സഹായി

SFI GEC PALAKKAD

MODULE - 3

AC CIRCUIT

(~~1) 10+j30~~

(1) Add two voltages $50\angle 30$ and $30\angle -45$ and show the result in polar form.

Ans: $50\angle 30 \rightarrow 43.30 + j25$ $\quad$ $30\angle -45 \rightarrow 21.21 - j21.21$

$= 64.51 + j3.79$

$= 64.62\angle 3.36^\circ$

(2) Perform the following operation and give the result in polar form.

$(10-j10)\ (10\angle -60^\circ)\ (10e^{j30})$

$(14.14\angle -45^\circ)\ (10\angle -60^\circ)\ (10\angle 30^\circ)$

$1414\angle -75$

-45
-60
$\overline{105}$
30

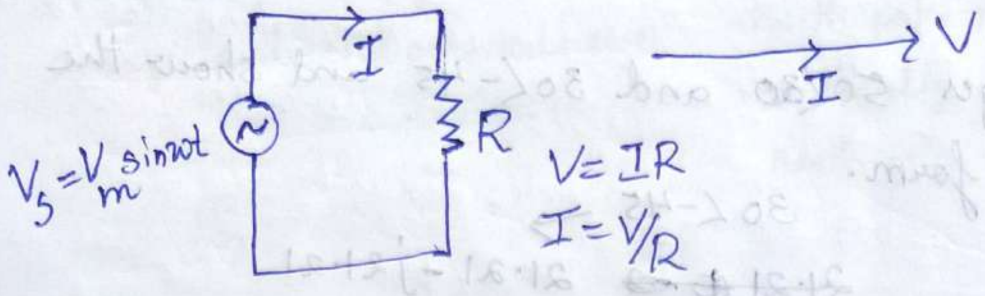
(3) $\dfrac{(5e^{j30})\ (2-j4)}{(3+j3)\ (2\angle -15)}$

Ans: $\dfrac{(5\angle 30)\ (4.47\angle -63.43)}{(4.24\angle 45)\ (2\angle -15)} = \dfrac{(22.35\angle -33.43)}{(8.48\angle 30)}$

$= (2.63\angle -63.43)$

AC Circuits Phasor diagram

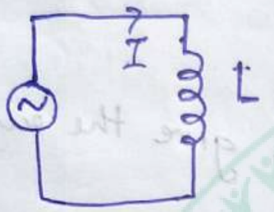
Circuit with resistance



$V = IR$

$I = V/R$

Circuit with inductor



$e = L\frac{dI}{dt}$, voltage ~~the~~ across the inductor

$V_m \sin \omega t = L\frac{di}{dt}$

$V_m \sin \omega t \, dt = L dI$

$\int \frac{V_m}{L} \sin \omega t \, dt = \int dI$

$I = \frac{V_m}{L} \int \sin \omega t \, dt$

$I = \frac{V_m}{L\omega} - \cos \omega t$

$I = \frac{V_m}{(L\omega)} - \cos \omega t$

$L\omega = X_L$, inductive reactance.

$\omega = 2\pi f$ (angular frequency)

$$I = \frac{V_m}{X_L} \cdot (-\cos \omega t)$$

$$I = I_m \sin(\omega t - \pi/2)$$

$$V_s = V_m \sin \omega t$$

Current lags by phase of 90° behind voltage.

Power in purely inductive circuit

$$p = VI$$

$$= V_m \sin \omega t \cdot -I_m \cos \omega t$$

$$= -V_m I_m \sin \omega t \cos \omega t$$

$$= -V_m I_m \frac{\sin 2\omega t}{2}$$

$$= -\frac{V_m I_m}{2} \cdot \sin 2\omega t$$

$$\text{Average power} = \frac{1}{2\pi} \int_0^{2\pi} -\frac{V_m I_m}{2} \sin 2\omega t \, d\omega t$$

$$= -\frac{V_m I_m}{4\pi} \int_0^{2\pi} \sin 2\omega t \, d\omega t$$

$$-\frac{V_m I_m}{4\pi} \left[\frac{-\cos 2\omega t}{2} \right]_0^{2\pi}$$

$$= -\frac{V_m I_m}{8\pi} \left[-\cos 4\pi - (-\cos 0) \right]$$

$$= -\frac{V_m I_m}{8\pi} \left[-1 + 1 \right] = 0$$

Average Power in purely inductive circuit is zero.

$V_m \sin \omega t = L \frac{dI}{dt}$

$V_m \sin \omega t \, dt = L \, dI$

$\int \frac{V_m}{L} \sin \omega t \, dt = \int dI$

$I = \frac{V_m}{L} \left(\int \sin \omega t \, dt \right)$

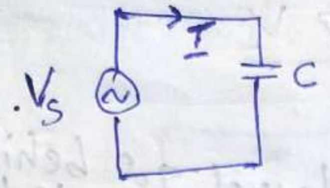
$I = \frac{V_m}{L\omega} \cdot -\cos \omega t$

$X_L = L\omega$

$I = \frac{V_m}{X_L} \cdot -\cos \omega t$

$I = I_m \sin(\omega t - \pi/2)$

Circuit with capacitance



$$i = \frac{dq}{dt}$$

$$q = CV$$

$$i = \frac{d(CV)}{dt}$$

$$i = C\frac{dv}{dt}$$

$$i = C\frac{d}{dt}V_m \sin \omega t$$

$$i = C \cdot V_m \frac{d}{dt}\sin \omega t$$

$$i = CV_m \cos \omega t \cdot \omega$$

$$i = CV_m \omega \cos \omega t$$

$$i = \frac{V_m}{1/C\omega} \sin(\omega t + 90)$$

$$\frac{1}{C\omega} = X_C$$

capacitive reactance

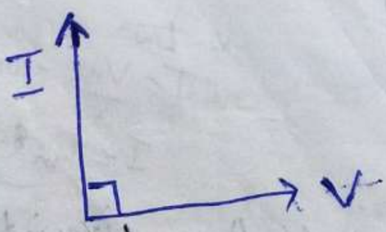
$$\omega = 2\pi f$$

Unit $= \Omega$

$$i = \frac{V_m}{X_C}\sin(\omega t + 90)$$

$$i = I_m \sin(\omega t + 90)$$

$$V = V_m \sin \omega t$$



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Power in Purely ~~inductive~~ capacitive Circuit

$P = V \cdot I$

$P = V_m \sin \omega t \; I_m \sin(\omega t + 90)$

$= V_m I_m \sin \omega t \cdot \cos \omega t$

$= V_m \cdot I_m \frac{\sin 2\omega t}{2}$

$= \frac{V_m \cdot I_m}{2} \cdot \sin 2\omega t$

Average power $= \frac{1}{2\pi} \int_0^{2\pi} \frac{V_m \cdot I_m}{2} \sin 2\omega t \, d\omega t$

