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Answer 1

BOX 1: 2 White and 8 Black
BOX 2: 4 White and 11 Black
BOX 3: 3 White and 9 Black

a)

By Complement Rule;

P{At least one of them is white.} = 1 - P{None of them is white.} = 1 - P{All of them are black.}

$$= 1 - (\frac{8}{10}).(\frac{11}{15}).(\frac{9}{12}) = 1 - \frac{792}{1800} = \frac{1008}{1800} = 0.56$$

$Answer$ = **0.56**

b)

Probability that all of the three balls are white is equal to multiplication of the probabilities that taking a white ball from each box since they are independent.
P{All of the three balls drawn are white.} $= (\frac{2}{10}).(\frac{4}{15}).(\frac{3}{12}) = \frac{24}{1800} = 0.0133333$

$Answer$ = **0.0133333**

c)

If I draw two balls from the,

BOX 1: P{get two white balls} $= (\frac{2}{10}).(\frac{1}{9}) = \frac{2}{90} = 0.022222$

BOX 2: P{get two white balls} $= (\frac{4}{15}).(\frac{3}{14}) = \frac{12}{210} = 0.05714$

BOX 3: P{get two white balls} $= (\frac{3}{12}).(\frac{2}{11}) = \frac{10}{132} = 0.04545$

As seen above, probability of getting two white balls is highest in the case I draw from the **BOX 2**, since $0.05714 > 0.04545 > 0.022222$.
Hence, I would draw from **BOX 2**.

Answer = **BOX 2**

d)

If I first draw from the **BOX 1** and then again from **BOX 1**: $P = (\frac{2}{10}).(\frac{1}{9}) = \frac{2}{90} = 0.022222$

If I first draw from the **BOX 1** and then from **BOX 2**: $P = (\frac{2}{10}).(\frac{4}{15}) = \frac{8}{150} = 0.053333$

If I first draw from the **BOX 1** and then from **BOX 3**: $P = (\frac{2}{10}).(\frac{3}{12}) = \frac{6}{120} = 0.05$

If I first draw from the **BOX 2** and then from **BOX 1**: $P = (\frac{4}{15}).(\frac{2}{10}) = \frac{8}{150} = 0.053333$

If I first draw from the **BOX 2** and then again from **BOX 2**: $P = (\frac{4}{15}).(\frac{3}{14}) = \frac{12}{210} = 0.05714$

If I first draw from the **BOX 2** and then from **BOX 3**: $P = (\frac{4}{15}).(\frac{3}{12}) = \frac{12}{180} = 0.066667$

If I first draw from the **BOX 3** and then from **BOX 1**: $P = (\frac{3}{12}).(\frac{2}{10}) = \frac{6}{120} = 0.05$

If I first draw from the **BOX 3** and then from **BOX 2**: $P = (\frac{3}{12}).(\frac{4}{15}) = \frac{12}{180} = 0.066667$

If I first draw from the **BOX 3** and then again from **BOX 3**: $P = (\frac{3}{12}).(\frac{2}{11}) = \frac{6}{132} = 0.04545$

So, probability of getting two white balls is the highest in the case I draw one ball from **BOX 2** and one ball from **BOX 3**.

As seen above, order does not matter.

Hence, First I would draw one ball from **BOX 2** and then draw one ball from **BOX 3**, or **vice versa** because order is not important.

Answer: (**BOX 2**, **BOX 3**) or (**BOX 3**, **BOX 2**)

e)

Let random variable X be the number of white balls.

I will list the possibilities in such an order:
For example;

WWB $\longrightarrow$ Write from BOX 1, Write from BOX 2, Black from BOX 3

Support of X = {0,1,2,3}

The Distribution of X

$$P\{X = 0\} = P\{BBB\} = (\frac{8}{10}).(\frac{11}{15}).(\frac{9}{12}) = \frac{792}{1800}$$

$$P\{X = 1\} = P\{WBB \cup BWB \cup BBW\} = (\frac{2}{10}).(\frac{11}{15}).(\frac{9}{12}) + (\frac{8}{10}).(\frac{4}{15}).(\frac{9}{12}) + (\frac{8}{10}).(\frac{11}{15}).(\frac{3}{12}) = \frac{750}{1800}$$

$$P\{X = 2\} = P\{BWW \cup WBW \cup WWB\} = (\frac{8}{10}).(\frac{4}{15}).(\frac{3}{12}) + (\frac{2}{10}).(\frac{11}{15}).(\frac{3}{12}) + (\frac{2}{10}).(\frac{4}{15}).(\frac{9}{12}) = \frac{234}{1800}$$

$$P\{X = 3\} = P\{WWW\} = (\frac{2}{10}).(\frac{4}{15}).(\frac{3}{12}) = \frac{24}{1800}$$

Now, I will calculate expected value of X:

$$E(X) = (\frac{792}{1800}).0 + (\frac{750}{1800}).1 + (\frac{234}{1800}).2 + (\frac{24}{1800}).3 = \frac{1290}{1800} = 0.716667$$

Answer = **0.716667**

f)

We are drawing a random ball from a random box.

