

CORRELATED BINARY DATA

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ABSTRACT. We define a class for describing data from toxicology experiments and implement fitting of a variety of existing models and trend tests.

"../R/CorrBin-package.R" 1≡

```
#'Nonparametrics for Correlated Binary and Multinomial Data
#'
#'This package implements nonparametric methods for analyzing exchangeable
#'binary and multinomial data with variable cluster sizes with emphasis on trend testing. The
#'input should specify the treatment group, cluster-size, and the number of
#'responses (i.e. the number of cluster elements with the outcome of interest)
#'for each cluster.
#'
#'\itemize{ \item The \code{\link{CBData}}/\code{\link{CMDData}} and \code{\link{read.CBData}}/\code{\link{read.CMDData}}
#'functions create a 'CBData' or 'CMDData' object used by the analysis functions.
#'\item \code{\link{ran.CBData}} and \code{\link{ran.CMDData}} can be used to generate random
#' binary or multinomial data using a variety of distributions.
#'\item \code{\link{mc.test.chisq}} tests the assumption of marginal compatibility
#'underlying all the methods, while \code{\link{mc.est}} estimates the
#'distribution of the number of responses under marginal compatibility.
#'\item Finally, \code{\link{trend.test}} performs three different tests for trend
#'along the treatment groups for binomial data. }
#'
#'\@name CorrBin-package
#'\@aliases CorrBin-package CorrBin
#'\@docType package
#'\@author Aniko Szabo
#'
#'\@Maintainer: Aniko Szabo <aszabo@mcw.edu>
#'\@references Szabo A, George EO. (2009) On the Use of Stochastic Ordering to
#'\@Test for Trend with Clustered Binary Data. \emph{Biometrika}
#'
#'\@Stefanescu, C. & Turnbull, B. W. (2003) Likelihood inference for exchangeable
#'\@binary data with varying cluster sizes. \emph{Biometrics}, 59, 18-24
#'
#'\@Pang, Z. & Kuk, A. (2007) Test of marginal compatibility and smoothing
#'\@methods for exchangeable binary data with unequal cluster sizes.
#'\@emph{Biometrics}, 63, 218-227
#'\@keywords package nonparametric
NULL
◇
```

Uses: `ran.CBData` [9b](#).

1. DEFINING `CBData` – A CLASS FOR **CLUSTERED BINARY Data**

We start with defining an S3 class describing data from toxicology experiments. The class is a data frame with the following columns:

Trt: a factor defining (treatment) groups

ClusterSize: an integer-valued variable defining the cluster size

NResp: an integer-valued variable defining the number of responses (1s)

Freq: an integer-valued variable defining frequency for each Trt/ClusterSize/NResp combination

`CBData` converts a data frame to a `CBData` object. `x` is the input data frame; `trt`, `clustersize`, `nresp` and `freq` could be strings or column indices defining the appropriate variable in `x` (`freq` can also be `NULL`, in which case it is assumed that each combination has frequency 1).

"../R/CBData.R" 2a≡

```
#'Create a 'CBdata' object from a data frame.
#
#'#The \code{CBData} function creates an object of class \dfn{CBData} that is
#'#used in further analyses. It identifies the variables that define treatment
#'#group, clustersize and the number of responses.
#
#'#@export
#'#importFrom stats aggregate
#'#@param x a data frame with one row representing a cluster or potentially a
#'#set of clusters of the same size and number of responses
#'#@param trt the name of the variable that defines treatment group
#'#@param clustersize the name of the variable that defines cluster size
#'#@param nresp the name of the variable that defines the number of responses in
#'#the cluster
#'#@param freq the name of the variable that defines the number of clusters
#'#represented by the data row. If \code{NULL}, then each row is assumed to
#'#correspond to one cluster.
#'#return A data frame with each row representing all the clusters with the
#'#same trt/size/number of responses, and standardized variable names:
#'#return \item{Trt}{factor, the treatment group}
#'#return \item{ClusterSize}{numeric, the cluster size}
#'#return \item{NResp}{numeric, the number of responses}
#'#return \item{Freq}{numeric, number of clusters with the same values}
#'#author Aniko Szabo
#'#seealso \code{\link{read.CBData}} for creating a \code{CBData} object
#'#directly from a file.
#'#keywords classes manip
#'#examples
#
#'#data(shelltox)
#'#sh <- CBData(shelltox, trt="Trt", clustersize="ClusterSize", nresp="NResp")
#'#str(sh)
#
#
#◇
```

File defined by [2ab](#), [3ab](#), [4](#), [5a](#), [6ab](#), [7ab](#), [8](#), [9ab](#), [11ab](#), [12](#).

