The Role of Energy Efficiency in Productivity: Evidence from Canada*

by Anil Gogebakan†

July 24, 2025

Abstract

This paper quantifies how the misallocation of energy, capital, and labor across provinces and sectors lowers productivity in Canada. Using annual provincial input-output data (2014–2020) and a standard Hsieh and Klenow-style misallocation framework, I decompose the loss into interprovincial (within-sector) and intersectoral (within-province) components and measure each input's contribution. Unlike most studies focused on firm-level variation within manufacturing, I examine the full economy at the province-sector level. Results show that Canada operates 15–32% below its efficient production potential, depending on the assumed substitutability of goods between provinces. Reallocating inputs within provinces optimally narrows this modestly to 14–30%, highlighting interprovincial misallocation as the primary source of loss. Interestingly, energy—which accounts for only about 8% of input costs—is responsible for 1–2.5% of the gap, making it the second largest contributor. Capital contributes about 1%, while labor appears to be allocated nearly optimally.

Keywords: Misallocation; Total Factor Productivity; Distortions; Energy; Canada.

JEL classification: O11, O13, O41, O47, O51, D24, D61.

*This project has been supported by Environment and Climate Change Canada's Economics and Environmental Policy Research Network (EEPRN).

†University of Calgary, anil.gogebakan@ucalgary.ca

1 Introduction

What is the productivity loss in Canada due to energy misallocation? While the inefficient use of resources has long been recognized as a major source of substantial economic output loss and low productivity (Hsieh and Klenow (2009), Restuccia and Rogerson (2017), Brandt et al. (2013), Bartelsman et al. (2013) Chen and Irarrazabal (2015)), most existing studies focus on capital and labor misallocation across firms within the manufacturing sector (Bartelsman et al. (2013), Chen and Irarrazabal (2015)). In contrast, energy—a key input in nearly all economic activities—has received relatively little attention (Asker et al. (2019), Choi (2020), Tombe and Winter (2015)), despite its growing relevance in both productivity and environmental policy debates.

Energy differs from capital and labor in ways that make its misallocation particularly relevant in the context of aggregate productivity analysis. First, energy is far less mobile than capital or labor, as its availability and cost vary significantly across provinces due to differences in natural endowments and infrastructure—challenges that are further amplified by interprovincial trade barriers. Second, energy markets are heavily shaped by regulation, ownership, and policy—resulting in persistent price gaps across provinces that do not adjust through market mechanisms, unlike wages or returns to capital. Third, although energy represents only about 8% of input costs, its misallocation contributes up to 2–3% of aggregate output loss—making it more distortionary per dollar than either labor or capital. Furthermore, improving the efficiency of energy use is not only economically beneficial but also environmentally strategic. Achieving higher output with the same energy input can reduce the economic cost of environmental regulations, making it easier to meet climate targets without sacrificing growth. These features make energy a critical, yet mostly disregarded, factor in understanding allocative efficiency.

This paper quantifies the productivity loss in Canada from the misallocation of energy in addition to capital and labor at the sector level across provinces. Using detailed annual provincial input-output data from Statistics Canada for the period 2014–2020, I extend the standard Hsieh and Klenow (2009) framework to incorporate energy as a third input alongside capital and labor. I measure the marginal revenue products of each input at the sector-province level and compare them to an efficient benchmark, allowing me to compute both the magnitude and sources of allocative inefficiency.

Canada is an especially relevant case for this analysis. Its provinces operate with substantial autonomy over energy policy, resulting in significant variation in prices, regulatory regimes, and energy mix. These differences, combined with fragmented infrastructure and limited interprovincial trade, make the Canadian economy particularly vulnerable to spatial misallocation of energy. Quantifying these inefficiencies is essential for designing better policies that promote both economic productivity and energy efficiency.

The results reveal substantial inefficiencies in the allocation of energy, capital, and labor across provinces and sectors. Using a constant elasticity of substitution parameter, I find that aggregate output could be up to 32% higher under a conservative elasticity assumption ($\sigma = 3$) and about 15% higher under a more elastic assumption ($\sigma = 7$). These elasticity values mirror the existing literature, which typically estimates substitutability across production units within sectors. Since the model here employs a more aggregated provincial–sectoral level data, these values should be interpreted as illustrative benchmarks rather than precise estimates: they span the plausible range of substitution possibilities, but actual substitutability across provinces may be lower or higher because of additional frictions and heterogeneity at this higher level of aggregation. Presenting results for both lower and higher elasticity scenarios allows us to quantify the bounds of the range of the potential efficiency gains and show how sensitive the estimates are to the assumed substitutability parameter.

I further decompose the potential gains into interprovincial and intersectoral components. Interprovincial misallocation—driven by regulatory fragmentation and limited energy trade—accounts for approximately 30% out of the total 32% loss, while within-province (intersectoral) misallocation is more modest: labor is nearly optimally allocated, capital misallocation accounts for 1–2% of the loss, and energy misallocation being 2–3%, highest among the production factors, despite making up just 8% of input costs. This highlights the more pronounced disproportionate role of energy and the importance of interprovincial factors—such as trade barriers and regulatory differences—in driving inefficiency.

To model interprovincial flows and their contribution to misallocation, I adopt an approach analogous to the Armington model of trade, which assumes imperfect substitutability between similar goods produced in different locations. This assumption reflects the observed persistence of cross-provincial price differences, suggesting that provincial outputs are not perfect substitutes and that interprovincial frictions limit reallocation. Framing the model in this way provides a theoretically consistent and empirically relevant basis for analyzing the role of interprovincial trade frictions in aggregate productivity losses. In short, findings from this paper highlight the need to integrate energy policy and interprovincial trade reforms more centrally into productivity-enhancing strategies.

This paper makes three main contributions. First, it provides the first comprehensive estimate of energy misallocation in Canada using sector-by-province data, offering insights that go beyond the manufacturing sector and firm-level analyses common in the literature. Second, it quantifies the welfare cost of energy distortions across geographic and sectoral dimensions, emphasizing the role of spatial frictions in depressing productivity. Third, it offers a tractable and generalizable framework for evaluating allocative efficiency in energy use, which can inform policy debates around energy pricing, interprovincial infrastructure, and climate policy.

The remainder of the paper is organized as follows: Section 2 describes the data and measurement approach. Section 3 presents the theoretical framework. Section 4 outlines the main findings. Section 5 concludes.

2 Data

This study examines the data from the Provincial Symmetric Input-Output Tables (Catalogue no. 15-211-X) published by Statistics Canada's Industry Accounts Division. These tables provide a comprehensive, annually consistent depiction of inter-industry transactions at the provincial level in Canada. Specifically, I utilize the detailed aggregation level for the years from 2014 to 2020 inclusive, which offers a high-resolution view of economic flows across provinces and sectors.

The symmetric input-output tables reformat the standard supply and use tables into an industry-by-industry framework, allowing for clearer identification of the production structure and intermediate demand relationships. The data captures all inter-sectoral purchases—including expenditures on imports, inventory withdrawals, and primary inputs—making them well-suited for structural and efficiency analyses. The final demand tables similarly record all purchases by final demand categories from provincial and imported sources.

The data used reflect Statistics Canada's most detailed industry classifications and are harmonized across years, enabling consistent cross-provincial and intertemporal comparisons. The version of the tables used in this study corresponds to the level of aggregation that was previously known as "Aggregation Level S," which was renamed "Detailed" in 2019.

For methodological transparency and further technical detail, the construction of these tables is documented by Statistics Canada and available through direct inquiry with the Industry Accounts Division.

3 Model

3.1 Aggregate Output and Sectoral Shares

I consider a standard model of monopolistic competition with heterogeneous provinces, indexed by i. I closely follow the framework of (Hsieh and Klenow, 2009) with a natural extension of energy as an input in the production function. In the economy, a single aggregate output Y is produced by aggregating all sector contributions at the national level:

$$Y = \prod_{s=1}^{S} Y_s^{\theta_s}, \quad \text{where } \sum_{s=1}^{S} \theta_s = 1. \tag{3.1}$$

θ_s is the share of each sector within the national economic output. Each sector's output Y_s is given by:

$$Y_s = \left(\sum_i Y_{si}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}. \tag{3.2}$$

This is the standard constant elasticity of substitution (CES) function over provinces with elasticity of substitution parameter σ.

