Student Information

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Question 1

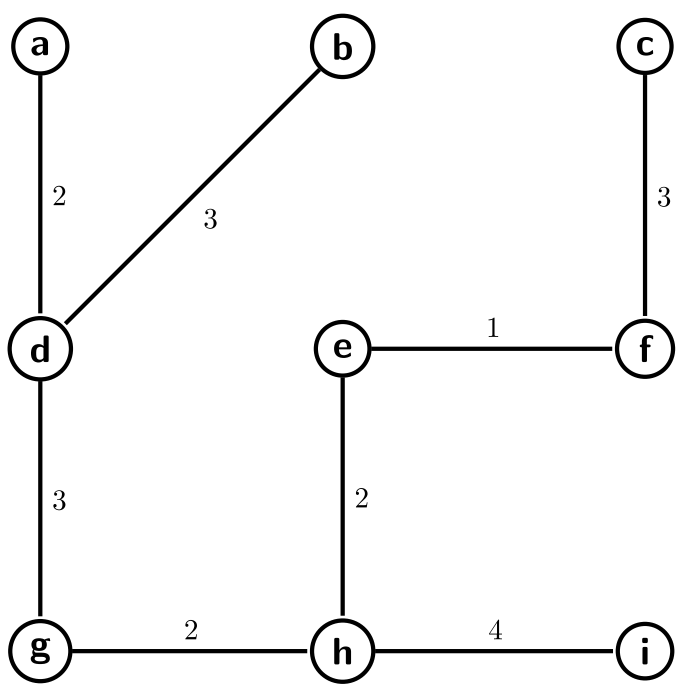
I will use **Kruskal's Algorithm** in this question.

a)

Adding Order	Edge	Weight
1	$\{e, f\}$	1
2	$\{a, d\}$	2
3	$\{g, h\}$	2
4	$\{e, h\}$	2
5	$\{d, g\}$	3
6	$\{c, f\}$	3
7	$\{b, d\}$	3
8	$\{h, i\}$	4

b)

Minimum Spanning Tree Of Graph G



c)

Answer to the first question (Is the minimum spanning tree unique for the graph G in Figure 1?) :

Yes, the minimum spanning tree for the graph G is unique.
Here is the weights of the minimum spanning tree for the graph G: $\{1, 2, 2, 2, 3, 3, 3, 4\}$.

All edges of weight 1 are included in the minimum spanning tree.
All edges of weight 2 are included in the minimum spanning tree.

If we could form another MST, we could do it by

- **Case1:** including the another edge of weight 3 which is $\{f, h\}$ and excluding the edge $\{h, i\}$ which is of weight 4
- **Case2:** including the another edge of weight 3 which is $\{f, h\}$ and excluding the edge $\{b, d\}$ which is of weight 3
- **Case3:** including the another edge of weight 3 which is $\{f, h\}$ and excluding the edge $\{d, g\}$ which is of weight 3
- **Case4:** including the another edge of weight 3 which is $\{f, h\}$ and excluding the edge $\{c, g\}$ which is of weight 3

But a circuit (e,h,f) occurs in each case, so these are not spanning trees. Therefore we cannot form another minimum spanning tree.

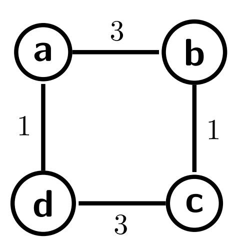
Hence, the minimum spanning tree for the graph G is unique.

Answer to the second question (In general, is the minimum spanning tree unique for any connected edge-weighted undirected graph?):

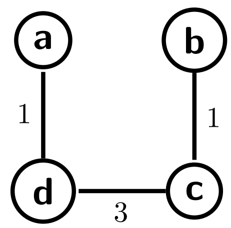
No, in general, the minimum spanning tree is not unique for any connected edge-weighted directed graph. If a graph includes edges of the same weight, then this graph **might** (i.e there is no guarantee) have multiple minimum spanning trees.

For example;

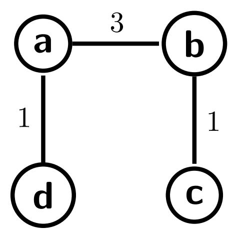
Graph H



the graph H, as given above, has two different minimum spanning trees which are



and



Hence, not all connected edge-weighted undirected graphs have unique minimum spanning tree.

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d)

Proof:
Let G be a weighted graph and let e be the unique minimum-weight edge of G.

Assume that T is a minimum spanning tree for G which does not contain the edge e.

Then consider the graph $T + e$. This graph must contain a circuit C that contains the edge e.