Let W = {Drawn ball is white.}

$$P\{W\} = \frac{1}{3}.\frac{2}{10} + \frac{1}{3}.\frac{4}{15} + \frac{1}{3}.\frac{3}{12} = \frac{43}{180}$$

Let $\{W_1\}$ = {Drawn ball from BOX 1 is white}

$$P\{W_1\} = \frac{1}{3}.\frac{2}{10} = \frac{2}{30} = \frac{1}{15}$$

The conditional probability that this ball was taken from BOX 1 given this ball is white is found as follows:

$$P = \frac{P\{W_1\}}{P\{W\}} == \frac{\frac{1}{15}}{\frac{43}{180}} = \frac{12}{43} = 0.279$$

Answer = **0.279**

Answer 2

a)

S = {Sam is corrupted.}
D = {The ring is destroyed.}

It is given that,

i) P{D | $\overline{S}$} = 0.9
ii)$P\{D|S\} = 0.5$
iii)$P\{S\} = 0.1$

By applying Complement Rule to iii) $\longrightarrow P\{\overline{S}\} = 1 - P\{S\} = 1 - 0.1 = 0.9$

I am required to find P{S | D}, which is the probability that Sam is corrupted given that the ring is destroyed.

Apply **Bayes Rule** and **Law of Total Probability**,

$$\text{P\{S | D\}} = \frac{P\{D|S\}.P\{S\}}{P\{D\}} = \frac{P\{D|S\}.P\{S\}}{P\{D|S)\}.P\{S\} + P\{D|\overline{S}\}.P\{\overline{S}\}} = \frac{(0.5).(0.1)}{(0.5).(0.1) + (0.9).(0.9)} = 0.05813953488$$

$Answer$ = **0.05813953488**

b)

Let F = {Frodo is corrupted.}

It is given that;

i) P{F} = 0.25
ii) P{D | F} = 0.2
iii) P{D |$\overline{S} \cap \overline{F}$} = 0.9
iv)$P\{D| \cap F\} = 0.05$

I am required to find P{S $\cap F|D\}$.

By **complement rule**; P{$\overline{F}$} $= 1 - P\{F\} = 1 - 0.25 = 0.75$

It is also given that the corruption of Frodo (F) and the corruption of Sam (S) are independent events. Therefore;

P{S $\cap F\} = P\{S\}.P\{F\} = (0.1).(0.25) = 0.025$

$P\{S \cap \overline{F}\} = P\{S\}.P\{\overline{F}\} = (0.1).(0.75) = 0.075$

$P\{\overline{S} \cap F\} = P\{\overline{S}\}.P\{F\} = (0.9).(0.25) = 0.225$

$P\{\overline{S} \cap \overline{F}\} = P\{\overline{S}\}.P\{\overline{F}\} = (0.9).(0.75) = 0.675$

By Bayes Rule and Total Law of Probability;

P{S $\cap F|D\}$

$$= \frac{P\{D|S\cap F\}.P\{S\cap F\}}{P\{D|S\cap F\}.P\{S\cap F\}+P\{D|S\cap \overline{F}\}.P\{S\cap \overline{F}\}+P\{D|\overline{S}\cap F\}.P\{\overline{S}\cap F\}+P\{D|\overline{S}\cap \overline{F}\}.P\{\overline{S}\cap \overline{F}\}}$$

We already know P{D | $\overline{S}\cap\overline{F}\}$ and P{D | S $\cap F\}$. **We will found** $P\{D|$S $\cap\overline{F}\}$ and P{D | $\overline{S}\cap F\}$.

Note that P{D $\cap S\} = P\{D\cap S\cap F\} + P\{D\cap S\cap \overline{F}\}$.

By using the fact **P{X** $\cap Y\} = P\{X|Y\}.P\{Y\},$ **we can rewrite the above equation as follows:**

$P\{D|S\}.P\{S\} = P\{D|S\cap F\}.P\{S\cap F\} + P\{D|S\cap \overline{F}\}.P\{S\cap \overline{F}$**}**

Putting known values;

(0.5) . (0.1) = (0.05) . (0.025) + $P\{D|S\cap\overline{F}\}$ **. (0.075)**

$\rightarrow P\{D|S\cap\overline{F}\} = 0.05$

Note that P{D $\cap F\} = P\{D\cap S\cap F\} + P\{D\cap \overline{S}\cap F\}$.

