Regularized Maximum Likelihood Methods for Black Hole Imaging

Andy Nilipour

Forward Modeling Approach to Radio Interferometry

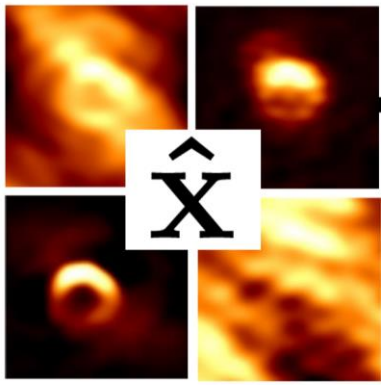
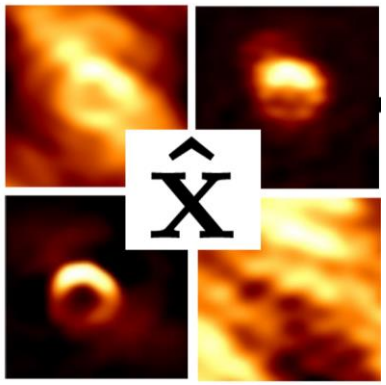


Image Model

Adapted from Chael 2024

Forward Modeling Approach to Radio Interferometry



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Forward Model

(NFFT, etc.)

$\hat{y}$

Simulated Observation Products

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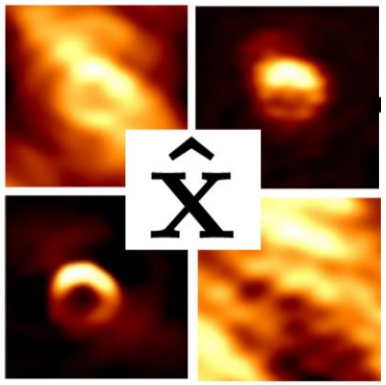

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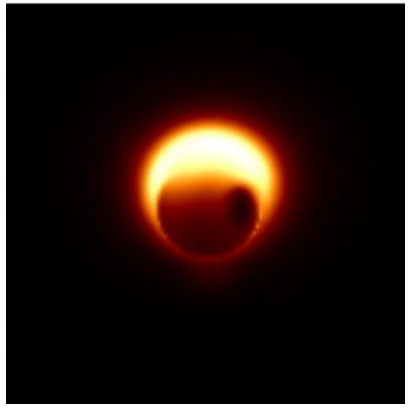
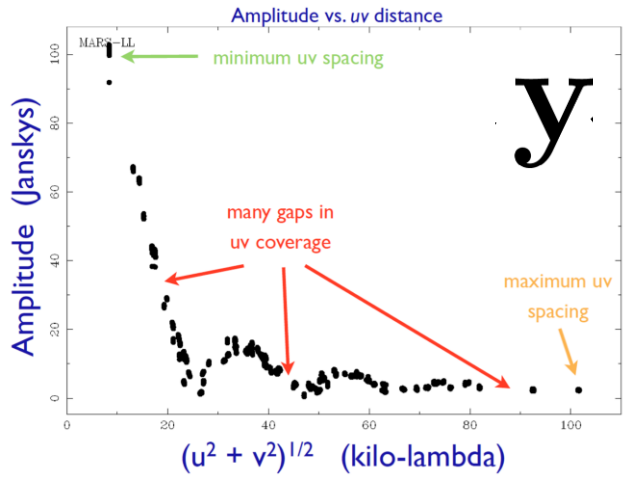


True Sky Brightness

Observation



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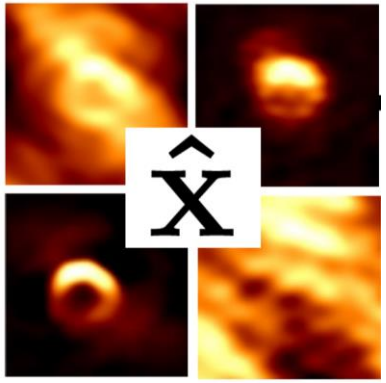

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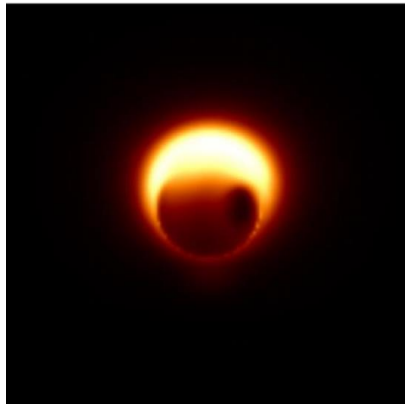
$\log P(\mathbf{y}|\hat{\mathbf{x}})$

