Chapter 4
Discrete System Simulation

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Discrete Event Simulation

- Discrete system is a system in which the changes are discontinuous.
- Each change in the state of system is called as an event.
- For Example, arrival or departure of a customer in a queue is an event.
- The simulation of a discrete system is referred to as discrete event simulation.

- Applications of discrete event simulation:
  - It is used by operation research workers to study large complex systems which cannot be studied by analytical method.
  - In inventory control.
  - Study of sea and airport.
  - Steel melting stuffs.
  - Telephone exchanges.
  - Production line.
  - Project scheduling etc.

Representation of time:

- There are two different models for moving a discrete system through time.
  - Fixed time step model or interval oriented model
  - Event to event(next event) model or event oriented model

Fixed time step model v/s next event model:

- In a fixed time step model a timer or clock is simulated by the computer.
- This clock is updated by a fixed time interval Ϫ and the system is examine to see if any event has taken place during this time interval.

- All events that place during this period are treated as if they occur simultaneously at the tail end of this interval.
- In the next event simulation model, the computer advances time to the occurrence of the next event. It shifts from event to event
- The system does not change in between only those points in time are kept track when something of interest happens to the system.

- Ex: To illustrate the difference between the two modules, let us assume that we are simulating the dynamics of population in a fish bowl staring with 10 fishes. If the fixed time step model is used with the interval “ τ= 1 day” then the fish bowl would see once every 24 hrs and any births and deaths that take place are presume to be during the last moment of this period. But the next event model is used, then we first find out when the next event birth and death is to be take place then advance the clock exactly to that time. The next event model is preferred because it does not waste any computed time is scanning those points in time when nothing take place. The only drawback of the next event model is that its implementation is more complicated than the fixed time step model.

- Another approach to represent the passage of time is called significant event simulation. • This method is applicable to continuous system in which there are quiescent periods. The interval between the events in an event oriented approach is a quiescent period. • This approach assumes that simple analytical function can be used to protect the span of a quiescent period. • The significant event is one with the least span. • Determining this event by simple comparisons of the projections allows the clock to be updated by an extended period of time.

