സഹായി

SFI GEC PALAKKAD

Module II

Moment

~~Rotating~~ In addition to the tendency to move a body, force may also tend to rotate a body about an axis.

The turning tendency of a force about a point is called the moment of the force about the point. It is measured by the pdt of the force & the ⊥r to distance of its line of action from the point.

$$M = F d$$

⇒ Moment is a vector quantity.

- Magnitude.
- Direction.
- Axis of rotation.

⇒ Unit : Nm

⇒ Depending upon the relative position of the force & moment centre, the moment of a force will be either clockwise or counter clockwise.

⇒ If a clockwise moment is taken as +ve, then the anticlockwise moment taken as -ve.

Resultant of co-planar ∥ force system

⇒ Line of action ∥ to each other.

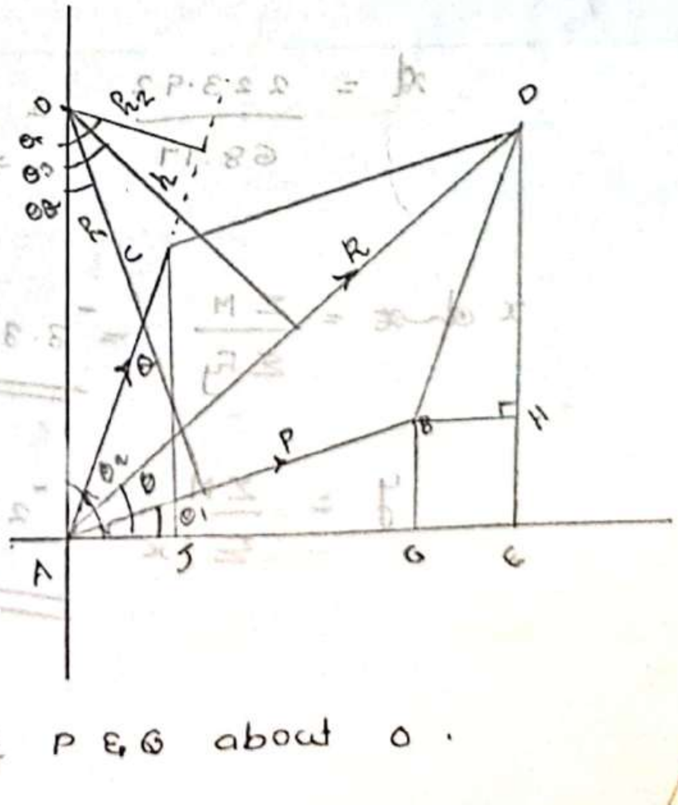
→ Magnitude of resultant can be found out by adding up all the forces using a sin convention for the sense of force.

$$R = \Sigma F.$$

⇒ Location of resultant can be determined by applying principle of moments (Varignon's theorem)

Principle of moments (Varignon's theorem)

The moment of the resultant of a system of non-current coplanar forces acting on a body about any point is equal to the algebraic sum of the moments of all the forces forming the system about the same point.



$AE = AG + GE = AG + BH$

$AD \cos\theta = AB \cos\theta_1 + BD \cos\theta_2$

$R\cos\theta = P\cos\theta_1 + Q \cos\theta_2$

Multip. by OA

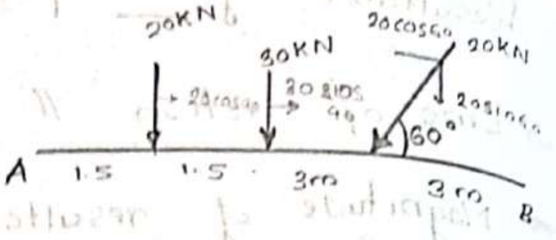
$R \cdot OA \cdot \cos\theta = P \cdot OA \cdot \cos\theta_1 +$
$Q \cdot OA \cdot \cos\theta_2$

$Rh = Ph_1 + Qh_2$

ie Moment of R about O = sum of moment of P & Q about O.

Hence proved.

Q. Determine the resultant & position of the resultant & direction of resultant.



$$\Sigma F_x = -20\cos 60 = -10\ N$$

$$\Sigma F_y = -20\sin 60 + -30\sin 90 - 20\sin 90.$$

$$= -20\frac{\sqrt{3}}{2} - 30 - 20$$

$$= -10\sqrt{3} - 50\ N$$

$$= -67.32\ N$$

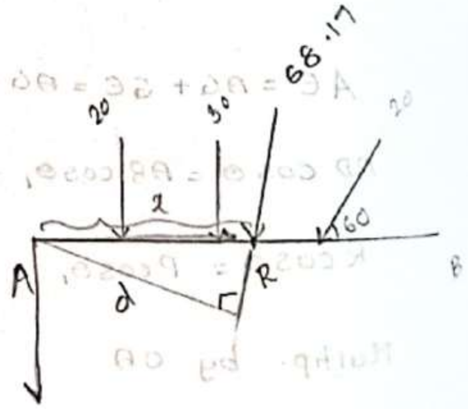
$$R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2} = \sqrt{-10^2 + (-67.32)^2}$$

$$= 68.14\ N$$

$$R\,d = 20\times 1.5 + 30\times 3 + 20 \sin 60 \times 6$$

$$= 223.92.$$

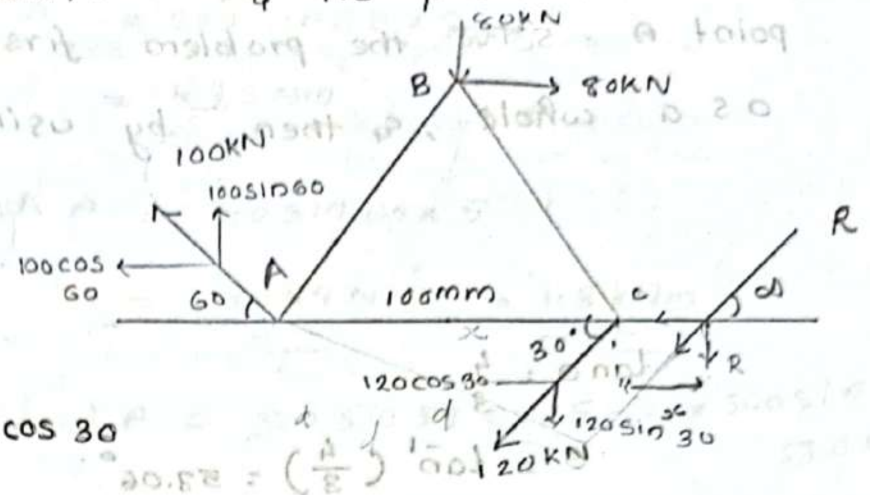
$$d = \frac{223.92}{68.17} = 3.28\ m$$



$$x = \frac{\Sigma M}{\Sigma F_y} = -3.32\ m.$$

$$y = \frac{\Sigma M}{\Sigma F_x} = -22.39\ M$$

Q) Find the value of resultant & its position.



$\Sigma F_x = 80\cos 0 + 80\cos 90 - 100\cos 60 - 120\cos 30$

$= -73.92\ N$

$\Sigma F_y = 80\sin 0 - 80\sin 90 + 100\sin 60 - 120\sin 30$

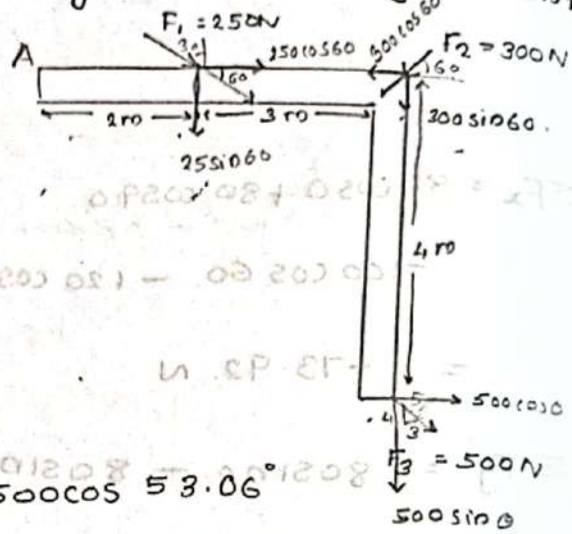
$= -53.39\ N$

$R = \sqrt{(-73.92)^2 + (-53.39)^2} = \underline{91.18\ N}$

$\alpha = \tan^{-1}\left(\frac{53.39}{73.92}\right) = \underline{35.84^\circ}$

~~Rd = 50 × 80 sin 90~~

Determine the moment of each of the 3 forces about point A. Solve the problem first by using each forces as a whole, & then by using principle of moments.