The sectoral profit maximization problem yields the aggregate price index P given by:

$$P = \prod_{s=1}^{S} \left(\frac{P_s}{\theta_s} \right)^{\theta_s} \tag{3.3}$$

Intuitively sectoral prices are scaled to their shares in the national economy and then aggregated based on the same shares.

Also, province- and sector-level profit maximization gives us the revenue equation for each province-by-sector level of revenue.

$$P_{si} Y_{si} = P_s Y_s^{\frac{1}{\sigma}} Y_{si}^{\frac{\sigma-1}{\sigma}} = P_{si}^{1-\sigma} P_s^{\sigma} Y_s. \tag{3.4}$$

Where the second part of the equality follows from simple algebra, where we take the power of σ on both sides of the first equality.

The sectoral expenditure minimization problem gives us the sectoral price index given by:

$$P_s = \left(\sum_i P_{si}^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \tag{3.5}$$

Now, I turn to the terms of the production function and productivity. I start with a usual profit maximization for sector s in province i. Define the production function as the Cobb-Douglas form with three inputs to production, namely capital (K), labor (L), and energy (E).

$$Y_{si} = A_{si} K_{si}^{\alpha_s} L_{si}^{\beta_s} E_{si}^{\gamma_s}, \quad \text{where } \alpha_s + \beta_s + \gamma_s = 1. \tag{3.6}$$

A_{si} represents total physical factor productivity (TFPQ). Each sector s in province i solves the following problem:

$$\max_{K_{si}, L_{si}, E_{si}} \quad P_{si} Y_{si} - (1 + \tau_{K_{si}}) r K_{si} - (1 + \tau_{L_{si}}) w L_{si} - (1 + \tau_{E_{si}}) p_E E_{si}. \tag{3.7}$$

Each input is subject to input distortions $\tau_{K_{si}}, \tau_{L_{si}}, \tau_{E_{si}}$, so that sectors in each province face distorted input prices.

By plugging Y_{si} and $P_{si}Y_{si}$ expressions above, we can solve this standard problem to get the marginal revenue product for each input.

$$MRPK_{si} = \alpha_s \frac{(\sigma - 1)}{\sigma} \frac{P_s Y_s^{\frac{1}{\sigma}} Y_{si}^{-\frac{1}{\sigma}}}{K_{si}} = \alpha_s \frac{(\sigma - 1)}{\sigma} \frac{P_{si} Y_{si}}{K_{si}} = (1 + \tau_{K_{si}}) r, \tag{3.8}$$

$$MRPL_{si} = \beta_s \frac{(\sigma - 1)}{\sigma} \frac{P_s Y_s^{\frac{1}{\sigma}} Y_{si}^{-\frac{1}{\sigma}}}{L_{si}} = \beta_s \frac{(\sigma - 1)}{\sigma} \frac{P_{si} Y_{si}}{L_{si}} = (1 + \tau_{L_{si}}) w, \tag{3.9}$$

$$MRPE_{si} = \gamma_s \frac{(\sigma - 1)}{\sigma} \frac{P_s Y_s^{\frac{1}{\sigma}} Y_{si}^{-\frac{1}{\sigma}}}{E_{si}} = \gamma_s \frac{(\sigma - 1)}{\sigma} \frac{P_{si} Y_{si}}{E_{si}} = (1 + \tau_{E_{si}}) p_E. \tag{3.10}$$

Where $MRPK_{si}, MRPL_{si}, MRPE_{si}$ are the Marginal Revenue Product of capital, labor, and energy, respectively.

Define the following,

$$TFPQ_{si} = A_{si} = \frac{Y_{si}}{K_{si}^{\alpha_s} L_{si}^{\beta_s} E_{si}^{\gamma_s}} \tag{3.11}$$

$$TFPR_{si} = P_{si} A_{si} = \frac{P_{si} Y_{si}}{K_{si}^{\alpha_s} L_{si}^{\beta_s} E_{si}^{\gamma_s}} \tag{3.12}$$

where TFPQ is total physical factor productivity, which naturally can be different for each sector and wouldn't mean any distortion. On the other hand TFPR indicates total factor revenue productivity, and it should be equalized across provinces and sectors if it were not for distortions. Any dispersion in TFPR would translate into lower output and would mean misallocation of resources.

It is straightforward to see that the geometric average of marginal revenue products would be proportional to TFPR, and also it is proportional to the geometric average of distortion (τ) terms.

Hence,

$$TFPR_{si} \propto (MRPK_{si})^{\alpha_s}(MRPL_{si})^{\beta_s}(MRPE_{si})^{\gamma_s} \propto (1+\tau_{K_{si}})^{\alpha_s}(1+\tau_{L_{si}})^{\beta_s}(1+\tau_{E_{si}})^{\gamma_s} \quad (3.13)$$

Defining sectoral weighted average marginal revenue product for inputs as follows

$$\overline{MRPK_s} = \frac{\sum_i K_{si} MRPKsi}{\sum_i K_{si}} \quad (3.14)$$

gives us

$$\frac{\overline{MRPK_s}}{MRPK_{si}} = \frac{1}{(1+\tau_{K_{si}})\sum_i \frac{1}{(1+\tau_{K_{si}})}\frac{P_{si}Y_{si}}{P_s Y_s}} \quad (3.15)$$

$$\frac{\overline{MRPL_s}}{MRPL_{si}} = \frac{1}{(1+\tau_{L_{si}})\sum_i \frac{1}{(1+\tau_{L_{si}})}\frac{P_{si}Y_{si}}{P_s Y_s}} \quad (3.16)$$

$$\frac{\overline{MRPE_s}}{MRPE_{si}} = \frac{1}{(1+\tau_{E_{si}})\sum_i \frac{1}{(1+\tau_{E_{si}})}\frac{P_{si}Y_{si}}{P_s Y_s}} \quad (3.17)$$

Intuitively, these are the deviations from the optimal allocation of resources across sectors and provinces.

With a bit more algebra, we arrive at the expression below.

$$A_s = \left[\sum_i \left(A_{si}\left(\frac{\overline{MRPK_s}}{MRPK_{si}}\right)^{\alpha}\left(\frac{\overline{MRPL_s}}{MRPL_{si}}\right)^{\beta}\left(\frac{\overline{MRPE_s}}{MRPE_{si}}\right)^{\gamma}\right)^{\sigma-1}\right]^{\frac{1}{\sigma-1}}. \quad (3.18)$$

Which is the total factor productivity at the sector level. To arrive at the output, we need to multiply each sector's productivity by based on their sector share θ_s to get the aggregate productivity level.

3.2 Aggregate output

Now we have all the ingredients to calculate aggregate output in the economy. If there were **no distortions** ($\tau_K = \tau_L = \tau_E = 0$), TFP_s would reach to its efficient level TFP_s^* - When distortions exist, provinces with higher distortions contribute less to output, reducing aggregate TFP.

$$A_s^* = TFP_s^* = \left(\sum_i A_{si}^{\sigma-1} \right)^{\frac{1}{\sigma-1}} \tag{3.19}$$

$$\frac{TFP_s}{TFP_s^*} = \left[\sum_i \left(\frac{A_{si}}{A_s^*} \left(\frac{\overline{MRPK_s}}{MRPK_{si}} \right)^{\alpha} \left(\frac{\overline{MRPL_s}}{MRPL_{si}} \right)^{\beta} \left(\frac{\overline{MRPE_s}}{MRPE_{si}} \right)^{\gamma} \right)^{\sigma-1} \right]^{\frac{1}{\sigma-1}}. \tag{3.20}$$

Finally, it is straightforward to compare the efficient level of aggregate output with the actual level of output.

$$\frac{Y}{Y^*} = \prod_s \left(\frac{TFP_s}{TFP_s^*} \right)^{\theta_s} \tag{3.21}$$

