CENG 371 - Scientific Computing
2023-1
Homework 2

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Question 2

a)

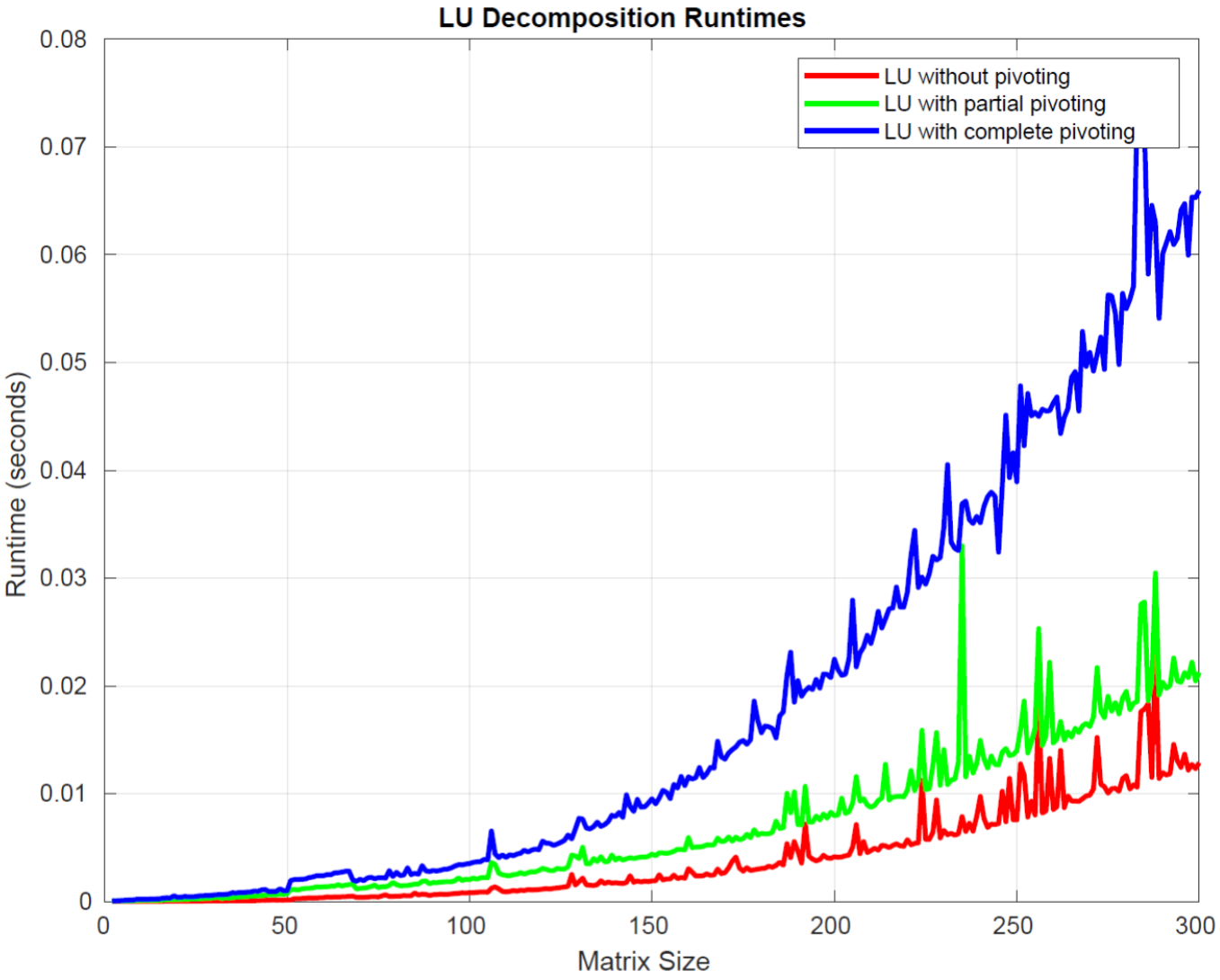

Figure 1: Runtimes of the algorithms

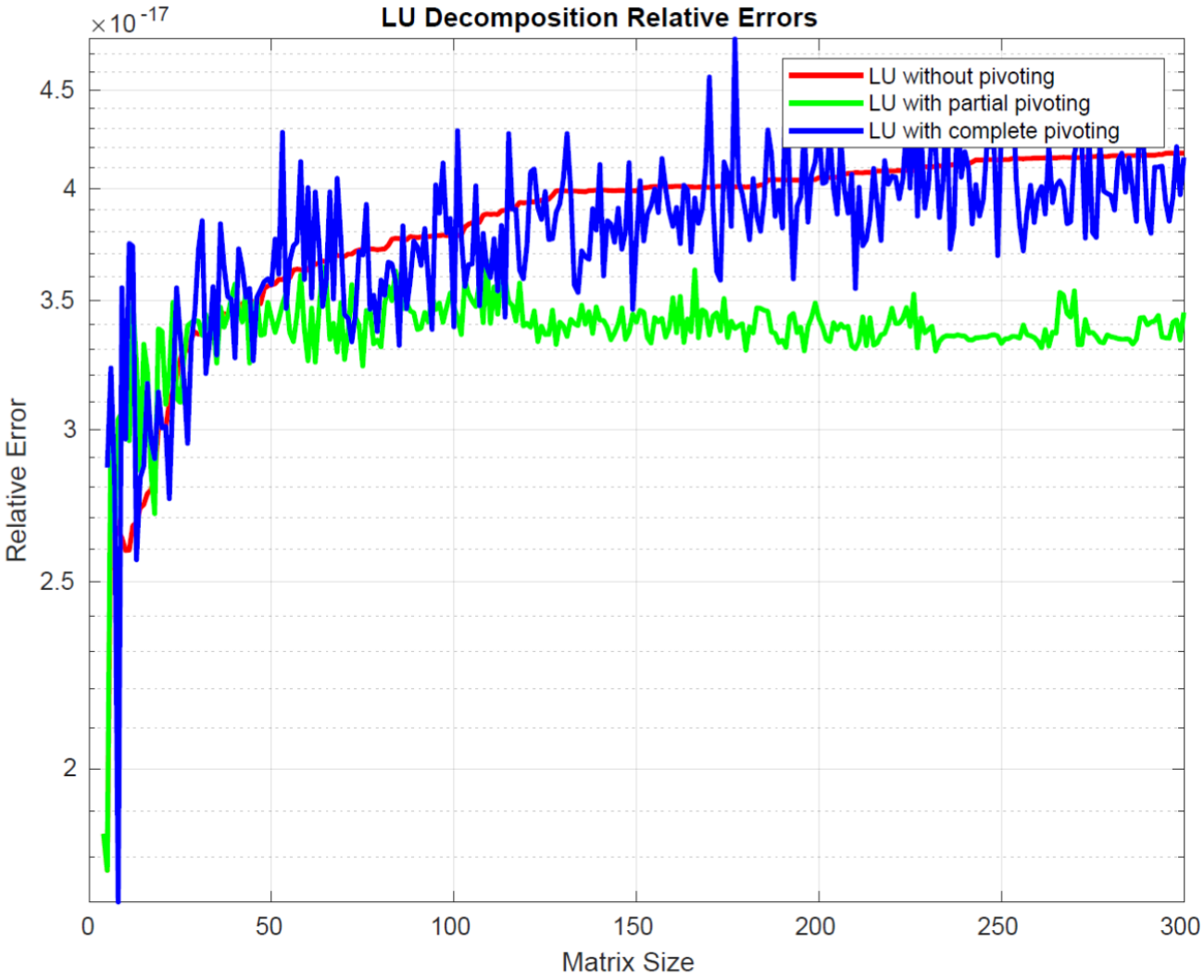

Figure 2: Relative errors of the algorithms

As it can be seen from the graphs above:

Runtimes of the algorithms can be compared as follows with respect to general trend:

Complete Pivoting > Partial Pivoting > Without Pivoting

LU Factorization Without Pivoting is the fastest one because it does not require any computation for pivoting, it calculates factorization directly on A. LU Factorization with Complete Pivoting is the slowest because it shuffles both rows and columns which require a lot of computations. LU Factorization with Partial Pivoting is the middle one becuase it only apply pivoting to rows.

Relative errors of the algorithms can be compared as follows with respect to general trend:

In the larger matrix sizes, it is crystal-clear that LU Factorization with partial pivoting has the smallest relative error. LU Factorization with complete pivoting fluctuates too much and LU Factorization without pivoting is more stable. However, if we take the average of LU with complete pivoting, it would be closer to LU without pivoting.

b) For the algorithm LU Factorization without pivoting, the elimination process gets stuck if the current diagonal entry is 0. That is,

$$\mathrm{A} = \begin{bmatrix} 1 & 2 & 5 & 8 \\ 0 & 0 & 5 & 9 \\ 0 & 3 & 3 & 8 \\ 0 & 7 & 6 & 7 \end{bmatrix}$$

Observe that, a_{22} is zero. So, you cannot get rid of a_{32} and a_{42} by subtracting 0 from them. Therefore, LU Factorization without pivoting fails in this case.

However, LU Factorization with partial pivoting and LU Factorization with complete pivoting can solve this issue by replacing rows (or columns) thanks to their pivoting capabilities. Hence, LU Factorization with partial pivoting and LU Factorization with complete pivoting can factorize any square matrix but LU Factorization without pivoting fails in some cases as described above.