$\tan\theta = \frac{4}{3}$

$\theta = \tan^{-1}\left(\frac{4}{3}\right) = \underline{\underline{53.06^\circ}}$

$\Sigma F_x = 250\cos 60 - 300\cos 60 + 500\cos 53.06^\circ$

$= 275.48\ N$

$\Sigma F_y = -250\sin 60 - 300\sin 60 - 500\sin 53.06$

$= -875.94\ N$

$R = \sqrt{(275.48)^2 + (875.94)^2} = \underline{\underline{918.24\ N}}$

$Rd = 250\sin 60 \times 2 + 300 \sin 60 \times 5 + 500\sin 53.06 \times 5 + 4 \times 500\cos 53.06$

$= \cancel{433.012} + \cancel{1299}.03 - \cancel{1080767.58}$

$= 433 + 1299 + \cancel{76}\ 796.2$

$= \underline{\underline{2528.2\ Nm}}$

$d = \frac{2528.2}{918.24} = \underline{\underline{2.75\ m}}$

Moment of F_1 about A $= 250 \sin 60 \times 2$

$= 433$ Nm

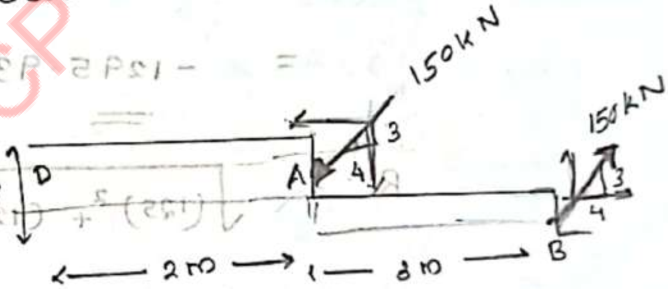
Moment of F_2 about A $= 300 \sin 60 \times 5$

$= 1299$ Nm $= 1.3$ kNm

Moment of F_3 about A $= 500 \sin 53.06 \times 5 - 4 \times 500 \sin 53.06$

$= 796.2$ Nm

a. Determine the moment of the couple acting on the machine member shown below.



$\tan\theta = \frac{3}{4}$

$\theta = \tan^{-1}\left(\frac{3}{4}\right) = 36.86°$

$\Sigma F_x = 150 \cos 36.86 - 150 \cos 36.86$

$= 0$

$\Sigma F_y = 150 \sin 36.86 - 150 \sin 36.86$

$= 0$

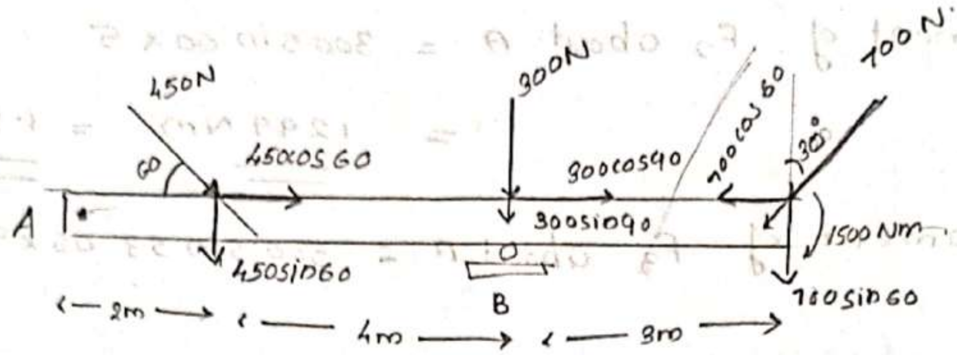
$R = 0$

$Rd = 150 \sin 36.86 \times 2 + 5 \times 150 \sin 36.86 + \cos 36.86$

$= 179.96 + 449.89 + 0.8$

$= 630.65$

Q. Replace 3 forces acting on the shaft beam by a single resultant force. Specify where the force acts, measured from end A.



$$\Sigma F_x = 450\cos 60 + 300\cos 90 - 700\cos 60$$

$$= -125\text{N}$$

$$\Sigma F_y = -450\sin 60 - 300\sin 90 - 700\sin 60$$

$$= -1295.93\text{ N}$$

$$R = \sqrt{(125)^2 + (1295.93)^2} = 1301.94\text{ N}$$

$$Rd = 2\times 450\sin 60 + 6\times 300\sin 90 + 9\times 700\sin 60 + 1500$$

$$= 779.42 + 1800 + 5455.96 + 1500$$

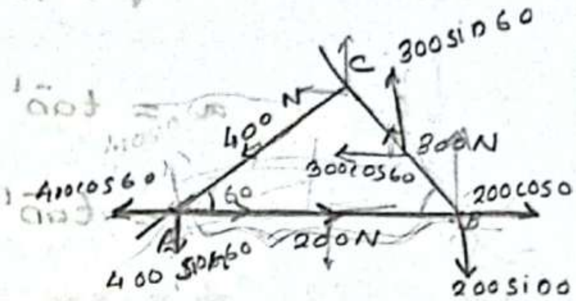
$$= 9535.38$$

$$d = \frac{9535.38}{1301.94} = 7.33\text{ m}$$

$$x = \frac{\Sigma M}{\Sigma F_y}$$

$$\theta = \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right) = 84.5^\circ$$

1. 3 forces 200N, 300N & 400N act along 3 sides of an equilateral triangle taken in order. Find the magnitude & line of action of the resultant force.



$\Sigma F_x = 200\cos 0 \mp 400\cos 60 - 300\cos 60$

$= -150\,N$

$\Sigma F_y = -200\sin 0 + 300\sin 60 \mp 400\sin 60$

$= -86.6\,N$

$R = \sqrt{(150)^2 + (86.6)^2} = 173.2\,N$

$\theta = \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right) = 30^\circ$

~~Rd =~~ $\Sigma M_A = -300\sin 60 \times AB$

$= -259.81 \times AB$.

$\Sigma M_A = Rx$

$x = \frac{\Sigma M_A}{R} = \frac{259.81\,AB}{173.2} =$

$\Sigma M_A = -300\sin 60 \times AB$

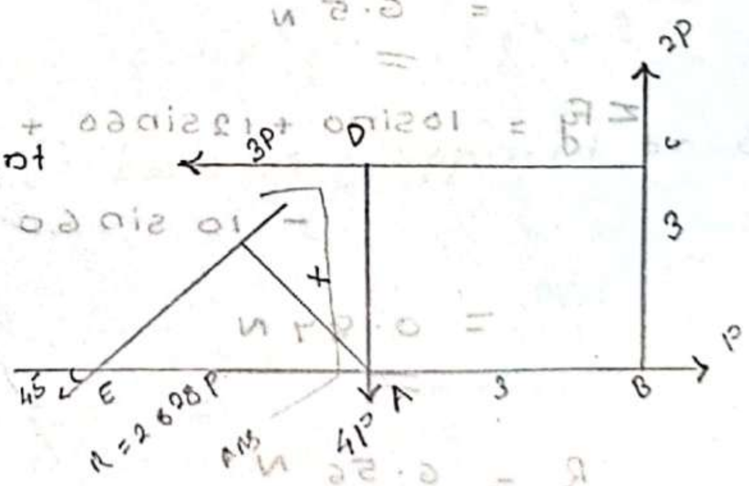
$= -259.81 \times AB$

$\Sigma M_A = Rx$

$x = \frac{\Sigma M_A}{R} = \frac{259.81\,AB}{173.2}$

$x = 1.5\,a$

2. Find out the resultant & its position



$\Sigma F_x = P\cos 0 - 3P\cos 0$

$= -2P$.

