Assignment-2

Problem Statement (1)

(1) $H_0: \mu = 25$ & $H_1: \mu \neq 25$

→ Correctly Stated as Null Hypothesis is about status quo

(2) $H_0: \sigma > 10$ & $H_1: \sigma = 10$

→ Wrong - Because Null Always takes Sign ($=, \geq, \leq$)

(3) $H_0: \bar{x} = 50$ & $H_1: \bar{x} \neq 50$

→ Correct - Null is 50 then Alternate → $\bar{x}$ is different from 50

(4) $H_0: p = 0.1$ & $H_1: p = 0.5$

→ Wrong:- we Cannot Assign ($=$) in both null and Alternate

(5) $H_0: s = 30$ & $H_1: s > 30$

→ Correct → Rightly stated as Alternate is Right tailed.

Problem Statement -2

Average Cost ($\bar{x}$) = 52

Standard Deviation = 4.5

H_0: Average Cost $\bar{x} = 52$

H_1: Average Cost $\bar{x} \neq 52$

$n = 100$ (Sample Size) $\mu = 52$

$\bar{x} = 52.80$ (Sample mean) $S = 4.5$

$$z = \frac{x - \mu}{S_x / \sqrt{n}} = \frac{52.80 - 52}{4.5 / \sqrt{100}}$$

$\boxed{z = 1.78}$ $\boxed{\text{At LOS } 5\% \text{ Critical Value } \pm 1.96}$

→ Since $z = 1.78$, which falls in the Acceptance zone, so we fail to Reject the null Hypothesis

Problem Statement 3   A Chemical pollutant

$\mu = 34$ $\sigma = 8$ $H_0 : \mu = 34$

LOS = 1% $n = 50$ $H_1 : \mu \neq 34$

$x = 32.5$

$$z = \frac{32.5 - 34}{8/\sqrt{50}} = \frac{-1.5}{1.13137} = -1.03258 = -1.33$$

Z critical values at 1% are -2.58 and +2.58

Z value of -1.33 is within Acceptance zone hence we fail to reject the null hypothesis.

Problem Statement 4   US population

family of 4 spends on Dental expenses (μ) = 1135

$n = 22$

LoS = 5% (LoS) (Alpha 0.5)

$\bar{x} = 1031.32$

$s = 240.37$

$H_0 : \mu = 1135$

$H_1 : \mu \neq 1135$

$$z = \frac{x - \mu}{s/\sqrt{n}} = \frac{1031.32 - 1135}{240.37/\sqrt{22}} = \frac{-103.68}{51.25} = -2.02$$

Z - critical ± 1.96

The Z value of -2.02 is out of Acceptance Zone hence Reject the null hypothesis and Accept the Alternate.

Scanned with CamScanner

Problem Statement 5

$n = 400$ $\quad LOS = 5\%$ $\quad H_0: \mu_0 = 48432$

$H_1: \mu \neq 48432$

$s = 2000$

$x = 48574$

$$Z = \frac{x - \mu}{s/\sqrt{n}} = \frac{48574 - 48432}{2000/\sqrt{400}}$$

$$Z = \frac{142 \times 20}{2000} = 1.42$$

Z critical value at 5% Level of Significance ± 1.96

Z critical value at 10% Level of Significance ± 1.64

Both at LoS of 5% and LoS of 10% the Z of 1.42 falls under the Acceptance Zone

Hence we fail to Reject the null hypothesis

So :- Average Annual Income of metropolis is 48432.

Problem Statement 6

$H_0: \mu = 32.28$ (Avg price/sq feet)

$H_1: \mu \neq 32.28$

$\mu = 32.28$ $\quad n = 19$

$\bar{x} = 31.67$ $\quad s = 1.29$

LOS - 5%

$$Z = \frac{x - \mu}{s/\sqrt{n}} = \frac{31.67 - 32.28}{1.29/\sqrt{19}} = \frac{-0.61 \times 4.35}{1.29}$$

$$= -2.061$$

Z critical value at 5% — ± 1.96

Z value of -2.06 falls out of Acceptance Zone and Hence we Reject null hypothesis.

