KTH Computer Science
and Communication

Benchmarking Human Solving Methods for Rubik's cube

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DD143X - Bachelor Thesis
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Abstract

This is a skeleton for KTH theses. More documentation regarding the KTH thesis class file can be found in the package documentation.

Sammanfattning

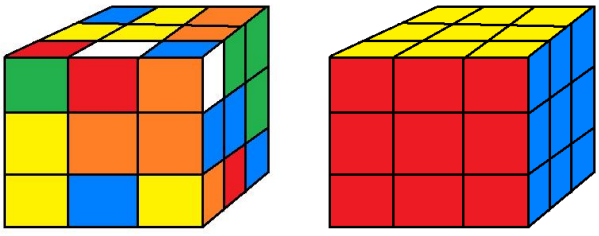
Denna fil ger ett avhandlingsskelett. Mer information om LaTeX-mallen finns i dokumentationen till paketet.

Contents

Chapter 1

Introduction

The Rubik's cube is an 3-D combination puzzle, where each side of the cube is covered with nine squares in six possible colours: white, red, blue, orange, green and yellow.
You start with a scrambled cube, meaning that the colored miniature cubes are randomly positioned by using random operations on the cube (fig 1.1a) .
The goal is to obtain the unique solution where each side of the cube are covered with only one colour per side (fig 1.1b). Different methods have been developed with series of notation to solve subproblems one at the time to reach the unique solution. The general idea which many methods are based on is to solve it one layer at the time.
If you were to randomly rotate the faces in an attempt to solve the cube, there is almost zero chance of achieving the solved state in your lifetime because of all the possible permutations of the cube. There are $4.3 * 10^{19}$ (or 43 quintillion) [5] different states. Assuming you get to a unique state every second it would take more than 130 billion years to test 10% of the cubes possible states [appendix A].
There are two major ways to compete in solving the Rubik's cube: the least amount of moves and solving the cube as fast as possible (speedcubing). The cube was invented by the professor of architecture Ernő Rubik as a teaching tool to help his student understand 3D objects. It was not until he scrambled his new cube and



(a) Scrambled cube (b) Solved cube

Figure 1.1

tried to restore it, that he realize that his creation was a puzzle.
The cube was originally called magic cube and was licensed to be sold by the american toy company Ideal Toy Company in 1980. [7].

1.1 Problem Definition

This thesis will explore two commonly known beginner methods for human solving of rubik's cubes to find the differences. Both methods are based on the idea to solve the cube one layer at the time which is the easiest way for the human mind to solve this kind of problem [6]. We will evaluate both methods operation variance, number of operations to solve, to find the most move-efficient of them both.

During the literature study there was no such comparison of solving-methods for 3x3x3 cube found. The information found about solving-methods for 3x3x3 cubes was tutorials of how to use them.

1.2 Problem Statement

Which of the beginner algorithms would be more effective for speedcubing?
Which of the beginner algorithms solves the cube with the least amount of moves?
Which beginner algorithm is easiest to learn and execute to someone inexperienced with the cube?

1.3 Purpose

The purpose of testing the methods is to find out which uses the least amount of moves and to find out which method suits speedcubing best by analyzing the usage of different operations. The reason the operations shows if the method is suitable for speedcubing is because some of the operations are more time-consuming than others. The inexperienced user will find this as a guideline as to which algorithm to start his/her journey towards solving the Rubik's cube.

1.4 Structure

The second section will introduce the reader to concepts necessary to understand the algorithms implemented and benchmarked. The third section will explain the methods used in this thesis. The fourth sections will explain the implementations made in detail and the difficulties.
The fifth section will be for presenting the results and the sixth section for discussion regarding the results. After that will there be a conclusions section that completes the circle of the thesis, answering the problem statements. Lastly the references

used to this thesis will be listed followed by appendix containing computations and graphs.

1.5 Terminology

Cubie a miniature cube

Scramble perfoming a amount of random operations on a solved cube to reach a non-solved state.

Layer contains one side of the cube and one row of the four neighbouring sides

Operation Rotating one layer of the cube

Notation a character that is a abbreviation of the operation-name.

