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MODULE IV

DYNAMICS

Dynamics deals with the motion of bodies under the action of forces. It has two distinct parts - kinematics and kinetics.

Equations of Kinematics

$$s = ut + \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

$$v = u + at$$

Kinetics:

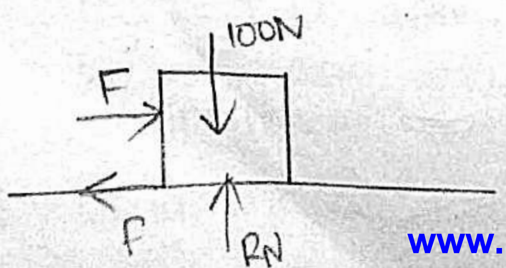
Kinetics is the study of the relation existing between the forces acting on a body, the mass of the body and the motion of the body.

Equations of motion (D'Alembert Principle/Newton's II law)

$$F = ma$$

$$F - ma = 0$$

Q: A block weighing 100 N rests on a horizontal plane. Find the magnitude of force required to give the box an acceleration of 2.5 ms^{-2}. The coefficient of kinetic friction between the block and the plane is 0.25.



$a = 2.5\,ms^{-2}$

$R_N = 100N$

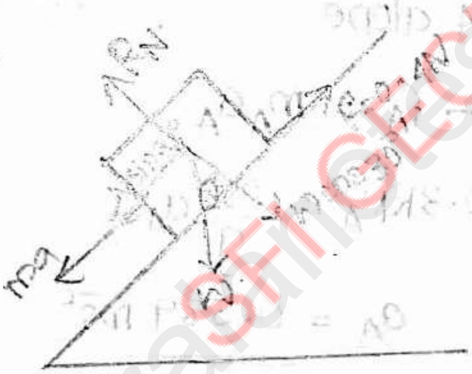
$F - f = ma$

$F - \mu R_N = ma$

$F - 0.25 \times 100 = \frac{100}{9.81} \times 2.5$

$F = 50.1\,N$

Q: A body of mass 50kg slides down a rough inclined plane inclined 30° to horizontal. coefficient of friction between plane and body is 0.4. Determine acceleration of the body.



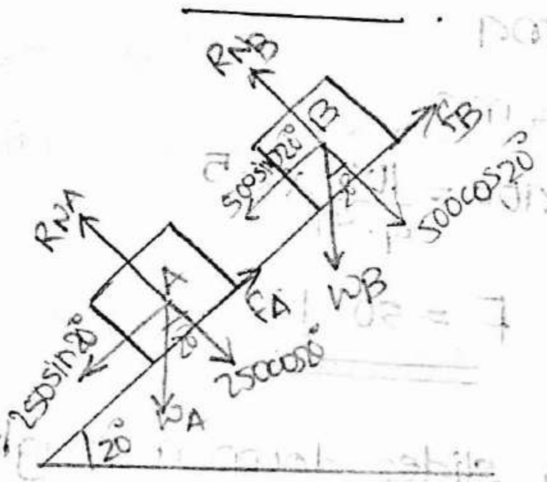
$-f + W\sin 30^\circ = ma$

$-0.4 \times 50 \times 9.81 \cos 30^\circ + 50 \times 9.81 \sin 30^\circ = 50a$

$a = \frac{75.336}{50}$

$= 1.51\,ms^{-2}$

Q: Two bodies A and B weighing 250N and 500N respectively are held stationary 10m apart and plane on a 20° inclined plane. coefficient of friction between A and plane is 0.3 while it is 0.2 between B and plane. If they are released simultaneously, calculate the time taken and the distance travelled by each block before they are at the verge of collision.



$$R_{NA} = 250\cos 20^\circ = 234.923\,N$$

$$R_{NB} = 500\cos 20^\circ = 469.846\,N$$

Consider motion of A alone

$$250\sin 20^\circ - f_A = m_A a_A$$

$$250\sin 20^\circ - 0.3R_{NA} = \frac{250}{g}a_A$$

$$\underline{a_A = 0.589\ ms^{-2}}$$

Consider motion of B alone

$$500\sin 20^\circ - 0.2R_{NB} = \frac{500}{g}a_B$$

$$\underline{a_B = 1.51\ ms^{-2}}$$

Let 'x' be the distance travelled by A in 't' seconds.

