

Stat Assignment - I

Problem-1

Data - 6, 7, 5, 7, 7, 8, 7, 6, 9, 7, 4, 10, 6, 8, 8, 9, 5, 6, 7, 8

Ascending \rightarrow 4, 4, 5, 5, 6, 6, 6, 7, 7, 7, 7, 7, 8, 8, 8, 8, 9, 9, 10

$$n = 20$$

$$\begin{aligned}\text{mean} &= \frac{2+4+2 \times 5+4 \times 6+5 \times 7+4 \times 8+2 \times 9+10}{20} \\ &= (8+10+24+35+32+18+10) / 20 \\ &= 137 / 20 = 6.85\end{aligned}$$

$$\text{median} = \frac{20}{2} = 10 \Rightarrow \text{median is average of } 10^{\text{th}} \text{ and } 11^{\text{th}}$$

$$\begin{array}{l}10^{\text{th}} \text{ number} = 7 \\ 11^{\text{th}} \text{ number} = 7\end{array} \rightarrow \boxed{\text{Median} = \frac{7+7}{2} = 7}$$

$$\text{Standard Deviation} = 1.63$$

mode \rightarrow highest frequency (7 has highest frequency)

$$\text{so mode} = 7$$

Problem-2 data \Rightarrow 28, 40, 68, 70, 75, 75, 75, 75, 80, 86, 89, 90, 90, 90, 97, 97, 100, 100, 100, 104, 104, 104, 109, 113, 120, 120, 120, 123, 123, 130, 140, 145, 170, 174, 194, 217

$$n (\text{Count}) = 35$$

$$\text{mean} = \frac{\text{Sum of all}}{35} = \frac{3763}{35} = 107.5143$$

$$\text{median} = \frac{35^{\text{th}}}{2} = 18^{\text{th}} \text{ term} = 100$$

$$\text{Mode} = 75 \text{ (highest frequency 4)}$$

$$\text{Standard Deviation} = \sqrt{\frac{\sum_{i=1}^{35} (x_i - \bar{x})^2}{(n-1)}} = \boxed{39.338}$$

Problem - 3 The number of times I go to the Gym in week is -

$$x = 0, 1, 2, 3, 4, 5$$

$$f(x) = 0.09, 0.15, 0.40, 0.25, 0.10, 0.01$$

$$\begin{aligned}\text{mean} &= 0 \times 0.09 + 1 \times 0.15 + 2 \times 0.40 + 3 \times 0.25 + 4 \times 0.10 + 5 \times 0.01 \\ &= 0 + 0.15 + 0.8 + 0.75 + 0.4 + 0.05 \\ &= 2.15\end{aligned}$$

Variance

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1} \quad n = 6$$

$$\begin{aligned}\sigma^2 &= \frac{(0-2.15)^2 + (1-2.15)^2 + (2-2.15)^2 + (3-2.15)^2 + (4-2.15)^2 + (5-2.15)^2}{6-1} \\ &= 3.65\end{aligned}$$

Problem - 4

D - diameter of hole drilled

~~Target Diameter = 12.5 mm~~

$$\begin{aligned}P(D=d) &= 20e^{-20(d-12.5)} \\ d &> 12.5\end{aligned}$$

Problem - 4

Target Diameter = 12.5 mm

$$P(D=d) = 20e^{(-20)(d-12.5)}, d > 12.5$$

Proportion of parts > 12.6 = ?

$$P(x > 12.6) = 1 - P(x \leq 12.6)$$

$$P(d > 12.6) = 1 - P(12.5 < d < 12.6)$$

$$P(12.5 < d < 12.6) = \int_{12.5}^{12.6} 20e^{-20(d-12.5)} (d-12.5) \, dx$$

$$\Rightarrow \left[-e^{-20(d-12.5)} \right]_{12.5}^{12.6}$$

$$\Rightarrow 0.865$$

$$P(d > 12.6) = (1 - 0.865) = 0.135$$

$$CDF(d=11) = 0$$

Conclusion

13.5% of parts will be scrapped

Q.5

$$\text{fault rate (p)} = 30\% = 0.3$$

$$\text{LED chosen (n)} = 6$$

$$x = 2$$

This is a
Binomial Distribution

Probability of having 2 faulty LED = $\frac{6!}{2!} (0.3)^2 (0.7)^4$

$$\Rightarrow \frac{6!}{4! 2!} \propto 0.09 \times 0.2401$$

$$\Rightarrow \frac{6 \times 5 \times 4!}{4! \times 2} \propto 0.021609$$

$$= 0.3241 \text{ or } 32.41\%$$

$$\boxed{\text{Mean} = np = 6 \times 0.2 = 1.2}$$

$$\boxed{\text{Std Deviation} (\sigma) = \sqrt{np(1-p)} = \sqrt{6 \times 0.3 \times 0.7} = 1.26}$$

