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SFI GEC PALAKKAD

MODULE -2

WAVE OPTICS

Huygen's wave theory

Light is propagated through a medium in the form of transverse wave. Each point of source of light is a centre of disturbance from which waves are propagated in all direction. Each point of this wave front is a source of secondary disturbance and wavelet emitting from these points spread out in all directions. The displacement of any particle at any instant, can be represented as,

$$y = A \sin \omega t$$

A is the amplitude and ωt is the phase

Period (T)

It is the time taken by the particle to make one complete vibration

Wavelength (λ)

It is the distance between two successive crest or trough. It is the distance travelled by the wave during T seconds.

Frequency(ν)

Number of vibrations made by the particle in one second

$$\nu = \frac{1}{T}$$

Velocity V is equal to distance travelled in one second

$$V = \frac{\lambda}{T} = \nu \lambda$$

Interference

It is the remodification of light energy due to the super position of two or more light waves having nearly same amplitude same frequency and constant phase difference.

Principle of super position

Resultant displacement of the particle acted by two or more waves is equal to algebraic sum of the displacement due to each wave.

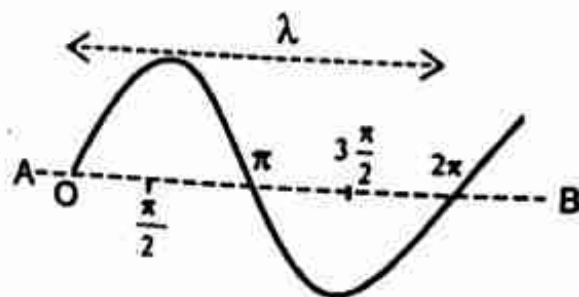
The superposition of waves resulting maximum intensity is called constructive interference and give minimum intensity called destructive interference.

There is no loss of energy due to interference, total energy remains constant only remodification of energy takes place.

Path difference and Phase difference

Path difference λ corresponds to phase difference 2π

$$\lambda \rightarrow 2\pi$$



Path difference x corresponds to phase difference ϕ

$$x \rightarrow \phi$$

$$\frac{\lambda}{x} = \frac{2\pi}{\phi}$$

$$\phi = \left(\frac{2\pi}{\lambda}\right) x$$

COHERENT SOURCE

The two source are said to be coherent if they emit light wave of nearly same amplitude, same frequency and constant phase difference.

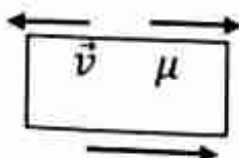
Eg:- Two slit illuminated by Monochromatic source of light.

CONDITIONS FOR INTERFERENCE

- Sources should be coherent
- Light wave from two coherent source should superimpose at the same time and same place
- The sources should be very close to each other.

OPTICAL PATH

A light travelling through a medium of refractive index μ and length d with a velocity v .



The time of travel in medium, $t = \frac{d}{v}$

During this time the distance travelled by the light in air = ct

$$= c \times \frac{d}{v}$$

$$= \frac{c}{v} \times d$$

$$= \mu d \left[\because \frac{c}{v} = \mu \right]$$

μd is the optical path.

CONDITION FOR MAXIMUM INTENSITY AND MINIMUM INTENSITY

Path difference between the wave is integral multiple of λ

Path difference = $n\lambda$ where $n=0,1,2,3,\dots$

i.e path difference is $= 0, \lambda, 2\lambda, \dots$

Condition for minimum intensity is half integral multiple of λ

i.e path difference = $(2n+1)\frac{\lambda}{2}$ where $n=0,1,2,3,\dots$

i.e path difference = $\frac{\lambda}{2}, 3\frac{\lambda}{2}, 5\frac{\lambda}{2}, \dots$ etc

The amplitude of two interfering waves must be nearly the same or equal

If $a_1 = a_2 = a$

$$I_{\max} = (a+a)^2 = 4a^2 = 4I_0$$

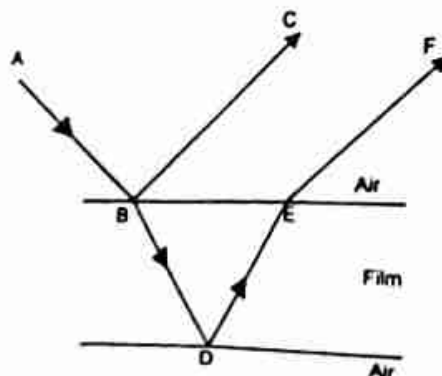
I_0 is intensity due to single wave.

$$I_{\min} = (a-a)^2 = 0$$

For constructive interference, the resultant intensity is 4 times the intensity of incident wave.