$\therefore$ Distance travelled by B in 't' sec is $(x+10)$.

$$S = ut + \frac{1}{2}at^2$$

$$x = \frac{1}{2} \times 0.589t^2$$

$$x+10 = \frac{1}{2} \times 1.51 \times t^2$$

$$\frac{x}{x+10} = \frac{0.589}{1.51}$$

$$1.51x = 0.589x + 5.89$$

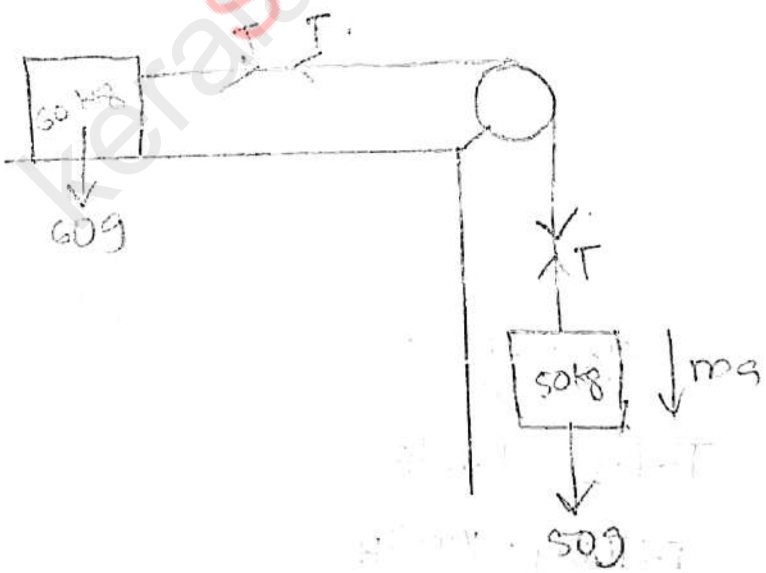
$$0.921x = 5.89$$

$$x = 6.4\,m$$

$$6.4 = \frac{1}{2} \times 0.589 t^2$$

$$\underline{\underline{t = 4.66s}}$$

Q: A mass of 60kg lies on a smooth horizontal plane. It is connected to a fine string passing through a smooth pulley at the edge of table to a mass 50kg hanging freely. Find tension in the string and acceleration of the system.



$$W - T = ma$$

$$50g - T = ma = 50a$$

$$T = 50(g-a) \longrightarrow (1)$$

$$T = ma = 60a \longrightarrow (2)$$

Sub ② in ①,

$$60a = 50g - 50a$$

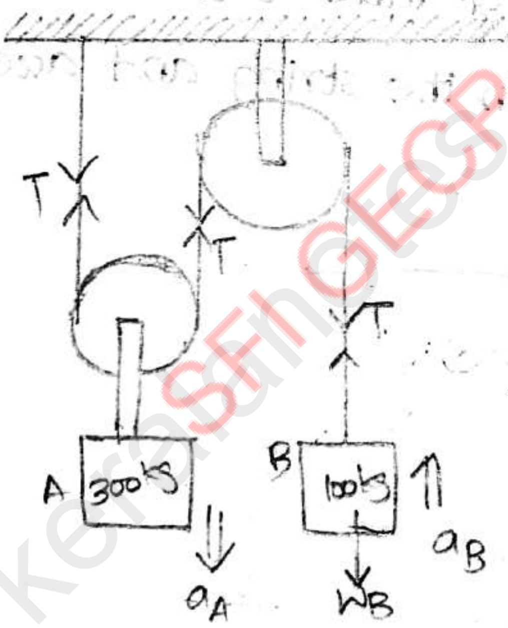
$$110a = 50g$$

$$a = \frac{5}{11}g = 4.46\ ms^{-2}$$

$$T = 60 \times 4.46$$

$$= \underline{267.55N}$$

Q: Determine tension in string and acceleration of two bodies of 300 kg and 100kg connected by a string by a frictionless, smoothbless pulley.