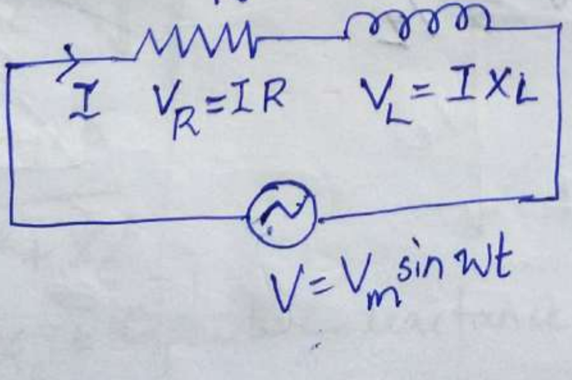
$= \frac{V_m I_m}{4\pi} \int_0^{2\pi} \sin 2\omega t \, d\omega t$

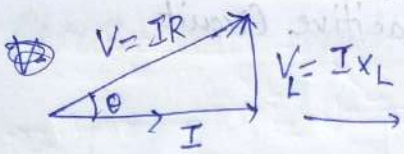
$= \frac{V_m I_m}{4\pi} \left[\frac{-\cos 2\omega t}{2} \right]_0^{2\pi}$

$= \frac{V_m I_m}{8\pi} \left[-\cos 4\pi + \cos 0 \right]$

$= \frac{V_m I_m}{8\pi} \times 0 = 0$

Series RL circuit





→ Voltage △le.

$$V = \sqrt{V_R^2 + V_L^2}$$

$$= \sqrt{(IR)^2 + (IX_L)^2}$$

$$= \sqrt{I^2(R^2 + X_L^2)}$$

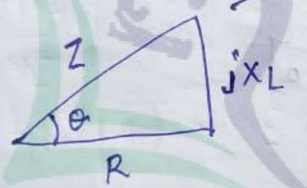
$$V = I\sqrt{R^2 + X_L^2}$$

Impedance unit Ω

$$Z = \sqrt{R^2 + X_L^2}$$

$$V = IZ$$

$$I = V/Z$$



→ Impedance △le

$|Z| = \sqrt{R^2 + X_L^2}$ → Magnitude

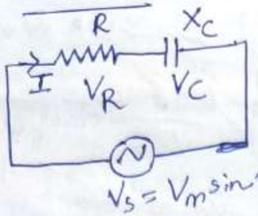
$Z = R + jX_L$ — Rectangular form

Power factor, $\cos\theta = R/Z$

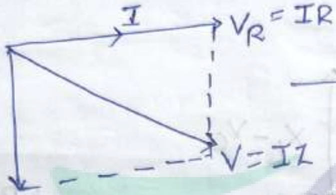
$X_L = L\omega$

where $\omega = 2\pi f$

RC Circuit



$V_s = V_m \sin \omega t$



→ Voltage Δle –

$V_C = IX_C$

$V = \sqrt{V_R^2 + V_C^2}$

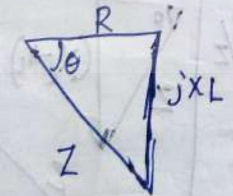
$V = \sqrt{(IR)^2 + (IX_c)^2}$

$V = \sqrt{I^2R^2 + I^2X_C^2}$

$V = I\sqrt{R^2 + X_C^2}$

$V = IZ$, where $Z = \sqrt{R^2 + X_C^2}$ Unit of impedance = Ohm

Impedance Δle

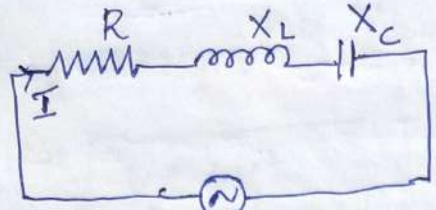


$Z = R - jX_C$

$Z = \sqrt{R^2 + X_C^2}$

X_C = Capacitive reactance $X_C = \frac{1}{C\omega}$

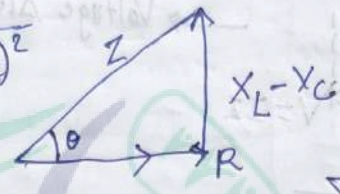
R-L-C Circuit

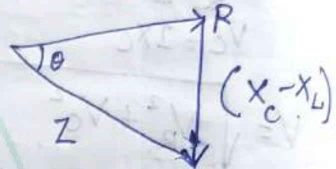


$V = V_m \sin \omega t$

$Z = R + j(X_L - X_C)$

$Z = \sqrt{R^2 + (X_L - X_C)^2}$





R-L Circuit

- Impedance $Z = R + jX_L$

$Z = \sqrt{R^2 + X_L^2}$

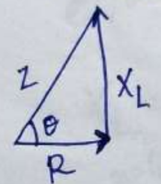
$X_L = L\omega = L \cdot 2\pi F$

- Current $I = V/Z$

Power factor

$\cos\theta = R/Z$

Impedance Δle



R-C Circuit

- $Z = R - jX_C$ X_C = capacitive reactance

$X_C = 1/C\omega$

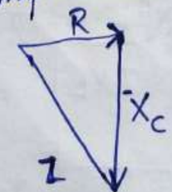
$Z = \sqrt{R^2 + X_C^2}$

$= \frac{1}{C 2\pi F}$

$I = V/Z$

Power factor $\cos\theta = R/Z$

Impedance Δle



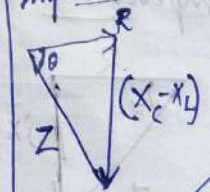
R-L-C Circuit

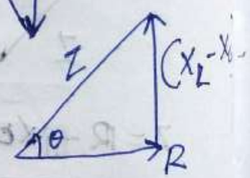
$Z = R + j(X_L - X_C)$

$Z = \sqrt{R^2 + (X_L - X_C)^2}$

$I = V/Z$

Impedance Δle





Power factor

$\cos\theta = R/Z$

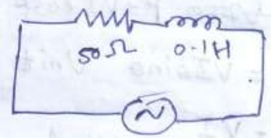
1) A resistor of 50 ohm and an inductor of 0.1 H are connected in series across a 200 V, 50 Hz supply. Find the :

(i) Impedance

(ii) Power factor

(iii) Active power

(iv) Reactive power



200V, 50 Hz

Ans: $R = 50\,\Omega$

$L = 0.1\,H$

$V = 200\,V$

$f = 50\,Hz$

$\omega = 2\pi f = 2 \times \pi \times 50 = 314.159$

$X_L = L\omega = 0.1 \times 314.159 = 31.42\,\Omega$

$V = IZ$

$I = V/Z$

$= \frac{200}{59.05} = 3.386\,A$

(i) $Z = \sqrt{R^2 + X_L^2} = \sqrt{(50)^2 + (31.42)^2} = 59.05\,\Omega$

(ii) Power factor, $\cos\theta = R/Z$

$\cos\theta = \frac{50}{59.05} = 0.846$

(iii) Active power, $P = VI\cos\theta = 200 \times 3.386 \times 0.846$

$= 572.91\,W$

(iv) Reactive power $= Q = VI\sin\theta = 200 \times 3.386 \times 0.532$

$= 360.27\,VAR$

$\left(\sin\theta = \frac{X_L}{Z}\right) = 0.532$

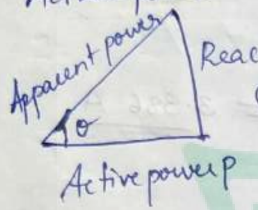
Power Δle

① Active power. ~~$P = VI\cos$~~ $P = VI\cos\theta$ Unit Watt W or kW

(2) Reactive power $Q = VI\sin\theta$ Unit Volt Ampere Reactive VAR or KVAR

(3) Apparent power $S = VI$ Unit = Volt Ampere VA or KVA

Active ~~power – It is actual power~~



power factor

$$\cos\theta = \frac{\text{Active power}}{\text{Apparent power}}$$

· Active power (P) It is the actual power dessipated in the circuit

· Reactive power (Q_1) – It is the power associated with the reactive components. (Inductance and Capacitance of the circuit)

