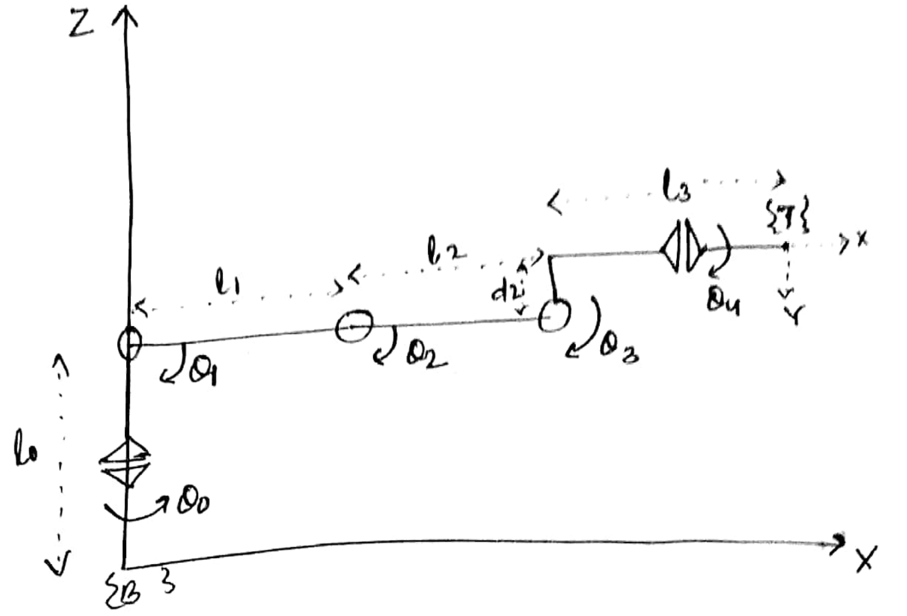
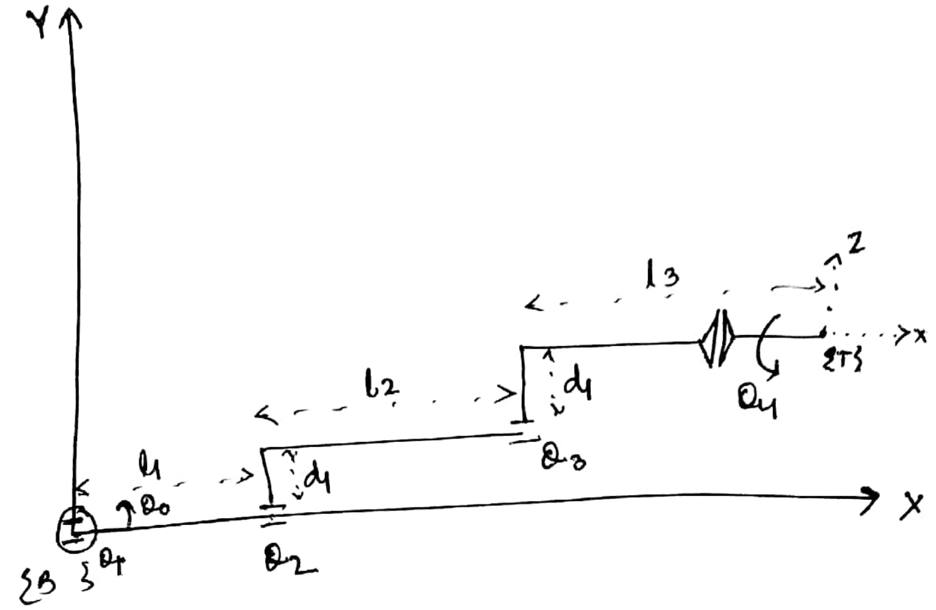
Q1)

①

Side view



Top view



we have to find ${}^{B}_{T}T$ for forward kinematics

i) Forward Kinematics

$$ {}^{B}_{T}T = {}^{B}_{0}T \; {}^{0}_{1}T \; {}^{1}_{2}T \; {}^{2}_{3}T \; {}^{3}_{4}T $$

Notations

$D_{Axis}(\text{Length})$: Translate along 'Axis' for 'Length'