$$a_A = \frac{a_B}{2}$$

$$T - W_B = m_B a_B$$

$$T - 100g = 100a_B$$

$$T = 100(g + a_B) \longrightarrow ①$$

$$300g - 2T = m_A a_A$$

$$2T = 300g - 300a_A$$

$$T = 150(g - a_A) \longrightarrow ②$$

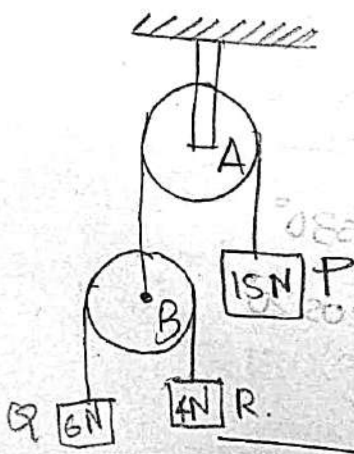
$$10g\sin 30^\circ - T = ma_1 = 10a_1$$
$$T - 5g\sin 20^\circ = ma_2 = 5a_2$$
$$a_1 = a_2 = a$$
$$5g - T = 10a$$
$$T - 5g\sin 20^\circ = 5a$$
$$5g - 10a = 5a + 5g\sin 20^\circ$$
$$15a = 5g - 5g\sin 20^\circ$$
$$3a = g - g\sin 20^\circ$$
$$\underline{\underline{a = 2.15\, ms^{-2}}}$$
$$T = 5g - 10a$$
$$= \underline{\underline{27.55\,N}}$$

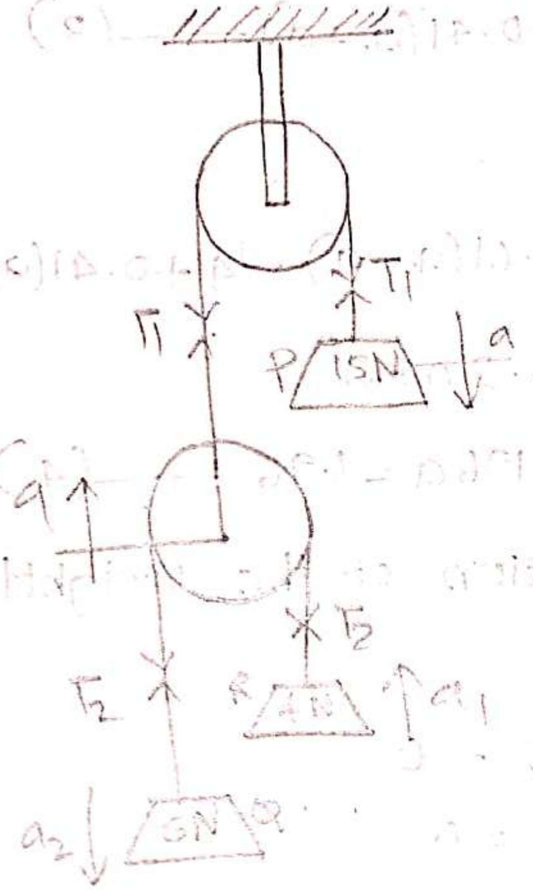
D'Alembert's Principle

It states that resultant of a system of force acting on a body is in dynamic equilibrium with inertia force.

$$F = ma$$
$$F + (-ma) = 0$$

Q: Find the acceleration of weights P, Q and R using D'Alembert's Principle.