Log Likelihood



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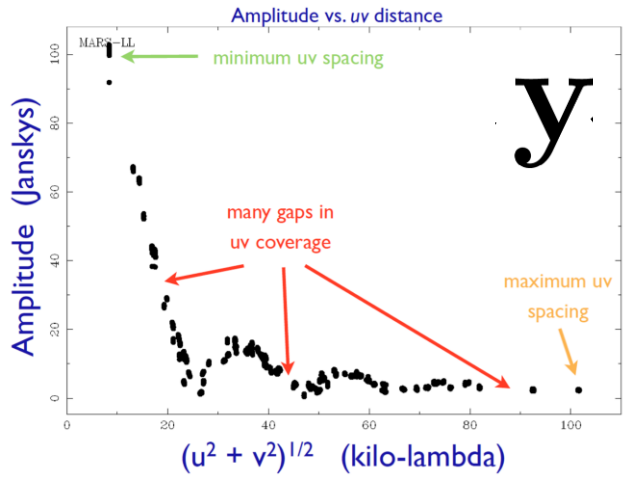

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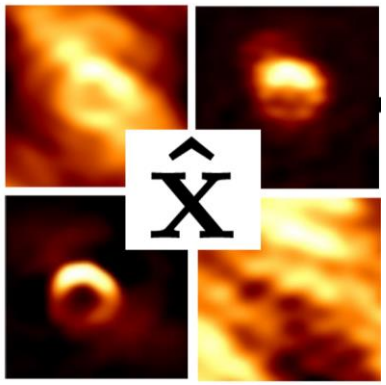

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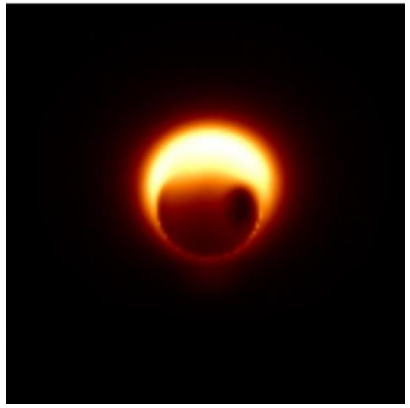

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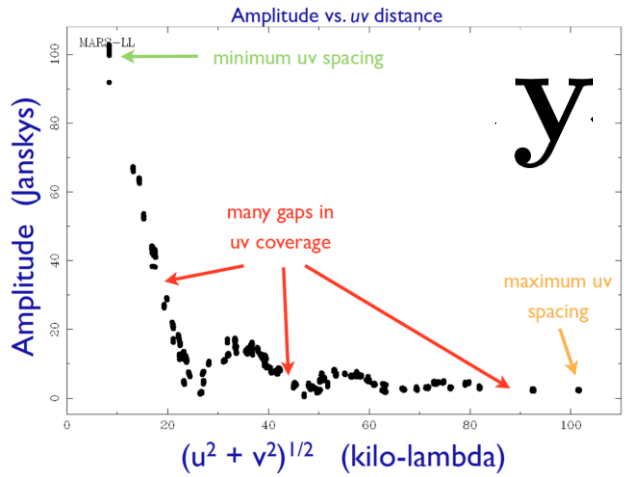

Observation Products

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Bayesian Imaging and Regularized Maximum Likelihood

Posterior **Likelihood** **Prior**

$$\log P(\hat{\mathbf{x}}|\mathbf{y}) = \log P(\mathbf{y}|\hat{\mathbf{x}}) + \log P(\hat{\mathbf{x}}) + c$$

Bayesian Imaging and Regularized Maximum Likelihood

Posterior Likelihood Prior

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Cost Data “error” Regularizers

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Cost Data “error” Regularizers

Julia Implementation of RML

VLBISkyRegularizers.jl Search Ctrl K Home Introduction Tutorials VLBISkyRegularizers API •••

