CENG 384 - Signals and Systems for Computer Engineers
Spring 2023
Homework 4

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June 8, 2023

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1. (a) $H(j\omega) = \dfrac{j\omega - 1}{j\omega + 1}$

   $\dfrac{Y(j\omega)}{X(j\omega)} = \dfrac{j\omega - 1}{j\omega + 1}$

   $(j\omega + 1)Y(j\omega) = (j\omega - 1)X(j\omega)$

   $y'(t) + y(t) = x'(t) - x(t)$

   (b) $H(j\omega) = \dfrac{j\omega - 1}{j\omega + 1} = 1 - \dfrac{2}{j\omega + 1}$

   Impulse response of this system is

   $h(t) = F^{-1}\{1 - \dfrac{2}{j\omega + 1}\} = F^{-1}\{1\} - F^{-1}\{\dfrac{2}{j\omega + 1}\} = \delta(t) - 2e^{-t}u(t)$

   (c) $X(j\omega) = \dfrac{1}{j\omega + 2}$

   $Y(j\omega) = X(j\omega).H(j\omega)$

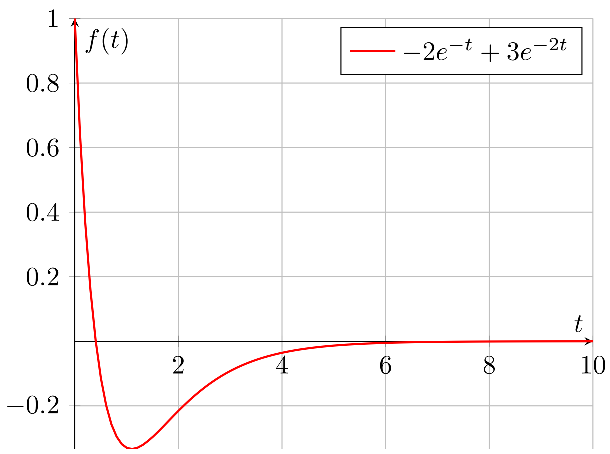
   $= \Big(\dfrac{j\omega - 1}{j\omega + 1}\Big)\Big(\dfrac{1}{j\omega + 2}\Big) = \dfrac{A}{j\omega + 1} + \dfrac{B}{j\omega + 2}$.Then

   $Aj\omega + 2A + Bj\omega + B = j\omega - 1$

   $A + B = 1$ and $2A + B = -1 \implies A = -2$ and $B = 3$.

   $Y(j\omega) = -\dfrac{2}{j\omega + 1} + \dfrac{3}{j\omega + 2}$

   $y(t) = F^{-1}\{Y(j\omega)\} = -2e^{-t}u(t) + 3e^{-2t}u(t)$

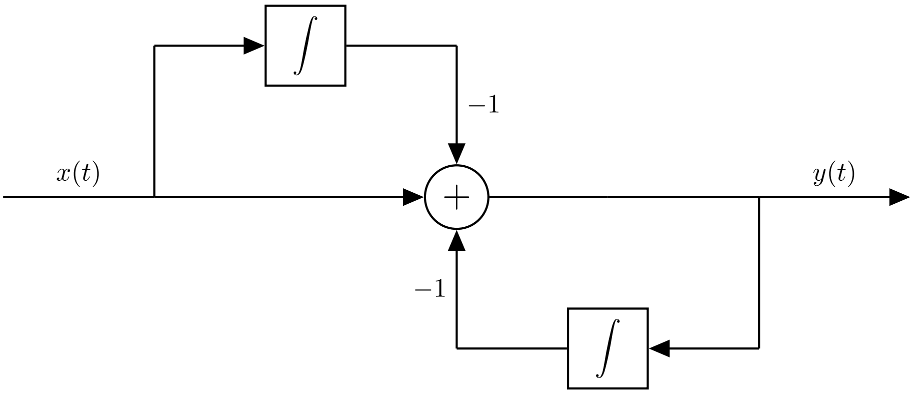


(d) Our equation was $y'(t)+y(t)=x'(t)-x(t)$

If we take integral of both side for time t,We will have:

