The Role of Energy Efficiency in Productivity: Evidence from Canada*

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Abstract

This paper quantifies how misallocation of energy, alongside capital and labor, across provinces and sectors reduces productivity. Using Canadian provincial input–output data (2014–2020) within a Hsieh–Klenow framework, I decompose productivity losses into interprovincial (within-sector) and intersectoral (within-province) components and estimate each input's contribution separately. Unlike most studies focused on the manufacturing sector, this is the first comprehensive analysis of energy misallocation covering the entire economy. Results suggest misallocation lowers aggregate productivity by 5–8%, with most of the gap driven by within-sector distortions. Energy, though only around 8% of input costs, accounts for up to 1.5% of the gap—comparable to capital and exceeding labor—highlighting its outsized role. The findings identify interprovincial barriers and energy market distortions as key areas for narrowing productivity gaps and guiding climate policy. Reallocating energy could significantly improve productivity while reducing emissions, delivering a 'double dividend.'

Keywords: Misallocation; Total Factor Productivity; Distortions; Energy.

JEL classification: O11, O13, O41, O47, O51, D24, D61.

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1 Introduction

What are the productivity losses from energy misallocation, and how do they compare with those from capital and labor? This paper addresses this question by quantifying the contribution of energy, capital, and labor misallocation to aggregate productivity losses across provinces and sectors. I argue that energy is not only an essential input to production, but also has unique characteristics that make its misallocation particularly relevant for understanding productivity losses. While energy represents a relatively small share of production inputs, I find that its misallocation has a disproportionately large impact on productivity.

The idea that resource misallocation depresses output is well established Hsieh and Klenow (2009); Restuccia and Rogerson (2017); Bartelsman et al. (2013). Most of the literature, however, focuses on capital and labor allocation across firms within manufacturing sectors Bartelsman et al. (2013); Chen and Irarrazabal (2015). In contrast, energy—measured as direct purchases from sectors such as electricity, oil and natural gas, petroleum, coal, and other fuels—has received little attention Asker et al. (2019); Choi (2020); Tombe and Winter (2015), despite its central role in production and its high exposure to regulation, infrastructure, and policy frictions. This paper brings energy to the forefront, treating it as a third factor of production alongside capital and labor, and quantifies how its misallocation reduces productivity across sectors and provinces.

Energy differs from capital and labor in several critical ways. First, it is geographically less mobile: electricity grids, pipelines, and refinery capacity create sharp interprovincial differences in cost and availability. Second, energy prices are shaped by provincial regulations, climate policy, ownership structures, and subsidies, which limit adjustment through competitive markets. Third, unlike capital or labor, most forms of energy must be consumed close to the time of production—electricity especially—making short-run misallocation particularly costly. Fourth, energy markets are highly exposed to global price shocks in oil, gas, and coal, which can propagate unevenly across provinces, unlike wages or capital rental rates, which generally adjust more slowly and with less interprovincial variation. Finally, energy is closely linked to both capital and labor, complementing other inputs, so its misallocation can have cascading effects throughout the economy. These distinctive features of energy markets make Canada, with its diverse provincial regulations and infrastructure constraints, a particularly compelling setting to examine energy misallocation.

In Canada, provincial governments are primary authorities over energy regulation, pricing, and infrastructure, leading to substantial variation in energy costs and supply across provinces and sectors. At the same time, interprovincial trade remains fragmented: barriers to transporting electricity, oil, and gas across borders persist, limiting the scope for arbitrage.

Some studies focusing on the Canadian economy estimate the productivity losses from internal trade barriers at 3–7% Albrecht and Tombe (2016); Alvarez et al. (2019), but the focus of their analysis is on final goods. In contrast, this paper focuses on input allocation, highlighting that frictions in energy markets—a key production input—are a significant and underexplored source of aggregate productivity losses. By positioning energy misallocation within the broader internal trade debate, this paper underscores its policy relevance: more efficient allocation can simultaneously boost productivity and reduce environmental damage.

This analysis is highly relevant to Canada's environmental challenges and ongoing policy debates. Productivity gains from energy efficiency reduce both production costs and emissions, generating a "double dividend" Goulder (1995). I find that energy misallocation accounts for up to 2% of aggregate productivity losses (combining within- and between-sector components), suggesting these potential efficiency gains could significantly lower the economic cost of achieving climate targets. In the context of varying provincial carbon pricing and renewable energy potentials Tombe and Winter (2015); MacNab (2017), reallocating energy to its most productive uses can help meet national climate goals more efficiently. Furthermore, improving interprovincial energy flows can reduce reliance on local sources with higher emissions, providing environmental benefits that GDP-based measures alone may fail to capture. These characteristics of the Canadian economy further highlight energy misallocation as both an economic and environmental concern, with direct implications for infrastructure planning and climate policy.

Methodologically, this paper extends the Hsieh–Klenow (2009) framework by incorporating energy as a third input and allowing for imperfect substitution across provinces using an Armington-style CES aggregator. This extension captures the persistence of interprovincial price differences, which imply that provincial outputs are not perfect substitutes. Using detailed Statistics Canada provincial input–output tables from 2014–2020, I measure marginal revenue products of energy, capital, and labor at the province–sector level and compare them with efficient benchmarks. This approach allows me to decompose aggregate productivity losses into within-sector (interprovincial) and within-province (intersectoral) components, and to quantify how much each input contributes to productivity losses separately.

The results reveal substantial efficiency losses. Depending on the elasticity of substitution, aggregate productivity could be 8–9.4% higher under efficient allocation. Most of this loss originates from interprovincial misallocation: within-sector distortions account for 3.4–4.3 percentage points of the gap, compared with 1.3–4.0 points for intersectoral distortions. Energy consistently emerges as a key driver, explaining up to 1.6 percentage points of the within-sector loss and rivaling or exceeding the contributions of labor in most years. This suggests that interprovincial frictions in energy allocation—driven by regulation, infrastructure, and policy barriers—emerge as a key contributor to productivity gaps.

This paper makes several contributions. First, it provides the first comprehensive estimates of energy misallocation using province–sector input-output tables, moving beyond firm-level and manufacturing-focused analyses. Second, it connects allocative inefficiency in energy use to interprovincial trade frictions and infrastructure barriers, complementing and extending the literature on internal trade. Third, it frames energy misallocation as both an economic and environmental challenge: reallocating energy more efficiently generates a "double dividend," boosting productivity while reducing emissions. Fourth, it incorporates a spatial dimension, accounting for interprovincial variation in energy allocation, which is novel in the context of energy misallocation. Together, the results show that more integrated energy markets, streamlined regulations, and infrastructure investments can improve interprovincial energy flows and enhance productivity.

The remainder of the paper proceeds as follows. Section 2 describes the data and measurement approach. Section 3 develops the theoretical framework. Section 4 presents the main results. Section 5 concludes.

2 Data

This study examines the data from the Provincial Symmetric Input-Output Tables (Catalogue no. 15-211-X) published by Statistics Canada's Industry Accounts Division. These tables provide a comprehensive, annually consistent depiction of inter-industry transactions at the provincial level in Canada. Specifically, I utilize the detailed aggregation level for the years from 2014 to 2020 inclusive, which offers a high-resolution view of economic flows across provinces and sectors. The analysis covers 234 sectors and 10 Canadian provinces, providing a comprehensive view of regional production structures and inter-industry dependencies.

The symmetric input-output tables restructure the standard supply and use tables into an industry-by-industry framework, allowing for clearer identification of the production structure and intermediate demand relationships. The data captures all inter-sectoral purchases—including expenditures on imports, inventory withdrawals, and primary inputs—making them well-suited for structural and efficiency analyses. The final demand tables similarly record all purchases by final demand categories from provincial and imported sources.

The data used reflect Statistics Canada's most detailed industry classifications and are harmonized across years, enabling consistent cross-provincial and intertemporal comparisons. The version of the tables used in this study corresponds to the level of aggregation that was previously known as "Aggregation Level S," which was renamed "Detailed" in 2019[1].

3 Model

3.1 Aggregate Output and Sectoral Shares

I consider a standard model of monopolistic competition with heterogeneous provinces, indexed by i. I closely follow the framework of (Hsieh and Klenow, 2009) with a natural extension of energy as an input in the production function. In the economy, Cobb-Douglas production technology produces a single final product Y:

$$Y = \prod_{s=1}^{S} Y_s^{\theta_s}, \quad \text{where} \sum_{s=1}^{S} \theta_s = 1. \tag{3.1}$$

$\theta_s = \frac{P_s Y_s}{PY}$ is the revenue share of the sector in the final national product. Here P_s refers to the price of product Y_s, and P refers to the price of final good. The output of each sector, Y_s, is assumed as a Constant Elasticity of Substitution (CES) aggregate of contributions from each province i, given by:

$$Y_s = \left(\sum_i Y_{si}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \tag{3.2}$$

σ is the elasticity of substitution between provinces and is assumed to be constant.

The sectoral profit maximization problem yields the aggregate price index P.[2]

$$P = \prod_{s=1}^{S} \left(\frac{P_s}{\theta_s} \right)^{\theta_s} \tag{3.3}$$

Intuitively, the aggregate price index can be interpreted as a geometric average of sectoral prices, weighted by their revenue shares. In this formulation, sectors that account for a larger share of national production have a proportionally greater influence on the overall price level.

Furthermore, profit maximization at the province level yields the optimal revenue at the province–sector level.[3]

$$P_{si} Y_{si} = P_s Y_s^{\frac{1}{\sigma}} Y_{si}^{\frac{\sigma-1}{\sigma}} = P_{si}^{1-\sigma} P_s^{\sigma} Y_s. \tag{3.4}$$

[1]For methodological transparency and further technical detail, the construction of these tables is documented by Statistics Canada and available through direct inquiry with the Industry Accounts Division.

[2]See Appendix B.1 for the derivation.

[3]See Appendix B.2 for the derivation.

The second equality follows from straightforward algebra, obtained by raising both sides of the first expression to the power σ and rearranging terms.

The cost minimization problem at the provincial level gives us the sectoral price index.[4]

$$P_s = \left(\sum_i P_{si}^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \tag{3.5}$$

Next, I introduce the terms of the production function and productivity. Consider the standard profit maximization problem for sector s in province i. The production function is assumed to be Cobb-Douglas with three inputs: capital (K), labor (L) and energy (E):

$$Y_{si} = A_{si} K_{si}^{\alpha_s} L_{si}^{\beta_s} E_{si}^{\gamma_s}, \quad \text{where } \alpha_s + \beta_s + \gamma_s = 1. \tag{3.6}$$

Here, A_{si} denotes total factor productivity in physical units (TFPQ). Input shares are allowed to vary across sectors but are held constant across provinces within a sector. Each sector s in province i then solves the following profit maximization problem.

$$\max_{K_{si}, L_{si}, E_{si}} \quad P_{si} Y_{si} - (1 + \tau_{K_{si}}) r K_{si} - (1 + \tau_{L_{si}}) w L_{si} - (1 + \tau_{E_{si}}) p_E E_{si} \tag{3.7}$$

Each input is subject to input distortions $\tau_{K_{si}}, \tau_{L_{si}}, \tau_{E_{si}}$, so that sectors in each province face distorted input prices. By substituting the expressions for Y_{si} and $P_{si}Y_{si}$ above, we solve this standard problem to obtain the marginal revenue products for each input.[5]

$$MRPK_{si} = \alpha_s \frac{(\sigma - 1)}{\sigma} \frac{P_s Y_s^{\frac{1}{\sigma}} Y_{si}^{-\frac{1}{\sigma}}}{K_{si}} = \alpha_s \frac{(\sigma - 1)}{\sigma} \frac{P_{si} Y_{si}}{K_{si}} = (1 + \tau_{K_{si}}) r, \tag{3.8}$$

$$MRPL_{si} = \beta_s \frac{(\sigma - 1)}{\sigma} \frac{P_s Y_s^{\frac{1}{\sigma}} Y_{si}^{-\frac{1}{\sigma}}}{L_{si}} = \beta_s \frac{(\sigma - 1)}{\sigma} \frac{P_{si} Y_{si}}{L_{si}} = (1 + \tau_{L_{si}}) w, \tag{3.9}$$

$$MRPE_{si} = \gamma_s \frac{(\sigma - 1)}{\sigma} \frac{P_s Y_s^{\frac{1}{\sigma}} Y_{si}^{-\frac{1}{\sigma}}}{E_{si}} = \gamma_s \frac{(\sigma - 1)}{\sigma} \frac{P_{si} Y_{si}}{E_{si}} = (1 + \tau_{E_{si}}) p_E. \tag{3.10}$$

Where $MRPK_{si}, MRPL_{si}, MRPE_{si}$ are the marginal revenue products of capital, labor, and energy, respectively.

