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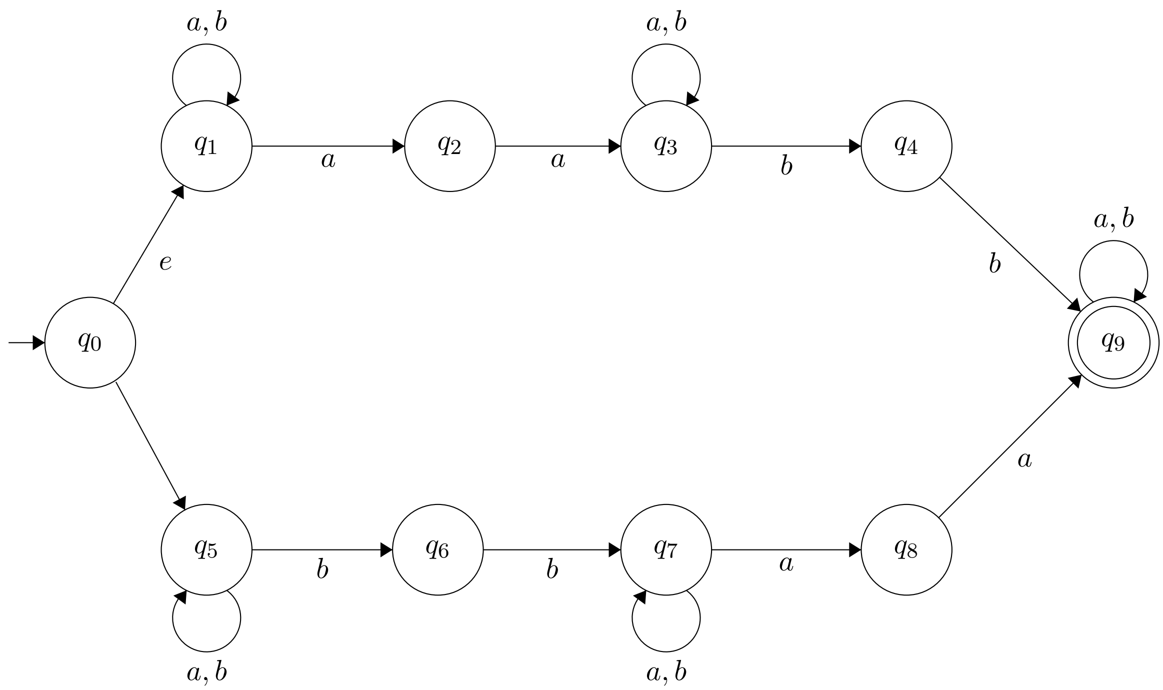
Answer 1

a)

$L_0 = [(a \cup b)^* aa(a \cup b)^* bb(a \cup b)^*] \cup [(a \cup b)^* bb(a \cup b)^* aa(a \cup b)^*]$

b)



M: The NFA recognizing the language L_0

Now, let's formally define M:

M = (K,Σ,δ,s,F) where ,

K = $\{q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_7, q_8, q_9\}$
$\Sigma = \{a, b\}$
s = q_0

F = $\{q_9\}$

and,

$\delta = \{(q_0, \epsilon, q_1), (q_0, \epsilon, q_5), (q_1, a, q_1), (q_1, b, q_1), (q_1, a, q_2)(q_2, a, q_3), (q_3, a, q_3), (q_3, b, q_3), (q_3, b, q_4), (q_4, b, q_9),$
$(q_5, a, q_5), (q_5, b, q_5), (q_5, b, q_6), (q_6, b, q_7), (q_7, a, q_7), (q_7, b, q_7), (q_7, a, q_8), (q_8, a, q_9), (q_9, a, q_9), (q_9, b, q_9)\}.$

c)

Let's construct a DFA, called $M^{'} = (K^{'}, \Sigma , \delta^{'}, s^{'}, F^{'})$, which is equivalent to the NFA in part b.

Let's apply the subset construction algorithm to M. Since M has 10 states, $M^{'}$ will have 2^{10} states. However, only few of these states will be relevant to the operation of $M^{'}$ i.e. those states that can be reached from state $s^{'}$ by reading some input string. Obviously, any state in $K^{'}$ that is not reachable from $s^{'}$ is irrelevant to the operation of $M^{'}$ and to the language accepted by it. We shall build this reachable part of $M^{'}$ by starting from $s^{'}$ and introducing a new state only when it is needed as the value of $\delta^{'}(q, x)$ for some state $q \in K^{'}$ already introduced and some $x \in \Sigma$.

Now, let's define ϵ-closure of each state in M.