Reactive power is not dessipated in circuit but stored and released during each AC circuit. It is also called Wattless component of power.

$Q = VI\sin\theta$

Unit = VAR

Volt Ampere Reactive.

· Apparent power (S) – It is the product of R.M.S Voltage and current in the circuit

$S = VI$ Unit VA or KVA

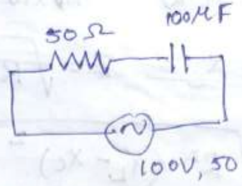
$$S = \sqrt{P^2 + Q^2}$$

Power factor

It is defined as the ratio of active power by apparent power. From the impedance Δle, it is also written as $\cos\theta = R/Z$

2) A resistor of $50\,\Omega$ and a capacitor of $100\,\mu F$ are connected in series across a 100V, 50 Hz supply. Find

(i) impedance
(ii) Current
(iii) Power factor
(iv) Voltage across resistor
(v) Voltage across capacitor.



$X_c = \frac{1}{C\omega} = \frac{1}{100\times10^{-6}\times 2\times\pi\times 50}$

$= \cancel{0.03}\ 31.83\,\Omega$

Ans: (i) $Z = \sqrt{R^2 + X_c^2}$

$= \sqrt{50^2 + 31.83^2} = \underline{\underline{59.27}}\,\Omega$

(ii) $I = V/Z = \frac{100}{59.27} = \underline{\underline{1.687}}\ A$

(iii) Power factor, $\cos\theta = R/Z = \frac{50}{59.27} = \underline{\underline{0.843}}$

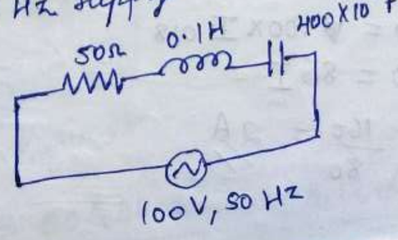
(iv) Voltage across resistor, $V = IR$

$V = 1.687 \times 50 = \underline{\underline{84.35}}\ V$

(v) Voltage across capacitor, $V = I X_C = 1.687 \times 31.83 = \underline{\underline{53.69\ V}}$

3) A resistor of $50\,\Omega$, an inductor of 0.1 H and a capacitor of $400\,\mu F$ are connected in series and the combination is connected across a 100V, 50 Hz supply. Find

(i) Power consumed
(ii) Power factor.



$X_L = L\omega$

$= L \times 2 \times \pi \times f = 0.1 \times 2 \times \pi \times 50 = \underline{31.42\ \Omega}$

$X_C = \frac{1}{C\omega} = \frac{1}{400 \times 10^{-6} \times 2 \times \pi \times 50} = \underline{79.6\ \Omega}$

$\cancel{X_L - X_C} \quad Z = \cancel{\sqrt{R^2 + (X_L - X_C)^2}} \ 55.$

$Z = \sqrt{R^2 + (X_L - X_C)^2} = \underline{69.4\ \Omega}$

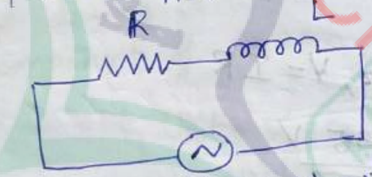
$I = V/Z = \frac{100}{69.4} = \underline{1.44\ A}$

Power factor, $\cos\theta = R/Z = \frac{50}{69.4} = 0.72$

$P = VI\cos\theta = 100 \times 1.44 \times 0.72 = \underline{103.7\ W}$

4) A series RL circuit takes 160 W at 0.8 power factor lagging from 100V, 50 Hz supply. What are the values of R and L?

Ans:



$V_s = V_m \sin\omega t$

100V, 50 Hz

$\cos\theta = 0.8$

$P = 160\ W$

$P = VI\cos\theta$

$160 = 100 \times I \times 0.8$

$160 = 80 I$

$I = \frac{160}{80} = \underline{2\ A}$

$\cancel{V=IR}$ $V = IZ$

$Z = V/I = \frac{100}{2} = 50\ \Omega$

$\cancel{V=IR}$ $\cos\theta = R/Z$

$0.8 = \frac{R}{50}$

$R = 0.8 \times 50 = 40\ \Omega$

$Z = \sqrt{R^2 + X_L^2}$

$Z = \sqrt{40^2 + X_L^2}$ $\quad Z^2 = R^2 + X_L^2$

$X_L^2 = Z^2 - R^2$

$X_L = \sqrt{Z^2 - R^2} = \sqrt{50^2 - 40^2} = 30\ \Omega$

$X_L = L\omega$

$L = \frac{X_L}{\omega}$

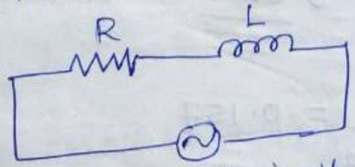
$L = \frac{30}{2 \times \pi \times 50} = 0.095\ H = 95.5\ mH$

5) A 50 Hz sinusoidal voltage $40+j30$ is applied across a series RL circuit resulting in sinusoidal current $4+j1$ A. Calculate

(i) The impedance of the circuit

(ii) Power consumed in the circuit

(iii) Power factor of the circuit



$V_s = V_m \sin \omega t$

$I = 4+j1$

$f = 50\,Hz \quad V = 40+j30 = 50\angle 36.86 \quad = 4.12\angle 14.03$

(i) $Z = \frac{V}{I} = \frac{50\angle 36.86}{4.12\angle 14.03} = 12.13\angle 22.83$

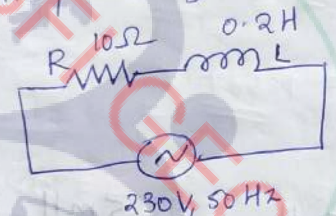
(ii) $P = VI\cos\theta$

$p = 50 \times 4.12 \times \cos 22.83$

$= 189.86\ W$

(iii) power factor, $\cos\theta = \cos 22.83 = 0.921$

(6) ~~A 230~~ A 230V, 50 Hz supply is applied to a coil of resistance $10\ \Omega$ and inductance 0.2 H. Calculate
(i) the reactance and impedance of the coil
(ii) Current and it's phase angle relative to applied voltage.



