

GENERALIZED LINEAR DENSITY RATIO MODEL (GLDRM) WITH FREQUENCY WEIGHTS

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The code is based on the `gldrm` package, with minor changes to incorporate frequency weights. The code is simplified a bit as well by removing some options:

- sampling weights are removed
- there is no variance/standard error calculation, and no inference against the null

"../R/wgldrm.R" 1≡

```
#' Main optimization function
#
# This function is called by the main \code{gldrm} function.
#
# @keywords internal
# @importFrom stats lm.fit lm.wfit weighted.mean
gldrm.control <- function(eps=1e-10, maxiter=100, returnfTiltMatrix=TRUE,
                          returnfOScoreInfo=FALSE, print=FALSE,
                          betaStart=NULL, f0Start=NULL)
{
  gldrmControl <- as.list(environment())
  class(gldrmControl) <- "gldrmControl"
  gldrmControl
}

gldrmFit <- function(x, y, linkfun, linkinv, mu.eta, mu0=NULL, offset=NULL, weights=NULL,
                    gldrmControl=gldrm.control(), thetaControl=theta.control(),
                    betaControl=beta.control(), f0Control=f0.control())
{
  ## Extract control arguments
  if (class(gldrmControl) != "gldrmControl")
    stop("gldrmControl must be an object of class \'gldrmControl\' returned by
         gldrmControl() function.")
  eps <- gldrmControl$eps
  maxiter <- gldrmControl$maxiter
  returnfTiltMatrix <- gldrmControl$returnfTiltMatrix
  returnfOScoreInfo <- gldrmControl$returnfOScoreInfo
  print <- gldrmControl$print
  betaStart <- gldrmControl$betaStart
  f0Start <- gldrmControl$f0Start

  ## Tabulation and summary of responses used in estimating f0
  n <- length(y)
  spt <- sort(unique(y)) # observed support
  ySptIndex <- match(y, spt) # index of each y value within support
  # sptFreq <- table(ySptIndex)
  # attributes(sptFreq) <- NULL

  # create a weight matrix for score.logT1 calculation
  weightsMatrix <- matrix(0, nrow=n, ncol=length(spt))
```

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weightsMatrix[cbind(1:n, ySptIndex)] <- weights

# weighted version of "sptFreq"
sptFreq.weighted <- colSums(weightsMatrix)

## Initialize offset
if (is.null(offset))
  offset <- rep(0, n)

## Initialize mu0 if not provided by user
if (is.null(mu0)) {
  mu0 <- weighted.mean(y, weights)
} else if (mu0 <= min(spt) || mu0 >= max(spt)) {
  stop(paste0("mu0 must lie within the range of observed values. Choose a different ",
    "value or set mu0=NULL to use the default value, weighted.mean(y,weights)."))
}

## Initialize f0
if (is.null(f0Start)) {
  # weighted version of initial value for baseline distribution
  f0 <- sptFreq.weighted / sum(sptFreq.weighted)
  if (mu0 != weighted.mean(y, weights))
    f0 <- getTheta(spt=spt, f0=f0, mu=mu0, weights=weights, ySptIndex=1, thetaStart=0,
      thetaControl=thetaControl)$fTilt[, 1]
} else {
  if (length(f0Start) != length(spt))
    stop("Length of f0Start should equal number of unique values in the response.")
  if (any(f0Start <= 0))
    stop("All values in f0Start should be strictly positive.")
  f0 <- f0Start / sum(f0Start)
  f0 <- getTheta(spt=spt, f0=f0, mu=mu0, weights=weights, ySptIndex=1, thetaStart=0,
    thetaControl=thetaControl)$fTilt[, 1]
}

## Initialize beta
## The starting values returned by lm.fit guarantee that all mu values are
## within the support range, even if there is no intercept.
## Offset could still create problems.
lmcoef <- stats::lm.wfit(x=x, y=linkfun(mu0) - offset, weights)$coef
if (is.null(betaStart)) {
  beta <- lmcoef
} else {
  if (length(betaStart) != ncol(x))
    stop("Length of betaStart should equal the number of columns in the model matrix.")
  beta <- betaStart
}

## Drop coefficients if x is not full rank (add NA values back at the end)
naID <- is.na(lmcoef)
beta <- beta[!naID]
x <- x[, !naID, drop=FALSE]
eta <- c(x %*% beta + offset)
mu <- linkinv(eta)
if (ncol(x) >= n)
  stop("gldrm requires n > p.")
if (any(mu < min(spt) | mu > max(spt)))
  stop("Unable to find beta starting values that do not violate convex hull condition.")