Chapter 2

Background

Providing the reader with knowledge of how the cube works and operations that you can perform on it as well as the algorithms that are in focus for this thesis.

2.1 Competitions

There are two types of competitions regarding the cube.

2.1.1 Speedcubing

When competing in an official event regulated by the World Cube Association (WCA), the competitor has at maximum 15 seconds of inspection time of the cube before the solve begins.[9] The time stops when the competitor have reached the unique solution.

2.1.2 Fewest moves

The competitors have 60 minutes without any inspection time and the competitor should also be able to hand in a written solution with the notations used in the correct format [9].

2.2 Rubik's Cube

Explanation of how the cube is constructed and the different notations for the operations.

2.2.1 Description

The cube consists of 26 cubies with three,two or one visible sides depending on the type of cubie. There is one core piece consisting of three axes which holds the center pieces together [8]. The corner and edge pieces (fig 2.1) are the cubies that

are movable to different edge and corner positions, the center pieces (fig 2.1) can only be moved according to the axis.

2.2.2 Notation

The notation describes the different move operations on the cube. This thesis uses the notation used by WCA. Explained below:
Clockwise 90 degrees:
F - Front face
B - Back face
R - Right face
L - Left face
U - Upper face
D - Bottom face
To denote the anti-clockwise 90 degrees rotation just put a single citation mark (‘) after the letter. For example F’ - move front face anti-clockwise 90 degrees.[10] To denote clockwise 180 degrees rotation just put (2) after the letters described above.

2.3 Algorithms

The algorithms are physically methods for solving the cube by hand but matches the mathematical definition of an algorithm. Unlike many of the “near-optimal solvers” (some of simulates several moves ahead to find the optimal move[4]), these algorithms can with little effort be taught to humans.

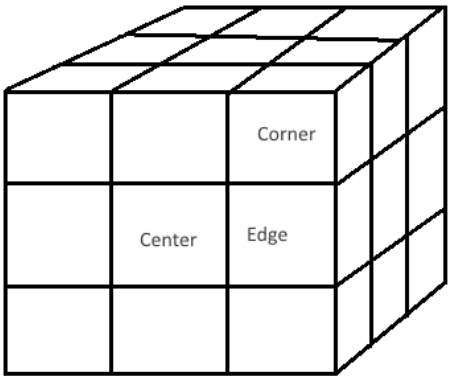
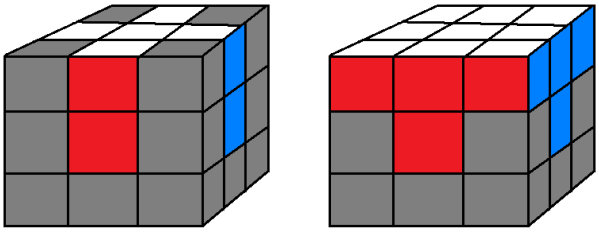


Figure 2.1: Cubie types



(a) The white cross done (b) First layer complete

Figure 2.2

2.3.1 Layer-by-layer using daisy method

The layer-by-layer (LBL) algorithms divides the cube into layers and makes it possible to solve the subproblems without breaking any pieces already made. The daisy method is to solve the white cross by first make a cross with white edges and a yellow center and then turn the white edges, completing the white cross in the bottom [2].

1. White cross

The goal here is to achieve a white cross, so that the white center-piece is aligned with its 4 white edge-pieces in the bottom. For it to be a completed step, the second color of the white edge-pieces must also align with it's center-piece counterpart as shown in fig 2.2a. This is done by first making a cross on the top with the yellow center-piece and then flip the edges over to the bottom one at the time using an operation, to make sure the edge-pieces on the vertical sides are aligned with the center-pieces.

2. White corners

With the white cross done the next step is to complete the first layer by positioning the white corner-pieces correct between the cross edges (fig 2.2b). This is achieved with three different operation-combinations depending on the colour positions, all focusing on the upper-front-left corner of the cube.

3. Middle layer edges

The next step is to solve the middle layer by moving down a cubie from the middle of the third layer to the correct position on the second layer (fig 2.3a).

