

Stats Assignment - I

Problem-1

Data — 6, 7, 5, 7, 7, 8, 7, 6, 9, 7, 4, 10, 6, 8, 8, 9, 5, 6, 4, 8

Ascending \rightarrow 4, 4, 5, 5, 6, 6, 6, 6, 7, 7, 7, 7, 7, 8, 8, 8, 8, 9, 9, 10

$$n = 20$$

$$\begin{aligned}\text{mean} &= \frac{2+4 + 2 \times 5 + 4 \times 6 + 5 \times 7 + 4 \times 8 + 2 \times 9 + 10}{20} \\ &= (8+10+24+35+32+18+10)/20 \\ &= 137/20 = 6.85\end{aligned}$$

$$\text{Median} = \frac{20}{2} = 10 \Rightarrow \text{Median is Average of 10th and 11th numbers}$$

$$10\text{th number} = 7$$

$$11\text{th number} = 7$$

$$\rightarrow \boxed{\text{Median} = \frac{7+7}{2} = 7}$$

$$\text{Standard Deviation} = 1.63$$

mode \rightarrow Highest frequency (7 has highest frequency)
So mode = 7

Problem-2

data \Rightarrow 28, 40, 68, 70, 75, 75, 75, 75, 80, 86, 89, 90, 90, 97, 97, 100, 100, 100, 104, 104, 109, 113, 120, 120, 122, 123, 123, 130, 140, 145, 170, 174, 194, 217

$$n (\text{Count}) = 35$$

$$\text{mean} = \frac{\text{Sum of all}}{35} = \frac{3763}{35} = 107.5143$$

$$\text{Median} = \frac{35\text{th}}{2} = 18\text{th term} = 100$$

$$\text{Mode} = 75 (\text{Highest frequency } 4)$$

$$\text{Standard Deviation} = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{(n-1)}} = \boxed{39.338}$$

problem - 3 The number of times I go to the gym in weeks

$$x = 0, 1, 2, 3, 4, 5$$

$$f(x) = 0.04, 0.15, 0.40, 0.25, 0.10, 0.01$$

$$\begin{aligned}\text{mean} &= 0 \times 0.04 + 1 \times 0.15 + 2 \times 0.4 + 3 \times 0.25 + 4 \times 0.1 + 5 \times 0.01 \\ &= 0 + 0.15 + 0.8 + 0.75 + 0.4 + 0.05 \\ &= 2.15\end{aligned}$$

$$\text{Variance } s^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1} \quad n=6$$

$$s^2 = \frac{(0-2.15)^2 + (1-2.15)^2 + (2-2.15)^2 + (3-2.15)^2 + (4-2.15)^2 + (5-2.15)^2}{(6-1)}$$

$$= 3.65$$

problem - 4

D - diameter of hole drilled

Target Diameter = 12.5 mm

$$PDF(d) = 20e^{-20(d-12.5)}$$

$$d > 12.5$$

problem - 4

Target Diameter = 12.5 mm

$$PDF(d) = 20e^{(-20)(d-12.5)}, d > 12.5$$

Proportion of parts $> 12.6 = ?$

$$P(d > 12.6) = 1 - P(12.5 < d < 12.6)$$

$$P(d > 12.6) = 1 - P(12.5 < d < 12.6)$$

$$P(12.5 < d < 12.6) = \int_{12.5}^{12.6} 20 \times e^{(-20)(d-12.5)}$$

$$= \left[-e^{-20(d-12.5)} \right]_{12.5}^{12.6}$$

$$\Rightarrow 0.865$$

$$P(d > 12.6) = (1 - 0.865) = 0.135$$

$$CDF(d=11) = 0$$

Conclusion

(13.5% of parts will be scrapped)

Q.5

Fault rate (p) = 30% = 0.3

LED chosen (n) = 6

$x = 2$

This is a Binomial Distribution

Probability of having 2 faulty LED = ${}^6C_2 (0.3)^2 (0.7)^4$

$$\Rightarrow \frac{6!}{4! 2!} \times 0.09 \times 0.2401$$

$$\Rightarrow \frac{6 \times 5 \times 4!}{4! \times 2} \times 0.021609 = \boxed{0.3241 \text{ or } 32.41\%}$$

$$\boxed{\text{Mean} = np = 6 \times 0.3 = 1.8}$$

$$\boxed{\text{Std Devial } (\sigma) = \sqrt{np(1-p)} = \sqrt{6 \times 0.3 \times 0.7} = 1.26}$$

