ITCS 6151/8151 Intelligent Robotics - Spring 2024

Assignment 3

Due: Apr 5, 23:59

Name:
UNCC Email:

Submission Format: Submit your solution as a single **.zip** file containing (1) a single PDF file and (2) the source code files for the implementation portions of the assignment. Your source code files should run without having to download any third-party package and should be able to reproduce all figures and results reported in your PDF file.

List of Collaborators and Acknowledgements: List the names of all people you have collaborated with and for which question(s).

Questions about this assignment should be directed to TA Dushyant Pawar, dpawar4@charlotte.edu

1. **(10pts)** Answer the following questions for the 4R planar open chain of Fig. 1

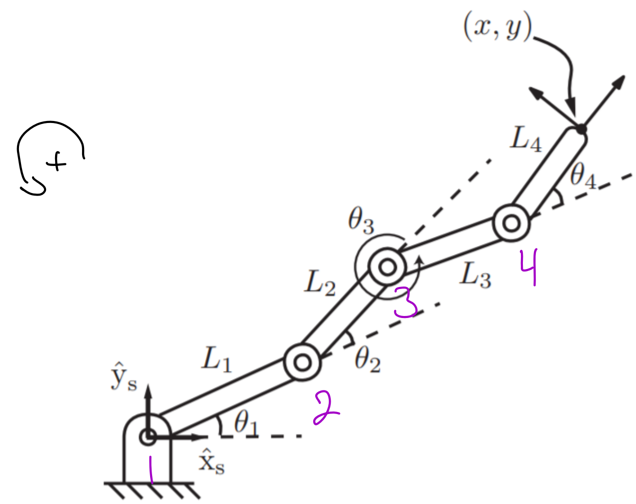


Figure 1: A 4R planar open chain.

   (a) (2pt) For the forward kinematics of the form

$$T(\theta) = e^{[\mathcal{S}_1]\theta_1} e^{[\mathcal{S}_2]\theta_2} e^{[\mathcal{S}_3]\theta_3} e^{[\mathcal{S}_4]\theta_4} M, \tag{1}$$

   write down $M \in$SE(2) and each $\mathcal{S}_i = (\omega_{zi}, v_{xi}, v_{yi}) \in \mathbb{R}^3$.

   (b) (2pt) Write down the body Jacobian.

   (c) (2pt) Suppose that the chain is in static equilibrium at the configuration $\theta_1 = \theta_2 = 0, \theta_3 = \pi/2, \theta_4 = -\pi/2$ and that a force $f = (10, 10, 0)$ and a moment $m = (0, 0, 10)$ are applied to the tip (both f and m are expressed with respect to the fixed frame). What are the torques experienced at each joint?

   (d) (2pt) Under the same conditions as (c), suppose that a force $f = (-10, 10, 0)$ and a moment $m = (0, 0, -10)$, also expressed in the fixed frame, are applied to the tip. What are the torques experienced at each joint?

   (e) (2pt) Find all kinematic singularities for this chain.

1

a)

	ω_i			v_i		
1	0	0	1	0	0	0
2	0	0	1	0	$-L_1$	0
3	0	0	1	0	$-L_1-L_2$	0
4	0	0	1	0	$-L_1-L_2-L_3$	0

$$M = \begin{bmatrix} R & p \\ 0 & 1 \end{bmatrix} \therefore \begin{bmatrix} \cos(\theta) & -\sin(\theta) & x \\ \sin(\theta) & \cos(\theta) & y \\ 0 & 0 & 1 \end{bmatrix}$$

$$M = \begin{bmatrix} 1 & 0 & L_4 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$e^{S_1} \begin{bmatrix} \cos(\theta_1) & -\sin(\theta_1) & 0 \\ \sin(\theta_1) & \cos(\theta_1) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$e^{S_2} \begin{bmatrix} \cos(\theta_1+\theta_2) & -\sin(\theta_1+\theta_2) & 0 \\ -\sin(\theta_1+\theta_2) & \cos(\theta_1+\theta_2) & -L_1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$e^{S_3} \begin{bmatrix} \cos(\theta_1+\theta_2+\theta_3) & -\sin(\theta_1+\theta_2+\theta_3) & 0 \\ -\sin(\theta_1+\theta_2+\theta_3) & \cos(\theta_1+\theta_2+\theta_3) & -(L_1+L_2) \\ 0 & 0 & 1 \end{bmatrix}$$

$$e^{S_4} \begin{bmatrix} \cos(\theta_1+\theta_2+\theta_3+\theta_4) & -\sin(\theta_1+\theta_2+\theta_3+\theta_4) & 0 \\ -\sin(\theta_1+\theta_2+\theta_3+\theta_4) & \cos(\theta_1+\theta_2+\theta_3+\theta_4) & -(L_1+L_2+L_3) \\ 0 & 0 & 1 \end{bmatrix}$$

$$B_i = \begin{pmatrix} \hat{\omega}_i \\ \hat{v}_i \end{pmatrix}$$

b)

