7204H	0.001927	1.61(4)	Freire et al. (2003)
2	J0514 - 4002A	0.14549547	2.453(14)	Freire et al. (2007)
3	J0621 + 1002	0.027026849	2.32(8)	Splaver et al. (2002)
4	B1516 + 02B	0.000646723	2.29(17)	Freire et al. (2008b)
5	J1748 - 2021A	0.0518649	1.97(15)	Freire et al. (2008b)
6	J1748 - 2021B	0.0002266235	2.92(20)	Freire et al. (2008a)
7	J1748 - 2446I	0.003658	2.17(2)	Ransom et al. (2005)
8	J1748 - 2446J	0.013066	2.20(4)	Ransom et al. (2005)
9	B1802 - 07	0.00945034	1.62(7)	Thorsett & Chakrabarty (1999)
10	J1824-2452C	0.006553	1.616(7)	Freire et al. (2008a)

Table 3. Millisecond pulsar binaries with constraints on the mass ratio

#	PSR Name	$f(m)~[{ m M}_{\odot}]$	q	Reference
1	J1311-3430	$3 \times 10^{-7}$	175(3)	Romani et al. (2015)
2	J1740 - 5340	0.002644	5.85(13)	Ferraro et al. (2003)
3	J1816+4510	0.0017607	9.54(0.21)	Kaplan et al. (2013)
4	B1957+20	$5 \times 10^{-6}$	69.2(8)	van Kerkwijk et al. (2011)

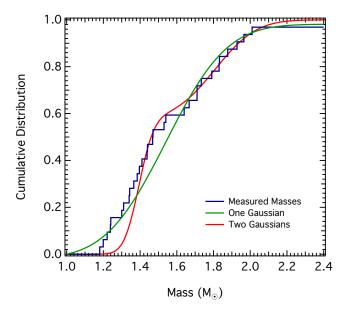


Figure 3. Cumulative histogram of MSP masses. The green curve shows the cumulative distribution for a single Gaussian, with parameters that correspond to the most likely values inferred from the data for this intrinsic distribution. The departure of the observed masses from a single Gaussian is already evident in this figure and the cumulative histogram for the two-Gaussian-component model shown in red provides a significantly better description of the data.

where  $P(\boldsymbol{\theta})$  is the prior for  $\boldsymbol{\theta}$  (see next section) and P(data) ensures proper normalization.

The posterior distribution for the parameter vector  $\boldsymbol{\theta}$  is sampled using a many-particle affine invariant Markov chain Monte Carlo (MCMC) sampler (Goodman & Weare 2010) as implemented in the python package emcee (Foreman-Mackey et al. 2013). We experimented with different number of samplers (from 4 to 800), thinning factors (0–100), and initialization strategies. The results were overall consistent with maximum differences of order 1% in the inferred marginalized median parameters and the location of the maximum likelihood in the posterior distribution. The values reported below were obtained using 800 samplers, a thinning factor of 50 and 2000 iterations per sampler. The samplers were initialized in a small sphere enclosing the preferred model parameters, after some iteration.

## 4. RESULTS

Before we apply the Bayesian statistical tools to infer the parameters of the various intrinsic models, we plot the cumulative distribution of MSP masses to assess visually the level of complexity that we would need to incorporate into the underlying distributions that can be supported by the data. In Fig. 3, we show a cumulative histogram of the most likely values for the MSP masses. If the data were described by a single Gaussian, the cumulative histogram would look like the curve

shown in green. However, the presence of multiple inflection points strongly suggests the presence of multiple components in the underlying distribution. It is evident already from this figure that the two Gaussian component model shown in red offers a better description of the data. We will now demonstrate this result quantitatively using the Bayesian inference method discussed in the previous section.

#### 4.1. Normal Distribution

We start by testing the normal distribution (henceforth Model I),

$$P(m_{\rm p}|\mu,\sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{(m_{\rm p}-\mu)^2}{2\sigma^2}\right]$$
(13)

as the simplest possible approximation. We implement very weak theoretical constraints and use a flat prior with  $1.0 \le \mu \le 2.5\,\mathrm{M}_\odot$  and  $0.0 < \sigma \le 1.0\,\mathrm{M}_\odot$ . We also restrict our estimates to  $0.8 \le m_\mathrm{p} \le 3.0\,\mathrm{M}_\odot$ .

The corresponding posterior probability distribution is shown in Figs. 4 and 5. The posterior samples yield  $\mu=1.542^{+0.054}_{-0.057}~{\rm M}_{\odot}$  and  $\sigma=0.260^{+0.061}_{-0.043}~{\rm M}_{\odot}$  for the median and 68% confidence levels (C.L.), which are slightly larger than the values reported in Özel et al. (2012). This is not surprising as a larger dispersion is required to accommodate for the additional high mass NSs measured after 2012. Nevertheless, this model also predicts a high number of stars with  $m_{\rm p}\simeq 1.55~{\rm M}_{\odot}$  and  $m_{\rm p}\leq 1.1~{\rm M}_{\odot}$ , which are not observed (Fig. 2). Qualitatively, this is a strong indication for asymmetry, either due to skewness or the presence of a second component at higher masses.

#### 4.2. Bimodal Normal Distribution

We next test for the existence of a second peak using the bimodal normal distribution (Model II),

$$P(m_{\rm p}|\mu_1, \sigma_1, \mu_2, \sigma_2, r) = (1-r)G(\mu_1, \sigma_1) + rG(\mu_2, \sigma_2)$$
(14)

where  $G(\mu_{1,2}, \sigma_{1,2})$  are the two normal components and r is their relative ratio. One useful property of this model is that it can also account for skewness in the case of a single-peaked distribution, e.g., when  $\mu_2 = (1+\epsilon)\mu_1$  and  $\sigma_2 > \sigma_1$ , with  $\epsilon \in \mathbb{R}$  being a small number.

Here we also use boxcar priors which are set as follows: since the two components are mutually interchangeable, we use  $1.0 \le \mu_1 \le 1.6$  and  $1.45 \le \mu_2 \le 2.8\,\mathrm{M}_\odot$ , as well as  $0.0 < \sigma_{1,2} \le 0.5\,\mathrm{M}_\odot$ . This ensures faster numerical convergence but still allows for a single-peaked distribution centered around  $\sim 1.55\,\mathrm{M}_\odot$ , as above. We adopt  $0 \le r \le 1$  for the relative ratio of the two components.

The MCMC (Figs. 6 & 7) yields a maximum likelihood at  $[\mu_1, \sigma_1, \mu_2, \sigma_2, r] =$ 

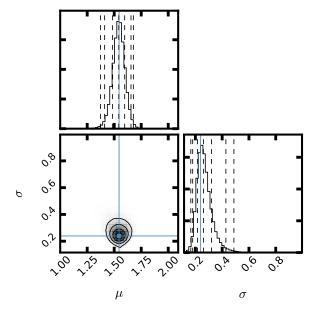


Figure 4. Histogram of the MCMC samples drawn from the posterior of the normal distribution. The contours enclose 68, 98 and 99% of the samples. The blue lines show the location of the maximum likelihood.

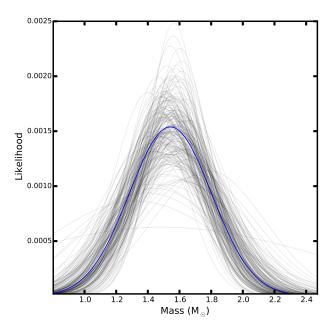


Figure 5. Realization of the inferred normal distribution. The blue line shows the model corresponding to the median values of the MCMC samples. Grey lines represent 1000 samples drawn randomly from the posterior and can be viewed as a measure of the uncertainty in the inferred parameters.

