Technical Report - Public transport rescheduling tool

Abstract

Public transportation networks play a crucial role in urban mobility by providing efficient, reliable, and sustainable transport options. One of the main challenges in bus network operations is minimizing passenger inconvenience by improving service connectivity and thus, minimizing waiting times. Timetable synchronization seeks to optimize the timing of bus arrivals at transfer nodes, ensuring minimal waiting times for passengers transferring between routes. This is particularly important in multimodal transport networks, where smooth connections between different transport modes significantly influence passenger satisfaction. The bus timetabling resynchronization problem allowing modifications to the initial timetable is a problem already studied in the literature. This study builds on the bus timetable resynchroniza- tion problem and expands it by introducing the concept of generalized passenger travel times, which reflects how increased in-vehicle passenger load leads to longer perceived passenger travel times

1. Mathematical Model

1.1. Initial Formulation

The bus timetable resynchronization problem considering passenger demand and in-vehicle passenger load is formulated in this section. The goal is to (a) re-synchronize the timetable of bus lines in order to maximize the synchronized transfers at the transferring nodes (b) to maximize the number of the passengers benefiting from the synchronization and (c) to investigate the impact of in-vehicle passenger load on the perceived passenger travel times.

Table 1 presents the notation, parameters and decision variables used to define the mathematical model.

Table 1: Notation, parameters and decision variables used in the mathematical model.

Sets:	
I	set of lines in the public transport network
Sl_i	set of stations of line i , $i \in I$
IL	set of trips of a line
B_{ij}	set of transfer stations common to lines i, j, $i, j \in I$
Parameters:	
Q_i	capacity of vehicle of line i, $i \in I$
h_i	headway of line i in the existing timetable, $i \in I$
h_i^{min}	minimum headway of line i allowed for synchronized timetable, $i \in I$
h_i^{max}	maximum headway of line i allowed for synchronized timetable, $i \in I$
$n_{i\,pb}^{j}$	actual number of passengers benefiting from a smooth transfer for p-th trip of line i to line j at node b when synchronized timetable is used, $i, j \in I, p \in IL, b \in B_{i,j}$
$\hat{n}_{i\,b}^{j}$	estimated number of passengers, each transferring from line i to line j at transfer node b when the initial timetable is used, $i, j \in I, b \in B_{i,j}$
q_i^{mp}	number of passengers that embark in the vehicle at station m of line i of p-th trip, $m \in Sl_i, i \in I$, $p \in Il_i$
qu_i^{mp}	number of passengers that disembark from the vehicle at station m of line i of p-th trip, $m \in Sl_i, i \in I$, $p \in Il_i$
η	the maximal value of deviation allowed from the departure times
NS	Number of seats in each vehicle
T	length of planning period in minutes; the planning period can be standardized as [0,T]
t_{ib}	travel time from depot of line i to node b in the planning period, $i \in I, b \in B_{ij}$
$ttr_{m-1,m}$	travel time from station m-1 to m of line i, $m \in Sl_i, i \in I$
w_b^{max}	maximum separation time between synchronized bus arrivals at node b, $b \in B_{i,j}$
w_b^{min}	minimum separation time between synchronized bus arrivals at node b, $b \in B_{i,j}$
$\hat{x}_{ip}$	departure time of the p-th trip of line i in the initial timetable; $x_{ip} = (p-1)h_i, \forall i \in I, p \in Il_i$
Variables:	
x_{ip}	departure time of the p-th trip of line i
p_i^{mp}	perceived arrival time of passengers at station m of p-th trip of line i, m $\in$Sl$_i$,p $\in IL$, $i \in I$
u_i^{mp}	actual arrival time of passengers at station m of p-th trip of line i, m $\in$Sl$_i$,p $\in IL$, $i \in I$
l_i^{mp}	in-vehicle load of p-th trip after leaving station m of line i, $m \in Sl_i, i \in I$
δ_i^{mp}	binary variable used for linearization, p $\in$IL,m$\in$Sl$_i$, i $\in$I
g_i^{mp}	continuous variable used for linearization, p $\in$IL,m$\in$Sl$_i$, i $\in$I
y_{ipd}^{jq}	binary variable, where $y_{ipd}^{jq} = 1$ if the p-th trip of line i arrives first at node b and synchronizes with the q-th trip of line j within time window $[w^{min}, w^{max}]$ and 0 otherwise

The initial mathematical formulation of the Bus Timetable Resynchronization Problem (BTRP) is presented and explained in the following passage.

$$\max \quad \sum_{i\in I}\sum_{p\in IL}\sum_{j\in I}\sum_{q\in IL}\sum_{b\in B^{ij}} y_{ipb}^{jq} \tag{1}$$