"../R/CBData.R" 2b≡

```
CBData <- function(x, trt, clustersize, nresp, freq=NULL){
  if (!is.data.frame(x)) stop("x has to be a data frame")
  nms <- names(x)
  process.var <- function(var){
    if (is.character(var)){
      if (var %in% nms) res <- x[[var]]
    }
  }
}
```

```

      else stop(paste("Variable '", var, "' not found"))
    }
    else {
      if (is.numeric(var)){
        if (var %in% seq(along=nms)) res <- x[[var]]
        else stop(paste("Column", var, " not found"))
      }
      else stop(paste("Invalid variable specification:",var))
    }
  }
  trtvar <- factor(process.var(trt))
  csvar <- process.var(clustersize)
  nrespvar <- process.var(nresp)
  if (is.null(freq)) freqvar <- rep(1, nrow(x))
  else freqvar <- process.var(freq)

  d <- data.frame(Trt=trtvar, ClusterSize=csvar, NResp=nrespvar, Freq=freqvar)
  d <- aggregate(d$Freq, list(Trt=d$Trt, ClusterSize=d$ClusterSize, NResp=d$NResp),sum)
  names(d)[4] <- "Freq"
  d$ClusterSize <- as.numeric(as.character(d$ClusterSize))
  d$NResp <- as.numeric(as.character(d$NResp))
  class(d) <- c("CBData", "data.frame")
  d}

```

◇
 File defined by [2ab](#), [3ab](#), [4](#), [5a](#), [6ab](#), [7ab](#), [8](#), [9ab](#), [11ab](#), [12](#).
 Defines: CBdata.data.frame Never used.

The `read.CBData` function reads in clustered binary data from a tab-delimited text file. The first column should give the treatment group, the second the size of the cluster, the third the number of responses in the cluster. Optionally, a fourth column could give the number of times the given combination occurs in the data.

"../R/CBData.R" 3a≡

```

#'Read data from external file into a CBData object
#'
#'A convenience function to read data from specially structured file directly
#'into a \code{CBData} object.
#'
#'@export
#'@importFrom utils read.table
#'@param file name of file with data. The first column should contain the
#'treatment group, the second the size of the cluster, the third the number of
#'responses in the cluster. Optionally, a fourth column could give the number
#'of times the given combination occurs in the data.
#'@param with.freq logical indicator of whether a frequency variable is present
#'in the file
#'@param ... additional arguments passed to \code{\link[utils]{read.table}}
#'@return a \code{CBData} object
#'@author Aniko Szabo
#'@seealso \code{\link{CBData}}
#'@keywords IO file
#'

```

◇
 File defined by [2ab](#), [3ab](#), [4](#), [5a](#), [6ab](#), [7ab](#), [8](#), [9ab](#), [11ab](#), [12](#).

"../R/CBData.R" 3b≡

```

read.CBData <- function(file, with.freq=TRUE, ...){
  d <- read.table(file, col.names=c("Trt","ClusterSize","NResp", if (with.freq) "Freq"), ...)
  if (!with.freq) d$Freq <- 1
  d <- aggregate(d$Freq, list(Trt=d$Trt, ClusterSize=d$ClusterSize, NResp=d$NResp),sum)
  names(d)[4] <- "Freq"
  d$ClusterSize <- as.numeric(as.character(d$ClusterSize))
  d$NResp <- as.numeric(as.character(d$NResp))
  d <- CBData(d, "Trt", "ClusterSize", "NResp", "Freq")
  d}

```

◇

File defined by [2ab](#), [3ab](#), [4](#), [5a](#), [6ab](#), [7ab](#), [8](#), [9ab](#), [11ab](#), [12](#).

Defines: `read.CBData` Never used.

The `[".CMDData]` function defines subsetting of `CMDData` objects. If the subsetting is only affecting the rows, then the appropriate attributes are preserved, and the unused levels of `Trt` are dropped. Otherwise the returned object does not have a `CMDData` class anymore.