Consider the downward motion of P.

$$F + (-ma) = 0$$

$$15 - T_1 - \frac{15}{9.81} \times a = 0$$

$$T_1 = 15 - 1.53a \quad \text{---(1)}$$

Consider the downward motion of Q.

$$F + (-ma) = 0$$

$$6 - T_2 - \frac{6}{9.81} \times (a_1 - a) = 0$$

$$T_2 = 6 - 0.61(a_1 - a) \quad \text{---(2)}$$

Conside the upward motion of R

$$F + (-ma) = 0$$

$$T_2 - 4 - \frac{4}{9.81}(a_1 + a) = 0$$

$$T_2 = 4 + 0.41(a_1 + a) \quad \text{——— (3)}$$

Eqn.s (2) and (3)

$$\Rightarrow T_2 = 6 - 0.61(a_1 - a) = 4 + 0.41(a_1 + a)$$

$$\Rightarrow 1.02a_1 - 0.2a = 2$$

$$a_1 - 0.196a = 1.96 \quad \text{——— (4)}$$

consider the motion of the weightless pulley B.

$$F + (-ma) = 0$$

$$F = 0$$

$$2T_2 - T_1 = 0$$

$$T_1 = 2T_2$$

From eqn.(1),

$$2T_2 = 15 - 1.53a$$

$$T_2 = 7.5 - 0.765a \quad \text{——— (5)}$$

Eqn.s. (2) and (5)

$$\Rightarrow 6 - 0.61(a_1 - a) = 7.5 - 0.765a$$

$$-a_1 + 2.25a = 2.46 \quad \text{——— (6)}$$

Adding (4) and (6)

$$2.054a = 4.42$$

$$a = 2.15 \text{ m/s}^2$$

From eqn. (4),

$\cancel{a_1+a}$

$$a_1 - 0.196a = 1.96$$

$$a_1 = 1.96 + 0.196 \times 2.15$$

$$= \underline{\underline{2.38 \ m/s^2}}$$

Acceleration of $P = a = 2.15 \ m/s^2$

Acceleration of $Q = a_1 - a = 2.38 - 2.15$

$$= \underline{\underline{0.23 \ m/s^2}}$$

Acceleration of $R = a_1 + a$

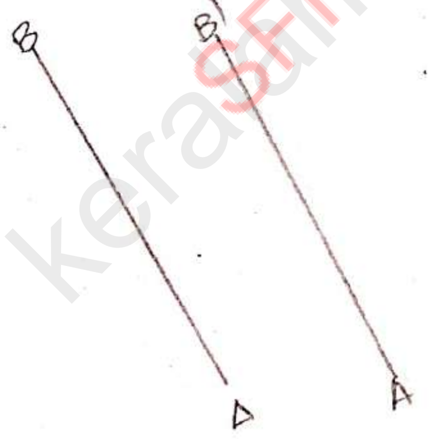
$$= 2.38 + 2.15$$

$$= \underline{\underline{4.53 \ m/s^2}}$$

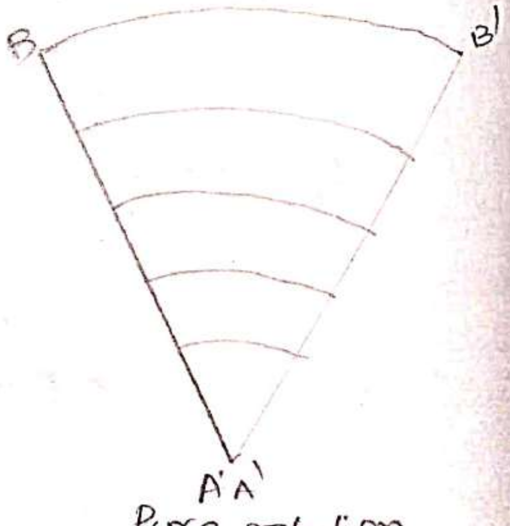
6/11/2019

	ANGULAR MOTION	RECTILINEAR MOTION
Initial velocity	ω_0	u
Final velocity	ω	v
Acceleration	α	a
	$V=\frac{d\theta}{dt}$	$V=\frac{ds}{dt}$
	$a=\frac{d\omega}{dt}$	$a=\frac{dv}{dt}$
	$\omega=\omega_0+\alpha t$	$v=u+at$
	$\omega^2=\omega_0^2+2\alpha\theta$	$v^2=u^2+2as$
	$\theta=\omega_0 t+\frac{1}{2}\alpha t^2$	$s=ut+\frac{1}{2}at^2$