$E(q_0) = \{q_0, q_1, q_5\}$	E$(q_5) = \{q_5\}$
$E(q_1) = \{q_1\}$	$E(q_6) = \{q_6\}$
$E(q_2) = \{q_2\}$	$E(q_7) = \{q_7\}$
$E(q_3) = \{q_3\}$	$E(q_8) = \{q_8\}$
$E(q_4) = \{q_4\}$	$E(q_9) = \{q_9\}$

Since $s^{'}$ = E$(q_0) = \{q_0, q_1, q_5\}$

(q_1, a, q_1), (q_1, a, q_2), (q_5, a, q_6) are all transition of the form (q,a,p) for some q $\in$ s$^{'}$. It follows that

$$\delta^{'}(s^{'}, a) = E(q_1) \cup E(q_2) \cup E(q_5) = \{q1, q2, q5\}$$

(q_1, b, q_1), (q_5, b, q_5), (q_5, b, q_6) are all transition of the form (q,b,p) for some q $\in$ s$^{'}$.

$$\delta^{'}(s^{'}, b) = E(q_1) \cup E(q_5) \cup E(q_6) = \{q1, q5, q6\}$$

Repeating this calculation for the newly introduced states, we have the following:

$$\delta^{'}(\{q_1, q_2, q_5\}, a) = \{q_1, q_2, q_3, q_5\}$$
$$\delta^{'}(\{q_1, q_2, q_5\}, b) = \{q_1, q_5, q_6\}$$

$$\delta^{'}(\{q_1, q_5, q_6\}, a) = \{q_1, q_2, q_5\}$$
$$\delta^{'}(\{q_1, q_5, q_6\}, b) = \{q_1, q_5, q_6, q_7\}$$

$$\delta^{'}(\{q_1, q_2, q_3, q_5\}, a) = \{q_1, q_2, q_3, q_5\}$$
$$\delta^{'}(\{q_1, q_2, q_3, q_5\}, b) = \{q_1, q_3, q_4, q_5, q_6\}$$

$$\delta^{'}(\{q_1, q_5, q_6, q_7\}, a) = \{q_1, q_2, q_5, q_7, q_8\}$$
$$\delta^{'}(\{q_1, q_5, q_6, q_7\}, b) = \{q_1, q_5, q_6, q_7\}$$

$$\delta^{'}(\{q_1, q_3, q_4, q_5, q_6\}, a) = \{q_1, q_2, q_3, q_5\}$$
$$\delta^{'}(\{q_1, q_3, q_4, q_5, q_6\}, b) = \{q_1, q_3, q_4, q_5, q_6, q_7, q_9\}$$

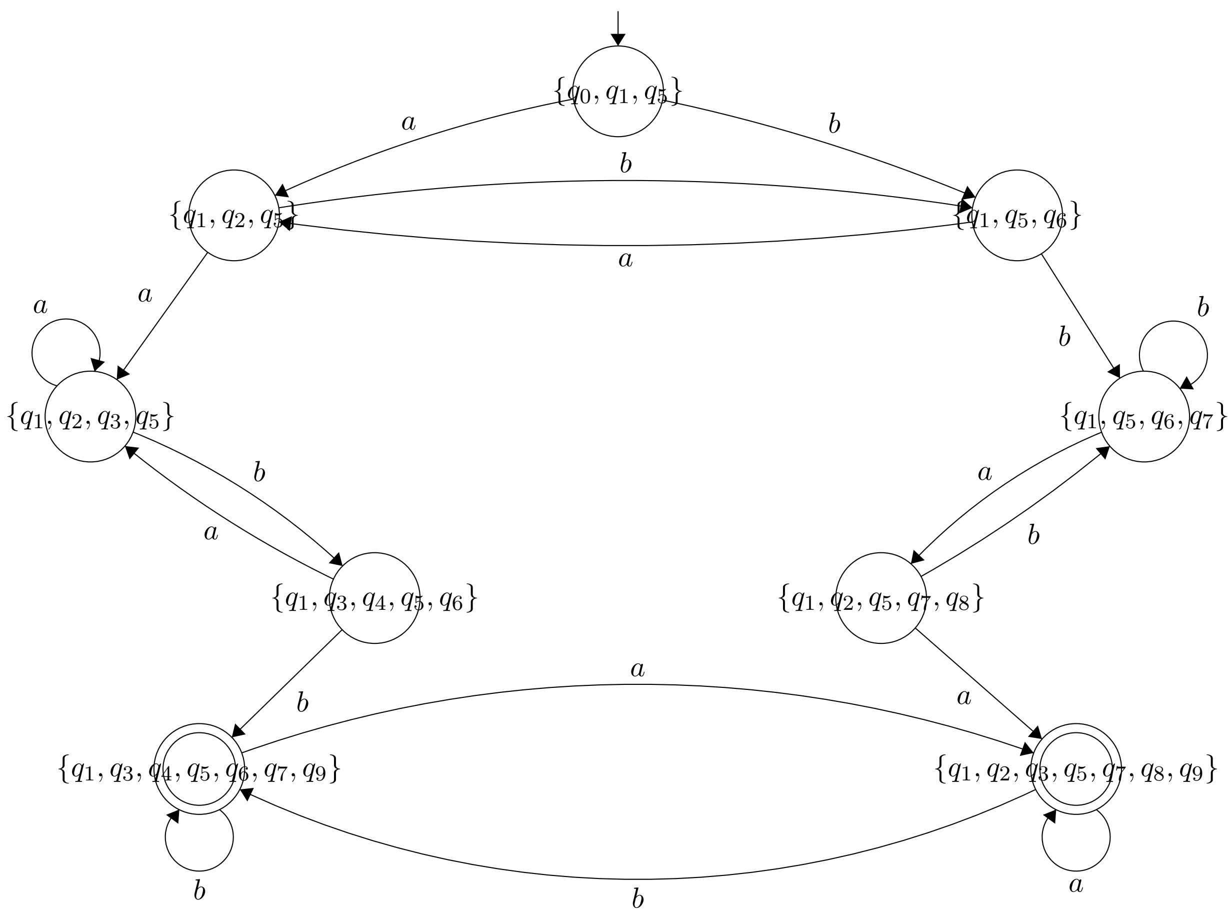
$\delta'(\{q_1, q_2, q_5, q_7, q_8\}, a) = \{q_1, q_2, q_3, q_5, q_7, q_8, q_9\}$
$\delta'(\{q_1, q_2, q_5, q_7, q_8\}, b) = \{q_1, q_5, q_6, q_7\}$