[1.396, 0.045, 1.84, 0.100, 0.389]. Overall, for the second peak the posterior likelihood yields  $\mu_2 = 1.807^{+0.081}_{-0.132}$  and  $\sigma_2 = 0.177^{+0.115}_{-0.072}$  for the median and 68% C. L. Finally, the marginalized posterior likelihood for the relative ratio of the two peaks has  $r = 0.425^{+0.150}_{-0.132}$ . A comparison of the two models follows in the remainder of this section.

#### 4.3. Model Selection

The summary statistics of the two models described above are given in Table 4. Qualitatively, Model II seems to provide a better description of the data as it accounts both for the apparent small number of pulsars with masses around  $\sim 1.55\,\mathrm{M}_\odot$  and those with  $m_\mathrm{p} < 1.1\,\mathrm{M}_\odot$  (see also Figs. 1 & 2).

As both models are only empirical approximations rather than true physical descriptions of the intrinsic mass distribution, we employ the second-order Akaike Information Criterion (Akaike 1974; Burnham & Anderson 2002),

$$AICc_i = -2\ln\hat{\mathcal{L}}_i + 2k\frac{n}{n-k-1}$$
 (15)

as a means to quantify the relative information loss and select the model that best describes the data.

The second term in Eq. 15 introduces a penalty for the complexity of each model, which depends on the number of free parameters k and the number of data points n. The relative likelihood can then be computed as

$$\delta \hat{\mathcal{L}}_{AICc} = \exp\left[\frac{AIC_{min} - AIC_{max}}{2}\right]$$
 (16)

In our case, AIC $_{\rm min}=452.4$  for Model II and AIC $_{\rm max}=461.1$  for Model I. This yields  $\delta\hat{\mathscr{L}}_{\rm AICc}=0.013$  for Model I, which means that Model II is highly favored, even after accounting for the larger number of parameters. In the following, we discuss the properties of Model II in more detail.

#### 4.4. Properties of the Bimodal Distribution

As briefly mentioned in Section 4.2, Model II does not necessarily imply the presence of a second component, as it can also account for skewness in the case of a single-peaked distribution. Indeed, as can be seen in Fig. 7 and Table 4, the marginalized likelihoods for  $\mu_2$ ,  $\sigma_2$  and r span a wide range of values, even allowing for a normal distribution peaking around 1.55 M $_{\odot}$  within the 99% C.L. To quantify the separation of the two components, we use the statistic (Ashman et al. 1994),

$$D = 2^{1/2} \frac{|\mu_1 - \mu_2|}{\sqrt{\sigma_1^2 + \sigma_2^2}} \tag{17}$$

According to Ashman et al. (1994), for a mixture of two normal distributions, a clean separation requires D > 2. The histogram for D inferred from the posterior is shown

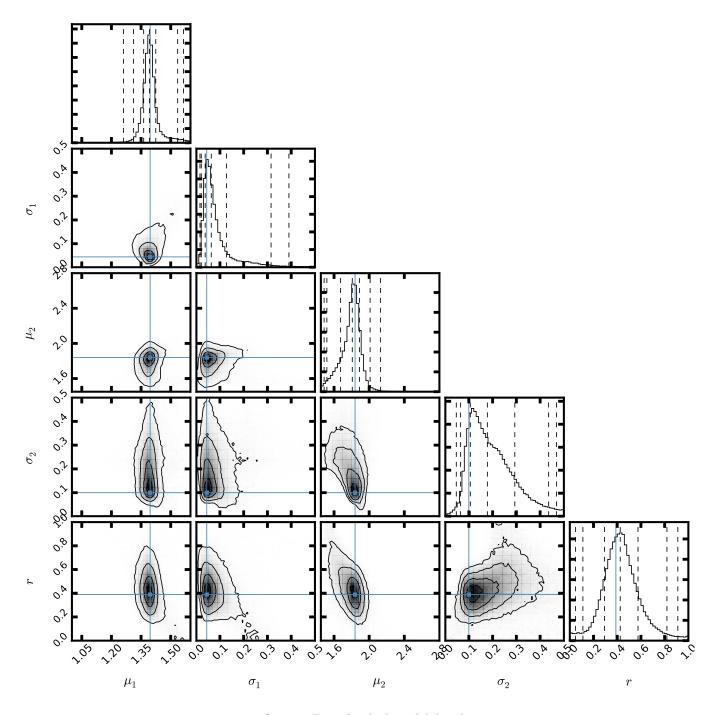


Figure 6. Same as Fig. 2 for the bimodal distribution.

 Table 4. Summary of Model Estimates

-	$\mu_1$ $\sigma_1$			$\mu_2$			$\sigma_2$			r				AICc		
CL	50%	68%	98% 50%		68%	98%	50%	68%	% 98%		68%	98% 50%		68%	98%	
Model I	1.542	1.485 -1.597	1.412 -1.655	0.260	0.216 - 0.320	0.180 - 0.427	-	-	-	-	-	-	-	-	-	461.1
Model II	1.393	1.363 - 1.424	1.312 - 1.535	0.064	0.038 - 0.128	0.021 - 0.128	1.807	1.674 - 1.889	1.518 - 2.011	0.178	0.106 - 0.293	0.065 - 0.436	0.425	0.293 - 0.841	0.110 - 0.819	452.4
	μ			σ			$\log \Delta M$			$\sigma_{\Delta M}$						
Model III	1.229	1.098 -1.382	1.020-1.496	0.121	0.058 - 0.228	0.042 - 0.325	-0.514	-0.8850.272	-1.1450.074	0.332	0.172 - 0.447	0.036 - 0.490		-	-	

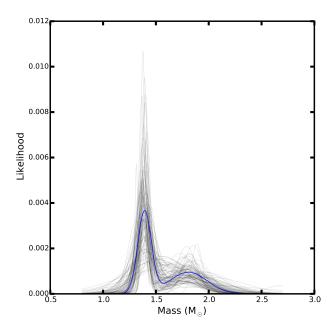


Figure 7. Same as Fig. 3 for the bimodal distribution.

in Fig. 8. We find  $D=3.12^{+1.95}_{-1.61}$ , with  $D\geq 2$  for 73% of the samples. Hence, albeit less favored, a uni-modal distribution with a significant positive skewness cannot be ruled out conclusively with the existing sample of MSP masses.

An alternative way to assert the likelihood of an asymmetric uni-modal distribution is to test whether mass accretion onto the NS can reproduce the observed sample, starting with normally distributed birth masses. To do this we assume that the masses of recycled pulsars can be described by,

$$P(m_{\rm p}|\mu,\sigma_{\mu}) = \int d\Delta M \int d\sigma_{\Delta M} P_{\rm b}(\mu - \Delta M, \sigma_{M}) P_{\rm a}(\Delta M, \sigma_{\Delta M}),$$
(18)

which we refer to as Model III in what follows. Here  $P_{\rm b}(\mu-\Delta M)$  follows Eq. 13 and represents the MSP birth-mass distribution, and

$$P_{\rm a}(\Delta M) = \frac{1}{\Delta M \ln(10) 2\pi (\sigma_{\Delta M}^2)^{1/2}} \times \exp \left[ -\frac{(\log \Delta M - \log \Delta M_0)^2}{2\sigma_{\Delta M}^2} \right]$$
(19)

is a log-normal distribution which we use to approximate the effect of mass accretion. We show the postrior likelihoods for the parameters of Model III in Fig. 9. Unlike Models I & II, the posterior is not well-localized and consequently, the selection criteria used above can

