# സഹായി

**SFI GEC PALAKKAD**

C NSA192004 Pages:4



Reg No.:_______________ Name:__________________________

**APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY**

FIRST SEMESTER B.TECH DEGREE EXAMINATION(2019 SCHEME), DECEMBER 2019

**Course Code: EST100**

**Course Name: ENGINEERING MECHANICS**

**(2019-Scheme)**

Max. Marks: 100 Duration: 3 Hours

## PART A

***(Answer all questions, each carries 3 marks.)***

1 A ladder of weight 30 kg is supported at wall and floor as shown in fig 1 below. A man of weight 72 kg stands on it vertically, 8 m above the floor level. There is a 100 kg force acting at top-most point of the ladder vertically. The mass distribution of the ladder is uniform. Considering all contact surfaces smooth, draw the free body diagram. (3)

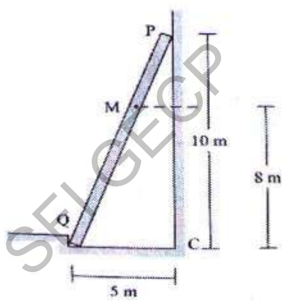



Fig 1

2 State and explain Varignon's theorem for concurrent coplanar forces. (3)

3 Briefly explain the analysis of forces acting on a wedge with a suitable example. (3)

4 A simply supported beam AB of span 4m is carrying point loads 10N, 6N & 4N at 1m, 2m & 3m respectively from support A. Calculate reactions at supports A and B. (3)

5 A force 2i+4j-3k is applied at the point A(1,1,-2). Find the moment of the force about the point (2,-1,2) (3)

6 Calculate the area moment of inertia of a rectangular cross-section of breadth 'b' and depth 'd' about the centroidal horizontal axis. (3)

7 A body is projected at an angle such that its horizontal displacement is 3 times that of maximum height. Find the angle of projection. (3)

8 The position of a particle moving along a straight line is defined by the relation (3)

$x = t^3 - 3t^2 - 9t + 12$

Determine the time taken by the particle when its velocity becomes zero.

9 A flywheel weighing 500N and having radius of gyration 0.4 m loses it speed from 300rpm to 180 rpm in 1 minute. Calculate the torque acting on it. (3)

10 Distinguish damped and undamped free vibrations. (3)

## PART B

*(Answer one full question from each module, each question carries 14 marks)*

### Module-I

11 a) A rope 9m long is connected at A and B, two points on the same level, 8m apart. A load of 300N is suspended from a point C on the rope, 3m from A. What load connected to a point D, on the rope, 2m from B is necessary to keep portion CD parallel to AB. (5)

b) Concurrent forces of 1,3,5,7,9,11 N are applied to the center of a regular hexagon acting towards its vertices as shown in **fig 2**. Determine the magnitude and direction of the resultant. (9)

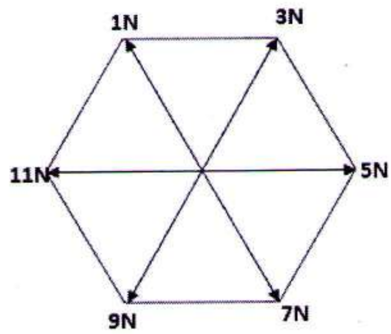



**Fig 2**



12 a) A uniform wheel 60 cm diameter weighing 1000 N rests against a rectangular obstacle 15 cm height as shown in **fig 3**. Find the least force required which when acting through the centre of the wheel will just turn the wheel over the corner of the block. (5)

b) The system of connected flexible cables shown in **Fig.4** is supporting two loads of 550 N and 600 N at points B and D, respectively. Determine the tensions in the various segments of the cable. (9)

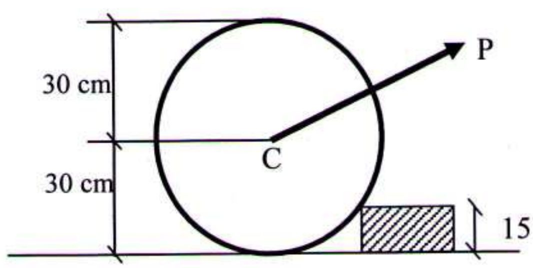



**Fig 3**

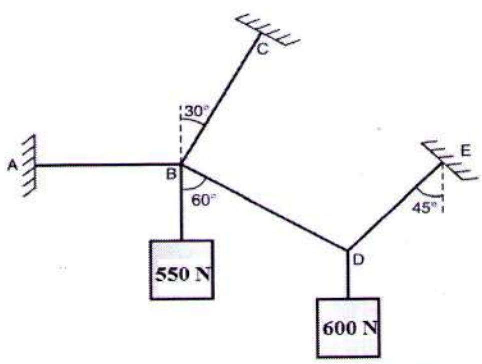



**Fig 4**

### Module-II

13 a) Find the force required to move a load of 30N up a rough inclined plane, applied parallel to the plane. The inclination of the plane is such that when the same body is kept on a perfectly smooth plane inclined at an angle, a force of 6N applied at an (7)

inclination of 30° to the plane keeps the same in equilibrium. Assume coefficient of friction between the rough plane and the load is equal to 0.3.

b) For the beam with loading shown in **Fig.5**, determine the reactions at the supports. (7)

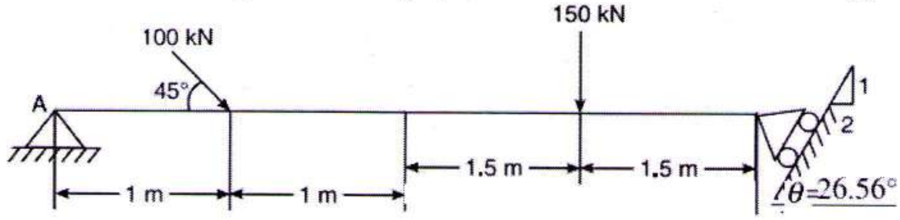
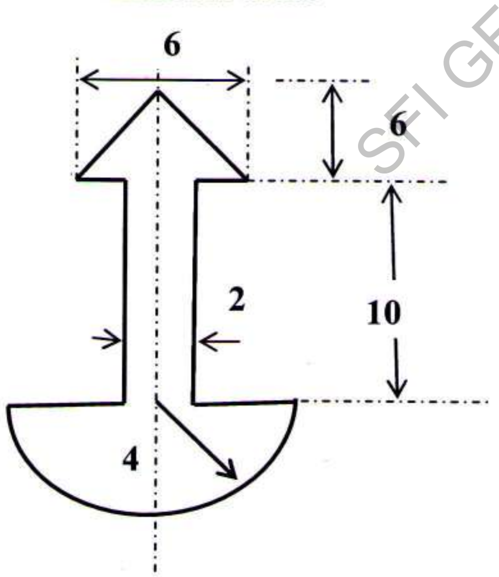



**Fig.5**

14 A uniform ladder 4 m long weighs 200 N. It is placed against a wall making an angle of 60° with the floor. The coefficient of friction between the wall and the ladder is 0.25 and that between the ground and the ladder is 0.35. The ladder in addition to its own weight, has to support a man of 1000 N at the top at B. Calculate: (i) The horizontal force *P* to be applied to the ladder at the ground level to prevent slipping. (ii) If the force *P* is not applied, what should be the minimum inclination of the ladder with the horizontal, so that it does not slip with the man at the top? (14)

## Module-III

15 Find the moment of inertia of shaded area about the horizontal and vertical centroidal axis. All dimensions in cm. (14 )






16 A force P is directed from a point A(4,1,4) meters towards a point B (-3,4,1)metres. Determine the moment of force P about x and y axis if it produces a moment of 1000Nm about z axis. (14)