"../R/CBData.R" 4≡

```

#'Extract from a CBData or CMDData object
#'
#'The extracting syntax works as for \code{\link{[.data.frame]}}, and in general the returned object is no
#'However if the columns are not modified, then the result is still a \code{CBData} or \code{CMDData} obje
#' and the unused levels of treatment groups dropped.
#'
#'@param x \code{CMDData} object.
#'@param i numeric, row index of extracted values
#'@param j numeric, column index of extracted values
#'@param drop logical. If TRUE the result is coerced to the lowest possible dimension.
#'The default is the same as for \code{\link{[.data.frame]}}: to drop if only one column is left, but not
#'@return a \code{CBData} or \code{CMDData} object
#'@author Aniko Szabo
#'@seealso \code{CBData}, \code{\link{CMDData}}
#'@keywords manip
#'@name Extract
#'
NULL

```

◇

File defined by [2ab](#), [3ab](#), [4](#), [5a](#), [6ab](#), [7ab](#), [8](#), [9ab](#), [11ab](#), [12](#).

```
"../R/CBData.R" 5a≡
```

```
#'@rdname Extract
#'@export
#'@examples
#'
#'data(shelltox)
#'str(shelltox[1:5,])
#'str(shelltox[1:5, 2:4])

"[.CBData" <- function(x, i, j, drop){
  res <- NextMethod("[")
  if (NCOL(res) == ncol(x)){
    res <- "[.data.frame"(x, i, )
    if (is.factor(res$Trt)) res$Trt <- droplevels(res$Trt)
    res
  }
  else {
    class(res) <- setdiff(class(res), "CBData")
  }
  res
}
◇
```

File defined by [2ab](#), [3ab](#), [4](#), [5a](#), [6ab](#), [7ab](#), [8](#), [9ab](#), [11ab](#), [12](#).

Defines: [.CBData Never used.

`unwrap.CBData` is a utility function that reformats a `CBData` object so that each row is one observation (instead of one cluster). A new 'ID' variable is added to indicate clusters. It is first defined as a generic function to allow generalization.

```
"../R/aaa-generics.R" 5b≡
```

```
#'Unwrap a clustered object
#'
#'\code{unwrap} is a utility function that reformats a CBData or CMDData object so
#'\that each row is one observation (instead of one or more clusters). A new
#'\ID' variable is added to indicate clusters. This form can be useful for
#'\setting up the data for a different package.
#'
#'@aliases unwrap unwrap.CBData
#'@export
#'@param object a \code{\link{CBData}} object
#'@param \dots other potential arguments; not currently used
#'@return For \code{unwrap.CBData}: a data frame with one row for each cluster element (having a binary
#'\outcome) with the following standardized column names
#'@return \item{Trt}{factor, the treatment group}
#'@return \item{ClusterSize}{numeric, the cluster size}
#'@return \item{ID}{factor, each level representing a different cluster}
#'@return \item{Resp}{numeric with 0/1 values, giving the response of the cluster
#'\element}
#'@author Aniko Szabo
#'@keywords manip
#'@examples
#'
#'data(shelltox)
#'ush <- unwrap(shelltox)
#'head(ush)
#'
```

```

unwrap <- function(object,...) UseMethod("unwrap")
◇
Uses: unwrap.CBData 6a.

```

"../R/CBData.R" 6a≡

```

#'@rdname unwrap
#'@method unwrap CBData
#'@export

unwrap.CBData <- function(object,...){
  freqs <- rep(1:nrow(object), object$Freq)
  cb1 <- object[freqs,]
  cb1$Freq <- NULL
  cb1$ID <- factor(1:nrow(cb1))
  pos.idx <- rep(1:nrow(cb1), cb1$NResp)
  cb.pos <- cb1[pos.idx,]
  cb.pos$Resp <- 1
  cb.pos$NResp <- NULL
  neg.idx <- rep(1:nrow(cb1), cb1$ClusterSize-cb1$NResp)
  cb.neg <- cb1[neg.idx,]
  cb.neg$Resp <- 0
  cb.neg$NResp <- NULL
  res <- rbind(cb.pos, cb.neg)
  res[order(res$ID),]
}

```

◇
 File defined by 2ab, 3ab, 4, 5a, 6ab, 7ab, 8, 9ab, 11ab, 12.
 Defines: unwrap.CBData 5b, 8.

2. RAO-SCOTT ADJUSTED COCHRAN-ARMITAGE TEST

The RS-adjusted CA test for trend is based on design-effect adjustment.

