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QUESTION 1:

Let K be the subset of the natural numbers(N). There is a least x element (smallest element) in the natural numbers set(N), by the Well-Ordering principle given in the question. Assume that there exists an $x \in N$ and $x<1$. Then, multiply both sides with x, to get $x^2<x$. We can do this multiplication because N is closed under multiplication and we get $x^2 \in N$. Then, this contradicts the assumption we did because the Well-Ordering principle said that x is the smallest element in natural numbers. But we saw that x^2 is the smallest element. Therefore, from the contradiction, we saw that x cannot be smaller than 1 and 1 is the smallest integer.

QUESTION 2

Part 1:

Basis:

1-) S(m,1) = Y1+Y2+Y3+…+Ym = 1

If Y1 = 1 and Y2, Y3, Y4 …, Ym are equal to 0

If Y2 = 1 and Y1, Y3, Y4 …, Ym are equal to 0

If Y3 = 1 and Y1, Y2, Y4 …, Ym are equal to 0

…

If Ym = 1 and Y1,Y2, Y3, Y4 …, Ym-1 are equal to 0

There are m different cases in S(m,1).

S(m,1) = m and it confirms the equation (n+m-1)! / (n! * (m-1)!)

2-) S (1, n) = Y1=n

That means Y1 can be only n and therefore it has only one case.

And it confirms the equation (n+m-1)! / (n! * (m-1)!)

Induction:

Assume that S (m, n+1) and S (m+1, n) is true.

Then, we can divide S (m+1, n+1) to two parts.

S (m+1, n+1) = X1+ X2+ X3+ … + Xm + Xm+1= n + 1

X1+ X2+ X3+ … + Xm= n + 1 => is the part where Xm+1 is equal to 0 and this part is equal to

S (m, n+1).

Then, we need to write the second part as Xm+1 > 0.

So, we can write it as:

X1+ X2+ X3+ … + Xm + Xm+1 = n because Xm+1 is {1, 2, 3, 4 ….., n,n+1) and we need to reduce it by one and that is why we got n instead of n+1 at the result part.

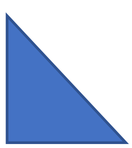
X1+ X2+ X3+ … + Xm + Xm+1 = n => is the S (m+1, n).

Therefore, S (m+1, n+1) = S (m+1, n) + S (m, n+1).

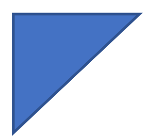
Additionally, (n+m+1)! / ((n+1)! * m!) = (n+m)! / (n! * m!) + (n+m)! / ((n+1)! * (m-1)!)

QUESTION 3

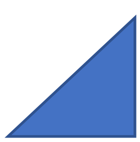
a) There are 4 different triangles in this question. And expect one of them we can place them in 21 different places. But one of them can be placed to 28 different places. Therefore we can place this triangle 91 different places.



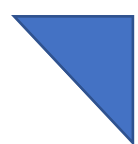
7*8/2 = 28



6*7/2 = 21



6*7/2 = 21



6*7/2 = 21

28+21+21+21 = 91

b)

If a set A has n elements and a set B has m elements, then the number of onto functions from A to B equals: m!S(n, m).

Here S(n, m) is Stirling number of the second kind, and m! means m factorial.

In your problem, n = 6 and m = 4.

Thus, the number of onto functions equals 4! S (6, 4). 4! = 24 and S(4, 3) = 65. Thus, there are 1560 onto functions.

QUESTION 4

a)

N is the length of the string.

We need to divide a_n to two parts. The first part is when we remove the last element, it will still include the consecutive elements, so that it will be equal to a_{n-1} and the last element will be independent from the first n-1 elements, so the last element can be one of the 3 digits {0,1,2}.

_ _ _ _ _ _ _ _ _ _ _ _ .... _ _ _ | _ So, this part has a_{n-1} * 3 different cases.

a_{n-1}

In this second part, the first n-1 element has no consecutive elements, so its count is $3^{n-1} – a_{n-1}$ where 3^{n-1} is the every case and where a_{n-1} is the consecutive cases. So we can say that $3^{n-1} – a_{n-1}$ is the non consecutive cases.

_ _ _ _ _ _ _ _ _ _ _ _ .... _ _ _ | _ This part has $3^{n-1} – a_{n-1}$

$3^{n-1} – a_{n-1}$

Therefore, $a_n = 3*a_{n-1} + 3^{n-1} – a_{n-1} = 3^{n-1} + 2*a_{n-1}$

b)

$n \geq 2$

$a_2 = 3 = (0,0)$, $(1,1)$, $(2,2)$

$a_3 = 15 = (0,0,0), (1,0,0), (2,0,0), (0,0,1), (0,0,2), (1,1,1), (0,1,1), (2,1,1), (1,1,0), (1,1,2), (2,2,2), (0,2,2), (1,2,2), (2,2,0), (2,2,1)$

c) $a_{n(general)} = a_{n(homogeneous)} + a_{n(particular)}$

we need to solve $a_{n(homogeneous)}$ first:

characteristic roots of the rec. rel. İs $3*r^2 – 6*r = 0$

$r^2 – 2r = 0$

$r * (r-2) = 0$

$r1 = 0, r2 = 2$

Therefore, $a_{n(homogeneous)}$ is equal to $= B * 0^n + A * 2^n$

$= A * 2^n$

Secondly, we need to find the particular part:
3^n is not the characteristic root of the $a_{n(homogeneous)}$ part, so we can directly sat that $a_{n(particular)} = C * 3^n$

Then,

$3* C * 3^n = 3^n + 6 *C * 3^{n-1}$

$3* C * 3^n = 3^n + 2 *C * 3^n$

$3* C * 3^n = 3^n (2*C + 1)$

$3*C = (2*C +1)$

$C=1$

Therefore, $a_{n(particular)} = C * 3^n = 3^n$

$a_n = A*2^n + 3^n$

For n = 1 a1 = A*2 + 3 = 0

For n=2 a2 = A*4 + 9 = 3

A = -3/2

Therefore,

$A_n = -3 * 2^{n-1} + 3^n$