

## 1: Probability Density Function (PDF)-

A **Probability Density Function (PDF)** is a function that tells us **how likely a continuous value is within a range**.

Key points:

- Used for **continuous data**
- Probability of an **exact value is zero**
- Probability is calculated over a **range of values**
- Probability = **area under the curve**

You **cannot say**: Probability that a person is exactly 170.235 cm

That probability = **0**

You **can say**: Probability that a person's height is between **165 cm and 175 cm**

This probability is: The **area under the PDF curve** between 165 and 175

## 2. Normal Distribution:

A **Normal Distribution** is a **continuous probability distribution** where:

- Data is **symmetrical**
- Most values are clustered around the **mean**
- The shape looks like a **bell**

In a normal distribution:

- Mean = Median = Mode

Example: **Exam scores**

- Most students score around the average
- Very low and very high scores are fewer

Suppose:

- Average exam score = 70
- Standard deviation = 10

Then:

- 68% students score between **60 and 80**
- 95% students score between **50 and 90**

If data is normally distributed:

Range	Meaning
$\pm 1$ standard deviation	~68% of data
$\pm 2$ standard deviations	~95% of data
$\pm 3$ standard deviations	~99.7% of data

**One-line intuition: Normal distribution describes natural variation around an average**

### 3. Bernoulli Distribution

A **Bernoulli Distribution** models an experiment that has **only two possible outcomes**:

- Success (1)
- Failure (0)

There is **only one trial**

#### Key Parameter

- $p$  = probability of success
- Probability of failure =  $1 - p$

Example: **Coin toss**

- Head  $\rightarrow$  success
- Tail  $\rightarrow$  failure

#### Important Properties

- Only **one trial**
- Only **two outcomes**
- Used as a **building block** for other distributions

## 4. Binomial Distribution

A **Binomial Distribution** models the **number of successes** in a **fixed number of independent Bernoulli trials**.

In short: Many yes/no experiments counted together

**Conditions (must all be true)**

1. Fixed number of trials (**n**)
2. Each trial has two outcomes (success/failure)
3. Probability of success (**p**) is constant
4. Trials are independent

Example: **Coin tosses**

- Toss a coin 10 times
- Count how many heads appear

## 5. Uniform Distribution

A **Uniform Distribution** is a probability distribution where **all outcomes are equally likely**.

There is **no bias** toward any value in the given range.

**Key Idea** : Every value has the **same probability**.

### Types

1. **Discrete Uniform Distribution**
2. **Continuous Uniform Distribution**

Examples: **1. Discrete Uniform**

- Rolling a fair die  
Outcomes: {1, 2, 3, 4, 5, 6}  
Probability of each =  $1/6$

**2. Continuous Uniform**

- Random number between 0 and 1  
Every value in this range is equally likely

## 6. Student's $t$ Distribution

The **Student's  $t$  distribution** is a **continuous probability distribution** that is used when:

- The **sample size is small**
- The **population standard deviation is unknown**
- Data is approximately **normally distributed**

It looks like a **normal distribution**, but with **fatter (heavier) tails**.

### Why do we need it?

When the sample size is small, estimates are **less certain**.

The  $t$  distribution accounts for this extra uncertainty.

<b>Normal Distribution</b>	<b><math>t</math> Distribution</b>
Large sample size	Small sample size
Population $\sigma$ known	Population $\sigma$ unknown
Thin tails	Thick tails

As sample size increases:  **$t$  distribution**  $\rightarrow$  **normal distribution**

### Example: Small survey

- Average salary from 15 people
- Use  $t$ , not normal

**One-line intuition:  $t$  distribution is a safer version of normal distribution for small samples**

## 7. Poisson Distribution

A **Poisson Distribution** models the **number of times an event occurs** in a **fixed interval of time or space**, when:

- Events happen **independently**
- Average rate of occurrence is **constant**
- Events are **rare** relative to the interval

**Key Parameter  $\lambda$  (lambda)** = average number of events per interval

**What kind of questions it answers** :How many times will something happen in a given period?

Example: **Call center**

- Average 5 calls per minute
- Probability of receiving exactly 3 calls in the next minute

## Simple Example

Suppose:

- On average, a customer service desk receives **2 complaints per hour**

Question:

What is the probability of receiving **exactly 1 complaint** in the next hour?

This situation follows a **Poisson Distribution**.

## Important Properties

- Used for **counts**, not measurements
- Interval must be fixed (time/area/volume)
- Mean = Variance =  $\lambda$

## Final Big Picture (Very Short)

- **Bernoulli** → One yes/no event
- **Binomial** → Count of yes/no events
- **Uniform** → All outcomes equal
- **Normal** → Natural variation
- **t** → Normal with small data
- **Poisson** → Event counts over time