"../R/CBData.R" 6b≡

```

#'Rao-Scott trend test
#'
#'\code{RS.trend.test} implements the Rao-Scott adjusted Cochran-Armitage test
#'\for linear increasing trend with correlated data.
#'
#'\The test is based on calculating a \dfn{design effect} for each cluster by
#'\dividing the observed variability by the one expected under independence. The
#'\number of responses and the cluster size are then divided by the design
#'\effect, and a Cochran-Armitage type test statistic is computed based on these
#'\adjusted values.
#'
#'\The implementation aims for testing for \emph{increasing} trend, and a
#'\one-sided p-value is reported. The test statistic is asymptotically normally
#'\distributed, and a two-sided p-value can be easily computed if needed.
#'
#'@export
#'@param cbdata a \code{\link{CBData}} object
#'@return A list with components
#'\@return \item{statistic}{numeric, the value of the test statistic}
#'\@return \item{p.val}{numeric, asymptotic one-sided p-value of the test}

```

File defined by 2ab, 3ab, 4, 5a, 6ab, 7ab, 8, 9ab, 11ab, 12.
Uses: GEE.trend.test 8, RS.trend.test 7a.

File defined by 2ab, 3ab, 4, 5a, 6ab, 7ab, 8, 9ab, 11ab, 12.
 Defines: RS.trend.test 6b, 7b.

```
#'GEE-based trend test
#'\code{GEE.trend.test} implements a GEE based test for linear increasing trend
#'\code{GEE.trend.test} for correlated binary data.
#'\code{GEE.trend.test} The actual work is performed by the \link[geepack]{geese} function of
#'\code{GEE.trend.test} the \link[geepack]{geepack} library, which is required for this feature to work.
#'\code{GEE.trend.test} This function only provides a convenient wrapper
#'\code{GEE.trend.test} to obtain the results in the same format as \link{RS.trend.test} and
#'\code{GEE.trend.test} \link{SO.trend.test}.
```

```

#'
#'The implementation aims for testing for \emph{increasing} trend, and a
#'one-sided p-value is reported. The test statistic is asymptotically normally
#'distributed, and a two-sided p-value can be easily computed if needed.
#'
#'\@export
#'\@importFrom stats binomial pnorm
#'\@param cbdata a \code{\link{CBData}} object
#'\@param scale.method character string specifying the assumption about the
#'\change in the overdispersion (scale) parameter across the treatment groups:
#'"fixed" - constant scale parameter (default); "trend" - linear trend for the
#'\log of the scale parameter; "all" - separate scale parameter for each group.
#'\@return A list with components
#'\@return \item{statistic}{numeric, the value of the test statistic}
#'\@return \item{p.val}{numeric, asymptotic one-sided p-value of the test}
#'\@author Aniko Szabo, aszabo@mcw.edu
#'\@seealso \code{\link{RS.trend.test}}, \code{\link{SO.trend.test}} for
#'\alternative tests; \code{\link{CBData}} for constructing a CBData object.
#'\@keywords htest models
#'\@examples
#'
#'\data(shelltox)
#'\if (require(geepack)){
#'\  GEE.trend.test(shelltox, "trend")
#'}

```

File defined by 2ab, 3ab, 4, 5a, 6ab, 7ab, 8, 9ab, 11ab, 12.

Uses: GEE.trend.test 8, RS.trend.test 7a.

"../R/CBData.R" 8≡

```

GEE.trend.test <- function(cbdata, scale.method=c("fixed", "trend", "all")){
  if (!requireNamespace("geepack", quietly = TRUE)) {
    stop("Please install geepack: install.packages('geepack')")
  }
  ucb <- unwrap.CBData(cbdata)
  scale.method <- match.arg(scale.method)
  if (scale.method=="fixed") {
    geemod <- geepack::geese(Resp~unclass(Trt), id=ucb$ID, scale.fix=FALSE, data=ucb,
                           family=binomial, corstr="exch") }
  else if (scale.method=="trend"){
    geemod <- geepack::geese(Resp~unclass(Trt), sformula=~unclass(Trt), id=ucb$ID, data=ucb,
                           family=binomial, sca.link="log", corstr="exch")}
  else if (scale.method=="all"){
    geemod <- geepack::geese(Resp~unclass(Trt), id=ucb$ID, sformula=~Trt, data=ucb,
                           family=binomial, sca.link="log", corstr="exch") }
  geesum <- summary(geemod)
  testres <- geesum$mean[2,"estimate"]/geesum$mean[2,"san.se"]
  p <- pnorm(testres, lower.tail=FALSE)
  list(statistic=testres, p.val=p)
}

```

File defined by 2ab, 3ab, 4, 5a, 6ab, 7ab, 8, 9ab, 11ab, 12.

Defines: GEE.trend.test 6b, 7b.

Uses: unwrap.CBData 6a.