So the mean price is not equal to 32.28

Hence Average price has ~~not~~ Changed.

Problem Statement 7

(9) Acceptance Region — $48.5 < \bar{x} < 51.5$

Sample size(n) = 10

LOS (α) = 5%

Df = 10-1 = 9

$t_{5\%}$ (Two tailed) = ± 2.262156

$\bar{x} - 2.26 \times SE < \bar{x} < \bar{x} + 2.26 \times S.E$

$$S.E = \frac{51.5 - 48.5}{2 \times 2.26} = 0.664$$

Beta Calculation

(i) Beta ($\mu = 52$) $\Rightarrow P\left(\frac{48.5-52}{0.664} < t < \frac{51.5-52}{0.664}\right)$

$= P(-5.27 < t < -0.75)$

$= \cancel{0.77 = 77\%}$ $\quad 0.23 - 0.00025 = 23\%$

There are 77% Chance that this test will fail to ~~detect~~ Reject null Hypothesis if True mean is 52

(ii) Beta ($\mu = 50.5$) $= P(-3.01 < t < 1.51)$

$\cancel{\beta = 0.9317 = 93\%}$

$\beta = 0.91 - 0.007 \approx 91\%$

Problem Statement 7 Continued

Acceptance Region	α	Sample size	β @ 52	β @ 50.5
$48.5 < \bar{x} < 51.5$	5%	10	$P(-5.27 < t < -0.75)$ 23%	$P(-3.01 < t < 1.51)$ 91%
$48 < \bar{x} < 52$	5%	10	$P(-4.12 < t < 0)$ $= 0.5 - 0.0007$ $\simeq 50\%$	$P(-2.32 < t < 1.7)$ $= 93.8 - 0.01$ $\simeq 93.7\%$
$48.8 < \bar{x} < 51.9$	5%	16	$P(-4.4 < t < -.14)$ $= 44.5 - .00025$ $\simeq 44\%$	$P(-2.29 < t < 1.92)$ $= 96.29 - 1.7$ $\simeq 94.3\%$
$48.42 < \bar{x} < 51.58$	5%	16.	$P(-4.83 < t < -.57)$ $= 28.85 - .00011$ $= 28\%$	$P(-2.84 < t < 1.45)$ $= 91.61 - 0.0066$ $= 91.6\%$

From the above Table we Can Conclude that when $48.8 < \bar{x} < 51.9$ and Actual mean is $\mu = 50.5$ then there is 94.3% probability that this test will fail to Reject the null Hypothesis when null Hypothesis is Actually false.

Problem Statement 8

$n = 16 \qquad \mu = 10 \qquad \bar{x} = 12 \qquad s = 1.5$

$$t\text{-score} = \frac{12-10}{1.5/\sqrt{16}} = \frac{2 \times 4}{1.5} = 5.33$$

Problem Statement 9

Sample mean – $\bar{x}$

Sample Size $(n) = 16$

t – Score at 99%

$D.f = 16 - 1 = 15$

$\boxed{t\text{-Score} - 2.947}$ Below which 99% of sample mean will fall

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Problem Statement – 10

Sample size $(n) = 25$ Sample mean $(\bar{x}) = 60$

Standard Deviation $= 4$ $Df = 24$

$\cancel{t\text{ Score} = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{60 - \mu}{4/\sqrt{25}} = \frac{5(60 - \mu)}{4}}$