Define the following productivity measures:

$$TFPQ_{si} = A_{si} = \frac{Y_{si}}{K_{si}^{\alpha_s} L_{si}^{\beta_s} E_{si}^{\gamma_s}} \tag{3.11}$$

[4]See Appendix B.3 for the derivation

[5]See Appendix B.4 for the derivation.

$$TFPR_{si} = P_{si}A_{si} = \frac{P_{si}Y_{si}}{K_{si}^{\alpha_s} L_{si}^{\beta_s} E_{si}^{\gamma_s}} \tag{3.12}$$

where TFPQ denotes total factor productivity in physical terms, which may naturally differ across sectors without implying any distortion. In contrast, TFPR indicates total factor revenue productivity, and it should be equalized across provinces and sectors if it were not for distortions. Thus, any observed dispersion in TFPR reflects misallocation of inputs and translates into a lower aggregate output.

It is straightforward to show that the geometric average of marginal revenue products is proportional to TFPR, and equivalently, to the geometric mean of the distortion terms (τ).[6]

Hence,

$$\begin{aligned} TFPR_{si} &\propto (MRPK_{si})^{\alpha_s}(MRPL_{si})^{\beta_s}(MRPE_{si})^{\gamma_s} \\ &\propto (1+\tau_{K_{si}})^{\alpha_s}(1+\tau_{L_{si}})^{\beta_s}(1+\tau_{E_{si}})^{\gamma_s} \end{aligned} \tag{3.13}$$

Defining the sectoral weighted average marginal revenue product for inputs as follows.

$$\overline{MRPK_s} = \frac{\sum_i K_{si} MRPK_{si}}{\sum_i K_{si}} \tag{3.14}$$

We can then calculate deviations from the weighted sectoral averages which capture the dispersion and, consequently, the degree of input misallocation.[7]

$$\frac{\overline{MRPK_s}}{MRPK_{si}} = \frac{1}{(1+\tau_{K_{si}})\sum_i \frac{1}{(1+\tau_{K_{si}})}\frac{P_{si}Y_{si}}{P_sY_s}} \tag{3.15}$$

$$\frac{\overline{MRPL_s}}{MRPL_{si}} = \frac{1}{(1+\tau_{L_{si}})\sum_i \frac{1}{(1+\tau_{L_{si}})}\frac{P_{si}Y_{si}}{P_sY_s}} \tag{3.16}$$

$$\frac{\overline{MRPE_s}}{MRPE_{si}} = \frac{1}{(1+\tau_{E_{si}})\sum_i \frac{1}{(1+\tau_{E_{si}})}\frac{P_{si}Y_{si}}{P_sY_s}} \tag{3.17}$$

Intuitively, these are the deviations from the optimal allocation of inputs across sectors and provinces.

By rearranging terms and simplifying, we arrive at the following expression:

$$A_s = \left[\sum_i \left(A_{si}\left(\frac{\overline{MRPK_s}}{MRPK_{si}}\right)^{\alpha}\left(\frac{\overline{MRPL_s}}{MRPL_{si}}\right)^{\beta}\left(\frac{\overline{MRPE_s}}{MRPE_{si}}\right)^{\gamma}\right)^{\sigma-1}\right]^{\frac{1}{\sigma-1}} \tag{3.18}$$

[6]See Appendix B.4 for the derivation.
[7]See Appendix B.5 for the derivation.

This expression represents total factor productivity at the sector level. It allows a direct comparison between the observed sectoral productivity and the efficient benchmark. In the absence of distortions ($\tau_K = \tau_L = \tau_E = 0$) A_s attains its efficient level A_s^*. When distortions are present, provinces with larger distortions contribute less than their optimal level to sectoral output, therefore reducing aggregate TFP.

$$A_s^* = \left(\sum_i A_{si}^{\sigma-1} \right)^{\frac{1}{\sigma-1}} \tag{3.19}$$

The ratio of observed productivity to its efficient benchmark defines the sectoral productivity gap, which can be expressed as:

$$\frac{A_s}{A_s^*} = \left[\sum_i \left(\frac{A_{si}}{A_s^*} \left(\frac{\overline{MRPK_s}}{MRPK_{si}} \right)^{\alpha} \left(\frac{\overline{MRPL_s}}{MRPL_{si}} \right)^{\beta} \left(\frac{\overline{MRPE_s}}{MRPE_{si}} \right)^{\gamma} \right)^{\sigma-1} \right]^{\frac{1}{\sigma-1}} \tag{3.20}$$

This formulation shows that sectoral TFP, A_s, is maximized when marginal products of capital, labor, and energy are equalized across provinces. In that efficient allocation, each province contributes proportional to its underlying productivity A_{si}, and sectoral TFP reaches A_s^*. When distortions drive wedges between marginal products, inputs are misallocated: some provinces operate with too little capital, labor, or energy while others have too much. This dispersion reduces the effective weight of distorted provinces in the CES aggregator, lowering observed sectoral TFP relative to the efficient benchmark.

3.2 Aggregate Productivity

To obtain aggregate productivity, I express total output in terms of sectoral outputs, using the Cobb-Douglas production technology at the sector level as assumed above, with sector shares θ_s. This formulation then leads to:[8]

$$\frac{A}{A^*} = \prod_s \left(\left(\frac{A_s}{A_s^*} \right) \left(\frac{k_s}{k_s^*} \right)^{\alpha_s} \left(\frac{l_s}{l_s^*} \right)^{\beta_s} \left(\frac{e_s}{e_s^*} \right)^{\gamma_s} \right)^{\theta_s} \tag{3.21}$$

This expression shows that aggregate productivity relative to its efficient benchmark depends jointly on sector-level productivity, $\frac{A_s}{A_s^*}$, and the allocation of capital, labor, and energy across sectors. Deviations of these factors from their efficient levels (k_s^*, l_s^*, e_s^*) reduce aggregate productivity, reflecting input misallocation both within and across sectors.

[8] See Appendix B.7 for the derivation.

3.3 Aggregate Productivity Decomposition

We decompose national-level productivity to identify how distortions in input allocation contribute to overall welfare loss. Let $\hat{x} = x/x^*$ denote the ratio of the observed variable to its efficient benchmark (i.e., in the absence of distortions). Aggregate productivity gap, $\hat{A}$, at the national level can be expressed as:

$$\hat{A} = \frac{A}{A^*} = \underbrace{\prod_s \left(\frac{A_s}{A_s^*}\right)^{\theta_s}}_{\text{Within-sector misallocation}} \times \underbrace{\prod_s \left(\left(\frac{k_s}{k_s^*}\right)^{\alpha_s}\left(\frac{l_s}{l_s^*}\right)^{\beta_s}\left(\frac{e_s}{e_s^*}\right)^{\gamma_s}\right)^{\theta_s}}_{\text{Between-sector misallocation}} \tag{3.22}$$

The first term captures *within-sector* misallocation, while the second captures *between-sector* misallocation. At the aggregate level, *within-sector* productivity gap, $\hat{A}_{within}$, can be written as:

$$\prod_s \left(\frac{\left[\sum_i \left(A_{si}\left(\frac{R_{si}/(1+\tau_{K_{si}})}{\sum_i R_{si}/(1+\tau_{K_{si}})}\right)^{\alpha_s}\left(\frac{R_{si}/(1+\tau_{L_{si}})}{\sum_i R_{si}/(1+\tau_{L_{si}})}\right)^{\beta_s}\left(\frac{R_{si}/(1+\tau_{E_{si}})}{\sum_i R_{si}/(1+\tau_{E_{si}})}\right)^{\gamma_s}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}}{\left[\sum_i A_{si}^{\sigma-1}\right]^{\frac{1}{\sigma-1}}} \right)^{\theta_s} \tag{3.23}$$

Here, $R_{si} = P_{si}Y_{si}/P_sY_s$ is the revenue share of province i in sector s. Similarly, *between-sector* misallocation, $\hat{A}_{between}$, is expressed as:

$$\prod_s \left(\underbrace{\left(\frac{\left(\frac{1}{\overline{1+\tau_{Ks}}}\right)\cdot\sum_s \alpha_s\theta_s}{\sum_s \frac{\alpha_s\theta_s}{\overline{1+\tau_{Ks}}}}\right)^{\alpha_s}}_{\text{Capital Misallocation}} \times \underbrace{\left(\frac{\left(\frac{1}{\overline{1+\tau_{Ls}}}\right)\cdot\sum_s \beta_s\theta_s}{\sum_s \frac{\beta_s\theta_s}{\overline{1+\tau_{Ls}}}}\right)^{\beta_s}}_{\text{Labor Misallocation}} \times \underbrace{\left(\frac{\left(\frac{1}{\overline{1+\tau_{Es}}}\right)\cdot\sum_s \gamma_s\theta_s}{\sum_s \frac{\gamma_s\theta_s}{\overline{1+\tau_{Es}}}}\right)^{\gamma_s}}_{\text{Energy Misallocation}} \right)^{\theta_s} \tag{3.24}$$

Where $\overline{1+\tau_{Ks}} = \left(\sum_i \frac{R_{si}}{1+\tau_{K_{si}}}\right)^{-1}$ denotes the harmonic mean of $1+\tau_{K_{si}}$ weighted by R_{si}.[9] The harmonic mean captures the effective price distortions weighted by revenue shares. This decomposition allows us to calculate the productivity loss from misallocation of various inputs, as well as the within-sector and between-provinces. Therefore, we can identify how much of the overall productivity gap is due to misallocation of each specific input, as well as inefficiencies within sectors across provinces.

[9]See Appendix B.7 for the derivation.

4 Results

I begin showing the dispersion of marginal revenue products for each input by using the regression residuals from the following equation.[10]

$$\ln(\frac{P_{si}Y_{si}}{rK_{si}}) = \beta_0 + \sum_s \beta_s \gamma_s + \epsilon_{si} \tag{4.1}$$

Here, the dependent variable is the ratio of revenue to capital spending. The term β_0 captures constant factors that are common across provinces, such as the province-level substitutability parameter σ, the rental rate of capital, the wage level, and the price of energy. The sector-level fixed effects, γ_s, reflect sector-specific averages. Finally, the residual term, ϵ_{si}, measures the dispersion of marginal revenue products from their sector-level averages after accounting for the constant terms and sector-level effects. In the empirical analysis below, the residual variances from the above regression are used to illustrate the degree of factor misallocation across sectors and provinces.

