DIFFRACTION

Diffraction:

Diffraction is the phenomena of **bending light around an obstacle** (a small opaque object, a sharp edge, or a narrow hole) **when the size of the obstacle is comparable to the wavelength of light**.

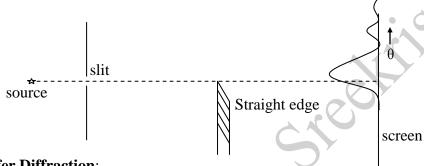
Or it is the encroachment of light into the geometrical shadow of an obstacle when the size of the obstacle is comparable to the wavelength of light.

Diffraction is classified into two based on the positions of the source of light and screen to the obstacle causing diffraction.

a) Fresnel Diffraction:

In Fresnel diffraction, the source of light and the screen are at a finite distance from the obstacle causing diffraction. Here the wavefront falling on the obstacle is spherical or cylindrical. Lenses are not used for producing diffraction.

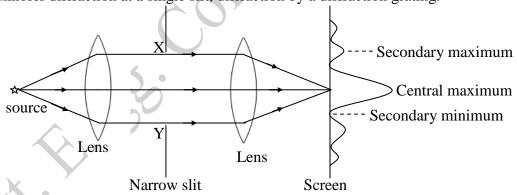
Eg: diffraction at a straight edge by a narrow linear source (slit) or point source of light.



b) **Fraunhofer Diffraction**:

In Fraunhofer diffraction, the source of light and the screen are at infinite distance from the obstacle causing diffraction. Here the wavefront falling on the obstacle is a plane. For practical purposes, convex lenses are used to produce plane wavefront from a nearby source and also to form an image on a nearby screen.

Eg: Fraunhofer diffraction at a single slit, diffraction by a diffraction grating.



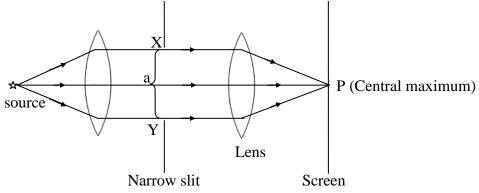
Fraunhofer diffraction at a single slit: (not in syllabus)

Let XY be a narrow slit of width 'a'. When a monochromatic light of wavelength λ is incident on this slit, instead of producing a sharp image on the screen, we get a diffraction pattern consisting of a central maximum and several secondary maxima and minima on either side of the central maximum.

Condition for Central Maximum:

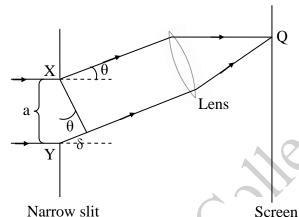
According to Huygen's principle, each point on a wavefront can be considered as secondary sources of light. These secondary sources send out secondary waves in all directions. Most of the secondary waves travel in the same direction of incident light (without diffraction). When focused by using a lens, they

will converge to a point at the centre of the screen. Since there is no path difference between these waves, they interfere constructively to produce maximum brightness at the centre called central maximum.



Condition for secondary minimum:

Consider a beam of secondary waves diffracted through an angle θ and reaching the point Q such that the path difference between the waves from extreme points of the slit (X and Y) is an integral multiple of wavelength ($\delta = n \lambda$, n = 1, 2, 3...), then most of the secondary waves interfere destructively producing a secondary minimum. Moving outwards from the centre intensity of secondary minimum increases.



Narrow slit

From Figure $\sin \theta = \frac{\delta}{a}$ or $\delta = a \sin \theta = n\lambda$ or $\sin \theta = \pm \frac{n\lambda}{a}$ where n = 1,2,3...

$$\sin \theta = \pm \frac{n\lambda}{a} \quad \text{where } n = 1, 2, 3....$$

Condition for secondary minimum:

Consider a beam of secondary waves diffracted through an angle θ and reaching the point Q such that the path difference between the waves from extreme points of the slit (X and Y) is an odd multiple of $\delta = (2n+1)\frac{\lambda}{2}$, n = 1, 2, 3..., then most of the secondary waves interfere constructively producing a secondary maximum. Moving outwards from the centre intensity of secondary maximum decreases.