Problem

Binomial Distribution

Gausare

Attempt (n) = 8

Success (p) = 0.75

~~Target~~

$$P(x=5) = {}^8C_5 (0.75)^5 (0.25)^3$$
$$= 20.76\%$$

$$P(x=4) = {}^8C_4 (0.75)^4 (0.25)^4 = 8.65\%$$

$$P(x=6) = {}^8C_6 (0.75)^6 (0.25)^2 = 31.14\%$$

Basakha

Attempt (n) = 12

Success (p) = 0.45

$$P(x=5) = {}^{12}C_5 (0.45)^5 (0.55)^7$$
$$= 22.25\%$$

$$P(x=4) = {}^{12}C_4 (0.45)^4 (0.55)^8 = 16.99\%$$

$$P(x=6) = {}^{12}C_6 (0.45)^6 (0.55)^6 = 21.24\%$$

Gausare

$$P(4) = 8.65\% \quad P(5) = 20.76\%$$

$$P(6) = 31.14\% \Rightarrow P(4 \leq x \leq 6) = 60.55\%$$

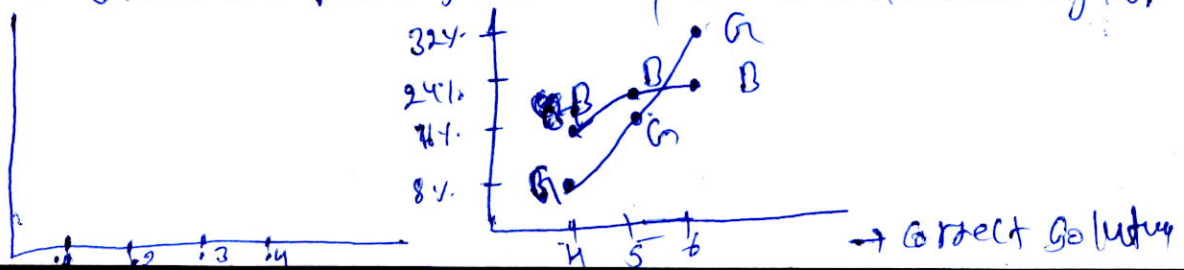
Basakha

$$P(4) = 16.99\% \quad P(5) = 22.25\%$$

$$P(6) = 21.24\% \Rightarrow P(4 \leq x \leq 6) = 60.48\%$$

Inference \rightarrow Both have same CDF ($4 \leq x \leq 6$)

Two factors \rightarrow Gausare has high Accuracy but Basakha has high speed.



Problem - 7 Arrival per hour = 72

$$\text{Poisson} = \frac{e^{-\mu} \cdot \mu^k}{k!}$$

$$\mu = 72$$

Arrival in 4 minutes -

$$\mu = \frac{72}{60} \times 4 = 4.8$$

$$P(k=4)$$

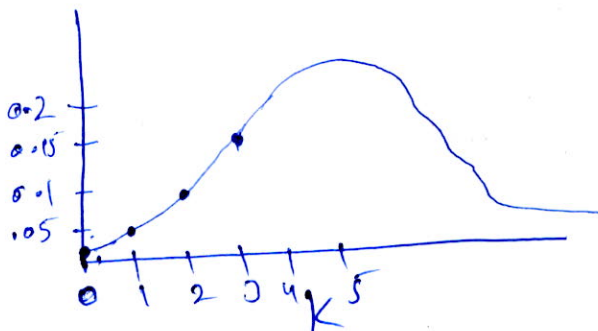
$$(a) P(k=5) = \frac{e^{-4.8} (4.8)^5}{5!} = \boxed{17.47\%}$$

$$(b) P(0) + P(1) + P(2) + P(3) = P(k \leq 3)$$

$$P(k \leq 3) = \frac{e^{-4.8} (4.8)^0}{0!} + \frac{e^{-4.8} (4.8)^1}{1!} + \frac{e^{-4.8} (4.8)^2}{2!} + \frac{e^{-4.8} (4.8)^3}{3!} = 0.0082297 + 0.0395027 + 0.094806686 + 0.15169064 = \boxed{0.29422}$$

$$(c) P(k > 3) = 1 - P(k \leq 3)$$

$$= 1 - 0.29422 = 0.70578 = \boxed{70.57\%}$$



problem - 8 entry per minute = 77

$$\text{error per minute} = \frac{6}{60} = \frac{1}{10}$$

Time taken to enter 455 words = 5.9 minute ($\frac{455}{77}$)