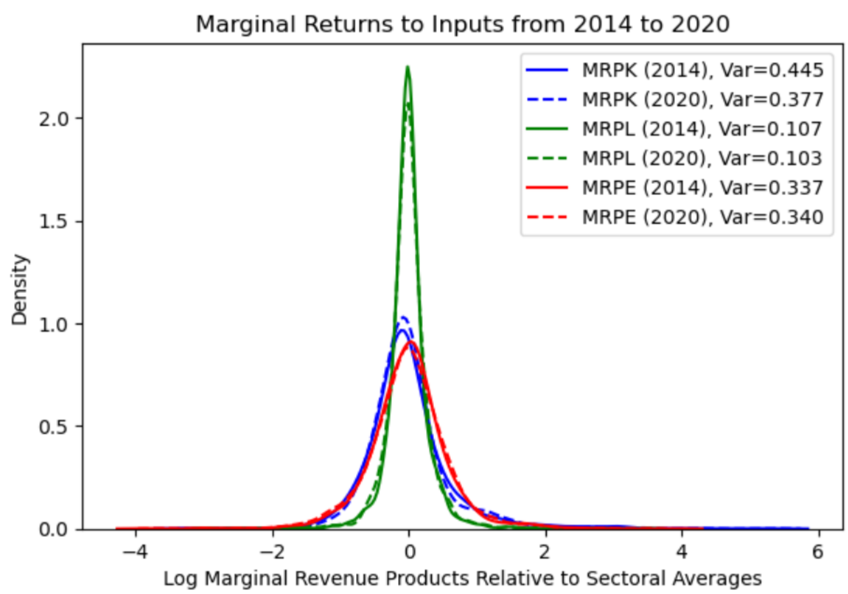

Figure 4.1: Marginal returns to inputs, 2014 vs. 2020.

Notes: The figure compares the distribution of marginal revenue products (MRPs) for capital, labor, and energy between 2014 and 2020. Residual variances from the regression specification in Equation B.49 provide a measure of input-specific misallocation. A tighter distribution implies more efficient allocation, whereas a wider spread reflects stronger distortions and greater potential gains from reallocation.

In Figure 4.1, I present the distribution of marginal returns to labor, capital, and energy for 2014 and 2020. The comparison reveals distinct patterns of input allocation across provinces

[10]See Appendix B.6

and sectors. Labor displays the lowest dispersion in both years, suggesting relatively limited misallocation. This is consistent with the literature, which finds that labor markets typically adjust more flexibly across regions and sectors. Capital also shows signs of improvement, with its distribution becoming more compressed over time, pointing to gains in allocative efficiency. Energy, however, shows a different pattern. Its dispersion remains wide and largely unchanged between 2014 and 2020, indicating that relative marginal returns to energy have not converged across provinces and sectors. Compared to labor and capital, this persistence suggests that energy is subject to more persistent distortions. As a result, while capital allocation appears to improve over time, the allocation of energy shows little evidence of adjustment, implying that differences in energy productivity remain an important source of overall factor misallocation.

Figure A.1 further examines changes in the dispersion of marginal revenue products across inputs between 2014 and 2020 at the sector level. For visual clarity, sectors are ranked by the magnitude of these changes, and the top 30 sectors with the largest shifts are plotted. A decline in variance indicates improved allocative efficiency, whereas an increase reflects a greater misallocation. The results reveal a heterogeneous pattern: some sectors exhibit notable efficiency gains, while others experience increasing dispersion, particularly for energy. It is also common for a sector to improve in one input while showing a deterioration in another. Labor exhibits the smallest changes in dispersion, whereas capital and energy account for the majority of variation over the period, with energy displaying the most persistent dispersion. This sectoral heterogeneity highlights that misallocation is not uniform across the economy but shaped by sector-specific characteristics and dynamics. In line with the broader misallocation literature, persistent dispersion often reflects frictions—regulatory, infrastructural, or institutional—that impede the reallocation of inputs to their most productive uses. In this context, the evidence indicates that energy is the input where such frictions are most pronounced.

4.1 Relative TFPR Dispersion by Province

Next, I examine the dispersion of total factor productivity revenue (TFPR) relative to sectoral averages. Higher dispersion reflects greater productivity losses due to misallocation across the economy. By definition, TFPR corresponds to the geometric average of marginal revenue products under a Cobb–Douglas production function, which allows for a straightforward expression of TFPR relative to its sectoral mean. To quantify the variation, I compute the variance of the logarithm of this ratio, capturing the percentage deviation from the sectoral average. The results are presented for the first and last years of the sample period, 2014 and 2020, at the province level.

$$\frac{TFPR_{si}}{\overline{TFPR_s}} = \left[\left(\frac{\overline{MRPK_s}}{MRPK_{si}} \right)^{\alpha_s} \left(\frac{\overline{MRPL_s}}{MRPL_{si}} \right)^{\beta_s} \left(\frac{\overline{MRPE_s}}{MRPE_{si}} \right)^{\gamma_s} \right]^{-1} \tag{4.2}$$

This equation implies the following explicit formula that we can use to illustrate the productivity gap at the provincial level.

$$\frac{TFPR_{si}}{\overline{TFPR_s}} = \frac{(1+\tau_{K_{si}})^{\alpha_s}(1+\tau_{L_{si}})^{\beta_s}(1+\tau_{E_{si}})^{\gamma_s}}{\left(\frac{1}{\sum_i (1+\tau_{K_{si}})\frac{P_{si}Y_{si}}{P_sY_s}}\right)^{\alpha_s}\left(\frac{1}{\sum_i (1+\tau_{L_{si}})\frac{P_{si}Y_{si}}{P_sY_s}}\right)^{\beta_s}\left(\frac{1}{\sum_i (1+\tau_{E_{si}})\frac{P_{si}Y_{si}}{P_sY_s}}\right)^{\gamma_s}} \tag{4.3}$$

This expression shows how a province's TFPR deviates from the sectoral average as a function of distortions in the use of capital, labor, and energy. Each input-specific distortion, denoted by $\tau_{K_{si}}, \tau_{L_{si}}, \tau_{E_{si}}$, amplifies or reduces the province's marginal revenue product relative to the sector mean. The exponents $\alpha_s, \beta_s, \gamma_s$ reflect the input shares in the production function, so the overall TFPR gap aggregates the effect of misallocation across all inputs. In essence, Equation (4.3) provides a clear, quantitative measure of how much a province's productivity is affected by misallocation compared to the sector benchmark, allowing us to compute and compare the relative efficiency across provinces.

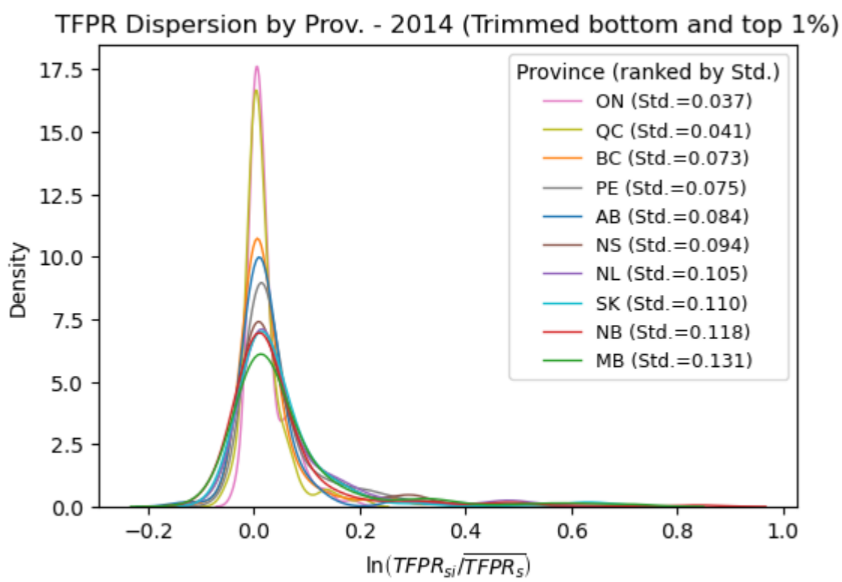

FIGURE 4.2: Relative TFPR dispersion by provinces in 2014

Notes: TFPR is expressed relative to the sectoral average and it reflects the log-distribution of 4.3. Provinces are ordered by deviation from the sectoral benchmark. Top and bottom 1% of observations are trimmed for clarity.

Figures 4.2 and 4.3 present the dispersion of TFPR relative to the sectoral benchmark

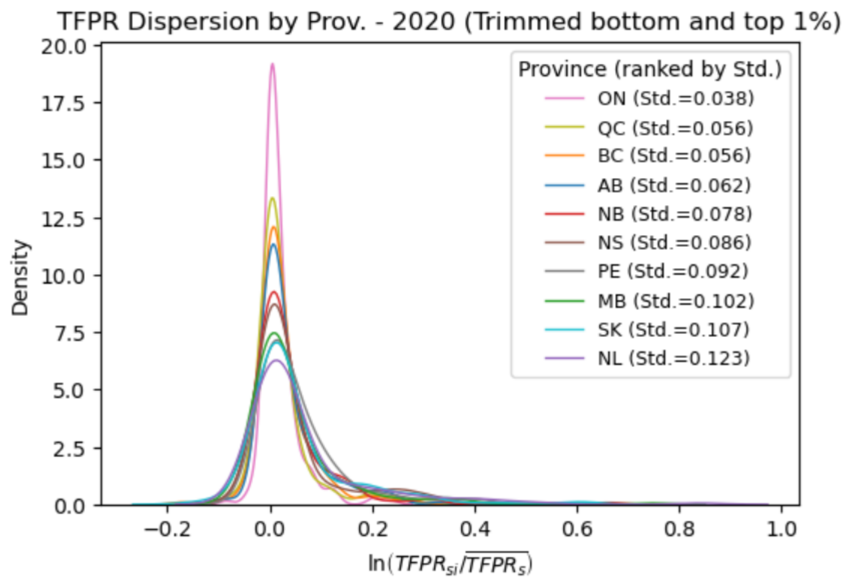

FIGURE 4.3: Relative TFPR dispersion by provinces in 2020

Notes: TFPR is expressed relative to the sectoral average and it reflects the log-distribution of 4.3. Provinces are ordered by deviation from the sectoral benchmark. Top and bottom 1% of observations are trimmed for clarity.

for 2014 and 2020 across provinces. The provinces in the legend are ordered by the degree of dispersion around the benchmark. Ontario and Quebec exhibit the lowest levels of misallocation-related productivity loss in both years, although Quebec's position deteriorates noticeably between 2014 and 2020. In contrast, Alberta and British Columbia show some improvement over the same period, whereas New Brunswick experiences a clear decline. Manitoba and Saskatchewan, despite some improvement, remain among the provinces most affected by misallocation. To ensure a clearer presentation of the distributions, I trim the top and bottom 1% of observations to remove outliers. Therefore, these figures highlight the relative positions of provinces in terms of efficiency and misallocation over time.

In general, the dispersion of TFPR relative to the benchmark can differ by as much as 20% from the sectoral average, suggesting that misallocation is a significant contributor to productivity losses in Canada.

4.2 Total Misallocation and Decomposition

I decompose aggregate total factor productivity (TFP) losses at the province-sector level into two components: Within-sector across provinces and between-sector within provinces, along with input-specific productivity loss measures from misallocation of capital, labor, and energy separately.

Tables 1 and 2 report these results for the years 2014-2020, under two reference elasticities of substitution between sectors, $\sigma = 3$ and $\sigma = 7$ respectively.