## Get initial theta and log likelihood

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```

th <- getTheta(spt=spt, f0=f0, mu=mu, weights=weights, ySptIndex=ySptIndex,
               thetaStart=NULL, thetaControl=thetaControl)
llik <- th$llik

conv <- FALSE
iter <- 0
while (!conv && iter <= maxiter)
{
  iter <- iter+1
  betaold <- beta
  f0old <- f0
  llikold <- llik

  ## update beta (mu) and theta, with fixed f0:
  bb <- getBeta(x=x, y=y, spt=spt, ySptIndex=ySptIndex, f0=f0,
               linkinv=linkinv, mu.eta=mu.eta, offset=offset, weights=weights,
               betaStart=beta, thStart=th,
               thetaControl=thetaControl, betaControl=betaControl)

  th <- bb$th
  llik <- bb$llik
  mu <- bb$mu
  beta <- bb$beta

  ## update f0 and theta, with fixed beta (mu)
  ff <- getf0(y=y, spt=spt, ySptIndex=ySptIndex,
              weights=weights, sptFreq.weighted=sptFreq.weighted, mu=mu, mu0=mu0, f0Start=f0, thSta
              thetaControl=thetaControl, f0Control=f0Control, trace=FALSE)

  th <- ff$th
  llik <- ff$llik
  f0 <- ff$f0

  ## Check convergence
  del <- abs((llik - llikold) / llik)
  if (llik == 0) del <- 0
  conv <- del < eps

  if (print) {
    cat("iteration ", iter,
        "\nrelative change in log-likelihood = ", del,
        " (eps = ", eps, ")\n")
  }
}

## Final values
eta <- linkfun(mu)
dmudeta <- mu.eta(eta)
llik <- ff$llik
theta <- th$theta
bPrime <- th$bPrime
bPrime2 <- th$bPrime2
fTilt <- th$fTilt[cbind(ySptIndex, seq_along(ySptIndex))]]

## Add NA values back into beta vector and varbeta if covariate matrix is not full rank
nBeta <- length(beta) + sum(naID)
betaTemp <- rep(NA, nBeta)
betaTemp[!naID] <- beta
beta <- betaTemp

```

```

## Return gldrm object
attributes(beta) <- NULL
attributes(f0) <- NULL

fit <- list(conv=conv, iter=iter, llik=llik,
            beta=beta, mu=mu, eta=eta, f0=f0, spt=spt, mu0=mu0,
            theta=theta, bPrime=bPrime, bPrime2=bPrime2, fTilt=fTilt, weights=weights)

if (returnfTiltMatrix)
  fit$fTiltMatrix <- t(th$fTilt)

if (returnf0ScoreInfo) {
  fit$score.logf0 <- ff$score.log
  fit$info.logf0 <- ff$info.log
}

class(fit) <- "gldrm"
fit
}
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```

File defined by 1, 4, 6, 10.