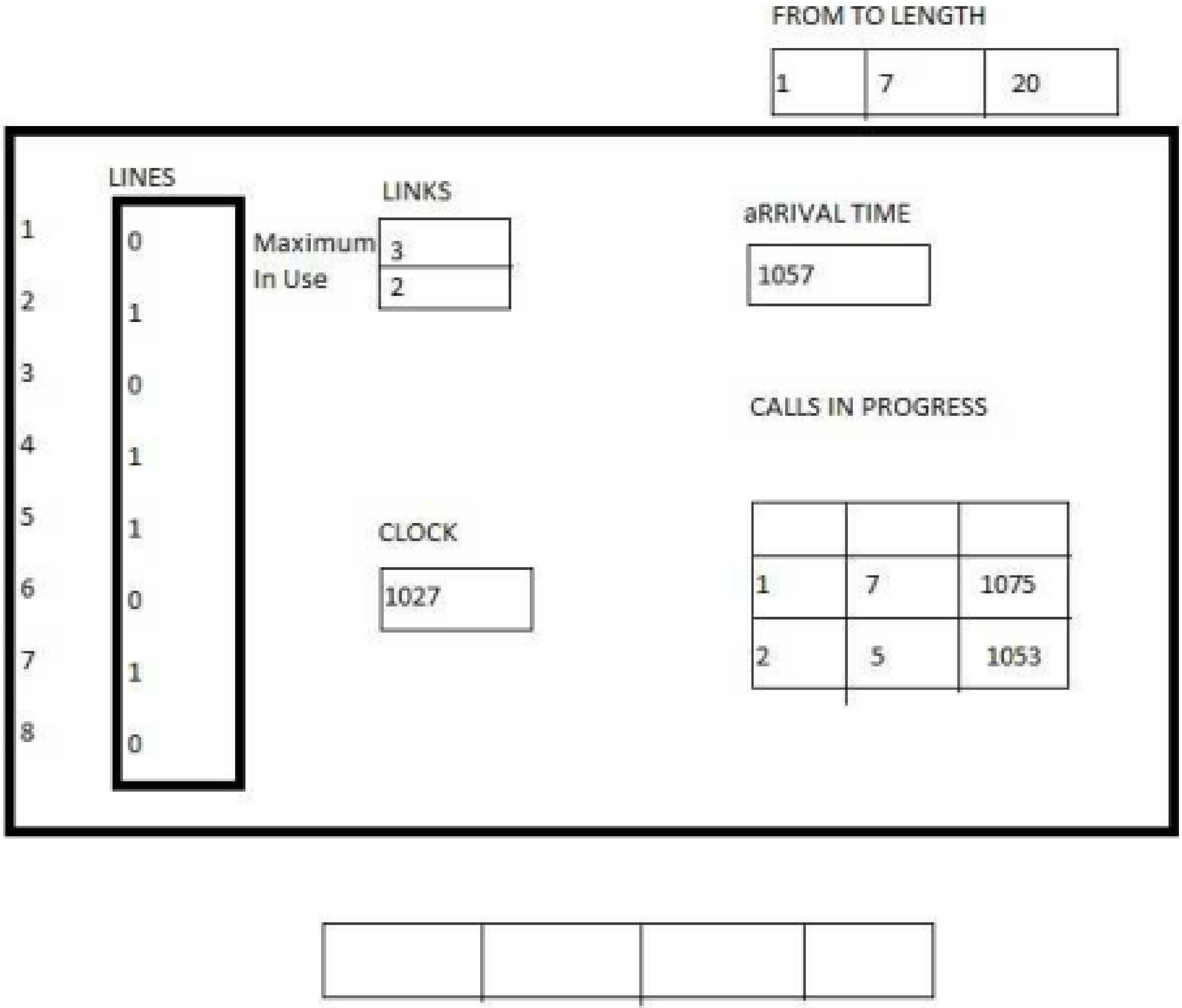
Generation of Arrival Patterns

- Trace driven simulation:
  - Here the sequence of inputs are generated from observations of a running system.
  - Programs monitors are attached to the running system to extract the data with little or no disturbances of the system operation.

- Bootstrapping:
- Here the arrival time of the next entity is immediately calculated from the inter-arrival time distribution.
- The term bootstrapping is used to describe this process of making one entity create its successor.
- The method requires keeping only the arrival time of the next entity.
- It is the most preferred method of generating arrival through computer simulation program.

Simulation of Telephone System

- The System has a number of Telephone connecting to a switch board by lines. A switch has a number of links which can be used to connect any two lines subject to the condition that only one connecting at a time can be made to each line. It assume that it lost call system i.e. any call that can be connected at time immediately it abandon. A call may be lost because no link is available it is called as a lost call. The object of the simulation is to process a given number of calls and determine what proportion are successfully completed block or busy. The current state of the system is that lne-2 is connected to line-5 and a line-4 is connected to line-7.Each line is treated as an entity and it's availability is an attribute a table of numbers is establish which show the current status of each line. A zero in the table means the line is free and where 1 means it is busy.


FROM TO LENGTH
1
7
20
LINES
1
2
3
4
5
6
7
8
0
1
0
1
1
0
1
0
LINKS
Maximum
In Use
3
2
aRRIVAL TIME
1057
CALLS IN PROGRESS
1
7
1075
2
5
1053
CLOCK
1027

- To keep track of event the number of representing currently the clock time 5 1027.
- The unit of time is taken to be in second. There is a list of call in progress showing which line call is connected and the time each call finishes.
- The arrival of call is generated by the boot strapping technique.
- There are 2 activities causing event.
    1. New calls can arrive
    2. Existing calls can finish

- There are 3-future events :
  - • The call between lines 2 and 5 is due to finish at time 1053.
  - • The call between line 4 and 7 is due to finish at time 1075.
  - • A new call is due to arrive at time 1057.

The simulation proceeds by exhibiting a cycle of state to simulate each event.

Steps:- 1. Find the next potential event.

2. Select an activity.

3. Test if the event can be executed.

4. Change the system event.

5. Gather statistics.

The simulation will be run until a given number of calls have been processed or until the simulation time has elapsed.