Path difference due to dissimilar reflection

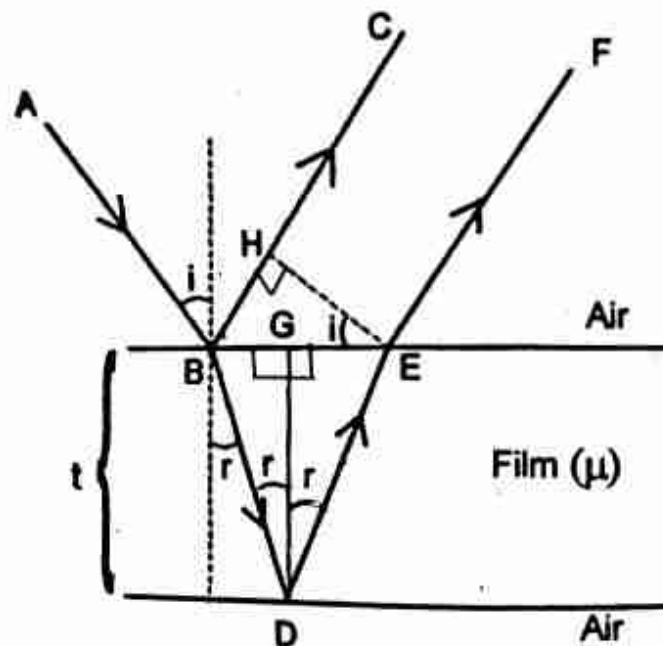
When reflection takes place at the surface of denser medium, there is a phase change of π or additional path difference $\frac{\lambda}{2}$ with respect to the reflection takes place at rarer medium.



BC undergo phase change π or additional path difference $\frac{\lambda}{2}$ (Actual path difference is $BC + \frac{\lambda}{2}$)

Colour of thin films by reflected light

Consider a thin transparent film of thickness, t and refractive index μ . The optical path difference between the light rays reflected from top to bottom surface of the film is given by,



Path difference = $(BD + DE)_{\text{film}} - BH_{\text{air}}$
 (Since BD and DE are in film equivalent length in air is $(BD + DE) \mu$)

$$\text{Optical path difference} = \mu (BD + DE) - BH \quad \text{---(1)}$$

Triangles BGD and $\triangle EGD$ are similar

$$\therefore BD = DE$$

From the $\triangle BGD$,

$$\begin{aligned} \cos r &= \frac{GD}{BD} \\ &= \frac{t}{BD} \\ BD &= \frac{t}{\cos r} \end{aligned} \quad \text{---(2)}$$

$$DE = \frac{t}{\cos r} \quad [\because BD = DE] \quad \text{---(3)}$$

Consider $\triangle BHE$,

$$\begin{aligned} \sin i &= \frac{BH}{BE} = \frac{BH}{BG + GE} = \frac{BH}{2BG} \quad [BG = GE] \\ BH &= 2BG \sin i \end{aligned}$$

Consider $\triangle BGD$,

$$\begin{aligned} \tan r &= \frac{BG}{DG} \\ \tan r &= \frac{BG}{t} \\ BG &= t \times \tan r \end{aligned}$$

$$\therefore BH = 2BG \sin i$$

$$BH = 2t \times \tan r \times \sin i \quad -(4)$$

(2), (3) and (4) are substituted in (1), optical path difference is

$$\mu \left[\left(\frac{t}{\cos r} \right) + \left(\frac{t}{\cos r} \right) \right] - 2t \times \tan r \sin i$$

(in film) (in air)

$$\begin{aligned} \text{opd} &= \mu \frac{2t}{\cos r} - 2t \times \frac{\sin r}{\cos r} \times \mu \sin r & \left[\frac{\sin i}{\cos r} = \mu \right] \\ &= \frac{2\mu t}{\cos r} [1 - \sin^2 r] \\ &= \frac{2\mu t}{\cos r} \times \cos^2 r & \sin^2 r + \cos^2 r = 1 \end{aligned}$$

$$\text{Opd} = 2\mu t \cos r$$

\therefore Actual path difference between reflected rays BC and EF is,

$$2\mu t \cos r - \frac{\lambda}{2}$$

• Condition for brightness (constructive interference)