By using the fact that, $P\{X\cap Y\} = P\{X|Y\}.P\{Y\},$ **we can rewrite the above equation as follows:**

$P\{D|F\}.P\{F\} = P\{D|S\cap F\}.P\{S\cap F\} + P\{D|\overline{S}\cap F\}.P\{\overline{S}\cap F$**}**

Putting the known values;

(0.2) . (0.25) = (0.05) . (0.025) + P{D | $\overline{S}\cap F\}$.**(0.225)**

$\rightarrow P\{D|\overline{S}\cap F\} = 0.216667$

Now, we know all the values we need to know to calculate P{S $\cap F|D\}$:

P{S $\cap F|D\}$

$$= \frac{P\{D|S\cap F\}.P\{S\cap F\}}{P\{D|S\cap F\}.P\{S\cap F\}+P\{D|S\cap \overline{F}\}.P\{S\cap \overline{F}\}+P\{D|\overline{S}\cap F\}.P\{\overline{S}\cap F\}+P\{D|\overline{S}\cap \overline{F}\}.P\{\overline{S}\cap \overline{F}\}}$$

$$= \frac{(0.05).(0.025)}{(0.05).(0.025)+(0.65).(0.075)+(0.216667).(0.225)+(0.9).(0.675)}$$

$= 0.001769$

$Answer$ **= 0.001769**

Answer 3

a)

A = {The number of snowy days in Ankara }
I = {The number of snowy days in Istanbul }

It is given that in the table;
Support of A = {1,2,3}
Support of B = {1,2}

The event "there are 4 snowy days in total" consists of
- "2 snowy days in Ankara and 2 snowy days in Istanbul" ,
- "3 snowy days in Ankara and 1 snowy days in Istanbul"

P{Four snowy days in total} = P{a = 2, i=2} + P{a = 3, i = 1} = 0.2 + 0.12 = 0.32

Hence, the probability that there are four snowy days in total is 0.32

Answer = 0.32

b)

For snowy days in Ankara and Istanbul to be independent,

P{A = a, I = i} = P{A = a} . P{I = i} (1)
must be satisfied for all a and i in supports of A and I, respectively. If it is not satisfied, then this means they are dependent.

I will calculate marginal probabilities by using Addition Rule given in the textbook.

P{A = 1} = P{A = 1,I = 1} + P{A = 1, I = 2} = 0.18 + 0.12 = 0.30
P{A = 2} = P{A = 2, I = 1} + P{A = 2, I = 2} = 0.30 + 0.20 = 0.50
P{A = 3} = P{A = 3, I = 1} + P{A = 3, I = 2} = 0.12 + 0.08 = 0.20

P{I = 1} = P{A = 1, I = 1} + P{A = 2, I = 1} + P{A = 3, I = 1} = 0.18 + 0.30 + 0.12 = 0.60
P{I = 2} = P{A = 1, I = 2} + P{A = 2, I = 2} + P{A = 3, I = 2} = 0.12 + 0.2 + 0.08 = 0.40

We have obtained,

a	P{A = a}
1	0.3
2	0.5
3	0.2

Table: Marginal Distribution of A

i	P{I = i}
1	0.6
2	0.4

Table: Marginal Distribution of I

Now, we'll check for (1)

$\mathbf{P\{A = 1, I = 1\} = 0.18 = P\{A = 1\} \ . \ P\{I = 1\} = (0.3).(0.6) = 0.18} \longrightarrow satisfied$

$P\{A = 1, I = 2\} = 0.12 = P\{A = 1\}.P\{I = 1\} = (0.3).(0.4) = 0.12 \longrightarrow satisfied$

$P\{A = 2, I = 1\} = 0.30 = P\{A = 1\}.P\{I = 1\} = (0.5).(0.6) = 0.30 \longrightarrow satisfied$

$P\{A = 2, I = 2\} = 0.20 = P\{A = 1\}.P\{I = 1\} = (0.5).(0.4) = 0.20 \longrightarrow satisfied$

$P\{A = 3, I = 1\} = 0.12 = P\{A = 1\}.P\{I = 1\} = (0.2).(0.6) = 0.12 \longrightarrow satisfied$

$P\{A = 3, I = 2\} = 0.08 = P\{A = 1\}.P\{I = 1\} = (0.2).(0.4) = 0.08 \longrightarrow satisfied$

As seen above, (1) has been satisfied for all a and i. Hence, the snowy days in Ankara, A, and the snowy days in Istanbul, I, are independent.