(i) $X_L = L\omega$

$X_L = 0.2 \times 2 \times \pi \times 50 = 62.83\ \Omega$

$Z = \sqrt{R^2 + X_L^2} = \sqrt{10^2 + 62.83^2} = 63.62\ \Omega$

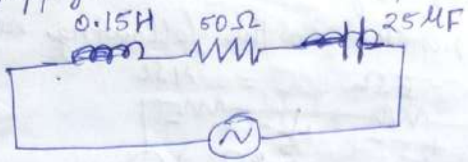
(ii) $V = IZ$

$I = V/Z = \frac{230}{63.62} = 3.62\ A$

$\cos\theta = R/Z = \frac{10}{63.62} = 0.157$

$\theta = \cos^{-1} 0.157 = 80.96^\circ$

(7) A series RLC circuit with a resistance of 50 Ω, a capacitance of 25 μF and an inductance of ~~0.5~~ 0.15 H is connected across 230 V, 50 Hz supply. Determine the impedance of the circuit.



$X_L = L\omega$

$= 0.15 \times 2\times\pi\times 50 = 47.12\ \Omega$

$X_C = \dfrac{1}{C\omega} = \dfrac{1}{25\times10^{-6}\times 2\times\pi\times 50} = \underline{\underline{127.32\ \Omega}}$

$Z = \sqrt{R^2 + (X_C - X_L)^2} = \sqrt{50^2 + (127.32 - 47.12)^2} = \underline{\underline{94.57\ \Omega}}$

(8) An alternating ~~current~~ voltage $160 + j120$ V is applied to a circuit when the current in the circuit is found to be $6+j8$. Find

(i) Impedance of the circuit

(ii) Phase angle

(iii) Power consumed

$V = 160 + j120 = 200\angle 36.86$

$I = 6 + j8 = 10\angle 53.13$

$Z = \dfrac{V}{I} = \dfrac{200\angle 36.86}{10\angle 53.13} = \underline{\underline{20\angle -16.27\ \Omega}}$

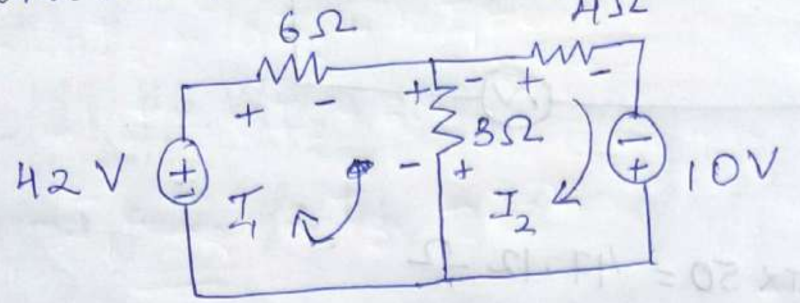
(iii) $P = VI\cos\theta$

$= 200 \times 10 \times \cos 16.27 = \underline{\underline{1919.9}}\ W$

~~(iii) 20∠−16.27 = 19.19 − j5.6~~ (ii) $\theta = \underline{\underline{16.27}}$

Remedial

1) By mesh analysis, find the currents flowing through 4Ω and 3Ω resistors in the following network



Loop - 1

$42 - 6I_1 - 3(I_1 - I_2) = 0$

$42 - 6I_1 - 3I_1 + 3I_2 = 0$

$42 - 9I_1 + 3I_2 = 0$

$42 = 9I_1 - 3I_2$

$14 = 3I_1 - I_2 \quad -(1)$

Loop - 2

$10 - 3(I_2 - I_1) - 4I_2 = 0$

$10 - 3I_2 + 3I_1 - 4I_2 = 0$

$10 - 7I_2 + 3I_1 = 0$

$10 = 7I_2 - 3I_1 \quad - (2)$

$\cancel{14 = 3I_1 - I_2}$ $\cancel{3I_1 - I_2 = 14 \quad \times(1)}$ $\cancel{-3I_1 + I_2 = -14}$

$\cancel{-3I_1 + 7I_2 = 10 \quad \times(3)}$

-147

$3I_1 - I_2 = 14$

$(+)\ -3I_1 + 7I_2 = 10$

$0 \quad 6I_2 = 24 \qquad I_2 = 24/6 = 4\,A$

$3I_1 - 4 = 14$

$3I_1 = 18$

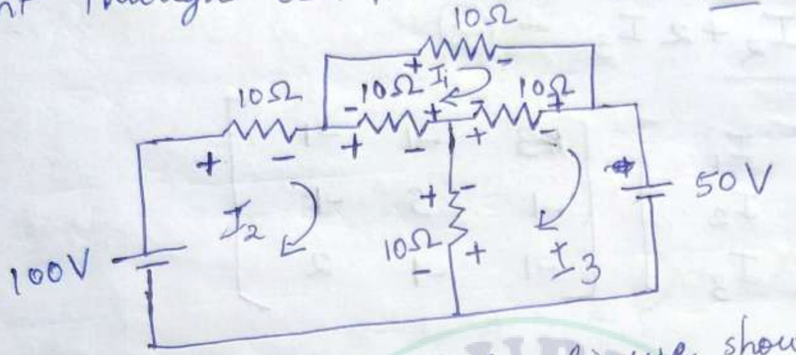
$I_1 = 18/3 = \underline{6\,A}$

Current through 4Ω resistor $= I_2 = 4A$

Current through 3Ω resistor $= (I_1 - I_2) = 6 - 4 = \underline{2\,A}$

Current through 6Ω resistor $= I_1 = \underline{6\,A}$

2)



Find the mesh current in the figure shown by mesh analysis.

-10 -10-10

Loop-1

$-10I_1 - 10(I_1 - I_3) - 10(I_1 - I_2) = 0$

$-10I_1 - 10I_1 + 10I_3 - 10I_1 + 10I_2 = 0$

-10-10-10

$-30I_1 + 10I_2 + 10I_3 = 0$

$0 = 30I_1 - 10I_2 - 10I_3$

$0 = 3I_1 - I_2 - I_3$ — ①

Loop-2

$100 - 10I_2 - 10(I_2 - I_1) - 10(I_2 - I_3) = 0$

$100 - 10I_2 - 10I_2 + 10I_1 - 10I_2 + 10I_3 = 0$

$100 - 30I_2 + 10I_1 + 10I_3 = 0$

$10 = -I_1 + 3I_2 - I_3$ — ②

$100 = -10I_1 + 30I_2 - 10I_3$

Loop - 3

$$-50 - 10(I_3 - I_2) - 10(I_3 - I_1) = 0$$

$$-50 - 10I_3 + 10I_2 - 10I_3 + 10I_1 = 0$$

$$-50 - 20I_3 + 10I_2 + 10I_1 = 0$$

$$-50 = -10I_1 - 10I_2 + 20I_3$$

$$-5 = -I_1 - I_2 + 2I_3 \quad - (3)$$

$$\begin{bmatrix} 0 \\ 10 \\ -5 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} \begin{bmatrix} 3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 2 \end{bmatrix}$$