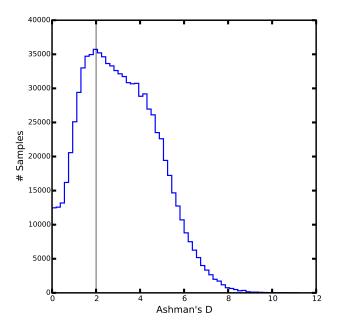


Figure 8. Histogram of the Ashman-D statistic for the MCMC samples drawn from the bimodal distribution. 73% of the samples have  $D \geq 2$ , favoring a bimodal distribution

not be applied directly. However, a qualitative comparison with Models I and II is still possible by examining the physical implications of the maximum likelihood model, located at  $[\mu - \Delta M, \sigma_M, \log_{10} \Delta M, \sigma_{\Delta M}] =$ [1.23, 0.040, -0.511, 0.464]. The "birth mass" distribution  $P_{\rm b}$  – likely constrained by low-mass NSs close to the range  $1.2-1.4\,\mathrm{M}_{\odot}$  – is narrowly distributed around an extremely low mass. This would imply that all massive NSs were born with an extremely low mass and subsequently accreted a substantial amount of material,  $\Delta M$ . As we discuss below in more detail, this assertion is disfavored by stellar evolution considerations in at least a subset of the cases, supported by the systems' observed properties (for instance, the nature of their companions). A more plausible model would require a relatively broad birth-mass distribution, and an accretion kernel that yields high probabilities for small  $\Delta M$ s. However, as can be seen in Figure 8, such models have low likelihood values compared to the preferred model. For this reason, in any further calculations, we focus only on Models I and II.

# 5. CONSTRAINTS ON THE NEUTRON STAR EQUATION OF STATE

One interesting implication of the inferred distribution is that 3% of NS must have masses above  $2.1\,\mathrm{M}_{\odot}$ . However, the true predictive power of the model is small, as the existence of such high-mass NSs depends primarily on the underlying EoS. Indeed, there exist com-

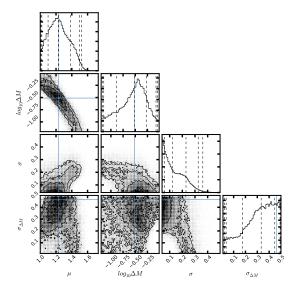


Figure 9. Posterior likelihoods over the parameters of a model that convolves a Gaussian birth-mass distribution with a log-normal kernel that represents the effects of accretion (Model III; see Section 4.4).

pelling theoretical evidence in favor of a relatively soft EoS, which can only support NSs with masses below  $\leq 2.1\,\mathrm{M}_{\odot}$ . This would introduce a high-mass cut off in the distribution. With the framework developed here, we can place constraints on the maximum NS mass by including an extra truncation parameter,  $M_{\mathrm{max}}$  so that

$$P(m_{\mathbf{p}}|\boldsymbol{\theta}) = \begin{cases} P(m_{\mathbf{p}}|\mu_1, \sigma_1, \mu_2, \sigma_2, r), & \text{if } m_{\mathbf{p}} \leq M_{\text{max}} \\ 0, & \text{otherwise.} \end{cases}$$
(20)

The posterior probability of this model is shown in Fig. 10 and yields

$$M_{\text{max}} \ge 2.018 \,\mathrm{M}_{\odot} \,(98\% \,\mathrm{C.L.}),$$
 (21)

or  $M_{\rm max} \geq 1.924\,{\rm M}_{\odot}$  at 99.98% C.L.. This is imposed mainly by the massive pulsar PSR J0348+0432. Interestingly, this limit appears to be almost insensitive to the choice of the underlying model for  $m_p \leq M_{\rm max}$ ; switching from Model II to I yields  $M_{\rm max} \geq 2.019\,{\rm M}_{\odot}$  (98% C.L.).

#### 5.1. Expectations for Future Pulsar Surveys

The detection of a high-mass truncation would be of high importance for nuclear physics calculations as it would provide direct constraints on the EoS at very high densities (Özel & Psaltis 2009, see). The inferred MSP mass distribution and Eq. 20 allow to assess the likelihood of measuring  $M_{\rm max}$  in the future.

Pulsar surveys planned for, e.g., the Square-Kilometre Array (SKA) in Phase 2, will provide a nearly complete census of the radio-pulsar population in the Galaxy (Keane et al. 2015). Follow-up observations of these dis-

coveries are expected to yield precision mass measurements for over 350 MSPs (e.g. Tauris et al. 2015b; Antoniadis et al. 2015, and references therein). Using our estimates above, we can simulate their underlying mass distribution, include a cut-off at a given  $M_{\rm max}$ , and then use Eq. 20 to assess its detectability.

The robustness of such a simulation depends critically on a realistic estimate for the expected precision of NS mass-measurements with future instruments, which is beyond the scope of this work. For masses derived with radio timing, it is safe to assume that the distribution of uncertainties will remain nearly constant. This is because both the survey sensitivity and timing precision scale similarly with the telescope size. For optical spectroscopy, uncertainties depend on the companion's brightness and therefore for a given telescope the achieved precision will scale (at least) with the square of the distance. Nevertheless, future optical telescopes such as LSST, E-ELT and TMT will significantly increase the Galaxy volume that can be probed with this technique.

For the estimates presented here we adopt a distribution of measurement uncertainties that is uniform between the minimum and maximum errors shown in Table 1.

For our simulation, we first use the median parameters for Model II introducing a cut-off mass  $M_{\rm max}$  at values ranging from 2.0 to  $2.5\,{\rm M}_{\odot}$ . We then draw a number of MSPs to which we assign uncertainties that follow the aforementioned uniform distribution. Finally, we apply Eq. 20 and examine the marginalized posterior probability for  $M_{\rm max}$ .

We show the results of two simulations for 350 and 500 MSPs in Fig. 11. In the former case, a high-mass truncation would be detectable up to  $\sim 2.15\,\mathrm{M}_\odot$  with a  $3\sigma$  precision of  $\sim 0.2\,\mathrm{M}_\odot$ . In the second scenario,  $M_{\mathrm{max}}$  can be detected up to  $2.25\,\mathrm{M}_\odot$ , a value that is above the predictions of a large number of EoS models.

## 5.2. Massive NSs in the known MSP population

Another question related to the maximum mass is whether if we can identify massive NSs among MSP systems that do not have any measured constraints on their total mass or mass ratio. There are currently  $\sim 200$  binary MSPs listed in the ATNF pulsar catalogue<sup>2</sup> (Manchester et al. 2005), excluding those listed in Tables 1-3.