problem 8

Binomial distribution

Gausdave

$$\text{Attempt (n)} = 8$$

$$\text{Success (P)} = 0.75$$

~~Target~~

$$P(x=5) = \frac{8!}{5!} (0.75)^5 (0.25)^3$$

$$= 20.76\%$$

$$P(x=4) = \frac{8!}{4!} (0.75)^4 (0.25)^4$$

Basakha

$$\text{Attempt (n)} = 12$$

$$\text{Success (P)} = 0.45$$

$$P(n=5) = \frac{12!}{5!} (0.45)^5 (0.55)^7$$

$$= \cancel{7776} \times 22.25\%$$

$$P(x=6) = \frac{12!}{6!} (0.45)^6 (0.55)^6 = 16.99\%$$

$$P(x=7) = \frac{12!}{7!} (0.45)^7 (0.55)^5 = 21.24\%$$

Gausdave

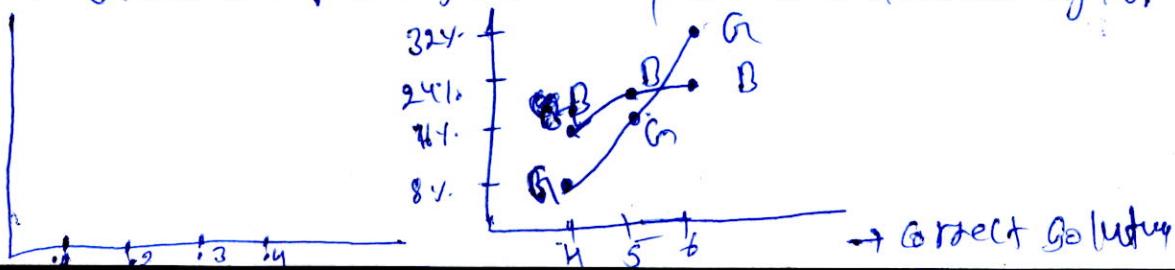
$$P(4) = 8.65\% \quad P(5) = 20.76\% \quad P(6) = 31.14\% \Rightarrow P(4 \leq x \leq 6) = 60.55\%$$

Basakha

$$P(4) = 16.99\% \quad P(5) = 22.25\% \quad P(6) = 21.24\% \Rightarrow P(4 \leq x \leq 6) = 60.48\%$$

Inference → Both have same CDF ($4 \leq x \leq 6$)

Two factors → Gausdave has high accuracy but Basakha has high speed.



→ Correct Solution

Problem - 7 Arrival per hour = 72

$$\text{Poisson} = \frac{e^{-\lambda} \cdot \lambda^k}{k!}$$

$$P(K=4)$$

~~λ = 72~~

arrival in 4 minutes -

$$\lambda_4 = \frac{72}{60} \times 4 = 4.8$$

(a) $P(K=5) = e^{-4.8} (4.8)^5 / 5! = 17.47\%$

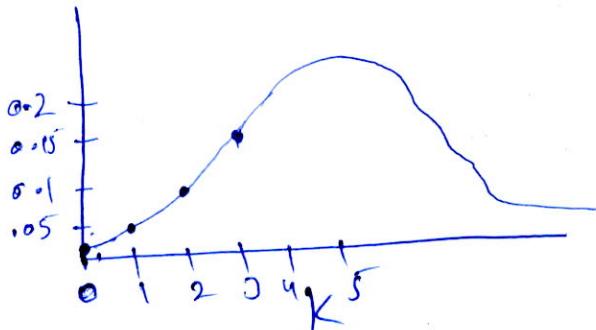
(b) $P(0) + P(1) + P(2) + P(3) = P(K \leq 3)$

$$P(K \leq 3) = \frac{e^{-4.8} (4.8)^0}{0!} + \frac{e^{-4.8} (4.8)^1}{1!} + \frac{e^{-4.8} (4.8)^2}{2!} + \frac{e^{-4.8} (4.8)^3}{3!} = 0.0082297 + 0.0395027 + 0.094806686 + 0.15169064$$

(c) $P(R \geq 3) = 1 - P(K \leq 3)$

$$0.29492$$

$$= 1 - 0.29492 = 0.7057 = 70.57\%$$



problem - 8

entry per minute = 77

error per minute = $6/60 = 1/10$

Time taken to enter 455 words = 5.9 minute ($\frac{455}{77}$)