$\Sigma F_y = -4P\sin 90 + 2P\sin 90$

$= -2P$

$\theta = 45$

$M = 3.5P$

$x = 2.5a$

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

$$= \sqrt{(-2P)^2 + (-2P)^2} = 2.828P$$

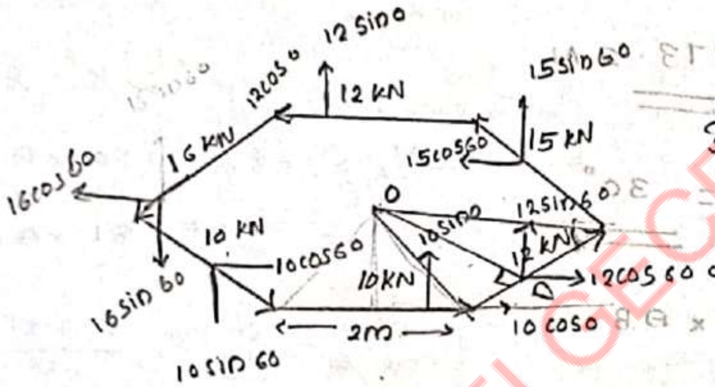
$$\alpha = \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right)$$

$$= \tan^{-1}\left(\frac{-2P}{-2P}\right) = 45^\circ$$

$$Rx = 5P \times a$$

$$x = \frac{5Pa}{2.828P} = 1.768a$$

?



Find the resultant force & its direction. Find position w.r. to centre of the hexagon.

$$\Sigma F_x = 10\cos 0 + 12\cos 60 - 15\cos 60 - 12\cos 0 - 16\cos 60 + 10\cos 60$$

$$= -6.5\ N$$

$$\Sigma F_y = 10\sin 0 + 12\sin 60 + 15\sin 60 + 12\sin 0 - 16\sin 60 - 10\sin 60$$

$$= 0.87\ N$$

$$R = 6.56\ N$$

$$\theta = \tan^{-1}\left(\frac{0.87}{-6.5}\right)$$

$$= \cancel{82.5^\circ} \quad -7.62^\circ$$

$\Sigma M_o = Rd = -(10 + 12 + 15 + 12 + 16 + 10) \times 2 \sin 60$

$= -129.75 \text{ NM}$

$$d = \frac{-129.75}{6.56} = 19.8 \text{ m}$$

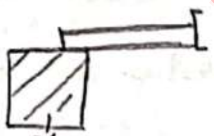
$$x = \frac{\Sigma M}{\Sigma F_y} = \frac{-129.75}{0.87} = 149.8 \text{ m}$$

$$y = \frac{\Sigma M}{\Sigma F_x} = \frac{-129.75}{-6.5} = 19.96 \text{ m.}$$

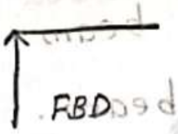
Types of Supports

1) Simple support.

If one end of the beam rests on a fixed support, the support is known as simple support.

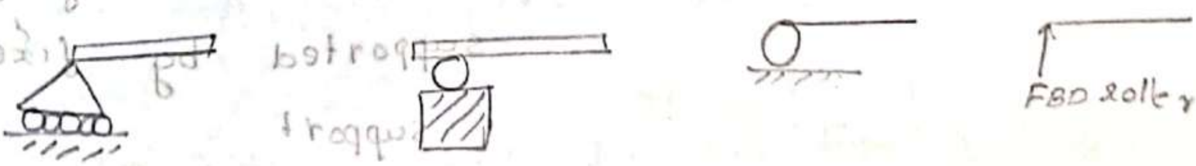


simple support



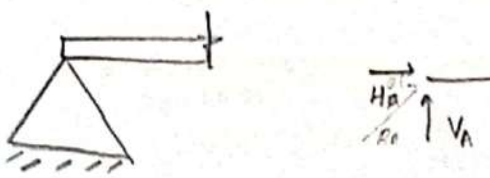
2) Roller support.

Here one end of the beam is supported on a roller.



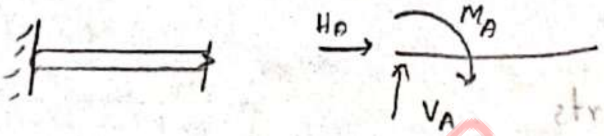
3) Hinged support.

The beam does not move either along or normal to the axis but can rotate.



4) Fixed Support.

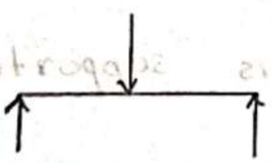
The beam is not free to rotate or slide along the length of the beam or in the direction normal to the beam. Therefore 3 reaction components can be observed. Also known as built-in support.



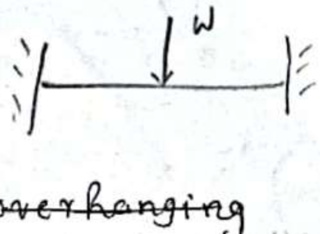
Types of beam

1) Simply supported beam.
2) fixed beam
3) overhanging beam
4) Cantilever beam.
5) Continuous beam.

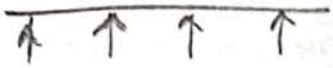
1) Simply supported beam: Supports are provided at both ends of beam.



2) Fixed beam : when both end of forces are supported by fixed ~~beam~~ support

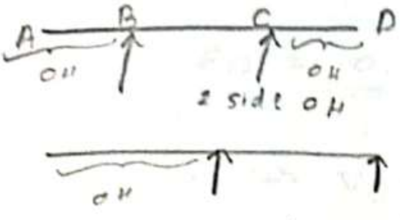


3) ~~overhanging~~ continuous : when beam is supported by ~~by~~ no more than 2 points.



4) Cantilever beam : one end of the beam is based & other end is kept free.

5) Overhanging beam : when the beam is projecting beyond the surface it is known as overhanging beam.



Types of loads

⇒ concentrated load or point load.

⇒ Uniformly distributed load (udl)

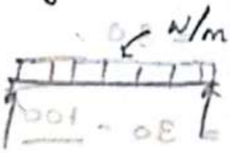
⇒ uniformly varying load (uvl)

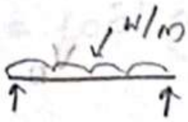
⇒ External moment.

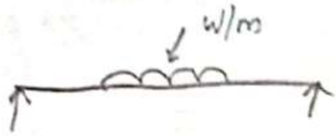
⇒ General loading

Concentrated load : when the loads are applied certain points in a wheel.

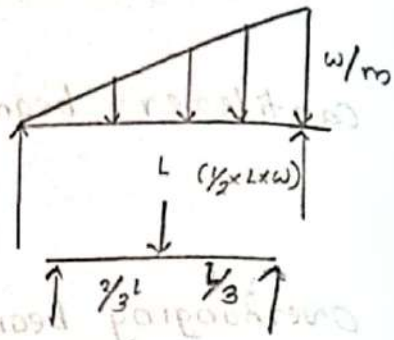
Uniformly distributed : The load which have same intensity of load over a certain ~~intensity~~ of ~~load~~ length of a beam





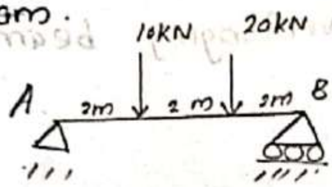


Uniformly varying load (UVL) : In this type of load, load intensity vary from one point to other.

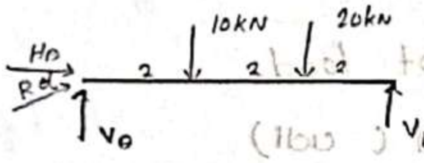


6/2/20

Q. Draw the diagram.



calculate support reaction at A & B.



$$\Sigma F_x = 0 \Rightarrow H_A = 0.$$

$$\Sigma F_y = 0 \Rightarrow V_A - 10 - 20 + V_B = 0$$

$$V_A + V_B - 30 = 0$$

$$\Sigma M = 0 \Rightarrow 10 \times 2 + 20 \times 4 - V_B \times 6 = 0$$

$$20 + 80 - 6V_B = 0$$

$$100 - 6V_B = 0$$

$$6V_B = 100$$

$$V_B = \frac{100}{6} = \underline{16.6\,N}$$

$$V_A + \frac{100}{6} = 30$$

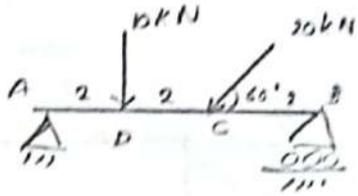
$$V_A = 30 - \frac{100}{6}$$

$$= \frac{80}{6} = \underline{13.3\,N}$$

$$R_A = \sqrt{0^2 + (13.3)^2} = \underline{13.3\,N}$$

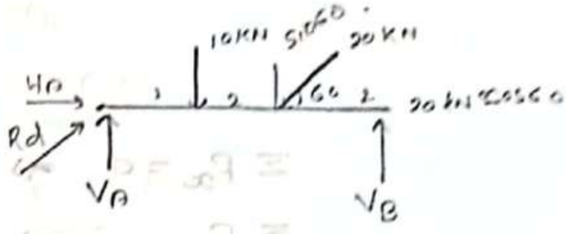
$$\theta = \tan^{-1}\left(\frac{H_A}{V_A}\right) = \underline{\underline{90°}} \text{ not defined.}$$