$y(t)+\int y(t)=x(t)-\int x(t)$. Then

$y(t)=x(t)-\int x(t)-\int y(t)$



2. (a) $Y(e^{j\omega})\Big(e^{j\omega}-\dfrac{1}{2}\Big)=X(e^{j\omega})e^{j\omega}$

$$H(e^{j\omega})=\frac{Y(e^{j\omega})}{X(e^{j\omega})}=\frac{e^{j\omega}}{e^{j\omega}-\frac{1}{2}}.$$

if we simplify the fractional expression with $e^{j\omega}$:

$$H(e^{j\omega})=\frac{1}{1-\frac{1}{2}e^{-j\omega}}$$

(b) $h[n]=F^{-1}\{H(j\omega)\}=\Big(\dfrac{1}{2}\Big)^n u[n]$

(c) $y[n]=F^{-1}\{Y(e^{j\omega})\}=F^{-1}\{X(e^{j\omega}).H(e^{j\omega})\}$

$$X(e^{j\omega})=F\{x[n]\}=\frac{1}{1-\frac{3}{4}e^{-j\omega}}$$

$$Y(e^{j\omega})=X(e^{j\omega}).H(e^{j\omega})=\Big(\frac{1}{1-\frac{3}{4}e^{-j\omega}}\Big)\Big(\frac{1}{1-\frac{1}{2}e^{-j\omega}}\Big)$$

$$Y(e^{j\omega})=\Big(\frac{1}{1-\frac{3}{4}e^{-j\omega}}\Big)\Big(\frac{1}{1-\frac{1}{2}e^{-j\omega}}\Big)=\frac{A}{1-\frac{3}{4}e^{-j\omega}}+\frac{B}{1-\frac{1}{2}e^{-j\omega}}.$$

$$A-\frac{A}{2}e^{-j\omega}+B-\frac{3B}{4}e^{-j\omega}=1$$

$A+B=1$ and $\dfrac{A}{2}+\dfrac{3B}{4}=0$. Then $A=3$ and $B=-2$.

$$Y(e^{j\omega})=\frac{3}{1-\frac{3}{4}e^{-j\omega}}-\frac{2}{1-\frac{1}{2}e^{-j\omega}}$$

$$y[n]=F^{-1}\{Y(e^{j\omega})\}=3\Big(\frac{3}{4}\Big)^n u[n]-2\Big(\frac{1}{2}\Big)^n u[n]$$

3. (a) Firstly lets calculate frequency response of overall system.

$$H(j\omega)=H_1(j\omega).H_2(j\omega)=\frac{1}{(j\omega)^2+3j\omega+2}$$

$$\frac{Y(j\omega)}{X(j\omega)}=\frac{1}{(j\omega)^2+3j\omega+2}$$

$$((j\omega)^2+3j\omega+2)Y(j\omega)=X(j\omega)$$

$$y''(t)+3y'(t)+2y(t)=x(t)$$

(b) $H(j\omega)=\dfrac{1}{(j\omega)^2+3j\omega+2}=\dfrac{A}{j\omega+1}+\dfrac{B}{j\omega+2}$

$$Aj\omega+2A+Bj\omega+B=1$$

$A + B = 0$ and $2A + B = 1$, then $A = 1$ and $B = -1$

$$H(j\omega) = \frac{1}{j\omega + 1} - \frac{1}{j\omega + 2}$$

From fourier inverse transform:

$$h(t) = e^{-t}u(t) - e^{-2t}u(t)$$

(c) $Y(j\omega) = X(j\omega).H(j\omega)$

$$= j\omega\Big(\frac{1}{(j\omega)^2 + 3j\omega + 2}\Big) = \frac{j\omega}{(j\omega)^2 + 3j\omega + 2} = \frac{A}{j\omega + 1} + \frac{B}{j\omega + 2}$$