Committing error (k) = 2

$$\lambda = \frac{1}{10} \times 5.9 = 0.59$$

$$P(K=2) = e^{-0.59} (0.59)^2 / 2! = 0.09648 = 9.6\%$$

Time for 250 words = $\frac{250}{77} = 3.25$ minute

Time for 1000 words = $\frac{1000}{77} = 12.99$ minute

Mean error in 3.25 minute = 0.325

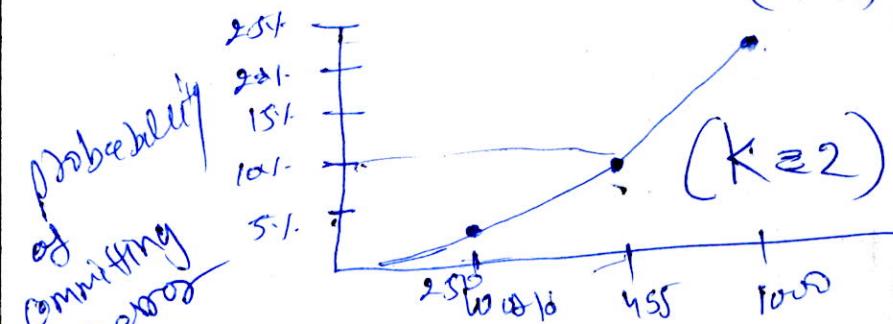
Mean error in 12.99 minute = 1.299

Probability of committing ~~one~~ 2 errors

(a) 455 word document $\lambda(k=2) = \boxed{1.64}$

(b) 250 word document $\lambda(k=2) = e^{-0.325} \frac{(0.325)^2}{2!} = 0.038 = \boxed{3.8\%}$

(c) 1000 word document $\lambda(k=2) = e^{1.299} \frac{(1.299)^2}{2!} = 0.23 = \boxed{23.01\%}$



① λ Increases as the number of words Increases

② PMF function will Increase with Number of words

problem - 9 Same as question | problem No - 4

(Repeated)

problem 10 (a) $P(Z > 1.96) = 10.38\%$ $- P(Z \leq -1.96) = 0$

$$P(Z < -0.86) = 19.49\%$$

$$P(Z > -1.37) = 91.47\%$$

$$P(1.25 < Z < 0.27) = 53.87\%$$

(b) $P(Z > z) = 0.05 \quad \boxed{z = 1.64}$

(c) $P(-2 < Z < z) = 0.99 \quad \boxed{-2.58 < z < 2.58}$

problem statement - 11

$$\mu = 10 \text{ mA}$$

$$\sigma = \sqrt{\mu (\text{mA})^2} = 2$$

$$P(9 < n < 11)$$

$$= P\left(\frac{9-10}{2} < z < \frac{11-10}{2}\right)$$

$$= P\left(-\frac{1}{2} < z < \frac{1}{2}\right)$$

$$= P(-0.5 < z < 0.5)$$

$$= \boxed{0.5987 = 0.6915}$$

$$= \boxed{100\% \times 69.15\% = 30.85\%}$$

$$= \boxed{38.3\%}$$

~~P($x > 13$)~~ Probability of exceeding
 $P(x > 13) = P(z > \frac{13-10}{2})$
 $= P(z > 1.5)$

$$= 1 - 0.9394$$

$$= \boxed{0.0606} \quad P(x > 13) = 6.06\%$$

$$\textcircled{1} \quad P(z < 0) = 0.98$$

$$P\left(z < \frac{n-10}{2}\right) = 0.98$$

$$P(z < 2.5) = 0.98$$

$$\Rightarrow 2.5 \leq \frac{n-10}{2}$$

$$\Rightarrow 5 \leq n-10$$

$$\Rightarrow n \geq 15$$

Current measurement is 15

problem statement - 12

Part 1 $\mu = 0.2508$

$$\sigma = 0.0005$$

$$\text{Lower Value} = 0.2500 - 0.0015 = 0.2485$$

$$\text{Upper Value} = 0.2500 + 0.0015 = 0.2515$$

$$P(0.2485 < n < 0.2515) = P(-3 < z <$$

$$= P\left(\frac{0.2485 - 0.2508}{0.0005} < z < \frac{0.2515 - 0.2508}{0.0005}\right)$$

$$= P(-4.6 < z < 1.4)$$

$$= 91.92\%$$

part 2 $\mu = 0.2500 \quad 0.2500$

$$\sigma = 0.0005$$

$$\text{Lower} = 0.2485$$

$$\text{Upper} = 0.2515$$

$$P(0.2485 < n < 0.2515) =$$

$$P\left(\frac{0.2485 - 0.2515}{0.0005} < z < \frac{0.2515 - 0.2508}{0.0005}\right)$$

$$= P(-3 < z < 3)$$

$$= 99.74\%$$