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SFI GEC PALAKKAD

Magnetism & electromagnetic theory

Electro-Magnetic theory

* **Gauss' theorem in Electrostatics**

It states that electric flux over a closed surface is equal to the $1/\epsilon_0$ times net charge enclosed by the surface.

OR,

Surface Integral of Electric field over a closed surface 'S' is $1/\epsilon_0$ times net charge enclosed by surface.

$$\boxed{\phi_E = \oint_S \vec{E} \cdot \vec{ds} = \frac{1}{\epsilon_0} \cdot \Sigma q}$$

* **Gauss' Law in Magnetostatics**

Magnetic flux enclosed by a closed surface 'S' is zero. $\boxed{\phi_B = \oint_S \vec{B} \cdot \vec{ds} = 0}$

Gauss' Law in magnetism states that the magnetic field lines going into the closed surface is exactly balanced by fieldlines coming out. It tells magnetic monopoles donot exist.

* **Ampere Circuital theorem**

The line integral of magnetic flux density is 'μ_0' times current enclosed by the path.

$$\boxed{\oint_c \vec{B} \cdot \vec{dl} = \mu_0 i_{en}}$$

(c → contour Integral)

* Faraday's Law of electromagnetic theory

when magnetic flux linked with the Circuit changes and EMF is induced in it. This induced EMF is equal to rate of change of magnetic flux linked with the Circuit. It always opposes the changes in magnetic flux.

$$\boxed{e = -\frac{d\phi}{dt}}$$

* Stoke's theorem

It connects line integral and surface integral.

$$\oint_C \vec{A}\cdot d\vec{l} = \iint_S (\nabla \times \vec{A})\cdot d\vec{s}$$

* Gauss' Divergance theorem

It connects surface Integral to volume integral.

$$\iint_S \vec{A}\cdot d\vec{s} = \iiint_V (\nabla \cdot A)\cdot d\vec{v}$$

* Magnetic field

It is the area around a magnet (or current carrying conductor) in which there is a magnetic force. Mag. field is a vector Quantity. it is represented by mag. flux density ($\vec{B}$) or intensity of magnetic field ($\vec{H}$)

* Magnetic flux density

Total no. of magnetic fieldlines passing ⊥rly through unit area. Let 'ϕ' is magnetic flux passing normally through an area 'A', then flux density = ϕ/A (w/m² = tesla). Unit of 'B' is w/m² or tesla

* Intensity of Magnetisation (M)

It is a measure of magnetisation of a magnetised specimen. It is defined as magnetic moment per volume. It is

$$M = \frac{m}{V} = \frac{2l \times p}{V}$$ where,

magnetic moment (m) is product of pole strength and total length of specimen, $m = 2l \times p$

* Susceptibility (χ)

It is a measure of how much a material become magnetized in an applied magnetic field. It is ratio of magnetization to intensity of magnetic field. $\boxed{\chi = \frac{M}{H}}$

* permeability (μ)

It is defined as the property of material to allow the magnetic lines of force to pass through it. It is ratio of magnetic flux density to intensity of magnetic field.

$$\mu = B/H$$

* Relative permeability

Relative permeability of material is the comparison of permeability with vacuum or free space.

$\mu_R = \mu/\mu_0$, $\mu \rightarrow$ permeability of medium

$\mu_0 \rightarrow$ permeability of free space.

$= 4\pi \times 10^{-7}$ H/m

* Relationship b/w Relative permeability & Susceptibility

* When a magnetic material is placed in a magnetic field of uniform intensity 'H',

magnetic flux pass through it due to magnetised field (H) and due to material being magnetised

Then, magnetic fluxdensity, $B = \mu_0 (\vec{H} + \vec{M})$

$$\mu_r = \mu/\mu_0 \quad ; \quad \chi = \frac{\vec{M}}{\vec{H}} \quad ; \quad \mu = \frac{B}{H}$$

$$\vec{B} = \mu_0 (\vec{H} + \vec{M})$$

$$\frac{\vec{B}}{\vec{H}} = \mu_0 \left(\frac{\vec{H}}{\vec{H}} + \frac{\vec{M}}{\vec{H}} \right) = \mu_0 (1 + \chi)$$

$$\mu = \mu_0 (1 + \chi)$$

$$\frac{\mu}{\mu_0} = (1 + \chi)$$

$$\boxed{\mu_r = (1 + \chi)}$$

Del operator (∇)

The vector operator (∇) is defined as

$$\nabla = \hat{i} \frac{\partial}{\partial x} + \hat{j} \frac{\partial}{\partial y} + \hat{k} \cdot \frac{\partial}{\partial z}$$, where $\hat{i}, \hat{j}, \hat{k}$ represents the unit vectors along x, y, z directions.