"../R/wgldrm.R" 4≡

```

#' Beta optimization routing
#'
#' @param x Covariate matrix.
#' @param y Response vector.
#' @param spt Vector of unique observed support points in the response.
#' @param ySptIndex Index of each \code{y} value within the \code{spt} vector.
#' @param f0 Current values of f0.
#' @param linkinv Inverse link function.
#' @param mu.eta Derivative of inverse link function.
#' @param offset Vector of known offset values to be added to the linear
#' combination (x' beta) for each observation. Mostly intended for likelihood ratio
#' and score confidence intervals.
#' @param sampprobs Optional matrix of sampling probabilities.
#' @param betaStart Starting values for beta (typically the estimates from the
#' previous iteration).
#' @param thStart Starting theta values. Needs to be a list of values matching
#' the output of the \code{getTheta} function.
#' @param thetaControl A "thetaControl" object returned from the \code{theta.control}
#' function.
#' @param betaControl A "betaControl" object returned from the \code{beta.control}
#' function.
#'
#' @return A list containing the following:
#' \itemize{
#' \item \code{beta} Updated values.
#' \item \code{mu} Updated mean for each observation.
#' \item \code{th} Updated list returned from the \code{getTheta} function.
#' \item \code{llik} Updated log-likelihood.
#' \item \code{iter} Number of iterations until convergence. (Will always be
#' one unless \code{maxiter} is increased to something greater than one using the
#' \code{betaControl} argument.)
#' \item \code{conv} Convergence indicator. (Will always be FALSE unless
#' \code{maxiter} is increased to something greater than one using the
#' \code{betaControl} argument.)

```

```

#' }
#'
#' @keywords internal
beta.control <- function (eps = 1e-10, maxiter = 1, maxhalf = 10)
{
  betaControl <- as.list(environment())
  class(betaControl) <- "betaControl"
  betaControl
}

getBeta <- function(x, y, spt, ySptIndex, f0, linkinv, mu.eta, offset, weights,
  betaStart, thStart,
  thetaControl=theta.control(), betaControl=beta.control())
{
  ## Extract control arguments
  if (class(betaControl) != "betaControl")
    stop("betaControl must be an object of class betaControl returned by betaControl() function.")
  eps <- betaControl$eps
  maxiter <- betaControl$maxiter
  maxhalf <- betaControl$maxhalf

  sptMin <- min(spt)
  sptMax <- max(spt)
  beta <- betaStart
  th <- thStart
  llik <- th$llik

  conv <- FALSE
  maxhalfreached <- FALSE
  iter <- 0
  while (!conv && !maxhalfreached && iter < maxiter)
  {
    iter <- iter+1

    ## Update mean vector and related quantities
    eta <- c(x %*% beta + offset)
    mu <- linkinv(eta)
    dmudeta <- mu.eta(eta)
    betaold <- beta
    muold <- mu
    thold <- th
    llikold <- llik

    ## Compute weighted least squares update
    w <- weights * dmudeta^2 / th$bPrime2
    ymm <- y - mu
    r <- ymm / dmudeta

    yeqmu <- which(abs(ymm) < 1e-15)
    w[yeqmu] <- 0 # prevent 0/0
    r[yeqmu] <- 0 # prevent 0/0

    if (any(w==Inf)) break

    betastep <- unname(coef(lm.wfit(x, r, w)))
    betastep[is.na(betastep)] <- 0

    ## Let q = b''*(theta) / b''(theta)
    ## W = diag{dmudeta^2 / b''(theta) * q}

```

```

## r = (y - b'*(theta)) / (q * dmudeta)
## We need to solve for beta such that I(betaHat) %*% beta = Score(betaHat),
## or equivalently, X'WX = X'Wr, or equivalently W^{1/2}X = W^{1/2}r.
## The linear system can be solved using qr.coef().

### Update beta and take half steps if log-likelihood does not improve
beta <- beta + betastep
eta <- c(x %*% beta + offset)
mu <- linkinv(eta)
if (min(mu)<sptMin || max(mu)>sptMax) {
  llik <- -Inf
} else {
  th <- getTheta(spt=spt, f0=f0, mu=mu, weights=weights, ySptIndex=ySptIndex,
    thetaStart=thold$theta, thetaControl=thetaControl)
  llik <- th$llik
}

nhalf <- 0
while ((llik<llikold) && (nhalf<maxhalf)) {
  nhalf <- nhalf + 1
  beta <- (beta + betaold) / 2
  eta <- c(x %*% beta + offset)
  mu <- linkinv(eta)
  if (min(mu)<sptMin || max(mu)>sptMax) {
    llik <- -Inf
  } else {
    th <- getTheta(spt=spt, f0=f0, mu=mu, weights=weights, ySptIndex=ySptIndex,
      thetaStart=thold$theta, thetaControl=thetaControl)
    llik <- th$llik
  }
}

if (llik < llikold) {
  beta <- betaold
  mu <- muold
  th <- thold
  llik <- llikold
  conv <- FALSE
  maxhalfreached <- TRUE
} else {
  del <- (llik - llikold) / llik
  if (llik == 0) del <- 0 # consider converged if model fit is perfect
  conv <- del < eps
}

}

return(list(beta=beta, mu=mu, th=th, llik=llik, iter=iter, conv=conv))
}

```

File defined by 1, 4, 6, 10.