$R_{axis}(\theta)$: rotation along 'axis' for 'θ'

$$ {}^{B}_{0}T = D_Z(l_0) \; R_Z(\theta_0) $$

$$ {}^{0}_{1}T = R_Y(\theta_1) $$

$$ {}^{1}_{2}T = D_X(l_1) \; R_Y(\theta_2) $$

$$ {}^{2}_{3}T = D_Y(d_1) \; D_X(l_2) \; R_Y(\theta_3) $$

$$ {}^{3}_{4}T = D_Y(d_1) \; D_Z(d_2) \; D_X(l_3) \; R_X(\theta_4) $$

Then define all the Translation and Rotation matrixes

$$ D_Z(l_0) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & l_0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad R_Z(\theta_0) = \begin{bmatrix} \cos\theta_0 & -\sin\theta_0 & 0 & 0 \\ \sin\theta_0 & \cos\theta_0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ R_Y(\theta_1) = \begin{bmatrix} \cos\theta_1 & 0 & \sin\theta_1 & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta_1 & 0 & \cos\theta_1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$D_x(l_1) = \begin{bmatrix} 1 & 0 & 0 & l_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad R_y(\theta_2) = \begin{bmatrix} \cos\theta_2 & 0 & \sin\theta_2 & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta_2 & 0 & \cos\theta_2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

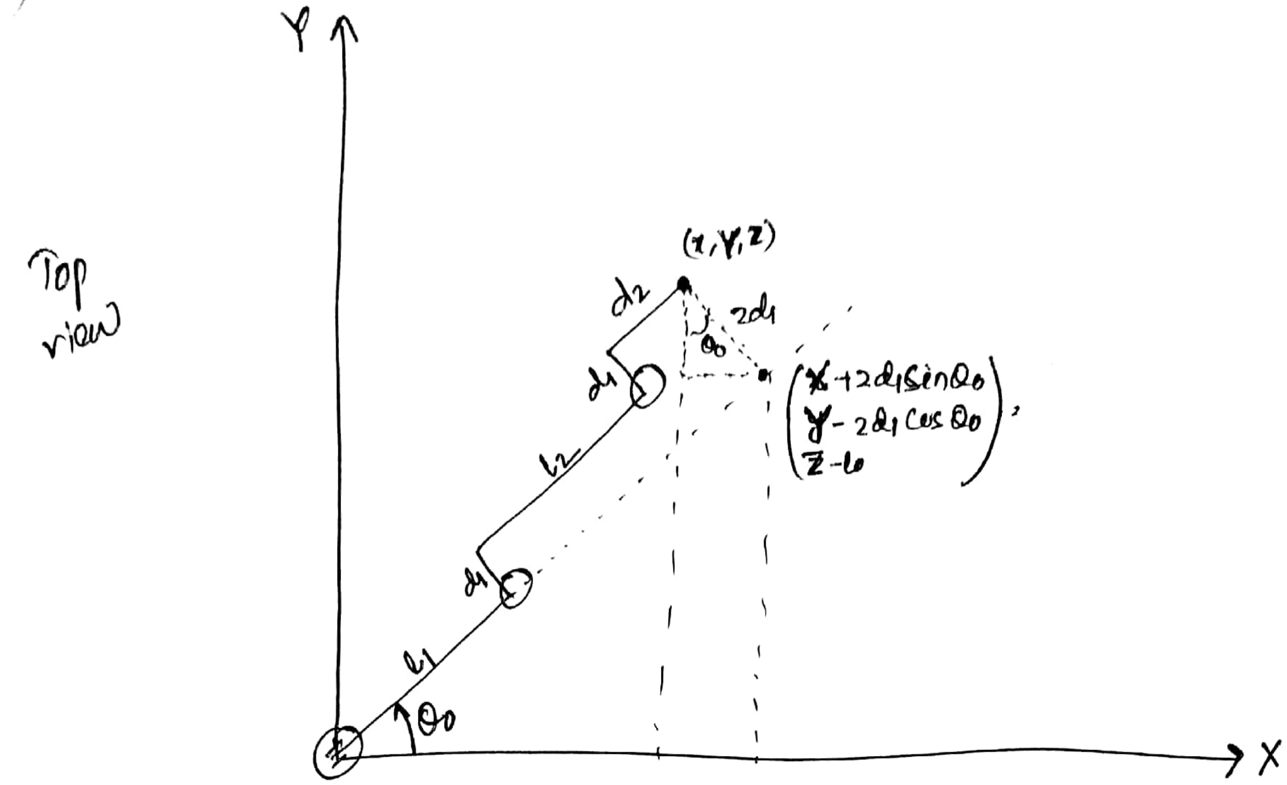
$$D_y(d_1) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad D_x(l_2) = \begin{bmatrix} 1 & 0 & 0 & l_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_y(\theta_3) = \begin{bmatrix} \cos\theta_3 & 0 & \sin\theta_3 & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta_3 & 0 & \cos\theta_3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$D_y(d_1) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad D_z(d_2) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$D_x(l_3) = \begin{bmatrix} 1 & 0 & 0 & l_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad R_x(\theta_4) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta_4 & -\sin\theta_4 & 0 \\ 0 & \sin\theta_4 & \cos\theta_4 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Q2) Inverse Kinematics