ω_i			v_i		
0	0	1	0	$L_2+L_3+L_4$	0
0	0	1	0	L_3+L_4	0
0	0	1	0	L_4	0
0	0	1	0	0	0

$$M = \begin{bmatrix} 1 & 0 & L_1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$e^{B_1} \begin{bmatrix} \cos(\theta_1+\theta_2+\theta_3+\theta_4) & -\sin(\theta_1+\theta_2+\theta_3+\theta_4) & 0 \\ -\sin(\theta_1+\theta_2+\theta_3+\theta_4) & \cos(\theta_1+\theta_2+\theta_3+\theta_4) & (L_1+L_2+L_3) \\ 0 & 0 & 1 \end{bmatrix}$$

$$e^{B_2} \begin{bmatrix} \cos(\theta_1+\theta_2+\theta_3) & -\sin(\theta_1+\theta_2+\theta_3) & 0 \\ -\sin(\theta_1+\theta_2+\theta_3) & \cos(\theta_1+\theta_2+\theta_3) & (L_1+L_2) \\ 0 & 0 & 1 \end{bmatrix}$$

$$e^{B_3} \begin{bmatrix} \cos(\theta_1+\theta_2) & -\sin(\theta_1+\theta_2) & 0 \\ -\sin(\theta_1+\theta_2) & \cos(\theta_1+\theta_2) & L_1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$e^{B_4} \begin{bmatrix} \cos(\theta_1) & -\sin(\theta_1) & 0 \\ \sin(\theta_1) & \cos(\theta_1) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

c) $\tau = T(\theta) \cdot \omega$

$$\omega = \begin{bmatrix} F \\ M \end{bmatrix}$$

$F = 10 \quad 10 \quad 0$

$M = 0 \quad 0 \quad 10$

```
clc
clear

syms L;
t1 = 0;
t2 = 0;
t3 = pi/2;
t4 = -pi/2;

f=[10 10 0]';
m=[0 0 10]';

s1= [cos(t1) -sin(t1) 0; sin(t1) cos(t1) 0; 0 0 1];
s2= [cos(t1+t2) -sin(t1+t2) 0; sin(t1+t2) cos(t1+t2) 0; 0 L 1];
s3= [cos(t1+t2+t3) -sin(t1+t2+t3) 0; sin(t1+t2+t3) cos(t1+t2+t3) 0; 0 L 1];
s4= [cos(t1+t2+t3+t4) -sin(t1+t2+t3+t4) 0; sin(t1+t2+t3+t4) cos(t1+t2+t3+t4) 0; 0 L 1];


m1 = s1*m;
m2 = s2*m;
m3 = s3*m;
m4 = s4*m;

disp(m1)
disp(m2)
disp(m3)
disp(m4)
```

Command Window

```
     0
     0
    10

 0
 0
10

 0
 0
10

 0
 0
10
```

$M_1 = 10\,Nm$

$M_2 = 10\,Nm$

$M_3 = 10\,Nm$

$M_4 = 10\,Nm$

D)

```
clc
clear

syms L;
t1 = 0;
t2 = 0;
t3 = pi/2;
t4 = -pi/2;

f=[-10 10 0]';
m=[0 0 -10]';

s1= [cos(t1) -sin(t1) 0; sin(t1) cos(t1) 0; 0 0 1];
s2= [cos(t1+t2) -sin(t1+t2) 0; sin(t1+t2) cos(t1+t2) 0; 0 L 1];
s3= [cos(t1+t2+t3) -sin(t1+t2+t3) 0; sin(t1+t2+t3) cos(t1+t2+t3) 0; 0 L 1];
s4= [cos(t1+t2+t3+t4) -sin(t1+t2+t3+t4) 0; sin(t1+t2+t3+t4) cos(t1+t2+t3+t4) 0; 0 L 1];


m1 = s1*m;
m2 = s2*m;
m3 = s3*m;
m4 = s4*m;

disp(m1)
disp(m2)
disp(m3)
disp(m4)
```