$\cancel{t(95\%) = 1.711 = \frac{5(60 - \mu)}{4}}$

$5(60 - \mu) = 6.844$

$\cancel{\mu = 60 - 1.3688 = 58.6312}$

$$\text{Standard Error (S.E)} = \frac{4}{\sqrt{25}} = \frac{4}{5} = 0.8$$

$$P(t < 95\%) = \pm 2.064$$

$$\text{Confidence Interval} = (60 + 2.064 \times 0.8,\ 60 + 2.064 \times 0.8)$$

$$= (58.35 < \bar{x} < 61.65)$$

$$\text{Probability}(-0.05 < t < 0.10) = 53.94\% - 48\% = 6\%$$

$$P(-0.05 \times 2.064 < t < 0.1 \times 2.064) = P(-0.1032 < t < 0.2064)$$

$$= 58.08 - 45.93 = 12\%$$

Problem Statement-11

Suppose :- Avg number of people Travelling from Bglr to Chennai $= \mu_1$

Avg - — — from Bglr to Hosur $= \mu_2$

$H_0: \mu_1 = \mu_2 \qquad H_1: \mu_1 \neq \mu_2$

$n_1 = 1200 \qquad n_2 = 800$

$x_1 = 452 \qquad x_2 = 523$

$s_1 = 212 \qquad s_2 = 185$

$$Z\text{-Score} = \cancel{\frac{452-523}{\sqrt{\frac{212}{1200}+\frac{1}{800}}}} \qquad \frac{452-523}{\sqrt{\frac{(212)^2}{1200}+\frac{(185)^2}{800}}}$$

$$= \frac{452-523}{\sqrt{37.45+42.78}} = \frac{-71}{8.957} = -7.92$$

Z (critical at 5% LOS) $= \pm 1.96$.

Z test Score $\neq$ Z (critical Score @ 95%)

which falls out of acceptance zone

Hence - mean population is not same for Two Samples

Problem Statement - 12

$H_0: \mu_1 = \mu_2 \qquad H_1: \mu_1 \neq \mu_2$

[People preferring duracell are different from]

Duracell	Energizer
$n_1 = 100$	$n_2 = 100$
$x_1 = 308$	$x_2 = 254$
$s_1 = 84$	$s_2 = 67$

$$Z = \frac{308-254}{\sqrt{\frac{84^2}{100}+\frac{67^2}{100}}}$$

$$Z = \frac{54 \times 100}{107.44} = 5.025$$

Z-critical ($\alpha = 5\%$) $= \pm 1.96$

Z-test is out of bound of Zcritical Hence we Reject the null Hypothesis and number of people is different for both population

~~Q13~~ problem Statement - 13

H_0: Avg Percentage Increase in price of Sugar do not differs when sold ~~on~~ at two different prices.

H_1: Differs.

Population 1	Population 2
price - 27.5	price - 20.
$n_1 = 14$	$n_2 = 9$
$x_1 = 0.317\%$	$x_2 = 0.21\%$
$s_1 = 0.12\%$	$s_2 = 0.11\%$

$$t \text{ score} = \frac{(0.317 - 0.21)}{\sqrt{\frac{(0.12)^2}{14} + \frac{(0.11)^2}{9}}}$$

$$= \frac{\cancel{\ }}{\sqrt{0.001028}} = \frac{0.107}{0.04871} = 2.196$$

t (critical at $\alpha - 5\%$ and $Df = (14+9-2) = 21$) $= 2.080$

which is less than 2.196

Hence we Reject the null hypothesis

and the Avg percentage change/Increase in prices is different at different prices.

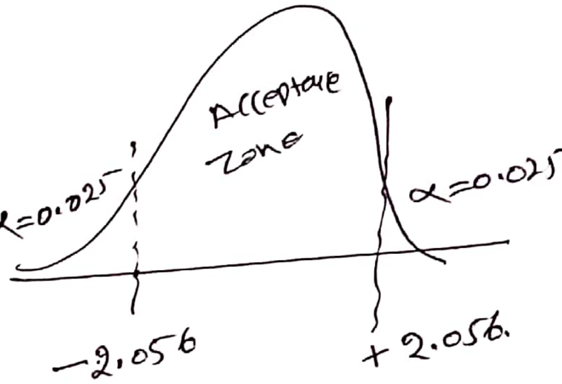
Problem Statement - 14

H_0: No Impact of price Reduction on Sale $(\mu_1 = \mu_2)$

H_1: There is Impact of price Reduction on Sale

Population-1 Before Reduction	Population-2 After Reduction
$n_1 = 15$	$n_2 = 12$
$\bar{x}_1 = 6598$	$\bar{x}_2 = 6870$
$s_1 = 844$	$s_2 = 669$

$$t = \frac{6598 - 6870}{\sqrt{\frac{844^2}{15} + \frac{669^2}{12}}} = -0.934$$



t (at $Df = 26, \alpha - 5\%$) $= 2.056$

t-score falls in the Acceptance zone hence we fail to reject the null hypothesis.