3.3 Productivity Decomposition

To understand which input distortion or which dimension (i.e., province or sector) contributes to welfare loss, we want to break down the equation. Let $\hat{x} = x/x^*$ be the comparison term between two levels of a variable. Here we are comparing the actual productivity level to the optimal level of productivity (i.e. no distortions). We start writing down the national level productivity TFP/TFP^* or A/A^*.

$$\frac{A}{A^*} = \underbrace{\prod_s \left(\frac{A_s}{A_s^*} \right)^{\theta_s}}_{\text{Within-sector misallocation}} \times \underbrace{\prod_s \left(\left(\frac{k_s}{k_s^*} \right)^{\alpha_s} \left(\frac{l_s}{l_s^*} \right)^{\beta_s} \left(\frac{e_s}{e_s^*} \right)^{\gamma_s} \right)^{\theta_s}}_{\text{Between-sector misallocation}} \tag{3.22}$$

Within component can be explicitly expressed as:

$$\left(\frac{A}{A^*}\right)_{within} = \prod_s \left(\frac{\left[\sum_i \left(A_{si} \left(\frac{R_{si}/(1+\tau_{K_{si}})}{\sum_i R_{si}/(1+\tau_{K_{si}})} \right)^{\alpha_s} \left(\frac{R_{si}/(1+\tau_{L_{si}})}{\sum_i R_{si}/(1+\tau_{L_{si}})} \right)^{\beta_s} \left(\frac{R_{si}/(1+\tau_{E_{si}})}{\sum_i R_{si}/(1+\tau_{E_{si}})} \right)^{\gamma_s} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}}{\left[\sum_i A_{si}^{\sigma-1} \right]^{\frac{1}{\sigma-1}}} \right)^{\theta_s} \tag{3.23}$$

Where $R_{si} = P_{si}Y_{si}/P_sY_s$. *Between* component can be expressed as

$$\left(\frac{A}{A^*}\right)_{\text{between}} = \prod_s \left(\underbrace{\left(\frac{\left(\frac{1}{\overline{1+\tau_{Ks}}}\right) \cdot \sum_s \alpha_s \theta_s}{\sum_s \frac{\alpha_s \theta_s}{1+\tau_{Ks}}} \right)^{\alpha_s}}_{\text{Capital Misallocation}} \times \underbrace{\left(\frac{\left(\frac{1}{\overline{1+\tau_{Ls}}}\right) \cdot \sum_s \beta_s \theta_s}{\sum_s \frac{\beta_s \theta_s}{1+\tau_{Ls}}} \right)^{\beta_s}}_{\text{Labor Misallocation}} \times \underbrace{\left(\frac{\left(\frac{1}{\overline{1+\tau_{Es}}}\right) \cdot \sum_s \gamma_s \theta_s}{\sum_s \frac{\gamma_s \theta_s}{1+\tau_{Es}}} \right)^{\gamma_s}}_{\text{Energy Misallocation}} \right)^{\theta_s} \tag{3.24}$$

where $\overline{1+\tau_{Ks}} = \left(\sum_i \frac{R_{si}}{1+\tau_{K_{si}}} \right)^{-1}$ denotes the harmonic mean of $1+\tau_{K_{si}}$ weighted by R_{si}. I present the full derivation of these expressions in the appendix section. Now, we are ready to calculate the productivity loss from misallocation of various inputs, as well as the within-sector and between-provinces. This breakdown allows us to identify how much of the overall productivity gap is due to misallocation of each specific input, as well as inefficiencies within sectors across provinces.

4 Results

I decompose aggregate total factor productivity (TFP) losses at the province-sector level into two components: within-sector across provinces (denoted $\hat{A}_{\text{within}}$) and between-sector within provinces ($\hat{A}_{\text{between}}$), along with input-specific productivity loss measures from misallocation of capital, labor, and energy. Table 1 reports these results for the years 2014–2020, under two benchmark elasticities of substitution across sectors, $\sigma = 3$ and $\sigma = 7$.

Under the conservative assumption of $\sigma = 3$, the aggregate productivity index ($\hat{A}$) ranges between 0.674 and 0.684, indicating an economy-wide TFP loss of approximately 32–33% relative to the efficient allocation benchmark. The within-sector component ($\hat{A}_{\text{within}}$) remains relatively stable around 0.69–0.70 (i.e., TFP loss around 30-31%), implying persistent interprovincial misallocation within the same sectors. These losses likely reflect frictions in factor mobility between provinces, such as regulatory inconsistency, limited trade integration,

Table 1: TFP Decomposition and Input Misallocation (2014–2020)

	2014	**2015**	**2016**	**2017**	**2018**	**2019**	**2020**
			$\sigma = 3$				
$\hat{A}$	0.674	0.675	0.684	0.681	0.680	0.682	0.679
$\hat{A}_{\text{between}}$	0.958	0.976	0.987	0.984	0.984	0.980	0.983
$\hat{A}_{\text{within}}$	0.702	0.690	0.693	0.692	0.691	0.695	0.691
$\hat{A}_{\text{capital}}$	0.996	0.988	0.987	0.989	0.990	0.988	0.988
$\hat{A}_{\text{labor}}$	0.986	1.00	1.00	1.00	1.00	1.00	1.00
$\hat{A}_{\text{energy}}$	0.975	0.988	0.991	0.988	0.989	0.983	0.991
			$\sigma = 7$				
$\hat{A}$	0.841	0.845	0.854	0.852	0.845	0.853	0.851
$\hat{A}_{\text{between}}$	0.958	0.976	0.987	0.984	0.984	0.980	0.983
$\hat{A}_{\text{within}}$	0.877	0.865	0.865	0.866	0.858	0.871	0.866
$\hat{A}_{\text{capital}}$	0.996	0.988	0.987	0.989	0.990	0.988	0.988
$\hat{A}_{\text{labor}}$	0.986	1.00	1.00	1.00	1.00	1.00	1.00
$\hat{A}_{\text{energy}}$	0.975	0.988	0.991	0.988	0.989	0.983	0.991

or barriers to interprovincial trade infrastructure.

By contrast, the between-sector component ($\hat{A}_{\text{between}}$) remains closer to the efficiency frontier, ranging from 0.958 to 0.987. While this suggests a relatively efficient allocation of resources across sectors within provinces, deeper examination reveals that inefficiencies are still meaningful and largely driven by distortions in capital and energy input use. These between-sector inefficiencies translate to roughly 2–4% annual productivity loss, highlighting significant inefficiencies for input allocation between sectors within each province. However, it is worth noting that over time allocation has improved and the productivity loss due to input misallocation between sectors has become closer to 2% suggesting an improvement in this end.

Taking a closer look at the between-term, we can comment on Input-specific misallocation patterns. Capital allocation is accounting for 1-2% of the productivity loss, with $\hat{A}$capital ranging from 0.987 to 0.996, contributing significantly to between-sector misallocation. Labor, meanwhile, is allocated with near perfection, with $\hat{A}$labor reaching 1.00 in most years. Notably, energy emerges as a key source of aggregate productivity loss. Despite its relatively small share in total input use (roughly 8%), the energy-specific efficiency term, $\hat{A}_{\text{energy}}$, ranges from 0.975 to 0.991—corresponding to a 1–2.5% productivity loss. This substantial impact underscores the disproportionate role of energy in driving misallocation and highlights the need to scrutinize energy use more closely as a critical contributor to economy-wide productivity gaps. These distortions suggest that energy is not flowing efficiently toward its most productive uses, potentially due to pricing rigidities, subsidies, or a lack of coordination between energy and industrial policy.

When the elasticity of substitution is raised to $\sigma = 7$, overall productivity losses decrease: $\hat{A}$ rises to 0.841–0.854. As expected, greater substitutability allows for more reallocation within sectors across provinces, improving both within- and between-sector efficiency. The within-sector component ($\hat{A}_{\text{within}}$) improves to 0.858–0.877, reflecting better resource reallocation (i.e., 12-15 % productivity loss) across provinces within the same sector. However, the between-sector term remains constant by construction and continues to reflect the persistent inefficiencies tied to factor-specific distortions.