Table 1 reports the estimated potential gains in total factor productivity (TFP) by eliminating input misallocation across provinces and sectors in Canada over the period 2014–2020 when $\sigma = 3$. The numbers are expressed as percentage deviations from the efficient benchmark, that is, $(1 - \frac{A}{A*}) \times 100$, where A denotes observed TFP and A* denotes counterfactual TFP under efficient input allocation. These values measure the extent to which distortions in capital, labor, and energy allocation lower aggregate productivity.

Table 1: Potential TFP Gains from Input Reallocation (in %), 2014–2020, $\sigma = 3$

Component	**2014**	**2015**	**2016**	**2017**	**2018**	**2019**	**2020**
Total Misallocation	8.05	6.46	4.90	4.84	5.28	5.74	5.08
Between-sector Misallocation	3.96	2.25	1.27	1.53	1.53	1.96	1.63
Capital	1.80	1.22	0.55	0.66	0.71	0.88	0.83
Labor	0.78	0.50	0.36	0.37	0.39	0.37	0.46
Energy	1.43	0.55	0.36	0.50	0.45	0.73	0.34
Within-sector Misallocation	4.26	4.31	3.67	3.37	3.81	3.86	3.50
Capital	1.33	1.27	1.75	1.36	1.71	1.85	1.33
Labor	2.55	2.76	1.26	1.54	1.69	1.54	1.73
Energy	1.53	1.67	1.14	0.93	1.09	0.98	0.81

Notes: Total potential TFP gains, $(1 - \hat{A})$ x 100, are decomposed into between-sector (within provinces) and within-sector (across provinces) components. By construction, the sum of within- and between-sector components equals the total potential gain with some rounding errors. Between-sector misallocation is further decomposed by input (capital, labor, energy); these input-specific contributions sum to the between-sector component with some rounding errors. Within-sector input-specific contributions are derived from counterfactuals in which distortions in one input are retained while distortions in the other two inputs are set to zero. Because of the non-linear structure in the expression, these input-specific within-sector measures do not sum to the within-sector component (see Appendix B.7, Equations B.82, B.83, B.84), it rather delivers the *relative* contributions from each input under within-sector component. Results are based on a conservative elasticity of substitution across provinces assumption, $\sigma = 3$, yielding lower-bound estimates of potential gains.

Under a conservative elasticity of substitution between provinces, $\sigma = 3$, the total potential productivity gains from eliminating input misallocation range from 8% in 2014 to 5% in 2020. This gradual decline suggests a modest but consistent improvement in allocative efficiency over the period. The decomposition of these potential gains into within and between sectors reveals a clear pattern: Most of the productivity loss is sourced from within-sector misallocation, that is, inefficient allocation of inputs across provinces within the same sector. In 2014, for example, within-sector misallocation accounted for 4.26% of the total 8.05% potential gain, while between-sector misallocation contributed 3.96%. This relationship persists throughout the sample period, highlighting the importance of interprovincial distortions within the same sectors.

Breaking down the sources of between-sector misallocation by input type reveals that capital is the largest contributor. In 2014, for example, capital misallocation alone accounted for 1.80 percentage points of the between-sector productivity loss. Energy followed closely with 1.43 percentage points, while labor accounted for 0.78 percentage points. These patterns are broadly consistent over time: capital misallocation remains the primary factor, contributing between 0.55 and 1.80 percentage points per year. However, despite making up only around 8% of total input use, energy is consistently the second-largest source of misallocation, with potential contributions ranging from 0.34 to 1.43 percentage points annually. This disproportionate impact highlights the critical role of energy in shaping allocative efficiency. In contrast, labor misallocation is relatively modest—typically below 0.5 percentage points—indicating a more efficient allocation of labor across sectors. The outsized role of energy, despite its relatively small input share, highlights why energy misallocation deserves more attention alongside capital in discussions of productivity and input use.

Within-sector misallocation presents a more complex picture, particularly in terms of input-specific contributions. Notably, the sum of the input-specific within-sector misallocation terms—capital, labor, and energy—does not equal the total within-sector misallocation. This is by construction: each input-specific figure results from a separate counterfactual scenario in which distortions in that particular input are retained, while distortions for other two inputs are set to zero. Due to the non-linear, complementary nature of input use in production, the total gains from removing all distortions simultaneously are not equal to the sum of individual gains. However, we observe several patterns. Labor misallocation plays a prominent role within sectors, especially in earlier years, indicating that labor mobility is limited due to provincial differences. For instance, inefficiencies in labor allocation contributed 2.55 percentage points to within-sector (across provinces) misallocation in 2014 and 2.76 points in 2015. Capital also remains a persistent and important source of within-sector inefficiency, particularly in later years in the sample—for example, accounting for 1.85 percentage points in 2019. Energy misallocation within sectors is also sizeable, despite its low input share, with contributions of 1.53 percentage points in 2014 and 0.83 % in 2015. While the results suggest that energy efficiency within sectors across provinces is improving, it remains a significant source of productivity loss, highlighting the need for further research.

Table 2: Potential TFP Gains from Input Reallocation (in %), 2014–2020, $\sigma = 7$

Component	2014	2015	2016	2017	2018	2019	2020
Total Misallocation	9.40	7.51	5.81	5.72	6.85	6.52	5.81
Between-sector Misallocation	3.96	2.25	1.27	1.53	1.53	1.96	1.63
Capital	1.80	1.22	0.55	0.66	0.71	0.88	0.83
Labor	0.78	0.50	0.36	0.37	0.39	0.37	0.46
Energy	1.43	0.55	0.36	0.50	0.45	0.73	0.34
Within-sector Misallocation	5.66	5.39	4.60	4.26	5.40	4.65	4.25
Capital	5.12	3.30	4.89	4.29	7.28	4.41	3.51
Labor	5.85	5.51	3.35	4.21	5.49	3.75	3.82
Energy	3.27	3.36	2.36	2.23	3.37	2.07	1.85

Notes: Total potential TFP gains, $(1 - \hat{A})$ x 100, are decomposed into between-sector (within provinces) and within-sector (across provinces) components. By construction, the sum of within- and between-sector components equals the total potential gain with some rounding errors. Between-sector misallocation is further decomposed by input (capital, labor, energy); these input-specific contributions sum to the between-sector component with some rounding errors. Within-sector input-specific contributions are derived from counterfactuals in which distortions in one input are retained while distortions in the other two inputs are set to zero. Because of the non-linear structure in the expression, these input-specific within-sector measures do not sum to the within-sector component (see Appendix B.7, Equations B.82, B.83, B.84), it rather delivers the *relative* contributions from each input under within-sector component. Results are based on a high elasticity of substitution across provinces assumption, $\sigma = 7$, yielding upper-bound estimates of potential gains.

To examine the range of potential productivity gains under a less conservative elasticity of substitution, I repeat the analysis assuming $\sigma = 7$. Table 2 reports the results of this exercise, indicating that potential aggregate productivity gains are higher, ranging from 5.81% in 2020 to 9.40% in 2014. This higher estimate arises because a larger σ allows for greater substitutability of inputs across provinces, which amplifies the productivity gap between the observed allocation and the efficient frontier. Under this higher elasticity, the within-sector misallocation component reaches to the range of 4.25 to 5.66 percentage points per year. In contrast, the between-sector component remains unchanged from the $\sigma = 3$ case. This is consistent with the model's structure, in which the between-sector term is independent of σ. The increase in potential aggregate productivity gains is, therefore, sourced entirely from larger distortions within sectors across provinces. I further break it down by input type to explore the role of each input; potential productivity gains from reallocating capital within sectors reach 7.28 percentage points in 2018. Labor-related distortions also remain sizable, accounting for 5.85 and 5.51 percentage points of within-sector misallocation in 2014 and 2015, respectively. While energy's share in production is relatively small, its misallocation is disproportionately large, with potential productivity gains peaking at 3.37 percentage points in 2018. These patterns indicate that capital and energy misallocation persistently account for a substantial share of the measured productivity gap when σ is higher.

The results indicate two key sources of aggregate productivity loss: (1) interprovincial input distortions within sectors, and (2) misallocation of energy use despite its low share of production as an input, highlighting the need to focus on energy as an important source of aggregate productivity loss. Labor appears to be more efficiently allocated across sectors within a province, whereas there is still significant room for improvement across provinces, highlighting the significance of interprovincial distortions in the labor market as well. The disproportionate role of energy in driving both within- and between-sector inefficiencies suggests that policies aimed at improving pricing, coordination, and investment in energy markets could generate sizable productivity gains.

5 Conclusion

This paper quantifies the productivity loss in Canada resulting from the misallocation of energy, labor, and capital across sectors and provinces. By extending the standard misallocation framework to include energy as a distinct input and using detailed provincial input–output data, I estimate the potential gains from reallocating inputs to their most productive uses at the province-by-sector level and decompose these gains into within-sector (inter-provincial) and between-sector (inter-sectoral) components, with both terms further broken down by input contributions relative to an efficient benchmark.

The results reveal significant inefficiencies. In the benchmark case, under conservative assumption of interprovincial substitutability ($\sigma = 3$), efficient reallocation of inputs between provinces and sectors could increase aggregate output by approximately 5- 8%. Within-sector (interprovincial) misallocation—driven by regulatory fragmentation and limited energy trade—accounts for roughly 3.4 to 4.3 percentage points of this loss, while between-sector (intersectoral) misallocation contributes about 1.3 to 4.0 percentage points. In the between-sector dimension, energy consistently emerges as the second-largest contributor—exceeding labor in all years and, in some cases, approaching capital's contribution—despite representing only about 8% of input costs. In the within-sector dimension, the role of energy remains substantial, accounting for 1.0 to 1.7 percentage points of total loss.

These results highlight two key points. First, energy plays a disproportionately large role in shaping aggregate productivity, despite its relatively small share in production—a characteristic that has often led to its omission in misallocation analyses. This highlights the need to recognize energy as a critical driver of aggregate productivity loss, and to incorporate it more systematically into future misallocation-related studies, while also integrating energy policy more centrally into broader productivity and growth strategies. Second, interprovincial fragmentation, including trade barriers and regulatory differences, remains a major obstacle to efficient input allocation, suggesting that greater coordination between provincial regulations and investment in interprovincial trade infrastructure could generate significant economic gains.

In conclusion, the results underscore the importance of considering both spatial and input-specific dimensions of misallocation when evaluating aggregate productivity. By explicitly incorporating energy as a distinct factor of production, this paper adds a critical layer to the misallocation literature and demonstrates that even relatively small cost inputs can generate sizable distortions when poorly allocated. The Canadian context—with its provincial regulatory heterogeneity and limited internal trade—further illustrates how institutional frictions can amplify inefficiencies. While focused on Canada, the analysis offers broader lessons for federal systems with fragmented energy regulation or limited interregional energy trade, such as the United States, Australia, or European countries. These insights provide a foundation for future research on province-sector-level misallocation and the role of coordination in national productivity strategies. Addressing these inefficiencies is not only essential for uncovering Canada's growth potential, but also for aligning economic and climate objectives in a coherent policy framework.

