CENG 424 - Logic for Computer Science
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Homework 2

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1. Let's construct truth tables for the given premises and conclusion r.

p	q	r	$\neg q$	$p \land q$	$p \land q \Rightarrow r$	$q \lor \neg q$	p
0	0	0	1	0	1	1	0
0	0	1	1	0	1	1	0
0	1	0	0	0	1	1	0
0	1	1	0	0	1	1	0
1	0	0	1	0	1	1	1
1	0	1	1	0	1	1	1
1	1	0	0	1	0	1	1
1	1	1	0	1	1	1	1

Table 1: Truth Table for Premises

The last three column on Table 1 above corresponds the truth table values of the given premises.

p	q	r	r
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

Table 2: Truth Table for the Conclusion

The last columnd on Table 2 above corresponds the truth table values of the conclusion.

Now, we need to eliminate all rows that do not satisfy premises in the Table 1 and we need to eliminate all rows that do not satisfy premises in the Table 2. Please go to the next page.

p	q	r	$\neg q$	$p \wedge q$	$p \wedge q \Rightarrow r$	$q \vee \neg q$	p
X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X
1	0	0	1	0	1	1	1
1	0	1	1	0	1	1	1
X	X	X	X	X	X	X	X
1	1	1	0	1	1	1	1

Table 3: Truth Table for Premises after elimination of rows

p	q	r	r
X	X	X	X
0	0	1	1
X	X	X	X
0	1	1	1
X	X	X	X
1	0	1	1
X	X	X	X
1	1	1	1

Table 4: Truth Table for the Conclusion after elimination of rows

As you can see from the Table 3 and Table 4, remaining rows in the first table are 5th, 6th and 8th rows and remaining rows in the second table 2nd, 4th, 6th and 8th rows. Since $\{5, 6, 8\}$ is not a subset of $\{2, 4, 6, 8\}$, given premises does **NOT** entail the conclusion.

2.

1. $p \Rightarrow q$	Premise
2. $q \Rightarrow r$	Premise
3. $(q \Rightarrow r) \Rightarrow (p \Rightarrow (q \Rightarrow r))$	II
4. $p \Rightarrow (q \Rightarrow r)$	MP: 2, 3
5. $(p \Rightarrow (q \Rightarrow r)) \Rightarrow ((p \Rightarrow q) \Rightarrow (p \Rightarrow r))$	ID
6. $(p \Rightarrow q) \Rightarrow (p \Rightarrow r)$	MP: 4, 5
7. $p \Rightarrow r$	MP: 1, 6
8. $(p \Rightarrow r) \Rightarrow ((p \Rightarrow \neg r) \Rightarrow \neg p)$	CR
9. $((p \Rightarrow \neg r) \Rightarrow \neg p)$	MP: 7, 8

3. Here is a formal proof for p from the premise $\neg\neg p$.

1.	$\neg\neg p$	Premise
2.	$\neg\neg p \Rightarrow (\neg p \Rightarrow \neg\neg p)$	II
3.	$\neg p \Rightarrow \neg\neg p$	MP: 1, 2
4.	$(\neg p \Rightarrow \neg p) \Rightarrow ((\neg p \Rightarrow \neg\neg p) \Rightarrow p)$	CR
5.	$(\neg p \Rightarrow ((\neg p \Rightarrow \neg p) \Rightarrow \neg p)) \Rightarrow ((\neg p \Rightarrow (\neg p \Rightarrow \neg p)) \Rightarrow (\neg p \Rightarrow \neg p))$	ID
6.	$(\neg p \Rightarrow ((\neg p \Rightarrow \neg p) \Rightarrow \neg p)$	II
7.	$(\neg p \Rightarrow (\neg p \Rightarrow \neg p)) \Rightarrow (\neg p \Rightarrow \neg p)$	MP: 5, 6
8.	$\neg p \Rightarrow (\neg p \Rightarrow \neg p)$	II
9.	$\neg p \Rightarrow \neg p$	MP: 7, 8
10.	$(\neg p \Rightarrow \neg\neg p) \Rightarrow p$	MP: 4, 9
11.	p	MP: 3, 10

If you want to see the assignments in each step in which I used a standard axiom schemata, you can find them below.

Step 2: II, $\varphi = \neg\neg p$, $\Psi = \neg p$

Step 4: CR, $\varphi = \neg p$, $\Psi = p$

Step 5: ID $\varphi = \neg p$, $\Psi = \neg p \Rightarrow \neg p$, $\chi = \neg p$

Step 6: II, $\varphi = \neg p$, $\Psi = \neg p \Rightarrow \neg p$

Step 8: II, $\varphi = \neg p$, $\Psi = \neg p$

4. Let's rewrite the sentence with paranthesis in order to avoid confusion by taking precedence of operators into account.

$$((p \vee q) \Rightarrow r) \Rightarrow (p \Rightarrow (q \Rightarrow r))$$

Now, let's apply INDO steps to write it in the clausal form.