Based on our models,  $\sim 40$  of these should have masses above  $1.8\,\mathrm{M}_{\odot}$ . However, these systems are impossible to identify directly without any further information. One possibility is to search among those systems that may

<sup>&</sup>lt;sup>2</sup> http://www.atnf.csiro.au/research/pulsar/psrcat/

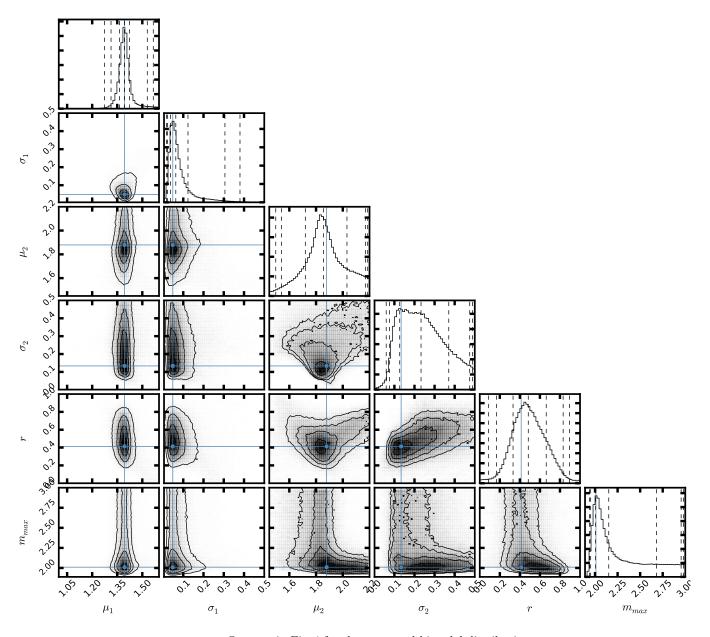
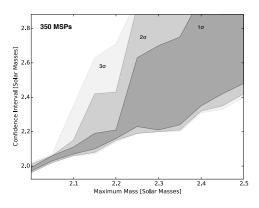
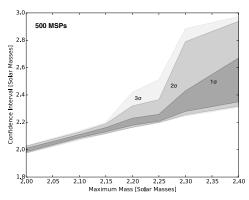


Figure 10. Same as in Fig. 4 for the truncated bimodal distribution.

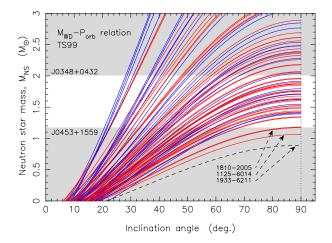




**Figure 11.** Projected precision on the localization of the maximum NS mass as a function of the number of MSP mass measurements.

have experienced a significant episode of mass transfer, e.g., those associated with eclipsing  $\gamma$ -ray systems (see Section 2). As we briefly discussed in the introduction, there is indeed accumulating evidence for massive NSs in these binaries. Unfortunately, precision measurements are hard to achieve with current methods, as the timing precision is relatively poor and the pulsar companions are not understood well enough to provide meaningful mass estimates using optical spectroscopy. Nevertheless, alternative methods such as those based on pulsar scintillometry (Pen et al. 2014) may help to overcome these limitations in the future.

Another approach is to employ constraints imposed by binary evolution theory. For example, the masses of He-core white dwarf companions to MSPs are known to correlate tightly with the orbital period, (e.g., Tauris & Savonije 1999; see also Fig. 2 in Tauris & van den Heuvel 2014 for a comparison with recent data). The main reason for this is the correlation between the radius and the degenerate core mass of the (sub)giant star



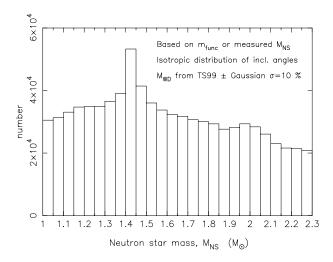


Figure 12. NS masses as a function of inclination (upper panel) and accumulated mass likelihoods (lower panel) for all known MSPs with He white dwarf companions, based on the  $M_{\rm WD}-P_{\rm b}$  relation of Tauris & Savonije (1999). The summary statistics can be found in Table 5. The red curves are for binaries with  $P_{\rm b} < 30$  days; blue curves are for systems wider than 30 days. PSR J1933–6211 (black curve) is an example of a system with a CO white dwarf which does not obey the  $M_{\rm WD}-P_{\rm b}$  relation, and thus it is not included in our statistical sample.

progenitor of the He white dwarf. Hence, from the measured orbital period  $P_{\rm b}$ , we can determine the mass of the He white dwarf,  $M_{\rm WD}$ , and thus, given the mass function of the observed system, obtain the mass of the pulsar as a function of orbital inclination angle.

The top panel of Fig.12 (the 'spaghetti plot') shows this result for all known MSPs with He white dwarf companions, calculated using the relation of Tauris & Savonije (1999) for solar chemical composition. The

red curves are for binaries with orbital periods of less than 30 days; blue curves are for systems wider than 30 days. There is a slight trend for wider systems having marginally more massive NSs (see discussions below).

Assuming an isotropic distribution of orbital inclinations as in Sections 2.3.2 & 2.3.3, we can calculate the mass likelihood for each pulsar as:

$$\mathcal{L}_{j}(m_{\mathrm{p}}|m_{\mathrm{c}}) = C_{j} \int dm_{\mathrm{c}} \int d(\cos i) \times \delta \left[ f_{0} - f(m_{\mathrm{c}}, m_{\mathrm{p}}, i) \right] \times \exp \left[ -\frac{(m_{c} - m_{0}^{j})^{2}}{2\sigma_{m_{0}^{j}}^{2}} \right], \quad (22)$$

where  $m_0$  is the theoretical prediction for the white dwarf mass. We show in the bottom panel of Fig. 12 the expected NS mass distribution marginalized over the inclination angle based on these likelihoods for all known MSPs with He white dwarf companions. Here, we calculated the white dwarf mass using the  $M_{\rm WD}-P_{\rm b}$  relation of Tauris & Savonije (1999), assumed a fractional uncertainty of 10% (see also Table 5 in the Appendix), and assumed that there are no selection effects in the observed sample. Only in cases where the MSPs mass is measured (cf. Table 1), we use this value and its associated uncertainty. We disregarded the solutions for NS masses outside the interval of  $1.0-2.3~{\rm M}_{\odot}$ .

At first sight, there seems to be a significant number of potentially massive NS candidates. Nevertheless, it should be stressed that these results depend on the applicability of the  $M_{\rm WD}-P_{\rm b}$  relation. For example, some systems may simply reflect a violation of required assumptions behind this relation, i.e., the evolution in LMXB systems with donor star masses  $\lesssim 2.3 \,\mathrm{M}_{\odot}$  (Tauris & Savonije 1999). An example of an MSP which did not evolve from an LMXB system is PSR J1933-6211 (plotted as a dashed black curve in the top panel of Fig. 12). This white dwarf companion has a CO core (Matteo et al., in prep.) and, thus, the system most likely evolved from an intermediate mass X-ray binary (like PSR J1614-2230, Tauris et al. 2011), i.e., under conditions in which a non-degenerate core developed in the white dwarf progenitor star. Therefore, this system will not obey the  $M_{
m WD}-P_{
m b}$  relation, which would have predicted an unrealistic NS mass of  $< 0.9 M_{\odot}$ . Interestingly enough, in the bottom panel of Fig. 12, a peak of a NS mass near 1.4  $M_{\odot}$  is seen in this distribution of MSP masses, similar to that shown in Fig. 7.

## 6. DISCUSSION

#### 6.1. Summary

In this work, we used the available ensemble of MSP mass measurements to examine the underlying recycled NS mass distribution. The main results can be broadly summarized as follows:

- A normal distribution does not seem to provide an adequate description of the observed MSP masses. Based on our Bayesian analysis method outlined in Section 3, we find strong evidence for asymmetry. More specifically, a bimodal distribution peaking at  $\sim 1.4$  and  $\sim 1.8\,\mathrm{M}_{\odot}$  is favored by the data, but a single-peaked distribution with strong positive skewness is allowed within 20% of the posterior likelihood and cannot be conclusively ruled out (see Section 4.3).
- Massive NSs seem to be more common than previously thought. In the inferred distributions, we find that approximately 20% of binary MSPs have masses above  $1.8\,\mathrm{M}_\odot$ . This number is independent of the assumption made for the shape of the underlying mass distribution.
- Including a high-mass truncation in our models yields a robust estimate on the maximum NS mass of  $M_{\rm max} \geq 2.018\,{\rm M}_{\odot}$  at 98% C.L. This result is again not sensitive to the adopted distribution model.
- As the number of mass measurements increases, it may become possible to precisely measure a maximum mass cut-off. More specifically, our simulation shows that with 350 MSP mass measurements following the currently favored bimodal distribution, it will be possible to localize an  $M_{\rm max}$  lower than  $2.15\,{\rm M}_{\odot}$  with a precision better than 5%.