VLBISkyRegularizers.jl

RML Methods for Comrade

Introduction View on Github

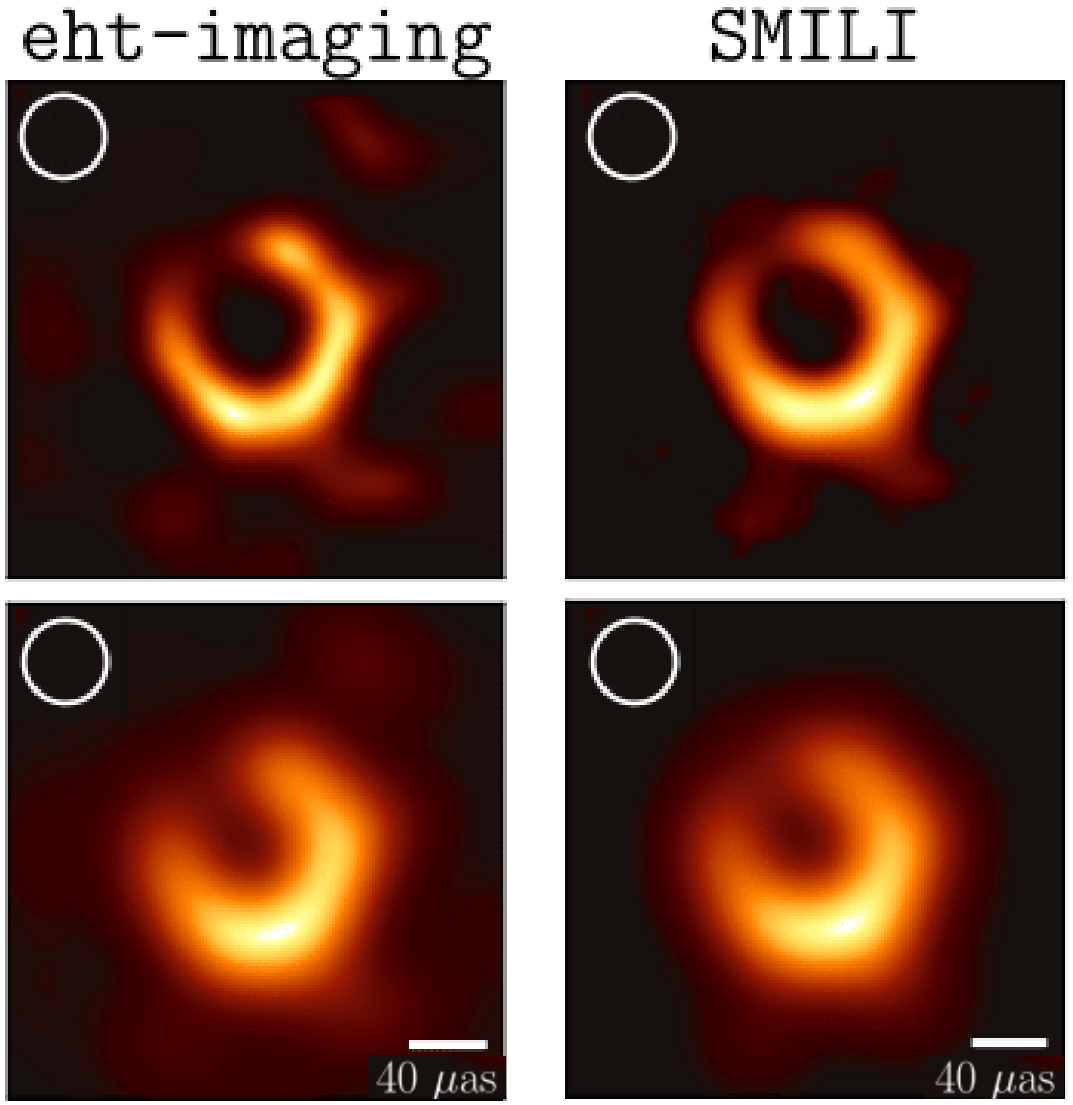
VLBISkyRegularizers.jl

- Implementation of popular spatial and polarization regularizers
- Can compose regularizers in different (linear and log) domains
- Allows for user custom regularizers

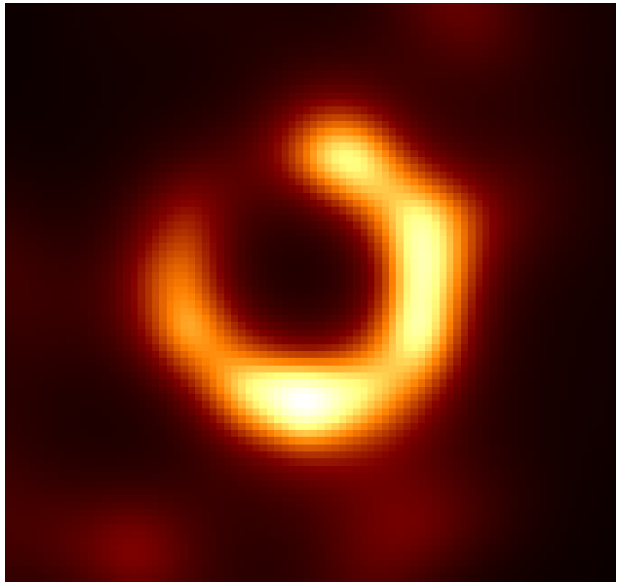
```
subtypes(AbstractRegularizer)
```

```
AddRegularizer
KLE
L1
PSDistance
TSV
TV
WaveletL1
WeightRegularizer
```

Sample Imaging on Real Data



EHT 2018 M87* Paper I (2024)



VLBISkyRegularizers.jl

Why add RML functionality to Comrade?

