CENG 424 - Logic for Computer Science
2023-1
Homework 1

Anıl Eren Göçer
e2448397@ceng.metu.edu.tr

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1. (a)

A	B	$\neg B$	$A \wedge \neg B$	$A \longrightarrow B$	$\neg(A \wedge \neg B)$
0	0	1	0	1	1
0	1	0	0	1	1
1	0	1	1	0	0
1	1	0	0	1	1

Observe that the last two columns which belongs to $A \longrightarrow B$ and $\neg(A \wedge \neg B)$ respectively are equal. So, these expressions are logical equivalences.

(b)

A	B	$\neg A$	$\neg B$	$\neg A \vee B$	$\neg B \vee A$	$A \longleftrightarrow B$	$(\neg A \vee B) \wedge (\neg B \vee A)$
0	0	1	1	1	1	1	1
0	1	1	0	1	0	0	0
1	0	0	1	0	1	0	0
1	1	0	0	1	1	1	1

Observe that the last two columns which belongs to $A \longleftrightarrow B$ and $(\neg A \vee B) \wedge (\neg B \vee A)$ respectively are equal. So, these expressions are logical equivalences.

(c)

A	B	$\neg A$	$\neg A \longrightarrow B$	$A \longrightarrow (\neg A \longrightarrow B)$	1
0	0	1	0	1	1
0	1	1	1	1	1
1	0	0	1	1	1
1	1	0	1	1	1

Observe that the last two columns which belongs to $A \longrightarrow (\neg A \longrightarrow B)$ and 1 respectively are equal. So, these expressions are logical equivalences.

(d)

A	B	C	$\neg A$	$\neg B$	$A \vee \neg B$	$\neg A \wedge B$	$(A \vee \neg B) \longrightarrow C$	$(\neg A \wedge B) \vee C$
0	0	0	1	1	1	0	0	0
0	0	1	1	1	1	0	1	1
0	1	0	1	0	0	1	1	1
0	1	1	1	0	0	1	1	1
1	0	0	0	1	1	0	0	0
1	0	1	0	1	1	0	1	1
1	1	0	0	0	1	0	0	0
1	1	1	0	0	1	0	1	1

Observe that the last two columns which belongs to $(A \vee \neg B) \longrightarrow C$ and $(\neg A \wedge B) \vee C$ respectively are equal. So, these expressions are logical equivalences.

2. In the following questions, $\Longrightarrow$ symbol is used to represent conversions.

(a) $A \wedge (\neg A \longrightarrow A) \implies$

$A \wedge (A \vee A) \implies$

$A \wedge A \implies$

A

The resulting expression is a literal, so it is a CNF. It can be easily converted to conjunctions of disjunctions of literals. For example, A is logically equivalent to $(A \vee A) \wedge (A \vee A)$.

(b) $(A \longrightarrow B) \longrightarrow ((A \longrightarrow \neg B) \longrightarrow \neg A) \implies$

$(A \longrightarrow B) \longrightarrow (\neg(A \longrightarrow \neg B) \vee \neg A) \implies$

$(A \longrightarrow B) \longrightarrow (\neg(\neg A \vee \neg B) \vee \neg A) \implies$

$(A \longrightarrow B) \longrightarrow ((A \wedge B) \vee \neg A) \implies$

$\neg(\neg A \vee B) \vee ((A \wedge B) \vee \neg A) \implies$

$(A \wedge \neg B) \vee ((A \wedge B) \vee \neg A) \implies$

$(A \wedge \neg B) \vee (A \wedge B) \vee \neg A \implies$

$[A \wedge (\neg B \vee B)] \vee \neg A \implies$

$[A \wedge \top] \vee \neg A \implies$

$A \vee \neg A \implies$

T

The resulting expression is true, so it is a CNF. It can be easily converted to conjunctions of disjunctions of literals. For example, $\top$ is logically equivalent to $(A \vee \neg A) \wedge (B \vee \neg B)$.

(c) $(A \longrightarrow (B \vee \neg C)) \wedge \neg A \wedge B \implies$

$(\neg A \vee (B \vee \neg C)) \wedge \neg A \wedge B \implies$

$(\neg A \vee B \vee \neg C) \wedge \neg A \wedge B$

The resulting expression is conjunction of disjunction of literals and literals itself, so it is a CNF. In order see clearly, we can convert it to conjunctions of disjunctions of litersals. For example, the resulting expression is logically equivalent to $(\neg A \vee B \vee \neg C) \wedge (\neg A \vee \neg A) \wedge (B \vee B)$.

