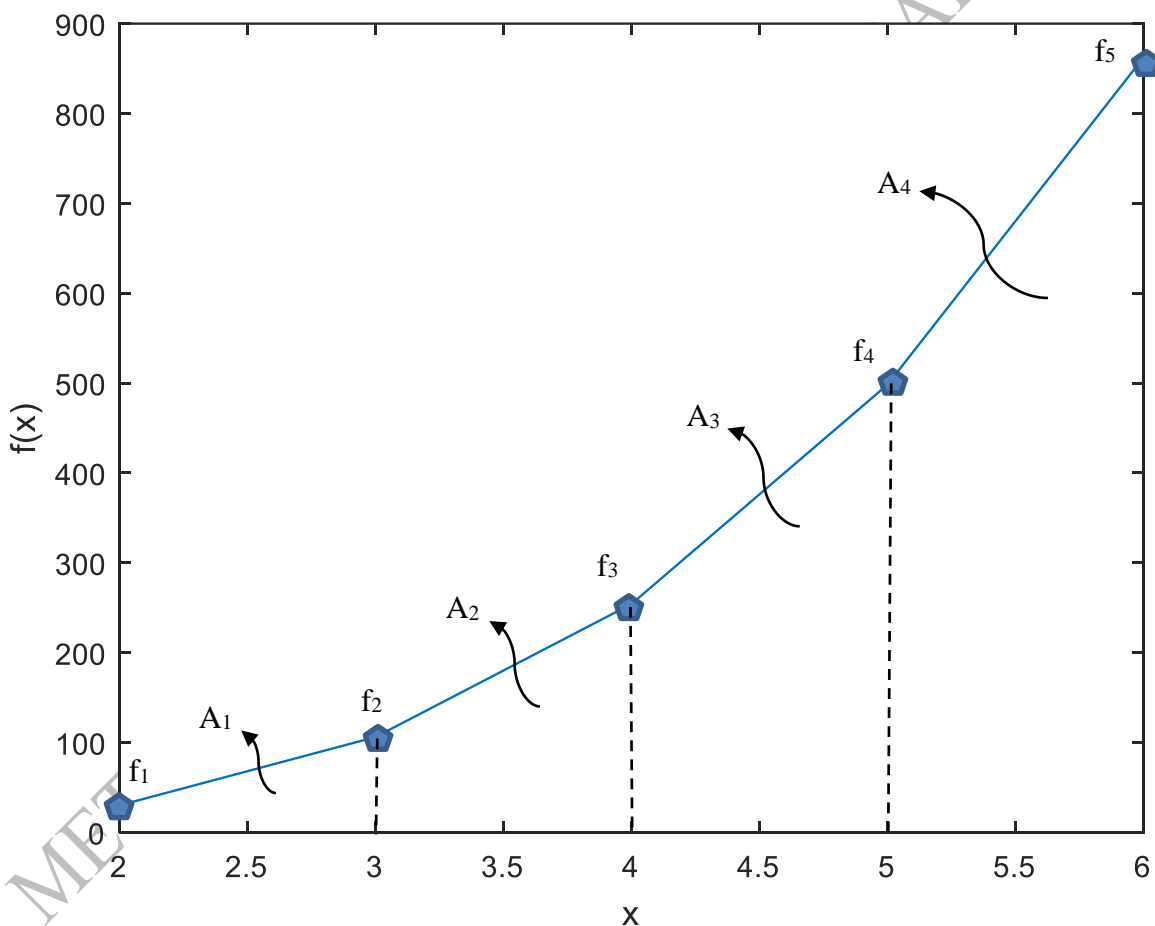


Homework #5

1. Consider the function $f(x) = 4x^3 - 2$. Create a vector 'x' bounded by [2,6] with an increment 1. Compute the function values at each discrete 'x' point and plot the f(x)-x diagram.

Point	x	f
f_1	2	30
f_2	3	106
f_3	4	254
f_4	5	498
f_5	6	862



In this figure, the area under each line is a trapezoid. Using a 'for loop' compute the areas and store them in a vector 'A' such that $A = [A_1 \ A_2 \ A_3 \ A_4]$.

Answer: $A = [68 \ 180 \ 376 \ 680]$

Hint: The area of the first trapezoid can be computed with the following command line:
 $A(1) = h \cdot (f(1) + f(2)) / 2$ where $h = 1$.