From Figure
$$\sin \theta = \frac{\delta}{a}$$
 or $\delta = a \sin \theta = (2n+1)\frac{\lambda}{2}$ or $\sin \theta = \pm (2n+1)\frac{\lambda}{2a}$ where $n = 1,2,3...$

Diffraction Grating:

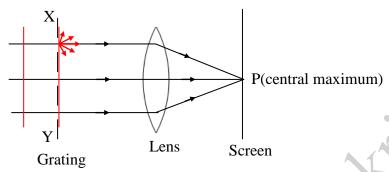
It is an arrangement of a large number of narrow rectangular slits of equal width separated by opaque portions (lines). All lines are also having the same width. The width of the slit is represented by 'a' and width of the line by 'b'. The length (a+b) is called a **grating element** and the points separated by an integral multiple of (a+b) are called **corresponding points**.

Diffraction grating used in the visible region contains nearly 5000 – 12000 lines /cm.

When monochromatic light is incident on this grating, it is transmitted through the slits and is obstructed (blocked) by the lines. Due to the diffraction effect, we get a central maximum and many principal maxima on either side of the central maximum.

Condition for Central maximum:

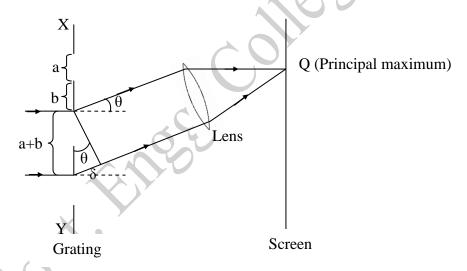
According to Huygen's principle, each part of the wavefront *incident on the grating and passing through the slits* sends out secondary waves in all directions. Most of the secondary waves travel in the same direction of incident light (without diffraction). When focused by using a lens, they will converge to a point at the centre of the screen. Since there is no path difference between these waves, they interfere constructively to produce maximum brightness at the centre called central maximum.



Note: Since the position of the central maximum is the same for all wavelengths, it has the same colour as that of the incident light.

Condition for Principal maxima:

Consider a beam of secondary waves diffracted through an angle θ and reaching the point Q such that the path difference between the waves from two nearby corresponding points is an integral multiple of wavelength ($\delta = n \lambda$, n = 1, 2, 3...), then all the secondary waves emerging from different corresponding points and diffracted through angle θ interfere constructively producing a principal maximum.



From Figure
$$\sin \theta = \frac{\delta}{a+b}$$
 or $\delta = (a+b)\sin \theta = n\lambda$ $(n = 1,2,3...)$

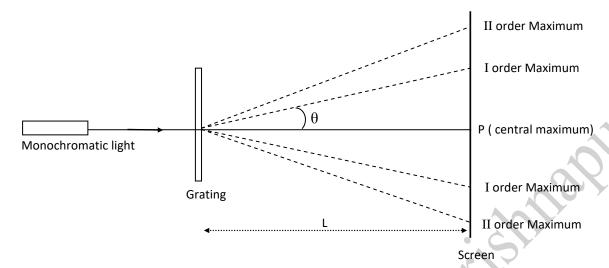
If N is the number of grating elements (lines) per unit length, then (a + b) N = 1 or $(a + b) = \frac{1}{N}$

∴
$$\sin \theta = nN\lambda$$
 (n = 1,2,3....). This is called **grating law** or **grating equation**.

If n = 1, it is called first order Principal maximum

n = 2, it is called second order Principal maximum, and so on.

Thus we get many principal maxima on either side of the central maximum.