Q. Calculate



$\Sigma F_x = 0 \Rightarrow H_A - 20\cos 60 = 0$

$H_A = \underline{\underline{10}}$



$\Sigma F_y = 0 \Rightarrow V_A - 10 - 20\sin 60 + V_B$

$V_A + V_B - 27.3 = 0.$

$V_A + V_B = \underline{\underline{27.3}}$

$\Sigma M = 0 \Rightarrow 2 \times 10 + 4 \times 20\sin 60 - 6V_B$

$20 + 40\sqrt{3} - 6V_B.$

$6V_B = 20 + 40\sqrt{3}$

$V_B = \underline{\underline{14.88 kN}}$

$V_A = 27.3 - 14.88$

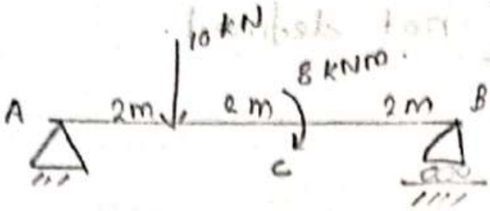
$\Rightarrow \underline{\underline{12.42 kN}}$

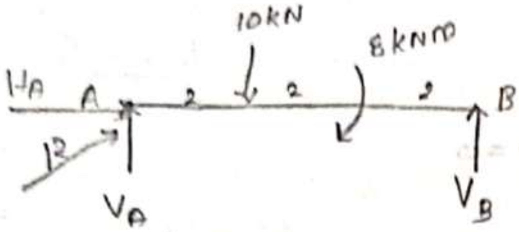
$R = \sqrt{H_A^2 + V_A^2} = \underline{\underline{15.95 kN}}$

$\theta = \tan^{-1}\left(\frac{\cancel{12.42}\ 10}{\cancel{10}\ 12.42}\right) = \underline{\underline{\cancel{51.20°}}}\ \cancel{38.79}°$ 51.11

$V_B = R_B = 14.88 kN$

Q.





$\Sigma F_x = 0 \Rightarrow H_A = 0$. $\quad R_B = V_B$

$\Sigma F_y = 0 \Rightarrow V_A - 10 + V_B = 0$

$V_A + V_B = 10$

$\Sigma M = 0 \Rightarrow 2 \times 10 + 8 - 6 \times V_B$

$20 + 8 - 6V_B$

$6V_B = 28$

$R_B = V_B = \frac{28}{6} = \underline{4.67 \text{ kN}}$

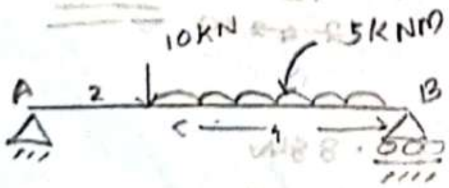
$V_A = 10 - 4.67$

$= \underline{5.34 \text{ kN}}$.

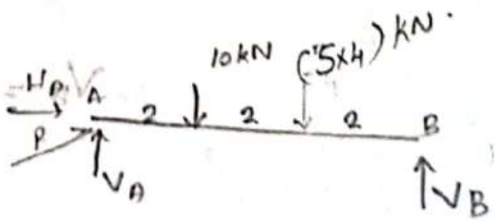
$R = \sqrt{(5.34)^2} = \underline{5.34 \text{ kN}}$

$\theta = \tan^{-1}\left(\frac{H_A}{V_A}\right) = \underline{\text{not defined}}$

Q.



$\Rightarrow$



$\Sigma F_x = 0 \Rightarrow H_A = 0$

$\Sigma F_y = 0 \Rightarrow V_A - 10 - 20 + V_B = 0$

$$V_A + V_B = 30$$

$\Sigma M_0 = 0 \Rightarrow 2 \times 10 + 4 \times 20 - 6 V_B$

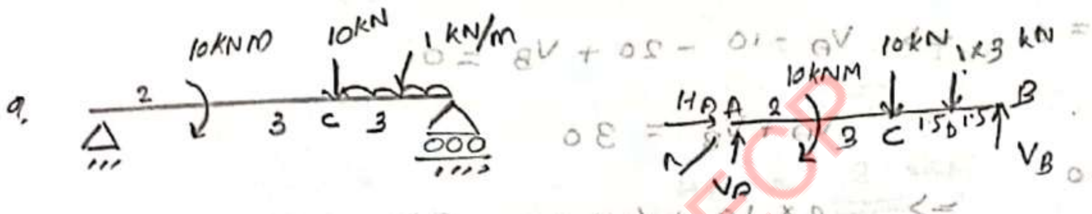
$$100 = 6 V_B$$

$$V_B = 16.6 \text{ N}$$

$$V_A = 13.3 \text{ N}$$

$$R_A = 13.3 \text{ N}$$

9.



$\Sigma F_x = 0 \Rightarrow$ ~~A_A~~ $H_A = 0.$

$\Sigma F_y = 0 \Rightarrow V_A$ ~~-10~~ $\mp 10 \mp 3 + V_B$

$$V_A + V_B \Rightarrow + \cancel{23}\ 13$$

$\Sigma M_0 = 0 \Rightarrow$ ~~2×10~~ $10 + 5 \times 10 + 6.5 \times 3 - 8 V_B$

$$10 + 50 + 19.5 - 8 V_B$$

$$8 V_B = 79.5$$

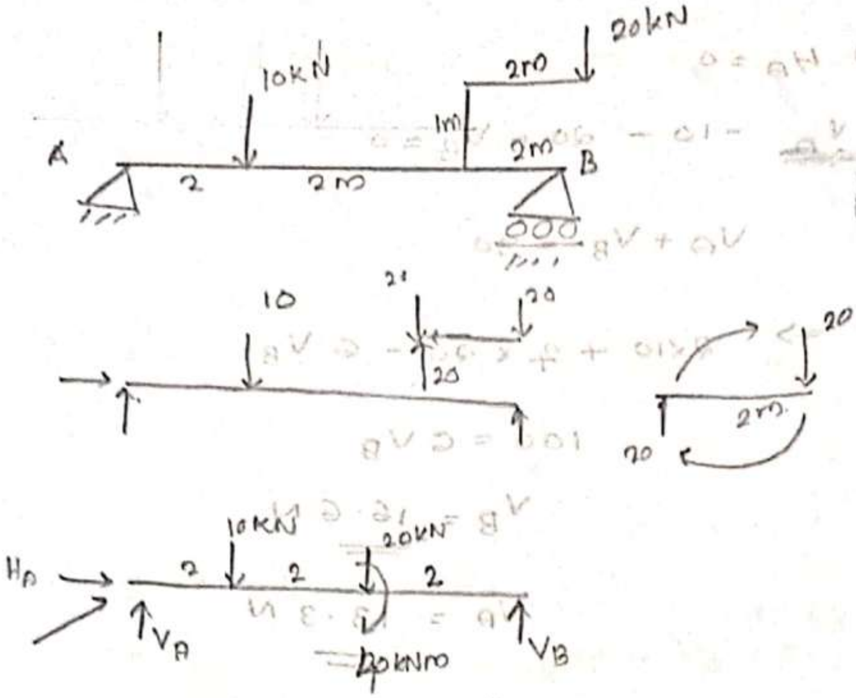
$$V_B = 9.94 \text{ N}$$

$$V_A = \overset{13}{\cancel{23}} - 9.94$$

$$= 13.06 \text{ N}$$

$$R = \sqrt{(13.06)^2} = 13.06$$

Q.



$$\Sigma F_x = 0 \Rightarrow H_{A} = 0$$

$$\Sigma F_y = 0 \Rightarrow V_A - 10 - 20 + V_B = 0$$

$$V_A + V_B = 30$$

$\Sigma M = 0$

$$\Rightarrow 2 \times 10 + 4 \times 20 - 6 V_B + 40 = 0$$

$$20 + 80 + 40 - 6V_B = 0$$

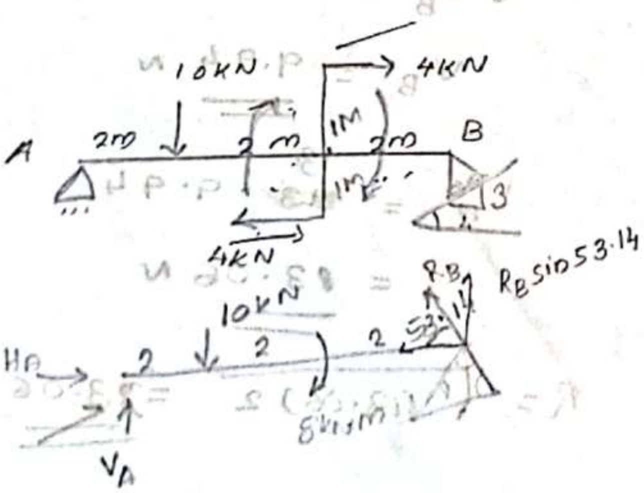
$$6 V_B = 140$$

$$V_B = \cancel{20}\ 23.3\ kN$$

$$R_A = V_A = \cancel{10}\ 6.7\ kN$$

$$R_B = 6.7\ kN$$

Q.



$$\tan\theta = \frac{3}{4}$$

$$\theta = 36.86$$

$\Sigma F_x = 0 \Rightarrow H_A - R_B \cos 53.14 = 0$. $\quad H_A - 0.59 R_B = 0$.

