Bayesian Inference for Multi-Messenger Nuclear Astrophysics

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I. SCIENTIFIC BACKGROUND

The August 2017 LIGO observation [1] of a neutron star merger has dramatically changed the study of neutron stars. In particular, neutron star mergers are now thought to be major contributors to r-process nucleosynthesis. Also, neutron star mergers offer an important way to constrain the properties of dense matter. The nature of neutron-rich dense matter, in turn, has been an important frontier for the nuclear physics and astrophysics communities for over five decades (see e.g. Refs. [2, 3]).

However, there are important limitations. The LIGO data set consists of many data points strain as a function of time, but this data must be combined to obtain constraints on the radii of the two individual neutron stars. This means that, for the purposes of neutron star structure, the LIGO data effectively contains only two data points. Because the most optimistic models for the equation of state (EOS) of dense matter, for example, contain at least three parameters, the LIGO data is not enough to fully constrain any model. This situation will change in the future, as LIGO obtains more data on future mergers. Depending on the (still unknown, see e.g. Ref. [4]) mass function for neutron stars which merge, future LIGO data may map out a significant part of the equation of state.

In the mean time, there is a growing body of electromagnetic data on neutron star structure. There are now at least seven neutron stars in globular clusters for which mass and radius data is available [5]. In addition, there are a handful of neutron stars which exhibit X-ray bursts that show evidence for photospheric radius expansion (e.g. Ref. [6]). No study has yet combined this electromagnetic data with the gravitational wave data from the 2017 merger.

Knowledge of the equation of state alone is insufficient to describe observed neutron star phenomenology. Neutron star cooling [7, 8] and pulsar glitches [9] both depend strongly on the role played by superfluidity and superconductivity [10]. On the other hand, neither neutron star mass and radius observations nor gravitational wave detections are likely to constrain superfluidity or superconductivity in the near future. Thus, a joint analysis of data which constrains the equation of state and pairing in dense matter is required to refine our understanding of neutron star observations.

In addition, determination of the equation of state does not ensure knowledge of the composition of dense matter. This was demonstrated in Ref. [11] where it was found that a mass-radius curve with neutrons and protons alone was nearly identical to that from a model containing deconfined quarks. Neutron star cooling, on the other hand, is sensitive to the neutron-to-proton ratio, since that controls the threshold for the direct Urca process [12].

This proposal thus aims to use XSEDE resources to perform two Bayesian inference calculations over the next year which combine X-ray data, gravitational wave data, and nuclear structure information to obtain the equation of state of dense matter, the nature of proton superconductivity and neutron superfluidity at high densities, and the proton-to-neutron ratio in the neutron star core. This research program is funded by an NSF CAREER award entitled “The Composition of Dense Matter and Observations of Neutron Stars”.

II. FIRST FULL INFERENCE OF GRAVITATIONAL WAVE AND ELECTROMAGNETIC DATA DESCRIBING NEUTRON STAR STRUCTURE (PROJECT A)

A. Research Objectives

Project A combines X-ray data on two kinds of neutron stars (quiescent low-mass X-ray binaries and photospheric radius expansion bursts) with the LIGO data on the August 2017 merger to obtain a complete picture of the equation of state (EOS) of dense matter and the neutron star mass-radius relation. While the EOS and the mass-radius relation goal for decades, the modern effort has used Bayesian inference to do this problem since 2010 [13, 14]. The PI's landmark publication on this topic [13] has over 500 citations. Our Progress document shows initial results on the mass-radius relation obtained during the startup allocation.

B. Research Plan

MCMC is a natural choice for the marginalization integrals which need to be performed to characterize the posterior distribution. Each MCMC point begins with a calculation of the equation of state (EOS) from a set of (typically about 10) parameters. Because there is a large degree of uncertainty in the EOS, the functional forms are quite simple, so computation of the EOS is fast. Then, one must solve Einstein's field equations for a static spherically symmetric star, called the Tolman-Oppenheimer-Volkov (TOV) equations (for a review, see Ref. [3]. A solution of the TOV equations for the full range of central pressures gives the neutron star mass-radius curve. We also require a mass parameter for each neutron star with X-ray data, giving us 10 new parameters. Given these 20 parameters, computing the likelihood from the X-ray data requires only to perform several linear interpolations on data provided by Heinke and Nattila (see Personnel list below). This part of the calculation is well-trodden, and has been a central part of Steiner's research program since Ref. [13] in 2010.