$$\text{Path difference} = n\lambda$$

$$2\mu t \cos r - \frac{\lambda}{2} = n\lambda$$

$$2\mu t \cos r = n\lambda + \frac{\lambda}{2}$$

$$2\mu t \cos r = (2n + 1) \frac{\lambda}{2} \quad n = 1, 2, 3, \dots$$

• Condition for darkness

$$\text{Path difference} = (2n+1) \frac{\lambda}{2}$$

$$2\mu t \cos r - \frac{\lambda}{2} = (2n+1) \frac{\lambda}{2}$$

$$2\mu t \cos r = (2n+1) \frac{\lambda}{2} + \frac{\lambda}{2}$$

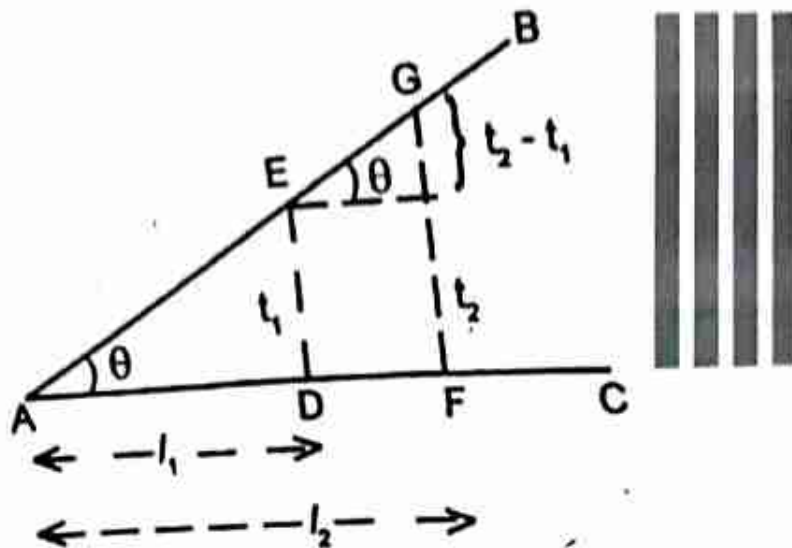
$$2\mu t \cos r = n\lambda$$

$$n = 0, 1, 2, 3, \dots$$

Colour of thin films

A thin transparent film viewed with white light shows different colour, the condition for darkness, $2\mu t \cos r = n\lambda$. A portion of the film satisfies the condition by substituting μ , t and r for wavelength λ . Then, the colour of that wavelength will be absent. In reflected light mixing of remaining colour will be seen. Eg. If red light satisfies above condition, that colour is absent and that film appears in blue-violet region.

Air Wedge



- 2 glass plate AB and AC are placed, they are contact in one end and separated by small distance at the other end.
- A wedge shaped air film formed between them
- When a beam of monochromatic light is incident on a glass plate normally the reflected rays from top and bottom surface of air film interfere each other.
- Equidistant, 11^{th} dark and bright bands are observed
- Angle between glass plate are called angle of air wedge θ , which is in radian
- Thickness of n^{th} and $(n+1)^{th}$ dark band t_1 and t_2 , and they are at distance l_1 and l_2 from the point of contact respectively.

$$\tan \theta = \frac{t_1}{l_1} = \frac{t_2}{l_2} = \frac{t_2 - t_1}{l_2 - l_1}$$

θ is very small, $\tan \theta \cong \theta$

$$\theta = \frac{t_2 - t_1}{\beta} \quad \text{--(1)}$$

Where, $\beta = l_2 - l_1$

Band width, β is the distance between 2 consecutive dark band or bright band.

• Condition for n^{th} dark band

$$2 \mu t_1 \cos r = n \lambda$$

For normal incidence $\cos r = 1$

For air $\mu = 1$

$$2t_1 = n \lambda \quad \text{--(2)}$$

• Condition for $(n+1)^{th}$ dark band

$$2t_2 = (n+1) \lambda \quad \text{--(3)}$$

(3)-(2) :-

$$2(t_2 - t_1) = \lambda$$

$$t_2 - t_1 = \frac{\lambda}{2}, \text{ substituting this in equation (1)}$$

$$(1) \Rightarrow \theta = \frac{\lambda}{2\beta}$$

If the medium between glass plate have refractive index μ , Then

$$\theta = \frac{\lambda}{2\mu\beta}$$

Applications

To find the diameter of wire

$$\tan \theta = \frac{d}{l}$$

$$\theta = \frac{d}{l}$$

$$\frac{\lambda}{2\beta} = \frac{d}{l}$$

$$d = \frac{\lambda l}{2\beta}$$



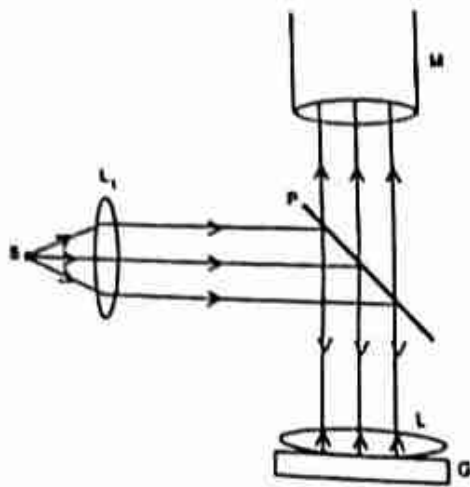
• Optical planeness of surface

The planeness of a surface can be tested by observing the nature of band obtained by air wedge experiment. If the surfaces are optically plane, the bands are parallel and equidistant. The bands are not have equal distant and parallel, the surfaces are not optically plane.

The given surface AB to be tested is placed on a optically plane glass plate AC which is called reference plate. A beam of monochromatic light allow to fall on air wedge. Then, if the interference pattern regular AB is optically plan otherwise it is not optically plane

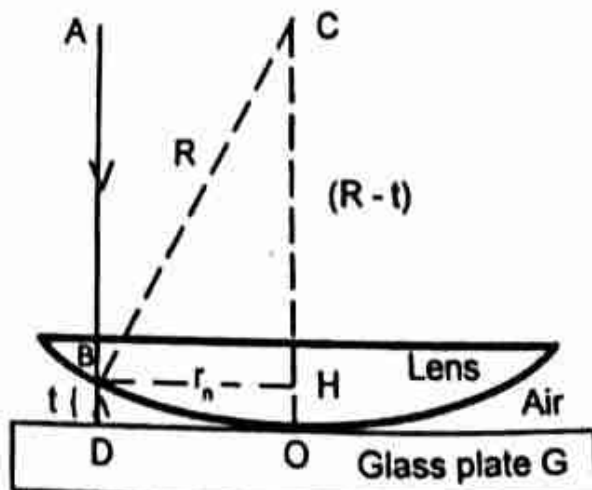
Newton's Ring

Circular interference fringes can be observed, if a very thin film of air or some other transparent medium of varying thickness is enclosed between a plane glass plate and plano convex lens of large radius of curvature.