Calculation of θ_0 :

$$\tan\theta_0 = \left(\frac{Y - 2d_1\cos\theta_0}{X + 2d_1\sin\theta_0}\right)$$

$$\Rightarrow \frac{\sin\theta_0}{\cos\theta_0} = \frac{Y - 2d_1\cos\theta_0}{X + 2d_1\sin\theta_0}$$

$$\Rightarrow X\sin\theta_0 + 2d_1\sin^2\theta_0 = Y\cos\theta_0 - 2d_1\cos^2\theta_0$$

$$\Rightarrow Y\cos\theta_0 - X\sin\theta_0 = 2d_1\underbrace{(\sin^2\theta_0 + \cos^2\theta_0)}_{1}$$

$$\Rightarrow Y\cos\theta_0 - X\sin\theta_0 = 2d_1$$ (to solve this equation)

$$a = \tan^{-1}\left(\frac{X}{Y}\right)$$

$$R = \sqrt{X^2 + Y^2}$$

$$\theta_0 = \cos^{-1}\left(2d_1/R\right) - a \qquad \text{--- (i)}$$

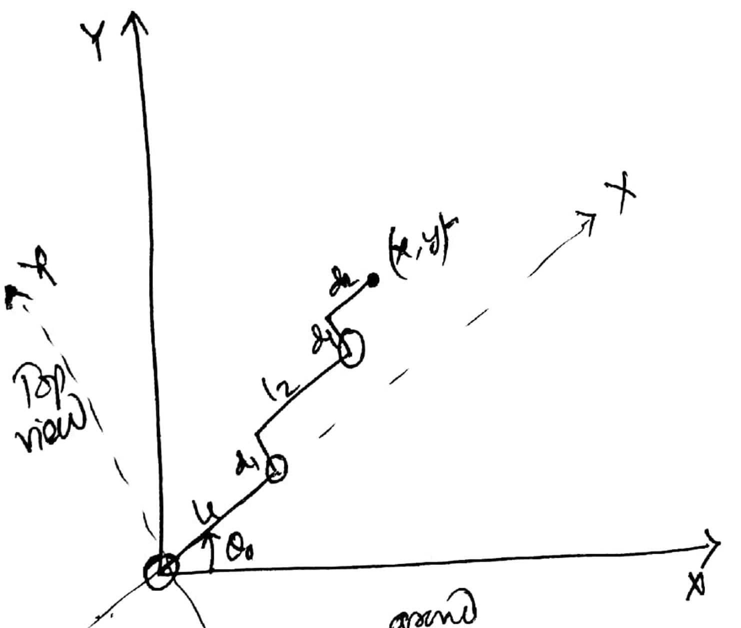
if $X = -ve$ & $Y = -ve$ or $Y = -ve$

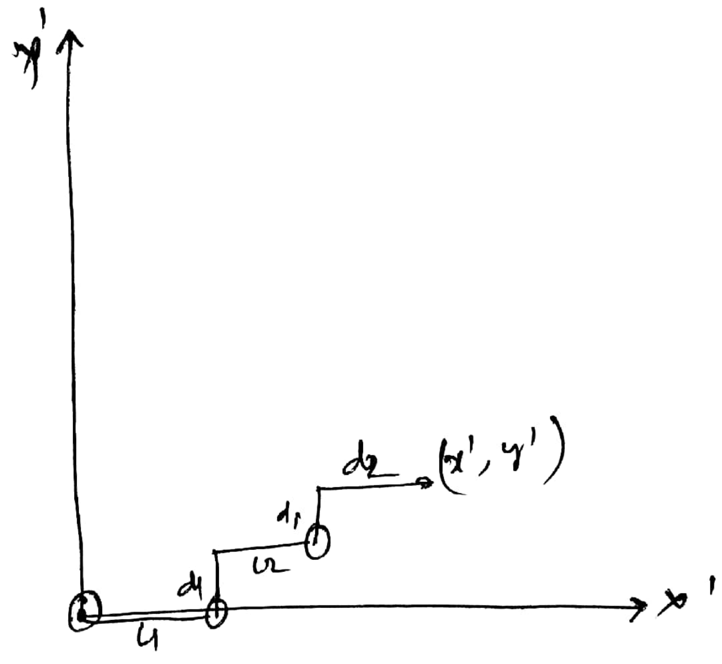
$$\theta_0 = \theta_0 - \pi \qquad \text{--- } (\theta_0 \text{ value})$$

To calculate θ_1 and θ_2 we have to rotate the base frame and align with the X axis of arm by θ_0 along Z axis to x', z', y' and ~~X~~ & Z coordinate will change

Before rotation θ_0 (x, y)

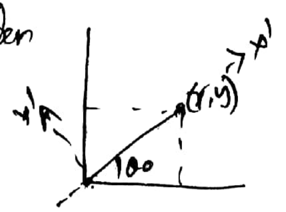
after rotation (x', y')