4. Yellow cross

With the second layer complete (fig 2.3b) it is time to work on the yellow cross. This is achieved by applying one of two operation-combinations or both depending

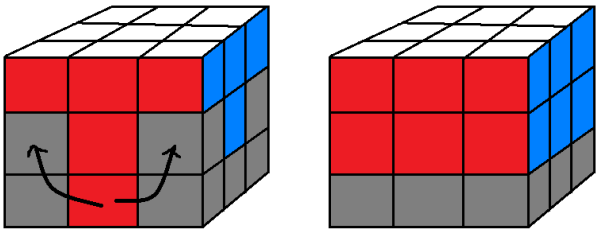
on how many white pieces that are correct positioned (fig 2.4).

5. Yellow corners

Positioning the yellow corners by applying different operations to move the edges depending och how many corners that are in position already.

6. Last layer permutation

Now when the corners of the last layer are in position so that the top is all yellow, the only thing left to do is to positioning the pieces of the last layer so they match with the colours of the vertical sides. This is achived by applying three different operation-combinations depending on if its edges or corners that should switch places.



(a) Technique for each side (b) Middle layer complete

Figure 2.3

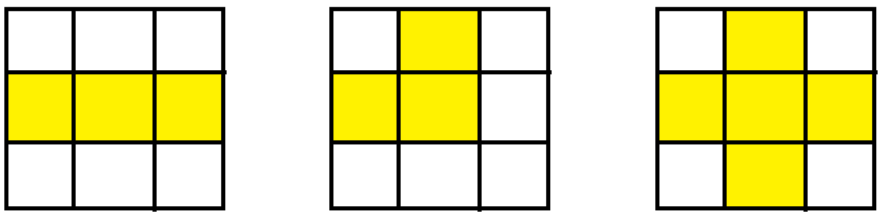


Figure 2.4: Different states to achive the yellow cross

2.3.2 Dedmore algorithm

The dedmore algorithm is also solving the cube layer-by-layer, but focusing on a corners-first solution[3]. It was one of the first solution guides to the Rubik's cube[1].

1. Top corners (the X)

Start off by finding a starting corner. For example aim to solve the blue top face first. Rotate the cube to obtain the blue side of the corner in top position. Then move the blue center-piece into position without moving your corner. When this is done, rotate the whole cube to obtain the corner in your top left on the front side, as in the fig 2.5a.

Next you'll search for the next corner that should be positioned in the top right front side. Depending on the position on this corner, you will use different operation sequences to get it in position like the fig 2.5b.

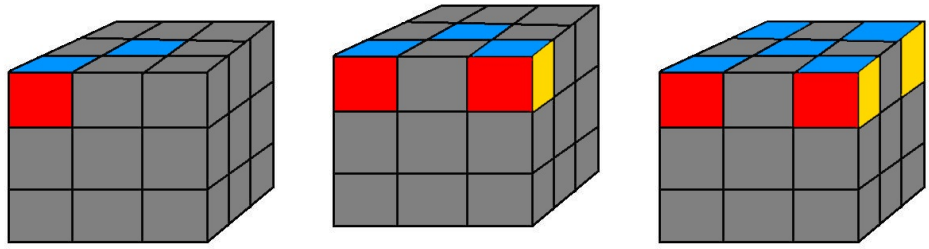
Simply rotate the cube and continue on. The goal of this step is to form an 'X' on the top face as in fig 2.5c.

2. Top edges

The goal of this step is to simply get the edges in place. Get them in place one by one. When this step is finished the cube looks like fig 2.6a.

3. Middle layer

The next step is to solve the middle layer by moving down a cubie from the middle of the third layer to the correct position on the second layer (fig 2.3a).



(a) First corner in position (b) Second corner in position (c) Top edges in position

Figure 2.5

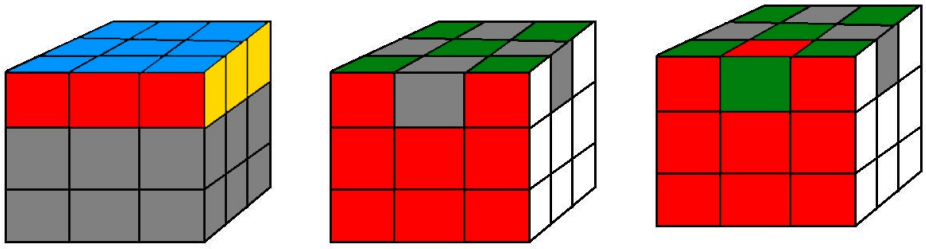
4. Bottom corners

Here the cube is turned upside down and the goal here is again to build a (green) X with the corners(fig 2.6b).