Command Window

```
     0
     0
   -10

 0
 0
-10

 0
 0
-10

 0
 0
-10
```

$M_1 = -10$

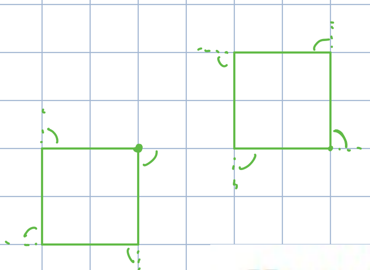
$M_2 = -10$

$M_3 = -10$

$M_4 = -10$

e) rank of jacobian < 3

@ $\pi/2$ and $-\pi/2$ for all values of theta



```
SB= exp(s1) + exp(s2) + exp(s3) + exp(s4);
rank(SB)
```

(d) (2pt) Under the same conditions as (c), suppose that a force $f = (-10, 10, 0)$ and a moment $m = (0, 0, -10)$, also expressed in the fixed frame, are applied to the tip. What are the torques experienced at each joint?

(e) (2pt) Find all kinematic singularities for this chain.

2. **(8pts)** Referring to Fig. 2, a rigid body, shown at the top right, rotates about the point (L, L) with angular velocity $\dot{\theta} = 1$.

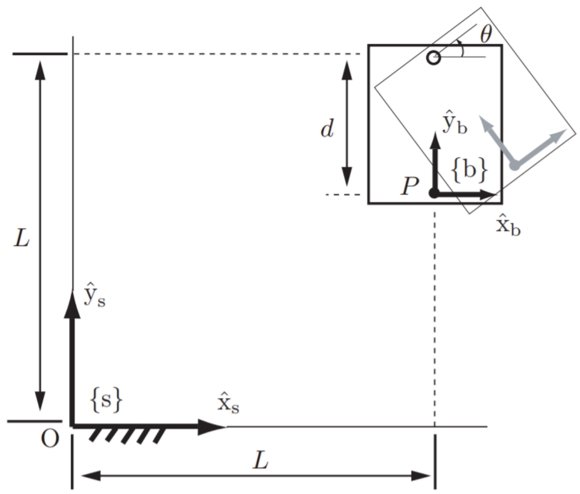


Figure 2: A 4R planar open chain.

(a) (1pts) Find the position of point P on the moving body relative to the fixed reference frame $\{s\}$ in terms of θ.

(b) (1pts) Find the velocity of point P in terms of the fixed frame.

(c) (1pts) What is T_{sb}, the configuration of frame $\{b\}$, as seen from the fixed frame $\{s\}$?

(d) (1pts) Find the twist of T_{sb} in body coordinates.

(e) (1pts) Find the twist of T_{sb} in space coordinates.

(f) (1pts) What is the relationship between the twists from (d) and (e)?

(g) (1pts) What is the relationship between the twist from (d) and $\dot{P}$ from (b)?

(h) (1pts) What is the relationship between the twist from (e) and $\dot{P}$ from (b)?

3. **(6pts)** The RRP robot in Fig. 3 is shown in its zero position.

(a) (3pts) Write down the screw axes in the space frame. Evaluate the forward kinematics when $\theta = (90°, 90°, 1)$. Hand-draw or use a computer to show the arm and the end-effector frame in this configuration. Obtain the space Jacobian J_s for this configuration.

(b) (3pts) Write down the screw axes in the end-effector body frame. Evaluate the forward kinematics when $\theta = (90°, 90°, 1)$ and confirm that you get the same result as in part (a). Obtain the body Jacobian J_b for this configuration.

4. **(5pts)** The RRRP chain of Fig. 4 is shown in its zero position. Let p be the coordinates of the origin of $\{b\}$ expressed in $\{s\}$.

(a) (3pts) Determine the body Jacobian $J_b(\theta)$ when $\theta_1 = \theta_2 = 0, \theta_3 = \pi/2, \theta_4 = L$.

②

2. **(8pts)** Referring to Fig. 2, a rigid body, shown at the top right, rotates about the point (L, L) with angular velocity $\dot{\theta} = 1$.

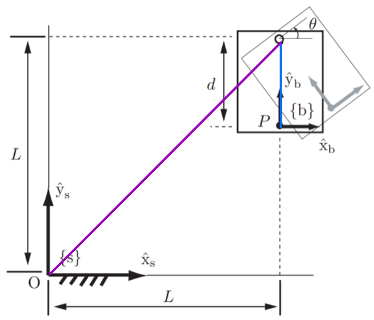


Figure 2: A 4R planar open chain.