Problem Statement - 15

Population 1 1980	Population 2 1985
$n_1 = 1000$	$n_2 = 100$
$x_1 = 53$	$x_2 = 43$
$p1 = 0.53$	$p2 = 0.53$
as P_1 is 53	as $x_2 = 43$
So $n_1 = 100$	so $P_2 = 0.43$

$$Z = \frac{P_1 - P_2}{\sqrt{P(1-P)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$P = \left(\frac{x_1 + x_2}{n_1 + n_2}\right)$$

$$Z = \frac{0.53 - 0.43}{\sqrt{(0.52 \times 0.48)\left(\frac{1}{100} + \frac{1}{100}\right)}} = \frac{0.1}{\sqrt{0.00499}} = 1.415$$

$$P = \left(\frac{53 + 43}{200}\right) = 0.48$$

$$\boxed{\begin{array}{l} Z = 1.415 \\ Z_{\alpha = 5\%} = 1.64 \end{array}}$$

Hence we fail to Reject Null Hypothes.

Problem Statement - 16

H_0: who buy checks with sweepstakes is ~~higher than~~ are same to who buy without Sweepstakes.

H_1: There is a Difference ($\mu_1 > \mu_2$)

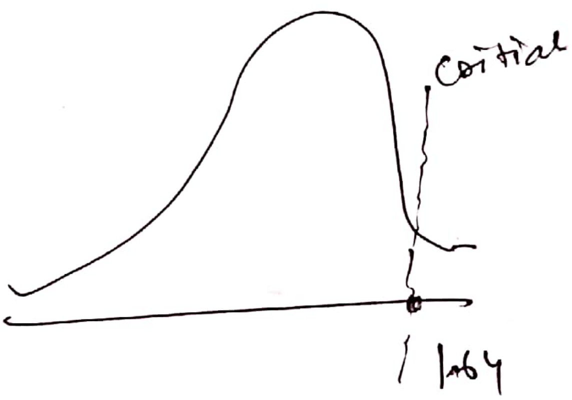
μ_1: Avg number of / proportion who are offered Sweepstakes

μ_2: Avg proportion who are not offered Sweepstakes.

with Sweepstakes.	No Sweepstakes
$n = 300$	$n_2 = 700$
$x_1 = 120$	$x_2 = 140$
$p_1 = 0.40$	$p_2 = 0.2$

$$p = \frac{120+140}{1000} = \frac{260}{1000} = 0.26$$

$$z = \frac{0.40 - 0.20}{\sqrt{(0.26 \times 0.74) \times \left(\frac{1}{300} + \frac{1}{700}\right)}} = \frac{0.2}{0.03} = 6.6$$



z (at α 5%) $= 1.64$

z - test $= 6.6$

Hence we Reject the null Hypothesis

Hence people who buy During Sweepstakes are higher than who buy without Sweepstakes.

problem 17

Die Thrown - 132

is Die unbiased ?

degree of freedom = $p-1$

Face	expected (E)	(O) Frequency	(O-E)	$\frac{(O-E)^2}{E}$
1	22	16	6	36/22
2	22	20	2	4/22
3	22	25	-3	9/22
4	22	14	-8	64/22
5	22	29	-7	49/22
6	22	28	6	36/22
				~~1980~~ 9.01

H_0 :- Die is unbiased

H_1 :- Die is not unbiased

$$Chi^2 = \sum\left(\frac{(O-E)^2}{E}\right) = 9.01$$

(~~α 5%~~) $DF = p-1 = 6-1 = 5$

Chi^2 (df=5 and Los 5%) = 11.07

Chi^2_Test < Chi^2 (critical)

Hence- we fail to Reject the null Hypothesis.

