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Q. 1

$(A \cup B)\backslash(A \cap B) = (A\backslash B) \cup (B\backslash A)$

$$
\begin{array}{rcc|l}
(A \cup B)\backslash(A \cap B) & \equiv & \{x | x \in (A \cup B) \wedge x \notin (A \cap B)\} & \text{Defn of set diff.} \\
 & \equiv & \{x | (x \in A \vee x \in B) \wedge x \notin (A \cap B)\} & \text{Defn of union} \\
 & \equiv & \{x | (x \in A \vee x \in B) \wedge \neg(x \in (A \cap B))\} & Defn\ of\ \notin \\
 & \equiv & \{x | (x \in A \vee x \in B) \wedge \neg(x \in A \wedge x \in B)\} & \text{Defn of intersection} \\
 & \equiv & \{x | (x \in A \vee x \in B) \wedge (x \notin A \vee x \notin B)\} & \text{De Morgan's law} \\
 & \equiv & \{x | ((x \in A \vee x \in B) \wedge (x \notin A)) \vee ((x \in A \vee x \in B) \wedge (x \notin B))\} & \text{Distributive laws} \\
 & \equiv & \{x | ((x \in A \wedge x \notin A) \vee (x \in B \wedge x \notin A)) \vee ((x \in A \wedge x \notin B) \vee (x \in B \wedge x \notin B))\} & \text{Complement laws} \\
 & \equiv & \{x | (\emptyset \vee (x \in B \wedge x \notin A)) \vee ((x \in A \wedge x \notin B) \vee \emptyset)\} & \text{Distributive laws} \\
 & \equiv & \{x | (x \in B \wedge x \notin A) \vee (x \in A \wedge x \notin B)\} & \text{Identity laws} \\
 & \equiv & (B\backslash A) \cup (A\backslash B) & \text{Defn of set diff. \& union} \\
 & \equiv & (A\backslash B) \cup (B\backslash A) & \text{commutative laws}
\end{array}
$$

Q. 2

$prove\ that\ the\ set\ \{f | f \subseteq N \times \{0,1\}\}\backslash\{f | f : \{0,1\} \rightarrow N, f\ is\ a\ function\}\ is\ uncountable.$

$Firstly,\ we\ need\ to\ prove\ that\ \{f | f \subseteq N \times \{0,1\}\}\ is\ uncountable.$

$1 \rightarrow \{0, x_1\}, \{1, x_2\}, \{2, x_3\}, \{3, x_4\}...$

$2 \rightarrow \{0, y_1\}, \{1, y_2\}, \{2, y_3\}, \{3, y_4\}...$

$3 \rightarrow \{0, z_1\}, \{1, z_2\}, \{2, z_3\}, \{3, z_4\}...$

$4 \rightarrow \{0, t_1\}, \{1, t_2\}, \{2, t_3\}, \{3, t_4\}...$

$a \in \{N \times \{0,1\}\}$

$a = \{0, a_1\}, \{1, a_2\}, \{2, a_3\}, \{3, a_4\}...$

$a_1 \neq x_1$

$a_2 \neq y_2$

$a_3 \neq z_3$

$a_4 \neq t_4$

....

$So\ there\ does\ not\ exist\ on\ enumeration\ counting\ each\ element\ in \{N \times \{0,1\}\}.$

$Secondly,\ we\ need\ to\ prove\ that\ \{f | f : \{0,1\} \rightarrow N\}\ is\ countable.$

$We\ know\ that\ N\ is\ a\ countable\ infinite\ set.\ And\ it's\ cardinality\ is |N|.\ When\ we\ look\ at\ our\ function\ \{f | f : \{0,1\} \rightarrow N\},\ its\ cardinality\ is |N| \times |N|.$

$As\ an\ example\ we\ can\ see\ that\ from\ b.$

$b \in \{N \times \{0,1\}\}$

$b = \{0, b_1\}, \{1, b_2\} and\ b1, b2 \in N$

$Since\ b\ is\ in\ the\ set\ given\ above,\ and\ it\ has\ |N| \times |N|\ cardinality.$

$We\ know\ that\ Cartesian\ product\ of\ countable\ sets\ is\ also\ countable\ because\ countable\ sets\ are\ bijective\ and\ Cartesian\ prod$

$Since,\ N\ is\ a\ countable\ infinite\ set,\ and\ our\ function's\ cardinality\ is |N| \times |N|,\ our\ set\ is\ countable\ too.$

$Finally,\ we\ need\ to\ prove\ that\ uncountable\ minus\ countable\ set\ is\ still\ uncountable.$

$A\ is\ \{f | f \subseteq N \times \{0,1\}\}$

$B\ is\ \{f | f : \{0,1\} \to N\}$

$We\ need\ to\ assume\ (A \backslash B)\ is\ countable.$

$And,\ we\ know\ that B\ is\ countable.$

$That\ means\ (A \backslash B) \cup B\ is\ countable.$

$But\ then A \subseteq (A \backslash B) \cup B. Therefore\ A\ cannot\ be subset\ of that\ set\ because\ A\ is\ uncountable\ and\ that\ set\ is\ countable.$

$That\ means\ (A \backslash B) is\ not\ countable.$

Q.3

$Question\ 3$
$Prove\ that\ the\ function\ f(n)\ =\ 4^n\ +\ 5n^2\ log\ n\ is\ not\ O(2^n).$

$Assume\ that\ (4^n + 5n^2 logn)\ \in\ O(2^n)$

$|4^n\ 5n^2 logn|\ \leq C|2^n|\ for\ \exists k\ \exists C\ and\ \forall n \geq k\ and\ k,\ C\ \in\ Z^+$

$|4^n + 5n^2 logn|\ \geq\ |4^n|$

$|4^n|\ >\ 0\ for\ all\ n$

$Therefore\ |4^n|\ =\ 4^n$

$4^n\ \leq C2^n$

$\log_2 2^n \leq \log_2 C$

$n \leq \log_2 C$

$Since\ c,\ k\ is\ a\ constant\ and\ \forall n\ n \geq k\ and\ n\ goes\ to\ infinite$

$n \leq \log_2 c\ cannot\ be\ true.$

$Therefore,\ n > \log_2 c$

$Thus,\ 4^n + 5n^2 logn \notin\ O(n^2)$

Q. 4

$prove\ that\ (2x-1)^n - x^2 = -x-1\ (mod(x-1))$

$2x-1\ \ mod(x-1) = 1$

$1^n - x^2 = -x-1\ (mod(x-1))$

$1^n = x^2 - x - 1\ (mod(x-1))$

$1^n = 1$

$x^2 - x - 1\ \ mod(x-1) =\ -1$

$1 = -1\ (mod(x-1))$

$2 = 0\ (mod(x-1))$

$Since\ 2\ is\ only\ divided\ by\ \ -2,\ -1,\ 1,\ 2\ and\ x\ is\ greater\ than\ 2,\ x-1\ can\ only\ be\ 2.$

$Therefore,\ x-1 = 2\ \rightarrow\ x = 3$