## Module-IV

17 An object of mass 5 kg is projected with a velocity of 20m/s at an angle of $60^0$ to the horizontal. At the highest point of its path the projectile explodes and breaks up into two fragments of masses 1kg and 4kg. The fragments separate horizontally after explosion. The explosion releases internal energy such that KE of the system at the highest point is doubled. Calculate the separation distance between two fragments when they reach the ground. (14)

18 A block of mass $M_1$ resting on an inclined plane is connected by a string and pulleys to another block of mass $M_2$ as shown in Fig.7. Find the tension in the string and acceleration of the blocks. Assume the coefficient of friction between the blocks $M_1$ and the plane to be 0.2. $M_1$ =1500N, $M_2$ = 1000N. Angle of inclined plane = $45^0$. (14)

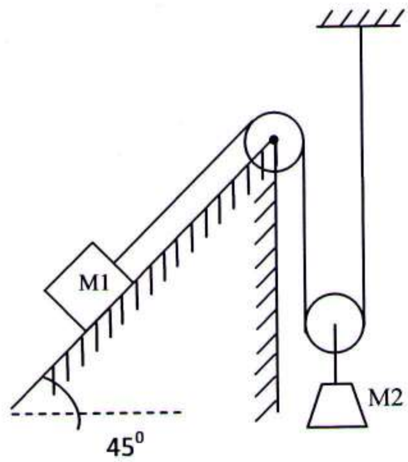



Fig 7

## Module-V

19 A rotor of an electric motor is uniformly accelerated to a speed of 1800 rpm from rest for 5 seconds and then immediately power is switched off and the motor decelerates uniformly. If the total time elapsed from start to stop is 12.5 sec, determine the number of revolutions made while (a) acceleration (b) deceleration. Also find the value of deceleration. (14)

20 a A spring stretches by 0.015m when a 1.75kg object is suspended from its end. How much mass should be attached to the spring so that its frequency of vibration is 3 Hz. (5)

b A particle moving with simple harmonic motion has velocities 8m/s and 4m/s when at the distance of 1m and 2m from the mean position. Determmine (a) amplitude (b) period (c) maximum velocity, and (d) maximum acceleration of the particle. (9)

****

# APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

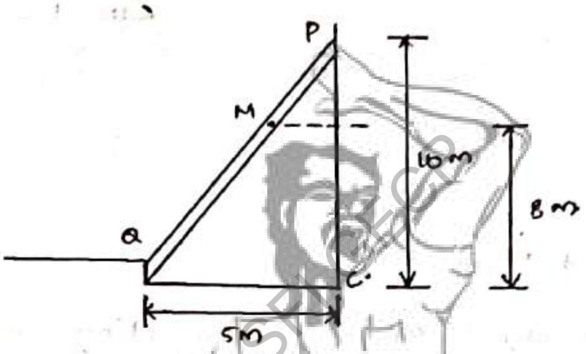
## FIRST SEMESTER BTECH DEGREE EXAMINATION (2019 SCHEME)

## EST100

## ENGINEERING MECHANICS

### PART - A

1. A ladder of weight 30 kg is supported at wall and floor as shown in fig. 1 below. A man of weight 72 kg stands on it vertically, 8 m above the floor level. There is a 100kg force acting at top-most point of the ladder vertically. The mass distribution of the ladder is uniform. Considering all contact surfaces smooth, draw the free body diagram.






Ans: Given data

$W = 30\,kg = 30 \times 9.81 = \underline{294.3\,N}$

$W_{man} = 72\,kg = 72 \times 9.81 = \underline{706.32\,N}$

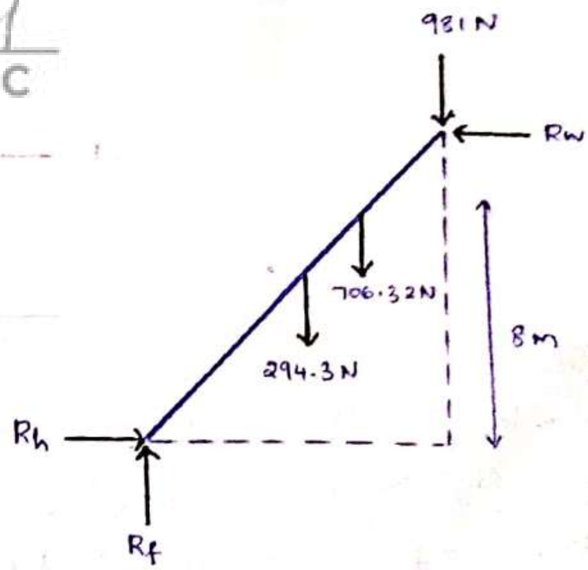
Force at ladder top acting vertically

$= 100\,kg$

$= 100 \times 9.81$

$= \underline{981\,N}$

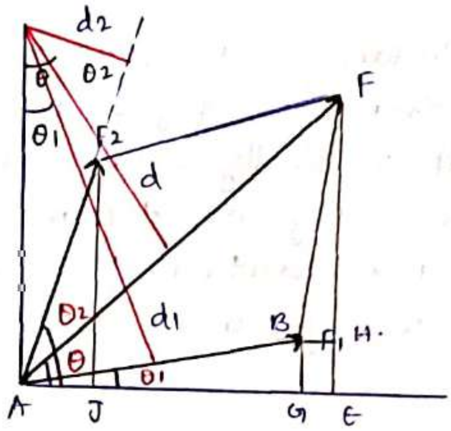
Smooth surface $\Rightarrow$ No friction.



2. State and explain Varignon's theorem for concurrent coplanar forces.

ans. Statement

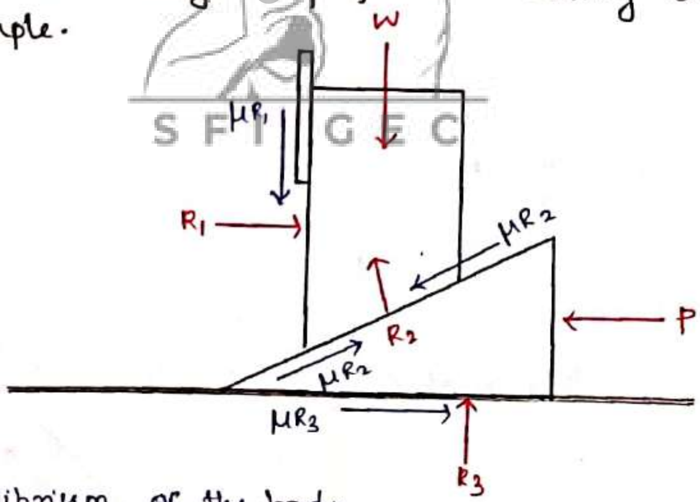
Varignon's theorem states that the moment of a force about any axis is equal to the sum of moments of its components about that axis.




- Consider a force acting at a point A.
- $F_1$ and $F_2$ are the components of F along any two directions.
- The moment of force F about any arbitary point O is $F \times d$. where d is the arm of force F.
- $d_1$ and $d_2$ are arm of forces of forces $F_1$ and $F_2$ respectively.
- Sum of moments of the components $F_1$ and $F_2$ about O is $F_1 d_1 + F_2 d_2$
- Then, $F \times d = F_1 d_1 + F_2 d_2$.

3. Briefly explain the analysis of forces acting on a wedge with a suitable example.

ans.