In sum, the decomposition points to two key inefficiency channels: persistent interprovincial frictions in input use within sectors, and misallocation across sectors that is driven largely by capital and especially energy inputs. While labor appears to be allocated efficiently, capital and energy distortions account for the bulk of the sectoral misallocation losses—approximately 2–3% of potential productivity each year. These findings elevate energy from a secondary concern to a central issue in misallocation, and highlight the need for more targeted reforms in provincial energy pricing, infrastructure, and industrial strategy.

5 Conclusion

This paper quantifies the productivity loss in Canada resulting from the misallocation of energy, labor, and capital across sectors and provinces. By extending the standard misallocation framework to include energy as a distinct input and using detailed provincial input-output data, I quantify the potential gains when the inputs are reallocated across provinces and sectors optimally and decompose this gains into interprovincial and intersectoral terms where intersectoral term is further decomposed into factors contribution from an efficient benchmark.

The findings reveal substantial inefficiencies. Under a conservative elasticity of substitution ($\sigma = 3$), reallocating inputs efficiently across provinces and sectors could increase aggregate output by up to 32%. Even with a more elastic assumption ($\sigma = 7$), the potential gain from optimal reallocation remains sizable at 15%. Decomposing these losses reveals that interprovincial misallocation—driven largely by regulatory fragmentation and limited energy trade—accounts for the majority of the productivity gap. Within provinces, labor appears to be nearly efficiently allocated, while capital misallocation contributes approximately 1% to the loss. Energy misallocation, however, accounts for 1–2.5% of the total, making it the most inefficiently used input despite its relatively small share in total input costs (around 8% on average).

These results highlight two key points. First, energy plays a disproportionately large role in shaping aggregate productivity, despite its relatively small share in production—a characteristic that has often led to its omission in misallocation analyses. This underscores the need to recognize energy as a critical driver of aggregate productivity loss and to incorporate it more systematically into future misallocation frameworks, while also integrating energy policy more centrally into broader productivity and growth strategies.. Second, interprovincial fragmentation—including trade barriers and regulatory differences—remains a major obstacle to efficient resource allocation, suggesting that greater coordination across provincial regulations and investment in interprovincial trade infrastructure could yield sizable economic gains.

In conclusion, the results underscore the importance of considering both spatial and input-specific dimensions of misallocation when evaluating aggregate productivity. By explicitly incorporating energy as a distinct factor of production, this paper adds a critical layer to the misallocation literature and demonstrates that even relatively small inputs can generate sizable distortions when poorly allocated. The Canadian context—with its provincial regulatory heterogeneity and limited internal trade—further illustrates how institutional frictions can amplify inefficiencies. While focused on Canada, the analysis offers broader lessons for federal systems with fragmented energy regulation or limited interregional energy trade, such as the United States, Australia, or European countries. These insights provide a foundation for future research on province-sector level misallocation and the role of coordination in national productivity strategies. Addressing these inefficiencies is not only essential for unlocking Canada's growth potential but also for aligning economic and climate objectives in a coherent policy framework.

References

Asker, J., A. Collard-Wexler, and J. De Loecker (2019). (mis) allocation, market power, and global oil extraction. *American Economic Review 109*(4), 1568–1615.

Bartelsman, E., J. Haltiwanger, and S. Scarpetta (2013). Cross-country differences in productivity: The role of allocation and selection. *American economic review 103*(1), 305–334.

Brandt, L., T. Tombe, and X. Zhu (2013). Factor market distortions across time, space and sectors in china. *Review of economic dynamics 16*(1), 39–58.

Chen, K. and A. Irarrazabal (2015). The role of allocative efficiency in a decade of recovery. *Review of Economic Dynamics 18*(3), 523–550.

Choi, B. (2020). Productivity and misallocation of energy resources: Evidence from korea's manufacturing sector. *Resource and Energy Economics 61*, 101184.

Hsieh, C.-T. and P. J. Klenow (2009). Misallocation and manufacturing tfp in china and india. *The Quarterly journal of economics 124*(4), 1403–1448.

Restuccia, D. and R. Rogerson (2017). The causes and costs of misallocation. *Journal of Economic Perspectives 31*(3), 151–174.

Tombe, T. and J. Winter (2015). Environmental policy and misallocation: The productivity effect of intensity standards. *Journal of Environmental Economics and Management 72*, 137–163.

Appendix

5.1 Derivation of Sectoral Shares

To determine θ_s, we solve the maximization problem:

$$\max_{Y_s} \quad PY - \sum_s P_s Y_s. \tag{5.1}$$

Plugging in the production function:

$$Y = \prod_{s=1}^{S} Y_s^{\theta_s}, \tag{5.2}$$

the first-order condition gives:

$$P\theta_s Y_s^{-1} \prod_{s=1}^{S} Y_s^{\theta_s} = P_s. \tag{5.3}$$

Multiplying both sides by Y_s yields:

$$P\theta_s \prod_{s=1}^{S} Y_s^{\theta_s} = P_s Y_s. \tag{5.4}$$

Solving for θ_s:

$$\theta_s = \frac{P_s Y_s}{PY}. \tag{5.5}$$

Now, plugging Y_s into the expression for Y would give us

$$Y = \prod_{s=1}^{S} \left(\frac{\theta_s PY}{P_s} \right)^{\theta_s} = PY^{\sum \theta_s} \prod_{s=1}^{S} \left(\frac{\theta_s}{P_s} \right)^{\theta_s} \tag{5.6}$$

As $\sum_{s=1}^{S} \theta_s = 1$, we get

$$P = \prod_{s=1}^{S} \left(\frac{P_s}{\theta_s} \right)^{\theta_s} \tag{5.7}$$

5.2 Province-Level Pricing and Revenue

Provinces solve:

$$\max_{Y_{si}} \quad P_s \left(\sum_i Y_{si}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} - \sum_i P_{si} Y_{si}. \tag{5.8}$$

FOC yields:

$$P_{si} = P_s Y_s^{\frac{1}{\sigma}} Y_{si}^{\frac{-1}{\sigma}}, \tag{5.9}$$

and thus,

$$P_{si} Y_{si} = P_s Y_s^{\frac{1}{\sigma}} Y_{si}^{\frac{\sigma-1}{\sigma}}. \tag{5.10}$$

5.3 Derivation of the Sectoral Price Index

We derive the sectoral price index P_s using the cost minimization problem.

The total output in sector s is given by a CES aggregator of individual variety outputs Y_{si}:

$$Y_s = \left(\sum_i Y_{si}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \tag{5.11}$$

where $\sigma > 1$ is the elasticity of substitution between varieties.

To derive P_s, consider a cost-minimizing province solving the problem:

$$\min_{Y_{si}} \sum_i P_{si} Y_{si} \quad \text{subject to} \quad Y_s = \left(\sum_i Y_{si}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}. \tag{5.12}$$

We form the Lagrangian:

$$\mathcal{L} = \sum_i P_{si} Y_{si} + \lambda_s \left(Y_s^{\frac{\sigma-1}{\sigma}} - \sum_i Y_{si}^{\frac{\sigma-1}{\sigma}} \right). \tag{5.13}$$

The first-order condition with respect to Y_{si} is:

$$P_{si} - \lambda_s \frac{\sigma-1}{\sigma} Y_{si}^{-\frac{1}{\sigma}} = 0. \tag{5.14}$$

Total costs are given by

$$\sum_i P_{si} Y_{si} = \sum_i \lambda_s \frac{\sigma-1}{\sigma} Y_{si}^{-\frac{1}{\sigma}} Y_{si} = \lambda_s \frac{\sigma-1}{\sigma} \sum_i Y_{si}^{\frac{\sigma-1}{\sigma}} = \lambda_s \frac{\sigma-1}{\sigma} Y_s^{\frac{\sigma-1}{\sigma}} \tag{5.15}$$

Rearranging yields the demand function,

$$Y_{si}^{\frac{\sigma-1}{\sigma}} = \left(\lambda_s \frac{1}{P_{si}} \frac{\sigma-1}{\sigma}\right)^{\sigma-1}. \tag{5.16}$$