$$Aj\omega + 2A + Bj\omega + B = j\omega$$

$A + B = 1$ and $2A + B = 0$. Then $A = -1$ and $B = 2$. Then

$$Y(j\omega) = -\frac{1}{j\omega + 1} + \frac{2}{j\omega + 2}$$

$$y(t) = F^{-1}\{Y(j\omega)\} = -e^{-t}u(t) + 2e^{-2t}u(t)$$

4. (a) The overall frequency response of the system is

$$H(e^{j\omega}) = H_1(e^{j\omega}) + H_2(e^{j\omega}) = \frac{3}{3 + e^{-j\omega}} + \frac{2}{2 + e^{-j\omega}} = \frac{5e^{-j\omega} + 12}{(e^{-j\omega})^2 + 5e^{-j\omega} + 6}$$

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{5e^{-j\omega} + 12}{(e^{-j\omega})^2 + 5e^{-j\omega} + 6}$$

$$Y(e^{j\omega})\big((e^{-j\omega})^2 + 5e^{-j\omega} + 6\big) = X(e^{j\omega})\big(5e^{-j\omega} + 12\big)$$

$$y[n-2] + 5y[n-1] + 6y[n] = 5x[n-1] + 12x[n]$$

(b) The overall frequency response of the system is

$$H(e^{j\omega}) = H_1(e^{j\omega}) + H_2(e^{j\omega}) = \frac{3}{3 + e^{-j\omega}} + \frac{2}{2 + e^{-j\omega}} = \frac{5e^{-j\omega} + 12}{(e^{-j\omega})^2 + 5e^{-j\omega} + 6}$$

(c) $H(e^{j\omega}) = \frac{3}{3 + e^{-j\omega}} + \frac{2}{2 + e^{-j\omega}} = \frac{1}{1 + \frac{1}{3}e^{-j\omega}} + \frac{1}{1 + \frac{1}{2}e^{-j\omega}}$