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problem Statement -18

Population = 1 million

Sample Size (n) = 10000

H_0: Gender and Voting are Independent

H_1: Gender & Voting are not Independent.

observed

	men	women	
voted	2792	3591	6383
not voted	1486	2136	3617
	4278	5722	10,000

expected value

	men	women
voted	2730.6	3652.4
not voted	1547.4	2069.6

$$\chi^2 = \frac{(O-E)^2}{E} =$$

	man	woman.
Voted	1.378	1.03
not Voted	2.43	1.81
		6.66

expected value of cell = $\frac{\text{Total of Rows} \times \text{Total of Colums.}}{\text{Grand Total.}}$

χ^2 (df = (2-1)(2-1) = 1 α = 5%) = 3.84 which is Less than 6.66 Hence we Reject Null Hypothesis

Problem Statement 19

Sample $(n) = 100$

Higgins	Reardon	White	Charlton
41	19	24	16

H_0: all Candidates are equally popular

H_1: all are not equally popular

	observed	expected	$\frac{(O-E)^2}{E}$
Higgins	41	25	10.24
Reardon	19	25	1.44
White	24	25	0.04
Charlton	16	25	3.24
	100	100	14.96

$\chi^2(df = 3, \alpha = 5\%) = 7.815$

$7.815 < \chi^2 < 14.96$

Hence we Reject the null Hypothesis.

Problem Statement 20

H_0: Age and photo prefence are Independent

H_1: Age and photo prefen are not Independent

observed (O)

Age	Photo A	Photo B	Photo C	
5-6 yrs	18	22	20	60
7-8 yrs	2	28	40	70
9-10 yrs	20	10	40	70
	40	60	100	200

expected (E)

Photo A	Photo B	Photo C
12	18	30
14	21	35
14	21	35

$(O-E)^2/E$

Photo			
3	0.88	3.33	
6.08	2.3	6.714	
2.57	5.76	.71	
			29.6

$\chi^2(\text{critical at } df = 2 \times 2 = 4, \alpha = 5\%) = 9.488$

$9.488 < \chi^2 < 29.6$

Hence we Reject the Null Hypothesis.

So Age and photo preference are not Independent

Problem Statement 21

Ho: no significant difference in "Support" & "no support" Condition

H1: significant difference in "support" & "no support" Condition.

Observed

	Support	no support	
Conform	18	40	58
not Conform	32	10	42
	50	50	100

expected

Support	no Support	
29	29	58
21	21	42
50	50	100

$(O-E)^2/E$

Support	no Support	
4.17	4.17	
5.76	5.76	
		19.86

χ^2 (Calculated) = 19.86

χ^2 (critical df=1, α=5%) = 3.84

Hence we Reject null Hypothesis.

Hence there is significant difference in "Support" and "no support" Condition.

Problem statement — 22

Ho: Height and leadership Quality are Independent

H1: Height and Leadership Quality are not Independent.

Observed (O)

	Short	Tall	
Leader	12	32	44
Follower	22	14	36
unclassified	9	6	15
	43	52	95

expected (E)

	Short	Tall	
	19.91	24.08	
	16.29	19.7	
	6.78	8.21	
			95

$(O-E)^2/E$

	Short	Tall	
	3.14	2.6	
	1.99	1.65	
	0.71	0.59	
	.		10.79

$df = (3-1) \times (2-1) = 2$

$\chi^2_{critical}$ (df=2, α=5%) = 10.71   1091 < χ^2 < 10.712

Hence we Reject the null Hypothesis.

Hence we Conclude that Height and Leadership Quality are dependent.

Problem Statement — 23

H_0: marital Status and Labor force Status are Independent

H_1: marital Status and Labor force Status are dependent

	married	widowed/Divorced separated	never married
employed	679	103	114
unemployed	63	10	20
not in Lab. for.	42	18	25

$$\chi^2 \text{ Critical } (df = 4, \alpha = 5\%) = 9.488$$

$$\chi^2 (\text{Calculated}) = 31.61$$

Hence we Reject the null Hypothesis.

So, we Conclude that marital Status & Labor force Status are not Independent

as χ^2 Calculated is greater than Critical value So null hypothesis is Rejected.