(a) (1pts) Find the position of point P on the moving body relative to the fixed reference frame $\{s\}$ in terms of θ.

(b) (1pts) Find the velocity of point P in terms of the fixed frame.

(c) (1pts) What is T_{sb}, the configuration of frame $\{b\}$, as seen from the fixed frame $\{s\}$?

(d) (1pts) Find the twist of T_{sb} in body coordinates.

(e) (1pts) Find the twist of T_{sb} in space coordinates.

(f) (1pts) What is the relationship between the twists from (d) and (e)?

(g) (1pts) What is the relationship between the twist from (d) and $\dot{P}$ from (b)?

(h) (1pts) What is the relationship between the twist from (e) and $\dot{P}$ from (b)?

$$\begin{bmatrix} 1 & 0 & L \\ 0 & 1 & L \\ 0 & 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} \cos(\dot{\theta}t - \frac{\pi}{2}) & -\sin(\dot{\theta}t - \frac{\pi}{2}) & 0 \\ \sin(\dot{\theta}t - \frac{\pi}{2}) & \cos(\dot{\theta}t - \frac{\pi}{2}) & -d \\ 0 & 0 & 1 \end{bmatrix}$$

ω_i			v_i			
0	0	1	$-L$	$-L$	0	$\therefore e^{S\theta} M T_{sb}$
$\hat{\omega}_i$			$\hat{v}_i$			
0	0	1	$-d$	0	0	$\therefore e^{B\theta} M T_{bs}$

a) $$\begin{bmatrix} L + d\cos(\dot{\theta}t - \pi/2) \\ L + d\sin(\dot{\theta}t - \pi/2) \\ 0 \end{bmatrix} \qquad \begin{bmatrix} L + d\cos(\theta - \pi/2) \\ L + d\sin(\theta - \pi/2) \\ 0 \end{bmatrix}$$

b) $$\dot{x} = d\dot{\theta}\sin(\theta)$$
$$\dot{y} = d\dot{\theta}\cos(\theta)$$
$$\dot{z} = 0$$

c) $$T_{sb} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 & L(1-\cos(\theta)) \\ \sin(\theta) & \cos(\theta) & 0 & L(1-\sin(\theta)) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

d) $= e^{B\theta}$ $\quad B = \begin{bmatrix} \hat{\omega}_i \\ \hat{v}_i \end{bmatrix}$

$[0\ 0\ 1\ \ L\ {-L}\ 0]'$

e) $= e^{S\theta}$ $\quad S = \begin{bmatrix} \omega_i \\ v_i \end{bmatrix}$

$[0\ 0\ 1\ \ d\ 0\ 0]'$

f) The twists both happen along the z axis, the relative velocities at each point are different by the distance relative to each frame and the components, the bodyframe in the home configuration shows has the twist axis aligned with the y axis while the space frame has relative x and y velocities.

g) The relative velocity is defined by the radius of rotation, but (d) only has a x component in the screw axis.

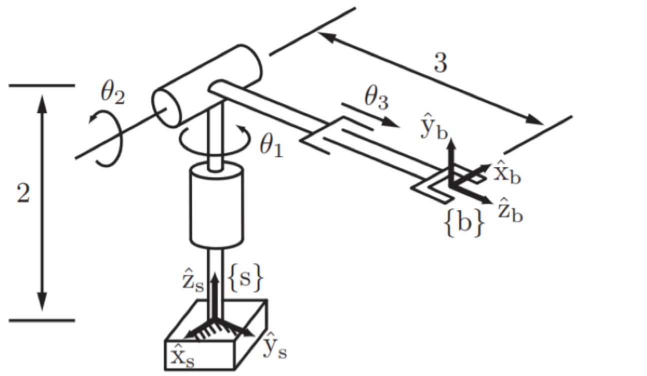
f) Relative velocities for both have x and y components, but are defined by different lengths of the radius of rotation in (d) and the L of the point from the space frame in (e)

③ **(6pts)** The RRP robot in Fig. 3 is shown in its zero position.

(a) (3pts) Write down the screw axes in the space frame. Evaluate the forward kinematics when $\theta = (90°, 90°, 1)$. Hand-draw or use a computer to show the arm and the end-effector frame in this configuration. Obtain the space Jacobian J_s for this configuration.