$\Sigma F_y = 0 \rightarrow V_A - 10 + R_B \sin 53.14 = 0$

$$V_A + R_B \sin 53.14 = 10$$

$$V_A + 0.8 R_B = 10.$$

$\Sigma M_0 = 0 \Rightarrow 20 + 8 - 6 \times R_B \sin 53.14$.

$$28 - 4.8 R_B = 0.$$

$$R_B = \frac{28}{4.8} = \underline{\underline{5.8\ kN}}$$

$$V_A = 10 - 0.8 \times 5.8$$

$$= \underline{\underline{5.34\ kN}}.$$

$$H_A = \underline{\underline{3.4\ kN}}$$

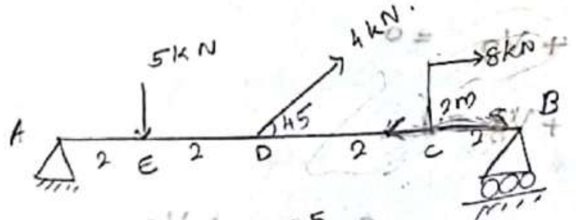
$$R_A = \sqrt{(H_A)^2 + (V_A)^2}$$

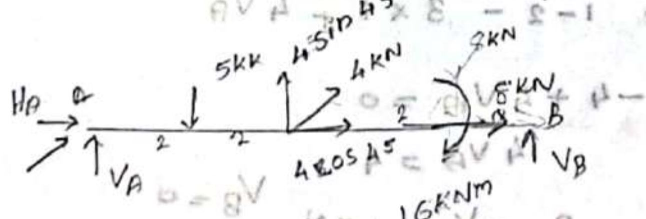
$$= \underline{\underline{6.3\ kN}}$$

$$R_B = 5.8\ kN.$$

$$\theta = \tan^{-1}\left(\frac{H_A}{V_A}\right) = \underline{\underline{32.48^\circ}}.$$

a.





$\Sigma F_x = 0 \Rightarrow H_A + 4\cos 45 = 0$

$8 + H_A + \frac{4}{\sqrt{2}} = 0$

$H_A = \underline{\underline{-10.83}}$

$\Sigma F_y = 0 \Rightarrow V_A - 5 + 4 \sin 45 + V_B = 0$

$V_A + V_B = 5 - \frac{4}{\sqrt{2}} = 2.17$

$\Sigma M_o = 0 \Rightarrow 10 \mp 4 \times 4 \sin 45 - 8V_B + 16 = 0.$

$10 + - 11.3 + 16 - 8V_B = 0$

$8V_B = \cancel{37.3}\ 14.7$

$R_B = V_B = \cancel{4.66}\ \underline{\underline{1.83\ kN}}$

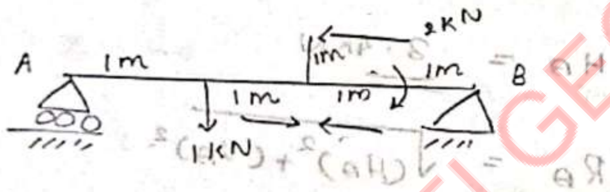
$V_A + V_B = 2.17$

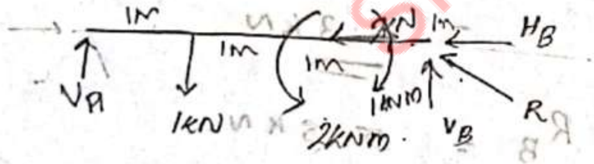
$\underline{\underline{V_A = 0.34\ kN}}$

$R_A = 10.84\ kN$

$\theta_A = (90 - 88.2)$

$= \underline{\underline{1.77^\circ}}$





$\Sigma F_x = 0 \Rightarrow -H_B - 2 = 0$

$H_B = -2.$

$\Sigma F_y = 0 \Rightarrow V_A - 1 + V_B = 0$

$V_A + V_B = 1$

$\Sigma M_o = 0 \Rightarrow \cancel{-4} \ 1 - 2 - 3 \times 1 + 4V_A$

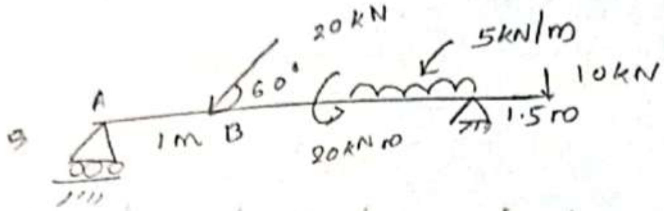
$-4 + 4V_A = 0$

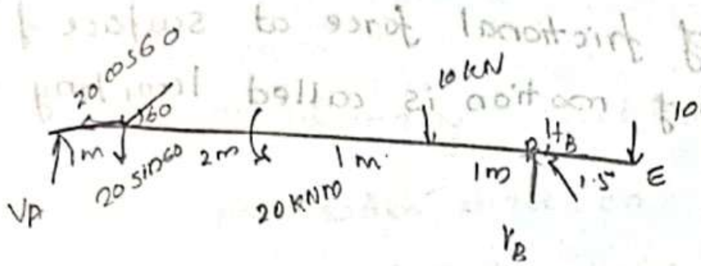
$4V_A = 4$

$\underline{\underline{R_A = V_A = 1\ kN}}$ $\quad \underline{\underline{V_B = 0}}$

$R_B = 2 \text{ kN}$

$\theta = \cancel{63.4}$ not defined.





$\Sigma F_x = 0 \rightarrow -H_B - 20\cos 60 = 0$

$H_B = -10\text{kN}$

$\Sigma F_y = 0 \Rightarrow V_A - 20\sin 60 - 10 + V_B - 10$

$V_A + V_B = 37.3$

$\Sigma M_0 = 0 \Rightarrow -10 \mp 20 \mp 4 \times 20 \sin 60 + 5V_A + 1.5 \times 10$

$R_A = \cancel{5} V_A = \cancel{99.2}\ -16.86 \text{ kN}$

$V_B = 20.44 \text{ KN}$

$R_B = 22.77 \text{ kN}$

$\theta = (90 - 26.06)$

$= \cancel{116.07}°\ 63.94°$

11/2/20

Friction

1) Dry friction
2) fluid friction
3) Internal friction

Limiting force of friction

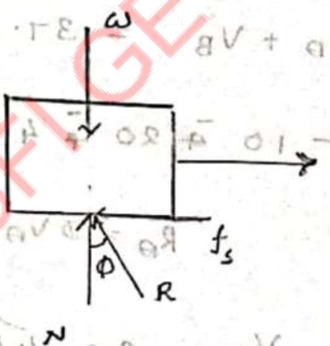
when The max. value of frictional force at surface of contact the body is a verge of motion is called limiting static frictional force

$$F_s = \mu_s N$$

F_s : limiting static frictional force

μ_s : coeffient of static friction.