$$\Delta = \begin{matrix} 3 & -1 & -1 & 3 & -1 \\ -1 & 3 & -1 & -1 & 3 \\ -1 & -1 & 2 & -1 & -1 \end{matrix} = (18 - 1 - 1) - (3 + 3 + 2)$$

$$= 16 - 8 = 8$$

$$\Delta_1 = \begin{bmatrix} 0 & -1 & -1 \\ 10 & 3 & -1 \\ -5 & -1 & 2 \end{bmatrix} = \begin{matrix} 0 & -1 & -1 & 0 & -1 \\ 10 & 3 & -1 & 10 & 3 \\ -5 & -1 & 2 & -5 & -1 \end{matrix}$$

$$= (0 - 5 + 10) - (15 + 0 - 20)$$

$$= 5 + 5 = 10$$

$$\Delta_2 = \begin{bmatrix} 3 & 0 & -1 \\ -1 & 10 & -1 \end{bmatrix} = \begin{matrix} 3 & 0 & -1 & 3 & 0 \\ -1 & 10 & -1 & -1 & 10 \\ -1 & -5 & 2 & -1 & -5 \end{matrix}$$

$= (60+0-5) - (10+15+0)$

$= 55 - 25 = \cancel{35}\ \underline{30}$

$\Delta_3 = \begin{bmatrix} 3 & -1 & 0 \\ -1 & 3 & 10 \\ -1 & -1 & -5 \end{bmatrix} = \begin{matrix} 3 & -1 & 0 & 3 & -1 \\ -1 & 3 & 10 & -1 & 3 \\ -1 & -1 & -5 & -1 & -1 \end{matrix}$

$-5+15=0$.

$3(-15+10) + 1(5+10)$

$= (-45+10+0) - (0-30-5) = \cancel{-5}\ -35 - -35$

$= -35 + 35 = 0$

$I_1 = \frac{\Delta_1}{\Delta} = \frac{10}{8} = 1.25\ A$

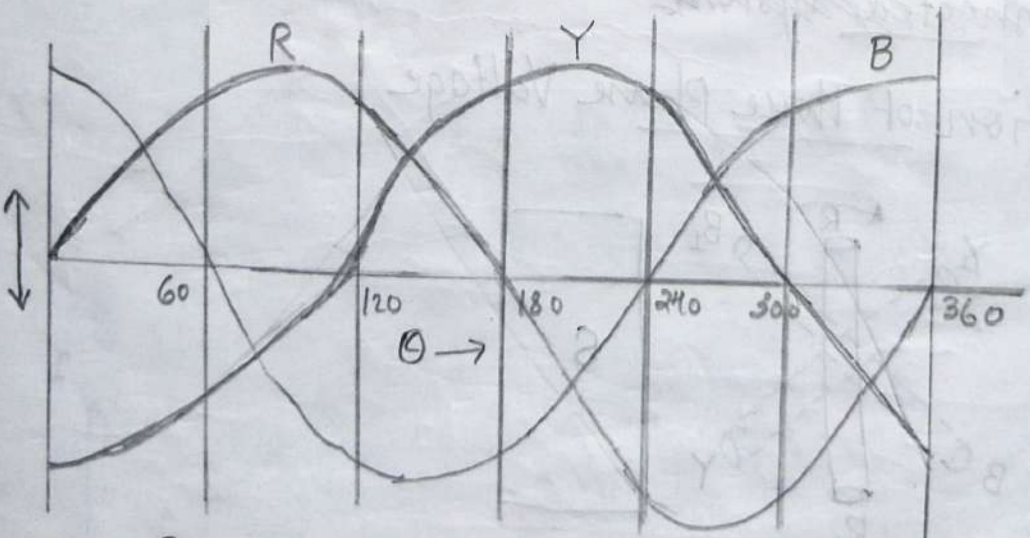
$I_2 = \frac{\Delta_2}{\Delta} = \frac{30}{8} = 3.75\ A$

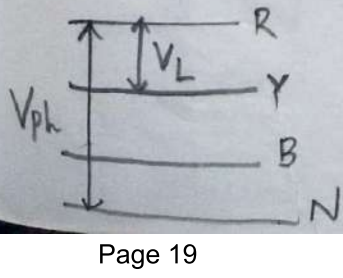
$I_3 = \frac{\Delta_3}{\Delta} = \frac{0}{8} = \underline{0\ A}$

180 360 = 180
120 300 = 180
60 240 = 180

Three-phase systems

WAVEFORMS OF THREE PHASE VOLTAGES





$V_{RY} = V_{YB} = V_{RB} = V_L$ line voltage

$V_{RN} = V_{BN} = V_{YN} = V_{ph}$ phase voltage

The voltage across any one winding is called phase voltage (V_{ph})

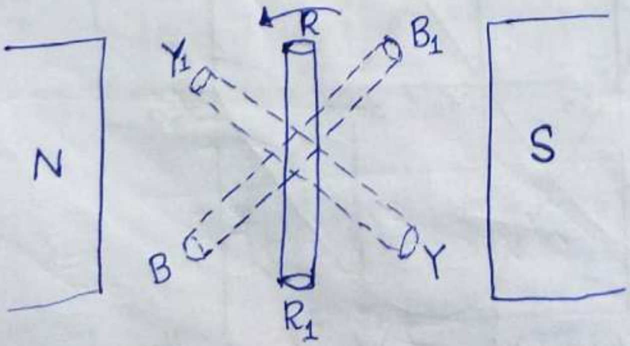
Advantages of three phase system (poly phase system) over single-phase system

- Generation of polyphase power is cheaper.
- Three phase machines has higher efficiency and higher power factor compared to a single phase system.
- For the same size, the output of a poly phase machine is greater than that of a single-phase machine, hence it is lighter and cheaper.
- Single phase motors are not self-starting whereas polyphase motors are self starting.
- The parallel operation of single phase alternator is not very smooth whereas three phase alternators run in parallel without any difficulty.

~~Connections of three phase systems~~

~~Star connected system~~

Production of Three Phase Voltage



Generation of three phase e.m.f.

. Let 3 coils ... rotate at uniform angular velocity in a uniform magnetic field b/w the poles N and S.

. Let R, Y, B be the start terminals and R_1, Y_1, B_1 be the finish terminals of these coils.

Voltage induced in coil $RR_1 = V_{RR_1}$

Voltage induced in coil $YY_1 = V_{YY_1}$

Voltage induced in coil $BB_1 = V_{BB_1}$

When the complete coil system rotates the emf induced in the three coils are all sinusoidal and equal in magnitude.

. However, since these coils are displaced $120°$ in space, the emfs V_{RR_1}, V_{YY_1} and V_{BB_1} also have $120°$ phase difference.