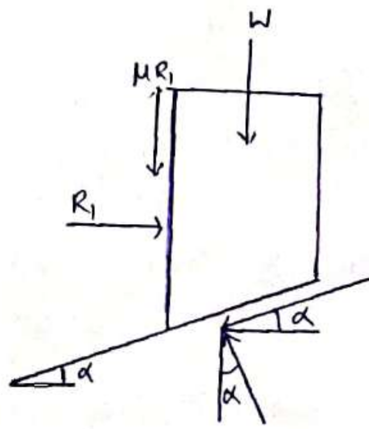





Consider the equillibrium of the body

Resolving the forces vertically,

$$R_2 \cos\alpha - \mu_2 R_2 \sin\alpha - \mu_1 R_1 - W = 0$$




Resolving forces horizontally

$$R_1 = \mu_2 R_2 \cos\alpha - R_2 \sin\alpha = 0.$$

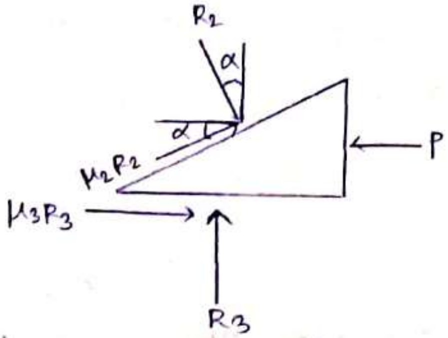
For equillibrium of the wedge

Resolving forces horizontally,

$$\mu_3 R_3 + \mu_2 R_2 \cos\alpha + R_2 \sin\alpha - P = 0$$

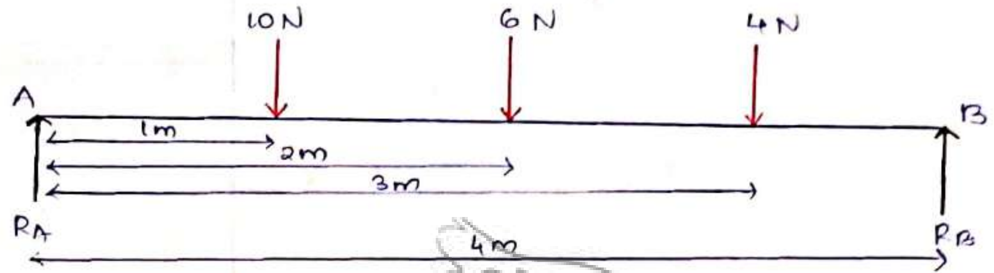
Resolving forces vertically,

$$R_3 + \mu_2 R_2 \sin\alpha - R_2 \cos\alpha = 0.$$




4. A simply supported beam AB of span 4m is carrying point loads 10N, 6N, and 4N at 1m, 2m and 3m respectively from support A. calculate reactions at supports A and B.

ans:




Consider the free body diagram of the beam

Equillibrium conditions.

$\Sigma F_v = 0$ and $\Sigma M = 0$

Applying $\Sigma F_v = 0$,

$$R_A + R_B - 10 - 6 - 4 = 0 \Rightarrow R_A + R_B = 20 \text{ —— (i)}$$

Applying $\Sigma M = 0$,

Taking moment about $A = 0$,

$$(10 \times 1) + (6 \times 2) + (4 \times 3) - (R_B \times 4) = 0$$

$$R_B = 34/4 = \underline{8.5 \text{ N}} \text{ —— (ii)}$$

$$\therefore R_A = 20 - 8.5 = 11.5 \text{ N} //$$

5. A force $2i + 4j - 3k$ is applied at the point A $(1, 1, -2)$. Find the moment of the force about the point $(2, -1, 2)$.

ans: Given

- Force $= \vec{F} = 2i + 4j - 3k$.
- Application point of force $= A(1, 1, -2)$
- Moment centre $= O(2, -1, 2)$.

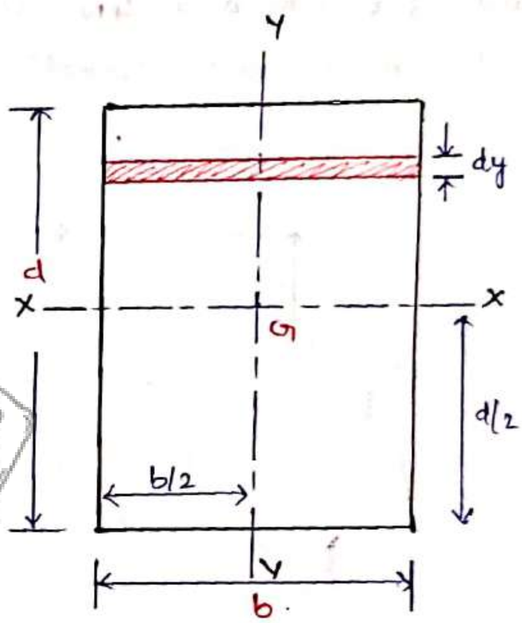
$\vec{r} = \vec{OA} = (1-2)\hat{i} + (1-{-1})\hat{j} + (-2-2)k = -\hat{i} + 2\hat{j} - 4k$

Moment of force

$$\vec{M} = \vec{r} \times \vec{F} = \begin{vmatrix} i & j & k \\ -1 & 2 & -4 \\ 2 & 4 & -3 \end{vmatrix} = 10\hat{i} - 11\hat{j} - 8k$$

6. Calculate the area moment of inertia of a rectangular cross section of breadth 'b' and depth 'd' about the centroidal horizontal axis.

ans: Moment of inertia about $x-x$ axis of a small horizontal strip




$$I_{xx} = \text{Area} \cdot y^2 = dA \cdot y^2 \quad \text{---} (1).$$

Moment of inertia of rectangle about $x-x$ axis will be

$$I_{xx} = \int dA \cdot y^2 \quad \text{---} (2)$$

Since $dA = b \cdot dy$ and the limits are $-d/2 \rightarrow d/2$,

$$(2) \Rightarrow \quad I_{xx} = \int_{-d/2}^{d/2} (b \cdot dy)\, y^2$$

$$I_{xx} = b \int_{-d/2}^{d/2} y^2\, dy.$$

$$I_{xx} = b \cdot \left[\frac{y^3}{3}\right]_{-d/2}^{d/2}$$

$$I_{xx} = b\left[\frac{d^3}{8 \times 3} + \frac{d^3}{8 \times 3}\right]$$

$$I_{xx} = \frac{2bd^3}{24}$$

$$\Rightarrow \quad I_{xx} = \frac{bd^3}{12} \quad //.$$

7. A body is projected at an angle such that its horizontal displacement is 3 times that of maximum height. Find the angle of projection.

an: <u>Given</u>

Range (R) = 3 × maximum height ($h_{max}$).

Since, $R = \dfrac{u^2 \sin 2\alpha}{g}$ $\qquad h_{max} = \dfrac{u^2 \sin^2\alpha}{2g}$

ie, $\dfrac{u^2 \sin 2\alpha}{g} = 3\,\dfrac{u^2 \sin^2\alpha}{2g}$

$\Rightarrow\ 2\sin\alpha\cos\alpha = \dfrac{3\sin^2\alpha}{2} = \tan\alpha = \dfrac{4}{3}$ //

$\alpha = \tan^{-1}(4/3)$

$\alpha = 53.13°$ //

8. The position of a particle moving along a straight line is defined by the relation

$x = t^3 - 3t^2 - 9t + 12$

Determine the time taken by the particle when its velocity becomes zero.

an: <u>Given</u>

$x = \cancel{8}t^3 - 3t^2 + 9t + 12$

Differentiating w.r.t time t

$v = \dfrac{dx}{dt} = 3t^2 - 6t - 9$

when velocity, $v = 0$, ie, $3t^2 - 6t - 9 = 0$.

Solving, $t = 3$, $t = -1$.

Since time cannot be a negative quantity.