We require three more parameters to describe the LIGO stars: the two masses and the velocity of the merger relative to the detector. To compute the tidal deformability at each value of the central pressure requires an additional differential equation to be solved (the documentation for our code explains this in more detail). The LIGO part of the likelihood is tabulated by O'Shaugnessy (see Personnel list) and requires a few more interpolations. The combination of the X-ray and LIGO data which measures neutron star structure is unique. Our collaboration will be the first to perform this kind of analysis.

In addition, as pioneered by Nattila in Ref. [6], we will add an additional systematic uncertainty parameter to all of the X-ray data points. Because the constraint from the X-ray data is fundamentally two-dimensional (mass and radius), the most efficient way to do this is to use FFTs. (See Progress document for a discussion on our use of FFTW.) This requires the addition of 10 new parameters and when combined with the calculation above, each successful MCMC point takes about 2 seconds (some rejections are much faster than this).

We propose to perform size MCMC simulations for Project A to allow us to explore our model assumptions. Different models for the high-density part of the EOS give significantly different results, so we choose Model A and C from Ref. [15]. We also want to vary the transition density from the low- to high-densty regimes, giving four models. Finally, for one model, we want to try simulating either the X-ray or LIGO data alone, giving a total of six MCMC simulations to be performed.

C. Allocation Plan

As the Performance and Scaling document shows, we obtain about 2300 points per SU, per model. Autocorrelation lengths for our simulations can be as large as 80 (again see Performance and Scaling document), so only 29 of those points are statistically independent. Our previous work has found 10^5 points are typically required for reasonable results. To obtain 10^5 points for six physical models we need 207k SUs. Our startup allocation found that both comet and bridges gave comparable results. However, the total available time in this allocation call is 80M hours on comet and 44M on bridges, so we keep to this ratio. Choosing 128 MPI tasks makes for a runtime of about 12 hours per model.

III. FIRST INFERENCE OF NEUTRON STAR STRUCTURE AND COOLING DATA (PROJECT B)

A. Research Objectives

Before this work, technical limitations have forced quantitative analyses of neutron star structure and neutron star thermal evolution to be almost entirely separate. In Project B, we will perform the first combined quantitative analysis of neutron star mass and radius data and temperature and age data for isolated neutron stars. In addition, we ensure our model reproduces the structure of heavy nuclei. This analysis will allow us to update constrains on the proton superconducting gap and neutron superfluid gap in dense matter [8]. More importantly, for the first time, we will be able to determine the proton-to-neutron (p/n) ratio in dense matter. Neutron star cooling is sensitive to the p/n ratio because the direct Urca process (the neutron star analog of neutron beta decay) can only occur in dense matter if the proton ratio is larger than about 10%. Our Progress document shows initial results on the cooling curves obtained during the startup allocation.

B. Research Plan

We presume that neutron stars contain only neutrons, protons, electrons, and muons and leave the description of neutron stars with exotic hadrons or deconfined quark matter to future work. We employ Bayesian inference and use MCMC to marginalize the posterior distribution.

In order to describe the thermal evolution, we require a more detailed equation of state than in Project A, since we require the composition of matter and the nucleon effective masses: all told this results in 19 parameters. Because neutrons are in a triplet superfluid phase and protons are in a superconducting phase, we need 6 more parameters to describe these pairing gaps and their dependence on density. We use the mass and radius data on quiescent low-mass X-ray binaries from 7 stars, requiring 7 more parameters which describe the neutron star masses and 7 more parameters which characterize the composition of the atmosphere, very similar to that done in Ref. [5].

We analyze the temperature data in a method taken from Ref. [16]. Ignoring the Carbon stars (because of the complexities of these objects), there are 16 stars. As above, these 16 stars require 16 mass parameters. For the younger stars, the atmosphere may vary, and so 9 more parameters are required to describe that variation. Thus our model has a total of 64 parameters. traditional Bayesian inference applied to this problem requires the solution of the TOV equations, the nuclear structure calculations, and several cooling curves (to handle the variation in mass and envelope composition) at every point. Each point requires about 5 minutes to compute on one processor. With a 64-dimensional parameter space, this computation is too costly to perform directly.