Now we solve for λ_s,

$$Y_s = \left(\sum_i Y_{si}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} = \left(\lambda_s \frac{\sigma-1}{\sigma}\right)^{\sigma} \left(\sum_i \left(\frac{1}{P_{si}}\right)^{\sigma-1}\right)^{\frac{\sigma}{\sigma-1}} \tag{5.17}$$

$$\lambda_s = \frac{\sigma}{\sigma-1} Y_s^{\frac{1}{\sigma}} \left(\sum_i \left(\frac{1}{P_{si}}\right)^{\sigma-1}\right)^{\frac{-1}{\sigma-1}} \tag{5.18}$$

Plugging this into our total cost expression yields

$$\sum_i P_{si} Y_{si} = \lambda_s \frac{\sigma-1}{\sigma} Y_s^{\frac{\sigma-1}{\sigma}} = Y_s^{\frac{1}{\sigma}} \left(\sum_i \left(\frac{1}{P_{si}}\right)^{\sigma-1}\right)^{\frac{-1}{\sigma-1}} Y_s^{\frac{\sigma-1}{\sigma}} = Y_s \left(\sum_i P_{si}^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \tag{5.19}$$

Therefore, based on the following equation

$$\sum_i P_{si} Y_{si} = P_s Y_s \tag{5.20}$$

we conclude that

$$P_s = \left(\sum_i P_{si}^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \tag{5.21}$$

5.4 Production Function

The Cobb-Douglas production function at the province level is:

$$Y_{si} = A_{si} K_{si}^{\alpha_s} L_{si}^{\beta_s} E_{si}^{\gamma_s}, \quad \text{where } \alpha_s + \beta_s + \gamma_s = 1. \tag{5.22}$$

5.5 Distortions in Input Markets

Each input is subject to a distortion $\tau_{K_{si}}, \tau_{L_{si}}, \tau_{E_{si}}$, so that firms face distorted input prices. The firm's problem is:

$$\max_{K_{si}, L_{si}, E_{si}} \quad P_s Y_s^{\frac{1}{\sigma}} Y_{si}^{\frac{\sigma-1}{\sigma}} - (1+\tau_{K_{si}}) r K_{si} - (1+\tau_{L_{si}}) w L_{si} - (1+\tau_{E_{si}}) p_E E_{si}. \tag{5.23}$$

We can rewrite this problem by using $Y_{si} = A_{si}K_{si}^{\alpha_s}L_{si}^{\beta_s}E_{si}^{\gamma_s}$:

$$\max_{K_{si},L_{si},E_{si}} \quad P_s Y_s^{\frac{1}{\sigma}} \left(A_{si}K_{si}^{\alpha_s}L_{si}^{\beta_s}E_{si}^{\gamma_s}\right)^{\frac{\sigma-1}{\sigma}} - (1+\tau_{K_{si}})rK_{si} - (1+\tau_{L_{si}})wL_{si} - (1+\tau_{E_{si}})p_E E_{si}. \tag{5.24}$$

The first-order conditions (FOCs) for optimal input choices are:

$$MRPK_{si} = P_s Y_s^{\frac{1}{\sigma}} \frac{(\sigma-1)}{\sigma} Y_{si}^{-\frac{1}{\sigma}} \alpha_s \frac{Y_{si}}{K_{si}} = (1+\tau_{K_{si}})r, \tag{5.25}$$

$$MRPL_{si} = P_s Y_s^{\frac{1}{\sigma}} \frac{(\sigma-1)}{\sigma} Y_{si}^{-\frac{1}{\sigma}} \beta_s \frac{Y_{si}}{L_{si}} = (1+\tau_{L_{si}})w, \tag{5.26}$$

$$MRPE_{si} = P_s Y_s^{\frac{1}{\sigma}} \frac{(\sigma-1)}{\sigma} Y_{si}^{-\frac{1}{\sigma}} \gamma_s \frac{Y_{si}}{E_{si}} = (1+\tau_{E_{si}})p_E. \tag{5.27}$$

Marginal revenue products:

$$MRPK_{si} = \alpha_s \frac{(\sigma-1)}{\sigma} \frac{P_s Y_s^{\frac{1}{\sigma}} Y_{si}^{-\frac{1}{\sigma}}}{K_{si}} = \alpha_s \frac{(\sigma-1)}{\sigma} \frac{P_{si}Y_{si}}{K_{si}} = (1+\tau_{K_{si}})r, \tag{5.28}$$

$$MRPL_{si} = \beta_s \frac{(\sigma-1)}{\sigma} \frac{P_s Y_s^{\frac{1}{\sigma}} Y_{si}^{-\frac{1}{\sigma}}}{L_{si}} = \beta_s \frac{(\sigma-1)}{\sigma} \frac{P_{si}Y_{si}}{L_{si}} = (1+\tau_{L_{si}})w, \tag{5.29}$$

$$MRPE_{si} = \gamma_s \frac{(\sigma-1)}{\sigma} \frac{P_s Y_s^{\frac{1}{\sigma}} Y_{si}^{-\frac{1}{\sigma}}}{E_{si}} = \gamma_s \frac{(\sigma-1)}{\sigma} \frac{P_{si}Y_{si}}{E_{si}} = (1+\tau_{E_{si}})p_E. \tag{5.30}$$

Now,

$$TFPQ_{si} = A_{si} = \frac{Y_{si}}{K_{si}^{\alpha_s}L_{si}^{\beta_s}E_{si}^{\gamma_s}} \tag{5.31}$$

$$TFPR_{si} = P_{si}A_{si} = \frac{P_{si}Y_{si}}{K_{si}^{\alpha_s}L_{si}^{\beta_s}E_{si}^{\gamma_s}} \tag{5.32}$$

Now, lets take the geometric average of Marginal revenue product of each input with their sector shares

$$\begin{aligned}
&(MRPK_{si})^{\alpha_s}(MRPL_{si})^{\beta_s}(MRPE_{si})^{\gamma_s} \\
&= (\alpha \frac{(\sigma-1)}{\sigma} \frac{P_{si}Y_{si}}{K_{si}})^{\alpha_s} (\beta \frac{(\sigma-1)}{\sigma} \frac{P_{si}Y_{si}}{L_{si}})^{\beta_s} (\gamma \frac{(\sigma-1)}{\sigma} \frac{P_{si}Y_{si}}{E_{si}})^{\gamma_s} \\
&= ((1+\tau_{K_{si}})r)^{\alpha_s}((1+\tau_{L_{si}})w)^{\beta_s}((1+\tau_{E_{si}})p_E)^{\gamma_s} \\
&= {\alpha_s}^{\alpha_s}{\beta_s}^{\beta_s}{\gamma_s}^{\gamma_s} \frac{(\sigma-1)}{\sigma} \frac{P_{si}Y_{si}}{K_{si}^{\alpha_s}L_{si}^{\beta_s}E_{si}^{\gamma_s}} \\
&= {\alpha_s}^{\alpha_s}{\beta_s}^{\beta_s}{\gamma_s}^{\gamma_s} \frac{(\sigma-1)}{\sigma} TFPR_{si}
\end{aligned} \tag{5.33}$$

Hence,

$$TFPR_{si} \propto (MRPK_{si})^{\alpha_s}(MRPL_{si})^{\beta_s}(MRPE_{si})^{\gamma_s} \propto ((1+\tau_K))^{\alpha_s}((1+\tau_L))^{\beta_s}((1+\tau_E))^{\gamma_s} \tag{5.34}$$

This formulation explicitly shows how TFPR is the geometric mean of marginal revenue products.

Now, to recover distortions we will go back to equations of marginal revenue product of inputs and normalize them in a specific way to get distortions explicitly so we can calculate it with data.