2. Woodall numbers has the following form (For further information about the Woodall numbers you can check: <http://mathworld.wolfram.com/WoodallNumber.html>).

$$W_n = 2^n * n - 1$$

Using a 'while loop' and an 'if condition' create an array 'W' for $n \geq 1$. The largest number of the array should not exceed 10^4 .

```
W = [ 1 7 23 63 159 383 895 2047 4607 ];
```

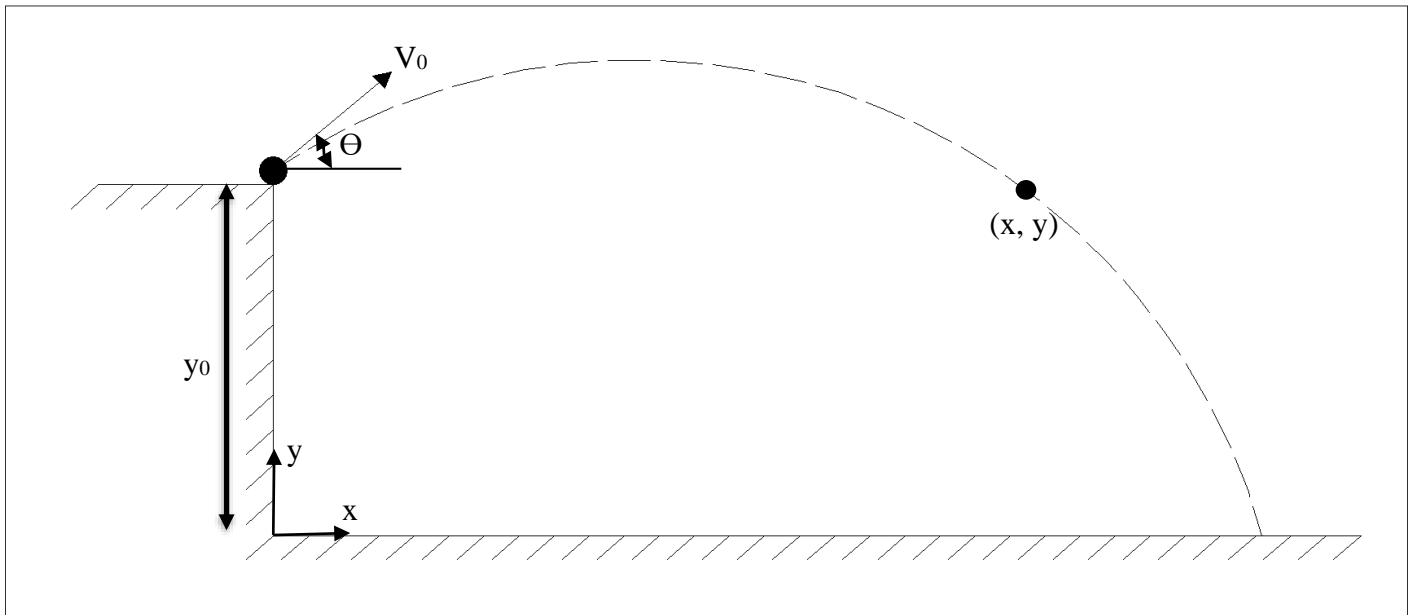
Then, using a 'for loop' compute the sum of all elements of W.

```
WS = 8185
```

Furthermore, compute the average of all elements of W.

```
WA = 909.4444
```

3. The projectile motion is illustrated in the figure below.



The position of the projectile at any time 't' is given with the following formulas:

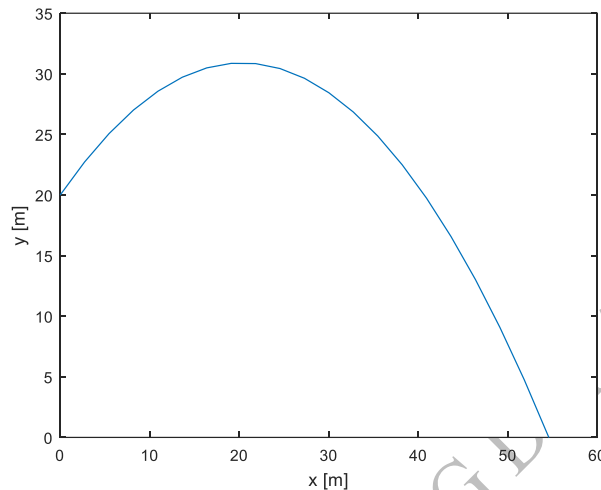
$$y = y_0 + V_0 * \sin\theta * t - \frac{1}{2} * g * t^2$$

$$x = x_0 + V_0 * \cos\theta * t$$

Create a time domain t which is bounded by $[0:4]$ with an increment 0.2 seconds. Take initial velocity $V_0 = 20$ m/s, initial height $y_0 = 20$ m, the slope $\Theta = 47$ degrees and gravitational acceleration $g = 9.81$ m/s². Using the given inputs, compute the position of the projectile at each 't' value. Plot the trajectory.

Hint: For degrees use `sind(x)` function.