"../R/wgldrm.R" 6≡

```

## Computes log(sum(exp(x))) with better precision
logSumExp <- function(x)
{
  i <- which.max(x)
  m <- x[i]

```

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    lse <- log1p(sum(exp(x[-i]-m))) + m
    lse
  }

## g function (logit transformation from appendix)
g <- function(mu, m, M) log(mu-m) - log(M-mu)

#' Control arguments for \eqn{\theta} update algorithm
#'
#' This function returns control arguments for the \eqn{\theta} update algorithm.
#' Each argument has a default value, which will be used unless a different
#' value is provided by the user.
#'
#' @param eps Convergence threshold for theta updates. Convergence is
#' evaluated separately for each observation. An observation has converged when
#' the difference between \eqn{b'(\theta)} and \eqn{\mu} is less than \code{epsTheta}.
#' @param maxiter Maximum number of iterations.
#' @param maxhalf Maximum number of half steps allowed per iteration if the
#' convergence criterion does not improve.
#' @param maxtheta Absolute value of theta is not allowed to exceed \code{maxtheta}.
#' @param logit Logical for whether logit transformation should be used. Use of
#' this stabilizing transformation appears to be faster in general. Default is TRUE.
#' @param logsumexp Logical argument for whether log-sum-exp trick should be used.
#' This may improve numerical stability at the expense of computational time.
#'
#' @return Object of S3 class "thetaControl", which is a list of control arguments.
#'
#' @keywords internal
theta.control <- function(eps=1e-10, maxiter=100, maxhalf=20, maxtheta=500,
                          logit=TRUE, logsumexp=FALSE)
{
  thetaControl <- as.list(environment())
  class(thetaControl) <- "thetaControl"
  thetaControl
}

#' getTheta
#' Updates theta. Vectorized but only updates observations that have not converged.
#'
#' @param spt Support of the observed response variable. (This is the set of
#' unique values observed, not the set of all possible values.)
#' @param f0 Values of the baseline distribution corresponding to the values of spt
#' @param mu The fitted mean for each observation. Note these values must lie
#' strictly within the range of the support.
#' @param sampprobs Matrix of sampling probabilities. The number of rows should
#' equal the number of observations, and the number of columns should equal
#' the number of unique observed support points.
#' @param ySptIndex Vector containing index of each observation's response value
#' within the \code{spt} vector. This is only needed to calculate the log-likelihood
#' after each update.
#' @param thetaStart Vector of starting values. One value per observation. If
#' \code{NULL}, zero is used as the starting value for each observation.
#' @param thetaControl Object of class \code{thetaControl}, which is a list of
#' control arguments returned by the \code{thetaControl} function.
#'
#' @return List containing the following:
#' \itemize{
#' \item \code{theta} Updated values.
#' \item \code{fTilt} Matrix containing the exponentially tilted distribution for each

```

```

#' observation, i.e.  $f(y|X=x)$ . Each column corresponds to an observation and sums to one.
#' \item \code{bPrime} Vector containing the mean of the exponentially tilted distribution
#' for each observation. Should match \code{mu} argument very closely.
#' \item \code{bPrime2} Vector containing the variance of the exponentially tilted
#' distribution for each observation.
#' matches \code{bPrime2}.
#' \item \code{llik} Semiparametric log-likelihood, evaluated at the current beta
#' and f0 values. If sampling weights are used, then the log-likelihood is conditional
#' on each observation being sampled.
#' \item \code{conv} Convergence indicator.
#' \item \code{iter} Number of iterations until convergence was reached.
#' }
#'
#' @keywords internal
getTheta <- function(spt, f0, mu, weights, ySptIndex, thetaStart=NULL, thetaControl=theta.control())
{
  ## Extract control arguments
  if (class(thetaControl) != "thetaControl")
    stop("thetaControl must be an object of class \'thetaControl\' returned by
         thetaControl() function.")
  logit <- thetaControl$logit
  eps <- thetaControl$eps
  maxiter <- thetaControl$maxiter
  maxhalf <- thetaControl$maxhalf
  maxtheta <- thetaControl$maxtheta
  logsumexp <- thetaControl$logsumexp