Using ∇ operator, we can do 3 vector operations. gradient, divergance and curl.

1. Gradient: The gradient of a scalar function $\phi(x y z)$ is defined as $\text{grad}\,\phi = \nabla \phi = \left[\hat{i} \frac{\partial}{\partial x} + \hat{j} \frac{\partial}{\partial y} + \hat{k} \frac{\partial}{\partial z} \right] \phi(x y z)$

$$= \hat{i} \frac{\partial \phi(x y z)}{\partial x} + \hat{j} \frac{\partial}{\partial y} \phi(x, y, z) + \hat{k} \frac{\partial \phi(x y z)}{\partial z}$$

The vector operator ∇ acting on a scalar function $\phi(x, y, z)$ gives a vector $\nabla \phi$. PHYSICAL SIGNIFICANCE (The direction of $\nabla \phi$ at any point is the direction in which we must move from that point to find the most rapid increase in function 'ϕ' with coordinates.), Q:3

Equation of Continuity

$\nabla \cdot \vec{J} = -\frac{\partial \rho}{\partial t}$, $\vec{J}$ is current density, ρ is charge density

Imagine a volume bounded by a closed surface. The amount of charge that comes out through the surface is equal to the rate of decrease of charge contained in the volume. This is Law of Conservation of Electric charge stated for time varying situations.

Let, '$\vec{J}$' be the current density (current/unit Area) at a point. $\nabla \cdot \vec{J}$ gives the net outflow of charge in unit time through closed surface, that encloses unit volume. charge contained in the unit volume is 'ρ'. Rate of decrease of charge in unit volume is $-\frac{\partial \rho}{\partial t}$. Acc to Eqn of Continuity

$$\nabla \cdot \vec{J} = -\frac{\partial \rho}{\partial t}$$

Curl

$\overline{A} = A_x \hat{i}$

curl of a vector valued function in a region is again a vector-valued function. Let,

$$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

$$\text{curl } \overline{A} = \nabla \times \vec{A}$$

$$= \left[\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right] \times \left[A_x \hat{i} + A_y \hat{j} + A_z \hat{k}\right]$$

$$\therefore \nabla \times \vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

$$= \hat{i}\left[\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right] + \hat{j}\left[\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right] + \hat{k}\left[\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right]$$

If $\nabla \times \vec{A} = 0$, then $\vec{A}$ is irrotational

physical Significance

Curl is also known as rotation of a vector. The magnitude of curl A $(|\nabla \times \vec{A}|)$ gives the line integral of $\vec{A}$ around a closed path that encloses unit area.

$$|\nabla \times \vec{A}| = \int_c \vec{A} \cdot d\vec{l}$$

direction of Curl A is given by right hand rule.

Maxwell's Equation

Gauss law in Electrostatics

$$\oint_s \vec{E} \cdot d\vec{s} = \frac{q_{en}}{\epsilon_0}$$

Applying Gauss divergence theorem,

$$\iint_s \vec{A} \cdot d\vec{s} = \iiint_v (\nabla \cdot \vec{A}) \cdot dv$$

$$\oint_s \vec{E} \cdot d\vec{s} = \iint_s \vec{E} \cdot d\vec{s} = \iiint_v (\nabla \cdot \vec{E}) \cdot dv$$

$$\rho = \frac{dq}{dv} \rightarrow dq = \rho \cdot dv$$

$$q_{en} = \int dq = \iiint_v \rho \cdot dv$$

$$\oint_s \vec{E} \cdot d\vec{s} = \frac{q_{en}}{\epsilon_0}$$

$$\iiint_V (\nabla\cdot\vec{E})\cdot dv = \frac{1}{\epsilon_0}\iiint_V \rho \cdot dv$$