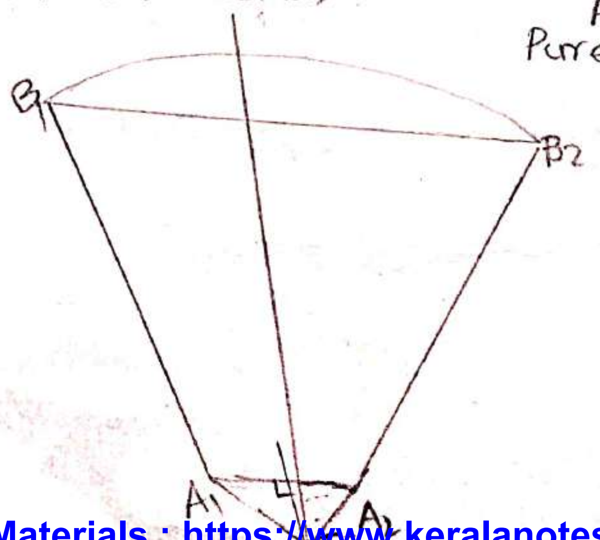
Combined Motions of Translation and Rotation.



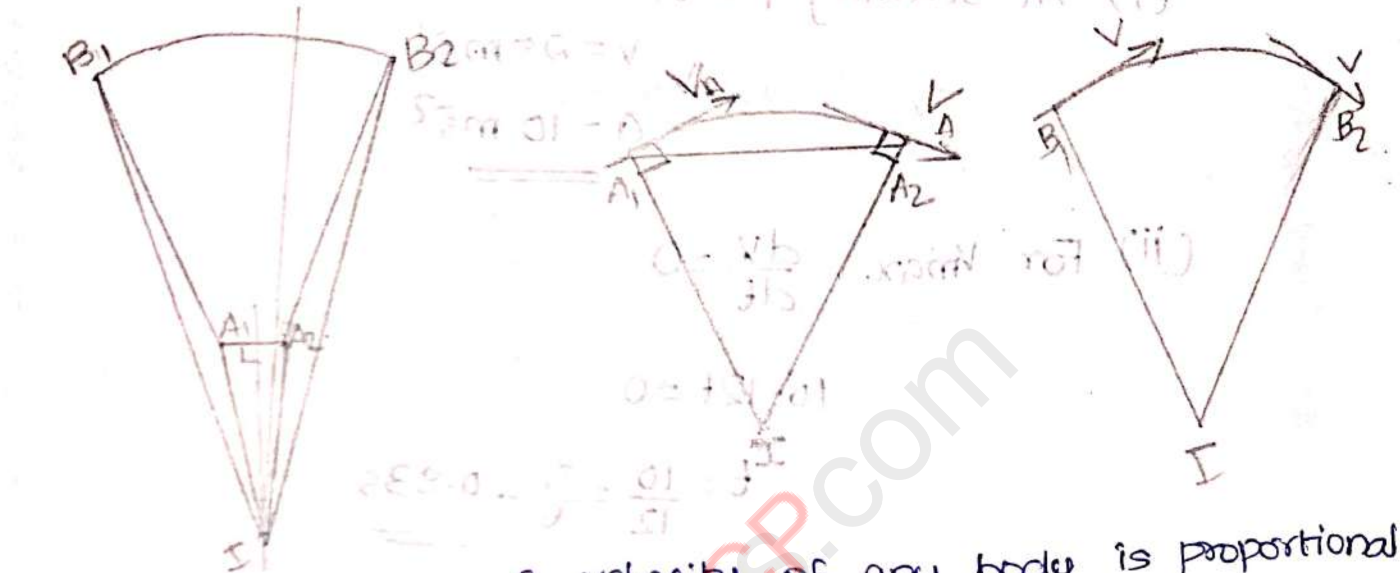
Pure translation



Pure rotation



combined motion of translation and rotation can be considered as a pure rotation about a single point at an instant. That point is called instantaneous centre.