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Appendix

A Figures

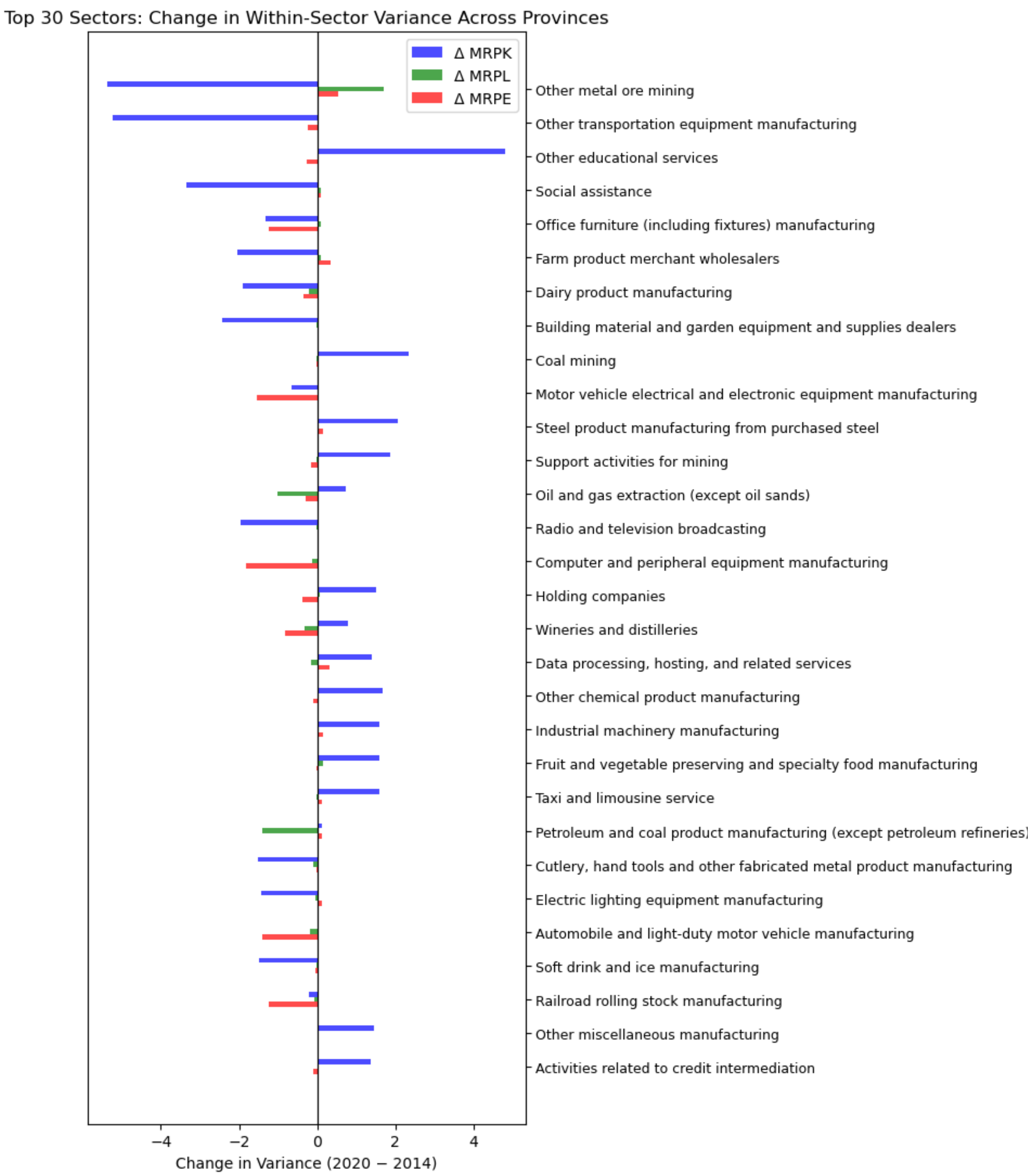

FIGURE A.1: Changes in the variation of marginal returns to inputs by sector, 2014 vs. 2020.

Notes: Changes in the dispersion of marginal revenue products by input and sector between 2014 and 2020. Sectors are ranked by the magnitude of change, and the top 30 are shown. A decline in variance indicates improved allocative efficiency, while an increase signals greater misallocation. Labor shows minimal change, whereas capital and energy account for most of the observed variation, with energy exhibiting the largest persistent dispersion.

B Mathematical Derivations

B.1 Derivation of Sectoral Expenditure Shares and the Aggregate Price Index

Consider the single final good defined as a Cobb–Douglas composite of sectoral outputs:

$$Y = \prod_{s=1}^{S} Y_s^{\theta_s}, \qquad \sum_{s=1}^{S} \theta_s = 1, \tag{B.1}$$

where $\theta_s \in (0,1)$ are the fixed expenditure (or technology) weights associated with each sector s. The corresponding expenditure minimization problem can be written as

$$\min_{\{Y_s\}_{s=1}^{S}} \sum_{s=1}^{S} P_s Y_s \quad \text{subject to} \quad Y = \prod_{s=1}^{S} Y_s^{\theta_s}, \tag{B.2}$$

where P_s denotes the sectoral price.

The first-order conditions of this problem imply that

$$\frac{P_s Y_s}{PY} = \theta_s, \tag{B.3}$$

so that the parameter θ_s coincides with the expenditure share of sector s in the total output. Economically, this reflects the well-known property of Cobb–Douglas preferences and technologies: a constant share of income is allocated to each component, independent of prices or income levels.

Isolating and substituting Y_s back into the aggregator yields the dual expression for the aggregate price index.

$$P = \prod_{s=1}^{S} \left(\frac{P_s}{\theta_s} \right)^{\theta_s}, \tag{B.4}$$

which is the exact price index associated with the Cobb–Douglas composite.

B.2 Province-Level Pricing and Revenue

Provinces solve:

$$\max_{Y_{si}} \quad P_s \left(\sum_i Y_{si}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} - \sum_i P_{si} Y_{si}. \tag{B.5}$$

The First Order Condition is

$$P_{si} = P_s Y_s^{\frac{1}{\sigma}} Y_{si}^{\frac{-1}{\sigma}}, \tag{B.6}$$

Multiplying both sides by Y_{si} to get an expression for province-level revenue yields:

$$P_{si}Y_{si} = P_s Y_s^{\frac{1}{\sigma}} Y_{si}^{\frac{\sigma-1}{\sigma}}. \tag{B.7}$$

This expression shows that the revenue of province i in sector s depends both on the output of the sector, Y_s, and on the province's own output, Y_{si}. The CES structure implies that demand reallocates toward provinces with relatively more competitive provinces (lower P_{si}), with the elasticity σ governing the strength of this substitution. In equilibrium, summing over provinces recovers total sectoral revenue P_sY_s, consistent with CES aggregation.

B.3 Derivation of the Sectoral Price Index

The sectoral price index P_s can be derived using the cost minimization problem. The total output in sector s is given by a CES aggregator of individual variety outputs Y_{si}:

$$Y_s = \left(\sum_i Y_{si}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}, \tag{B.8}$$

where $\sigma > 1$ is the elasticity of substitution between varieties. This functional form captures the idea that output combines provincial outputs with imperfect substitutability.

To derive P_s, consider a representative cost-minimizing province solving the problem:

$$\min_{Y_{si}} \sum_i P_{si}Y_{si} \quad \text{subject to} \quad Y_s = \left(\sum_i Y_{si}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}. \tag{B.9}$$

This ensures that the price index reflects the minimum expenditure required to obtain one unit of Y_s. The Lagrangian can be written as:

$$\mathcal{L} = \sum_i P_{si}Y_{si} + \lambda_s \left(Y_s^{\frac{\sigma-1}{\sigma}} - \sum_i Y_{si}^{\frac{\sigma-1}{\sigma}}\right). \tag{B.10}$$

The first-order condition with respect to Y_{si} is:

$$P_{si} - \lambda_s \frac{\sigma-1}{\sigma} Y_{si}^{-\frac{1}{\sigma}} = 0. \tag{B.11}$$

Total costs are given by:

$$\sum_i P_{si}Y_{si} = \sum_i \lambda_s \frac{\sigma-1}{\sigma} Y_{si}^{-\frac{1}{\sigma}} Y_{si} = \lambda_s \frac{\sigma-1}{\sigma} \sum_i Y_{si}^{\frac{\sigma-1}{\sigma}} = \lambda_s \frac{\sigma-1}{\sigma} Y_s^{\frac{\sigma-1}{\sigma}} \tag{B.12}$$

Rearranging yields the demand function:

$$Y_{si}^{\frac{\sigma-1}{\sigma}} = \left(\lambda_s \frac{1}{P_{si}} \frac{\sigma-1}{\sigma}\right)^{\sigma-1}. \tag{B.13}$$

This shows that demand for a province's output is decreasing in its own price, with the elasticity governed by σ.

Substituting into the CES aggregator and solving for λ_s gives:

$$Y_s = \left(\sum_i Y_{si}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} = \left(\lambda_s \frac{\sigma-1}{\sigma}\right)^{\sigma} \left(\sum_i \left(\frac{1}{P_{si}}\right)^{\sigma-1}\right)^{\frac{\sigma}{\sigma-1}} \tag{B.14}$$

$$\lambda_s = \frac{\sigma}{\sigma-1} Y_s^{\frac{1}{\sigma}} \left(\sum_i \left(\frac{1}{P_{si}}\right)^{\sigma-1}\right)^{\frac{-1}{\sigma-1}} \tag{B.15}$$

Plugging this back into the total cost expression yields:

$$\sum_i P_{si} Y_{si} = \lambda_s \frac{\sigma-1}{\sigma} Y_s^{\frac{\sigma-1}{\sigma}} = Y_s^{\frac{1}{\sigma}} \left(\sum_i \left(\frac{1}{P_{si}}\right)^{\sigma-1}\right)^{\frac{-1}{\sigma-1}} Y_s^{\frac{\sigma-1}{\sigma}} = Y_s \left(\sum_i P_{si}^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \tag{B.16}$$

Therefore, based on the following equation.

$$\sum_i P_{si} Y_{si} = P_s Y_s \tag{B.17}$$

we conclude that

$$P_s = \left(\sum_i P_{si}^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \tag{B.18}$$

The CES price index P_s is therefore the standard aggregator of province-level prices, reflecting how cheaper provinces receive larger expenditure shares, with σ governing the substitutability across provinces.

B.4 Distortions in Input Markets

Suppose each input market is subject to a distortion $\tau_{K_{si}}, \tau_{L_{si}}, \tau_{E_{si}}$, so that firms face distorted input prices. The firm's profit maximization problem is:

$$\max_{K_{si}, L_{si}, E_{si}} \quad P_s Y_s^{\frac{1}{\sigma}} Y_{si}^{\frac{\sigma-1}{\sigma}} - (1+\tau_{K_{si}}) r K_{si} - (1+\tau_{L_{si}}) w L_{si} - (1+\tau_{E_{si}}) p_E E_{si}. \tag{B.19}$$

To make this explicit, a Cobb–Douglas production function at the province-sector level is assumed: $Y_{si} = A_{si}K_{si}^{\alpha_s}L_{si}^{\beta_s}E_{si}^{\gamma_s}$, where $\alpha_s + \beta_s + \gamma_s = 1$.

$$\max_{K_{si},L_{si},E_{si}} \quad P_s Y_s^{\frac{1}{\sigma}} \left(A_{si}K_{si}^{\alpha_s}L_{si}^{\beta_s}E_{si}^{\gamma_s}\right)^{\frac{\sigma-1}{\sigma}} - (1+\tau_{K_{si}})rK_{si} - (1+\tau_{L_{si}})wL_{si} - (1+\tau_{E_{si}})p_E E_{si}. \tag{B.20}$$