(b) (3pts) Write down the screw axes in the end-effector body frame. Evaluate the forward kinematics when $\theta = (90°, 90°, 1)$ and confirm that you get the same result as in part (a). Obtain the body Jacobian J_b for this configuration.

$$\mathcal{V}_s = J_s(\theta)\dot{\theta} \qquad \mathcal{V}_b = J_b(\theta)\dot{\theta}$$

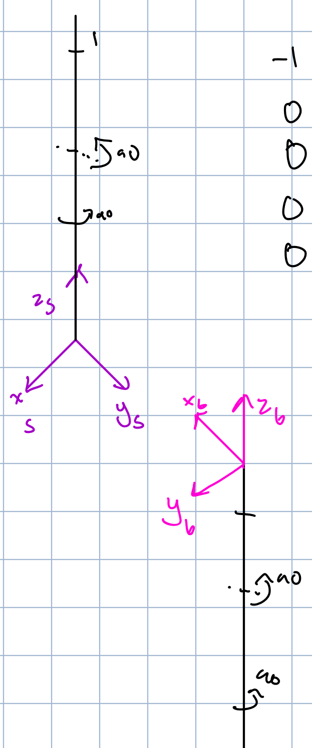


$$i \quad \omega_i: \begin{matrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{matrix} \qquad v_i: \begin{matrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 1 & 0 \end{matrix}$$

$$i \quad \hat{\omega}_i: \begin{matrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{matrix} \qquad \hat{v}_i: \begin{matrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{matrix}$$

$$S_i : \begin{bmatrix} \omega_i \\ v_i \end{bmatrix} \qquad B_i : \begin{bmatrix} \hat{\omega}_i \\ \hat{v}_i \end{bmatrix}$$

$$\left[\exp(J_1)\exp(J_2)\exp(J_3)\right] S_i : J \qquad \left[\exp(B_1)\exp(B_2)\exp(B_3)\right] B_i : J_B$$



$$\begin{matrix} -1 & 0 & 0 & \cos(\theta_1)(3\cos(\theta_2)+\theta_3) \\ 0 & 0 & 1 & \sin(\theta_1)(3\cos(\theta_2)+\theta_3) \\ 0 & 1 & 0 & \\ 0 & 0 & 1 & 3\sin(\theta_2)+\theta_3 \\ 0 & 0 & 0 & 1 \end{matrix}$$

$$\begin{matrix} \theta_1 & 90 \\ \theta_2 & 90 \\ \theta_3 & 1 \end{matrix} \rightarrow \begin{bmatrix} 0 \\ 0 \\ 6 \end{bmatrix}$$

$$\begin{matrix} 0 & 1 & 0 & (3+\theta_3)\cos(\theta_1+\theta_2) \\ -1 & 0 & 0 & (3+\theta_3)\sin(\theta_1+\theta_2) \\ 0 & 0 & 1 & (3+\theta_3)\sin(\theta_2) \end{matrix}$$

$$\begin{matrix} 90 \\ 90 \\ 1 \end{matrix} \rightarrow \begin{bmatrix} 0 \\ 0 \\ -6 \end{bmatrix}$$

4

4. **(5pts)** The RRRP chain of Fig. 4 is shown in its zero position. Let p be the coordinates of the origin of $\{b\}$ expressed in $\{s\}$.

   (a) (3pts) Determine the body Jacobian $J_b(\theta)$ when $\theta_1 = \theta_2 = 0, \theta_3 = \pi/2, \theta_4 = L$.

   (b) (2pts) Find $\dot{p}$ when $\theta_1 = \theta_2 = 0, \theta_3 = \pi/2, \theta_4 = L$ and $\dot{\theta}_1 = \dot{\theta}_2 = \dot{\theta}_3 = \dot{\theta}_4 = 1$.

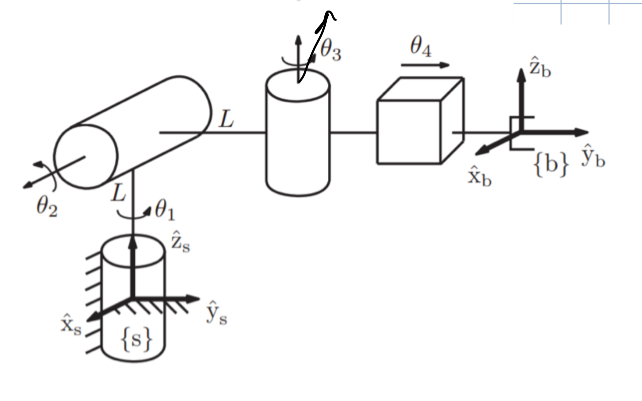


Figure 4: An RRRP spatial open chain.