- RML remains a popular choice for VLBI imaging
  - Yet other packages, such as eht-imaging (Python) and SMILI (Fortran), are not nearly as flexible nor as fast
- Modular regularizer structure composes well with the advantages of Comrade.jl
  - Hybrid modeling: geometric model, non-parametric image, large-scale Gaussian for intra-site baselines can be fit simultaneously
- Can work well with simultaneous developments for other Comrade extensions (time domain, multi-frequency)

Benefits of Julia

- Multiple dispatch and type hierarchy
  - Enables easy integration with the many libraries of the Comrade.jl ecosystem
  - Simplifies implementation of user-customizable regularizers

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```
subtypes(AbstractDomain)
```

```
ALRDomain
CLRDomain
LinearDomain
LogDomain
ParameterDomain
PolarizationDomain
VLBISkyRegularizers.PSDDomain
```

Benefits of Julia

- Autodifferentiation through Enzyme.jl
  - Speed-up compared to other RML packages
  - Eliminates need for handwritten gradients, enabling custom regularizers
  - Previous implementation with Zygote.jl sometimes required custom rules

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```python
def tv_alpha_grad(imvec, nx, ny, psize, norm_reg=NORM_REGULARIZER, beam_size=None):
    """Total variation gradient
    """
    if beam_size is None: beam_size = psize
    if norm_reg:
        norm = len(imvec)*psize / beam_size
    else:
        norm = 1

    im = imvec.reshape(ny,nx)
    impad = np.pad(im, 1, mode='constant', constant_values=0)
    im_l1 = np.roll(impad, -1, axis=0)[1:ny+1, 1:nx+1]
    im_l2 = np.roll(impad, -1, axis=1)[1:ny+1, 1:nx+1]
    im_r1 = np.roll(impad, 1, axis=0)[1:ny+1, 1:nx+1]
    im_r2 = np.roll(impad, 1, axis=1)[1:ny+1, 1:nx+1]

    #rotate images
    im_r1l2 = np.roll(np.roll(impad,  1, axis=0),-1, axis=1)[1:ny+1, 1:nx+1]
    im_l1r2 = np.roll(np.roll(impad, -1, axis=0), 1, axis=1)[1:ny+1, 1:nx+1]

    #add together terms and return
    g1 = (2*im - im_l1 - im_l2) / np.sqrt((im - im_l1)**2 + (im - im_l2)**2 + EPSILON)
    g2 = (im - im_r1) / np.sqrt((im - im_r1)**2 + (im_r1l2 - im_r1)**2 + EPSILON)
    g3 = (im - im_r2) / np.sqrt((im - im_r2)**2 + (im_l1r2 - im_r2)**2 + EPSILON)

    #mask the first row column gradient terms that don't exist
    mask1 = np.zeros(im.shape)
    mask2 = np.zeros(im.shape)
    mask1[0,:] = 1
    mask2[:,0] = 1
    g2[mask1.astype(bool)] = 0
    g3[mask2.astype(bool)] = 0

    # add terms together and return
    out= -(g1 + g2 + g3).flatten()
    return out/norm
```

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```
26      function solve_opt(post::VLBIPosterior, opttype=Optimisers.Adam(), adtype=Optimization.AutoEnzyme();
27          ntrials=5, maxiters=10_000, init_params=nothing, verbose=false, stride=1000)
28          tpost = asflat(post)
29          mapout = map(1:ntrials) do i
30              verbose && @info("$i/$(ntrials): Start Imaging")
31              g = OptimizationFunction(tpost, adtype)
```

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    im_l2 = np.roll(impad, -1, axis=1)[1:ny+1, 1:nx+1]
    out = -np.sum(np.sqrt(np.abs(im_l1 - im)**2 + np.abs(im_l2 - im)**2 + EPSILON))

    return out/norm
```

```julia
tv_base(x::AbstractArray) = @tullio tvF = sqrt(abs2(x[i,j] - x[i+1,j]) + abs2(x[i,j] - x[i,j+1]))
```

Benefits of Julia

- Convenience
  - Vectorization and broadcasting (Tullio.jl)
  - Macros (benchmarking, profiling, nice integration with VS code)
  - Parallelization and multi-threading

Summary

- Implementation of RML methods in Julia built on the Bayesian imaging package Comrade.jl, with all its advantages
- Faster and more flexible than other RML packages used by the Event Horizon Telescope community, enabled by Julia
- Julia offers many small and large benefits to package development