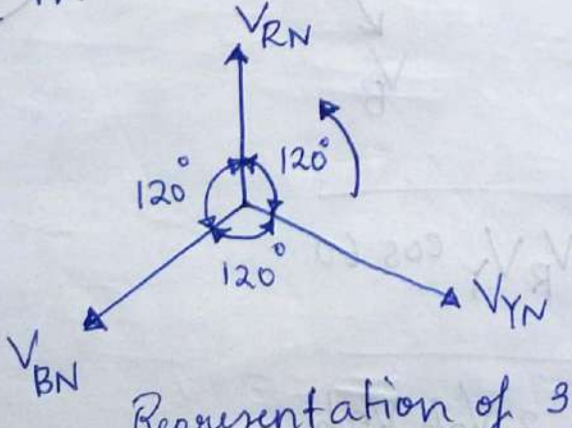
. This system of voltages so obtained are called 3 phase voltages.

. The instantaneous values of generated emfs in coils RR_1 (phase R), YY_1 (phase Y) and BB_1 (Phase B) are given by

$$V_{RR_1} = V_m \sin\theta$$

$$V_{YY_1} = V_m \sin(\theta - 120°)$$

$$V_{BB_1} = V_m \sin(\theta - 240°) \text{ or } V_m \sin(\theta + 120°)$$



Representation of 3 emfs by phasors

Where V_m is the maximum value of generated emf in each of the coils and θ is the position of the coil RR_1 from it's initial position

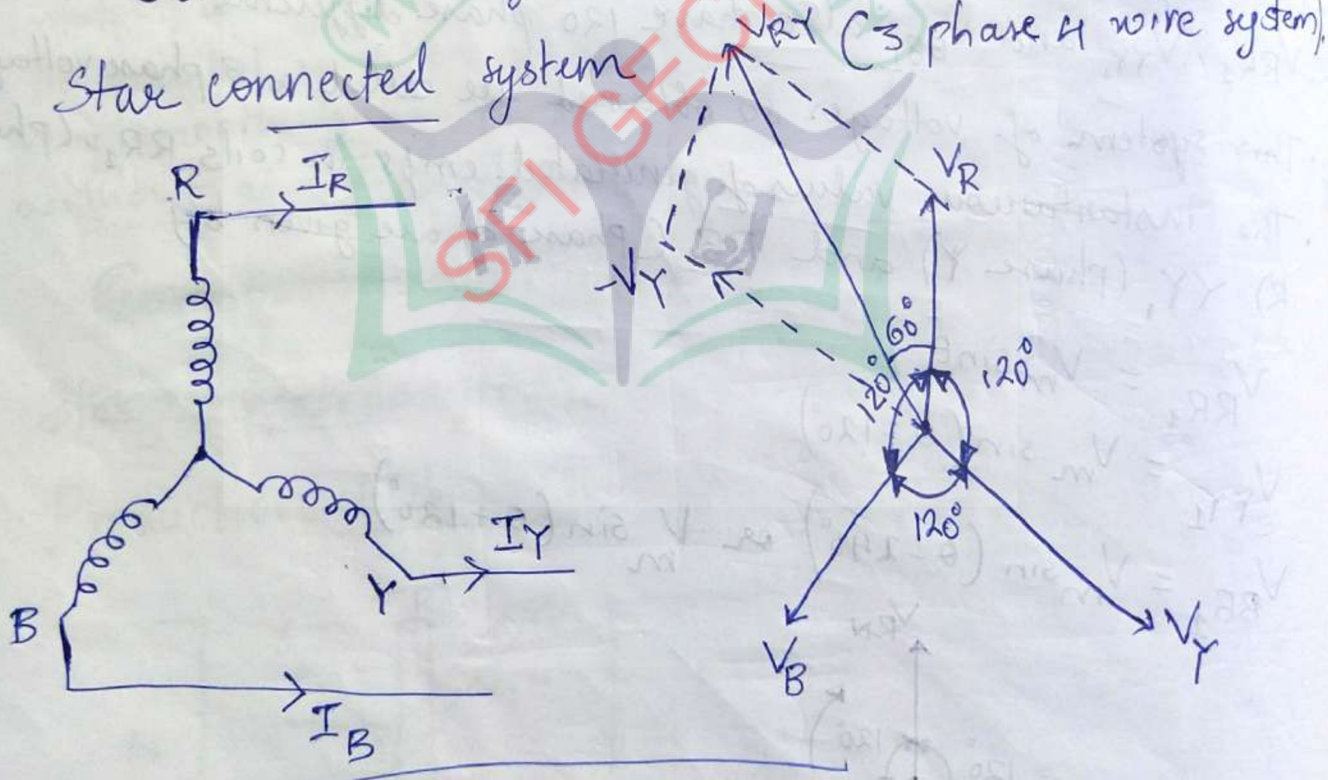
Phase sequence

The order in which the phase voltage of three phase system attain their peak or maximum positive value is called the phase sequence of the system.

- when the voltage of the red phase is at its positive maximum value, the voltage of the Y phase will be 120° behind its positive maximum value and that of B phase 240° behind its positive maximum value.
- The phase sequence of the voltages is RYB and the phasors are assumed to rotate in an anti-clockwise direction.

Connections of three phase systems

Star connected system



$$V_{RY} = \sqrt{(V_R)^2 + (V_Y)^2 \pm 2V_R V_Y \cos 60}$$

$$V_L = \sqrt{V_{ph}^2 + (-V_{ph})^2 + 2V_{ph}V_{ph}}$$

$$V_L = \sqrt{3V_{ph}^2}$$

$$\boxed{V_L = \sqrt{3}\ V_{ph}}$$

$$\boxed{I_L = I_{ph}}$$

Power in Star Connected system

$P = 3 \times$ Per phase Power

$3 \times V_{ph} I_{ph} \cos\phi = \sqrt{3} \times \sqrt{3} \times V_{ph} I_{ph} \cos\phi$

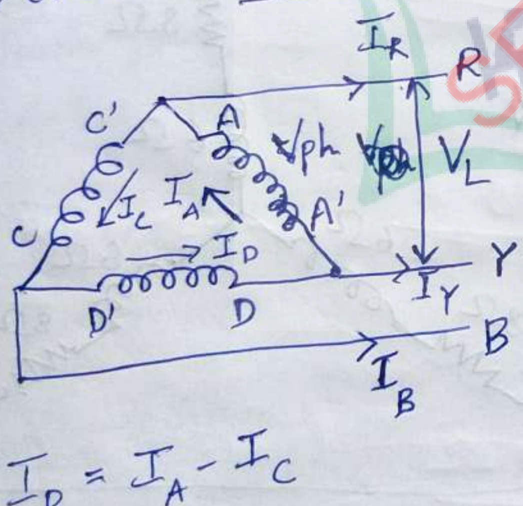
$P = \sqrt{3}\ V_L I_L \cos\phi$

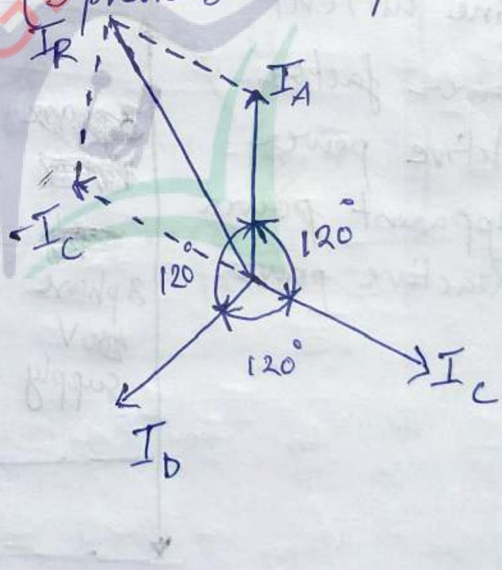
Active power $P = \sqrt{3}\ V_L I_L \cos\phi$