Let f be an edge of C different from e, and

set $T^* = T + e - f$

Then T^* is also a spanning tree for G, **but**

$wt(T^*) = wt(T + e - f) = wt(T)$ + $wt(e)$ - $wt(f)$ and $wt(e) < wt(f)$.

This implies $wt(T^*) < wt(T)$, which is an contradiction to T being a minimum spanning tree.

Therefore, no such minimum spanning tree T (i.e without e) can exist.

Hence, for a weighted graph, if the minimum-weight edge pf a graph is unique, then this edge is included in any minimum spanning tree for that graph.

(Note that $wt(x)$ gives the weight of the edge x if x is an edge, and gives the total weight of the graph x if x is a graph.)

Question 2

Yes, they are isomorphic.

Let's define an one-to-one and onto function f from the set of vertices $\{a, b, c, d, e, f\}$ to the set of vertices $\{m, n, o, p, r, q\}$ with

f(a) = n
f(b) = q
f(c) = o
f(d) = r
f(e) = m
f(f) = p

Observe that

- a is adjacent to the vertices $\{b, d, c\}$ and f(a) = n is adjacent to the vertices f(b) = q, f(d) = r, f(c) = o.
- b is adjacent to the vertices $\{a, c, e, f\}$ and f(b) = q is adjacent to the vertices f(a) = n, f(c) = o, f(e) = m, f(f) = p.
- c is adjacent to the vertices $\{a, b\}$ and f(c) = 0 is adjacent to the vertices f(a) = n, f(b) = q.
- d is adjacent to the vertices $\{a, e\}$ and f(d) = r is adjacent to the vertices f(a) = n, f(e) = m.
- e is adjacent to the vertices $\{b, d, f\}$ and f(e) = m is adjacent to the vertices f(b) = q, f(d) = r, f(f) = p.
- f is adjacent to the vertices $\{b, e\}$ and f(f) = 0 is adjacent to the vertices f(b) = q, f(e) = m.

As seen above, for all pairs (x,y), x $\neq$ y in $\{a, b, c, d, e, f\}$, if and only if x and y are connected, f(x) and f(y) in $\{m, n, o, p, r, q\}$ are connected .

Hence, f **is an isomorphism** .

Because f is an isomorphism, graphs G and H are isomorphic.

Question 3

a)

The number of vertices is **7**.
The number of edges is **6**.
The height of T is **3**.

b)

postorder: q,s,u,v,t,r,p

inorder: q,p,s,r,u,t,v

preorder: p,q,r,s,t,u,v

c)

Yes, T is a full binary tree because each of its internal vertices (p,r,t) has two children.

- p has children q and r
- r has children s and t
- t has children u,v

d)

No, T is not a complete binary tree.

For T to be a complete binary tree, it should be completely filled in every level, except possibly the last. And nodes are as far left as. But level 2 of T, i.e level including s:19 and t:43, is not completely filled although it is not the last level. Also, the last level is not filled starting from left. Because of these reasons, T is not a complete binary tree.

e)

No, T is not a balanced binary tree.

For a binary tree to be balanced, all leaves should be at the levels h and h-1 where h is the last level.

T is not a balanced binary tree because T has a leaf , q:13 , at level 1 which is h-2.

f)

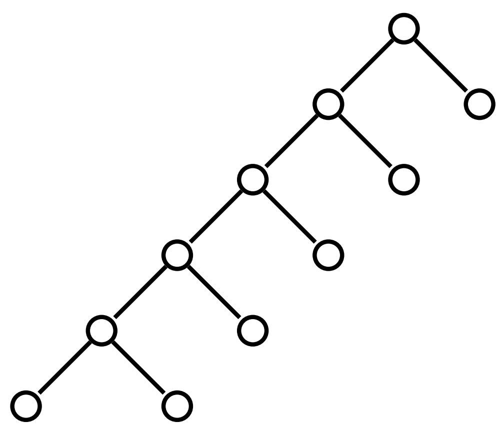
No, T is not a binary search tree (BST) .

For a binary tree to be a BST, for each node r in the tree, nodes in the right subtree of the r must have keys greater than r's key, and nodes in the left subtree of r must have keys less than the r's key.

However, for the node r:24, it has u:23 in its right subtree and $23 < 24$, so T is not a BST.

g)

The minimum number of nodes for a full binary tree with height 5 is **11**.
The full binary tree of height 5 with minimum number of nodes is in the form of



As seen above, the minimum number of nodes for a full binary tree with height 5 is **11**.