  ## Define value from inputs
  sptN <- length(spt)
  m <- min(spt)
  M <- max(spt)
  n <- length(mu)

  ## Format arguments
  spt <- as.vector(spt)
  f0 <- as.vector(f0)
  mu <- as.vector(mu)
  if (!is.null(thetaStart)) {
    thetaStart <- as.vector(thetaStart)
  } else {
    thetaStart <- rep(0, n)
  }

  ## Argument checks
  if (length(f0) != sptN)
    stop("spt and f0 must be vectors of equal length.")
  if (any(f0 < 0))
    stop("f0 values cannot be negative.")
  if (min(mu)<m || max(mu)>M)
    stop("mu starting values must lie within the range of spt.")
  if (length(thetaStart) != n)
    stop("thetaStart must be a vector with length equal length(mu)")

  ## Value does not change
  gMu <- g(mu, m, M)

  ## Initialize values
  theta <- thetaStart # initial values required
  thetaOld <- bPrimeErrOld <- rep(NA, n)

```



```

conv <- rep(FALSE, n)
maxedOut <- rep(FALSE, n)

if (logsumexp) {
  logf0 <- log(f0)
  logfUnstd <- logf0 + tcrossprod(spt, theta)
  logb <- apply(logfUnstd, 2, logSumExp)
  fTilt <- exp(logfUnstd - rep(logb, each=sptN))
  normfact <- colSums(fTilt)
  normss <- which(normfact != 1)
  fTilt[, normss] <- fTilt[, normss, drop=FALSE] / rep(normfact[normss], each=sptN)
} else {
  fUnstd <- f0 * exp(tcrossprod(spt, theta)) # |spt| x n matrix of tilted f0 values
  b <- colSums(fUnstd)
  fTilt <- fUnstd / rep(b, each=sptN) # normalized
}

bPrime <- colSums(spt*fTilt) # mean as a function of theta
bPrime2 <- colSums(outer(spt, bPrime, "-")^2 * fTilt) # variance as a function of theta
bPrimeErr <- bPrime - mu # used to assess convergence

## Update theta until convergence
conv <- (abs(bPrimeErr) < eps) |
  (theta==maxtheta & bPrimeErr<0) |
  (theta==maxtheta & bPrimeErr>0)
s <- which(!conv)
iter <- 0
while(length(s)>0 && iter<maxiter) {
  iter <- iter + 1
  bPrimeErrOld[s] <- bPrimeErr[s] # used to assess convergence

  ## 1) Update theta
  thetaOld[s] <- theta[s]
  if (logit) {
    tPrimeS <- (M-m) / ((bPrime[s]-m) * (M-bPrime[s])) * bPrime2[s]
    # t'(theta) from paper: temporary variable
    # only needed for the subset of observations that have not converged
    tPrimeS[is.na(tPrimeS) | tPrimeS==Inf] <- 0
    # If bPrime is on the boundary, then bPrime2 should be zero.
    # Exceptions are due to rounding error.
    thetaS <- theta[s] - (g(bPrime[s], m, M) - gMu[s]) / tPrimeS
  } else {
    thetaS <- theta[s] - bPrimeErr[s] / bPrime2[s]
  }
  thetaS[thetaS > maxtheta] <- maxtheta
  thetaS[thetaS < -maxtheta] <- -maxtheta
  theta[s] <- thetaS