$$h[n] = \big(-\tfrac{1}{3}\big)^n u[n] + \big(-\tfrac{1}{2}\big)^n u[n]$$

5. **Secret Message: I have a dream.**

```
import numpy as np
import matplotlib.pyplot as plt
from scipy.io import wavfile


def my_fft(x):
    N = len(x)

    if N <= 1:
        return x

    even = my_fft(x[::2])
    odd = my_fft(x[1::2])

    T = np.exp(-2j * np.pi * np.arange(N // 2) / N)
    return np.concatenate([even + T * odd, even - T * odd])


def reverse(signal):
    negative=signal[:len(signal)//2]
    positive=signal[len(signal)//2:]
    return np.concatenate([np.flip(negative),np.flip(positive)])


def my_ifft(x):
    N = len(x)

    if N <= 1:
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        return x

    even = my_ifft(x[::2])
    odd = my_ifft(x[1::2])

    T = np.exp(2j * np.pi * np.arange(N // 2) / N)
    return np.concatenate([even + T * odd, even - T * odd])

def fft(x):
    return my_fft(x)

def ifft(x):
    return my_ifft(x)/len(x)

def decode_audio_file(file_path):
    sample_rate, data = wavfile.read(file_path)
    fourier = fft(data)
    N = len(fourier)
    reversed_fourier=reverse(fourier)
    decoded_data = np.real(ifft(reversed_fourier)).astype('int16')

    plt.figure(1)
    plt.title("Time domain Encoded Audio Signal")
    plt.plot(data)
    plt.figure(2)
    plt.title("Time Domain Decoded Audio Signal")
    plt.plot(decoded_data)
    plt.figure(3)
    plt.title("Frequency Domain Encoded Audio Signal")
    plt.plot(np.absolute(fourier))


    wavfile.write('decoded.wav', sample_rate, decoded_data)
def encode_audio_file(file_path):
    sample_rate, data = wavfile.read(file_path)
    fourier = fft(data)
    N = len(fourier)
    reversed_fourier=reverse(fourier)
    encoded_data = np.real(ifft(reversed_fourier)).astype('int16')

    plt.figure(4)
    plt.title("Time domain Decoded Audio Signal")
    plt.plot(data)
    plt.figure(5)
    plt.title("Time domain Encoded Audio Signal")
    plt.plot(encoded_data)
    plt.figure(6)
    plt.title("Frequency Domain decoded Audio Signal")