$\mathcal{V}_b = J_b(\theta)\dot{\theta}$

i	ω_i			v_i		
	0	0	1	$-2L$	0	0
	1	0	0	0	0	$2L$
	0	0	1	$-L$	0	0
	0	0	0	0	L	0

$$\begin{bmatrix} 0 & -1 & 0 & -L\cos(\theta_3) - L \\ 1 & 0 & 0 & L\cos(\theta_1) \\ 0 & 0 & 1 & L + \cancel{L\sin\theta_2} \\ 0 & 0 & 0 & 1 \end{bmatrix} \therefore \begin{bmatrix} 0 & -1 & 0 & -L \\ 1 & 0 & 0 & -L \\ 0 & 0 & 1 & -L \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\dot{p}_{sb} = R_{sb}(\theta)\mathcal{V}_b \qquad \mathcal{V}_b = \begin{bmatrix} \hat{\omega}_i \\ \hat{v}_i \end{bmatrix} = J_b(\theta)\begin{pmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \\ \dot{\theta}_4 \end{pmatrix}$$

$$R_{sb} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\dot{p} = \sum_{i=1}^{4} S_i \dot{\theta}_i \qquad \dot{p} = \begin{matrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ -2L & 0 & -L & 0 \\ 0 & 0 & 0 & L \\ 0 & 2L & 0 & 0 \end{matrix} \therefore \begin{bmatrix} 1 \\ 0 \\ 2 \\ -3L \\ L \\ 2L \end{bmatrix}$$

$\dot{\omega}_x = 1$
$\dot{\omega}_y = 0$
$\dot{\omega}_z = 2$
$\dot{v}_x = -3L$
$\dot{v}_y = L$
$\dot{v}_z = 2L$

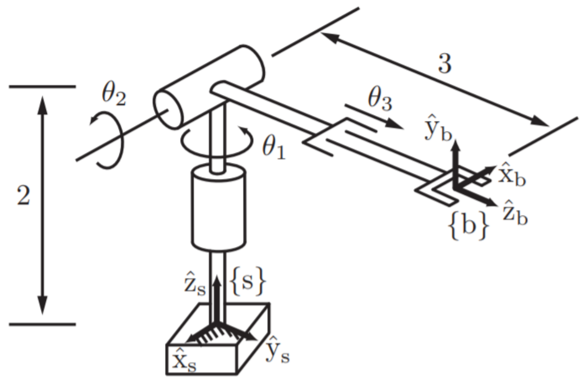


Figure 3: RRP robot shown in its zero position.

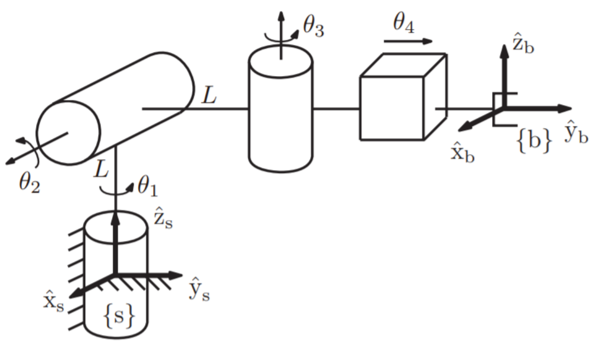


Figure 4: An RRRP spatial open chain.

(b) (2pts) Find $\dot{p}$ when $\theta_1 = \theta_2 = 0, \theta_3 = \pi/2, \theta_4 = L$ and $\dot{\theta}_1 = \dot{\theta}_2 = \dot{\theta}_3 = \dot{\theta}_4 = 1$.

5. **(11pts)** Consider the kinematics of the 7R WAM robot given in Assignment 2, with a detailed description on the Barrett website: `https://web.barrett.com/files/B2576_RevAC-00.pdf` and Example 4.7 (Page 148, Modern Robotics textbook). We will use the joint conventions from this document. The arm is shown in its all-zeros configuration, and the red arrows show the positive directions for each joint J1-J7.

   In our imagined scenario, the WAM is mounted upright to a tabletop whose surface is 1 meter above the floor. With respect to the origin (on the floor), the back-right corner of the robot (the location of the "Base" frame in the Barrett PDF) is located at x = 0.75m, y = 0.5m, z = 1.0m. The front of the robot is facing the positive y direction.