4. GENERATING RANDOM DATA

`ran.CBData` generates a random `CBData` object from a given two-parameter distribution. `sample.sizes` is a dataset with variables `Trt`, `ClusterSize` and `Freq` giving the number of clusters to be generated for each `Trt/ClusterSize` combination. `p.gen.fun` and `rho.gen.fun` are functions that generate the parameter values for each treatment group ($g = 1$ corresponds to the lowest group, $g = 2$ to the second, etc). `pdf.fun` is a function(`p`, `rho`, `n`) generating the pdf of the number of responses given the two parameters `p` and `rho`, and the cluster size `n`.

```
"../R/CBData.R" 9a≡
```

```
#'Generate random correlated binary data
#
#'\code{ran.mc.CBData} generates a random \code{\link{CBData}} object from a
#given two-parameter distribution.
#
#'\dfn{p.gen.fun} and \dfn{rho.gen.fun} are functions that generate the
#parameter values for each treatment group; \dfn{pdf.fun} is a function
#generating the pdf of the number of responses given the two parameters
#\dfn{p} and \dfn{rho}, and the cluster size \dfn{n}.
#
#'\code{p.gen.fun} and \code{rho.gen.fun} expect the parameter value of 1 to
#represent the first group, 2 - the second group, etc.
#
#'\@export
#'\@importFrom stats rmultinom
#'\@param sample.sizes a dataset with variables Trt, ClusterSize and Freq giving
#the number of clusters to be generated for each Trt/ClusterSize combination.
#'\@param p.gen.fun a function of one parameter that generates the value of the
#first parameter of \code{pdf.fun} (\emph{p}) given the group number.
#'\@param rho.gen.fun a function of one parameter that generates the value of
#the second parameter of \code{pdf.fun} (\emph{rho}) given the group number.
#'\@param pdf.fun a function of three parameters (\emph{p}, \emph{rho}, \emph{n}) giving the
#PDF of the number of responses in a cluster given the two parameters
#(\emph{p}, \emph{rho}), and the cluster size (\emph{n}). Functions implementing two
#common distributions: the beta-binomial (\code{\link{betabin.pdf}}) and
#q-power (\code{\link{qpower.pdf}}) are provided in the package.
#'\@return a CBData object with randomly generated number of responses with
#sample sizes specified in the call.
#'\@author Aniko Szabo, aszabo@mcw.edu
#'\@seealso \code{\link{betabin.pdf}} and \code{\link{qpower.pdf}}
#'\@keywords distribution
#'\@examples
#
# set.seed(3486)
# ss <- expand.grid(Trt=0:3, ClusterSize=5, Freq=4)
# #Trt is converted to a factor
# rd <- ran.CBData(ss, p.gen.fun=function(g)0.2+0.1*g)
# rd
#
◇
```

File defined by [2ab](#), [3ab](#), [4](#), [5a](#), [6ab](#), [7ab](#), [8](#), [9ab](#), [11ab](#), [12](#).

Uses: `ran.CBData` [9b](#).

```
"../R/CBData.R" 9b≡
```

```
ran.CBData <- function(sample.sizes, p.gen.fun=function(g)0.3,
```

```

                                rho.gen.fun=function(g)0.2, pdf.fun=qpower.pdf){
ran.gen <- function(d){
# d is subset(sample.sizes, Trt==trt, ClusterSize==cs)
  cs <- d$ClusterSize[1]
  trt <- unclass(d$Trt)[1]
  n <- d$Freq[1]
  p <- p.gen.fun(trt)
  rho <- rho.gen.fun(trt)
  probs <- pdf.fun(p, rho, cs)
  tmp <- rmultinom(n=1, size=n, prob=probs)[,1]
  cbind(Freq=tmp, NResp=0:cs, ClusterSize=d$ClusterSize, Trt=d$Trt)}

sst <- if (is.factor(sample.sizes$Trt)) sample.sizes$Trt else factor(sample.sizes$Trt)
a <- by(sample.sizes, list(Trt=sst, ClusterSize=sample.sizes$ClusterSize), ran.gen)
a <- data.frame(do.call(rbind, a))
a$Trt <- factor(a$Trt, labels=levels(sst))
a <- a[a$Freq>0, ]
class(a) <- c("CBData", "data.frame")
a
}
◇

```

File defined by [2ab](#), [3ab](#), [4](#), [5a](#), [6ab](#), [7ab](#), [8](#), [9ab](#), [11ab](#), [12](#).
 Defines: `ran.CBData` [1](#), [9a](#), [11a](#).