∴ Time taken by the particle when its velocity becomes zero = 3s //

9. A flywheel weighing 500N and having radius of gyration 0.4m loses it speed from 300 rpm to 180 rpm in 1 minute. Calculate the torque acting on it.

a. Given

Weight of flywheel $W = 500\,N$

Initial speed $N_0 = 300$ rpm.

Final speed $N = 180$ rpm

Time $t = 1\,min = 60s$

Radius of gyration $k = 0.4\,m$

Torque, $T = I\alpha = mk^2\alpha$

But $\Rightarrow \omega = \omega_0 + \alpha t$.

$\omega_0 = 2\pi N_0/60 = \frac{2\pi \times 300}{60} = 31.42$ rad/s //

$\omega = 2\pi N/60 = \frac{2\pi \times 180}{60} = 18.85$ rad/s //

Substituting, $18.85 = 31.42 + \alpha \times 60$

$\Rightarrow \alpha = 0.21$ rad/s$^2$ //

$\therefore T = I\alpha = mk^2\alpha = \{500/9.81\}(0.4)^2 \times -0.21$

$T = -1.708$ Nm



10. Distinguish damped and undamped free vibrations.

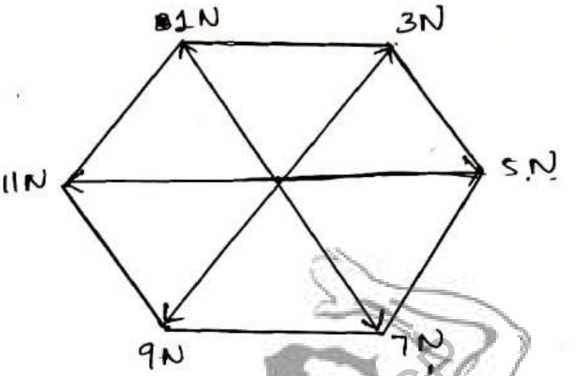
ans:

| DAMPED VIBRATION | UNDAMPED VIBRATION |
|---|---|
| • Object oscillate experiences resistive forces. | • The object oscillates freely without acting any resistive force acting against its motion. |
| • The total energy of the oscillating object decreases over time. | • The sum of kinetic and potential energies and potential energies always gives the total energy of the oscillating object, and value of the total energy does not change. |
| • Vibrating object loses its energy to the surroundings. | • Energy of the vibrating object does not get dissipated to surroundings. |

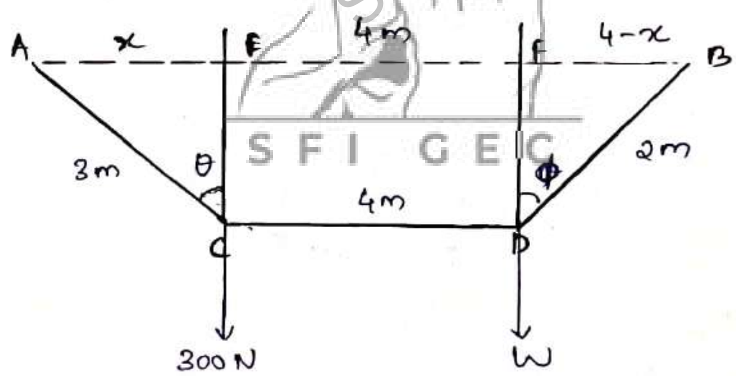
## PART-B

11. a) A rope 9m long is connected at A and B, two points on the same level, 8m apart. A load of 300 N is suspended from a point C on the rope, 3m from A. What load connected to a point D, on the rope, 2m from B is necessary to keep portion CD parallel to AB.

b) Concurrent forces of 1, 3, 5, 7, 9, 11 are applied to the center of a regular hexagon acting towards its vertices as shown in fig 2. Determine the magnitude and direction of the resultant.




an: a)






From the $\Delta ACE$,

$$y^2 = 3^2 - x^2.$$

From the $\Delta BDF$,

$$y^2 = 2^2 - (4-x)^2$$

$$3^2 - x^2 = 2^2 - (4-x)^2$$

$$9 - x^2 = 4 - 16 + 8x - x^2$$

$$8x = 21 \Rightarrow x = 2.625 \text{ m}$$

$\sin\theta = x/3 \Rightarrow \frac{2.625}{3} = 0.875 \Rightarrow \theta = \sin^{-1}(0.875)$

$\therefore \theta = 61.04°$ //

$\sin\phi = \frac{4-x}{2} = \frac{4-2.625}{2} = 0.6875 \Rightarrow, \phi = \sin^{-1}(0.6875)$

$\therefore \phi = 43.43°$ //

consider the equillibrium point C,

Resolving vertically,

$T_{AC}\cos\theta - 300 = 0$

$T_{AC} = \frac{300}{\cos 61.04} = \underline{\underline{619.58N}}$

Resolving horizontally,

$T_{CD} - T_{AC}\sin\theta = 0$

$T_{CD} = T_{AC}\sin\theta = 619.58 \sin 61.04$

$T_{CD} = 542.11N$ //

consider the equillibrium point D,

Resolving horizontally,

$T_{DB}\sin\phi - T_{CD} = 0$



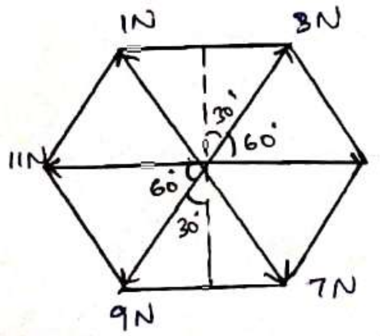
$T_{DB} = \frac{542.11}{\sin 43.43} = 788.56N$ //

Resolving vertically,

$T_{DB}\cos\phi - W = 0$

$W = T_{DB}\cos\phi = 788.56 \cos 43.43$

$W = 572.66N$ //




Resultant force

$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$

Direction

$\theta = \tan^{-1}\left|\frac{\Sigma F_y}{\Sigma F_x}\right|$

$\Sigma F_x = 5 + 3\cos 60 - 1\cos 60 - 11 - 9\cos 60 + 7\cos 60$

$\Sigma F_x = -6N$ //

$\Sigma F_y = 3\sin 60 + 1\sin 60 - 9\sin 60 - 7\sin 60$

$\Sigma F_y = -10.39\,N$ //

$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2} = \sqrt{(-6)^2 + (-10.39)^2}$

$R = 12\,N$ //

Direction of R

$\theta = \tan^{-1}\left|\frac{\Sigma F_y}{\Sigma F_x}\right| = \tan^{-1}\left(\frac{10.39}{6}\right)$

$\theta = 60°$ //

12 a) A uniform wheel 60 cm diameter weighing 1000 N rests against a rectangular obstacle 15 cm height as shown in fig 3. Find the least force required which when acting through the centre of the wheel will just turn the wheel over the corner of the block.

b) The system of connected flexible cables shown in fig 4 is supporting two loads of 550 N and 600 N at points B and D. Determine the tension in the various segments of the cable.