Implications out:

$$(\neg(p \vee q) \vee r) \Rightarrow (p \Rightarrow (q \Rightarrow r))$$
$$(\neg(p \vee q) \vee r) \Rightarrow (\neg p \vee (q \Rightarrow r))$$
$$(\neg(p \vee q) \vee r) \Rightarrow (\neg p \vee (\neg q \vee r))$$
$$\neg(\neg(p \vee q) \vee r) \vee (\neg p \vee (\neg q \vee r))$$

Negations in:

$$((p \vee q) \wedge \neg r) \vee (\neg p \vee (\neg q \vee r))$$

Distribution:

$$((p \wedge \neg r) \vee (q \wedge \neg r)) \vee (\neg p \vee \neg q \vee r)$$
$$(p \wedge \neg r) \vee (q \wedge \neg r) \vee \neg p \vee r \vee \neg q$$
$$(p \wedge \neg r) \vee (q \wedge \neg r) \vee (\neg p \vee r) \vee \neg q$$
$$T$$

Operators out:

$$\{\}$$

We have already obtained $\{\}$, so we showed that the sentence is valid.

5 (old question). First, I will write given premises in CNF form.

$$\begin{aligned}\phi_1 &= p \Rightarrow (q \vee r)\\ &= \neg p \vee (q \vee r)\\ &= \neg p \vee q \vee r\\ &= \{\neg p, q, r\}\end{aligned}$$

$$\begin{aligned}\phi_2 &= r \vee s \Rightarrow t\\ &= (r \vee s) \Rightarrow t\\ &= \neg(r \vee s) \vee t\\ &= (\neg r \wedge \neg s) \vee t\\ &= (t \vee \neg r) \wedge (t \vee \neg s)\\ &= \{t, \neg r\}, \{t, \neg s\}\end{aligned}$$

$$\begin{aligned}\phi_3 &= \neg(p \wedge q) \Rightarrow t\\ &= \neg(\neg(p \wedge q)) \vee t\\ &= (p \wedge q) \vee t\\ &= (p \vee t) \wedge (q \vee t)\\ &= \{p, t\}, \{q, t\}\end{aligned}$$

$$\begin{aligned}\phi_4 &= q \Rightarrow (s \wedge r)\\ &= \neg q \vee (s \wedge r)\\ &= (\neg q \vee s) \wedge (\neg q \vee r)\\ &= \{\neg q, s\}, \{\neg q, r\}\end{aligned}$$

$$\begin{aligned}\phi_5 &= \neg(\neg q \Rightarrow t)\\ &= \neg(\neg(\neg q) \vee t)\\ &= \neg(q \vee t)\\ &= \neg q \wedge \neg t\\ &= \{\neg q\}, \{\neg t\}\end{aligned}$$

Now, I will use these CNF forms in the following parts.

(a)

Observe that our Negated goal is $q \vee t$ which yields $\{q, t\}$.

1. $\{\neg q, s\}$	Premise, ϕ_4
2. $\{\neg q, r\}$	Premise, ϕ_4
3. $\{\neg q\}$	Premise, ϕ_5
4. $\{\neg t\}$	Premise, ϕ_5
5. $\{q, t\}$	Negated Goal
6. $\{t\}$	3, 5
7. $\{\}$	4, 6

We obtained $\{\}$ at step 7, so we showed $\{\phi_4, \phi_5\} \vdash \neg(q \vee t)$.

(b)

Observe that our Negated goal is $q \wedge \neg t$ which yields $\{q\}, \{\neg t\}$.

1. $\{\neg p, q, r\}$	Premise, ϕ_1
2. $\{t, \neg r\}$	Premise, ϕ_2
3. $\{t, \neg s\}$	Premise, ϕ_2
4. $\{p, t\}$	Premise, ϕ_3
5. $\{q, t\}$	Premise, ϕ_3
6. $\{\neg q, s\}$	Premise, ϕ_4
7. $\{\neg q, r\}$	Premise, ϕ_4
8. $\{q\}$	Negated Goal
9. $\{\neg t\}$	Negated Goal
10. $\{\neg r\}$	2, 9
11. $\{\neg s\}$	3, 9
12. $\{\neg q\}$	6, 11
13. $\{\}$	8, 12

We obtained $\{\}$ at step 13, so we showed $\{\phi_1, \phi_2, \phi_3, \phi_4\} \vdash (\neg q \vee t)$.

(c) Let's calculate negated goal

$$\neg(\neg q \vee (q \wedge s \wedge t)) = q \wedge \neg(q \wedge s \wedge t) = q \wedge (\neg q \vee \neg s \vee \neg t)$$

and it yields $\{q\}$, $\{\neg q, \neg s, \neg t\}$

1. $\{\neg q, s\}$	Premise, ϕ_4
2. $\{\neg q, r\}$	Premise, ϕ_4
3. $\{q\}$	Negated Goal
4. $\{\neg q, \neg s, \neg t\}$	Negated Goal
5. $\{s\}$	1, 3
6. $\{\neg q, t\}$	1, 4
7. $\{r\}$	2, 3
8. $\{\neg q, \neg s\}$	4, 6
9. $\{\neg s\}$	3, 8
10. $\{\}$	5, 9

We obtained $\{\}$ at step 10, so we showed $\{\phi_4\} \vdash \neg q \vee (q \wedge s \wedge t)$.