## 6.2. Selection Effects

Before discussing the ramifications of these results in more detail, it is necessary to consider possible selection effects that may bias our findings.

First, our dataset is intrinsically biased in the sense that it only contains NSs in binary systems. As binarity can significantly affect the outcome of massive star evolution (Langer 2012), the inferred mass distribution may differ substantially from that of isolated NSs. Currently, there exist very little information for the masses of single NSs. A recent study of micro-lensing events toward the Galactic Bulge in the OGLEIII survey (Wyrzykowski et al. 2015) suggests that the data are consistent with a uniform mass distribution of single compact objects but the uncertainties in the individual mass measurements using this method are still very large. Furthermore, it is possible that some of these lensing sources are normal stars located close to the lensed background objects. The ongoing Galactic survey by GAIA (Belokurov & Evans 2002), as well as the 4th phase of the OGLE survey (Udalski et al. 2015) and the KMTNet experiment (Henderson et al. 2014) will significantly improve on the aforementioned result and may even make it possible,

for the first time, to identify photometric or astrometric micro-lensing events caused by radio pulsars (Dai et al. 2015). If the distance and transverse velocity of a lensing pulsar can be determined, then it may become possible to measure its mass with a precision of order 10% (Dai et al. 2015).

A second bias when discussing NS masses is that in this work we have only considered MSPs, most of which have likely experienced a long-term period of mass accretion from their binary companion. The extent to which accretion shifts the current masses away from the NS birth masses depends purely on the ability of the NS to accumulate mass during the LMXB phase. As we will argue in more detail below, observational evidence suggest that, in most cases, the accretion efficiency is low, possibly smaller than  $\sim 10\%$ . Hence, even though it is not possible to completely deconvolve the impact of mass gain, MSP masses can still serve as a meaningful probe of the SN mechanism and the structure of their progenitors, as we discuss in more detail in Section 6.4.

Another point of caution is the small number of MSPs employed in the analysis: we use 32 MSPs of which only 18 have precisely measured masses. This makes our result sensitive to small number statistical fluctuations. For instance, the identification of the high-mass peak component in the distribution of MSP masses is only possible due to the recent precision measurements of a few high-mass NSs like PSRs J0348+0432 and J1949+3106, in addition to PSR J1614-2230, already considered in previous studies by Özel et al. (2012) and Kiziltan et al. (2013). Even though these measurements are robust, a measurement bias not reflected in the formal uncertainties may alter the posterior probability distribution, in particular the location and dispersion of the high-mass peak component.

Finally, one must also consider possible selection effects in the MSP sample itself, in particular those stars with precise mass measurements (Table 1). Masses derived from Shapiro delay are easier to achieve in short orbital period binaries with high inclinations. For longer period binaries, the signal is weaker but it also becomes logistically more difficult to achieve the required observing cadence. Similarly, the optical spectroscopy method is more relevant to compact systems. Here, fine orbital sampling and homogeneity are equally important, but there also exists an anti-correlation between the binary orbital period and the intrinsic luminosity of the white dwarf companion. This is because the stellar envelope thickness, which determines the cooling rate, depends sensitively on the white dwarf mass, which in turn scales with the orbital separation. Nevertheless, within our sample, there is no evident correlation between the mass of a binary MSP and its direct observational properties. For instance, the relativistic pulsars PSRs J1738+0333,

J0751+1807 and J0348+0432 have similar spin and orbital properties, but very different masses. An exception to this is possible for the eclipsing MSPs in Table 3 which may be massive as a class, although any conclusion is premature with current data. Here however, we only used their mass ratios in our analysis, which results in very broad likelihoods for their masses. Omitting this class entirely leads to almost identical posterior likelihoods and, therefore, does not impact any of the conclusions.

## 6.3. Asymmetry in the MSP mass distribution: Accretion vs NSs born massive

One of our key results is the detection of asymmetry in the MSP mass distribution, most likely caused by the presence of a high mass component centred around  $1.8 \, \mathrm{M}_{\odot}$ .

As briefly discussed above, one mechanism to produce an asymmetric mass distribution is accretion from a binary companion. In Section 4.4, we tested this hypothesis using a simple empirical model to simulate the effect of mass accretion. We found that the highest likelihood model implies that all NSs, including those with high masses, had birth masses close to  $1.1 - 1.3 \,\mathrm{M}_{\odot}$ . If this were the case and the birth masses of MSPs were similar, then we would expect a tight correlation between the observed masses and companion type, which is a direct probe of the magnitude and duration of the mass exchange episode. This is clearly not the case. For instance, PSR J1614-2230 has a CO white dwarf companion and most likely formed via a Case A Rochelobe overflow from at least a  $\sim 4.0\,\mathrm{M}_{\odot}$  main sequence donor. Consequently, the NS in this system did not experience a long-term accretion episode and, therefore, its birth mass must have been at least  $1.7\,\mathrm{M}_{\odot}$  (Tauris et al. 2011; Lin et al. 2011). Similarly, systems like PSRs J1918-0642 and J1738+0333 have very low NS masses, but otherwise appear to be fully recycled.

Hence, we conclude that the high number of massive NSs most likely reflects differences in the NS birth masses and that at least some of them must have been born with a mass larger than  $1.6\,\mathrm{M}_\odot$ . This again connects back to the core-collapse mechanism and the properties of the high-mass progenitor stars prior to the SN explosion.

Possibly the largest determining factor for the mass and nature of the compact remnant is the size of the progenitor's iron core at the onset of core collapse. The iron core mass depends on whether carbon burning proceeds convectively or radiatively, and thus it is sensitive to the  $^{12}{\rm C}/^{16}{\rm O}$  ratio at the depletion of central helium burning. This ratio depends primarily on the initial stellar ZAMS mass and whether the star evolves in isolation/wide-orbit binary or in a close binary system

(Wellstein & Langer 1999; Brown et al. 2001; Podsiad-lowski et al. 2004; see Tauris et al. 2011 for a summary).

In close binaries, which are more relevant to our MSP sample, the progenitor star of the NS may lose its hydrogen-rich envelope at an early stage, thereby reducing the growth of its helium core, resulting in a larger <sup>12</sup>C/<sup>16</sup>O ratio – similar to what is expected for the lower-mass end of massive isolated stars. In more massive stars  $(M > 20 \,\mathrm{M}_{\odot})$ , or somewhat less massive stars evolving in isolation), however, the destruction of carbon via the  $^{12}C(\alpha, \gamma)^{16}O$  reaction tends to dominate over its creation via the  $3\alpha$  process. Therefore, the net central carbon abundance is lower, leading to relatively fast radiative carbon burning, leaving behind a high central entropy and production of a more massive iron core. A large fraction of these stars are expected to form BHs via fallback. However, it is possible some of them will produce high-mass NSs instead, depending on the degree of stripping of envelope mass (Tauris et al. 2015a), the SN explosion physics (Ugliano et al. 2012; Pejcha & Thompson 2015; Müller et al. 2016) and the EoS. Other factors such as metallicity, stellar winds, rotational mixing and angular momentum transport, B-fields, and the location of the outermost oxygen burning shell, may influence the nature of the final remnant as well (Woosley et al. 2002; Heger et al. 2003).