4.1. Parametric pdf generating functions. `betabin.pdf` and `qpower.pdf` provide two classic distributions – beta-binomial and q-power – for generating correlated binary data. Either can be used in `ran.CBData`.

```
"../R/CBData.R" 11a≡
```

```
#'Parametric distributions for correlated binary data
#
#'\code{qpwr.pdf} and \code{betabin.pdf} calculate the probability
#distribution function for the number of responses in a cluster of the q-power
#and beta-binomial distributions, respectively.
#
#The pdf of the q-power distribution is \deqn{P(X=x) =
#\frac{\sum_{k=0}^x (-1)^k \binom{x}{k} q^{(n-x+k)\gamma}}{\sum_{k=0}^n (-1)^k C(x,k) q^{(n-x+k)\gamma}},} \eqn{x=0,\ldots,n}, where
#'\eqn{q=1-p}, and the intra-cluster correlation \deqn{\rho =
#\frac{q^{2\gamma}-q^2}{q(1-q)}}. \rho = (q^{2\gamma}-q^2)/(q(1-q)).}
#
#The pdf of the beta-binomial distribution is \deqn{P(X=x) = \binom{n}{x}
#\frac{B(\alpha+x, n+\beta-x)}{B(\alpha,\beta)},} \eqn{x=0,\ldots,n}, where \eqn{\alpha=
#p\frac{1-\rho}{\rho}}, \eqn{\alpha=
#(1-p)\frac{1-\rho}{\rho}}, \eqn{\beta=p(1-\rho)/\rho}, and \eqn{\beta=(1-p)(1-\rho)/\rho}.
#
# @export
# @name pdf
# @aliases qpwr.pdf betabin.pdf
# @param p numeric, the probability of success.
# @param rho numeric between 0 and 1 inclusive, the within-cluster correlation.
# @param n integer, cluster size.
# @return a numeric vector of length \eqn{n+1} giving the value of \eqn{P(X=x)}
# for \eqn{x=0,\ldots,n}.
# @author Aniko Szabo, aszabo@mcw.edu
# @seealso \code{\link{ran.CBData}} for generating an entire dataset using
# these functions
# @references Kuk, A. A (2004) Litter-based approach to risk assessment in
# developmental toxicity studies via a power family of completely monotone
# functions \emph{Applied Statistics}, 52, 51-61.
#
# Williams, D. A. (1975) The Analysis of Binary Responses from Toxicological
# Experiments Involving Reproduction and Teratogenicity \emph{Biometrics}, 31,
# 949-952.
# @keywords distribution
# @examples
#
# the distributions have quite different shapes
# with q-power assigning more weight to the "all affected" event than other distributions
# plot(0:10, betabin.pdf(0.3, 0.4, 10), type="o", ylim=c(0,0.34),
# ylab="Density", xlab="Number of responses out of 10")
# lines(0:10, qpwr.pdf(0.3, 0.4, 10), type="o", col="red")
#
◇
```

File defined by [2ab](#), [3ab](#), [4](#), [5a](#), [6ab](#), [7ab](#), [8](#), [9ab](#), [11ab](#), [12](#).
 Uses: [ran.CBData](#) [9b](#).

```
"../R/CBData.R" 11b≡
```

```
betabin.pdf <- function(p, rho, n){
  a <- p*(1/rho-1)
  b <- (1-p)*(1/rho-1)
  idx <- 0:n
  res <- choose(n, idx)*beta(a+idx, b+n-idx)/beta(a,b)
  res
}
```

}

◇
File defined by [2ab](#), [3ab](#), [4](#), [5a](#), [6ab](#), [7ab](#), [8](#), [9ab](#), [11ab](#), [12](#).

"../R/CBData.R" 12≡

```
#'@rdname pdf
#'@export
qpower.pdf <- function(p, rho, n){
  .q <- 1-p
  gamm <- log2(log(.q^2+rho*.q*(1-.q))/log(.q))
  res <- numeric(n+1)
  for (y in 0:n){
    idx <- 0:y
    res[y+1] <- choose(n,y) * sum( (-1)^idx * choose(y,idx) * .q^((n-y+idx)^gamm))
  }
  res <- pmax(pmin(res,1),0) #to account for numerical imprecision
  res
}
```

◇
File defined by [2ab](#), [3ab](#), [4](#), [5a](#), [6ab](#), [7ab](#), [8](#), [9ab](#), [11ab](#), [12](#).