Answer:

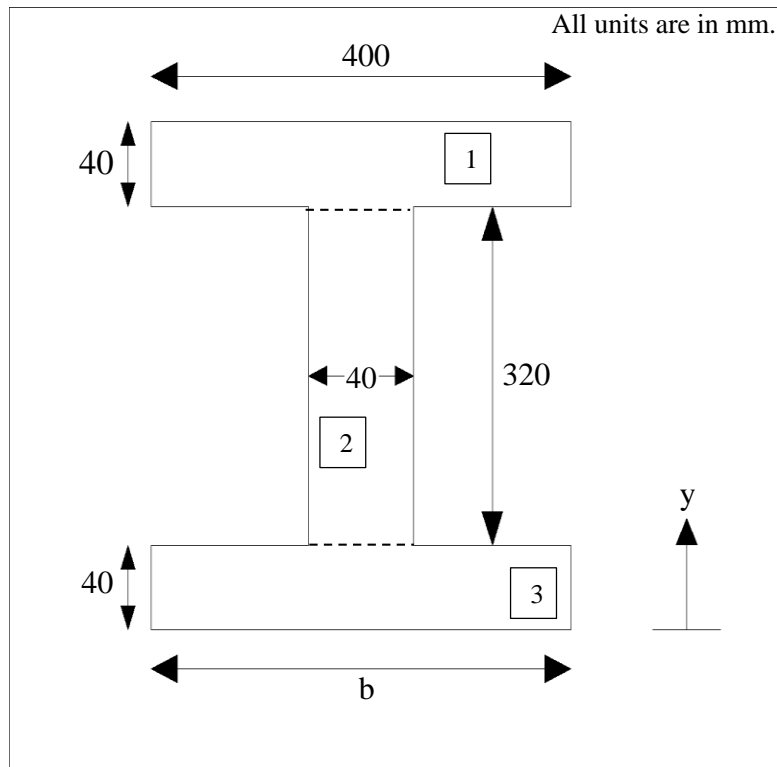


4. Create an array 'F' of Fibonacci numbers using a 'for loop'. Use the first 10 elements of the Fibonacci numbers. These numbers start with 1,1.. and previous two terms are added to give the third term such that $F(1) = 1$, $F(2) = 1$, $F(3) = F(1) + F(2) = 2$, $F(4) = F(2) + F(3) = 3$, $F(5) = F(3) + F(4) = 5$ and so on. (For further information about the Fibonacci numbers you can check: <http://mathworld.wolfram.com/FibonacciNumber.html>).

Answer: $F = [1 \ 1 \ 2 \ 3 \ 5 \ 8 \ 13 \ 21 \ 34 \ 55]$

5. Consider the function $f(x) = 1/x^2$. For an approximation with Taylor Series Expansion, we need the derivatives of function $f(x)$. The first three derivatives of this function are $f^{(1)} = -(2)/x^3$, $f^{(2)} = (2)*(3)/x^4$ and $f^{(3)} = -(2)*(3)*(4)/x^5$. First, express the n^{th} derivative of this function using the provided pattern (Not with MATLAB). Then, write a script that takes 'n' and 'x' as inputs to compute the n^{th} derivative of $f(x)$ at point x. Check the validity of your script with the provided answers.

n	x	$f^{(n)}$
3	1	-24
5	2	-5.625
4	2	1.875



6. In the figure above, the dimensions of an I-section are given. Create a vector ' b ' bounded by [400,600] with an increment 20. Compute the distance of the neutral axis from the base and the moment of inertia of the section for each element of ' b '.

Hint: Separate the section into 3 pieces as shown in the figure.

b (mm)	y_n (mm)	I (mm ⁴)
400	200.0000	1.150E+09
420	196.8421	1.176E+09
440	193.7931	1.201E+09
460	190.8475	1.224E+09
480	188.0000	1.247E+09
500	185.2459	1.270E+09
520	182.5806	1.291E+09
540	180.0000	1.312E+09
560	177.5000	1.333E+09
580	175.0769	1.352E+09
600	172.7273	1.371E+09

7. $\sqrt{2}$ can be approximated with the following iterative way.

$$a_{k+1} = \frac{a_k}{2} + \frac{1}{a_k}$$

Take the initial value $a_0 = 1$, take tolerance 10^{-5} , use approximate absolute relative error and iteratively compute $\sqrt{2}$.

$$\varepsilon_R = \left| \frac{a_{k+1} - a_k}{a_{k+1}} \right|$$

Answer:

n	$a \approx \sqrt{2}$	err
0	1.000000	-
1	1.500000	3.333E-01
2	1.416667	5.882E-02
3	1.414216	1.733E-03
4	1.414214	1.502E-06

8. Following formula relates the annual payment (A) and the present value (P) of an equipment or a service.

$$A = P * \frac{i * (1 + i)^t}{(1 + i)^t - 1}$$

In this notation i represents the interest rate and t represents the payment duration in years. Assume someone borrows 40000 TL and pays 12000 TL annually for 5 years. Compute the interest rate using the bisection method. The rate is bounded with $[0.05, 0.2]$. Take tolerance 10^{-2} and use approximate absolute relative error.

Answer: $i = 0.152$

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