Then the first-order conditions (FOCs) for optimal input choices are:

$$MRPK_{si} = P_s Y_s^{\frac{1}{\sigma}} \frac{(\sigma-1)}{\sigma} Y_{si}^{-\frac{1}{\sigma}} \alpha_s \frac{Y_{si}}{K_{si}} = (1+\tau_{K_{si}})r, \tag{B.21}$$

$$MRPL_{si} = P_s Y_s^{\frac{1}{\sigma}} \frac{(\sigma-1)}{\sigma} Y_{si}^{-\frac{1}{\sigma}} \beta_s \frac{Y_{si}}{L_{si}} = (1+\tau_{L_{si}})w, \tag{B.22}$$

$$MRPE_{si} = P_s Y_s^{\frac{1}{\sigma}} \frac{(\sigma-1)}{\sigma} Y_{si}^{-\frac{1}{\sigma}} \gamma_s \frac{Y_{si}}{E_{si}} = (1+\tau_{E_{si}})p_E. \tag{B.23}$$

Intuitively, distortions act as wedges between the value of the marginal product of an input and the common price of that input. Using the fact that $P_{si}Y_{si} = P_s Y_s^{\frac{1}{\sigma}} Y_{si}^{\frac{\sigma-1}{\sigma}}$, marginal revenue products can be rewritten as:

$$MRPK_{si} = \alpha_s \frac{(\sigma-1)}{\sigma} \frac{P_s Y_s^{\frac{1}{\sigma}} Y_{si}^{-\frac{1}{\sigma}}}{K_{si}} = \alpha_s \frac{(\sigma-1)}{\sigma} \frac{P_{si}Y_{si}}{K_{si}} = (1+\tau_{K_{si}})r, \tag{B.24}$$

$$MRPL_{si} = \beta_s \frac{(\sigma-1)}{\sigma} \frac{P_s Y_s^{\frac{1}{\sigma}} Y_{si}^{-\frac{1}{\sigma}}}{L_{si}} = \beta_s \frac{(\sigma-1)}{\sigma} \frac{P_{si}Y_{si}}{L_{si}} = (1+\tau_{L_{si}})w, \tag{B.25}$$

$$MRPE_{si} = \gamma_s \frac{(\sigma-1)}{\sigma} \frac{P_s Y_s^{\frac{1}{\sigma}} Y_{si}^{-\frac{1}{\sigma}}}{E_{si}} = \gamma_s \frac{(\sigma-1)}{\sigma} \frac{P_{si}Y_{si}}{E_{si}} = (1+\tau_{E_{si}})p_E. \tag{B.26}$$

Next, I define productivity measures. Physical productivity is:

$$TFPQ_{si} = A_{si} = \frac{Y_{si}}{K_{si}^{\alpha_s}L_{si}^{\beta_s}E_{si}^{\gamma_s}} \tag{B.27}$$

while revenue productivity incorporates prices:

$$TFPR_{si} = P_{si}A_{si} = \frac{P_{si}Y_{si}}{K_{si}^{\alpha_s}L_{si}^{\beta_s}E_{si}^{\gamma_s}} \tag{B.28}$$

Thus, $TFPR_{si}$ reflects how distortions affect the revenue side of productivity. However, differences in physical productivity, $TFPQ_{si}$, is natural and does not imply any misallocation. Finally, taking the geometric average of the marginal revenue products (with sector-level input shares) gives:

$$
\begin{aligned}
&(MRPK_{si})^{\alpha_s}(MRPL_{si})^{\beta_s}(MRPE_{si})^{\gamma_s} \\
&= (\alpha\frac{(\sigma-1)}{\sigma}\frac{P_{si}Y_{si}}{K_{si}})^{\alpha_s}(\beta\frac{(\sigma-1)}{\sigma}\frac{P_{si}Y_{si}}{L_{si}})^{\beta_s}(\gamma\frac{(\sigma-1)}{\sigma}\frac{P_{si}Y_{si}}{E_{si}})^{\gamma_s} \\
&= ((1+\tau_{K_{si}})r)^{\alpha_s}((1+\tau_{L_{si}})w)^{\beta_s}((1+\tau_{E_{si}})p_E)^{\gamma_s} \\
&= {\alpha_s}^{\alpha_s}{\beta_s}^{\beta_s}{\gamma_s}^{\gamma_s}\frac{(\sigma-1)}{\sigma}\frac{P_{si}Y_{si}}{K_{si}^{\alpha_s}L_{si}^{\beta_s}E_{si}^{\gamma_s}} \\
&= {\alpha_s}^{\alpha_s}{\beta_s}^{\beta_s}{\gamma_s}^{\gamma_s}\frac{(\sigma-1)}{\sigma}TFPR_{si}
\end{aligned}
\tag{B.29}
$$

Hence,

$$
\begin{aligned}
TFPR_{si} &\propto (MRPK_{si})^{\alpha_s}(MRPL_{si})^{\beta_s}(MRPE_{si})^{\gamma_s} \\
&\propto (1+\tau_{K_{si}})^{\alpha_s}(1+\tau_{L_{si}})^{\beta_s}(1+\tau_{E_{si}})^{\gamma_s}
\end{aligned}
\tag{B.30}
$$

This formulation shows that $TFPR_{si}$ is proportional to the geometric mean of marginal revenue products and, equivalently, of input distortions. Under an efficient allocation with zero distortions, all provinces within a sector would face the same $TFPR_{si}$; therefore, any observed variation directly reflects misallocation.

B.5 Sector-Level Productivity

Having incorporated province-level distortions, we next construct sector-level productivity measures by comparing each province's observed marginal revenue products (MRPs) to sector-level weighted averages across provinces.

Starting with capital, the sector-level weighted average MRP in sector s is:

$$
\overline{MRPK_s} = \frac{\sum_i K_{si}MRPKsi}{\sum_i K_{si}} = \frac{\sum_i \alpha_s\frac{\sigma-1}{\sigma}P_{si}Y_{si}}{\sum_i \alpha_s\frac{\sigma-1}{\sigma}\frac{P_{si}Y_{si}}{r(1+\tau_{K_{si}})}} = \frac{\sum_i P_{si}Y_{si}}{\sum_i \frac{P_{si}Y_{si}}{r(1+\tau_{K_{si}})}}
\tag{B.31}
$$

Here, each province's MRP is weighted by its capital usage, capturing the aggregate contribution of capital in the sector.

Given sectoral revenue $P_sY_s = \sum_i P_{si}Y_{si}$, this simplifies to:

$$
\overline{MRPK_s} = \frac{r}{\sum_i \frac{1}{(1+\tau_{K_{si}})}\frac{P_{si}Y_{si}}{P_sY_s}}
\tag{B.32}
$$

Intuitively, $\overline{MRPK_s}$ reflects both the factor price r and the distribution of distortions across provinces.

Comparing the province-level $MRPK_{si}$ to the sector-level weighted average ($\overline{MRPK_s}$):

$$\frac{\overline{MRPK_s}}{MRPK_{si}} = \frac{\frac{r}{\sum_i \frac{1}{(1+\tau_{K_{si}})} \frac{P_{si}Y_{si}}{P_sY_s}}}{r(1+\tau_{K_{si}})} = \frac{1}{(1+\tau_{K_{si}}) \sum_i \frac{1}{(1+\tau_{K_{si}})} \frac{P_{si}Y_{si}}{P_sY_s}} \tag{B.33}$$

and similarly for labor and energy:

$$\frac{\overline{MRPL_s}}{MRPL_{si}} = \frac{1}{(1+\tau_{L_{si}}) \sum_i \frac{1}{(1+\tau_{L_{si}})} \frac{P_{si}Y_{si}}{P_sY_s}} \tag{B.34}$$

$$\frac{\overline{MRPE_s}}{MRPE_{si}} = \frac{1}{(1+\tau_{E_{si}}) \sum_i \frac{1}{(1+\tau_{E_{si}})} \frac{P_{si}Y_{si}}{P_sY_s}} \tag{B.35}$$

Notice that in the absence of distortions ($\tau = 0$), these ratios are equal to 1, consistent with efficient allocation.

Given that we have explicit formulas for distortions and marginal revenue products compared to sector-weighted averages we can move forward to calculate the output implications of these. Taking the geometric average across all factors K, L, and E:

$$\begin{aligned}&\left(\frac{\overline{MRPK_s}}{MRPK_{si}}\right)^{\alpha} \left(\frac{\overline{MRPL_s}}{MRPL_{si}}\right)^{\beta} \left(\frac{\overline{MRPE_s}}{MRPE_{si}}\right)^{\gamma} \\ &\qquad = \left(\frac{\sum_i K_{si} MRPK_{si}}{MRPK_{si} \sum_i K_{si}}\right)^{\alpha} \left(\frac{\sum_i L_{si} MRPL_{si}}{MRPL_{si} \sum_i L_{si}}\right)^{\beta} \left(\frac{\sum_i E_{si} MRPE_{si}}{MRPE_{si} \sum_i E_{si}}\right)^{\gamma}\end{aligned} \tag{B.36}$$

Given $\sum_i K_{si} MRPK_{si} = \sum_i L_{si} MRPL_{si} = \sum_i E_{si} MRPE_{si} \propto P_sY_s = \sum_i P_{si}Y_{si}$, and $\sum_i K_{si} = K_s, \sum_i L_{si} = L_s, \sum_i E_{si} = E_s$, this geometric average is proportional to $TFPR_{si} = P_{si}A_{si}$.

Notice that,

$$\left(\frac{\overline{MRPK_s}}{MRPK_{si}}\right)^{\alpha} \left(\frac{\overline{MRPL_s}}{MRPL_{si}}\right)^{\beta} \left(\frac{\overline{MRPE_s}}{MRPE_{si}}\right)^{\gamma} = \frac{P_sY_s}{P_{si}A_{si}K_s^{\alpha_s} L_s^{\beta_s} E_s^{\gamma_s}} = \frac{P_sA_s}{P_{si}A_{si}} \tag{B.37}$$

Finally, we recover sector-level TFP, A_s by using the sector level price index derived earlier:

We have

$$P_s = \left(\sum_i P_{si}^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \tag{B.38}$$

To isolate A_s we can multiply the expression by A_{si} and take the power of $(\sigma - 1)$ and some over provinces.

$$\sum_i \left(\frac{\cancel{A_{si}} P_s A_s}{P_{si} \cancel{A_{si}}}\right)^{\sigma-1} = P_s^{(\sigma-1)} A_s^{(\sigma-1)} \underbrace{\sum_i P_{si}^{(1-\sigma)}}_{P_s^{1-\sigma}} = A_s^{\sigma-1} \tag{B.39}$$

If we take the power of $1/(\sigma-1)$ we arrive at $TFP_s = A_s$ by applying the same operations to the left-hand side of B.37 we get an expression for A_s

$$A_s = \left[\sum_i \left(A_{si} \left(\frac{\overline{MRPK_s}}{MRPK_{si}}\right)^{\alpha} \left(\frac{\overline{MRPL_s}}{MRPL_{si}}\right)^{\beta} \left(\frac{\overline{MRPE_s}}{MRPE_{si}}\right)^{\gamma}\right)^{\sigma-1}\right]^{\frac{1}{\sigma-1}}. \tag{B.40}$$

Finally, we need an expression for A_{si} to bring this model into data. Recall that province-sector level revenues are given by:

$$P_{si} Y_{si} = P_s (Y_s)^{\frac{1}{\sigma}} Y_{si}^{\frac{\sigma-1}{\sigma}} \tag{B.41}$$

At the province level, this implies:

$$Y_{si} = (P_s (Y_s)^{\frac{1}{\sigma}})^{\frac{-\sigma}{1-\sigma}} (P_{si} Y_{si})^{\frac{\sigma}{\sigma-1}} \tag{B.42}$$

Given that $Y_{si} = A_{si} K_{si}^{\alpha_s} L_{si}^{\beta_s} E_{si}^{\gamma_s}$ we can write:

$$A_{si} = \frac{(P_s Y_s)^{\frac{-1}{\sigma-1}}}{P_s} \frac{(P_{si} Y_{si})^{\frac{\sigma}{\sigma-1}}}{K_{si}^{\alpha_s} L_{si}^{\beta_s} E_{si}^{\gamma_s}} \tag{B.43}$$

So,

$$A_{si} \propto \frac{(P_{si} Y_{si})^{\frac{\sigma}{\sigma-1}}}{K_{si}^{\alpha_s} L_{si}^{\beta_s} E_{si}^{\gamma_s}} \tag{B.44}$$

which provides the empirical link between observed revenue, inputs, and productivity. Intuitively, sector-level TFP aggregates province-level outputs while adjusting for both input allocation and distortions, capturing the efficiency of the sector as a whole.