Reactive power $Q = \sqrt{3}\ V_L I_L \sin\phi$

Apparent power $S = \sqrt{3}\ V_L I_L$

Delta Connected system (3 phase 3 wire system)





$I_R = I_A - I_C$

$I_Y = I_D - I_A$

$I_B = I_C - I_D$

$I_R = \sqrt{I_A^2 + (-I_C)^2 + 2 \times I_A \times I_C \cos 60}$

$= \sqrt{I_{ph}^2 + I_{ph}^2 + I_{ph}^2} \rightarrow$ $\boxed{I_L = \sqrt{3}\ I_{ph} \quad V_L = V_{ph}}$

Power in Delta connected system

$P = 3$ per phase power

$= 3 V_{ph} I_{ph} \cos\theta$

$= \sqrt{3} \times \sqrt{3} V_{ph} I_{ph} \cos\theta$

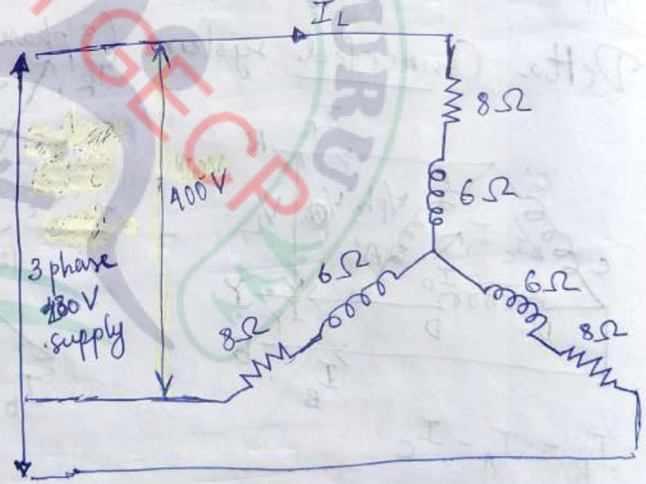
$P = \sqrt{3} \times V_L I_L \cos\theta$

$Q = \sqrt{3} \times V_L I_L \sin\theta$

$S = \sqrt{3} V_L I_L$

1) A balanced Y connected load of $(8 + j6)\ \Omega$ per phase is connected to a 3 phase 400 V supply. Find

(i) Line current
(ii) Power factor
(iii) Active power
(iv) Apparent power
(v) Reactive power



Impedance per phase, $Z_{ph} = (8 + j6)$

Line voltage $V_L = 400\text{V}$

$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230.94\text{ V} = 231\text{ V}$

$|Z_{ph}| = \sqrt{8^2+6^2} = 10\,\Omega$

phase current $I_{ph} = V_{ph}/Z_{ph} = 230.94/10 = 23.094\,A$

$I_{ph} = I_L$

i) $I_L = 23.094\,A$

ii) Power factor, $\cos\phi = (R/z) = 8/10 = 0.8$

iii) $P = \sqrt{3}\,V_L I_L \cos\phi = \sqrt{3} \times 400 \times 23.094 \times 0.8 = 12799.9\,W = 12\,KW$

iv) $S = \sqrt{3}\,V_L I_L = \sqrt{3} \times 400 \times 23.094 = 15999.9\,VA = 16\,KVA$

v) $Q = \sqrt{3}\,V_L I_L \sin\phi = \sqrt{3} \times 400 \times 23.094 \times 0.6$

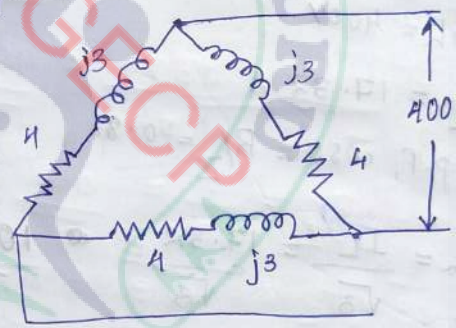
$= 9599.9\,VAR = \cancel{96\,KVAR}\ 9.6\,KVAR$

2) 3 impedances of $4+j3$ are connected in delta and connected across a 3 phase 400 V. supply. Find

(i) Line current

(ii) Power factor

(iii) Active power



$V_L = V_{ph}$

$I_L = \sqrt{3}\,I_{ph}$

Ans: $V_L = 400V$

$V_{ph} = 400\,V$

$|Z_{ph}| = \sqrt{4^2+3^2} = 5\,\Omega$

$I_{ph} = \dfrac{V_{ph}}{Z_{ph}} = \dfrac{400}{5} = 80\,A$

(i) $I_L = \sqrt{3} \times 80 = 138.56\,A$

(ii) PF, $\cos\theta = R/Z$

$$\cos\theta = \frac{4}{5} = 0.8$$

(iii) $P = \sqrt{3}\, V_L I_L \cos\phi = \sqrt{3} \times 400 \times 138.56 \times 0.8$

$$= 76797.7\ W$$

$$= 76.79\ KW$$

3) A three phase delta connected balanced load when connected 400 V supply ~~when connected to~~ draws 17.32 A at 0.8 power factor lagging. Each phase has a resistor and an inductor connecting in series. Find the resistance and reactance per phase.

$V_L = 400V$

$V_{ph} = 400V$

$I_L = 17.32\ A$

p.f, $\cos\theta = R/Z = 0.8$

$I_L = \sqrt{3}\, I_{ph}$

$I_{ph} = \frac{I_L}{\sqrt{3}}$

$$I_{ph} = \frac{I_L}{\sqrt{3}} = \frac{17.32}{\sqrt{3}} = 10\ A$$

$V_{ph} = I_{ph} Z_{ph}$

$$Z_{ph} = \frac{V_{ph}}{I_{ph}} = \frac{400}{10} = 40\ \Omega$$

$\cos\theta = R/Z$

$R = \cos\theta \times Z = 0.8 \times 40 = 32\ \Omega$

$Z = \sqrt{R^2 + X_L^2}$

$Z^2 = R^2 + X_L^2$ $\qquad X_L^2 = Z^2 - R^2$

$X_L = \sqrt{Z^2 - R^2}$

$X_L = \sqrt{40^2 - 32^2} = \underline{24\ \Omega}$

(4) Three impedances each having resistance 20 Ω and an inductive reactance of 15 Ω are connected in star across a 400 V, 3 phase, AC supply. Calculate (i) line current (ii) Power factor (iii) Total Power.