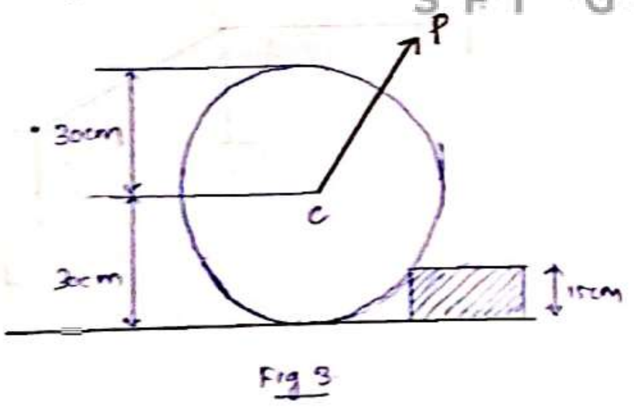
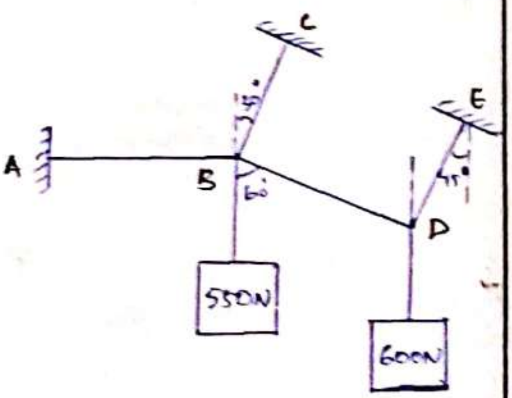


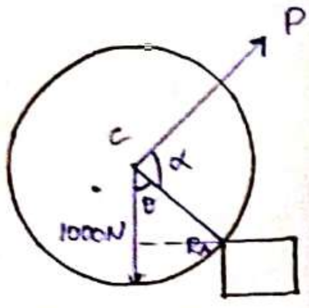

Fig 3




ans a) When the roller is turned about A, contact at B breaks and hence no reaction at B.

Let $R_A$ be the reaction at contact point A.

Inclination of force P with reaction $R_A$ be $\alpha$.



$\cos\theta = 15/30 \Rightarrow \theta = \cos^{-1}(1/2) \Rightarrow \theta = 60°$ //

Turning about point A, Forces = P, W, $R_A$

Taking moment about A = 0

$$P\sin\alpha \times AC - (1000 \times AC \sin 60°) = 0.$$

$$P\sin\alpha \, AC = 1000 \, AC \sin 60°$$

$$P\sin\alpha = 1000 \sin 60°$$

$$P = \frac{866}{\sin\alpha}$$

Least force means, P to be minimum, $\sin\alpha$ should be maximum,

$$\therefore P = 866N \text{ //}$$

To find $R_A$,

Resolve forces along AC,

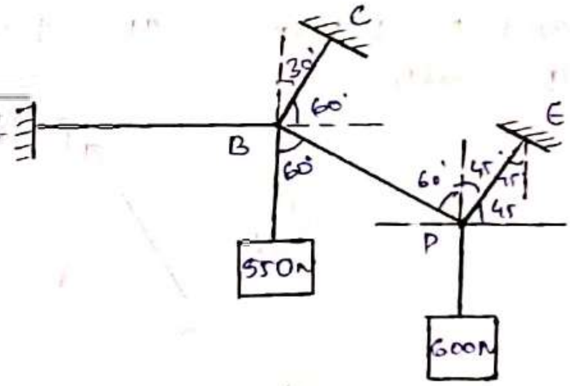
$$R_A - P\cos\alpha - 1000\cos\theta = 0$$

$$R_A = 866\cos 90 + 1000\cos 60$$

$$\therefore R_A = 500N \text{ //}$$

b) Consider equillibrium of D

Applying Lami's theorem




$$\frac{600}{\sin(60+45)} = \frac{T_{DE}}{\sin(90+30)} = \frac{T_{DB}}{\sin(90+45)}$$

$$\therefore T_{DE} = \frac{600}{\sin(60+45°)} \times \sin 120°$$

$$= \underline{537.94 \text{ N}}$$

$$\therefore T_{DB} = \frac{600}{\sin(60+45)} \times \sin 135°$$

$$= \underline{439.23 \text{ N}}$$

Consider equillibrium of B.

Resolving forces vertically,

$$T_{BC}\cos 30 - 550 - T_{BD}\cos 60 = 0$$

$$T_{BC} = \frac{(550 + 439.23\cos 60)}{\cos 30} = 888.675\text{ N}$$

Resolving forces horizontally,

$$T_{BD}\cos 30 + T_{BC}\cos 60 - T_{BA} = 0$$

$$\therefore T_{BA} = T_{BD}\cos 30 + T_{BC}\cos 60^\circ$$

$$= 439.23\cos 30 + 888.675\cos 60^\circ$$

$$T_{BA} = 824.72\text{ N}$$

## Module - II

13. a) Find the force required to move a load of 30 N up a rough inclined plane, applied parallel to the plane. The inclination of the plane is such that when the same body is kept on a perfectly smooth plane inclined at an angle, a force of 6 N applied at an inclination of 30° to the plane keeps the same in equillibrium. Assume the co-efficient of friction between the rough plane and the load is equal to 0.3.

b) For the beam with loading shown in fig 5. Determine the reactions at the supports.

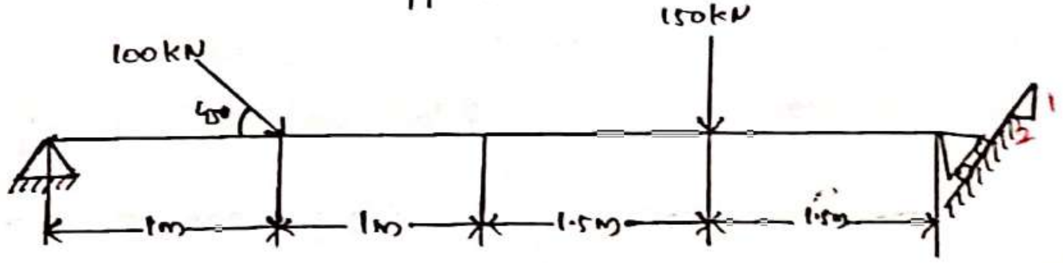
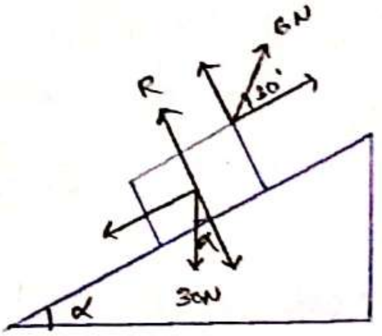



Fig 5

ans: Case : 1



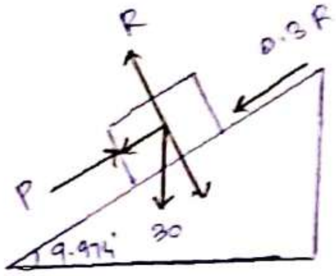

consider the body of weight 30 N, placed on a smooth inclined plane with an angle α and load 6N.

Resolving forces along the inclined plane,

$$6\cos 30 - 30\sin\alpha = 0$$

$$\therefore \alpha = \sin^{-1}\left(\frac{6\cos 30}{30}\right)$$

$$\underline{\alpha = 9.974^\circ}$$




Case : 2

Consider the body of weight 30N placed on a rough inclined plane with an inclination $\alpha = 9.974^\circ$

Resolving perpendicular to the inclined plane

$$R - 30\cos 9.974 = 0$$

$$R = 29.54\,N$$

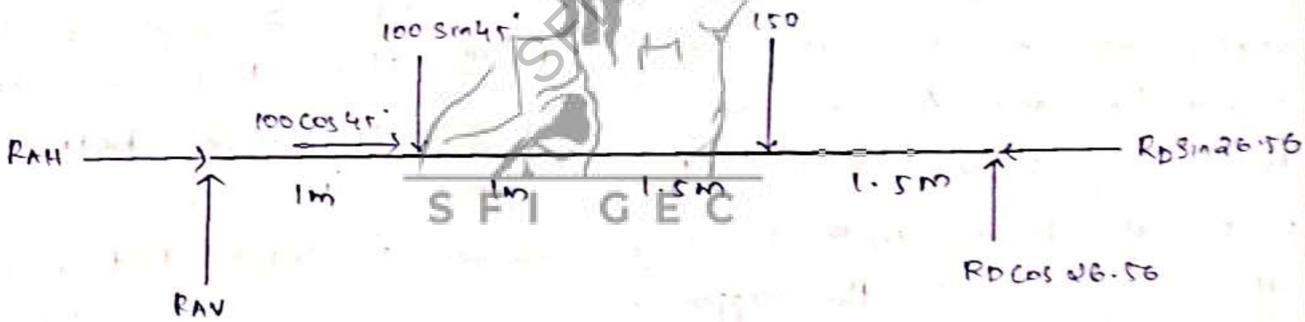
Resolving along inclined plane,

$$P - 0.3R - 30\sin 9.974 = 0$$

$$P = (0.3 \times 29.54) + 30\sin 9.974$$

$$P = 14.06\,N$$

b)






consider the free body diagram of the beam.