  ## 2) Update fTilt, bPrime, and bPrime2 and take half steps if bPrimeErr not improved
  ss <- s
  nhalf <- 0
  while(length(ss)>0 && nhalf<maxhalf) {
    ## 2a) Update fTilt, bPrime, and bPrime2
    if (logsumexp) {
      logfUnstd[, ss] <- logf0 + tcrossprod(spt, theta[ss])
      logb[ss] <- apply(logfUnstd[, ss, drop=FALSE], 2, logSumExp)
      fTilt[, ss] <- exp(logfUnstd[, ss, drop=FALSE] - rep(logb[ss], each=sptN))
      normfact <- colSums(fTilt[, ss, drop=FALSE])
      normss <- which(normfact != 1)
      fTilt[, ss[normss]] <- fTilt[, ss[normss], drop=FALSE] / rep(normfact[normss], each=sptN)
    }
  }
}

```

```

    } else {
      fUnstd[, ss] <- f0*exp(tcrossprod(spt, theta[ss])) # |spt| x n matrix of tilted f0 values
      b[ss] <- colSums(fUnstd[, ss, drop=FALSE])
      fTilt[, ss] <- fUnstd[, ss, drop=FALSE] / rep(b[ss], each=sptN) # normalized
    }
    bPrime[ss] <- colSums(spt*fTilt[, ss, drop=FALSE]) # mean as a function of theta
    bPrime2[ss] <- colSums(outer(spt, bPrime[ss], "-")^2 * fTilt[, ss, drop=FALSE]) # variance a
    bPrimeErr[ss] <- bPrime[ss] - mu[ss] # used to assess convergence

    ## 2b) Take half steps if necessary
    ss <- ss[abs(bPrimeErr[ss]) > abs(bPrimeErrOld[ss])]
    if (length(ss) > 0) nhalf <- nhalf + 1
    theta[ss] <- (theta[ss] + thetaOld[ss]) / 2
  }
  ## If maximum half steps are exceeded, set theta to previous value
  maxedOut[ss] <- TRUE
  theta[ss] <- thetaOld[ss]

  ## 3) Check convergence
  conv[s] <- (abs(bPrimeErr[s]) < eps) |
    (theta[s]==maxtheta & bPrimeErr[s]<0) |
    (theta[s]==-maxtheta & bPrimeErr[s]>0)
  s <- s[!conv[s] & !maxedOut[s]]
}

## Calculate log-likelihood
fTilt.extracted <- fTilt[cbind(ySptIndex, seq_along(ySptIndex))]
llik <- sum(weights[fTilt.extracted>0] * log(fTilt.extracted[fTilt.extracted>0]))

list(theta=theta, fTilt=fTilt, bPrime=bPrime, bPrime2=bPrime2,
      llik=llik, conv=conv, iter=iter)
}

```

File defined by 1, 4, 6, 10.

"../R/wgldrm.R" 10≡

```

#' Control arguments for f0 update algorithm
#'
#' This function returns control arguments for the \eqn{f_0} update algorithm.
#' Each argument has a default value, which will be used unless a different
#' value is provided by the user.
#'
#' @param eps Convergence threshold. The update has converged when the relative
#' change in log-likelihood between iterations is less than \code{eps}.
#' absolute change is less than \code{thesh}.
#' @param maxiter Maximum number of iterations allowed.
#' @param maxhalf Maximum number of half steps allowed per iteration if
#' log-likelihood does not improve between iterations.
#' @param maxlogstep Maximum optimization step size allowed on the
#' \code{log(f0)} scale.
#'
#' @return Object of S3 class "f0Control", which is a list of control arguments.
#'
#' @keywords internal
f0.control <- function(eps=1e-10, maxiter=1000, maxhalf=20, maxlogstep=2)

```

```

{
  f0Control <- as.list(environment())
  class(f0Control) <- "f0Control"
  f0Control
}