Recall that

$$\alpha_s \frac{(\sigma-1)}{\sigma}\frac{P_{si}Y_{si}}{rK_{si}} = (1+\tau_{K_{si}}) \tag{5.35}$$

$$\beta_s \frac{(\sigma-1)}{\sigma}\frac{P_{si}Y_{si}}{wL_{si}} = (1+\tau_{L_{si}}) \tag{5.36}$$

$$\gamma_s \frac{(\sigma-1)}{\sigma}\frac{P_{si}Y_{si}}{p_E E_{si}} = (1+\tau_{E_{si}}) \tag{5.37}$$

Take the average of both sides and set the average distortions to 0 to get.

$$\sum_i \tau_{K_{si}} = \sum_i \tau_{L_{si}} = \sum_i \tau_{E_{si}} = 0 \tag{5.38}$$

To identify province-level distortions relative to a sectoral benchmark, we normalize the average distortion to zero:

$$\frac{1}{I}\sum_i^I \alpha_s \frac{(\sigma-1)}{\sigma}\frac{P_{si}Y_{si}}{rK_{si}} = \frac{1}{I}\sum_i^I (1+\tau_{K_{si}}) = 1 \tag{5.39}$$

$$\frac{1}{I}\sum_i^I \beta_s \frac{(\sigma-1)}{\sigma}\frac{P_{si}Y_{si}}{wL_{si}} = \frac{1}{I}\sum_i^I (1+\tau_{L_{si}}) = 1 \tag{5.40}$$

$$\frac{1}{I}\sum_i^I \gamma_s \frac{(\sigma-1)}{\sigma}\frac{P_{si}Y_{si}}{p_E E_{si}} = \frac{1}{I}\sum_i^I (1+\tau_{E_{si}}) = 1 \tag{5.41}$$

This allows us to interpret each $\tau_{K_{si}}, \tau_{L_{si}}, \tau_{E_{si}}$ as a deviation from the sectoral mean, effectively treating the average province as undistorted.

Now,

$$\frac{\alpha_s \frac{(\sigma-1)}{\sigma}\frac{P_{si}Y_{si}}{rK_{si}}}{\frac{1}{I}\sum_i^I \alpha_s \frac{(\sigma-1)}{\sigma}\frac{P_{si}Y_{si}}{rK_{si}}} = (1+\tau_{K_{si}}) \tag{5.42}$$

$$\frac{\frac{P_{si}Y_{si}}{rK_{si}}}{\frac{1}{I}\sum_i^I \frac{P_{si}Y_{si}}{rK_{si}}} = (1+\tau_{K_{si}}) \tag{5.43}$$

$$\frac{\frac{P_{si}Y_{si}}{wL_{si}}}{\frac{1}{I}\sum_i^I \frac{P_{si}Y_{si}}{wL_{si}}} = (1+\tau_{L_{si}}) \tag{5.44}$$

$$\frac{\frac{P_{si}Y_{si}}{p_E E_{si}}}{\frac{1}{I}\sum_i^I \frac{P_{si}Y_{si}}{p_E E_{si}}} = (1+\tau_{E_{si}}) \tag{5.45}$$

With this simple trick we get rid of σ and sector share constants $(\alpha_s, \beta_s, \gamma_s)$

As we are already having distortions the next step is to calculate sector-level weighted averages of marginal revenue products.

we can start with

$$\overline{MRPK_s} = \frac{\sum_i K_{si} MRPKsi}{\sum_i K_{si}} = \frac{\sum_i \alpha_s \frac{\sigma-1}{\sigma} P_{si}Y_{si}}{\sum_i \alpha_s \frac{\sigma-1}{\sigma}\frac{P_{si}Y_{si}}{r(1+\tau_{K_{si}})}} = \frac{\sum_i P_{si}Y_{si}}{\sum_i \frac{P_{si}Y_{si}}{r(1+\tau_{K_{si}})}} \tag{5.46}$$

Given that sectoral revenue $P_sY_s = \sum_i P_{si}Y_{si}$ we can write that

$$\overline{MRPK_s} = \frac{r}{\sum_i \frac{1}{(1+\tau_{K_{si}})}\frac{P_{si}Y_{si}}{P_sY_s}} \tag{5.47}$$

Finally,

$$\frac{\overline{MRPK_s}}{MRPK_{si}} = \frac{\frac{r}{\sum_i \frac{1}{(1+\tau_{K_{si}})}\frac{P_{si}Y_{si}}{P_sY_s}}}{r(1+\tau_{K_{si}})} = \frac{1}{(1+\tau_{K_{si}})\sum_i \frac{1}{(1+\tau_{K_{si}})}\frac{P_{si}Y_{si}}{P_sY_s}} \tag{5.48}$$

Similar algebra yields,

$$\frac{\overline{MRPL_s}}{MRPL_{si}} = \frac{1}{(1+\tau_{L_{si}})\sum_i \frac{1}{(1+\tau_{L_{si}})}\frac{P_{si}Y_{si}}{P_sY_s}} \tag{5.49}$$

$$\frac{\overline{MRPE_s}}{MRPE_{si}} = \frac{1}{(1+\tau_{E_{si}})\sum_i \frac{1}{(1+\tau_{E_{si}})}\frac{P_{si}Y_{si}}{P_sY_s}} \tag{5.50}$$

It is straightworward to see these equaitons are equal to 1 if there were no distortions. Given that we have explicit formulas for distortions and marginal revenue products compared to sector averages we can move forward to calculate the output implications of these.

Now recall that we have derived the expression

$$P_{si}Y_{si} = P_sY_s^{\frac{1}{\sigma}}Y_{si}^{\frac{\sigma-1}{\sigma}}. \tag{5.51}$$

if we divide each side by P_sY_s we would get

$$\frac{P_{si}Y_{si}}{P_sY_s} = \left(\frac{Y_{si}}{Y_s}\right)^{\frac{\sigma-1}{\sigma}}, \tag{5.52}$$

Taking the geometric average across all factors K, L, and E:

$$\begin{aligned}\left(\frac{\overline{MRPK_s}}{MRPK_{si}}\right)^{\alpha}\left(\frac{\overline{MRPL_s}}{MRPL_{si}}\right)^{\beta}\left(\frac{\overline{MRPE_s}}{MRPE_{si}}\right)^{\gamma}\\ = \left(\frac{\sum_i K_{si}MRPK_{si}}{MRPK_{si}\sum_i K_{si}}\right)^{\alpha}\left(\frac{\sum_i L_{si}MRPL_{si}}{MRPL_{si}\sum_i L_{si}}\right)^{\beta}\left(\frac{\sum_i E_{si}MRPE_{si}}{MRPE_{si}\sum_i E_{si}}\right)^{\gamma}\end{aligned} \tag{5.53}$$

Now recalling the formulas for Marginal revenue products we can see that

$$\sum_i K_{si}MRPK_{si} = \sum_i L_{si}MRPL_{si} = \sum_i E_{si}MRPE_{si} \propto \sum_i P_{si}Y_{si} = P_sY_s \tag{5.54}$$

as $\alpha_s, \beta_s, and \gamma_s$ sums up to 1, we have P_sY_s in numerator. Also, note that

$$\sum_i K_{si} = K_s, \sum_i L_{si} = L_s, \sum_i E_{si} = E_s, \tag{5.55}$$

finally, the geometric average of marginal revenue products is proportional to $TFPR_{si} = P_{si}A_{si}$

Then we have,

$$\begin{aligned}\left(\frac{\overline{MRPK_s}}{MRPK_{si}}\right)^{\alpha}\left(\frac{\overline{MRPL_s}}{MRPL_{si}}\right)^{\beta}\left(\frac{\overline{MRPE_s}}{MRPE_{si}}\right)^{\gamma}\\ = \frac{P_sY_s}{P_{si}A_{si}K_s^{\alpha_s}L_s^{\beta_s}E_s^{\gamma_s}}\\ = \frac{P_sA_s}{P_{si}A_{si}}\end{aligned} \tag{5.56}$$

We have

$$P_s = \left(\sum_i P_{si}^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \tag{5.57}$$

To isolate A_s we can multiply the expression by A_{si} and take the power of $(\sigma - 1)$ and some over provinces.