The thickness of air film is zero at the point of contact between the lens glass plate and gradually increases towards the edge of the lens. The light from monochromatic source is made parallel by a convex lens and it is made to fall on a glass plate, inclined at 45° to the incident beam. The light is reflected normally on to the system of plate and lens light reflected from top and bottom surface of air film superimpose each other and corresponding bright and dark rings are observed due to their path differences. The fringes are circular, because the thickness of the film is symmetric about the point of contact.(is constant over a circle)

To find the radius of n^{th} dark ring



OH	$=$	t
OC	$=$	R
CH	$=$	$R-t$
BH	$=$	r_n

From ΔCHB ,

$$R^2 = r_n^2 + (R-t)^2$$

$$R^2 = r_n^2 + R^2 - 2Rt + t^2$$

t is very small, t^2 is neglected

$$r_n^2 = 2Rt \quad \text{---(1)}$$

Condition for dark ring,

$$2\mu t \cos r = n\lambda$$

$$\text{For a medium of } R-I \mu \quad r_n = \sqrt{\frac{nRP}{\mu}}$$

$$\cos r = 1 \quad [\text{for normal incidence}]$$

$$\text{For air } \mu = 1$$

$$2t = n\lambda$$

$$(1) \Rightarrow r_n^2 = nR\lambda \quad r_n \propto \sqrt{n}$$

$$r_n = \sqrt{nR\lambda}$$

Radius of dark ring is proportional to square root of natural numbers.

Find the radius of bright ring.

$$2\mu t \cos r = (2n+1)\frac{\lambda}{2}$$

For normal incidence, $\cos r = 1$, for air $\mu = 1$

Incidence,

$$2t = (2n+1)\frac{\lambda}{2}$$

$$(1) \Rightarrow r_n^2 = 2Rt$$

$$r_n^2 = \frac{(2n+1)R\lambda}{2}$$

$$r_n^2 = \sqrt{\frac{(2n+1)R\lambda}{2}}$$

To find the wavelength of monochromatic source of light

$$r_n^2 = nR\lambda$$

$$\text{We have, } r_n = \frac{D_n}{2}$$

$$\therefore \left(\frac{D_n}{2}\right)^2 = nR\lambda$$

$$D_n^2 = 4nR\lambda$$

$$D_{n+k}^2 = 4(n+k)R\lambda$$

$$D_{n+k}^2 - D_n^2 = 4(n+k)R\lambda - 4nR\lambda$$

$$= 4nR\lambda + 4kR\lambda - 4nR\lambda$$

$$= 4kR\lambda$$

$$\lambda = \frac{D_{n+k}^2 - D_n^2}{4KR}$$

In the case of bright ring

$$r_n^2 = \frac{(2n+1)R\lambda}{2}$$

We have, $r_n = \frac{D_n}{2}$

$$\left(\frac{D_n}{2}\right)^2 = \frac{(2n+1)R\lambda}{2}$$

$$D_n^2 = \frac{4}{2}(2n+1)R\lambda$$

$$D_n^2 = 2(2n+1)R\lambda$$

$$D_{n+k}^2 = 2[2(n+k)+1]R\lambda$$

$$D_{n+k}^2 - D_n^2 = 2[2(n+k)+1]R\lambda - 2(2n+1)R\lambda$$

$$= 4KR\lambda$$

$$\lambda = \frac{D_{n+k}^2 - D_n^2}{4KR}$$

To find the refractive index of the medium (liquid)

Let a liquid of refractive index μ be introduced between the glass plate and the lens.

Let d_{n+k} and d_n be the diameter of $(n+k)$ and n^{th} dark rings.

$$r_n = \sqrt{\frac{nR\lambda}{\mu}}$$

$$\left(\frac{d_n}{2}\right)^2 = \frac{nR\lambda}{\mu}$$

$$d_n^2 = \frac{4nR\lambda}{\mu}$$

$$d_{n+k}^2 = \frac{4(n+k)R\lambda}{\mu}$$

$$d_{n+k}^2 - d_n^2 = \frac{4KR\lambda}{\mu}$$

$$\therefore \lambda = \mu \frac{d_{n+k}^2 - d_n^2}{4KR}$$

$$d_{n+k}^2 - d_n^2 = \frac{4KR\lambda}{\mu} \quad \text{---(1)}$$

We know that, for air, medium

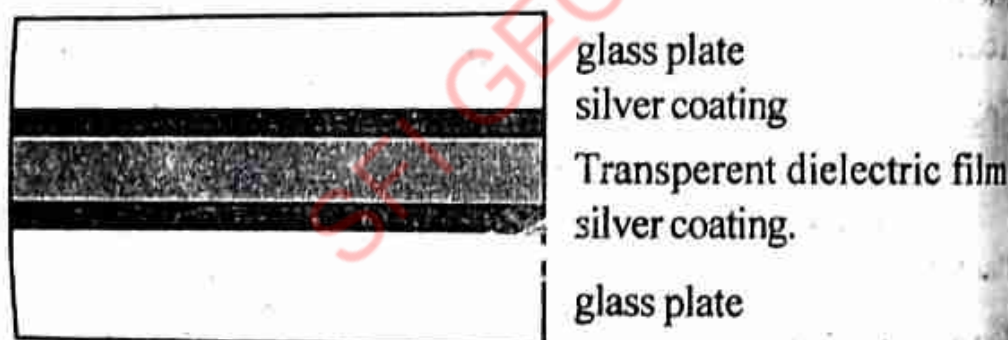
$$D_{n+k}^2 - D_n^2 = 4KR\lambda \quad -(2)$$

$$\frac{(2)}{(1)} \Rightarrow \frac{D_{n+k}^2 - D_n^2}{d_{n+k}^2 - d_n^2} = \mu$$

Interference Filter

The arrangement for filtering out a monochromatic beam from the incident white light is called interference filter.