In order to make this problem computationally tractable we do not fully compute every point, instead we populate the parameter space with a library of exact calculations and then use an emulator (based on either inverse-distance weighted or Gaussian process interpolation) to do the final inference.

C. Allocation Plan

During the startup allocation, we found that about 30 points are generated per SU (acceptances take only 5 minutes but rejections are often fast), thus we estimate that an additional 33K SU's will be required to generate our goal of a training data set with 10^6 points. Work during the startup allocation shows that 30K SUs will be required for the final inference to produce the fidelity necessary for reasonable constrains on the superfluid gaps and the p/n ratio. We request these SUs on comet because the configuration there seems a bit easier to compile our Fortran/C++ code.

IV. PERSONNEL

The PI, Andrew W. Steiner, is an assistant professor at the University of Tennessee, Knoxville, with a 25% joint appointment at Oak Ridge National Laboratory. His expertise is in the nuclear physics aspects of neutron stars. Steiner is the recipient of an NSF CAREER award (2016-2021) entitled "The Composition of Dense Matter and Observations of Neutron Stars" and the proposed allocation will support this research effort. This allocation will form an integral part of the thesis work of graduate student Mohammad Al-Mamun (Mamun). Steiner used the startup allocation resources to train Mamun in HPC, MPI, OpenMP, Globus and the use of XSEDE resources.

One of the strengths of Project A is the collaboration among several researchers who contribute to different aspects of the effort. There are no current plans for them to use the XSEDE allocation referred to in this proposal. They have separate computing resources which they may use for data analysis related to this project. All of these collaborators are aware of this XSEDE proposal and have indicated their interested in the project (we have started a new slack channel for this purpose).

- **Ingo Tews (U. Washington) and Stefano Gandolfi (LANL):** Tews and Gandolfi are experts in the nuclear physics of neutron-rich matter at low-densities. Their work is being employed in all of our MCMC models. A previous collaboration on a similar topic with Gandolfi led to an article in Phys. Rev. Lett. in 2012 [17].

- **Craig Heinke (U. Alberta):** Heinke is an expert in the analysis of quiescent low-mass X-ray binaries and he is pre-processing the X-ray data to obtain information on neutron star masses and radii. Steiner and Heinke finished a large analysis of this data earlier this year [5] Heinke's postdoc, Aarran Shaw will also participate, since he was a critical part of the analysis of the data from the neutron star in the M13 globular cluster [18].

- **Richard O'Shaugnessy (RIT; LIGO):** O'Shaughnessy is a member of the LIGO collaboration and is providing the LIGO data products from the August 2017 neutron star merger which are being used in our MCMC. An RIT graduate student will also assist in processing the input LIGO data.

- **Sophia Han (Ohio Univ.):** Han is an expert in the analysis of neutron star cooling [16, 19] and her expertise will be required for Project B.

- **Joonas Nattila (NORDITA, Stockholm):** Nattila is an expert in the analysis of photospheric radius expansion X-ray bursts [6, 20] and he pre-processes the X-ray data for use in our MCMC. Nattila and I recently finished a publication on this topic earlier this year.

Project B is a collaboration between the PI, Sophia Han (see above) and also Od Odbadrakh:

- **Od Odbadrakh (UTK and ORNL):** Odbadrakh's involvement began when Steiner was awarded software engineering support on an internal University of Tennessee competition sponsored by UTK's Advanced Computing Facility. He is assisting with the improving the implementation of the Newton-Raphson solver in the neutron star cooling code.

V. ACCESS TO OTHER RESOURCES

The PI, Steiner, has access to computing resources on UTK's Advanced Computing Facility, but the systems are much smaller and cannot handle a project of this size. The ACF provided some software engineering support through the involvement of Odbadrakh, but has promised no significant amount of computing time.

The PI is also funded by a DOE SciDAC award, which focuses on detailed models of the equation of state and neutrino interactions for large-scale simulations of core-collapse supernovae and neutron star mergers. The SciDAC collaboration uses significant computing resources provided by ORNL and the DOE. However, Steiner's work for this collaboration does not require any high-performance computing resources. This research program is separate from the NSF-funded research described above and no XSEDE resources will be used for SciDAC research.

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