$$
\begin{aligned}
\text{s.t.} \quad & x_{i1} \le h_i^{max} && \forall i \in I && (2)\\
& T - h_i^{max} \le x_{if^i} \le T && \forall i \in I && (3)\\
& h_i^{min} \le x_{ip} - x_{i(p-1)} \le h_i^{max} && \forall i \in I, \quad p \in IL && (4)\\
& -\eta \le x_{ip} - \hat{x}_{ip} \le \eta && \forall i \in I, \quad p \in IL && (5)\\
& n_{ipb}^{j} = \hat{n}_{ib}^{j} + \frac{\hat{n}_{ib}^{j}}{h_i}(x_{ip} - x_{i(p-1)} - h_i) && \forall (i,j) \in I, \quad b \in B_{ij}, \quad p \in IL && (6)\\
& (x_{jq} + t_{jb}) - (x_{ip} + t_{ib}) \ge w_b^{min} - M(1 - y_{ipb}^{jq}) && \forall (i,j) \in I, \quad (p,q) \in IL, \quad b \in B_{ij} && (7)\\
& (x_{jq} + t_{jb}) - (x_{ip} + t_{ib}) \le w_b^{min} - M(1 - y_{ipb}^{jq}) && \forall (i,j) \in I, \quad (p,q) \in IL, \quad b \in B_{ij} && (8)\\
& \sum_{q=1}^{q=|IL|} y_{ipb}^{jq} \le 1 && \forall (i,j) \in I, \quad p \in IL, \quad b \in B_{ij} && (9)\\
& l_i^{mp} = l_i^{m-1p} + q_i^{mp} - qu_i^{mp} && \forall i \in I, \quad m \in Sl_i, \quad p \in IL && (10)\\
& p_i^{mp} = u_i^{mp} + (p_i^{m-1,p} - u_i^{m-1,p}) + ttr_{i,m} d_i^{m-1,p} - ttr_{i,m} && \forall i \in I, \quad m \in Sli, \quad p \in IL && (11)\\
& x_{ip} \in \{0, 1, ..., T\} && \forall i \in I, \quad p \in IL && (12)\\
& y_{ipb}^{jq} \in \{0, 1\} && \forall (i,j) \in I, \quad (p,q) \in IL, \quad b \in B_{ij} && (13)\\
& && && (14)
\end{aligned}
$$

The objective function (1) maximizes the synchronized arrivals at the transfer nodes. Constraint (2) ensures that the first trip of each line departs at the beginning of the planning period, while constraint (3) forces the last trip of each line to depart at the end of the planning period. Constraint (4) ensures that the headways between consecutive trips of each line i is in the range of $[h_i^{min},h_i^{max}]$. Constraint (5) defines the maximal value of deviation of the departure times, relative to the initial timetable. Following, the number of bikes loaded on the vehicle after serving station j is calculated (7). The number of bikes loaded on the vehicle should not exceed vehicle capacity (8). Constraints (9) and (15) calculate the number of available bikes to load/unload in every station i. Constrains (16)-(19) determine whether a station is a load ($g_i^k = 1$) or unload ($d_i^k = 1$) station and connects this amount with the total available bikes. Next, constraint (20) ensures that a station can be either a load or an unload station. The limitations of parking station capacity are taken into account using constraint (21), where η_i^0 is the initial load (status) of a station i. The status of a station after being served by vehicle k is denoted by η_i in constraint (**??**). Finally, (**??**)-(**??**) are non-negativity constraints.

Let $0 \leq \rho \leq 1$ represent the parameter value corresponding to the in-vehicle loading point beyond which travel time is affected. When in-vehicle crowding levels fall within the range $[0, \rho Q_i]$ for a given line trip i, the perceived passenger travel times remain equal to actual travel times. However, if the in-vehicle load l_i^m of vehicle k upon departing station m of line i exceeds ρQ_i, the perceived in-vehicle travel time may be increased by a progressive travel time penalty $\gamma(l_i^m - \rho Q_i)$. Here, $\gamma \geq 0$ represents the marginal increase in perceived travel time per unit increase in in-vehicle crowding beyond the "trigger point" ρQ_i. This relationship is illustrated as a piecewise linear function.

From the piecewise linear function, the travel time penalty of the vehicle of line i after leaving the station m on the p-th trip is:

$$d_i^{mp} = 1 + \gamma \max\{l_i^{mp} - \rho Q_i, 0\} \forall i \in I, \quad p \in IL, \quad m \in Sl_i \tag{15}$$

The travel time penalty d_i^{mp} is provided by a nonlinear function. To linearize it, we introduce binary variables δ_i^{mp} and continuous variables g_i^{mp}. Then, the nonlinear expression is replaced by Eqs. :

$$
\begin{aligned}
& d_i^{mp} = 1 + \gamma g_i^{mp} && \forall i \in I, \quad p \in IL,, \quad m \in Sl_i && (16)\\
& g_i^{mp} \geq l_i^{mp} - \rho Q_i && \forall i \in I, \quad m \in Sl_i, \quad p \in IL && (17)\\
& g_i^{mp} \geq 0 && \forall i \in I, \quad m \in Sl_i, \quad p \in IL && (18)\\
& g_i^{mp} \leq l_i^{mp} - \rho Q_i + M\delta_i^{mp} && \forall i \in I, \quad m \in Sl_i, \quad p \in IL && (19)\\
& g_i^{mp} \leq M(1 - \delta_i^{mp}) && \forall i \in I, \quad m \in Sl_i, \quad p \in IL && (20)\\
& \delta_i^{mp} \in \{0, 1\} && \forall i \in I, \quad m \in Sl_i, \quad p \in IL && (21)\\
& && && (22)
\end{aligned}
$$

Acknowledgment

This project has received funding from the European Union's Horizon 2020 research and innovation program SUM under grant agreement No. 101103646.