$$\sum_i \left(\frac{\cancel{A_{si}} P_s A_s}{P_{si} \cancel{A_{si}}}\right)^{\sigma-1} = (P_s A_s)^{(\sigma-1)} \sum_i P_{si}^{(1-\sigma)} = A_s^{\sigma-1} \tag{5.58}$$

if we take the power of $1/(\sigma - 1)$ we arrive at $TFP_s = A_s$ by applying the same operations to the left-hand side we get an expression for TFP_s

$$A_s = \left[\sum_i \left(A_{si} \left(\frac{\overline{MRPK_s}}{MRPK_{si}}\right)^{\alpha} \left(\frac{\overline{MRPL_s}}{MRPL_{si}}\right)^{\beta} \left(\frac{\overline{MRPE_s}}{MRPE_{si}}\right)^{\gamma}\right)^{\sigma-1}\right]^{\frac{1}{\sigma-1}}. \tag{5.59}$$

Finally we get an expression for A_{si} to bring this model into data. Recall that

$$P_{si} Y_{si} = P_s (Y_s)^{\frac{1}{\sigma}} Y_{si}^{\frac{\sigma-1}{\sigma}} \tag{5.60}$$

this implies

$$Y_{si} = (P_s (Y_s)^{\frac{1}{\sigma}})^{\frac{-\sigma}{1-\sigma}} (P_{si} Y_{si})^{\frac{\sigma}{\sigma-1}} \tag{5.61}$$

therefore,

$$A_{si} = \frac{(P_s Y_s)^{\frac{-1}{\sigma-1}}}{P_s} \frac{(P_{si} Y_{si})^{\frac{\sigma}{\sigma-1}}}{K_{si}^{\alpha_s} L_{si}^{\beta_s} E_{si}^{\gamma_s}} \tag{5.62}$$

So

$$A_{si} \propto \frac{(P_{si} Y_{si})^{\frac{\sigma}{\sigma-1}}}{K_{si}^{\alpha_s} L_{si}^{\beta_s} E_{si}^{\gamma_s}} \tag{5.63}$$

5.6 Productivity Decomposition

We begin by defining sector-level total factor productivity A_s using a Cobb-Douglas production function, where output Y_s is produced using capital K_s, labor L_s, and energy E_s:

$$A_s = \frac{Y_s}{K_s^{\alpha_s} L_s^{\beta_s} E_s^{\gamma_s}} \tag{5.64}$$

Sectoral output Y_s aggregates province-level outputs Y_{si} through a constant elasticity of substitution (CES) aggregator with elasticity σ:

$$Y_s = \left(\sum_i Y_{si}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \tag{5.65}$$

Substituting province-level production functions $Y_{si} = A_{si} K_{si}^{\alpha_s} L_{si}^{\beta_s} E_{si}^{\gamma_s}$ into the CES aggregator, we express sectoral TFP as:

$$\Rightarrow A_s = \frac{\left(\sum_i \left(A_{si} K_{si}^{\alpha_s} L_{si}^{\beta_s} E_{si}^{\gamma_s}\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}}{K_s^{\alpha_s} L_s^{\beta_s} E_s^{\gamma_s}} \tag{5.66}$$

$$= \left[\sum_i \left(A_{si} \left(\frac{K_{si}}{K_s}\right)^{\alpha_s} \left(\frac{L_{si}}{L_s}\right)^{\beta_s} \left(\frac{E_{si}}{E_s}\right)^{\gamma_s}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \tag{5.67}$$

Note that this expression is just normalizing province-level inputs by the sector total, for which we know explicitly what they are.

$$k_{si} = \frac{K_{si}}{K_s} = \frac{K_{si}}{\sum_i K_{si}} = \frac{\frac{\alpha_s}{r} \frac{\sigma-1}{\sigma} \frac{P_{si} Y_{si}}{(1+\tau_{K_{si}})}}{\sum_i \frac{\alpha_s}{r} \frac{\sigma-1}{\sigma} \frac{P_{si} Y_{si}}{(1+\tau_{K_{si}})}} = \frac{R_{si}/(1+\tau_{K_{si}})}{\sum_i R_{si}/(1+\tau_{K_{si}})} \tag{5.68}$$

$$\text{Let revenue shares } R_{si} = \frac{P_{si} Y_{si}}{P_s Y_s}, \quad k_{si} = \frac{K_{si}}{K_s}, \quad l_{si} = \frac{L_{si}}{L_s}, \quad e_{si} = \frac{E_{si}}{E_s} \tag{5.69}$$

$$\Rightarrow A_s = \left[\sum_i \left(A_{si} k_{si}^{\alpha_s} l_{si}^{\beta_s} e_{si}^{\gamma_s}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \tag{5.70}$$

It is straightforward to see that the fully efficient allocation yields a sector-level TFP A_s^* expressed as:

$$A_s^* = \left[\sum_i A_{si}^{\sigma-1}\right]^{\frac{1}{\sigma-1}} \tag{5.71}$$

Dividing the actual level of TFP A_s to efficient level of TFP A_s^* we get:

$$\frac{A_s}{A_s^*} = \frac{\left[\sum_i \left(A_{si}\left(\frac{R_{si}/(1+\tau_{K_{si}})}{\sum_i R_{si}/(1+\tau_{K_{si}})}\right)^{\alpha_s}\left(\frac{R_{si}/(1+\tau_{L_{si}})}{\sum_i R_{si}/(1+\tau_{L_{si}})}\right)^{\beta_s}\left(\frac{R_{si}/(1+\tau_{E_{si}})}{\sum_i R_{si}/(1+\tau_{E_{si}})}\right)^{\gamma_s}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}}{\left[\sum_i A_{si}^{\sigma-1}\right]^{\frac{1}{\sigma-1}}} \tag{5.72}$$

Moving to the national level, we express national TFP (i.e., A) as:

$$A = \frac{Y}{K^{\bar{\alpha}}L^{\bar{\beta}}E^{\bar{\delta}}} = \frac{\prod_s Y_s^{\theta_s}}{K^{\bar{\alpha}}L^{\bar{\beta}}E^{\bar{\delta}}}, \quad \bar{\alpha} = \sum_s \alpha_s \theta_s \tag{5.73}$$

$$= \prod_s \left(\frac{A_s K_s^{\alpha_s} L_s^{\beta_s} E_s^{\gamma_s}}{K^{\alpha_s} L^{\beta_s} E^{\gamma_s}}\right)^{\theta_s} \tag{5.74}$$

$$= \prod_s A_s^{\theta_s}\left(\frac{K_s}{K}\right)^{\alpha_s\theta_s}\left(\frac{L_s}{L}\right)^{\beta_s\theta_s}\left(\frac{E_s}{E}\right)^{\gamma_s\theta_s} \tag{5.75}$$

$$= \prod_s \left(A_s\left(\frac{K_s}{K}\right)^{\alpha_s}\left(\frac{L_s}{L}\right)^{\beta_s}\left(\frac{E_s}{E}\right)^{\gamma_s}\right)^{\theta_s} \tag{5.76}$$

$$K = \sum_s K_s = \sum_s \sum_i K_{si} \tag{5.77}$$

$$k_s = \frac{K_s}{K} = \frac{\sum_i K_{si}}{\sum_s \sum_i K_{si}}, \quad = \frac{\sum_i \frac{\alpha_s}{r}\left(\frac{\sigma-1}{\sigma}\right)\frac{P_{si}Y_{si}}{1+\tau_{K_{si}}}}{\sum_s \sum_i \frac{\alpha_s}{r}\left(\frac{\sigma-1}{\sigma}\right)\frac{P_{si}Y_{si}}{1+\tau_{K_{si}}}} = \frac{\sum_i \alpha_s \frac{P_{si}Y_{si}}{1+\tau_{K_{si}}}}{\sum_s \sum_i \alpha_s \frac{P_{si}Y_{si}}{1+\tau_{K_{si}}}} \tag{5.78}$$

$$= \frac{\alpha_s \sum_i \frac{P_{si}Y_{si}}{1+\tau_{K_{si}}}}{\sum_s \alpha_s \sum_i \frac{P_{si}Y_{si}}{1+\tau_{K_{si}}}} = \frac{\alpha_s P_s Y_s \sum_i \frac{R_{si}}{1+\tau_{K_{si}}}}{\sum_s \alpha_s P_s Y_s \sum_i \frac{R_{si}}{1+\tau_{K_{si}}}} \tag{5.79}$$

as $R_{si} = P_{si}Y_{si}/P_sY_s$

$$\frac{K_s}{K} = \frac{\alpha_s P_s Y_s \sum_i R_{si}/(1+\tau_{K_{si}})}{\sum_s \alpha_s P_s Y_s \sum_i R_{si}/(1+\tau_{K_{si}})} = \frac{\alpha_s P_s Y_s \sum_i R_{si}/(1+\tau_{K_{si}})}{\sum_s \alpha_s P_s Y_s \sum_i R_{si}/(1+\tau_{K_{si}})} \quad \text{(Actual } k_s\text{)} \tag{5.80}$$