$$\iiint_V (\nabla\cdot\vec{E})\cdot dv = \iiint_V \frac{\rho}{\epsilon_0}\cdot dv$$

$$\nabla\cdot\vec{E} = \frac{\rho}{\epsilon_0}$$

$$\nabla \epsilon_0 \vec{E} = \rho$$

$$\epsilon_0 \cdot \vec{E} = \vec{D}$$

$\nabla\cdot\vec{D} = \rho$, it is called first Maxwell's equation

→ Electric flux Eq^n

→ Differential form of Gauss' law in electrostatics.

* In free space, charge density = 0, then

$$\nabla\cdot\vec{D} = 0$$

differential form Gauss' Law in magnetostatics

$$\oint_S \vec{B}\cdot d\vec{s} = 0$$

Applying Gauss' divergence Law,

$$\iint_S \vec{B}\cdot d\vec{s} = \iiint_V (\nabla \vec{B})\, dv$$

$$\iiint_V (\vec{\nabla}\cdot\vec{B})\cdot dv = 0$$

$\nabla\cdot\vec{B} = 0$, → magnetic flux Eq^n

→ Second maxwell's Eq^n

→ differential form of Gauss' Law in magnetostatics.

1. $\nabla\cdot\vec{D} = \rho$
2. $\nabla\cdot\vec{B} = 0$

} They are known as maxwell's divergence Eq^n.

MAGNETIC FIELD

It is a vector field generated by moving electric charge. Magnetic field is area around a magnet in which there is magnetic force, or it is the force experienced by unit positive charge in motion. It is a vector quantity can be represented either by magnetic flux density B or magnetic field intensity H.

Magnetic Flux density or Magnetic Induction B:

Magnetic flux density B is defined as the total number of magnetic field lines passing perpendicularly through unit area.

If ϕ is the magnetic flux passing normally through an area A, then

Magnetic Flux density B= $\frac{\phi}{A}$

Unit of B is weber/m^2

Gauss's law for magnetic flux

By Gauss's law the surface integral of magnetic flux over a closed surface is equal to zero

$$\oint \vec{B}.\,\vec{ds} = 0$$

[Flux linked with a closed surface in magnetic field is zero].

Consider a small vector area of element ds of a closed surface S. The Magnetic flux though this area passing normally is BCos$\theta.\,ds$ = $\vec{B}.\vec{ds}$

The total magnetic flux passing normally though the closed surface S=$\oint d\phi = \oint \vec{B}.\,\vec{ds}$

By Gauss's *law* $\oint \overrightarrow{B}.\overrightarrow{ds} = 0$

This means that the magnetic flux through a closed surface is always zero because all the magnetic field lines going in to the closed surface are exactly balanced by field lines coming out ie.it is also indicate magnetic mono pole do not exist, there is no starting point and ending point for magnetic flux.

Ampere's Circuital Law

Oersted observed that a magnetic field is always produced around a conductor carrying current. It states that a line integral of magnetic flux density B for a closed path is equal to μ_0 times the net current I enclosed by the path

$$\oint B.dl = \mu_0 I$$

μ_0 is the permeability of free space

$\mu_0 = 4\pi\ x\ 10^{-7}$ H/m

Faraday's law of electromagnetic induction

1.Whenever the magnetic flux linked with a circuit changes an e.m f is induced in the circuit. The induced e.m.f last as long as the flux changes.

2.The magnitude of induced em.f is equal to the rate of change of magnetic flux.

Induced e.m.f $e = \frac{d\phi}{dt}$

Lens's Law

It states that the induced current produced in a circuit flows in such a direction that it opposes the change.

$e = \frac{-d\phi}{dt}$ The -ve sign shows that induced emf opposes the change of magnetic flux.

