Student Information

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Q. 1

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\begin{array}{rcl|l}
\neg(p \land q) \leftrightarrow (\neg q \to p) & \equiv & (\neg p \lor \neg q) \leftrightarrow (\neg q \to p) & \text{De Morgan's Law} \\
& \equiv & (\neg p \lor \neg q) \leftrightarrow (q \lor p) & \text{implication equivalency} \\
& \equiv & ((\neg p \lor \neg q) \to (q \lor p)) \land ((q \lor p) \to (\neg p \lor \neg q)) & \text{biconditional equivalency} \\
& \equiv & (\neg(p \lor q) \lor (q \lor p)) \land \neg((\neg q \lor \neg p) \lor (\neg p \lor \neg q)) & \text{implication equivalency} \\
& \equiv & ((p \land q) \lor (q \lor p)) \land ((\neg q \land \neg p) \lor (\neg p \lor \neg q)) & \text{De Morgan's Law} \\
& \equiv & ((p \land q) \lor q) \lor ((p \land q) \lor p) \land ((\neg q \land \neg p) \lor \neg p) \lor ((\neg q \land \neg p) \lor \neg q) & \text{distributive laws} \\
& \equiv & (q \lor (p \land q)) \lor (p \lor (p \land q)) \land (\neg p \lor (\neg q \land \neg p)) \lor (\neg q \lor (\neg q \land \neg p)) & \text{commutative laws} \\
& \equiv & (q \lor p) \land (\neg p \lor \neg q) & \text{absorption laws} \\
& \equiv & (p \lor q) \land (\neg p \lor \neg q) & \text{commutative laws}
\end{array}
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Q. 2

a. $\forall x \forall z \forall y((x \neq z) \land I(x,y) \land I(z,y)) \to \forall t \neg(E(x,t) \land E(z,t))$

b. $\forall y \exists x \forall z((I(x,y) \land S(x,x) \land (S(x,z) \to (x = z)))$

c. $\forall x \forall y \forall z \forall t((I(x, medicine) \land I(y, medicine) \land I(z, medicine) \land A(x,t) \land A(y,t) \land A(z,t) \land J(t, medicine)) \to ((x = z) \lor (z = y) \lor (x = y)))$

Q. 3

a.

$$p \lor \neg q, v \lor r \vdash (\to q) \to p$$

1.	$p \lor \neg q$	*premise*
2.	$v \lor r$	*premise*
3.	$r \to q$	*assumption*
4.	p	*assumption*
5.	p	*copy* 4
6.	r	*assumption*
7.	q	$3.6 \ \to e$
8.	$\neg q$	assumption
9.	$\perp$	$7,8 \ \neg e$
10.	p	$\perp \ 9$
11.	p	assumption
12.	p	*copy* 11
13.	p	$1, 4-5, 6-10, 11-12 \ \lor e$
14.	$(r \to q) \to p$	$3, 13 \ \to i$

b.

$$\vdash ((q \to p) \to q) \to q$$

1. $(q \to p) \to q$ — Assumption
2. $q \vee \neg q$ — lemma
3. q — assumption
4. q — copy
5. $\neg q$ — assumption
6. q — assumption
7. $\perp$ — $5, 6 \neg e$
8. p — $7 \perp e$
9. $q \to p$ — $6 - 8 \to e$
10. q — $1, 9 \to e$
11. q — 2,3-4,5-10
12. $((q \to p) \to q) \to q$ — $1 - 11 \to i$

Q. 4

a.

$$\neg \forall x (P(x) \to Q(x)) \vdash \exists x (P(x) \wedge \neg Q(x))$$

1. $\neg \forall x (P(x) \to Q(x))$ — *premise*
2. $x0$
3. $\neg(P(x0) \to Q(x0))$ — $1, 2 \quad \forall xe$
4. $\neg(P(x0) \wedge \neg Q(x0))$ — *Assumption*
5. $P(x0)$ — *Assumption*
6. $\neg Q(x0)$ — *Assumption*
7. $P(x0) \wedge \neg Q(x0)$ — $5, 6$
8. $\perp$ — $4, 7 \quad \neg e$
9. $Q(x0)$ — $6, 8 \quad \neg i$
10. $P(x0) \to Q(x0)$ — $5, 9 \quad \to i$
11. $\perp$ — $3, 10 \quad \neg e$
12. $P(x0) \wedge \neg Q(x0)$ — $4, 11 \quad \neg i$
13. $\exists x (P(x) \wedge \neg Q(x))$ — $2, 11 \quad \exists xi$

b.

$$\forall x \forall y(P(x,y) \to \neg P(y,x)), \forall x \exists y P(x,y) \vdash \neg \exists v \forall z P(z,v)$$

1. $\forall x \forall y(P(x,y) \to \neg P(y,x))$	premise		
2. $\forall x \exists y P(x,y)$	premise		
3. z			
4. v			
5. x0/x			
6. y0/y			
7. $\forall y(P(x0/x,y) \to \neg P(y,x0/x))$	$1,5$	$\forall xe$	
8. $(P(x0/x,y0/y) \to \neg P(y0/y,x0/x))$	$6,7$	$\forall xe$	
9. $\exists y P(x0/x,y)$	$2,5$	$\exists xe$	
10. $P(x0/x,y0/y)$	$6,9$	$\exists xe$	$(assumption)$
11. $\neg P(y0/y,x0/x)$	$8,10$	$\to e$	
12. $\exists z \neg P(z,x0/x)$	$3,11$	$\exists xi$	
13. $\forall z P(z,x0/x)$	assumption		
14. z0/z			
15. $\neg P(z0/z,x0/x)$	$12,14$	$\exists xe$	
16. $P(z0/z,x0/x)$	$12,13$	$\forall xe$	
17. $\perp$	$15,16$	$\neg e$	
18. $\neg \forall z P(z,x0/x)$	$13,17$	$\neg i$	
19. $\forall v \neg \forall z P(z,v)$	$4,18$	$\forall xi$	
20. $\exists v \forall z P(z,v)$	$Assumption$		
21. v0/v			
22. $\forall z P(z,v0/v)$	$20,21$	$\exists xe$	
23. $\neg \forall z P(z,v0/v)$	$19,21$	$\forall xe$	
24. $\perp$	$22,23$	$\neg e$	
25. $\neg \exists v \forall z P(z,v)$	$20,24$	$\neg i$	