Committing error (k) = 2

$$\mu = \frac{1}{10} \times 5.9 = 0.59$$

$$P(k=2) = \frac{e^{-0.59} (0.59)^2}{2!} = 0.09648 = 9.6\%$$

$$\text{Time for 250 words} = \frac{250}{77} = 3.25 \text{ minute}$$

$$\text{Time for 1000 words} = \frac{1000}{77} = 12.99 \text{ minute}$$

$$\text{mean error in 3.25 minute} = 0.325$$

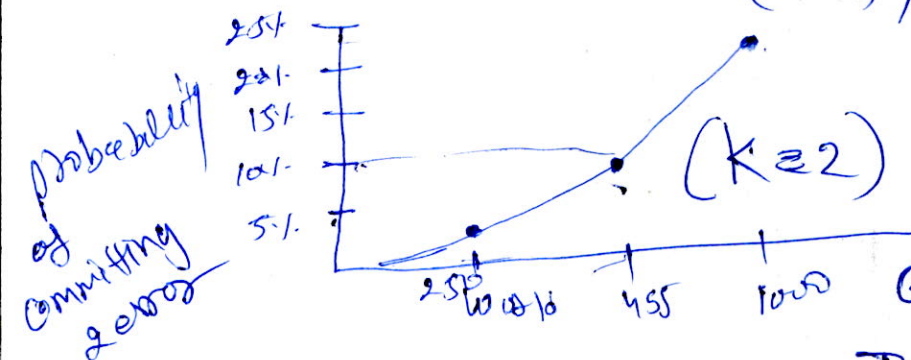
$$\text{mean error in 12.99 minute} = 1.299$$

Probability of committing 2 errors

(a) 455 word document $\lambda(k=2) = \boxed{11.64\%}$

(b) 250 word document $\lambda(k=2) = e^{-0.325} \frac{(0.325)^2}{2!} = 0.038 = \boxed{3.81\%}$

(c) 1000 word document $\lambda(k=2) = e^{-1.299} \frac{(1.299)^2}{2!} = 0.23 = \boxed{23.01\%}$



① λ increases as the number of words increases

② PMF function will increase with number of words

problem 9 same as question / problem No- 4
(Repeated)

problem 10 (a) $P(Z > 1.26) = 10.38\%$ $P(Z \leq -4.6) = 0$
 $P(Z < -0.86) = 19.49\%$
 $P(Z > -1.37) = 91.47\%$ $P(1.25 < Z \leq 0.37) = 53.97\%$

(b) $P(Z > 2) = 0.05$ $\boxed{Z = 1.64}$

(c) $P(-2 < Z < 2) = 0.99$ $\boxed{-2.58 < Z < 2.58}$

problem Statement - 11

$$\mu = 10 \text{ mA}$$

$$\sigma = \sqrt{4 (\text{mA})^2} = 2$$

①

$$P(9 < x < 11)$$

$$= P\left(\frac{9-10}{2} < z < \frac{11-10}{2}\right)$$

$$= P\left(-\frac{1}{2} < z < \frac{1}{2}\right)$$

$$= P(-0.5 < z < 0.5)$$

$$= 0.5487 = 0.4013$$

$$= 1 - 0.4013 = 0.5987 = 59.87\%$$

$$= 38.3\%$$

~~P(x < 13)~~ Probability of exceeding

$$P(x > 13) = P\left(z > \frac{13-10}{2}\right)$$

$$= P(z > 1.5)$$

$$= 1 - 0.9394$$

$$= 0.0606 = 6.06\%$$

$$P(x < x) = 0.98$$

$$P\left(z < \frac{x-10}{2}\right) = 0.98$$

$$P(z < 2.5) = 0.98$$

$$\Rightarrow 2.5 \leq \frac{x-10}{2}$$

$$\Rightarrow 5 \leq x-10$$

$$\Rightarrow x \geq 15$$

Current measurement is 15

problem Statement - 12

Part 1 $\mu = 0.2508$

$$\sigma = 0.0005$$

$$\text{Lower Value} = 0.2500 - 0.0005 = 0.2495$$

$$\text{Upper Value} = 0.2500 + 0.0005 = 0.2505$$

$$P(0.2495 < x < 0.2505) = P\left(\frac{0.2495-0.2508}{0.0005} < z < \frac{0.2505-0.2508}{0.0005}\right)$$

$$= P\left(\frac{-0.0013}{0.0005} < z < \frac{-0.0003}{0.0005}\right)$$

$$= P(-2.6 < z < -0.6)$$

$$= 91.92\%$$

Part 2 $\mu = 0.2500$

$$\sigma = 0.0005$$

$$\text{Lower} = 0.2495$$

$$\text{Upper} = 0.2505$$

$$P(0.2495 < x < 0.2505) =$$

$$P\left(\frac{0.2495-0.2500}{0.0005} < z < \frac{0.2505-0.2500}{0.0005}\right)$$

$$= P(-1 < z < 1)$$

$$= 99.74\%$$