5. Bottom edges

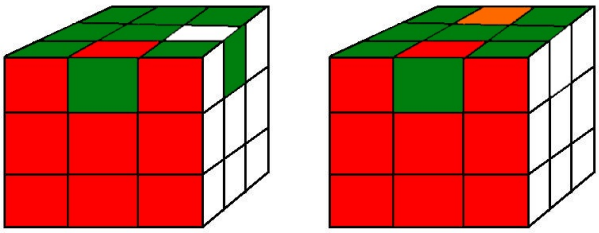
In this step, at least one edge is already in the correct place (the color might be switched as in fig 2.6c). Find that edge and put it on the front side. Then position every other edge correctly by using a series of operations.

When every edge are correctly positioned, there are 2 possible states left. Each requires a different sequence of operations to solve. The 'fish' pattern (fig 2.7a) and the 'H' pattern (fig 2.7b). Execute the sequence for the pattern and you'll then have solved the cube.



(a) Top edges in position (b) The green X (c) One edges correctly placed

Figure 2.6



(a) Fish pattern (b) H pattern

Figure 2.7

Chapter 3

Method

For this thesis there was two major methods used.

3.1 Implementation and data collection

Both of the algorithms are implemented in the same language and with the same built-in functions and data structures for as long as possible, to rule out differences to obtain a realistic comparison in the end. We need to run the algorithms a number of times to obtain a good set of data for the comparison. The data will be obtained during the runs by saving informations about the runs on files.

3.2 Analyze and representation

Based on the test data we will be able to evaluate each methods use of different operations, number of operations used and if suitable for speedcubing. The differences in used operations will provide information to evaluate if the times to solve are realistic for human solving. This depends on the difficulty of the operations.

Chapter 4

Implementation

Explanations of how the algorithms where implemented with pseudocode as well as how the cube was represented.

4.1 Cube representation

The cube is represented with a side-representation, only have to move the colours to the correct side and location when an operation is performed. Each side consists of nine positions each of which has a color. There exists no relation between positions in different sides unlike the physical cube, where for example an edge has two colors.

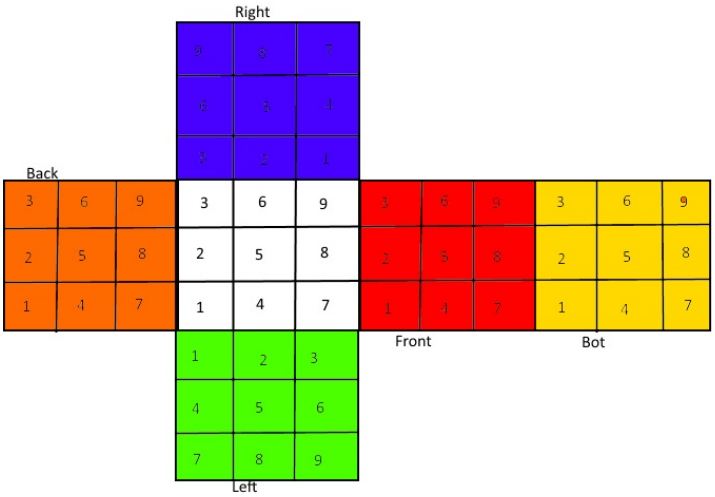


Figure 4.1: Side numberings

A disadvantage with the side-representation is that the only way to keep the rules of the physical cube (correct colour-combination of a cubie) is to enforce them during the operations.