#' f0 optimization routine
#'
#' @param y Vector of response values.
#' @param spt Vector of unique observed support points in the response.
#' @param ySptIndex Index of each \code{y} value within \code{spt}.
#' @param sptFreq.weighted Vector containing weighted frequency of each \code{spt} value.
#' @param mu Fitted mean for each observation. Only used if \code{sampprobs=NULL}.
#' @param mu0 Mean constraing for f0.
#' @param f0Start Starting f0 values. (Typically the estimate from the previous
#' iteration.)
#' @param thStart Starting theta values. Needs to be a list of values matching
#' the output of the \code{getTheta} function.
#' @param thetaControl A "thetaControl" object returned from the \code{theta.control}
#' function.
#' @param f0Control An "f0Control" object returned from the \code{f0.control}
#' function.
#' trace Logical. If TRUE, then progress is printed to terminal at each iteration.
#'
#' @return A list containing the following:
#' \itemize{
#' \item \code{f0} Updated values.
#' \item \code{llik} Updated log-likelihood.
#' \item \code{th} Updated list returned from the \code{getTheta} function.
#' \item \code{conv} Convergence indicator.
#' \item \code{iter} Number of iterations until convergence.
#' \item \code{nhalf} The number of half steps taken on the last iteration if the
#' initial BFGS update did not improve the log-likelihood.
#' \item \code{score.log} Score function with respect to log(f0) at convergence.
#' \item \code{info.log} Information matrix with respect to log(f0) at convergence.
#' }
#'
#' @keywords internal
getf0 <- function(y, spt, ySptIndex, weights, sptFreq.weighted, mu, mu0, f0Start, thStart,
  thetaControl=theta.control(), f0Control=f0.control(), trace=FALSE)
{
  ## Extract theta control arguments
  if (class(f0Control) != "f0Control")
    stop("f0Control must be an object of class f0Control returned by f0Control() function.")
  eps <- f0Control$eps
  maxiter <- f0Control$maxiter
  maxhalf <- f0Control$maxhalf
  maxlogstep <- f0Control$maxlogstep

  f0 <- f0Start # assumes sum(f0Start) = 1 and sum(f0Start * spt) = mu0
  th <- thStart
  llik <- th$llik
  score.log <- NULL
  smm <- outer(spt, mu, "-")
  ymm <- y - mu
  yeqmu <- which(abs(ymm) < 1e-15)

  conv <- FALSE

```

```

iter <- 0
while (!conv && iter<maxiter) {
  iter <- iter + 1

  # Score calculation
  score.logOld <- score.log

  fTiltSums <- rowSums(th$fTilt)
  fTiltSumsWeighted <- apply(th$fTilt, MARGIN=1, function(x) sum(weights * x))
  smmfTilt <- smm * th$fTilt
  ystd <- ymm / th$bPrime2
  ystdWeighted <- weights * ystd
  ystd[yeqmu] <- 0 # prevent 0/0
  ystdWeighted[yeqmu] <- 0 # prevent 0/0

  score.logT1 <- sptFreq.weighted
  score.logT2 <- fTiltSumsWeighted
  score.logT3 <- c(smmfTilt %*% ystdWeighted)
  score.log <- score.logT1 - score.logT2 - score.logT3

  # Inverse info, score step, and f0 step are on the log scale (score is not)
  if (iter == 1) {
    d1 <- min(fTiltSumsWeighted) # max inverse diagonal of first information term, on log scale
    d2 <- max(abs(score.log)) / maxlogstep
    d <- max(d1, d2)
    infoinvBFGS.log <- diag(1/d, nrow=length(f0))
  } else {
    scorestep.log <- score.log - score.logOld
    # f0step.log <- log(f0) - log(f0old)
    # to prevent the 0/0 situation
    ratiof0f0old <- f0 / f0old
    ratiof0f0old[is.na(ratiof0f0old)] <- 1
    f0step.log <- log(ratiof0f0old)

    sy <- sum(f0step.log * scorestep.log)
    yiy <- c(crossprod(scorestep.log, infoinvBFGS.log %*% scorestep.log))
    iys <- tcrossprod(infoinvBFGS.log %*% scorestep.log, f0step.log)
    infoinvBFGS.log <- infoinvBFGS.log + ((yiy - sy) / sy^2) * tcrossprod(f0step.log) - (1 / sy)
  }
  logstep <- c(infoinvBFGS.log %*% score.log)

  # Cap log(f0) step size
  logstep.max <- max(abs(logstep))
  if (logstep.max > maxlogstep)
    logstep <- logstep * (maxlogstep / logstep.max)