- Delayed Call
  - Suppose the telephone system is modified so that cannot be connected are not loosed instead the wait until they can be connected, they have called as delayed calls.
  - To keep records of these delayed calls a delayed call list is made.
  - When a call completely ends, it is necessary to check the delayed call list to see if a waiting call can be connected.

Gathering statistics

- The statistics of interested variables has to be gathered during simulation and the selection variables depend on the nature of our simulation studies.
- For example
  - Counts,
  - Summary measures,
  - Utilization, Occupancy
  - Transit times etc.
  - Counts: are number of entities or events. e.g. No. of devices, No. of links, Blocked call events etc.
  - Summary measures: Extreme values, Mean and standard deviations etc. For example Maximum links in telephone system.
  - Utilization: Fraction or percentage of time that some entities get engaged.
  - Occupancy: - Links available to use. - Fractions or percentage of a group of entities on average.
  - Distribution: Distribution of important values, such as waiting time in SSQM.
  - Transit time: Time taken for an entity to move from one part of the system to the other part. Fig. Gathering statistics during simulation

Counter and Summary statistics

– Counters are the basis for most statistics.

– Counters are used to accumulate totals or recorded current values of some level. In Telephone System Simulation (TSS),

Examples:

(a) Maxima/Minima

– Whenever new value is generated, it can be compared with the maxima or minima. If the generated value is new maxima/minima, the previous record have to be changed.

(b) Mean

$$Mean\ (m) = \frac{1}{N}\sum_{r=1}^{N} X_r$$

Where, N is the total number of observations and X_r is the r^{th} observed value.

(c) Standard deviation (from mean)

$$\sigma = \left[\frac{1}{N-1}\sum_{r=1}^{N}(m - X_r)^2\right]^{\frac{1}{2}}$$

Measuring utilization and occupancy

- Utilization is a statistics that gives a measure of the load on some entity (Variables).
- For example, in TSS what fraction of item i.e. individual line was engaged or busy during the simulation run.
- In single server queuing model, the server utilization i.e. the proportion of time the server is busy.

Let, a line becomes busy at time t_b and becomes free at time t_f where $t_f < t_b$, then the interval $t_f - t_b$ is added to a counter. By the end of simulation run, we can

calculate the utilization (U) using the ratio of accumulated total to the total time (T) i.e.