- The magnitude of velocity of any body is proportional to distance from instantaneous centre and is equal to angular velocity times the distance.
- Direction of velocity is $\perp^{lar}$ to line joining point and instantaneous centre.

$$V_A = \omega_A I A_1 \quad V_B = \omega I B_1$$

Rectilinear Motion

Q: Motion of a particle along a straight line is defined as $s = 25t + 5t^2 - 2t^3$, where, s is in m and t in sec. Find:

(i) velocity and acceleration at starting point.

(ii) time the particle reaches maximum velocity and the maximum velocity of the particle.

$$S = 25t + 5t^2 - 2t^3$$

$$v = \frac{ds}{dt} = 25 + 10t - 6t^2$$

$$a = 10 - 12t$$

(i) At starting point, $t = 0$

$$v = 25\ ms^{-1}$$

$$a = 10\ ms^{-2}$$

(ii) For $V_{max.}$, $\frac{dv}{dt} = 0$

$$10 - 12t = 0$$

$$t = \frac{10}{12} = \frac{5}{6} = 0.83s$$

$$V_{max.} = 25 + 10(0.83) - 6(0.83)^2$$

$$= 25 + 8.3 - 4.13$$

$$= 29.17\ ms^{-1}$$

Q: A point is moving in a straight line with acceleration given by $a = 15t - 20$. It passes through a reference point at $t = 0$ and another point 30m away after an interval of 5sec. Calculate displacement, velocity and acceleration of the point after a further interval of 5 sec.

$$a = 15t - 20$$

$$\frac{dv}{dt} = 15t - 20$$

$$dV = 15t\,dt - 20\,dt$$

$$V = \frac{15t^2}{2} - 20t + C$$

$$\frac{dx}{dt} = \frac{15t^2}{2} - 20t + C$$

$$x = \frac{15t^3}{2\times3} - \frac{20t^2}{2} + Ct + D$$

$$x = \frac{5t^3}{2} - 10t^2 + Ct + D$$

At $t=0,\ x=0$

$$D = 0$$

At $t=5,\ x=30$

$$30 = \frac{5(5)^3}{2} - 10(5)^2 + (5)C$$

$$30 = \frac{625}{2} - 250 + 5C$$

$$280\times2 = 625 + 10C$$

$$10C = -65$$

$$C = -6.5$$

$$x = 2.5t^3 - 10t^2 - 6.5t$$

$$V = 17.5t^2 - 20t - 6.5$$

At $t = 10\,s$

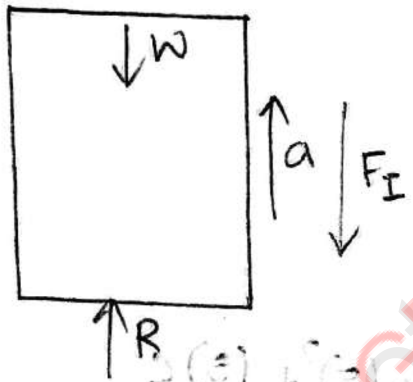
$$x = 2.5(10)^3 - 10(10)^2 - 6.5(10)$$

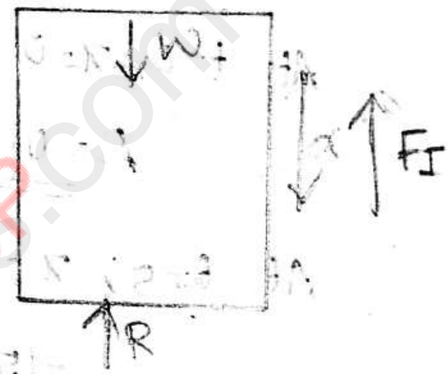
$$= 1435\,m$$

$$V = 7.5(10)^2 - 20(10) + 6.5$$
$$= \underline{\underline{543.5\ ms^{-1}}}$$

$$a = 15t - 20$$
$$= 15(10) - 20$$
$$= \underline{\underline{130\ ms^{-2}}}$$

MOTION OF LIFT





$$F = ma$$
$$F + F_I = 0$$
$$F + (-ma) = 0$$
$$(R - W) - \frac{W}{g}a = 0$$
$$R = W\left[1 + \frac{a}{g}\right].$$

$$F = ma$$
$$F + F_I = 0$$
$$F + (-ma) = 0$$
$$(W - R) - \frac{W}{g}a = 0$$
$$R = W\left[1 - \frac{a}{g}\right].$$