$\delta'(\{q_1, q_3, q_4, q_5, q_6, q_7, q_9\}, a) = \{q_1, q_2, q_3, q_5, q_7, q_8, q_9\}$
$\delta'(\{q_1, q_3, q_4, q_5, q_6, q_7, q_9\}, a) = \{q_1, q_3, q_4, q_5, q_6, q_7, q_9\}$

$\delta'(\{q_1, q_2, q_3, q_5, q_7, q_8, q_9\}, a) = \{q_1, q_2, q_3, q_5, q_7, q_8, q_9\}$
$\delta'(\{q_1, q_2, q_3, q_5, q_7, q_8, q_9\}, b) = \{q_1, q_3, q_4, q_5, q_6, q_7, q_9\}$

F', the set of final states, contains each set of states of which q_9 is a member, since q_9 is the sole member of F. So, $\{q_1, q_3, q_4, q_5, q_6, q_7, q_9\}$, $\{q_1, q_2, q_3, q_5, q_7, q_8, q_9\}$ are final states.

State Diagram of the M'



d)

Trace on NFA:

w' is accepted if and only if there is at least one sequence of moves terminating at a final state.

There are 8 possible sequence of moves which this NFA can follow when it is given w'. We need to check all of them.

1) $(q_0, bbabb) \vdash (\mathrm{q}_1, bbabb) \vdash (\mathrm{q}_1, babb) \vdash (\mathrm{q}_1, abb) \vdash (\mathrm{q}_1, bb) \vdash (\mathrm{q}_1, b) \vdash (\mathrm{q}_1, \epsilon)$

The NFA terminates at q_1, which is not a final state.

2) $(q_0, bbabb) \vdash (\mathrm{q}_1, bbabb) \vdash (\mathrm{q}_1, babb) \vdash (\mathrm{q}_1, abb) \vdash (\mathrm{q}_2, bb)$

The NFA gets stuck at q_2.

3) $(q_0, bbabb) \vdash (\mathrm{q}_5, bbabb) \vdash (\mathrm{q}_5, babb) \vdash (\mathrm{q}_5, abb) \vdash (\mathrm{q}_5, bb) \vdash (\mathrm{q}_5, b) \vdash (\mathrm{q}_5, \epsilon)$

The NFA terminates at q_5, which is not a final state.

4) $(q_0, bbabb) \vdash (\mathrm{q}_5, bbabb) \vdash (\mathrm{q}_5, babb) \vdash (\mathrm{q}_5, abb) \vdash (\mathrm{q}_5, bb) \vdash (\mathrm{q}_5, b) \vdash (\mathrm{q}_6, \epsilon)$

The NFA terminates at q_6, which is not a final state.

5) $(q_0, bbabb) \vdash (\mathrm{q}_5, bbabb) \vdash (\mathrm{q}_5, babb) \vdash (\mathrm{q}_5, abb) \vdash (\mathrm{q}_5, bb) \vdash (\mathrm{q}_6, b) \vdash (\mathrm{q}_7, \epsilon)$

The NFA terminates at q_7, which is not a final state.

2) $(q_0, bbabb) \vdash (\mathrm{q}_5, bbabb) \vdash (\mathrm{q}_5, babb) \vdash (\mathrm{q}_6, abb) \vdash (\mathrm{q}_2, bb)$

The NFA gets stuck at q_6.