Intensity of Magnetisation M

It is a measure of magnetisation of a magnetised material or extent to which a specimen is magnetised when placed in a magnetised field. It is defined as magnetic moment per unit volume of maternal. Its unit is A/m.

$$M = \frac{Magnetic\ moment}{Volume}$$

$$M = \frac{P\ x\ 2l}{V}$$

P = Pole strength

2 l= length of magnet.

Magnetic Susceptibility χ

It is a dimension less proportionality constant that indicates the degree of magnetization of a material in response to a applied magnetic field. It is the ratio of magnetisation M to the applied magnetising field intensity.

$$\chi = \frac{M}{H}$$

$$M = \chi H$$

Magnetic permeability μ

It is the property of the material to allow magnetic lines of force to pass through it. The permeability of the, material is equal to ratio of the flux density (B) in the medium to the field intensity (H)

$$\mu = \frac{B}{H}$$

Its Value denotes the ease with which the magnetic field lines pass through the material.

The relative permeability of the material is the comparison of permeability with the permeability of air or vacuum

Relative permeability $\mu_r = \frac{\mu}{\mu_o}$

Let B be the value of magnetic induction in a material when a magnetizing field H is applied, and B_o be the magnetic induction at someplace when material is removed.

$$B_0 \,\alpha\, H \,, \quad B = \mu_0 H$$

$$B \,\alpha\, H \,, \quad B = \mu H$$

$$\boxed{\frac{B}{B_0} = \frac{\mu}{\mu_0} = \mu_r}$$

When the magnetic material is placed in magnetic field intensity H, magnetic flux pass through it and material being magnetised. Magnetic flux density B is seen inside magnet is the result of applied magnetic field H and intensity of internal magnetisation M.

The flux density $B = \mu_0[M + H]$

$$=\mu_0[\chi H + H] \qquad \left[\because \chi = \frac{M}{H}\right]$$

$$B = \mu_0 H[1+\chi]$$

$$\mu_0 H[1+\chi] = \mu H$$

$$\boxed{\mu = \mu_0[1+\chi]}$$

Where μ is the permeability of the medium

In terms of relative permeability

$$\mu_r = \frac{\mu}{\mu_0} = \frac{\mu_0(1+\chi)}{\mu_0} = 1+\chi$$

$$\mu_r = 1+\chi$$

$$\boxed{\chi = \mu_r - 1}$$

Susceptibility can be zero, positive or negative

Classification of the Magnetic material

Magnetic Properties are induced in the material in presence of external magnetic field. On the basis of different magnetic properties materials are classified into different types.

1.Dimagnetism

2.Paramagnetism

3.Ferromagnetism

1.Diamagnetism

- If a diamagnetic material is exposed to a magnetizing field, it develops a magnetic moment in material in opposite direction, so that net magnetic field decreases.
- They are the substance which have tendency to move from stronger to weaker magnetic field.
- In diamagnetic substances, the resultant magnetic moment due to all atomic current loop is zero in the absence of an external magnetic field. But in the presence of an external magnetic field, weak magnetic dipole moments are produced in the atoms. The resultant of all the induced magnetic dipole moments produces a feeble net magnetic moment in the material to a direction opposite to that of the external field. This magnetic moment disappears when the external field is removed. Thus, a diamagnetic material placed near to a magnet is repelled by it.
- If a rod of diamagnetic material is freely suspended horizontally in uniform magnetic field it will orient perpendicular to field.
- Susceptibility is a small negative value
- Relative permeability is less than1.
- When diamagnetic material placed in a magnetic field, magnetic lines of force expelled from the material. The phenomenon of a perfect diamagnetism in super conductors is called Meissner effect
- The diamagnetism is independent of temperature

  Eg. Water, Hydrogen, Bismuth, Gold, Silver

In the presence of an external field the atomic magnetic dipole moments align opposite to the applied field.