Interference filter consists of a very thin film of transparent dielectric material like magnesium fluoride, cryolite etc, placed in between two optically plane and flat glass plates. A very thin metallic coating of silver or aluminum is deposited by evaporation on the inner surfaces of the glass plates where the dielectric film is in contact.



When a narrow beam of white light is incident normally on this filter, it undergoes multiple reflections in the dielectric, it loses a major part of its intensity and finally emerge from the filter as a very narrow band. Reducing the thickness of the dielectric film, a particular wavelength (colour) can be filtered out.

Anti reflection coating.

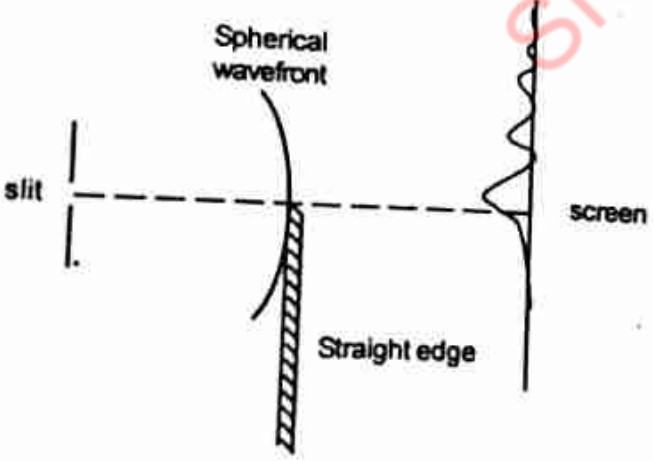
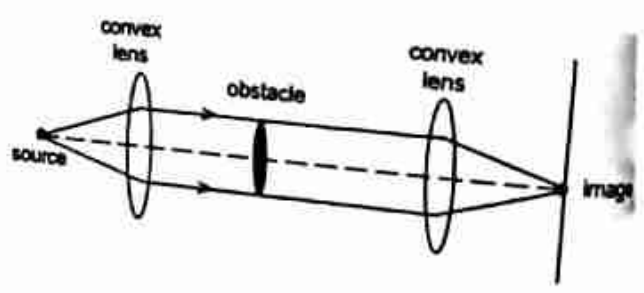
This is an important application of thin film interference and it is used to reduce the loss of intensity of the incident beam of light by reflection. In so many optical instruments like telescopes, microscopes, camera etc, the incident light undergoes several reflections from many reflecting surfaces.

Such a loss of intensity due to reflection can be reduced to a large extent or even can be eliminated by coating the reflecting surfaces of prisms, lenses etc. with a suitable transparent dielectric material such as calcium fluoride, magnesium fluoride, cryolite etc. The refractive index of such dielectric material must be in between that of air and glass. Such a film is called non-reflecting film or an

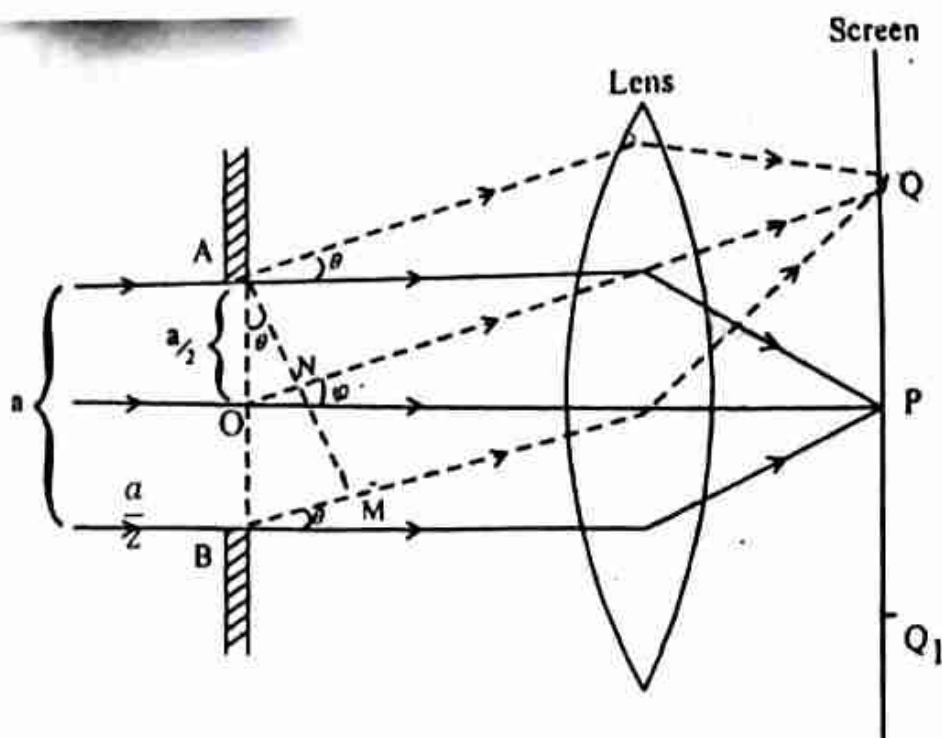
DIFFRACTION

It is the phenomena of bending of light around the corners of an obstacle or encroachment of light into geometrical shadow region. It occurs when the size of the object comparable with wavelength of light used.