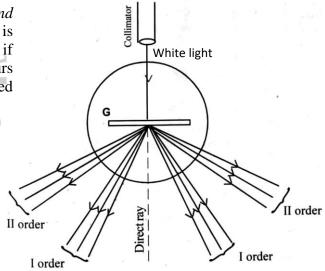


Grating spectrum with white light:

The grating equation shows that, for a given grating and each value of n, the angle of principal maximum (θ) is different for different wavelengths (colours). Hence if white light is used, it gets split up into different colours and a spectrum is formed. Violet has deviated less and red has deviated more in each order.

If n = 1, it is called first order spectrum

n = 2, it is called second order spectrum, and so on.



Difference between interference and diffraction:

Interference	Diffraction
1. Due to the superposition of waves from two	1. Due to the superposition of secondary waves
coherent sources.	from different parts of the same wavefront.
2. The intensity of maxima (bright bands) are	2. The intensity of maxima are not the same.
almost the same.	
3. The intensity of minima are almost zero (dark).	3. The minima are not dark.
4. Bands may be of equal width.	4. Bands are never of equal width.

Difference between Prism spectrum and grating spectra:

Prism Spectrum	Grating spectra
1. Produced by dispersion.	1. Produced by diffraction.
2. Only one spectrum is produced.	2. Grating spectra forms different orders on
	either side of the central maximum.

3. Brighter because the whole light is concentrated	3. Less bright because the light is distributed
in one spectrum.	among various orders.
4. Violet (low wavelength) is deviated more and	4. violet (low wavelength) is deviated less and
red the least.	red the most.
5. Resolving power is less	5. Very high resolving power.
6. Spectrum depends on the material of the prism	6. Independent of the material of the grating.

Resolving Power:

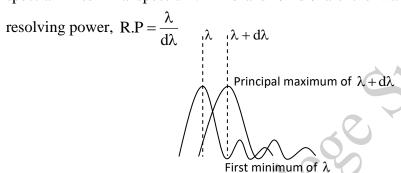
The resolving power of an optical instrument (telescope, microscope, eye, etc) is its ability to form distinct (separate) images of two neighbouring objects.

Rayleigh's Criterion for Resolution of Spectral Lines:

According to Rayleigh's criterion for resolution of spectral lines, two neighboring spectral lines (wavelengths λ and $\lambda + d\lambda$) are just resolved (just visible as separate) when the principal maximum of one wavelength in any order falls on the first minimum of the other in the same order.

Resolving Power of a Grating:

The resolving power of a grating is its ability to form distinct (separate) images of two neighboring spectral lines in a spectrum. If λ and $\lambda + d\lambda$ are the wavelengths of two nearby spectral lines, the



Condition for a principal maximum of wavelength $\lambda + d\lambda$ in n^{th} order is $(a + b) \sin \theta = n(\lambda + d\lambda)$

Similarly, condition for the first minimum of wavelength λ in n^{th} order is $(a+b)\sin\theta = n\lambda + \frac{\lambda}{N}$,

where N_1 is the total number of lines on the grating.

$$\therefore n(\lambda + d\lambda) = n\lambda + \frac{\lambda}{N_1} \text{ or } nd\lambda = \frac{\lambda}{N_1} \text{ and } \frac{\lambda}{d\lambda} = nN_1. \quad \text{(derivation not needed)}$$

Resolving power of the grating, $\frac{\lambda}{d\lambda} = nN$

$$\frac{\lambda}{d\lambda} = nN_1$$

Dispersive Power of a Grating:

In a grating different wavelengths are diffracted through different angles. Dispersive power of a grating is the ratio of change in the angle of diffraction to the corresponding change in wavelength. Let λ and $\lambda + d\lambda$ are the wavelengths of two nearby spectral lines which are diffracted through angles

$$\theta$$
 and $\theta + d\theta$ respectively, then $D.P = \frac{d\theta}{d\lambda}$.

The condition for nth order principal maximum of wavelength λ is $\sin \theta = nN \lambda$ Differentiating both sides, $\cos\theta d\theta = nN d\lambda$

:. Dispersive power of the grating,

$$\frac{d\theta}{d\lambda} = \frac{nN}{\cos\theta}$$