If we define (harmonic mean of sector-level distortions),

$$\overline{1+\tau_{Ks}} = \frac{1}{\sum_i \frac{R_{si}}{1+\tau_{K_{si}}}} \quad \text{(harmonic mean)} \tag{5.81}$$

and recalling that $\theta_s = P_s Y_s / PY$ then we can write

$$k_s = \frac{K_s}{K} = \frac{\alpha_s \theta_s / (\overline{1 + \tau_{Ks}})}{\sum_s \alpha_s \theta_s / (\overline{1 + \tau_{Ks}})} \tag{5.82}$$

Also, as there is no distortions in optimal allocation we can simply write

$$k_s^* = \frac{K_s^*}{K^*} = \frac{\alpha_s \theta_s}{\sum_s \alpha_s \theta_s} \tag{5.83}$$

$$\Rightarrow \frac{k_s}{k_s^*} = \frac{\left(\frac{1}{\overline{1+\tau_{Ks}}}\right) \cdot \sum_s \alpha_s \theta_s}{\sum_s \frac{\alpha_s \theta_s}{\overline{1+\tau_{Ks}}}} \tag{5.84}$$

Similar algebra yields,

$$\Rightarrow \frac{l_s}{l_s^*} = \frac{\left(\frac{1}{\overline{1+\tau_{Ls}}}\right) \cdot \sum_s \beta_s \theta_s}{\sum_s \frac{\beta_s \theta_s}{\overline{1+\tau_{Ls}}}} \tag{5.85}$$

$$\Rightarrow \frac{e_s}{e_s^*} = \frac{\left(\frac{1}{\overline{1+\tau_{Es}}}\right) \cdot \sum_s \gamma_s \theta_s}{\sum_s \frac{\gamma_s \theta_s}{\overline{1+\tau_{Es}}}} \tag{5.86}$$

Now, if we divide the actual national TFP A to efficient level of national TFP A^* we get:

$$\frac{A}{A^*} = \prod_s \left(\left(\frac{A_s}{A_s^*}\right) \left(\frac{k_s}{k_s^*}\right)^{\alpha_s} \left(\frac{l_s}{l_s^*}\right)^{\beta_s} \left(\frac{e_s}{e_s^*}\right)^{\gamma_s} \right)^{\theta_s} \tag{5.87}$$

$$\frac{A}{A^*} = \underbrace{\prod_s \left(\frac{A_s}{A_s^*}\right)^{\theta_s}}_{\text{Within-sector misallocation}} \times \underbrace{\prod_s \left(\left(\frac{k_s}{k_s^*}\right)^{\alpha_s} \left(\frac{l_s}{l_s^*}\right)^{\beta_s} \left(\frac{e_s}{e_s^*}\right)^{\gamma_s} \right)^{\theta_s}}_{\text{Between-sector misallocation}} \tag{5.88}$$

$$\frac{A}{A^*} = \prod_s \left(\frac{A_s}{A_s^*}\right)^{\theta_s} \times \prod_s \left[\underbrace{\left(\frac{k_s}{k_s^*}\right)^{\alpha_s}}_{\substack{\text{Capital} \\ \text{misallocation}}} \cdot \underbrace{\left(\frac{l_s}{l_s^*}\right)^{\beta_s}}_{\substack{\text{Labor} \\ \text{misallocation}}} \cdot \underbrace{\left(\frac{e_s}{e_s^*}\right)^{\gamma_s}}_{\substack{\text{Energy} \\ \text{misallocation}}} \right]^{\theta_s} \tag{5.89}$$

We can write the *within* portion of the expression explicitly as:

$$\left(\frac{A}{A^*}\right)_{within} = \prod_s \left(\frac{\left[\sum_i \left(A_{si}\left(\frac{R_{si}/(1+\tau_{K_{si}})}{\sum_i R_{si}/(1+\tau_{K_{si}})}\right)^{\alpha_s}\left(\frac{R_{si}/(1+\tau_{L_{si}})}{\sum_i R_{si}/(1+\tau_{L_{si}})}\right)^{\beta_s}\left(\frac{R_{si}/(1+\tau_{E_{si}})}{\sum_i R_{si}/(1+\tau_{E_{si}})}\right)^{\gamma_s}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}}{\left[\sum_i A_{si}^{\sigma-1}\right]^{\frac{1}{\sigma-1}}} \right)^{\theta_s} \tag{5.90}$$

We can also write the *between* portion explicitly as:

$$\left(\frac{A}{A^*}\right)_{between} = \prod_s \left(\left(\frac{\left(\frac{1}{1+\tau_{Ks}}\right)\cdot\sum_s \alpha_s\theta_s}{\sum_s \frac{\alpha_s\theta_s}{1+\tau_{Ks}}}\right)^{\alpha_s} \left(\frac{\left(\frac{1}{1+\tau_{Ls}}\right)\cdot\sum_s \beta_s\theta_s}{\sum_s \frac{\beta_s\theta_s}{1+\tau_{Ls}}}\right)^{\beta_s} \left(\frac{\left(\frac{1}{1+\tau_{Es}}\right)\cdot\sum_s \gamma_s\theta_s}{\sum_s \frac{\gamma_s\theta_s}{1+\tau_{Es}}}\right)^{\gamma_s} \right)^{\theta_s} \tag{5.91}$$

To further decompose the between term to find each input misallocation contribution we can write the following expressions.

$$\left(\frac{A}{A^*}\right)_{capital} = \prod_s \left(\left(\frac{\left(\frac{1}{1+\tau_{Ks}}\right)\cdot\sum_s \alpha_s\theta_s}{\sum_s \frac{\alpha_s\theta_s}{1+\tau_{Ks}}}\right)^{\alpha_s} \right)^{\theta_s} \tag{5.92}$$

$$\left(\frac{A}{A^*}\right)_{labor} = \prod_s \left(\left(\frac{\left(\frac{1}{1+\tau_{Ls}}\right)\cdot\sum_s \beta_s\theta_s}{\sum_s \frac{\beta_s\theta_s}{1+\tau_{Ls}}}\right)^{\beta_s} \right)^{\theta_s} \tag{5.93}$$

$$\left(\frac{A}{A^*}\right)_{energy} = \prod_s \left(\left(\frac{\left(\frac{1}{1+\tau_{Es}}\right)\cdot\sum_s \gamma_s\theta_s}{\sum_s \frac{\gamma_s\theta_s}{1+\tau_{Es}}}\right)^{\gamma_s} \right)^{\theta_s} \tag{5.94}$$

Therefore,

$$\begin{aligned}\frac{A}{A^*} &= \left(\frac{A}{A^*}\right)_{\text{within}} \times \left(\frac{A}{A^*}\right)_{\text{between}} \\ &= \left(\frac{A}{A^*}\right)_{\text{within}} \times \left(\frac{A}{A^*}\right)_{\text{capital}} \times \left(\frac{A}{A^*}\right)_{\text{labor}} \times \left(\frac{A}{A^*}\right)_{\text{energy}}\end{aligned} \tag{5.95}$$

or more compactly,

$$\hat{A} = \hat{A}_{within} \times \hat{A}_{between} \tag{5.96}$$

Or,

$$\hat{A} = \hat{A}_{within} \times \hat{A}_{capital} \times \hat{A}_{labor} \times \hat{A}_{energy} \tag{5.97}$$