Student Information

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Answer 1

a)

Size of Monte Carlo simulation, N, that will guarantee an error not exceeding ε with high probability $1-\alpha$ can be calculated using Normal Approximation as follows:

$$N \geq 0.25.(\frac{z_{\alpha/2}}{\varepsilon})^2$$

parameters ϵ and α can be calculated from the given information in the question as follows:

$$\varepsilon = 0.02 \text{ , } 1-\alpha = 0.99 \longrightarrow \alpha = 0.01$$

Then,

$$N \geq (0.25).(\frac{z_{0.01/2}}{\varepsilon})^2$$

$$N \geq (0.25).(\frac{z_{0.005}}{\varepsilon})^2$$

$$N \geq (0.25).(\frac{2.575}{0.02})^2$$

$$N \geq 4144.140625$$

By rounding up, size of my Monte Carlo simulation, N, is found as **4145**.

Answer: 4145

(Note: The value of $z_{0.005}$ is taken from the TABLE A4 as 2.575)

b)

- The weight of each automobile, X_A, is a Gamma distributed random variable with $\alpha = 190$, and $\lambda = 0.15$. The expected value for the weight of an automobile is found as:

$$E(X_A) = \frac{\alpha}{\lambda} = \frac{190}{0.15} = 1266.6667 \text{ kilograms}$$

Answer: 1266.6667 kilograms

- The weight of each truck, X_T, is a Gamma distributed random variable with $\alpha = 110$, and $\lambda = 0.01$. The expected value for the weight of an automobile is found as:

$$E(X_T) = \frac{\alpha}{\lambda} = \frac{110}{0.01} = 11000 \text{ kilograms}$$

Answer: 11100 kilograms

- The number of automobiles that pass over the bridge on a day, Y_A, is Poisson random variable with $\lambda = 50$. The expected value for the number of automobiles passing over the bridge on a day is found as:

$$E(Y_A) = \lambda = 50$$

Then, expected value for the total weight of all automobiles that pass over the bridge on a day is found as:

$$E(Y_A).E(X_A) = 50.(1266.6667) = 63333.335 \text{ kilograms}$$

Answer: 63333.335 kilograms

- The number of trucks that pass over the bridge on a day, Y_T, is Poisson random variable with $\lambda = 10$. The expected value for the weight of an automobile is found as:

$$E(Y_T) = \lambda = 10$$

Then, expected value for the total weight of all trucks that pass over the bridge on a day is found as:

$$E(Y_T).E(X_T) = 10.11000 = 110000 \text{ kilograms}$$

Answer: 110000 kilograms

Answer 2

Answers to questions in Q2

- To estimate the probability that the total weight of all the vehicles that pass over the bridge on a day is more than 200 tons, I have just used mean function and outputed it using below lines:

```
        p_est = mean(TotalWeight > 200000);
        fprintf('Estimated probability = %f\n',p_est);
```

  For a sample run of the code, it outputs the probability as:

```
Estimated probability = 0.225090
```

- To estimate the total weight, X, of all the vehicles passing over the bridge on a day, I have just find mean of all steps of the simulation and outputed it using below lines:

```
    expectedWeight = mean(TotalWeight);
    fprintf('Expected weight = %f\n',expectedWeight);
```

  For a sample run of the code, it outputs the expected total weight as:

```
Expected weight = 173419.027428
```

  As you can see, it is very close to calculations we have done in Q1, which is $110000 + 63333.335 = 173333.335$.

- For the $Std(X)$, I have just used std function and outputed it using below lines:

```
    stdWeight = std(TotalWeight);
    fprintf('Standard deviation = %f\n',stdWeight);
```

  For a sample run of the code, it outputs the standard deviation as:

```
Standard deviation = 36178.337403
```

  Comment on the accuracy of our estimator, X: Remember that, in the beginning, I have calculated the size of Monte Carlo study of size 4145 so that error will not exceed 0.02 with the probability 0.99. Thanks to this calculation, the value I have calculated, 173333.335, and the value coming from the simulation, 173419.027428, are very close to each other. The error is $\varepsilon = 0.0005$, which is **NOT** exceeding 0.02. So, it satisfies our claims.

If you want to see the output as a whole, here it is:

```
>> hw4
Estimated probability = 0.225090
Expected weight = 173419.027428
Standard deviation = 36178.337403
fx >>
```

Brief Description of My Monte Carlo Study

First, I have used the size of simulation that I have found in Q1-a), which is 4145.

```
N = 4145; % size of the Monte Carlo simulation with alpha = 0.01 and epsilon =
    0.02
```

Then, I set the parameters given in the question.

```
% Parameters
n_lambda_automobile = 50; % lambda of the number of automobiles passing over
n_lambda_truck = 10; % lambda of the number of trucks passing over
w_alpha_automobile = 190; % alpha of the weight of each automobile
w_lambda_automobile = 0.15; % lambda of the weight of each automobile
w_alpha_truck = 110; % alpha of the weight of each truck
w_lambda_truck = 0.01; % lambda of the weight of each truck
```

After this, I created a vector that keeps the total weight of vehicles passing over the bridge for each run of Monte Carlo simulation, which are 4145 times in this case.

```
TotalWeight = zeros(N,1);
```

Then, for each turn of this loop:

```
for k=1:N;

end;
```

First, I sampled the number of automobiles and the number of trucks from the Poisson Distribution.

```
    % generate n_automobile by sampling from the Poisson Distribution
    U = rand;
    i = 0;
    F = exp(-n_lambda_automobile);
    while (U >= F);
        i = i + 1;
        F = F + exp(-n_lambda_automobile) * n_lambda_automobile^i/gamma(i+1);
    end;
    n_automobile = i;

    % generate n_truck by sampling from the Poisson Distribution
    U = rand;
    i = 0;
    F = exp(-n_lambda_truck);
    while (U >= F);
        i = i + 1;
        F = F + exp(-n_lambda_truck) * n_lambda_truck^i/gamma(i+1);
    end;
    n_truck = i;
```

Second, I sampled the weight of each automobile and truck from Gamma distribution.

```
% generate w_automobile from the Gamma Distribution
w_automobile = 0;
for j=1:n_automobile;
    w_automobile = w_automobile + sum(-1/w_lambda_automobile * log(rand(w_alpha_automobile,1)));
end;

% generate w_truck from the Gamma Distribution
w_truck = 0;
for j=1:n_truck;
    w_truck = w_truck + sum(-1/w_lambda_truck * log(rand(w_alpha_truck,1)));
end;
```

Lastly, I have set the value of total weight of this turn to corresponding place in the TotalWeight vector by:

```
TotalWeight(k) = w_automobile + w_truck;
```