B.6 Measuring Input-Specific Distortions

To measure input-specific distortions, I begin by expressing the marginal revenue product of input under perfect competition and a Cobb–Douglas production technology assumptions.

Recall that

$$MRPK_{si} = \alpha_s \frac{(\sigma-1)}{\sigma} \frac{P_s Y_s^{\frac{1}{\sigma}} Y_{si}^{-\frac{1}{\sigma}}}{K_{si}} = \alpha_s \frac{(\sigma-1)}{\sigma} \frac{P_{si} Y_{si}}{K_{si}} = (1+\tau_{K_{si}})r \tag{B.45}$$

Taking logarithms and subtracting $ln(r)$ yields

$$\ln(MRPK_{si}) - \ln(r) = \ln(\alpha_s \frac{(\sigma-1)}{\sigma}) + \ln(\frac{P_{si} Y_{si}}{K_{si}}) - \ln(r) = \ln(1+\tau_{K_{si}}) \tag{B.46}$$

or equivalently,

$$\ln(MRPK_{si}) - \ln(r) = \ln(\alpha_s \frac{(\sigma-1)}{\sigma}) + \ln(\frac{P_{si} Y_{si}}{rK_{si}}) = \ln(1+\tau_{K_{si}}) \tag{B.47}$$

This can be rearranged as

$$\underbrace{\ln(MRPK_{si})}_{\epsilon_{si}} \underbrace{- \ln(r) - \ln(\frac{\sigma-1}{\sigma})}_{\beta_0} - \underbrace{\ln(\alpha_s)}_{\sum_s \beta_s \gamma_s} = \ln(\frac{P_{si} Y_{si}}{rK_{si}}) \tag{B.48}$$

This expression motivates the following regression to recover the dispersion of marginal revenue products.

$$\ln(\frac{P_{si} Y_{si}}{rK_{si}}) = \beta_0 + \sum_s \beta_s \gamma_s + \epsilon_{si} \tag{B.49}$$

The interpretation of the regression is intuitive. The dependent variable measures the ratio of revenue to capital expenditure. The intercept term captures common parameters, including the rent of capital and the elasticity of substitution. Sector-fixed effects absorb sector-level averages, while the error term reflects deviations from these averages. These residuals represent the unexplained variation and therefore provide information on the distribution of marginal revenue products. Formally, this implies $Var(\ln(MRPK_{si})) = Var(\ln(\hat{\epsilon}_{si}))$. This expression provides an estimate of the extent of misallocation. In the absence of distortions, marginal revenue products would be equalized within each sector (across provinces), implying a variance approaching zero. A larger residual variance, therefore, signals greater misallocation and larger potential productivity gains from reallocation.

The same procedure can be extended to labor and energy to obtain measures of misallocation across all major inputs.

B.7 Productivity Decomposition

I start by defining sector-level total factor productivity (TFP) A_s using a Cobb-Douglas production function, where output Y_s is produced with capital K_s, labor L_s, and energy E_s:

$$A_s = \frac{Y_s}{K_s^{\alpha_s} L_s^{\beta_s} E_s^{\gamma_s}} \tag{B.50}$$

Sectoral output Y_s aggregates province-level outputs Y_{si} through a constant elasticity of substitution (CES) aggregator with elasticity σ:

$$Y_s = \left(\sum_i Y_{si}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \tag{B.51}$$

This captures substitutability: more productive provinces contribute more to sector output.

Substituting province-level production functions $Y_{si} = A_{si} K_{si}^{\alpha_s} L_{si}^{\beta_s} E_{si}^{\gamma_s}$ into the CES aggregator, we express sectoral TFP, A_s, as:

$$\Rightarrow A_s = \frac{\left(\sum_i \left(A_{si} K_{si}^{\alpha_s} L_{si}^{\beta_s} E_{si}^{\gamma_s} \right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}}{K_s^{\alpha_s} L_s^{\beta_s} E_s^{\gamma_s}} \tag{B.52}$$

or more compactly,

$$= \left[\sum_i \left(A_{si} \left(\frac{K_{si}}{K_s} \right)^{\alpha_s} \left(\frac{L_{si}}{L_s} \right)^{\beta_s} \left(\frac{E_{si}}{E_s} \right)^{\gamma_s} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \tag{B.53}$$

This expresses sector TFP as a function of province-level productivity and allocation. Note that this expression is just normalizing province-level inputs by the sector total, for which we know explicitly what they are. We can explicitly write input shares and we can define the revenue shares to get a more compact expression. Let revenue and input shares be:

$$R_{si} = \frac{P_{si} Y_{si}}{P_s Y_s}, \quad k_{si} = \frac{K_{si}}{K_s}, \quad l_{si} = \frac{L_{si}}{L_s}, \quad e_{si} = \frac{E_{si}}{E_s} \tag{B.54}$$

By plugging in the terms in the marginal revenue products and a bit of algebra we arrive at an expression below. The expression shows that the capital share k_{si} allocated to province i in sector s is proportional to its revenue R_{si}, adjusted by the distortion $(1 + \tau_{K_{si}})$. Intuitively, provinces with larger revenues attract more capital, while higher distortions reduce their share. This captures how capital is distributed across provinces based on their revenue generating potential.

$$k_{si}=\frac{K_{si}}{K_s}=\frac{K_{si}}{\sum_i K_{si}}=\frac{\frac{\alpha_s}{r}\frac{\sigma-1}{\sigma}\frac{P_{si}Y_{si}}{(1+\tau_{K_{si}})}}{\sum_i \frac{\alpha_s}{r}\frac{\sigma-1}{\sigma}\frac{P_{si}Y_{si}}{(1+\tau_{K_{si}})}}=\frac{R_{si}/(1+\tau_{K_{si}})}{\sum_i R_{si}/(1+\tau_{K_{si}})} \tag{B.55}$$

$$\Rightarrow A_s=\left[\sum_i\left(A_{si}k_{si}^{\alpha_s}l_{si}^{\beta_s}e_{si}^{\gamma_s}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \tag{B.56}$$

It is straightforward to see that the fully efficient allocation yields an efficient sector-level benchmark TFP, A_s^*, expressed as:

$$A_s^*=\left[\sum_i A_{si}^{\sigma-1}\right]^{\frac{1}{\sigma-1}} \tag{B.57}$$

The ratio of observed sector level productivity term to efficient benchmark productivity term, $\frac{A_s}{A_s^*}$, isolates productivity losses from misallocation within sector s:

$$\frac{A_s}{A_s^*}=\frac{\left[\sum_i\left(A_{si}\left(\frac{R_{si}/(1+\tau_{K_{si}})}{\sum_i R_{si}/(1+\tau_{K_{si}})}\right)^{\alpha_s}\left(\frac{R_{si}/(1+\tau_{L_{si}})}{\sum_i R_{si}/(1+\tau_{L_{si}})}\right)^{\beta_s}\left(\frac{R_{si}/(1+\tau_{E_{si}})}{\sum_i R_{si}/(1+\tau_{E_{si}})}\right)^{\gamma_s}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}}{\left[\sum_i A_{si}^{\sigma-1}\right]^{\frac{1}{\sigma-1}}} \tag{B.58}$$

I will use B.58 to derive the national level productivity loss. At the national level, I can express national TFP, A, as:

$$A=\frac{Y}{K^{\bar{\alpha}}L^{\bar{\beta}}E^{\bar{\delta}}}=\frac{\prod_s Y_s^{\theta_s}}{K^{\bar{\alpha}}L^{\bar{\beta}}E^{\bar{\delta}}},\quad \bar{\alpha}=\sum_s \alpha_s\theta_s \tag{B.59}$$

$$\Rightarrow A=\prod_s\left(\frac{A_sK_s^{\alpha_s}L_s^{\beta_s}E_s^{\gamma_s}}{K^{\alpha_s}L^{\beta_s}E^{\gamma_s}}\right)^{\theta_s} \tag{B.60}$$

$$\Rightarrow A=\prod_s A_s^{\theta_s}\left(\frac{K_s}{K}\right)^{\alpha_s\theta_s}\left(\frac{L_s}{L}\right)^{\beta_s\theta_s}\left(\frac{E_s}{E}\right)^{\gamma_s\theta_s} \tag{B.61}$$

$$\Rightarrow A=\prod_s\left(A_s\left(\frac{K_s}{K}\right)^{\alpha_s}\left(\frac{L_s}{L}\right)^{\beta_s}\left(\frac{E_s}{E}\right)^{\gamma_s}\right)^{\theta_s} \tag{B.62}$$

Following the same logic as for sector-level input shares, I calculate national-level input shares by aggregating across both sectors and provinces. This allows me to obtain the total national input and directly compare it with sector-level inputs as follows.

$$K = \sum_s K_s = \sum_s \sum_i K_{si} \tag{B.63}$$

$$k_s = \frac{K_s}{K} = \frac{\sum_i K_{si}}{\sum_s \sum_i K_{si}}, \quad = \frac{\sum_i \frac{\alpha_s}{r}\left(\frac{\sigma-1}{\sigma}\right)\frac{P_{si}Y_{si}}{1+\tau_{K_{si}}}}{\sum_s \sum_i \frac{\alpha_s}{r}\left(\frac{\sigma-1}{\sigma}\right)\frac{P_{si}Y_{si}}{1+\tau_{K_{si}}}} = \frac{\sum_i \alpha_s \frac{P_{si}Y_{si}}{1+\tau_{K_{si}}}}{\sum_s \sum_i \alpha_s \frac{P_{si}Y_{si}}{1+\tau_{K_{si}}}} \tag{B.64}$$

$$= \frac{\alpha_s \sum_i \frac{P_{si}Y_{si}}{1+\tau_{K_{si}}}}{\sum_s \alpha_s \sum_i \frac{P_{si}Y_{si}}{1+\tau_{K_{si}}}} = \frac{\alpha_s P_s Y_s \sum_i \frac{R_{si}}{1+\tau_{K_{si}}}}{\sum_s \alpha_s P_s Y_s \sum_i \frac{R_{si}}{1+\tau_{K_{si}}}} \tag{B.65}$$

where $R_{si} = P_{si}Y_{si}/P_sY_s$. Then the observed level of capital, k_s, can be expressed as:

$$k_s = \frac{K_s}{K} = \frac{\alpha_s P_s Y_s \sum_i R_{si}/(1+\tau_{K_{si}})}{\sum_s \alpha_s P_s Y_s \sum_i R_{si}/(1+\tau_{K_{si}})} = \frac{\alpha_s P_s Y_s \sum_i R_{si}/(1+\tau_{K_{si}})}{\sum_s \alpha_s P_s Y_s \sum_i R_{si}/(1+\tau_{K_{si}})} \tag{B.66}$$

Define the harmonic mean of sector-level distortions as follows:

$$\overline{1+\tau_{Ks}} = \frac{1}{\sum_i \frac{R_{si}}{1+\tau_{K_{si}}}} \quad \text{(harmonic mean)} \tag{B.67}$$

