ANIL İÇEN 2448488

QUESTION 1:

$\sum_{i=1}^{\infty} a_n x^n = \sum_{i=1}^{\infty} a_{n-1} x^n + \sum_{i=1}^{\infty} 2^n x^n$

$= x * \sum_{i=1}^{\infty} a_{n-1} x^{n-1} + 2x * \sum_{i=1}^{\infty} 2x^{n-1}$

$A(x) – a_0 = x*A(x) + 2x * 1 / (1-2x)$

$= x*A(x) + 2x / (1-2x)$

$A(x) * (1-x) = 2x / (1-2x) + 1$

$A(x) * (1-x) = 1 / (1-2x)$

$A(x) = 1 / ((1-2x) * (1-x))$

$((1-2x) * (1-x)) = A/(1-2x) + B/(1-x)$

$A+B = 1$ $\quad$ $-A – 2B = 0$

$A= 2$ $\quad$ $B = -1$

$A(x) = 2 / (1-2x) + -1/(1-x)$

$2/ (1-2x) = (2, 4, 8, 16, 32, …, 2^n)$

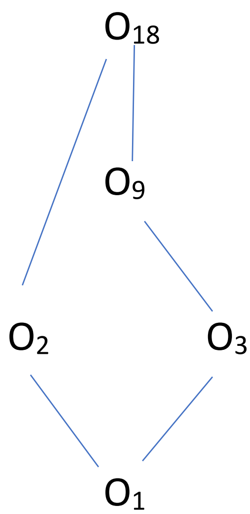
$-1/(1-x) = (-1, -1, -1, -1, …, -1)$

$A (x) = (1,3,7,15,31, …, 2^n-1)$

$A(n) = 2^n-1$

QUESTION 2:

A)



B)

$$
\begin{vmatrix}
1 & 1 & 1 & 1 & 1 \\
0 & 1 & 0 & 0 & 1 \\
0 & 0 & 1 & 1 & 1 \\
0 & 0 & 0 & 1 & 1 \\
0 & 0 & 0 & 0 & 1
\end{vmatrix}
$$

C)

A partial order relation is a lattice if for every pair of elements there is a unique Lower Upper bound and a unique Greater lower bound.

Then, in our graph, we can see that every pair of elements has a unique lower upper bound and unique greater lower bound.

Therefore, it is lattice.

D)

$$M_r = \begin{matrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{matrix} \qquad M_r^{-1} = \begin{matrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \end{matrix}$$

$$M_{S(R)} = \begin{matrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{matrix} + \begin{matrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \end{matrix} = \begin{matrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{matrix}$$

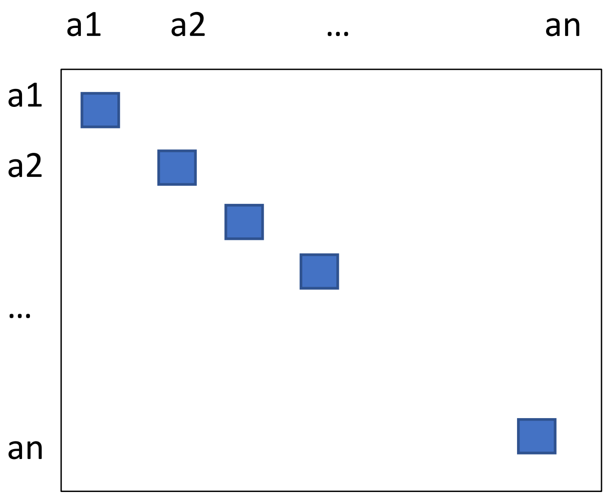
E)

The 2 and 9 are not comparable because 2 cannot divide 9, and 9 cannot divide 2. The 3 and 18 are comparable, because 3 can divide 18.

QUESTION 3:

A) A= {a1, a2, …, an}

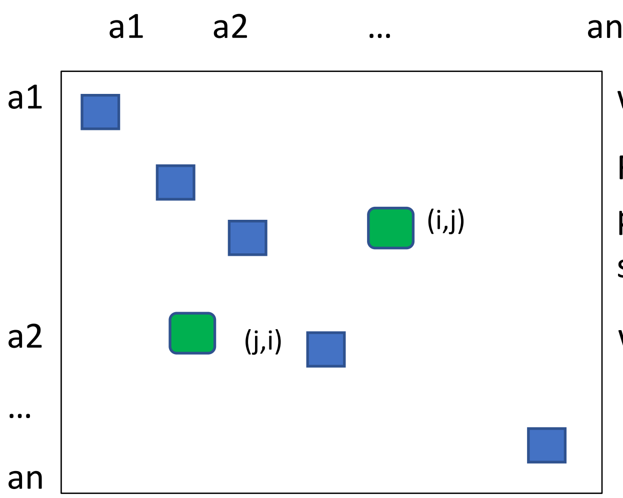
R $\subseteq$ A x A



a_{11}, a_{22}, a_{33}, …, a_{nn} can be 1 or 0

Therefore, we have 2^n different relations

from the diagonal part.



we have n*(n-1) / 2 pairs.

For each possible pair we have only 3 possible interpretations that makes it symmetric. That's why we have 3 options.

we get $3^{n*(n-1)/2}$ option from here.

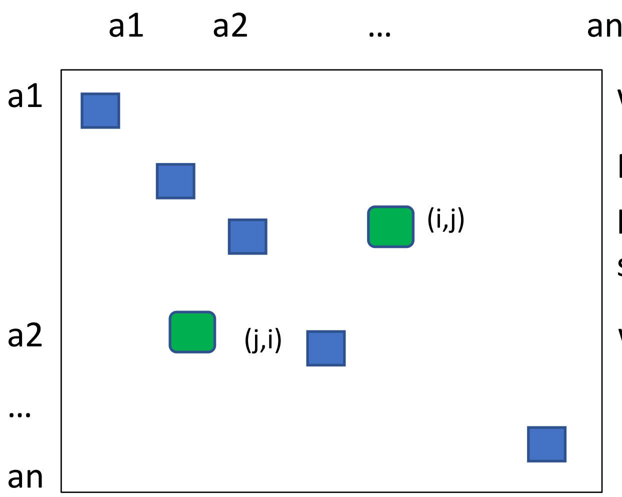
Therefore, there are $2^n * 3^{n*(n-1)/2}$ different relations on A for anti-symmetric relations.

B)
A= {a1, a2, …, an}

R $\subseteq$ A x A

This question is different but similar to first part because we need to find reflexive and anti-symmetric relations on A. The only difference is the diagonal part. In reflexive relations diagonal of the matrix must be 1. Therefore, we need to use the second part of the previous part.

And, that is:



we have n*(n-1) / 2 pairs.

For each possible pair we have only 3 possible interpretations that makes it symmetric. That’s why we have 3 options.

we get $3^{n*(n-1)/2}$ option from here.

Here, the blue squares are 1. Therefore, we can just change the green ones.

Therefore, there are $2^{n} * 3^{n*(n-1)/2}$ different relations on A for reflexive and anti-symmetric relations.