    plt.plot(np.absolute(fourier))
    wavfile.write('encoded.wav', sample_rate, encoded_data)
decode_audio_file('encoded.wav')
encode_audio_file("decoded.wav")
```
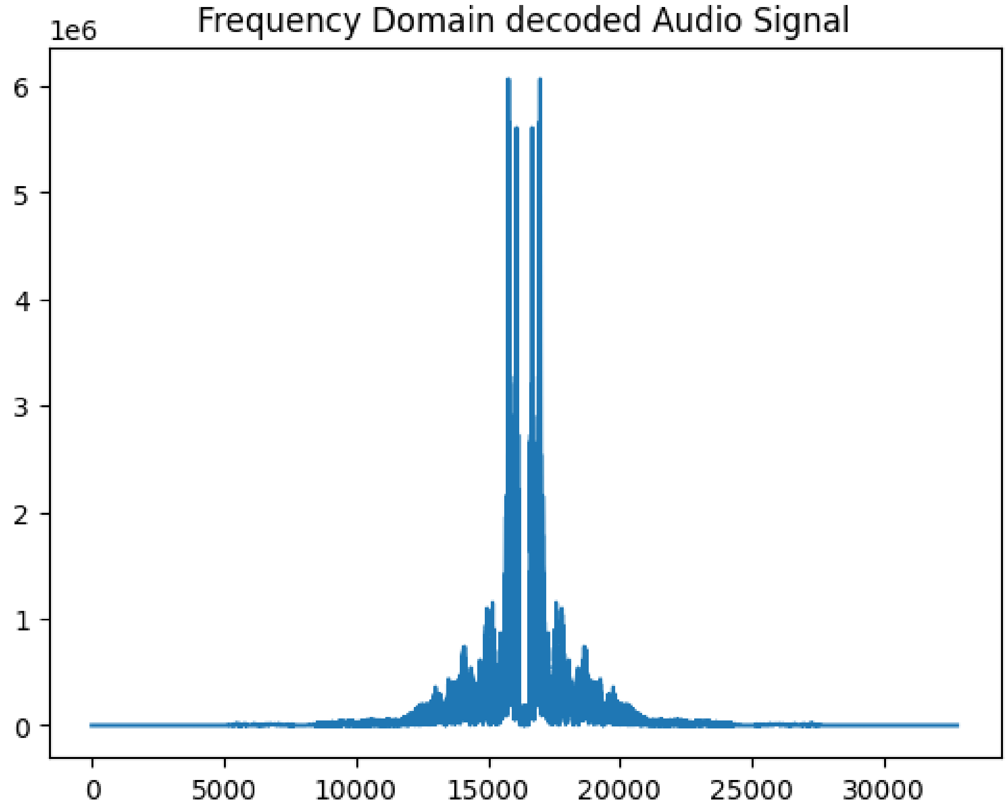
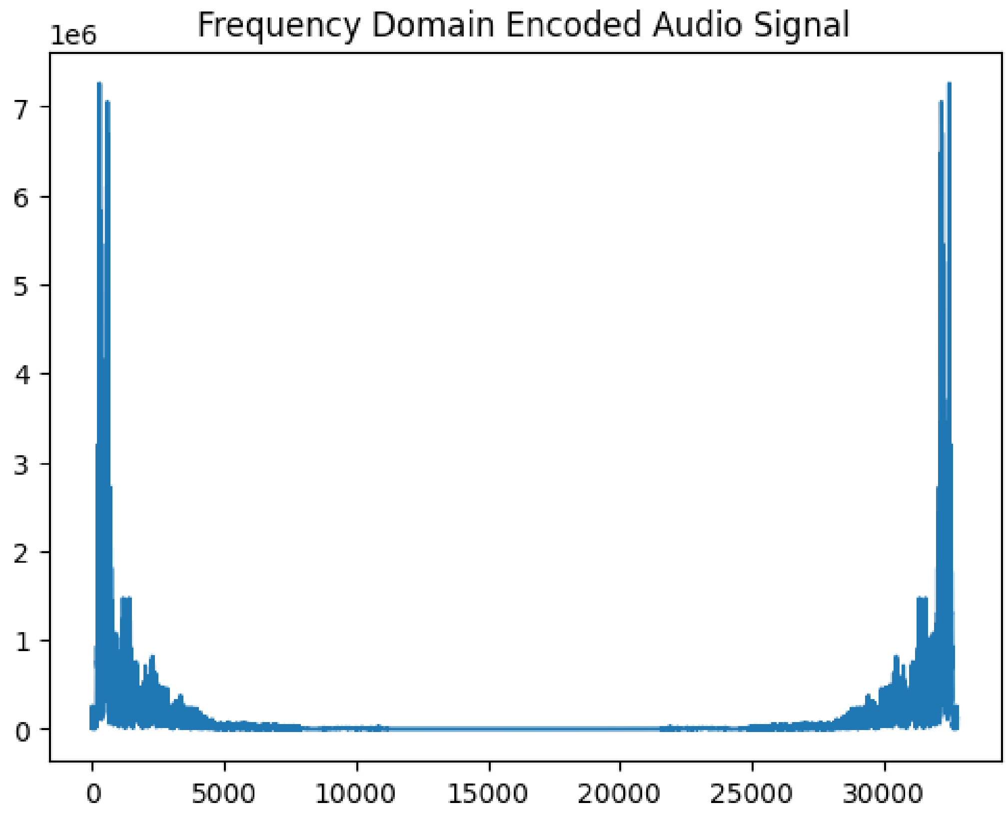
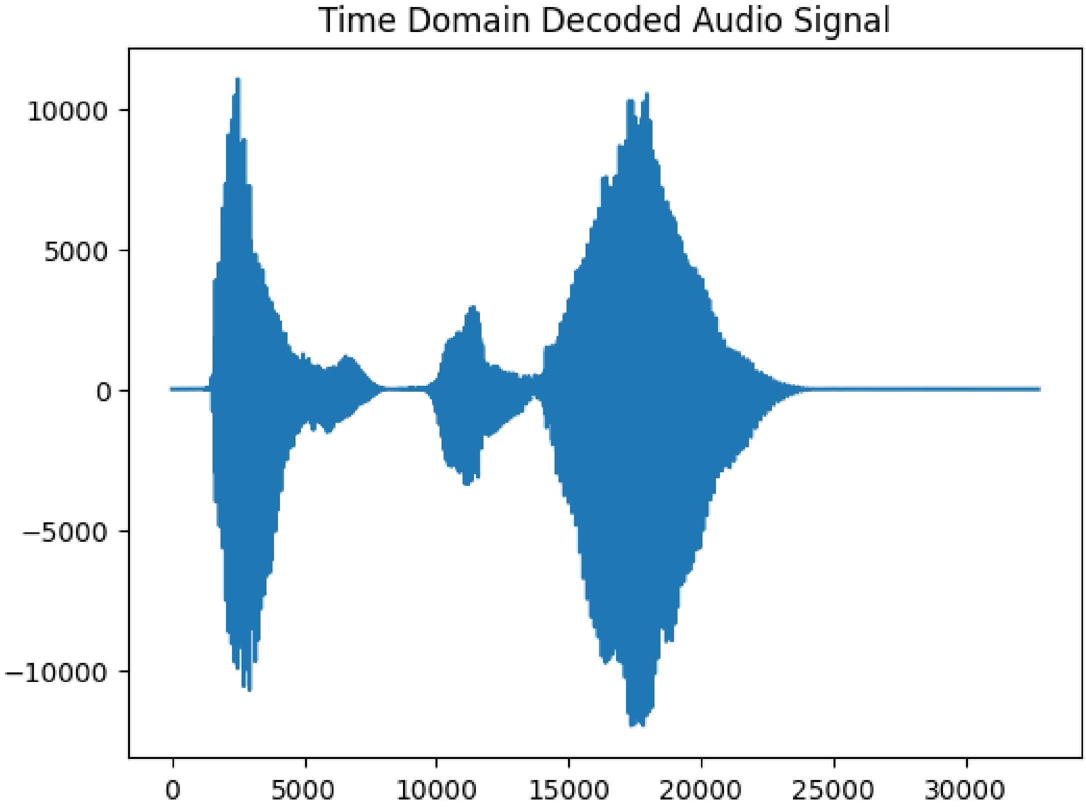
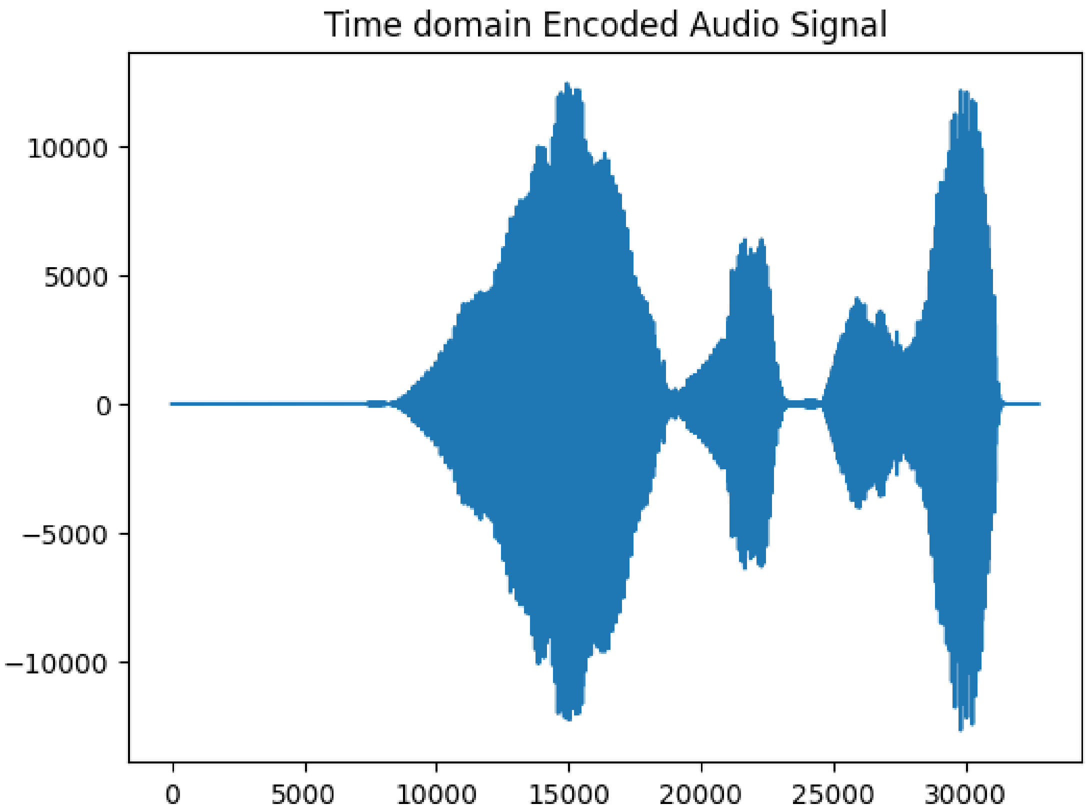

As it can be seen, both frequency domain encoded audio signals and frequency domain decoded audio signals are the two split and combined versions of each other.This is because of reverse process.

Because of reverse function, encoded and decoded signals on time domain are revers of each other.


Frequency Domain decoded Audio Signal
1e6
6
5
4
3
2
1
0
0
5000
10000
15000
20000
25000
30000


Frequency Domain Encoded Audio Signal
1e6
7
6
5
4
3
2
1
0
0
5000
10000
15000
20000
25000
30000


Time Domain Decoded Audio Signal
10000
5000
0
−5000
−10000
0
5000
10000
15000
20000
25000
30000


Time domain Encoded Audio Signal
10000
5000
0
−5000
−10000
0
5000
10000
15000
20000
25000
30000