There are 2 classes of diffraction depending on the position of source and screen with respect to obstacle.

Fresnel diffraction	Fraunhofer diffraction
Either the source or the screen are at finite distance from the obstacle	Source or the screen are at infinite distance
Wave front is spherical or cylindrical	Wave front is plane
Lenses are not used	Lenses are used to produce parallel rays.
Eg: Diffraction at straight edge	Plane transmission grating.
 <p>The diagram illustrates Fresnel diffraction at a straight edge. On the left, a vertical line represents a 'slit' source. A dashed horizontal line represents the central axis. A curved line labeled 'Spherical wavefront' originates from the slit. This wavefront passes a vertical line labeled 'Straight edge'. To the right of the edge, the wavefront is shown bending around the corner. A vertical line on the far right is labeled 'screen', where the diffracted light is observed.</p>	 <p>The diagram illustrates Fraunhofer diffraction. A 'source' on the left emits light rays that pass through a 'convex lens'. The rays then pass through an 'obstacle' (represented by a vertical bar). After the obstacle, the rays pass through a second 'convex lens' and are focused onto a vertical line on the right labeled 'image'.</p>

Fraunhofer diffraction at single slit



Let AB be a narrow slit of width a . A plane wave front of monochromatic light of wavelength λ is falling normally on the slit. According to Huygen's principle each point on the wave front behaves like secondary waves. Almost all the waves proceeding from the source are travelling parallel to OP and focus at P. These rays are covering equal path and super impose constructively and producing maximum brightness at P. This is called central maxima. Consider another point Q above P where the waves are travelling at an angle θ with OP. The waves producing from A and B of the slit are reach at Q. Then AM is the perpendicular. BM is the path difference between the waves.

From the ΔAMB ,

$$\sin \theta = \frac{BM}{AB}$$

$$BM = AB \sin \theta$$

$$BM = a \sin \theta$$

The point Q will be bright or dark depending on the path difference.

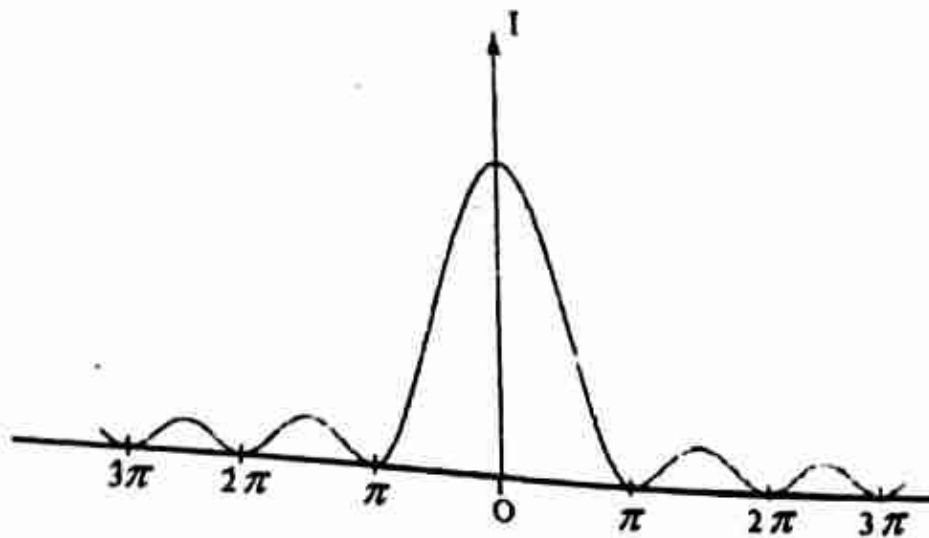
The general condition for minima,

$$a \sin \theta_n = n\lambda$$

The condition for secondary maxima,

$$a \sin \theta_n = (2n + 1) \frac{\lambda}{2}$$

Where θ is the diffracting angle. The intensity distribution for diffraction at a single slit is shown.