Langevin Theory of diamagnetism

Langevin theoretically calculated the susceptibility of a diamagnetic material taking the magnetic moment produced due to orbital motion of electron.

$$\chi = \frac{\mu_0 e^2 n z}{6m} < r >$$

μ_0-Permeability of free space

n - number of atoms per unit volume

z-Atomic number

e-electronic charge

m-mass of electrons

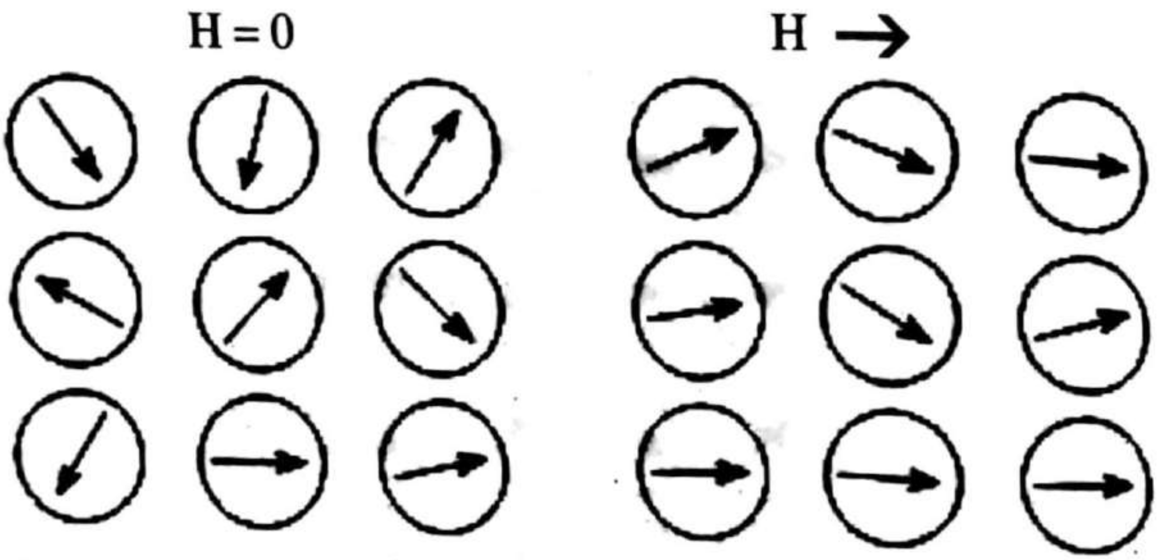
<r>-mean square distance of electrons from nucleus.

2. Para magnetism

- The material which possess a little net magnetic moment in the direction of applied external magnetic field are para magnetic material.
- When paramagnetic substance are placed in an external magnetic field they are feebly magnetised and magnetic moments are aligned in direction of magnetic field.
- They show tendency to move from weak magnetic field to strong magnetic field when kept in non-uniform magnetic field but induced magnetic field is weak.
- Paramagnetism is due to the presence of unpaired electrons in atoms or molecules.
- Paramagnets do not retain any magnetisation in the absences of externally applied magnetic field
- When para magnetic rod is freely suspended in magnetic field, it comes to rest in the direction of field.
- Susceptibility is positive and small. It is of the order of 10^{-3} to 10^{-5}
- Relative permeability is slightly greater than one
- The paramagnetic susceptibility depends on temperature
- According to Curies law magnetic susceptibility of paramagnetic material is inversely proportional to its temperature.
- $\chi \alpha \frac{I}{T}$

  $\chi = \frac{C}{T}$ Where C is Curse's constant

Eg Li, Mg, Cu, Cr. Pt,



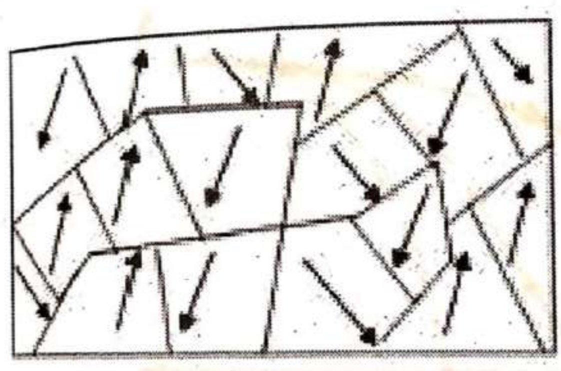
The atomic magnetic dipole moments are oriented randomly in the absence of an external field.

The atomic magnetic dipole moments are alined in the direction of the field.