Equillibrium conditions : $\Sigma F_V = 0$, $\Sigma F_H = 0$, $\Sigma M = 0$.

$\Sigma F_V = 0 \Rightarrow$ $R_{AV} + R_D\cos 26.56 - 100\sin 45 - 150 = 0$ —— (1).

$\Sigma F_H = 0 \Rightarrow$ $R_{AH} + 100\cos 45 - R_D\sin 26.56 = 0$ —— (2)

$\Sigma M = 0 \Rightarrow$ Taking moment about A,

$$(100\sin 45 \times 1) + (150 \times 3.5) - (R_D\cos 26.56 \times 5) = 0$$

$$\therefore R_D = 133.19\,kN$$

substituting value of $R_D$ in eqn (1),

$$R_{AH} = -100\cos 45 + R_D\sin 26.56$$

$\therefore R_{AH} = -11.15\ kN$ //

Subtituting value of $R_D$ in eqn (2),

$$R_{AV} = -R_D \cos 26.56 + 100 \sin 45 + 150$$

$$\therefore R_{AV} = 101.57\ kN \ //$$

Therefore Resultant Reaction at A,

$$= \sqrt{\Sigma R_{AH}^2 + \Sigma R_{AV}^2} = \sqrt{11.15^2 + 101.57^2}$$

$$= \underline{\underline{102.18\ kN}}$$

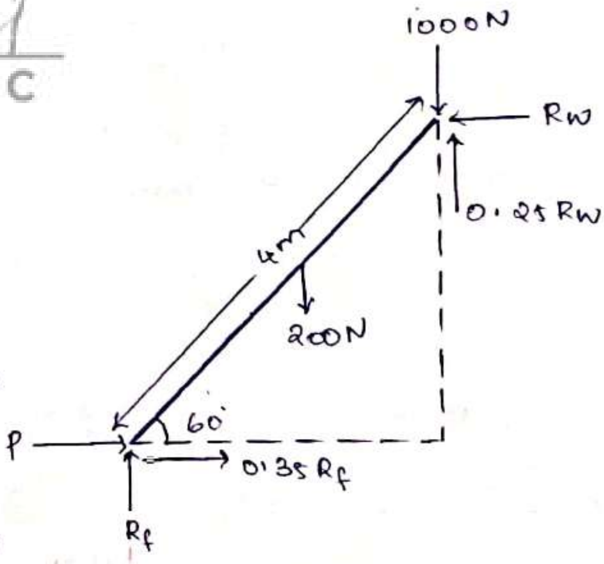
14. A uniform ladder 4m long weighs 200N. It is placed against a wall making an angle of 60° with the floor. The co-efficient of friction between the wall and the ladder is 0.25 and that between the ground and the ladder is 0.35. The ladder in addition to its own weight, has to support a man of 1000N at the top at B. calculate

i) The horizontal force P to be applied to the ladder at the ground level to prevent slipping.

(ii) If the force P is not applied what should be the minimum inclination of the ladder with the horizontal, so that it does not slip with the man at the top.



ans: Given data




Length of ladder = 4m

Weight of the ladder $W = 200N$

$\mu_w = 0.25$

$\mu_f = 0.35$

Inclination of ladder with floor = 60

Load at top = 1000N.

(i) consider the equillibrium of ladder

$\Sigma F_x = 0$

$$P + 0.35 R_f - R_w = 0 \quad \text{——— (1)}$$

$\Sigma F_y = 0$

$$R_f + 0.25 R_w - 200 - 1000 = 0$$

$$R_f + 0.25 R_w = 1200 \quad \text{——— (2)}$$

$\Sigma M = 0$, Taking Moment about point A.

$$(200 \times 2\cos 60) + (1000 \times 4\cos 60) - (0.25 R_w \times 4\cos 60) - (R_w \times 4\sin 60) = 0$$

$$200 + 2000 - 0.5 R_w - 3.464 R_w = 0$$

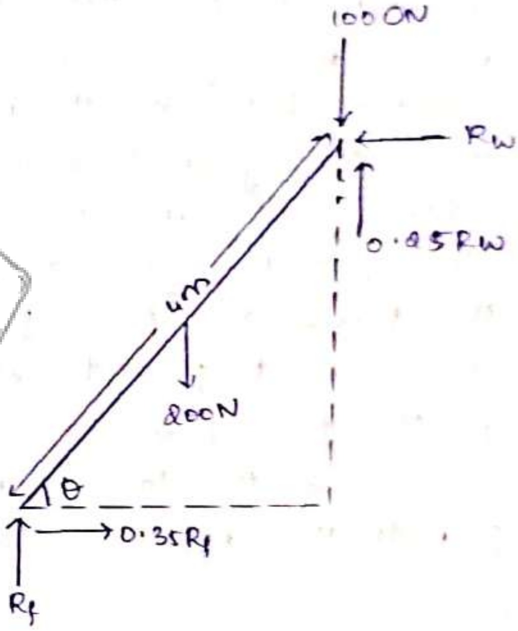
$$(0.5 + 3.464) R_w = 2200$$

$$\therefore R_w = 554.98\ N$$

From (2) $R_f = 1200 - 0.25 R_w = 1061.26 N$

(1) $P = R_w - 0.35 R_f = 183.5 N$

(ii) Consider the equillibrium of ladder




$\Sigma F_x = 0$

$0.35 R_f - R_w = 0$

$R_w = 0.35 R_f$ ——— (1)

$\Sigma F_y = 0$

$R_f + 0.25 R_w - 200 - 1000 = 0$

$R_f + 0.25 R_w = 1200$

$R_f + 0.25 \times 0.35 R_f = 1200$

$\therefore R_f = 1103.45 N$

$R_w = 0.35 \times 1103.45 = 386.2 N$

$\Sigma M = 0$, Taking Moment about A

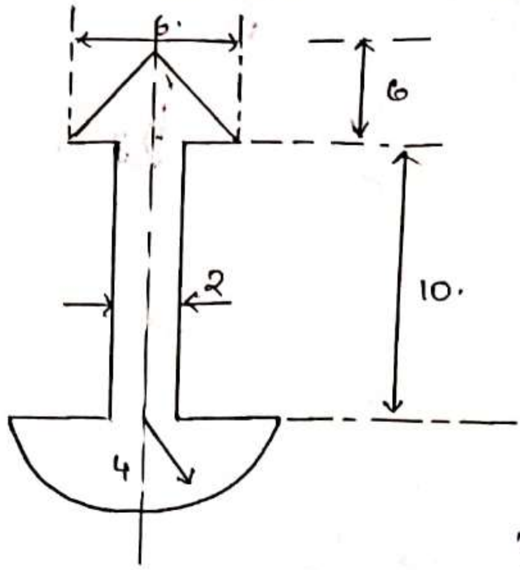
$$(200 \times 2\cos\theta) + (1000 \times 4\cos\theta) - (0.25 R_w \times 4\cos\theta) - (R_w \times 4\sin\theta) = 0$$

$$\cos\theta(400 \times 4000 - 0.25 \times 4 \times 386.2) = 4 \times 386.2 \sin\theta$$