#### 6.3.1. Further evidence for NSs born massive

In the previous section we interpreted the diverse characteristics of high-mass MSPs as a strong indicator for NSs born massive. Further evidence supporting this claim can be found in other binary NS types, where the effect of mass accretion is less severe. For instance, some high-mass X-ray binaries (HMXBs) like Vela X-1 ( $M_{\rm NS}=2.1(1)\,{\rm M}_{\odot}$ ; Falanga et al. 2015) and  $4U\,1700-37~(M_{\rm NS}=2.4(3)\,{\rm M}_{\odot};\,{\rm Kaper\ et\ al.\ 2006})~{\rm may}$ host massive NSs. For these systems, the maximum mass that could have been accreted from their companion is bound by the combination of a short evolution timescale of their high-mass donor ( $\tau_{\rm nuc} \sim 10\,{\rm Myr}$ ) and the Eddington limit for accretion onto a NS ( $M_{\rm Edd} \simeq$ a few  $10^{-8} \,\mathrm{M}_{\odot} \,\mathrm{yr}^{-1}$ ). In reality, the actual accretion rate is likely much smaller, as evident by the relatively low X-ray luminosities of most known HMXBs (Grimm et al. 2002), which imply that the total amount of accreted mass probably does not exceed  $\Delta M \simeq 0.01 \,\mathrm{M}_{\odot}$ (Tauris et al., in prep.).

DNSs are another example of binaries where mass accretion onto the NS is not important for their final mass. All known DNSs host NSs with masses between  $1.23-1.44\,\rm M_{\odot}$  with the exception of the recently discovered PSR J0453+1559 (Martinez et al. 2015), in which a  $1.56\,\rm M_{\odot}$  NS orbits a  $1.17\,\rm M_{\odot}$  companion. While there can be several episodes of mass transfer between the

HMXB and DNS phases, the net mass gain on the first born NS is likely to be less than  $\sim 0.01\,\mathrm{M}_\odot$ , part of which is accreted during the common envelope phase. Consequently, if PSR J0453+1559 is indeed a DNS, this 1.56  $M_\odot$  pulsar must have been born with a mass close to the observed one.

## 6.4. Long-term accretion efficiency in LMXBs

Assuming that the intrinsic MSP mass distribution is indeed bimodal, another insightful finding is the small dispersion for the low-mass component, implied by the posterior likelihood. This is similar to that inferred for DNSs by Ozel et al. (2012). All of the low-mass MSPs in our sample appear to be fully recycled, as evident by their companion types, spin periods and magnetic fields. The ZAMS progenitors of the white dwarf companions had initial masses between  $\sim 1.0$  and  $2.3 \,\mathrm{M}_{\odot}$ (see Antoniadis et al. 2012, and references therein). Therefore, the total mass transfer during the LMXB phase was of order  $0.6-2.1\,\mathrm{M}_{\odot}$  to generate the observed  $\sim 0.16-0.4\,\mathrm{M}_{\odot}$  white dwarf companions. Efficient accretion would imply that most of the NSs in these systems had initial masses below  $1.0 \,\mathrm{M}_{\odot}$ , which is highly unlikely, and such mass transfer (if close to conservative) would also produce a larger dispersion in the NS masses than seen in the observed data. Therefore, it is again likely that the birth masses of the MSPs were not much different from those observed today, typically smaller by  $\leq 0.1~M_{\odot}$ . We note that such a relatively small amount of accreted material is indeed sufficient to recycle the MSPs to spin periods of a few ms (Tauris et al. 2012). Our conclusion (see also Antoniadis et al. 2012) that the accretion during the LMXB phase is highly inefficient, corresponding to average accretion efficiencies of only  $\sim 5-20$  %, means that the far majority of the transferred matter, despite sub-Eddington mass-transfer rates in many cases, is lost from the LMXB system via accretion disk instabilities (van Paradijs 1996; Dubus et al. 2001) and/or propeller effects (see, e.g., Illarionov & Sunyaev 1975).

## 6.5. Correlation between MSP mass and orbital period?

Progenitors of NSs in wide orbits are less stripped prior to their explosion compared to those in close systems (Yoon et al. 2010). Hence, it is possible that the resulting NS masses could be somewhat larger in wider systems. To probe such a relation is difficult because it is masked by the subsequent LMXB accretion phase, which recycles the NS to become an MSP. Based on binary stellar evolution theory, Tauris & Savonije (1999) argued for an anti-correlation between the amount of mass accreted by the NS,  $\Delta M_{\rm NS}$  and orbital period. Despite this effect, however, we still find evidence for slightly more massive MSPs in wider sys-

tems. Using the sample of MSPs with He white dwarfs studied in Section 5.2, we find that the overall average NS mass is larger by  $0.06 \pm 0.01 \, \mathrm{M_\odot}$  in systems with  $P_\mathrm{b} > 30$  days, compared to systems with  $P_\mathrm{b} < 30$  days. Hence, we would expect the mass difference to be even larger at their birth, thereby supporting the evidence for the hypothesis that wide binaries, in general, produce more massive NSs at birth. Larger MSP masses in wider orbits can also help explain the results of Stairs et al. (2005), who found it difficult to reconcile the  $M_\mathrm{WD} - P_\mathrm{b}$  relation with observational data based on a statistical analysis assuming the same MSP mass in all binaries.

#### 6.6. Constraints on the Maximum Mass

High-mass NSs place stringent constraints on the EoS beyond the nuclear saturation density. The most massive known NS with a precisely measured mass is PSR J0348+0432 with  $M=2.01(4)\,\mathrm{M}_\odot$ , which is commonly adopted at face value as the limit for the maximum NS mass. Here, we demonstrate an alternative method which relies on Bayesian inference of the MSP mass distribution properties.

Kiziltan et al. (2013), who took a similar approach, identified the maximum NS mass with the tail of their inferred MSP mass distribution. In reality, this limit has very little physical relevance. This is because the simple empirical models used to fit the observed masses are not likely to be true representations of the underlying mass distribution and any statistical model can have an unaccounted for mass cut-off at the high mass end. Consequently, extrapolation to high masses is of limited value, as there is no guarantee that the observed masses carry information for the true maximum NS mass. Even if this were the case, the  $3\sigma$  limit of the mass distribution, adopted by Kiziltan et al. (2013) does not prevent the existence of NSs with larger masses.

Here, we have made use of the fact that the EoS can introduce a high-mass cut-off in the NS distribution. A search for a truncation in the currently observed masses with our Bayesian inference method, yields a limit for the maximum mass of  $M_{\rm max} \geq 2.018\,{\rm M}_{\odot}$  at 98% C.L or  $1.923\,{\rm M}_{\odot}$  at 99.98% C.L. Interestingly, the former appears to be insensitive to our choice for the model distribution. This method yields a more robust constraint, in the sense that it is derived from the likelihoods of all massive NSs.

# 6.7. Accelerating the discovery of massive NSs and prospects for measuring the maximum NS mass

The Bayesian framework used in our study also allows us to estimate the number of mass measurements required for a precision localization of a high-mass truncation in the underlying mass distribution. Our es-

timates suggest that if the maximum mass is smaller than  $\sim 2.15\,\mathrm{M}_\odot$ , then the measurement of 350 MSP masses following the inferred distribution suffices to localize  $M_{\mathrm{max}}$  with a precision of  $\sim 5\%$ . This number of inferred MSP masses should be possible with the upcoming SKA surveys.

