

Student Information

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Answer 1

a)

Let X be the number of at least one white drawn from every box.

$$P\{X = 1\} = P\{W,B,B\} + P\{B,W,B\} + P\{B,B,W\} =$$

$$(2/10)(11/15)(9/12) + (8/10)(4/15)(9/12) + (8/10)(11/15)(3/12) = 5/12$$

$$P\{X = 2\} = P\{W,W,B\} + P\{W,B,W\} + P\{B,W,W\} =$$

$$(2/10)(4/15)(9/12) + (2/10)(11/15)(3/12) + (8/10)(4/15)(3/12) = 13/100$$

$$P\{X = 3\} = P\{W,W,W\} =$$

$$(2/10)(4/15)(3/12) = 1/75$$

$$P = 5/12 + 13/100 + 1/75 = 14/25$$

b)

$$P\{X = 3\} = P\{W,W,W\} =$$

$$(2/10)(4/15)(3/12) = 1/75$$

c)

I would choose the second box because white density of the second box is greater than the others.

probability of picking two whites for a = $1/45$

probability of picking two whites for b = $2/35$

probability of picking two whites for a = $1/22$

d)

I would choose the second and third box because firstly, second box has the most white density, but after drawing one white from the second box, it becomes $3/14$ while third box having $3/12$ and first box having $2/10$ density. Therefore, I would draw from second box and third box respectively.

e)

expected value = $\mu = E(X) = \sum_x xP(x)$ Let X be the number of at least one white drawn from every box.

$$P\{X = 1\} = P\{W,B,B\} + P\{B,W,B\} + P\{B,B,W\} =$$

$$(2/10)(11/15)(9/12) + (8/10)(4/15)(9/12) + (8/10)(11/15)(3/12) = 5/12$$

$$P\{X = 2\} = P\{W,W,B\} + P\{W,B,W\} + P\{B,W,W\} =$$

$$(2/10)(4/15)(9/12) + (2/10)(11/15)(3/12) + (8/10)(4/15)(3/12) = 13/100$$

$$P\{X = 3\} = P\{W,W,W\} =$$

$$(2/10)(4/15)(3/12) = 1/75$$

$$\begin{aligned}\mu &= 1 * P(1) + 2 * P(2) + 3 * P(3) = 1 * (5/12) + 2 * (13/100) + 3 * (1/75) \\ &= 43/60\end{aligned}$$

f)

$P(B_1)$ = probability of choosing Box 1

$P(W)$ = probability of choosing a white ball

From Bayes Rule, we get $P(B_1|W) = \frac{P(W|B_1)P(B_1)}{P(W)}$

$P(B_1) = 1/3$ (Choosing randomly from 3 boxes)

$P(W) = P(B_1)P(W|B_1) + P(B_2)P(W|B_2) + P(B_3)P(W|B_3)$

$= (1/3)(2/10) + (1/3)(4/15) + (1/3)(3/12) = 43/180$

$P(B_1|W) = \frac{(1/3)(2/10)}{(1/3)(2/10) + (1/3)(4/15) + (1/3)(3/12)} = 12 / 43$

This is the probability of that white ball was taken from BOX 1.

Answer 2

a) $P(C)$ = probability of Sam's corruption = 0.1

$P(D)$ = probability of Ring's destruction = $0.9*0.9 + 0.1*0.5 = 0.86$

$P(D|C)$ = probability of Ring's destruction given that Sam is corrupted = 0.5

From Bayes Rule, we get $P(C|D) = \frac{P(D|C)P(C)}{P(D)}$

$= (0.5)(0.1)/(0.86) = 5/86 \simeq 0.0581$

b) $P(C)$ = probability of Sam's and Frodo's corruption

$P(D)$ = probability of Ring's destruction

From Bayes Rule, we get $P(C|D) = \frac{P(D|C)P(C)}{P(D)}$

$= \frac{(0.05)(0.25)(0.1)}{((0.05)(0.25)(0.1)) + (0.2)(0.25)(0.9) + (0.5)(0.75)(0.1) + (0.9)(0.9)(0.75)}$
 $= 0.001808$

	1	2	3	
1	0.18	0.30	0.12	0.60
2	0.12	0.20	0.08	0.40
	0.30	0.50	0.20	

a) There are only two options that there are 4 snowy days. Those are the 2 snowy days for Ankara and 2 snowy days for Istanbul with 0.2 probability. Moreover, 3 snowy days for Ankara and 1 snowy day for Istanbul with 0.12 probability. When we add them we get 0.32 probability for 4 snowy days.

b) The random variables are independent only if the multiplication of the probability of the Ankara and Istanbul gives the joint probability. In our case the multiplications of the probabilities gives us the joint probabilities as seen in above table. (For example Ankara's 1 snowy day probability is 0.3 and Istanbul's 1 snowy day probability is 0.6 and $[1,1] = 0.18$) Therefore, they are independent.