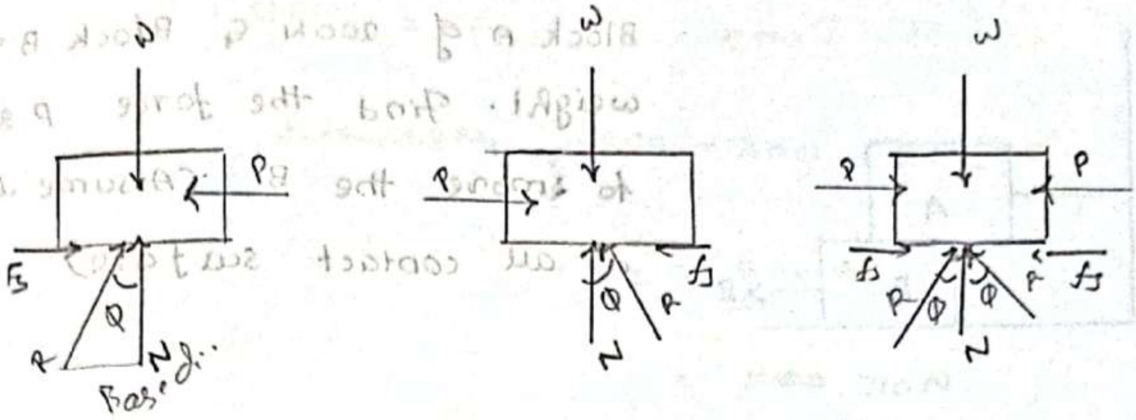
N : Normal reaction.



$$\mu_s = \frac{\text{limiting friction force}}{\text{normal reac}^n \text{ to 2 bodies}}$$

Angle of friction (ϕ): Angle made by the resultant of normal reaction & the friction force to the normal reac^n.

Cone of friction: The right circular cone with vertex at the point of contact of 2 surfaces, axis in the direction of the normal reac^n ($\vec{N}$) & semi-vertical angle is equal angle of friction ϕ.

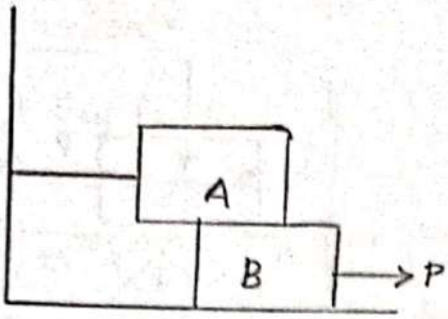


Consider the limiting equilibrium of body kept on a horizontal plane. when direction of external force is change, the direction of resultant b. change. But the angle b/w normal reaction & resultant should be same. when the direc^n of external force is gradually change 360° the resultant are generate a circular cone with a semi cone angle is equal to ϕ. The axis of this cone is called cone of friction. The axis of this cone will be the normal reaction & generators are the resultant force & base radius is equal to limiting frictional force.

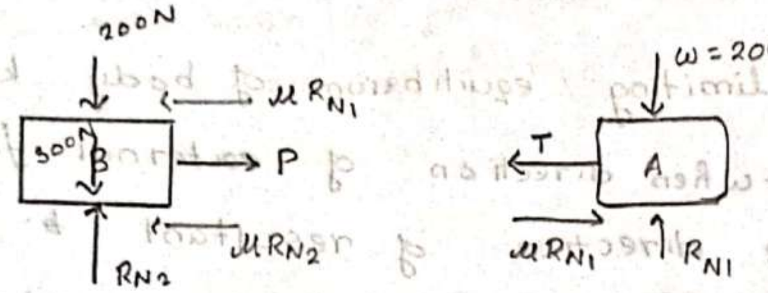
Angle of ~~free~~ re force

Max. inclination of the plane at which the body remain in equilibrium over the incline plane by ~~exi~~ assistance of frictional force alone.

Q.



Block A of 200N & Block B of 300N weight. Find the force P required to move the B. (Assume $\mu = 0.3$ at all contact surface)



Consider body A,

$$\Sigma F_x = 0 \Rightarrow 200\cos 90 - T\cos 0 + \mu R_{N1}\cos 0 - R_{N1}\cos 90$$

$$\cancel{R\cos}\ T = \mu R_{N1}$$

$$\Sigma F_y = 0 \Rightarrow -200\sin 90 - T\sin 90 + \mu R_{N1}\sin 0 + R_{N1}\sin 90$$

$$-200 - \cancel{T}\ R_{N1} = 0$$

$$\underline{R_{N1} = 200\text{ N}}$$

$$\underline{\underline{T = 60\text{ N}}}$$

Consider body B,

$$\Sigma F_x = 0 \Rightarrow P\cos 0 - \mu R_{N1}\cos 0 + 200\cos 90 + 300\cos 90 + R_{N2}\cos 90 - \mu R_{N2}\cos 0 = 0.$$

$$P - \mu R_{N1} - \mu R_{N2} = 0$$

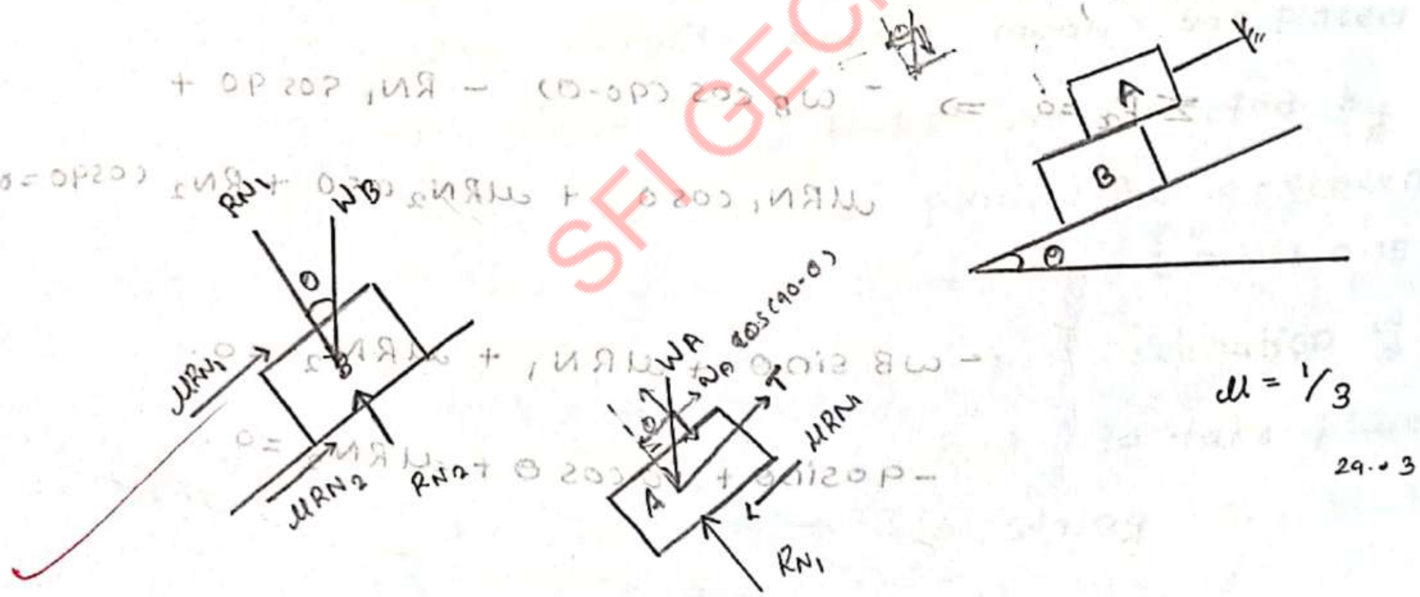
$P - \mu R N_2 = 60$

$\Sigma F_y = 0 \Rightarrow$, $\cancel{R - \mu R N_1}$ $- 200 - 300 + R N_2 = 0$.

$R N_2 = \underline{\underline{500\ N}}$

$P = \cancel{500}\ \underline{\underline{210\ N}}$

Imp Q. What should be the value of angle θ for the motion of the block B weighing 90 N to impend down the plane. The co-efficient of friction for all surfaces of contact is 1/3. Block A weights 30N. The block A is held in position as show in fig.



Consider A,

$\Sigma F_x = 0 \Rightarrow T \cos 0 - W_A \cos(90-\theta) + R_{N_1} \cos 90$

$- \mu R_{N_1} \cos 0 = 0$.

$T - W_A \sin\theta - \mu R_{N_1} = 0$.

$T - 30 \sin\theta - \mu R_{N_1} = 0$.