4.2 Algorithms

```
Input: A cube, Number of operations to be made
Output: A scrambled cube
Let rand be a Random object.
for Number of operations do
    int op = rand.nextInt(Tot num of op)
    switch op do
        All operations have a number, performce the operation with the
        number op
    endsw
end
```

Algorithm 1: Scramble

4.3 Scramble

The idea of the scrambler is to always start with a solved cube. The scramble are then simulated by making a large amount of operations in pseudo-random sequence (Algorithm 1). The scrambled cube is saved to a list awaiting the two solvers to pick and solve it.

4.4 Difficulty

Our first idea was to represent the cube by 26 cubies in a layer-by-layer system, but this was problematic due to which colour-representation of the cubies should face which side when different operations where performed. A pro for this first idea was that the colors of a cubie had an relation, the only thing keeping the colours correct now is the operations implemented.

Translating the methods for physical sovling of the cube to code was not always easy. Because of how intuative some of the steps are when solving by hand, this made it difficult implement in a good way.

Chapter 5

Results and Analyze

Containing average amount of moves and operation usage statistics.

5.1 Data

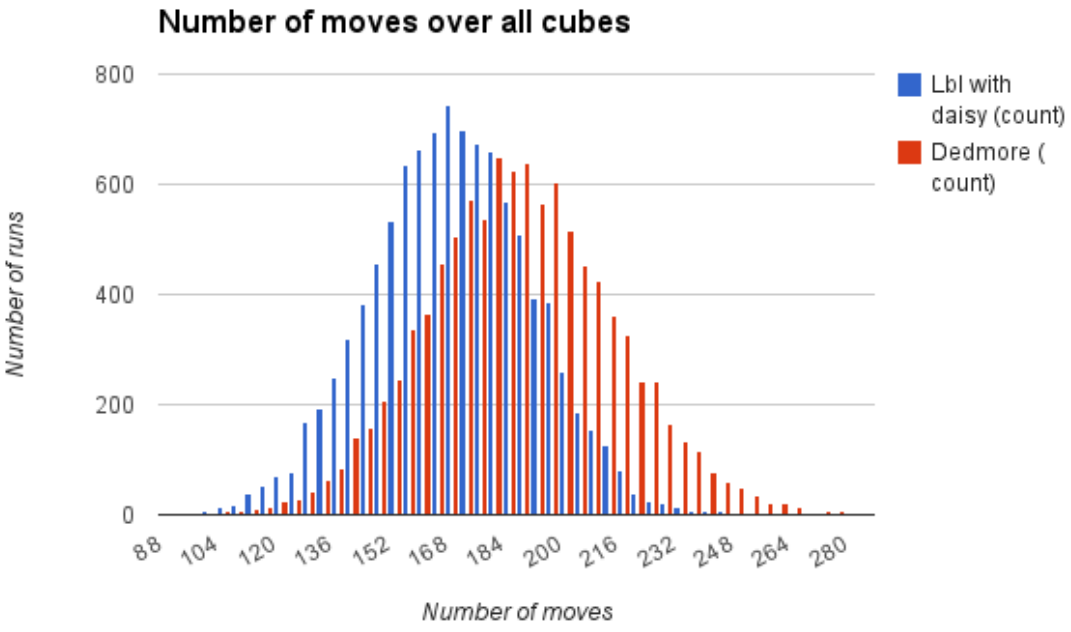


Figure 5.1

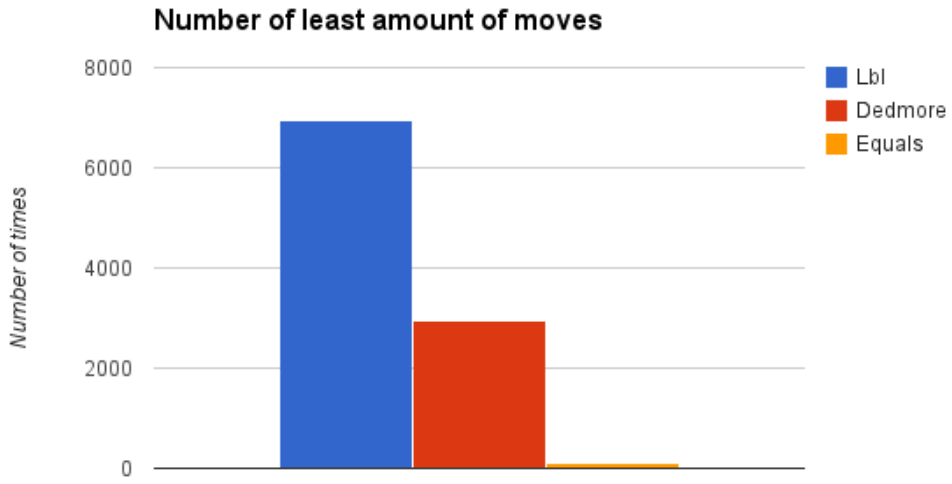


Figure 5.2

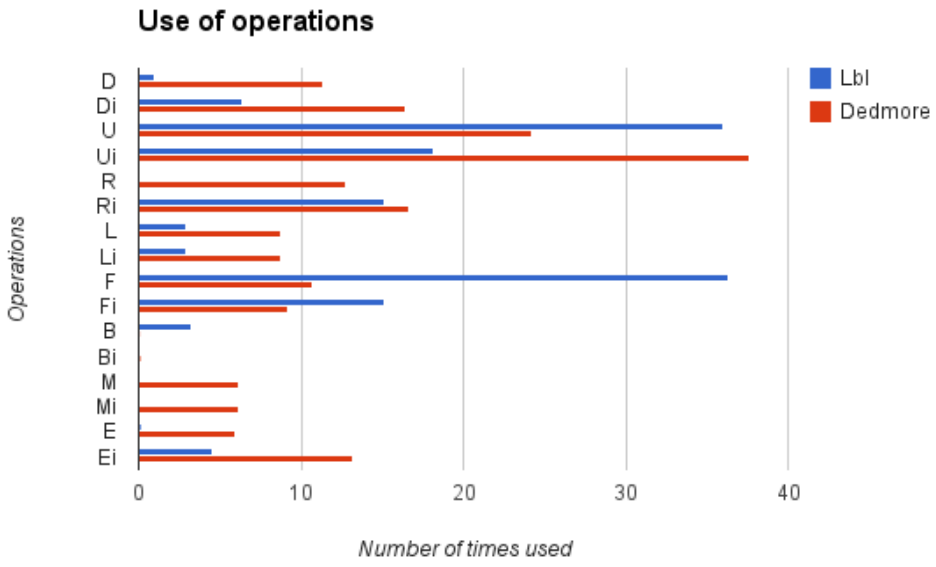