Obviously, the most important factor impacting the detection of  $M_{\rm max}$  is its actual value. Constraints on the NS radius from bursting and quiescent LMXB sources currently favor softer EoSs which cannot support NSs with masses much greater than  $2\,M_{\odot}$  (Ozel & Freire 2016). Hence, it is possible that stringent constraints on  $M_{\rm max}$  can be achieved sooner. Improving overall on measurement uncertainties, e.g., by increasing the observing cadence may help as well.

Another possible strategy would be to focus only on those NSs occupying the high-mass tail of the distribution. In Section 5.2, we argued that, for MSPs with white dwarf companions, it is possible to identify potential high-mass candidates by making use of the  $M_{
m WD}-P_{
m b}$  correlation for post-LMXB systems. Of the binaries shown in Table 5, some 20 have high probability ( $\gtrsim 80\%$ ) for having a mass above 1.8 M<sub> $\odot$ </sub>. Finally, a complementary approach would be to focus on special types of systems such as eclipsing MSPs and DNSs. For the former, existing mass constraints could be improved by exploring alternative methods, such as high resolution wide-band spectroscopy, or scintillometry (Pen et al. 2014). DNSs such as the double pulsar (Kramer et al. 2006) on the other hand, may make it possible to identify "special" NSs, such as those formed through an electron-capture SN (Podsiadlowski et al. 2004) or an ultra-stripped iron core-collapse SN (Tauris et al. 2013, 2015a), which have the potential to place direct constraints on the NS gravitational binding energy and consequently on the EoS. In addition, for the double pulsar, the measurement of the Lense-Thirring precession may soon result in the first measurement of a NS moment of inertia (Kramer & Wex 2009; Kehl et al. 2016), which results in direct constraints on the NS radius and the EoS (Raithel et al. 2016).

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#### **APPENDIX**

### A. THE MASS OF PSR J1012+5307

PSR J1012+5307 is a 5.3 ms pulsar with a low-mass He-white dwarf companion in a 14.4 h orbit. A spectroscopic analysis of the white dwarf was performed by two groups (van Kerkwijk et al. 1996; Callanan et al. 1998), who found different values for both the atmospheric properties and radial velocities. A subsequent analysis of the results revealed that the differences in the velocities were caused by a bias in the van Kerkwijk et al. (1996) analysis. A reanalysis of the high S/N Keck data from van Kerkwijk et al. (1996) yields  $K_{\rm WD} = 199 \pm 10 \,\mathrm{km \, s^{-1}}$  for the semi-amplitude of the white dwarf's orbital radial velocity, and  $q \equiv m_{\rm p}/m_{\rm c} = 10.0 \pm 0.7$  for the mass ratio, in agreement with Callanan et al. (1998). The discrepancy on the value of the surface gravity was traced to slight differences in the input physics of the atmospheric models used by the two teams (van Kerkwijk et al. 2005).

Interestingly, both studies find the same mass for the pulsar,  $m_{\rm p}=1.6(2)\,{\rm M}_{\odot}$ , but again due to the different white dwarf mass-radius relations adopted in their analysis. These models did not properly account for finite-temperature corrections nor the effect of an extended hydrogen envelope. A follow-up study by Driebe et al. (1998), based on the (biased) values of van Kerkwijk et al. (1996), using appropriate input physics, found  $m_{\rm c}=0.19(2)\,{\rm M}_{\odot}$  implying  $m_{\rm p}=1.9(3)\,{\rm M}_{\odot}$ .

Another important effect that became evident after these early He-white dwarf studies, is a bias in 1-D atmospheric models for relatively cool white dwarfs (Tremblay et al. 2011). Recent work demonstrates that this effect is caused by the imperfect scheme used to model convective transport in 1D models, with full corrections based on 3D DA atmospheres now available for the entire parameter space relevant to MSP companions (Tremblay et al. 2013, 2015). Finally, the recent detection of pulsational instabilities in white dwarfs with similar temperatures and masses, allows us to place further constraints on the mass of the system: the surface gravity reported by Callanan et al. (1998) would place PSR J1012+5307 in the middle of the instability strip, as derived empirically by Gianninas et al. (2015). Such pulsations are not detected (Kilic et al. 2015), implying that the true (1D) atmospheric parameters must be close to those reported by van Kerkwijk et al. (1996).

To derive the mass estimate reported in Table 1, we start with a simulated distribution of atmospheric parameters with  $T_{\rm eff} = 8550(50)\,\rm K$  and  $\log_{10}g = 6.75(1)\,\rm dex$ , following van Kerkwijk et al. (1996), but with slightly increased error estimates, to account for possible remaining uncertainties, and because we do not have access to the full covariance matrix of their atmospheric fit. We then map these samples to 3D-corrected values, using the relations of Tremblay et al. (2015), and then to a mass-radius distribution using the models of Althaus et al. (2013), which have been shown to yield reliable parameters for similar He-white dwarfs (e.g., Antoniadis et al. 2016, and references therein). Finally, we derive the mass of the pulsar,  $m_{\rm p} = 1.83(11)\,\rm M_{\odot}$  using the mass-ratio estimate discussed above. A follow-up spectroscopic study of PSR J1012+5307 to verify this estimate is in progress (Gemini project: GN-2016A-Q-70).

#### B. MASS CONSTRAINTS FOR MSPS WITH HE-WHITE DWARF COMPANIONS

Table 5 shows the predictions for the masses of MSPs with He white dwarf companions described in Section 5.2. The companion masses, orbital periods, dispersion measures and inferred distances based on the NE 2001 (Cordes & Lazio 2002) model for the distribution of free electrons in the Galaxy are also shown. Finally, we also calculate the pulsar mass for fixed inclination angles ( $i = 30^{\circ}$  and  $60^{\circ}$ ), and the inclination corresponding to a "canonical" pulsar mass of  $1.4 \,\mathrm{M}_{\odot}$ . The last column shows the likelihood for the pulsar to have a mass above  $1.8 \,\mathrm{M}_{\odot}$ .

Name	$P_0$	DM	d	$P_{\rm b}$	$\rm m_c^{ts99}$	$\rm m_p^{30^o}$	$\rm m_p^{60^o}$	$i(m_{\rm p}=1.4{\rm M}_{\odot})$	$\mathscr{L}(m_{\rm p} > 1.8{\rm M}_{\odot})$
	(s)	$(\mathrm{cm}^{-3}\mathrm{pc})$	(kpc)	(days)	${\rm M}_{\odot}$	${ m M}_{\odot}$	${\rm M}_{\odot}$	О	
$\operatorname{PSR}\operatorname{J0034-0534}$	0.0019	13.77	0.98	1.59	0.21	0.75	1.98	44.82	0.73
$\mathrm{PSR}\mathrm{J}0101\text{-}6422$	0.0026	11.93	0.73	1.79	0.21	0.64	1.74	49.74	0.66
PSR J0218+4232	0.0023	61.25	3.15	2.03	0.22	0.57	1.57	54.00	0.55
$\operatorname{PSR}\operatorname{J0437-4715}$	0.0058	2.64	0.16	5.74	0.24	0.94	2.45	38.46	0.62
PSR J0557 + 1550	0.0026	102.57	5.65	4.85	0.24	0.50	1.44	58.50	0.23