$\Sigma F_y = 0 \Rightarrow$ $T \sin\theta - \omega_B \sin(90-\theta) + R_{N_1} \sin 90$

$+ \mu R_{N_1} \sin\theta$

$-\omega_B \cancel{\sin} \cos\theta + R_{N_1} = 0.$

$R_{N_1} = 30\cos\theta.$

$$T - 30\cancel{\cos\theta}^{\sin\theta} - \frac{1}{3} 30\cos\theta = 0.$$

$\cancel{T - 40\cos\theta = 0}$

$\cancel{T = 40\cos\theta}$

Consider B,

$\Sigma F_x = 0 \Rightarrow$ $-\omega_B \cos(90-\theta) - R_{N_1} \cos 90 +$

$\mu R_{N_1} \cos\theta + \mu R_{N_2} \cos\theta + R_{N_2} \cos 90 = 0$

$-\omega B \sin\theta + \mu R_{N_1} + \mu R_{N_2} = 0.$

$-90\sin\theta + 10\cos\theta + \mu R_{N_2} = 0.$

$\Sigma F_y = 0 \Rightarrow$. $-\omega_B \sin(90-\theta) - R_{N_1} \sin 90$

$+ \mu R_{N_1} \sin\theta + \mu R_{N_2} \sin\theta + R_{N_2} \sin 90 = 0$

$-\omega_B \cos\theta - R_{N_1} + R_{N_2} = 0.$

$-90\cos\theta - 30\cos\theta + R_{N_2} = 0.$

$R_{N_2} = 120\cos\theta.$

$-90\sin\theta + 10\cos\theta + 40\cos\theta = 0$

$\frac{1}{3} \times 120$ 4

$-90\sin\theta + 50\cos\theta = 0$

$-90\sin\theta = 50\cos\theta$

$\frac{-90}{50}$

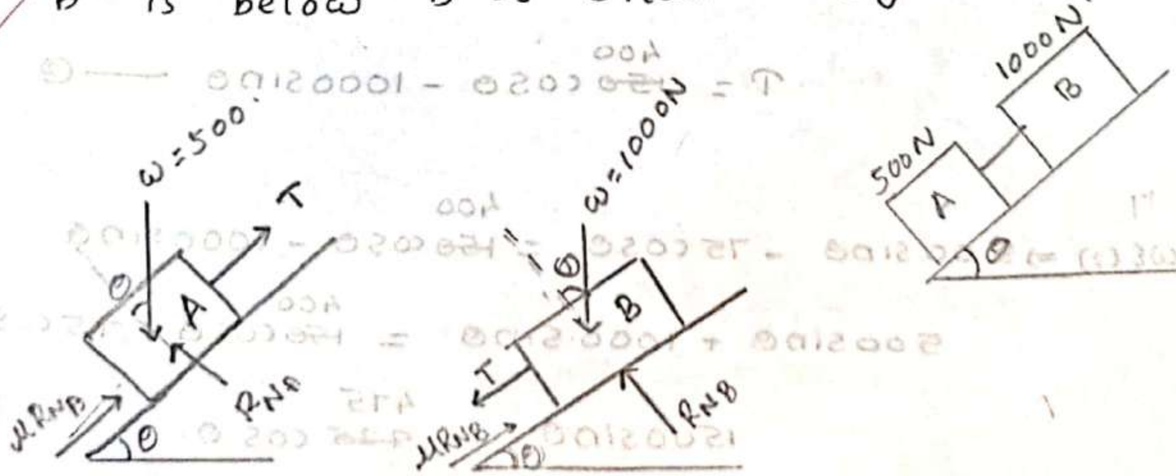
$\frac{\sin\theta}{\cos\theta} = \frac{-50}{90}$

$\tan\theta = \frac{-5}{9}$

$\theta = \tan^{-1}\left(\frac{5}{9}\right)$

$= 29.05^\circ$

Q 2 bodies A & B of weights 500N & 1000N are placed on an inclined plane. The blocks are connected by a string parallel to inclined plane. The co-efficent of friction b/w the inclined planed block A is 0.15 & that for the block B 0.4. Find the inclination of plane when the motion is about to take place. Also calculate the tension in string. The block A is below B as show in fig.



Block A

$$\Sigma F_x = 0 \Rightarrow P\cos\theta - 500\cos(90-\theta) + \mu R_{NA}\cos\theta + R_{NA}\cos 90$$

$$T - 500\sin\theta + \mu R_{NA} = 0.$$

$$T + \mu R_{NA} = 500\sin\theta.$$

$$\Sigma F_y = 0 \Rightarrow T\sin\theta - 500\sin(90-\theta) + \mu R_{NA}\sin\theta + R_{NA}\sin 90$$

$$-500\cos\theta + R_{NA} = 0$$

$$\underline{\underline{R_{NA} = 500\cos\theta}}.$$

$$T = 500\sin\theta - 0.15\times 500\cos\theta.$$

$$T = 500\sin\theta - 75\cos\theta. \quad \text{---} (1)$$

Block B

$$\Sigma F_x = 0 \Rightarrow -1000\cos(90-\theta) - T\cos\theta + \mu R_{NB}\cos\theta + R_{NB}\cos 90 = 0$$

$$-1000\sin\theta - T + \mu R_{NB} = 0$$

$$T = \mu R_{NB} - 1000\sin\theta$$

$$\Sigma F_y = 0 \Rightarrow -1000\sin(90-\theta) - T\sin\theta + \mu R_{NB}\sin\theta + R_{NB}\sin 90 = 0$$

$$-1000\cos\theta + R_{NB} = 0$$

$$R_{NB} = 1000\cos\theta.$$

$$T = \overset{400}{\cancel{150}}\cos\theta - 1000\sin\theta \quad \text{---} (2)$$

From (1) & (2) $\Rightarrow 500\sin\theta - 75\cos\theta = \overset{400}{\cancel{150}}\cos\theta - 1000\sin\theta.$

$$500\sin\theta + 1000\sin\theta = \overset{400}{\cancel{150}}\cos\theta + 75\cos\theta$$

$$1500\sin\theta = \overset{475}{\cancel{225}}\cos\theta.$$

$$\tan\theta = \frac{\cancel{225}\ 475}{1500}$$

$$\cancel{\theta = \tan^{-1}\left(\frac{3}{20}\right)}$$

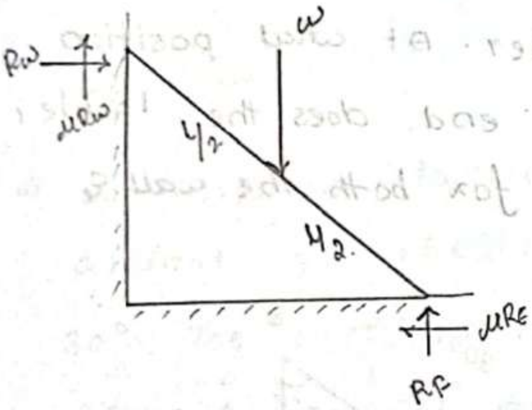
$$\theta = \tan^{-1}\left(\frac{19}{60}\right)$$

$$\theta = \cancel{16}\ 17.57^\circ$$

$$T = 500\sin(17.57) - 75\cos(17.57)$$

$$= 79.43\text{ N}$$

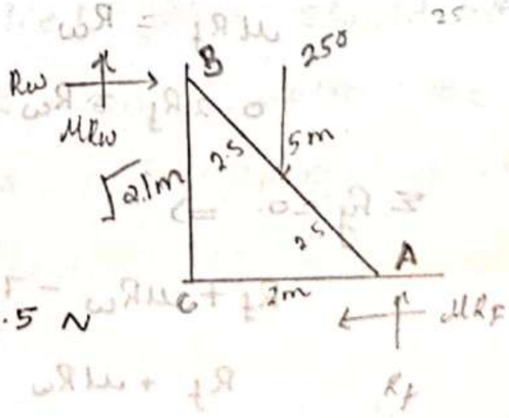
Q. Lader friction



Q. A uniform of lader of ~~250~~ 5m long weighting 250N is placed against a smooth vertical wall with a lower end 2m from the wall. The co-effient of friction b/w ladder & wall is 0.25. ST the lader remain in equilibrium in this ~~pot~~ position.