   A whiteboard marker has been attached rigidly to the robot's end plate, such that the marker is vertical and centered on the end plate when all joints are at zero (as shown in the PDF). The marker is 12cm long, so the drawing marker tip is 12cm from the end plate.

   (a) (3pts) Give the numerical body Jacobian J_b when all joint angles are $\pi/2$. Separate the Jacobian matrix into an angular-velocity portion J_ω (the joint rates act on the angular velocity) and a linear-velocity portion J_v (the joint rates act on the linear velocity).

   (b) (3pts) For this configuration, calculate the directions and lengths of the principal semi-axes of the three-dimensional angular-velocity manipulability ellipsoid (based on J_ω) and the directions and lengths of the principal semi-axes of the three-dimensional linear-velocity manipulability ellipsoid (based on J_v).

(c) (5pts) Recall the joint trajectory given in the file Theta_data.txt. Each line is a point on the joint trajectory; the file should have 836 points. Each point consists of seven space-separated numbers, one for each joint J1-J7. Joint values are given in radians. Assuming each joint i is moving with piece-wise constant velocity for all time steps $t = 1, \ldots, 836s$ with $\dot{\theta}_i^t = \frac{\theta_i^{t+1} - \theta_i^t}{\Delta t}$ and $\Delta t = 1$s. Create a program which implements the velocity kinematics of the 7-dof robot. Use this program to compute the linear velocity of the marker tip relative to the fixed world origin frame. Save the marker tip linear velocity in a similar format (one line per point, three space-separated values vx, vy, vz) and include a figure of the visualized velocity vector at each time step on the x-y-z trajectory of the marker tip. Submit your code as well.

5)

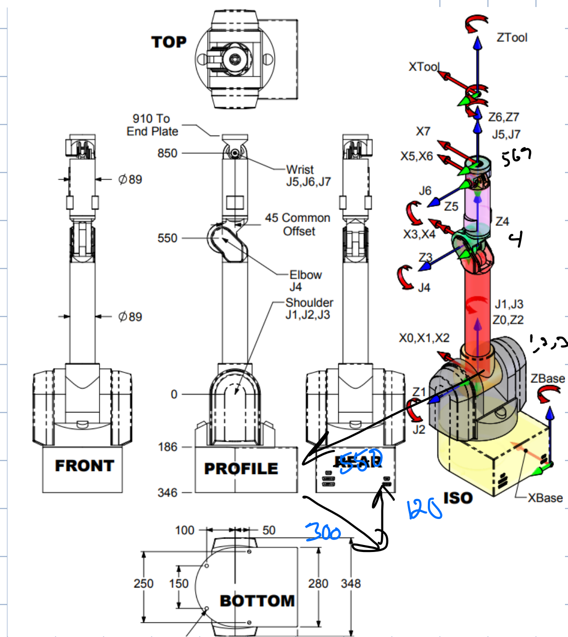
	ω_i			v_i		
1	1	0	0	0	0	0
2	0	1	0	0	0	0
3	0	0	1	0	0	0
4	0	0	1	0	-55	0
5	1	0	0	0	-85	0
6	0	1	0	85	0	0
7	0	0	1	0	0	0

1) $\theta_{1-7} = 90$

$V_6 = \begin{pmatrix} \omega_i \\ v_i \end{pmatrix} = J(\pi/2)(\dot{\theta}_i) \in \mathbb{R}^6$

$$\boxed{\begin{matrix} R_{56} \cdot v_i \\ R_{56} \cdot \omega_i \end{matrix}}$$

$R_{56} = \begin{matrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{matrix}$



b)

$$T(\pi/2) = \begin{bmatrix} 0 & -1 & 0 & 55 \\ 1 & 0 & 0 & -30 \\ 0 & 0 & 1 & 12 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$J J' = A$ eigs(A) = major, minor, inter axis