The harmonic mean appears naturally here because capital allocation depends on the inverse of the distortion. Intuitively, sectors with higher revenues R_{si} and lower distortions $(1+\tau_{K_{si}})$ weigh more heavily in determining the effective sector-level distortion. Using the harmonic mean ensures that provinces with high distortions contribute less, while those with low distortions and large revenues contribute the aggregate measure more. With $\theta_s = P_sY_s/PY$ denoting the sector's revenue share, we can then express the sector-level distortion in a compact form.

$$k_s = \frac{K_s}{K} = \frac{\alpha_s\theta_s/(\overline{1+\tau_{Ks}})}{\sum_s \alpha_s\theta_s/(\overline{1+\tau_{Ks}})} \tag{B.68}$$

Also, as there are no distortions in optimal allocation we can simply write

$$k_s^* = \frac{K_s^*}{K^*} = \frac{\alpha_s\theta_s}{\sum_s \alpha_s\theta_s} \tag{B.69}$$

Comparison of the observed, k_s, to efficient, k_s^*, level of input gives us:

$$\Rightarrow \frac{k_s}{k_s^*} = \frac{\left(\frac{1}{\overline{1+\tau_{Ks}}}\right)\cdot \sum_s \alpha_s\theta_s}{\sum_s \frac{\alpha_s\theta_s}{\overline{1+\tau_{Ks}}}} \tag{B.70}$$

Similar algebra for labor and energy yields,

$$\Rightarrow \frac{l_s}{l_s^*} = \frac{\left(\frac{1}{1+\tau_{Ls}}\right) \cdot \sum_s \beta_s \theta_s}{\sum_s \frac{\beta_s \theta_s}{1+\tau_{Ls}}} \tag{B.71}$$

$$\Rightarrow \frac{e_s}{e_s^*} = \frac{\left(\frac{1}{1+\tau_{Es}}\right) \cdot \sum_s \gamma_s \theta_s}{\sum_s \frac{\gamma_s \theta_s}{1+\tau_{Es}}} \tag{B.72}$$

Finally, the ratio of the observed national TFP, A, to the efficient level of national TFP, A^*, gives us the productivity loss due to misallocation:

$$\frac{A}{A^*} = \prod_s \left(\left(\frac{A_s}{A_s^*}\right) \left(\frac{k_s}{k_s^*}\right)^{\alpha_s} \left(\frac{l_s}{l_s^*}\right)^{\beta_s} \left(\frac{e_s}{e_s^*}\right)^{\gamma_s} \right)^{\theta_s} \tag{B.73}$$

$$\frac{A}{A^*} = \underbrace{\prod_s \left(\frac{A_s}{A_s^*}\right)^{\theta_s}}_{\text{Within-sector misallocation}} \times \underbrace{\prod_s \left(\left(\frac{k_s}{k_s^*}\right)^{\alpha_s} \left(\frac{l_s}{l_s^*}\right)^{\beta_s} \left(\frac{e_s}{e_s^*}\right)^{\gamma_s} \right)^{\theta_s}}_{\text{Between-sector misallocation}} \tag{B.74}$$

$$\frac{A}{A^*} = \prod_s \left(\frac{A_s}{A_s^*}\right)^{\theta_s} \times \prod_s \left(\underbrace{\left(\frac{k_s}{k_s^*}\right)^{\alpha_s}}_{\substack{\text{Capital}\\ \text{misallocation}}} \cdot \underbrace{\left(\frac{l_s}{l_s^*}\right)^{\beta_s}}_{\substack{\text{Labor}\\ \text{misallocation}}} \cdot \underbrace{\left(\frac{e_s}{e_s^*}\right)^{\gamma_s}}_{\substack{\text{Energy}\\ \text{misallocation}}} \right)^{\theta_s} \tag{B.75}$$

We can write the *within*, $\left(\frac{A}{A^*}\right)_{within}$, portion of the expression explicitly by plugging B.58:

$$\prod_s \left(\frac{\left[\sum_i \left(A_{si} \left(\frac{R_{si}/(1+\tau_{K_{si}})}{\sum_i R_{si}/(1+\tau_{K_{si}})}\right)^{\alpha_s} \left(\frac{R_{si}/(1+\tau_{L_{si}})}{\sum_i R_{si}/(1+\tau_{L_{si}})}\right)^{\beta_s} \left(\frac{R_{si}/(1+\tau_{E_{si}})}{\sum_i R_{si}/(1+\tau_{E_{si}})}\right)^{\gamma_s} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}}{\left[\sum_i A_{si}^{\sigma-1}\right]^{\frac{1}{\sigma-1}}} \right)^{\theta_s} \tag{B.76}$$

To understand each input's role in within sector productivity loss we can use the efficient benchmark for each input and can conclude each input's contribution to within term. To do so, we use the standard result from perfect competition case under Cobb-Douglas production technology.

$$P_{si} = \frac{A_s}{A_{si}} \tag{B.77}$$

by rearranging B.41 we can get

$$Y_{si} = Y_s \left(\frac{P_{si}}{P_s} \right)^{-\sigma} \tag{B.78}$$

Plugging P_{si} term in gives us:

$$Y_{si} = Y_s \left(\frac{A_s}{P_s} \right)^{-\sigma} A_{si}^{\sigma} \tag{B.79}$$

Finally, we can express the revenue shares $R_{si} = \frac{P_{si}Y_{si}}{P_sY_s}$ in terms of A_{si} by plugging P_{si} and Y_{si} terms into the numerator.

$$R_{si} = \frac{P_{si}Y_{si}}{P_sY_s} = \frac{\left(\frac{A_s}{A_{si}}\right)\left(Y_s\left(\frac{A_s}{P_s}\right)^{-\sigma} A_{si}^{\sigma}\right)}{P_sY_s} \tag{B.80}$$

It implies that $R_{si} \propto A_{si}^{\sigma-1}$ So, the efficient benchmark for revenue shares R_{si}^{*} can be expressed as:

$$R_{si}^{*} = \frac{R_{si}}{\sum_j R_{sj}} = \frac{A_{si}^{\sigma-1}}{\sum_j A_{sj}^{\sigma-1}} \tag{B.81}$$

Given this relationshio between revenue shars and productivity terms, we can write each input's relative contribution in the within term by assuming the other two inputs allocated efficiently. Therefore, following expressions can be written for $\left(\frac{A}{A^*}\right)_{withinK}$, $\left(\frac{A}{A^*}\right)_{withinL}$, and $\left(\frac{A}{A^*}\right)_{withinE}$ respectively.

$$\prod_s \left(\frac{\left[\sum_i \left(A_{si} \left(\frac{R_{si}/(1+\tau_{K_{si}})}{\sum_i R_{si}/(1+\tau_{K_{si}})} \right)^{\alpha_s} \left(\frac{A_{si}^{\sigma-1}}{\sum_j A_{sj}^{\sigma-1}} \right)^{\beta_s} \left(\frac{A_{si}^{\sigma-1}}{\sum_j A_{sj}^{\sigma-1}} \right)^{\gamma_s} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}}{\left[\sum_i A_{si}^{\sigma-1} \right]^{\frac{1}{\sigma-1}}} \right)^{\theta_s} \tag{B.82}$$

$$\prod_s \left(\frac{\left[\sum_i \left(A_{si} \left(\frac{A_{si}^{\sigma-1}}{\sum_j A_{sj}^{\sigma-1}} \right)^{\alpha_s} \left(\frac{R_{si}/(1+\tau_{L_{si}})}{\sum_i R_{si}/(1+\tau_{L_{si}})} \right)^{\beta_s} \left(\frac{A_{si}^{\sigma-1}}{\sum_j A_{sj}^{\sigma-1}} \right)^{\gamma_s} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}}{\left[\sum_i A_{si}^{\sigma-1} \right]^{\frac{1}{\sigma-1}}} \right)^{\theta_s} \tag{B.83}$$

$$
\prod_s \left(\frac{\left[\sum_i \left(A_{si} \left(\frac{A_{si}^{\sigma-1}}{\sum_j A_{sj}^{\sigma-1}} \right)^{\alpha_s} \left(\frac{A_{si}^{\sigma-1}}{\sum_j A_{sj}^{\sigma-1}} \right)^{\beta_s} \left(\frac{R_{si}/(1+\tau_{E_{si}})}{\sum_i R_{si}/(1+\tau_{E_{si}})} \right)^{\gamma_s} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}}{\left[\sum_i A_{si}^{\sigma-1} \right]^{\frac{1}{\sigma-1}}} \right)^{\theta_s} \tag{B.84}
$$

As for the *between* term, we can write the *between*, $\left(\frac{A}{A^*}\right)_{between}$ portion explicitly as:

$$
\prod_s \left(\left(\frac{\left(\frac{1}{1+\tau_{Ks}}\right) \cdot \sum_s \alpha_s \theta_s}{\sum_s \frac{\alpha_s \theta_s}{1+\tau_{Ks}}} \right)^{\alpha_s} \left(\frac{\left(\frac{1}{1+\tau_{Ls}}\right) \cdot \sum_s \beta_s \theta_s}{\sum_s \frac{\beta_s \theta_s}{1+\tau_{Ls}}} \right)^{\beta_s} \left(\frac{\left(\frac{1}{1+\tau_{Es}}\right) \cdot \sum_s \gamma_s \theta_s}{\sum_s \frac{\gamma_s \theta_s}{1+\tau_{Es}}} \right)^{\gamma_s} \right)^{\theta_s} \tag{B.85}
$$

To further decompose the between term to find each input misallocation contribution we can write the following expressions.

$$
\left(\frac{A}{A^*}\right)_{betweenK} = \prod_s \left(\left(\frac{\left(\frac{1}{1+\tau_{Ks}}\right) \cdot \sum_s \alpha_s \theta_s}{\sum_s \frac{\alpha_s \theta_s}{1+\tau_{Ks}}} \right)^{\alpha_s} \right)^{\theta_s} \tag{B.86}
$$

$$
\left(\frac{A}{A^*}\right)_{betweenL} = \prod_s \left(\left(\frac{\left(\frac{1}{1+\tau_{Ls}}\right) \cdot \sum_s \beta_s \theta_s}{\sum_s \frac{\beta_s \theta_s}{1+\tau_{Ls}}} \right)^{\beta_s} \right)^{\theta_s} \tag{B.87}
$$

$$
\left(\frac{A}{A^*}\right)_{betweenE} = \prod_s \left(\left(\frac{\left(\frac{1}{1+\tau_{Es}}\right) \cdot \sum_s \gamma_s \theta_s}{\sum_s \frac{\gamma_s \theta_s}{1+\tau_{Es}}} \right)^{\gamma_s} \right)^{\theta_s} \tag{B.88}
$$

Therefore,

$$
\begin{aligned}
\frac{A}{A^*} &= \left(\frac{A}{A^*}\right)_{\text{within}} \times \left(\frac{A}{A^*}\right)_{\text{between}} \\
&= \left(\frac{A}{A^*}\right)_{\text{within}} \times \left(\frac{A}{A^*}\right)_{\text{betweenK}} \times \left(\frac{A}{A^*}\right)_{\text{betweenL}} \times \left(\frac{A}{A^*}\right)_{\text{betweenE}}
\end{aligned} \tag{B.89}
$$

or more compactly,

$$
\hat{A} = \hat{A}_{within} \times \hat{A}_{between} = \hat{A}_{within} \times \hat{A}_{betweenK} \times \hat{A}_{betweenL} \times \hat{A}_{betweenE} \tag{B.90}
$$