$l = 5\text{m}.$

$\omega = 250\text{N}.$



$\Sigma F_x = 0 \Rightarrow \quad R_\omega = \mu R_F = 62.5\text{ N}$

$\Sigma F_y = 0 \Rightarrow \quad R_F = 250\text{ N}$

Moment about B,

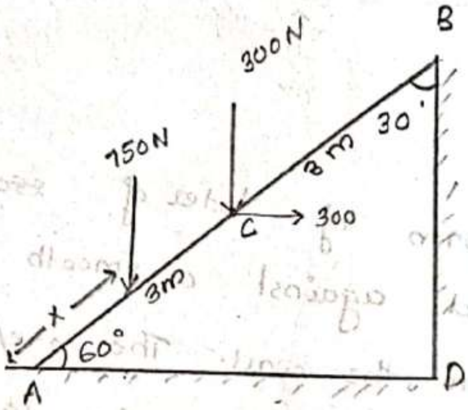
$$R_f \times BC - 250 \times 1 - F \times BC = 0$$

$$250 \times 2 - 250 = F \times 4.58$$

$$F = 54.59N$$

Since the frictional force at A is less than the limiting frictional force of 62.5 N, the ladder will remain in equilibrium.

Q. A uniform ladder 6m long weighing 300N, is resting against a wall with which it makes 30°. A man weighing 750N climbs up the ladder. At what position along the ladder from the bottom end does the ladder slips? The co-efficient of friction for both the wall & the ground with the ladder is 0.2.



$\mu = 0.2.$

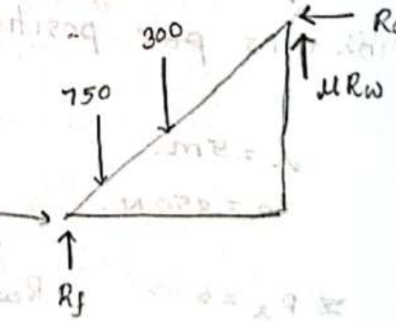
~~$\Sigma F_x = 0$~~ $\Sigma F_x = 0$

~~$R_f - \mu R_w = 0$~~.

$\mu R_f - R_w = 0$

$\mu R_f = R_w$

$0.2 R_f = R_w.$



$\Sigma F_y = 0. \implies$

$R_f + \mu R_w - 750 - 300 = 0$

$R_f + \mu R_w = 1050$

$\frac{R_w}{0.2} + 0.2 R_w = 1050$

$5.2 R_\omega = 1050$

$R_\omega = \underline{\underline{201.92 N}}$

$R_f = \underline{\underline{1009.6 N}}$

$\Sigma M = 0 \implies$

$750 \times x \cos 60 + 300 \times 3 \cos 60 - .2 R_\omega \times 6 \cos 60$

$- R_\omega \, 6 \sin 60 .$

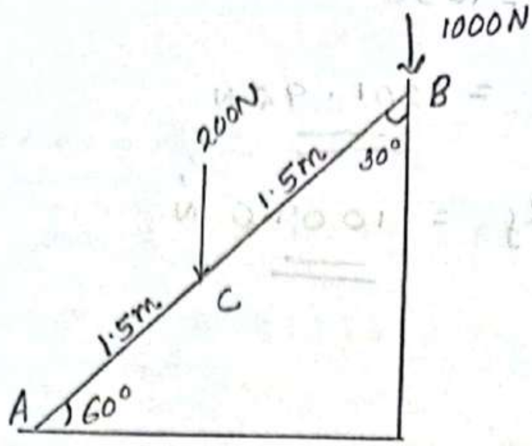
$\frac{750x}{2} + 450 - 0.6 R_\omega - 5.19 R_\omega$

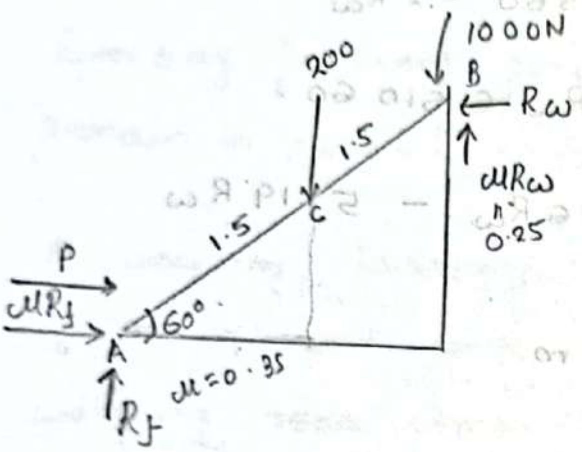
$x = \underline{\underline{1.92 \, m.}}$

Q. A uniform ladder 8m long weighs 200N. If is placed against a vertical wall with which it makes an angle of 30°. The co-efficient of friction b/w the wall & the ladder is 0.25 & that b/w the floor & ladder is 0.35. The ladder in addition to its own weight has to support a load of 1000N at its top end. Find

(i) The horizontal force P to be applied to the ladder at the floor level to prevent slipping.

(ii) If the force P is not applied what should be the minimum inclination of the ladder with the horizontal so that there is no slipping of it with the load at its top end.





$\Sigma F_x = 0 \Rightarrow \mu R_f + P - R_\omega = 0$

$0.35 R_f + P - R_\omega = 0.$

$\Sigma F_y = 0 \Rightarrow R_f + \mu R_\omega - 200 = 1000$

$R_f + 0.25 R_\omega = \cancel{200}\ 1200$

$0.35 R_f - R_\omega = -P \quad (1)$

$R_f + 0.25 R_\omega = 1200 \quad (2)$

$\Sigma M = 0, \quad -R_\omega \times 3 \sin 60 + -\mu R_\omega \times 3 \cos 60 + 1000 \times 3 \cos 60$

$200 \times 1.5 \cos 60$

$= -2.59 R_\omega - 0.375 R_\omega + 1500$

$+ 150.$

$= -2.965 R_\omega + 1650$

$2.965 R_\omega = 1650$

$R_\omega = \underline{\underline{554.49\ N}}$

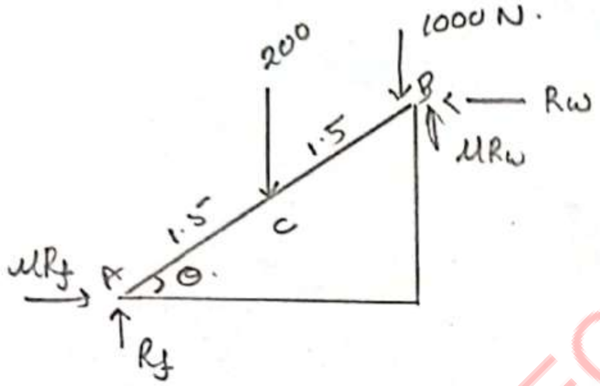
$R_f = 1200 - 139.12$

$\simeq \underline{\underline{1060.88\ N}}$

$0.35 \times 1060.88 - 554.49 = -P$

$-183.182 = -P$

$P = \underline{\underline{183.18\ N}}$



$\Sigma F_x = 0 \Rightarrow \mu R_f - R_\omega = 0$

$\mu R_f = R_\omega$. $\quad 0.35 R_f = R_\omega$

$\Sigma F_y = 0 \Rightarrow R_f + \mu R_\omega - 1200 = 0$

$R_f + \mu R_\omega = 1200$

$2.85 R_\omega + 0.25 R_\omega = 1200$

$3.1 R_\omega = 1200$

$R_\omega = \underline{\underline{387.09\ N}}$

$R_f = \underline{\underline{1105.97\ N}}$

$\Sigma M = 0 \Rightarrow$

$- R_\omega \times 3\sin\theta - \mu R_\omega\, 3\cos\theta + 1000 \times 3\cos\theta + 200 \times 1.5\cos\theta = 0.$

$-1161.27\sin\theta - 290.3175\cos\theta + 3000\cos\theta + 300\cos\theta = 0$

$$-1167.27\sin\theta = -3009.6825\cos\theta.$$

$$\frac{\sin\theta}{\cos\theta} = \frac{3009.6825}{1167.27}$$

$$= 2.578$$

$$\theta = \tan^{-1}(2.578)$$

$$= 68.8^\circ$$

Module 3

Square $= a^2$.

Cube $= a^3$

Cuboid $= lbh$

Sphere $= \frac{4}{3}(\pi r^3)$

cylinder $= \frac{1}{3}\pi r^2 h$.

hollow cylinder $= \frac{\pi}{4} \times h(D^2 - d^2)$

Diff. b/w centre of gravity & centroid

⇒ The term centre of gravity applies to bodies with mass & weight. centroid applies to plane area.

⇒ centre of gravity of a body is the point through which the resultant gravitational force acts for any orientation of body whereas centroid is the point is in the plane area such that the moment of area about any axis through that point is zero.