7) $(q_0, bbabb) \vdash (\mathrm{q}_5, bbabb) \vdash (\mathrm{q}_6, babb) \vdash (\mathrm{q}_7, abb) \vdash (\mathrm{q}_7, bb) \vdash (\mathrm{q}_7, b) \vdash (\mathrm{q}_7, \epsilon)$

The NFA terminates at q_7, which is not a final state.

8) $(q_0, bbabb) \vdash (\mathrm{q}_5, bbabb) \vdash (\mathrm{q}_6, babb) \vdash (\mathrm{q}_7, abb) \vdash (\mathrm{q}_8, bb)$

The NFA gets stuck at q_8.

Therefore, there is no sequence of moves terminating at a final state. Hence, $w^{'}$ is **not accepted** by the NFA.

Trace on DFA:

$$\begin{aligned}(\{q_0, q_1, q_5\}, bbabb) &\vdash (\{\mathrm{q}_1, q_5, q_5\}, babb)\\ &\vdash (\{\mathrm{q}_1, q_5, q_6, q_7\}, abb)\\ &\vdash (\{\mathrm{q}_1, q_2, q_5, q_7, q_8\}, bb)\\ &\vdash (\{\mathrm{q}_1, q_5, q_6, q_7\}, b)\\ &\vdash (\{\mathrm{q}_1, q_5, q_6, q_7\}, \epsilon)\end{aligned}$$

As you can see, the DFA terminates at the state $(\{q_1, q_5, q_6, q_7\}$, which is not a final (accept) state. Hence, $w^{'}$ is **not accepted** by the DFA.

Answer 2

a)

Assume that L_1 is regular.

Let p be the **pumping length** for L_1 given by the pumping lemma.

Let s = $a^{p+1}b^p$. Then s can be split into s = xyz, satisfying the conditions of the pumping lemma which are as follows:

(1) For each i ≥ 0, xy$^i z \in$ L$_1$.
(2) $|y| > 0$
(3) $|xy| \leq p$

By condition 3 of the pumping lemma, $|xy| \leq p$, y consists only of a's.

The pumping lemma states that $xy^i z \in$ L$_1$ even when i = 0, so let's consider string $xy^0 z = xz$. Removing string y decreases the number of a's in s because of condition 2 of pumping lemma, $|y| > 0$. Recall that s has just one more a than b. Therefore, xz cannot have more a's than b's, so it cannot be a member of L_1. Thus, we obtain a contradiction.

Hence, L_1 is not regular.

Remember that class of regular languages is closed under complementation. So, a language A is regular if and only if $\overline{A}$ is regular. This means that class of non-regular languages is also closed under complementation, so a languages A is non-regular if and only if $\overline{A}$ is non regular.

As seen abover, we have proven that L_1 is non-regular. Thus, $\overline{L_1}$ is also non-regular.

Hence, $L_2 = \overline{L_1}$ is **not regular** .

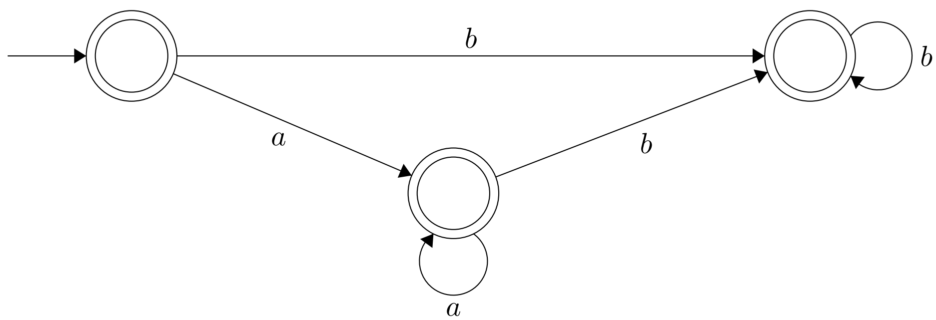
b)

Note that L_4 is a subset of L_5, namely $L_4 \subseteq L_5$.

Thus, $L_4 \cup L_5 \equiv L_5$

Observe that $L_5 = \{\epsilon, a, b, aa, bb, ab, ...\} = a^*b^*$, which is a regular expression showing that L_5 is regular.

Also, L_5 is recognized by the finite automaton given below:



Thus, L_5 is regular.

Now, consider $L_6 = b^*a(ab^*a)^*$. Because L_6 is generated by a regular expression, L_6 is regular.

Note that $L_4 \cup L_5 \cup L_6 \equiv L_5 \cup L_6$.

We have shown that L_5 and L_6 are regular.

Since regular languages are closed under union. $L_5 \cup L_6$ is regular.

Thus, $L_4 \cup L_5 \cup L_6$ is **regular** .