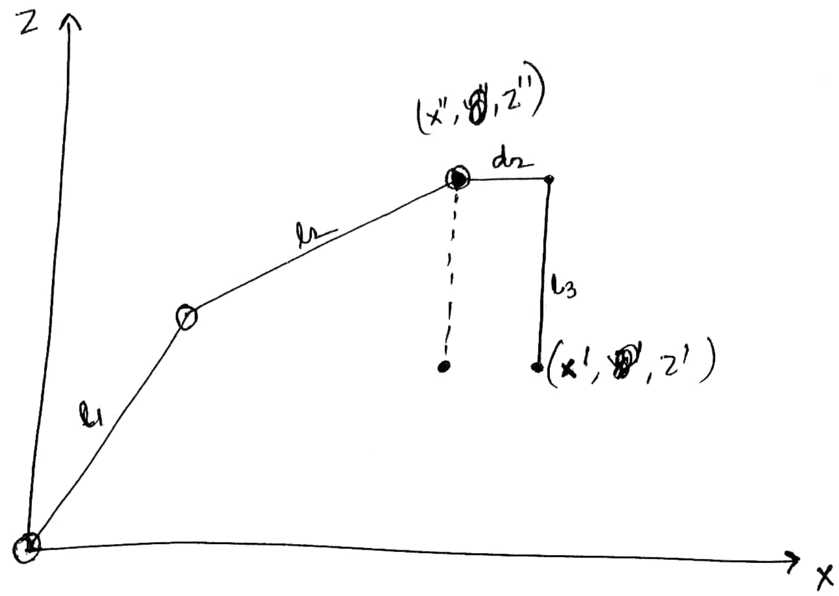
θ_0 rotated around Z axis

apply the projection rule



$x' = x\cos\theta_0 + y\cos$

side view

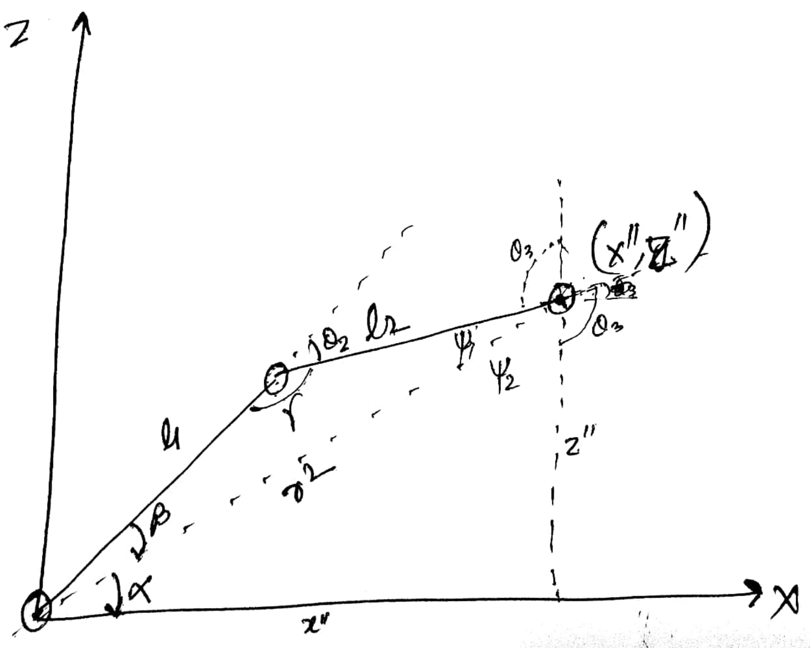


after simplifying l_0 and rotation around z axis by θ_0

$$x' = x\cos\theta + y\sin\theta$$
$$z' = z - l_0$$

then simplifying d_2 and l_3

$$x'' = x\cos\theta + y\sin\theta - d_2$$
$$z'' = z - l_0 + l_3$$



$\theta_1 = \alpha + \beta$

θ

$$r = \sqrt{(x'')^2 + (z'')^2}$$

$$= \sqrt{(x\cos\theta + y\sin\theta - d_2)^2 + (z - l_0 + l_3)^2}$$

Calculate α & β

$$\alpha = -\tan^{-1}\left(\frac{z''}{x'}\right)$$

$$\beta = -\cos^{-1}\left(\frac{l_1^2 + r^2 - l^2}{2 l_1 r}\right)$$

Calculate θ_1

~~theta~~ $\theta_1 = \alpha + \beta \longrightarrow$ (θ_1 value)

Calculate γ

$$\gamma = \cos^{-1}\left(\frac{l_1^2 + l_2^2 - r^2}{2 \cdot l_1 \cdot l_2}\right)$$

Calculate θ_2

$\theta_2 = \pi - \gamma \longrightarrow$ (θ_2 value)

Calculate θ_3