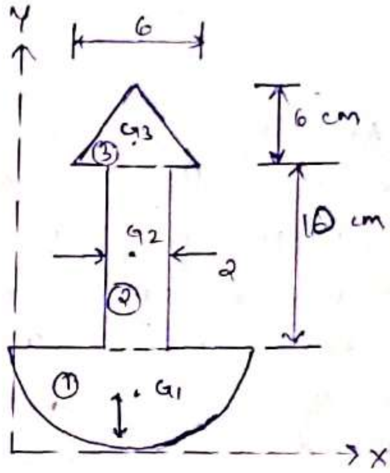
$$\therefore \theta = 68.95°$$

## Module - III

15\. Find the Moment of inertia of shaded area about the horizontal and vertical centroidal axis. All dimensions are in cm.




ans: Centroid




$$\bar{x} = \frac{a_1x_1 + a_2x_2 + a_3x_3}{a_1 + a_2 + a_3}$$

$$\bar{x} = \frac{\frac{1}{2}\times\pi\times4^2\times4 + 2\times10\times4 + \frac{1}{2}\times6\times6\times4}{\frac{1}{2}\pi\times4^2 + 2\times10 + \frac{1}{2}\times6\times6}$$

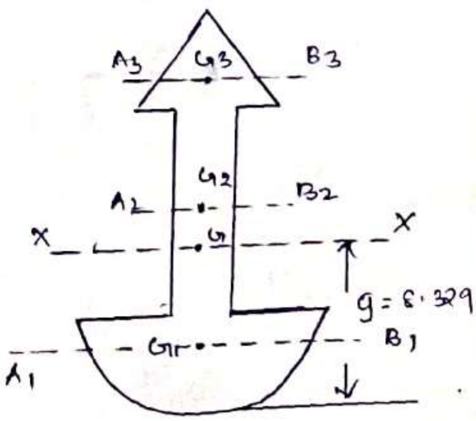
$\bar{x} = 4\,cm$ //

$$\bar{y} = \frac{a_1y_1 + a_2y_2 + a_3y_3}{a_1 + a_2 + a_3}$$

$$\bar{y} = \frac{\frac{1}{2}\pi\times4^2\times\left(4 - \frac{4\times4}{3\pi}\right) + 2\times10\times9 + \frac{1}{2}\times6\times6\times16}{\frac{1}{2}\pi4^2 + 2\times10 + \frac{1}{2}\times6\times6}$$

$\bar{y} = 8.329\,cm$ //






$$I_{Gxx} = I_{xx1} + I_{xx2} + I_{xx3}$$

$$I_{xx1} = I_{A_1B_1} + Ah^2$$

$$= 0.11R^4 + \frac{1}{2}\pi R^2\left(8.329 - \left(4 - \frac{4\times4}{3\pi}\right)\right)^2$$

$$I_{xx2} = \frac{2\times10^3}{12} + 2\times10\times(9 - 8.329)^2$$

$$I_{xx3} = \frac{6\times6^3}{36} + \frac{1}{2}\times6\times6\,(16 - 8.329)^2$$

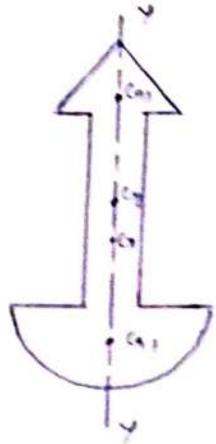
$I_{Gxx} = 2185.56\ cm^4$ //

$I_{Gyy} = I_{yy1} + I_{yy2} + I_{yy3}$

$I_{yy1} = \frac{\pi R^4}{8} = \frac{\pi \times 4^4}{8}$

$I_{yy2} = \frac{10 \times 2^3}{12}$

$I_{yy3} = \frac{6 \times 6^3}{12}$

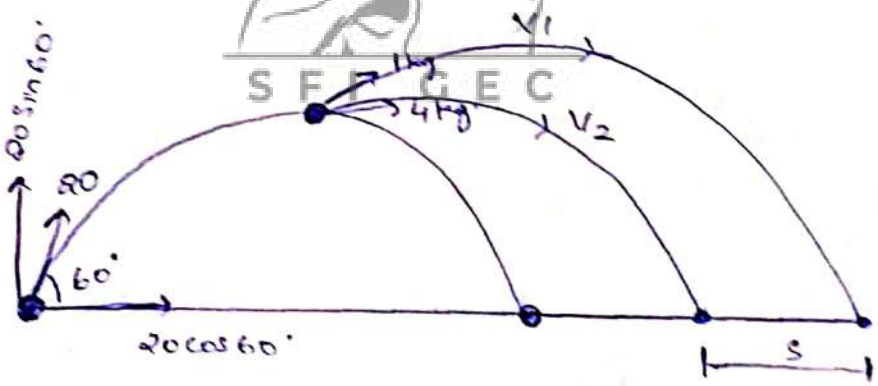


$I_{Gyy} = \underline{110.1667 cm^4}$

No $Ah^2$ term.

## Module - IV

17. An object of mass 5 kg is projected with a velocity of 20 m/s at an angle of 60° to the horizontal. At the highest point of its path the projectile explodes and breaks up into two fragments of masses 1 kg and 4 kg. The fragments separates horizontally after explosion. The explosion releases internal energy such that KE of the system at the highest point is doubled. Calculate the separation distance between two fragments when they reach the ground.




$V_x = 20\cos 60° = 10 m/s$ //     $V_y = 20\sin 60 = 10\sqrt{3} m/s$ //

$T = \frac{2u\sin\alpha}{g} = \frac{2 \times 20\sin 60}{9.81}$

$\therefore T = 3.53 s$ //

Time to reach maximum height $= \frac{3.53}{2} = 1.765 s$ //

Maximum height, $H = \frac{u^2 \sin^2 \alpha}{2g} = \frac{20^2 \sin^2 60}{2 \times 9.81}$

$H = 15.29$ m //

Initial kinetic energy of the object, $KE_1 = \frac{1}{2} m V_x^2$

$= \frac{1}{2} \times 5 \times (10)^2$

$= 250$ J //

Kinetic energy at maximum height, $KE_2 = 2 \times KE_1$

$= 2 \times 250$

$= 500$ J //

Since at the highest point, the object separates into two fragments each of mass $m_1$ and velocity $V_1$, and mass of $m_2$ and velocity $V_2$.

$\therefore \quad 500 = \frac{1}{2} m_1 V_1^2 + \frac{1}{2} m_2 V_2^2$

Since $m_1 = 1$ kg $\quad m_2 = 4$ kg

ie, $500 = \frac{1}{2} \times 1 \times V_1^2 + \frac{1}{2} \times 4 \times V_2^2$

ie, $V_1^2 + 4V_2^2 = 1000$ —— (1)



Applying conservation of momentum along x-axis,

$m_1 V_1 + m_2 V_2 = M V_x$

$1 \times V_1 + 4 \times V_2 = 5 \times 10$

$V_1 + 4V_2 = 50$ —— (2)

Solving (1) and (2) we get

$V_1 = 30$ m/s and $V_2 = 5$ m/s //

Distance travelled by the first fragment $= V_1 \times t = 30 \times 1.765$

$= 52.95$ m //

Distance travelled by the second fragment $= V_2 \times t = 5 \times 1.765$

$= 8.825$ m //

Separation of fragments $= 52.95 - 8.825 = 44.125$ m //

16

P from A (4, 1, 4) to B (-3, 4, 1)

$M_z = 1000$ Nm.