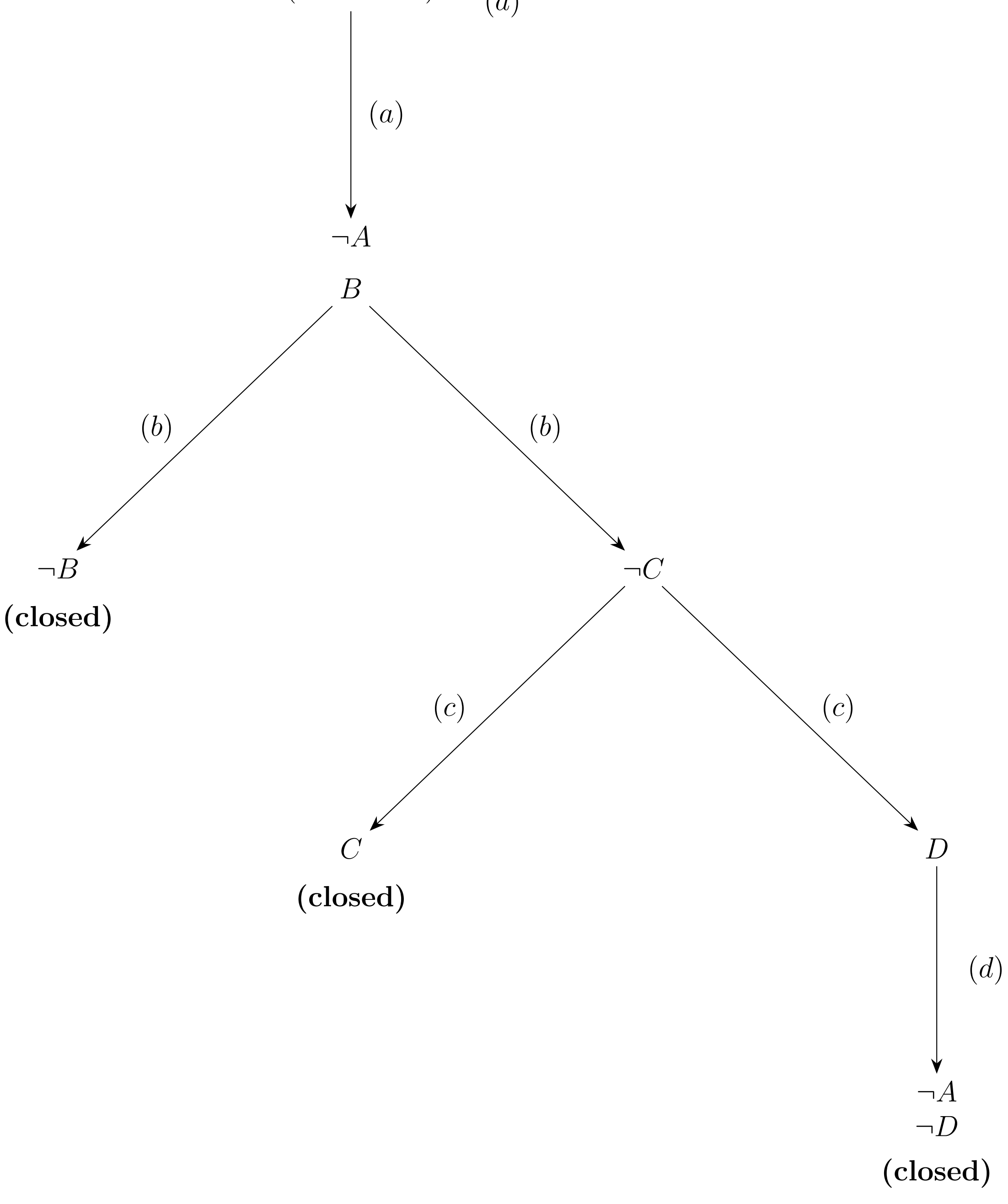
3.

$$\neg A \wedge B \qquad (a)$$

$$\neg(B \wedge C) \qquad (b)$$

$$C \vee D \qquad (c)$$

$$\neg(\neg A \longrightarrow D) \qquad (d)$$



All branches are **closed**, so given logical forms are **NOT** mutually consistent.

4. (bonus question)

(a)

1. $(A \longrightarrow C) \vee (B \longrightarrow C)$	premise
2. $A \wedge B$	assumption
3. $A \longrightarrow C$	assumption
4. A	$\wedge$ elimination, 2
5. C	$\longrightarrow elimination, 3, 4$
6. $B \longrightarrow C$	assumption
7. B	$\wedge elimination, 2$
8. C	$\longrightarrow elimination, 6, 7$
9. C	$\vee$ elimination 1, 3-5, 6-8
10. $(A \wedge B) \longrightarrow C$	$\longrightarrow$ introduction, 2-9

(b)

1. $\neg A \vee \neg B$	premise
2. $A \wedge B$	assumption
3. $\neg A$	assumption
4. A	$\wedge$ elimination, 2
5. $\perp$	$\neg$ elimination, 3,4
6. $\neg B$	assumption
7. B	$\wedge$ elimination, 2
8. $\perp$	$\neg$ elimination, 6,7
9. C	$\perp$ elimination 1,5,8
10. $(A \wedge B) \longrightarrow C$	$\longrightarrow$ introduction, 2-9