$$\psi_2 = \pi/2 - \alpha$$

$$\psi_1 = \cos^{-1}\left(\frac{l_2^2 + r^2 - l_1^2}{2 \cdot l_2 \cdot r}\right)$$

$$\theta_3 = \psi_2 - \psi_1 \longrightarrow (\theta_3 \text{ value})$$

θ_5 & θ_4 is given $0°$. $\longrightarrow$ (θ_4 & θ_5 value)

Q3) System Identification

$$\tau = I\ddot{\theta} + B\dot{\theta} + G(\theta)$$

I : inertia

B : viscous friction

G : gravity

Find G :

i) keeping $\ddot{\theta} = \dot{\theta} = 0$

$\tau_G = G(\theta)$ [τ_G found : 0.882] — (i)

Force provided to balance gravity.

Found to be

Find B :

to find B we have to move ~~the~~ state with constant angular velocity $\dot{\theta}$, so $\ddot{\theta} = 0$

$$\tau_B + \tau_G = \cancel{I\ddot{\theta}} + B\dot{\theta} + G(\theta)$$ [τ_B applied 12000.0]

$$B\dot{\theta} = \tau_G$$

$$B = \tau_G / \dot{\theta}$$ [B found = 0.079994.] — (ii)

Find I :

$$\tau_I = I\ddot{\theta}$$ [τ_I applied = 1.0]

$$I = \tau_I / \ddot{\theta}$$ [I found = 0.052051]