Q: A lift has an upward acceleration of $1.2\ ms^{-2}$. What force will a man weighing 750N exert on the floor of the lift? Also find the force exerted if it is moving with a downward acceleration $1.2\ ms^{-2}$. Find the upward acceleration of lift which cause a weight to exert a force 900N on the floor.

$$R = 750\left[1 + \frac{1.2}{9.8}\right]$$

$$= \frac{750 \times 11}{9.8} = \frac{8250}{9.8}$$

$$= \underline{841.8\,N}$$

$$R = 750\left[1 - \frac{1.2}{9.8}\right]$$

$$= \frac{750 \times 8.6}{9.8}$$

$$= \underline{658.16\,N}$$

$$900 = 750\left[1 + \frac{a}{9.8}\right]$$

$$\frac{900}{750} - 1 = \frac{a}{9.8}$$

$$\frac{150}{750} = \frac{a}{9.8}$$

$$a = \frac{3}{15} \times 9.8 = \underline{19.6\,ms^{-2}}$$

Q: Calculate the work done in pulling up a block weighing 20 kN for a length of 5m on a smooth plane inclined 20° with horizontal

$$W = F \cdot S$$
$$= 20\times10^3 \times 5 \sin 20^\circ$$
$$= 34.2\times10^3\ Nm$$

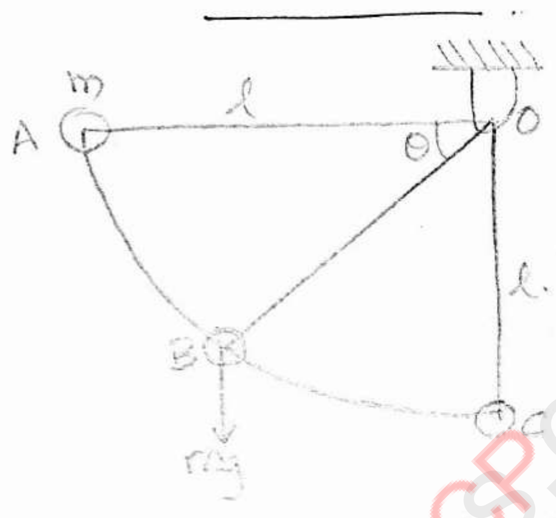
Impulse - Momentum.

$$Ft = mV_2 - mV_1$$

Q: An automobile weighing 25kN is moving at a speed of 60 km/hr. When the brakes are fully applied causing all four wheels to speed up determine the time required to stop the automobile. coefficient of friction between road and tyre is 0.5.

$$V_1 = 60\ km/hr = \left(60\times\frac{5}{18}\right) ms^{-1}$$
$$V_2 = 0$$
$$F = \mu R_N = 0.5\times25\times10^3$$
$$0.5\times25\times10^3\times t = \frac{25}{9}\left(60\times\frac{5}{18}\right)$$
$$t = \frac{300}{18\times9.8\times0.5\times10^3}$$
$$= 3.4\times10^{-3}\ s$$

Q: A simple pendulum is ~~rest~~ released from rest at A with the strings horizontal and swings downward. Express the velocity of ball as a function of angle 'θ'. Also obtain the expression for angular velocity of ball when the string is in vertical position.



A-B Workdone $= mgl\sin\theta$

Change in K.E $= \frac{1}{2}mV_B^2 - \frac{1}{2}mV_A^2$

$$V_B = \sqrt{2gl\sin\theta}$$

$$V_C = \sqrt{2gl\sin 90^\circ}$$

$$= \sqrt{2gl}$$

$$\omega = \frac{V}{r}$$

$$= \frac{\sqrt{2gl}}{l}$$

$$= \sqrt{\frac{2g}{l}}$$