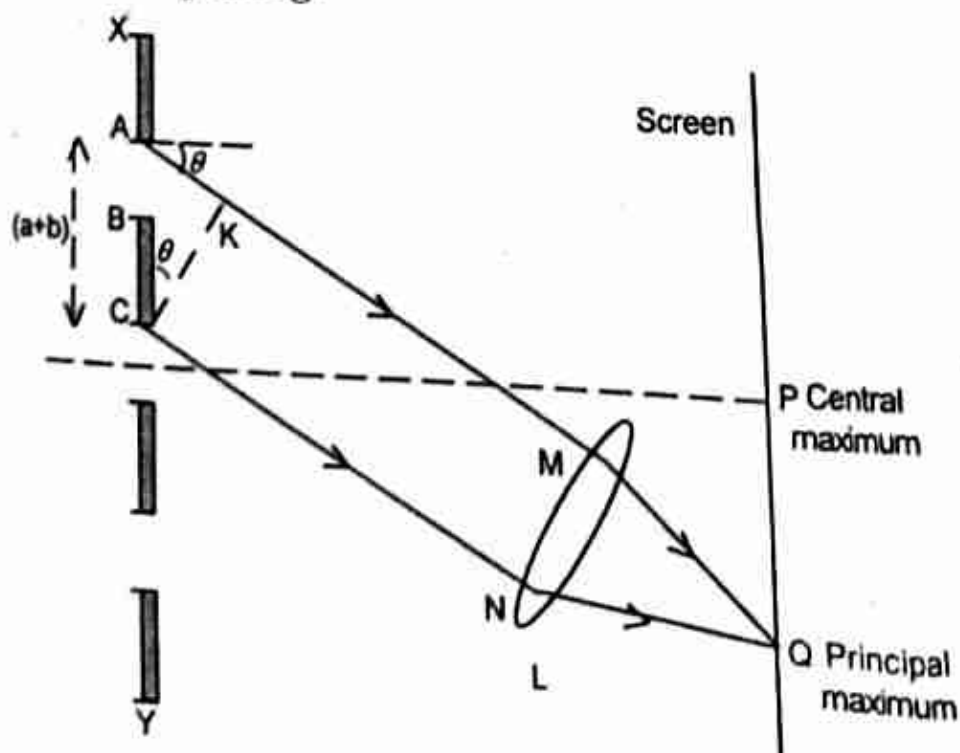
Plane transmission grating

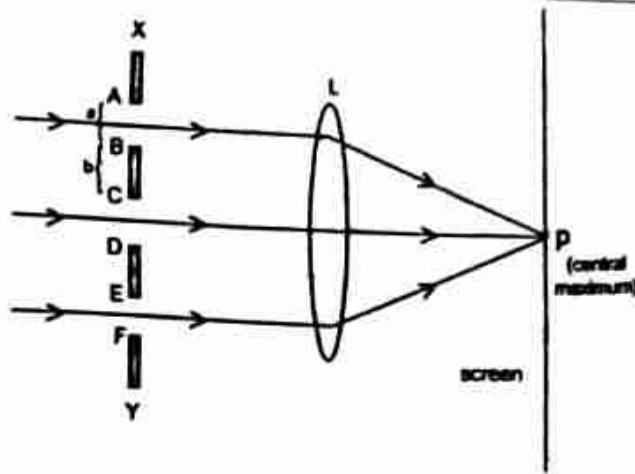
A plane transmission grating is a plane glass plate containing a large number of equidistant parallel lines drawn using fine diamond point. The space between the lines act as a narrow slit through which the light is transmitted. The lines are opaque to light. In the visible region grating contains 5000 lines to 12000 lines per cm.

Conditions for good quality grating.

1. Number of lines per cm should be large.
2. Spacing between the lines should be equal.

Theory of diffraction grating.





XY – represents a plane transmission grating, AB represents a slit width a BC represents a line width b . AC represents $(a+b)$ grating element or grating constant. Let a plane wavefront incident normally on a grating. Most of the waves travel in the direction of incident light and focused at P, and give a line of maximum intensity which has the colour of incident light. It is called central maximum. When the width of the slit is in the order of wavelength λ , light gets diffracted.

Consider two waves diffracted from A and C and θ be the angle of diffraction. Path difference is AK. From the ΔAKC

$$\sin \theta = \frac{AK}{AC}$$

$$\sin \theta = \frac{AK}{a+b}$$

$$AK = (a+b)\sin \theta$$

The path difference equal to $n\lambda$, the 2 waves interfere constructively.

$$\text{ie, } (a+b)\sin \theta = n\lambda \quad \text{---(1)}$$

where $n=0,1,2,3,\dots$

All the waves starting from different corresponding points superimpose each other and give a bright line Q called principle maxima. If $n=1$, called first order maxima or first order spectrum, $n=2$ called 2nd order spectrum. On either side of central maxima, number of principal maxima are obtained. If there are N lines/unit length, Then,