Figure 5.3: Avarage use of operations

5.2 Comparison

As shown in fig 5.1 we see that the Lbl with daisy method had a lower average (170) amount of moves than the dedmore method (188). Turning the whole cube in any

direction was not counted for as a move.

In the graph in fig 5.2 we compared the algorithms against each other on every cube. The algorithm which solved the cube in the least amount of moves got its counter incremented. This shows that over ten thousand scrambled cubes the Lbl with daisy method won 69% of the time, they got equal amount of moves 1% of the time and dedmore method had the fewest 30% of the time.

Chapter 6

Discussion

6.1 Comparison

6.2 Errors

Chapter 7

Conclusion

[8]

References

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Appendix A

Calculation of randomly rotate faces

Assumption: Every second you get to a new state.
There are $4.3 * 10^{19}$ different states (rounded down) [5]. 10% of the different states:
$4.3 * 10^{1}9 * 0.1 = 4.3 * 10^{18}$
4.31018 seconds to years = 136.4109 years [11]

Appendix B

Data

Avarage use of operations.

Operations	Lbl	Dedmore
D	1.0365	11.3665
E	0.1103	6.0217
F	36.4211	10.7248
B	3.2228	0.0718
Ri	15.1558	16.7828
L	2.8949	8.8148
M	0	6.1479
Ui	18.122	37.7467
U	36.0845	24.2097
Bi	3.2228	0.0955
Mi	0	6.1479
Li	2.8949	8.7691
R	25.2278	12.7341
Ei	4.5369	13.2933
Fi	15.1163	9.2282
Di	6.3726	16.5212

Number of times the algorithms had least amount of moves.

Lbl	Dedmore	Equals
6935	2960	105