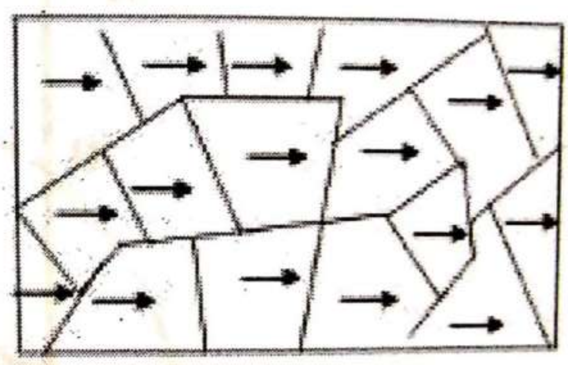
3. Ferro magnetism

- When ferromagnetic substance are placed in a magnetic field , they are strongly magnetized in the direction of magnetic field. Eg. Iron, cobalt, nickel, steel etc
- In ferromagnetic materials, the magnetisation exists even after the removal of magnetizing field.
- They get their magnetic property not only because of their atomic magnetic moment but also due to certain regions called magnetic domains. In each domain a very large number of atoms are aligned parallel to each other. So, the magnetic force with the domain is very strong.

- Before magnetization the magnetic domains are randomly oriented relative to each other .When strong magnetic field is applied all domains within the material are aligned known as magnetically saturated.
- Susceptibility is positive and very high
- Relative permeability is very high
- When placed in a magnetic filed lines of force have high concentration in materials.
- When ferromagnetic substances are suspended in a magnetic field, it comes to rest in direction of field.
- On increasing the temperature, the ferro magnetism gradually decreases and at a particular temperature called curie point the ferro magnetic properties of material disappear and it becomes paramagnetic. The temperature is called Curie temperature.



Ferromagentic Domains each domain has a net magnetization even in the absence of an external field



In the presence of an external field, the magnetic moments of domains alin in the field direction

Curie Weiss Law

According to curie Weiss law at above curie temperature the magnetic susceptibility of a ferromagnetic material is inversely proportional to $T - T_c$

$$\chi = \frac{C}{T - T_c}$$ C is Curie constant

T is the absolute temperature

T_c is the Curie temperature

Comparison

Diamagnetic	Paramagnetic	Ferromagnetic
The individual atoms or molecules have not net magnetic dipole moment in the absence of an external field	The individual atoms or molecules have a net magnetic dipole moment in the absence of an external field	The individual atoms, or molecules have a net magnetic dipole moment and these atomic magnets organises into domains in the absence of an external field.
They are weakly repelled by a magnet	They are weakly attracted by a magnet	They are strongly attracted by a magnet
They try to expel the magnetic field lines when placed in an external field and the resultant field within the material is reduced	They try to concentrate the magnetic field lines within them when placed in an external field and the resultant field within the material is enhanced.	The magnetic field lines are highly concentrated within them when placed in an external field and the resultant field is strongly enhanced.
They tend to move from a region of strong field to a region of weak field when placed in non uniform field.	They tend to move from a region of weak field to a region of strong field when placed in a non uniform field.	They tend to move from a region of weak field to a region of strong field when placed in a non uniform field.

Susceptibility is negative	Susceptibility is small and positive	Susceptibility is large positive value
Susceptibility is independent of temperature and the substances does not obey Curie 's law	Susceptibility varies inversely with temperature and the substance obeys Curie' law	Susceptibility Varies inversely with temperature and above Curie temperature the ferromagnetic substance becomes paramagnetic above Curie temperature, the substances obeys Curie-Weiss law.
Relative permeability is less than unity	Relative permeability is slightly greater than unity	Relative permeability is greater that unity
Do not exhibit the phenomenon of Hysteresis	Do not exhibit the phenomenon of Hysteresis	Exhibits the phenomenon of Hystereis
Examples: Gold, Copper, Antimony, Bismuth, Lead, Quartz, Air, Hydrogen, Water, Alcohol, Sodium Cholride etc.	Examples: Platinum, Aluminium, Lithinum, Magnesium, Chromium, Copper Chloride, etc	Examples: Iron, Nickel, Cobalt, Steel, Alnico etc