  # Save values from previous iteration
  f0old <- f0
  thold <- th
  llikold <- llik

  # Take update step
  f0 <- exp(log(f0) + logstep)
  # Scale and tilt f0
  f0 <- f0 / sum(f0)
  f0 <- getTheta(spt=spt, f0=f0, mu=mu0, weights=weights, ySptIndex=1,
                 thetaStart=0, thetaControl=thetaControl)$fTilt[, 1]
  # Update theta and likelihood

```

```

thold <- th
llikold <- llik
th <- getTheta(spt=spt, f0=f0, mu=mu, weights=weights, ySptIndex=ySptIndex,
               thetaStart=th$theta, thetaControl=thetaControl)
llik <- th$llik
conv <- abs((llik - llikold) / (llik + 1e-100)) < eps

# If log-likelihood does not improve, change step direction to be along gradient
# Take half steps until likelihood improves
# Continue taking half steps until log likelihood no longer improves
nhalf <- 0
if (llik < llikold) {
  llikprev <- -Inf
  while ((llik < llikold || llik > llikprev) && nhalf < maxhalf) {
    nhalf <- nhalf + 1

    # Set previous values
    llikprev <- llik
    thprev <- th
    f0prev <- f0
    infoinvBFGS.logprev <- infoinvBFGS.log

    f0 <- exp((log(f0) + log(f0old)) / 2)
    f0 <- f0 / sum(f0)
    f0 <- getTheta(spt=spt, f0=f0, mu=mu0, weights=weights, ySptIndex=1,
                  thetaStart=0, thetaControl=thetaControl)$fTilt[, 1]
    th <- getTheta(spt=spt, f0=f0, mu=mu, weights=weights, ySptIndex=ySptIndex,
                  thetaStart=th$theta, thetaControl=thetaControl)
    llik <- th$llik
    infoinvBFGS.log <- infoinvBFGS.log / 2
  }

  if (llik < llikprev) {
    nhalf <- nhalf - 1
    llik <- llikprev
    th <- thprev
    f0 <- f0prev
    infoinvBFGS.log <- infoinvBFGS.logprev
  }

  conv <- abs((llik - llikold) / (llik + 1e-100)) < eps
}

if (llik < llikold) {
  f0 <- f0old
  th <- thold
  llik <- llikold
  conv <- TRUE
}

if (trace) {
  printout <- paste0("iter ", iter, ": llik=", llik)
  if (nhalf > 0)
    printout <- paste0(printout, "; ", nhalf, " half steps")
  cat(printout, "\n")
}

# Final score calculation

```

```

    smm <- outer(spt, th$bPrime, "-")
    ymm <- y - th$bPrime
    yeqmu <- which(abs(ymm) < 1e-15)

    fTiltSums <- rowSums(th$fTilt)
    fTiltSumsWeighted <- apply(th$fTilt, MARGIN=1, function(x) sum(weights * x))
    smmfTilt <- smm * th$fTilt
    ystd <- ymm / th$bPrime2
    ystdWeighted <- weights * ystd
    ystd[yeqmu] <- 0 # prevent 0/0
    ystdWeighted[yeqmu] <- 0 # prevent 0/0
    ystd[yeqmu] <- 0 # prevent 0/0

    score.logT1 <- sptFreq.weighted
    score.logT2 <- fTiltSumsWeighted
    score.logT3 <- c(smmfTilt %*% ystdWeighted)
    score.log <- score.logT1 - score.logT2 - score.logT3

    # Final info calculation
    info.logT1 <- diag(fTiltSumsWeighted)
    info.logT2 <- tcrossprod(th$fTilt)
    info.logT3 <- tcrossprod(smmfTilt, smmfTilt * rep(ystdWeighted, each=nrow(smmfTilt)))
    info.log <- info.logT1 - info.logT2 - info.logT3

    list(f0=f0, llik=llik, th=th, conv=conv, iter=iter, nhalf=nhalf,
         score.log=score.log, info.log=info.log)
  }
  ◇

```

File defined by [1](#), [4](#), [6](#), [10](#).