$x = \sqrt{1.03}$

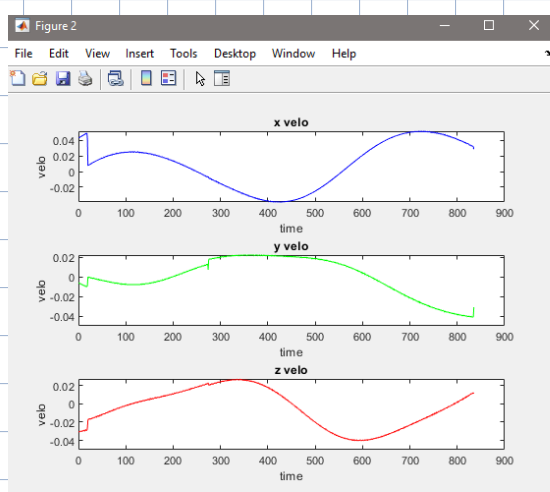
$y = \sqrt{.72}$

$z = \sqrt{.30}$

c) hw3_xyz.txt

intel roboticshw3.m

question 4



axes of the three-dimensional angular-velocity manipulability ellipsoid (based on J_ω) and the directions and lengths of the principal semi-axes of the three-dimensional linear-velocity manipulability ellipsoid (based on J_v).

(c) (5pts) Recall the joint trajectory given in the file Theta_data.txt. Each line is a point on the joint trajectory; the file should have 836 points. Each point consists of seven space-separated numbers, one for each joint J1-J7. Joint values are given in radians. Assuming each joint i is moving with piece-wise constant velocity for all time steps $t = 1, \ldots, 836s$ with $\dot{\theta}_i^t = \frac{\theta_i^{t+1} - \theta_i^t}{\Delta t}$ and $\Delta t = 1$s. Create a program which implements the velocity kinematics of the 7-dof robot. Use this program to compute the linear velocity of the marker tip relative to the fixed world origin frame. Save the marker tip linear velocity in a similar format (one line per point, three space-separated values vx, vy, vz) and include a figure of the visualized velocity vector at each time step on the x-y-z trajectory of the marker tip. Submit your code as well.

Question. HW 2.5 (10+5pts)

Create a forward kinematics implementation for a common 7-dof robotic arm: the Barrett WAM™. A description of arm's kinematics is available on Barrett website: https://web.barrett.com/files/B2576_RevAC-00.pdf and Example 4.7 (Page 148, Modern Robotics textbook). We will use the joint conventions from this document. The arm is shown in its all-zeros configuration, and the red arrows show the positive directions for each joint J1-J7.

In our imagined scenario, the WAM is mounted upright to a tabletop whose surface is 1 meter above the floor. With respect to the origin (on the floor), the back-right corner of the robot (the location of the "Base" frame in the Barrett PDF) is located at x = 0.75m, y = 0.5m, z = 1.0m. The front of the robot is facing the positive y direction.

A whiteboard marker has been attached rigidly to the robot's end plate, such that the marker is vertical and centered on the end plate when all joints are at zero (as shown in the PDF). The marker is 12cm long, so the drawing marker tip is 12cm from the end plate.

A whiteboard is mounted nearby (in some unknown position and orientation).

a. (3pts) In the all-zeros configuration, what is the location of the marker tip, given with respect to the world origin? (You should be able to answer this without any code.)

   The robot has decided to draw something! Its joint trajectory is given in the file Theta_data.txt. Each line is a point on the joint trajectory; the file should have 836 points. Each point consists of seven space-separated numbers, one for each joint J1-J7. Joint values are given in radians. The robot is drawing for the entire trajectory.

b. (7pts) Create a program which implements the forward kinematics of the 7-dof Barrett WAMTM. Use this program to convert the above joint trajectory into the x-y-z trajectory of the marker tip in the world origin frame. Save the marker tip trajectory in a similar format (one line per point, three space-separated values x, y, z). Submit your code as well.
c. (extra 3pts) Where is the whiteboard? Give the location of any point on its surface, along with a vector normal to it (out of the board).
d. (extra 2pts) What did the robot write? Include a figure of the drawing.

a) $\begin{bmatrix} 610 \\ 720 \\ 2376 \end{bmatrix}$

b) intelrobotics hw3.m

HW2_xyz.txt

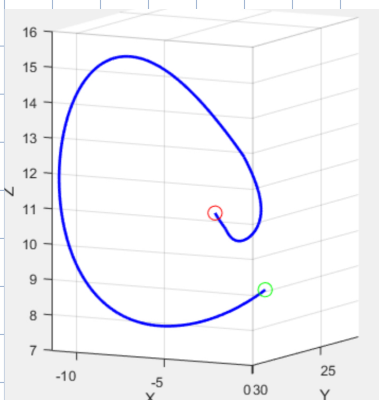
Point

c) [10.13, 29.99, 9.53]

normal vector

[-24.75, 2.43, 33.95]

d)



it looks like a C