$$(a+b)N = 1 \text{ (unit length)}$$

$$\therefore N = \frac{1}{a+b}$$

$$\text{Eqn-(1)} \Rightarrow \frac{\sin \theta}{N} = n\lambda$$

$$\sin \theta = Nn\lambda$$

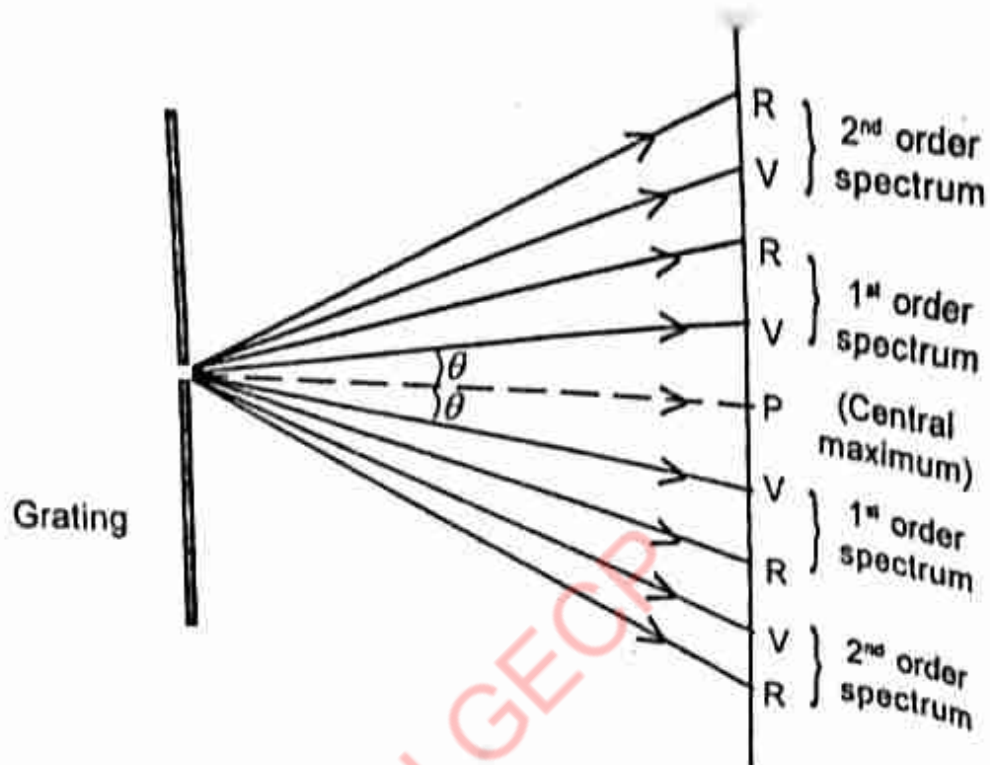
Grating equation or grating law.

Where $\sin \theta = \frac{\lambda}{d} \sin \theta$

N – no. of lines / unit length
 n – order of spectrum.

Grating spectrum with white light.

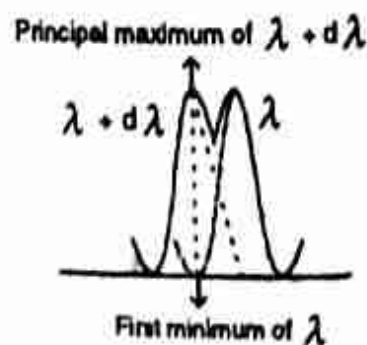
For grating spectrum with white light, θ is different for different colour, according to the equation $\sin \theta = Nn\lambda$



If the white light is used, it splits into different colour. For each value of n different colours are diffracted at different angle.

Rayleigh's criteria for resolution of spectral lines

According to this two neighboring spectral lines will be just resolved when the principle maximum of one falls on minima of the other with the same order.



λ and $\lambda + d\lambda$ be the wave length of 2 neighboring spectral lines with the same order. These two lines will be visible as separate when principle maxima of $\lambda + d\lambda$ falls on a minimum of λ

Resolving power of grating

It is the ability to show two neighbouring spectral lines as separate. It is the ratio of wavelength λ to change in wavelength $d\lambda$

$$\text{Resolving power} = \frac{\lambda}{d\lambda} \text{ where } \lambda = \frac{\lambda_1 + \lambda_2}{2}, d\lambda = \lambda_1 - \lambda_2$$

Suppose, there are N slits in a grating path difference between the waves from the extreme slit change by λ . Therefore the path difference between the waves from adjacent slits change by $\frac{\lambda}{N}$.

According to Rayleigh's criteria two spectral lines will be resolved when principle maximum of $\lambda + d\lambda$ falls on minimum of λ

Consider, maximum condition for $\lambda + d\lambda$

$$(a + b)\sin\theta = n(\lambda + d\lambda)$$

Consider minimum condition for λ

$$(a + b)\sin\theta = n\lambda + \frac{\lambda}{N}$$

$$\therefore n(\lambda + d\lambda) = n\lambda + \frac{\lambda}{N}$$

$$nd\lambda = \frac{\lambda}{N}$$

$$\frac{\lambda}{d\lambda} = nN$$

$$R.P = \frac{\lambda}{d\lambda} = nN$$

Dispersive power of grating

It is the ratio of change in angle of diffraction to change in wavelength.

$$DP = \frac{d\theta}{d\lambda}$$

Consider 2 waves λ and $\lambda + d\lambda$ diffracted through θ and $\theta + d\theta$

$$(a + b)\sin\theta = n\lambda$$

Differentiating on both sides

$$(a + b)\cos\theta d\theta = n\lambda d\lambda$$

$$\frac{d\theta}{d\lambda} = \frac{n}{(a + b)\cos\theta} = \frac{nN}{\cos\theta}$$

$$\left[\because N = \frac{1}{a + b} \right]$$

$$D.P = \frac{d\theta}{d\lambda} = \frac{nN}{\cos\theta}$$

Interference band	Diffraction band
1. Formed by superposition of waves from two coherent sources.	1. Formed by superposition of waves from different part of same wave front
2. Bands may be of equal width	2. Are never equal width
3. Bands of minimum intensity are almost dark	3. Bands of minimum intensity are not dark
4. Intensity of bright band is almost same	4. Not same

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