$\operatorname{PSR}\operatorname{J0613-0200}$	0.0031	38.78	0.90	1.20	0.21	0.85	2.20	41.43	0.65
PSR J0614-3329	0.0031	37.05	2.96	53.58	0.32	0.39	1.29	64.65	0.00
PSR J1017-7156	0.0023	94.22	0.26	6.51	0.24	0.55	1.57	54.11	0.51
PSR J1045-4509	0.0075	58.17	0.23	4.08	0.23	0.71	1.91	46.27	0.74
PSR J1056-7117	0.0263	93.04	5.27	9.14	0.25	1.23	3.14	32.43	0.59
$\operatorname{PSR}\operatorname{J1125-5825}$	0.0031	124.79	2.98	76.40	0.33	0.47	1.51	56.35	0.46
$\operatorname{PSR}\operatorname{J}1216\text{-}6410$	0.0035	47.40	1.71	4.04	0.23	0.73	1.97	45.25	0.75
$\operatorname{PSR}\operatorname{J}1231\text{-}1411$	0.0037	8.09	0.45	1.86	0.21	0.47	1.34	62.78	0.00
$\operatorname{PSR}\operatorname{J}1232\text{-}6501$	0.0883	239.40	10.00	1.86	0.21	0.76	2.00	44.57	0.71
$\operatorname{PSR}\operatorname{J1327-0755}$	0.0027	27.91	2.17	8.44	0.25	0.42	1.27	65.99	0.00
$\operatorname{PSR}\operatorname{J}1405\text{-}4656$	0.0076	13.88	0.74	8.96	0.25	0.48	1.41	59.58	0.12
$\operatorname{PSR}\operatorname{J}1431\text{-}5740$	0.0041	131.46	4.07	2.73	0.22	0.68	1.83	47.71	0.71
$\operatorname{PSR}\operatorname{J}1455\text{-}3330$	0.0080	13.57	0.74	76.17	0.33	0.52	1.61	53.39	0.58
PSR J1543-5149	0.0021	50.93	1.46	8.06	0.25	0.41	1.25	67.32	0.00
$\operatorname{PSR}\operatorname{J}1545\text{-}4550$	0.0036	68.39	2.01	6.20	0.24	0.82	2.17	42.05	0.65
$\mathrm{PSR}\mathrm{J}16003053$	0.0036	52.33	2.40	14.35	0.27	0.55	1.60	53.45	0.54
$\operatorname{PSR}\operatorname{J}1622\text{-}6617$	0.0236	88.02	4.66	1.64	0.21	1.56	3.83	27.96	0.58
PSR J1640 + 2224	0.0032	18.43	1.19	175.46	0.37	0.67	2.01	45.27	0.68
$\operatorname{PSR}\operatorname{J}1643\text{-}1224$	0.0046	62.41	0.42	147.02	0.36	2.40	5.94	21.73	0.56
$\mathrm{PSR}\mathrm{J}1708\text{-}3506$	0.0045	146.73	3.50	149.13	0.36	1.45	3.78	29.37	0.57
PSR J1709+2313	0.0046	25.35	1.83	22.71	0.28	0.33	1.12	77.73	0.00
$\operatorname{PSR}\operatorname{J1711-4322}$	0.1026	191.50	4.17	922.47	0.48	1.53	4.10	28.58	0.00
PSR J1713 + 0747	0.0046	15.97	1.05	67.83	0.33	0.41	1.36	61.51	0.02
$\operatorname{PSR}\operatorname{J}1732\text{-}5049$	0.0053	56.82	1.81	5.26	0.24	0.59	1.65	51.85	0.59
$\mathrm{PSR}\mathrm{J}1745\text{-}0952$	0.0194	64.47	2.38	4.94	0.24	1.44	3.58	29.52	0.58
$\operatorname{PSR}\operatorname{J1751-2857}$	0.0039	42.81	1.44	110.75	0.35	0.98	2.68	36.93	0.60
$\mathrm{PSR}\mathrm{J}1801\text{-}3210$	0.0075	177.71	5.08	20.77	0.28	1.24	3.18	32.35	0.59
$\operatorname{PSR}\operatorname{J}1803\text{-}2712$	0.3344	165.50	3.62	406.78	0.42	2.24	5.65	22.84	0.00
$\operatorname{PSR}\operatorname{J}1804\text{-}2717$	0.0093	24.67	1.17	11.13	0.26	0.55	1.58	53.97	0.52
$\operatorname{PSR}\operatorname{J}1811\text{-}2405$	0.0027	60.60	1.70	6.27	0.24	0.35	1.11	80.04	0.00
$\operatorname{PSR}\operatorname{J}1813\text{-}2621$	0.0044	112.52	3.37	8.16	0.25	0.58	1.65	52.08	0.58
$\operatorname{PSR}\operatorname{J}1825\text{-}0319$	0.0046	119.50	3.26	52.63	0.31	0.97	2.62	37.28	0.60
$\operatorname{PSR}\operatorname{J}1835\text{-}0114$	0.0051	98.00	2.67	6.69	0.24	0.63	1.74	49.86	0.66
PSR J1841 + 0130	0.0298	125.88	3.19	10.47	0.26	1.99	4.87	24.07	0.57
PSR J1844 + 0115	0.0042	148.22	3.45	50.65	0.31	1.48	3.78	28.97	0.57
PSR J1850 + 0124	0.0036	118.89	2.97	84.95	0.34	0.57	1.72	50.72	0.65
PSR J1853 + 1303	0.0041	30.57	1.60	115.65	0.35	0.65	1.92	46.70	0.74
PSR J1857 + 0943	0.0054	13.30	0.90	12.33	0.26	0.38	1.19	71.33	0.00
PSR J1901 + 0300	0.0078	253.89	5.50	2.40	0.22	0.84	2.21	41.43	0.65
PSR J1904+0412	0.0711	185.90	4.01	14.93	0.27	0.48	1.44	58.49	0.28
PSR J1910 + 1256	0.0050	38.07	1.95	58.47	0.32	0.85	2.35	40.18	0.62
PSR J1911-1114	0.0036	30.98	1.59	2.72	0.22	1.09	2.77	35.18	0.60
$\mathrm{PSR}\mathrm{J}1918\text{-}0642$	0.0076	26.55	1.40	10.91	0.26	0.38	1.20	70.32	0.00
${\rm PSR}{\rm J}1935{+}1726$	0.0042	61.60	3.11	90.76	0.34	0.73	2.10	43.70	0.66
PSR J1955 + 2908	0.0061	104.50	5.39	117.35	0.35	1.15	3.07	33.66	0.59
${\rm PSR}{\rm J2016{+}1948}$	0.0649	33.81	1.83	635.02	0.45	0.68	2.12	44.11	0.00
${\rm PSR}{\rm J2017}{+}0603$	0.0029	23.92	1.32	2.20	0.22	0.52	1.47	57.28	0.35
$\operatorname{PSRJ2019}{+2425}$	0.0039	17.20	0.91	76.51	0.33	0.32	1.16	73.39	0.00

PSR J2033+1734	0.0059	25.08	1.37	56.31	0.32	0.88	2.42	39.36	0.62
PSR J2043 + 1711	0.0024	20.71	1.13	1.48	0.21	0.53	1.48	57.05	0.34
$\operatorname{PSR}\operatorname{J2129-5721}$	0.0037	31.85	0.40	6.63	0.24	1.07	2.76	35.41	0.60
PSR J2229 + 2643	0.0030	23.02	1.43	93.02	0.34	2.08	5.19	23.63	0.56
$\operatorname{PSR}\operatorname{J2236-5527}$	0.0069	20.00	2.03	12.69	0.26	0.45	1.36	61.74	0.00
PSR J2317 + 1439	0.0034	21.91	1.89	2.46	0.22	0.56	1.55	54.64	0.48

Table B1. Predictions for MSPs with He white-dwarf companions

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