$$U = \frac{1}{T}\sum_{r=1}^{N}\left(t_f - t_b\right)_r$$

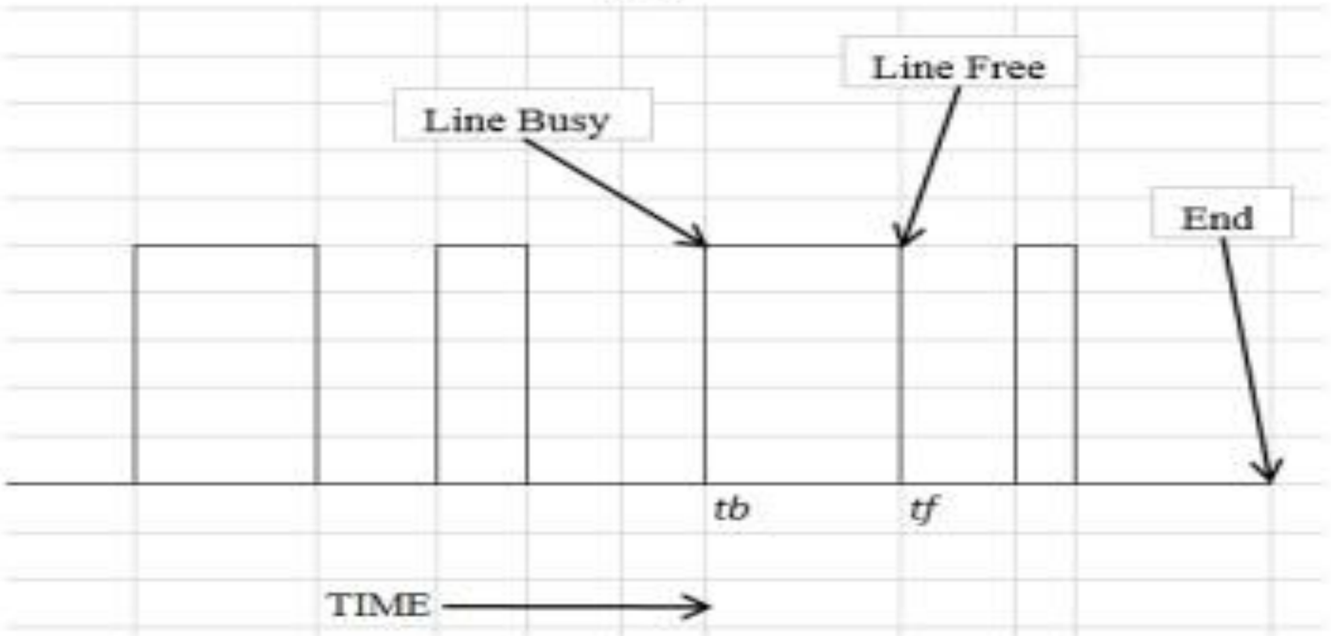


Fig. Utilization of a link in TSS

Furthermore, among m number of links, we may be interested to calculate the average number of links in use. To obtain this, we must have to keep records of currently busy links and the time when it get changed. If the busy number changes at time t_r to the value n_r then, at the time of the next change t_{r+1}, the quantity $n_r(t_{r+1} - t_r)$ must be calculated and added to the accumulated total. The average number in use (A) then calculated over the period T is calculated from:

$$A = \frac{1}{T}\sum_{r=1}^{N} n_r\left(t_{r+1} - t_r\right)$$

Recording distribution and Transit

After recording simulation statistics, we may be interested in finding the distribution of concerned variables from their counts i.e. how many times the value of the variable falls within the specified class or intervals. To obtain this, we need to accumulate the count of the variables in predefined intervals. This is done by adding 1 to the count of its interval at every new observation.

Normally, some uniform intervals are defined and tabulated with the following specifications:

(a) The lower limit of tabulation
(b) The interval size
(c) The number of intervals

The above terms are illustrated in the following diagram.

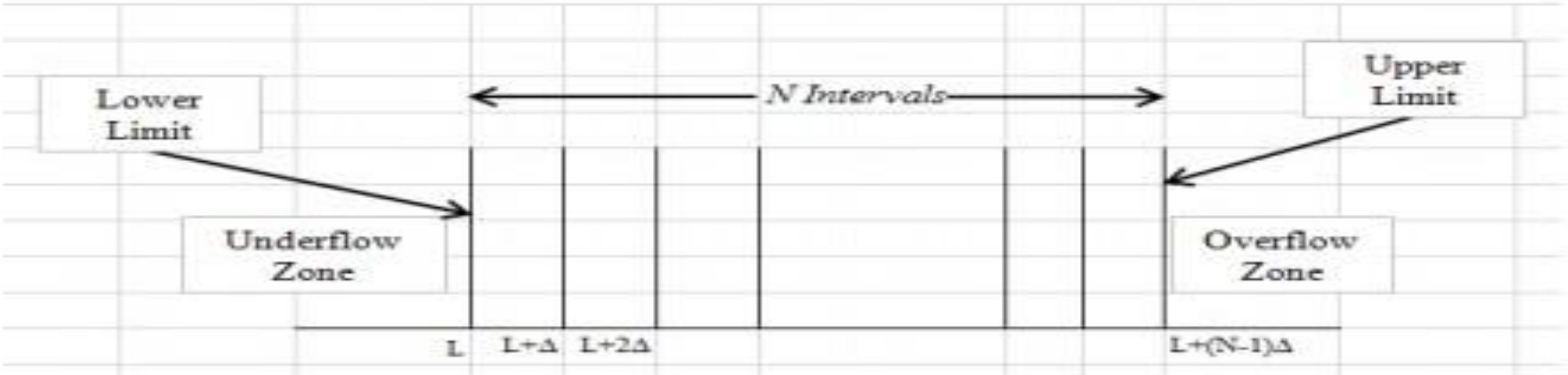


Fig. Definition of distribution table