$AB = -7\hat{i} + 3\hat{j} - 3\hat{k}$

$\Rightarrow \quad P = \frac{|P|}{\sqrt{67}} \times -7\hat{i} + 3\hat{j} - 3k$

$M_z = xF_y - yF_x = 1000$ Nm.

$= 4 \times \frac{|P| \times 3}{\sqrt{67}} - 1 \times \frac{-7 \times |P|}{\sqrt{67}} = 1000$ Nm

$= \frac{19\,|P|}{\sqrt{67}} = 1000$ Nm $\Rightarrow |P| = \underline{\underline{430.808 \text{ N}}}$

$M_x = yF_z - zF_y$

$= \frac{1 \times |P| \times -3}{\sqrt{67}} - \frac{4 \times |P| \times 3}{\sqrt{67}}$

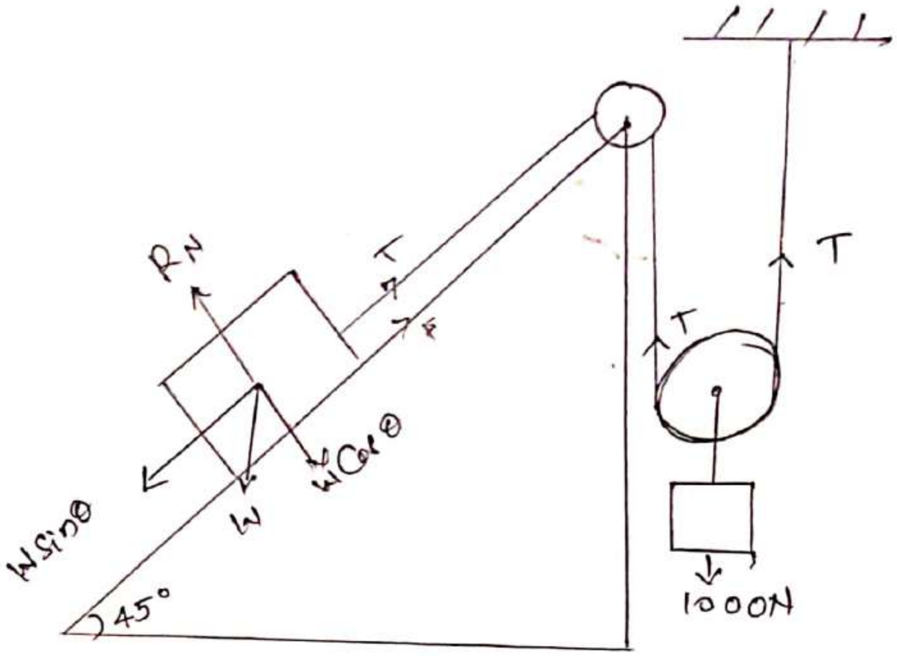
$= \underline{\underline{-789.473 \text{ Nm}}}$

$M_y = -xF_z + zF_x$

$= -4 \times \frac{|P| \times -3}{\sqrt{67}} + \frac{4 \times |P| \times -7}{\sqrt{67}}$

$= \underline{\underline{-842.105 \text{ Nm}}}$

(18)




Force on $M_1$,

$$1000 - 2T = \frac{1000}{g} \times a \rightarrow (1)$$

Consider $M_2$,

$$T - 1500 \sin 45 - 0.2 \times 1500 \cos 1500 = \frac{1500}{g} \times 2a \rightarrow (2)$$

By solving (1) & (2)

$$T - 848.5281 = \frac{3000}{g} \times a$$

$$a = -0.97687 \ m/s^2$$ ('−' indicates accl. is opposite direction.)

Acceleration of $M_1 = 1.95374 \ m/s^2$

Acceleration of $M_2 = 0.97687 \ m/s^2$.

From eq: (1),

$$T = 500 - \frac{500}{9.8} \times 0.97687$$

$$T = 549.7895 \ N$$

Scanned with CamScanner

19 a) $n = 1800$ rpm

$= \frac{1800}{60}$ rps

$n = 30$ rps

then $\omega = 2\pi n$

$= 2\pi \times 30$

$= 60\pi$ rad/s

Initially, the rotor accelerates for $t = 5s$

then, $\theta = \left(\frac{\omega_0 + \omega_f}{2}\right) t$

$= \left(\frac{0 + 60\pi}{2}\right) 5$

$\theta = 150\pi$ rad

then number of revolution made during acceleration

$n = \frac{\theta}{2\pi} = \frac{150\pi}{2\pi} = \underline{75}$

b) Here, $\omega_0 = 2\pi \times 30$ rad/s

$= 60\pi$ rad/s



and $\omega_f = 0$

The rotor decelerates for $t = 7.5s$

then $\theta = \left(\frac{\omega_0 + \omega_f}{2}\right) 7.5$

$= \left(\frac{60\pi + 0}{2}\right) 7.5$

$\theta = 225\pi$

$\therefore$ the number of revolutions made during deceleration

$n = \frac{\theta}{2\pi} = \frac{225\pi}{2\pi} = \underline{112.5}$

Also, we know

$$w_f = w_0 + \alpha t$$

here, $0 = 60\pi + \alpha \times 7.5$

$$\frac{-60\pi}{7.5} = \alpha$$

Deceleration value, $\alpha = -8\pi \ rad/s^2$

**20** a) Here, $x = 0.015 m$

$m_1 = 1.75 kg$

but $m_1 g = kx$

i.e., $k = \frac{m_1 g}{x}$

$$= \frac{1.75 \times 9.81}{0.015} = 1144.5 \ N/m$$

$f = 3 Hz$

i.e., $3 = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

$$9 = \frac{1}{4\pi^2} \frac{k}{m}$$

$$m = \frac{k}{4\pi^2 9} = \frac{1144.5}{4 \times (3.14)^2 \times 9}$$

$m = 3.22 kg$

b) when $v = 8 m/s$ , $x = 1m$
when $v = 4 m/s$ , $x = 2m$

we know, $v = \omega\sqrt{A^2 - x^2}$

then $8 = \omega\sqrt{A^2 - 1^2}$ - - - [1]

$4 = \omega\sqrt{A^2 - 2^2}$ - - - - [2]

Dividing eqn. [1] by [2]

$$\frac{8}{4} = \frac{\sqrt{A^2 - 1^2}}{\sqrt{A^2 - 2^2}}$$

Squaring both sides

$$4 = \frac{A^2 - 1}{A^2 - 4}$$

$$4A^2 - 16 = A^2 - 1$$

$$3A^2 = 15$$

$$A = \sqrt{5}\ m$$

(a) Amplitude, $A = \sqrt{5}\ m = 2.23m$

Substituting value of 'A' in eqn.[1]

$$8 = \omega\sqrt{5 - 1}$$

$$8 = \omega \times 2$$

$$\omega = 4\ rad/s$$

$$\omega = \frac{2\pi}{T_p} \Rightarrow T_p = \frac{2\pi}{\omega}$$

$$= \frac{2\pi}{4}$$

(b) $T_p = \pi/2\ s = 1.57s$

(c) Max. velocity $v_{max} = \omega A$

$$= 4\sqrt{5}$$

$$= \underline{\underline{8.94 \text{ m/s}}}$$

(d) Max. acceleration $a_{max} = \omega^2 A$

$$= 4^2\sqrt{5}$$

$$= \underline{\underline{35.68 \text{ m/s}^2}}$$