Student Information

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Answer 1

a)If the outcome of one event does not affect the outcome of the other event, the events are independent.

Joint probability density function is:

$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \ f(x,y) \ dxdy = \mathbf{1}$

In continuous probability, if f(x,y) = f(x)f(y), then it means they are independent. From part b, we get $f_X(x) = \frac{2}{\pi}\sqrt{1-x^2} \ and \ f_Y(y) = \frac{2}{\pi}\sqrt{1-y^2}$

$f(x)f(y) = \frac{4}{\pi^2}\sqrt{1-x^2}\sqrt{1-y^2}$

And f(x,y) is given $\frac{1}{\pi}$

We can clearly see that f(x,y) $\neq$f(x)f(y)

Therefore, they are not independent.

b) The marginal PDF of X can be found as follows:

$f_X(x) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)\,dy = \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \frac{1}{\pi}\,dy = \frac{2}{\pi}\sqrt{1-x^2}, -1 \leq x \leq 1$

The marginal PDF of Y can be found as follows:

$f_Y(y) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)\,dx = \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} \frac{1}{\pi}\,dx = \frac{2}{\pi}\sqrt{1-y^2}, -1 \leq y \leq 1$

c) Expected value of x is equal to:

E(X) $= \int_{-1}^{1} x f_X(x)\,dx$

When we write f(x) to its place we get:

E(X) $= \int_{-1}^{1} x \frac{2}{\pi}\sqrt{1-x^2}\, dx$

The integral's result is an odd function. When we try to give symmetric values to it, it gives 0.

Therefore E(X) = 0.

d) Var(X) $= \int_{-1}^{1} x^2 f_X(x)\, dx$ **-** $E(X)^2$

$= \int_{-1}^{1} x^2 \frac{2}{\pi}\sqrt{1-x^2}\, dx$

=1/4 - 0 = 1/4

Answer 2

a)
Firstly, we write the joint density function as,

$$\mathbf{f(Ta,Tb)} = \begin{cases} \mathbf{C^*(1/100\ ^*1/100),} & \mathbf{if}\ 0 \le Ta \le 100\ and\ 0 \le Tb \le 100 \\ \mathbf{0,} & \mathbf{otherwise} \end{cases}$$

We need to find C from F(Ta,Tb) known as joint cumulative density function.

$\mathbf{f}_{(Ta,Tb)}\mathbf{(Ta,Tb)} = \frac{\partial^2}{\partial Ta \partial Tb}\mathbf{F}_{(Ta,Tb)}$ **(Ta,Tb)**
f(Ta,Tb) $= \frac{1}{100} * \frac{1}{100}$

f(Ta) $= \int_0^{100} 1/10000\, dTb =$ **1/100**

f(Tb) $= \int_0^{100} 1/10000\, dTb =$ **1/100**

And F(∞) must be equal to 1.

F(Ta,Tb) $= \iint_V f(Ta,Tb)\, dTa\, dTb = \int_0^{100}\int_0^{100} C\ \ 1/100 * 1/100\ \ dTa\ \ dTb =$ **1**

= C = 1

F(Ta,Tb)= $\int_0^{Tb}\int_0^{Ta}\ 1/100 * 1/100\ \ dTa\ \ dTb$

b) We need to find the probability of pushing in first 10 seconds for a and the last 10 seconds. So, we can use $f_T a(10) * (1 - f_T b(10))$ **because Ta and Tb are independent.**

$f_T a(10) = F_T a(X < 10) =$ **1/10**

$(1 - f_T b(90)) = F_T b(X > 90) =$ **1 - 9/10 = 1/10**

$f_T a(10) * (1 - f_T b(90))$ =$F_T a(X < 10)$*$F_T b(X > 90)$ =**1 / 100**

c) We get that, A pushes the button at most 20 seconds after subject B. That means A can push the button before B and most 20 seconds after B. Then, For example when B pushes the button at 10th second. A can push the button in an interval which is [0,30]. Therefore we can write an equation with two parts. First part will include the first 80 seconds because A will get 20 seconds more than B's push time.

1/100*20.5/100+1/100*21.5/100+ ... +1/100*99.5/100 =48/100

Or we can draw the graph as 2D to find that (Ta+20)/10000 = Tb and from there we will find that first part is equal to

$\int_0^{80} (Ta + 20)/10000 \; dTb =$ **48/100**

Second part is a bit tricky because when B pushes the button after 80th second, A can take all the values, Therefore, we 100/100 probability for A pushing the button.

1/100*100/100+1/100*100/100+ ... +1/100*100/100 =20/100

Or we can draw the graph as 2D to find that Ta = 1/100 and from there we will find that second part is equal to

$\int_{80}^{100} 1/100 =$ **20/100**

When we add these two parts, we get 68/100. And that's the probability that subject A pushes the button at most 20 seconds after subject B.

d)

We need to find their elapsed time not differ by more than 30 seconds. To find this probability we need to draw the 2D graph A to B. From there first part where A is between [0,30] comes as x+30 = y/10000

That means $\int_0^{30} x + 30/10000$ =**13.5/100**

Second part where A is between [30,70] comes as x+30-(x-30) = 60 /10000

That means $\int_{30}^{70}\ 60/10000$ **=24/100**

For the last part where A is between [70,100] comes as 100-(x-30) = 130 -x

That means $\int_{70}^{100}\ 60/10000$ **=13.5/100**

When we add up all the parts we get 51/100.

Answer 3

a) $F_T(y) = P(T \leq y)$

$= 1 - P(T \geq y)$

$= 1 - P(min\{X_1, X_2, ..., X_n\} \geq y)$

$= 1 - P(X_1 \geq y, X_2 \geq y, ...X_n \geq y)$

$= 1 - P(X_1 \geq y)P(X_2 \geq y)...P(X_N \geq y)$

$= 1 - e^{-\lambda_1 y - \lambda_2 y ... - \lambda_n y}$

$1 - e^{-\sum_{i=1}^{n} \lambda_i y} \qquad y > 0$

b) we are given that for every different computer $\lambda_i = n/10\ \ 0 \leq n \leq 10$**.**
In the exponential distribution expected value is (E(X)) equal to 1/λ**. Therefore, we need to find the** λ **for all values and add them. That is 1/10+2/10+...+10/10 = 55/10**

Then expected value (E(X))is equal to $\frac{1}{5.5}$ **= 0.1818**

Answer 4

According to Normal Approximation to Binomial Distribution using the central limit theorem for both parts :

a) $P\{X \geq 70\} = P\{X > 69.5\} = P\{\frac{X - 74}{\sqrt{74.0.26}}\} < P\{\frac{69.5 - 74}{\sqrt{74.0.26}}\} = \Phi(-1.02591)$

$\Phi(-1.02591) = 0.1515$

$1 - 0.1515 = 0.8485$

b)

$P\{X \leq 5\} = P\{X < 5.5\} = P\{\frac{X - 10}{\sqrt{10.0.9}}\} < P\{\frac{5.5 - 10}{\sqrt{10.0.9}}\} = \Phi(-1.5)$

$\Phi(-1.5) = 0.066807$