$R = 20\ \Omega$ $\quad V_L = \sqrt{3}\, V_{ph}$

$X_L = 15\ \Omega$ $\quad I_L = I_{ph}$

$V_L = 400\ V$

$|Z_{ph}| = \sqrt{R^2 + X_L^2} = \sqrt{20^2 + 15^2} = \underline{25\ \Omega}$

$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = \underline{231\ V}$

$V_{ph} = Z_{ph} \cdot I_{ph}$

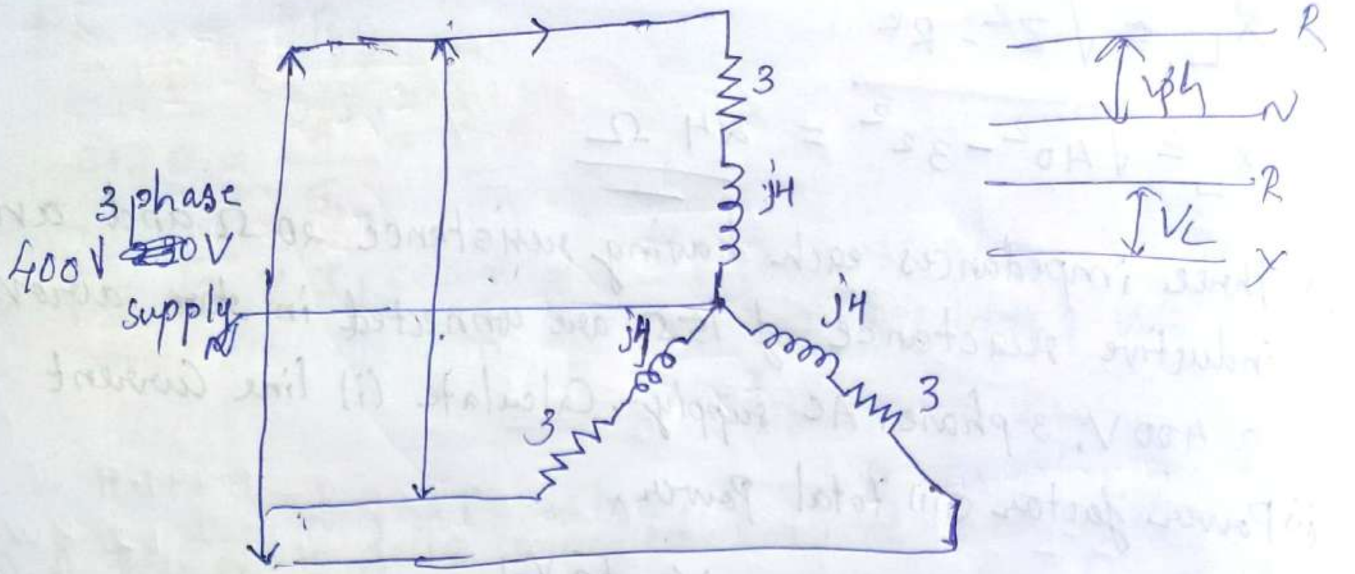
$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{231}{25} = \underline{9.24\ A}$

(i) $I_L = \underline{9.24\ A}$

(ii) $\cos\theta = R/Z = \frac{20}{25} = 0.8$

(iii) $P = \sqrt{3}\, V_L I_L \cos\theta = \sqrt{3} \times 400 \times 9.24 \times 0.8 = \underline{5.12\ KW}$

(5) Find the line current, power factor and the total power for the given star circuit.



$V_{ph} = \cancel{230V}$ 230V $\quad I_L = I_{ph}$ $\quad V_L = \sqrt{3}\, V_{ph}$

$Z = 3 + j4$ $\quad I_L = I_{ph}$

~~$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{230}{\sqrt{3}} = 132.8\,V$~~ $\quad V_L = \sqrt{3} \times 230 = 398.37\,V$

$|Z_{ph}| = \sqrt{3^2 + 4^2} = 5\,\Omega$ $\quad I_L = 46\,A$

$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{230}{5} = 46\,A$

$\cos\theta = R/Z = \frac{3}{5} = 0.6$

$P = \sqrt{3}\, V_L I_L \cos\theta = \sqrt{3} \times 398.37 \times 46 \times 0.6 = 19.04\,KW$

(6) A star connected 3 phase load consist of 3 identical impedances. When the load is connected to a 3 phase, 400 V supply the line current is 23.09 A and PF is 0.8 lagging. Calculate the total power taken by the load? If the load were reconnected in delta and supplied from the same 3 phase supply, Calculate the current flow in each line.

$V_L = 400\,V$ $\quad V_L = \sqrt{3} V_{ph}$

$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 231\,V$ $\quad I_L = I_{ph}$

$I_L = 23.09\,A$

$I_{ph} = 23.09\,A$

$\cancel{V_{ph} = I_{ph} \times Z_{ph}}$ $\quad P = \sqrt{3}\, V_L I_L \cos\theta$

$= \sqrt{3} \times 400 \times 23.09 \times 0.8$

$\cancel{Z_{ph} = \frac{V_{ph}}{I_{ph}} = \frac{231}{23.09} =}$ $\quad = 12.79\,KW$

$V_L = 400\,V$

$V_{ph} = 400\,V$ $\quad V_{ph} = I_{ph} \times Z_{ph}$

$Z_{ph} = \frac{V_{ph}}{I_{ph}} = \frac{231}{23.09} = 10.004\,\Omega$

$Z_{ph} = 10.004\,\Omega$

$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{400}{10.004} = 39.98\,A$

$I_L = \sqrt{3} \times 39.98 = 69.24\,A$

Admittance

Admittance is the reciprocal of impedance and is denoted by Y. For an AC circuit Admittance, $Y = \frac{1}{Z}$

$= \frac{RMS\ I}{RMS\ V}$

If the impedance is represented by $R+jX$ then the admittance can be ~~repre~~ found as follows.

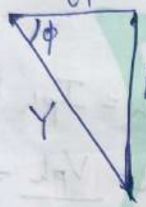
$$\text{Admittance, } Y = \frac{1}{Z} = \frac{1}{R+jX} = \frac{R-jX}{(R+jX)(R-jX)} = \frac{R-jX}{R^2+X^2}$$

$$= \frac{R}{R^2+x^2} - j\frac{X}{R^2+X^2}$$

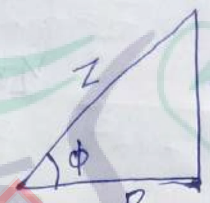
1) Conductance (G) given by $\frac{R}{R^2+X^2}$

(2) Susceptance (B) given by $\frac{X}{R^2+X^2}$

Unit for admittance, conductance and susceptance is Siemens (S) ohm^{-1} or mho.



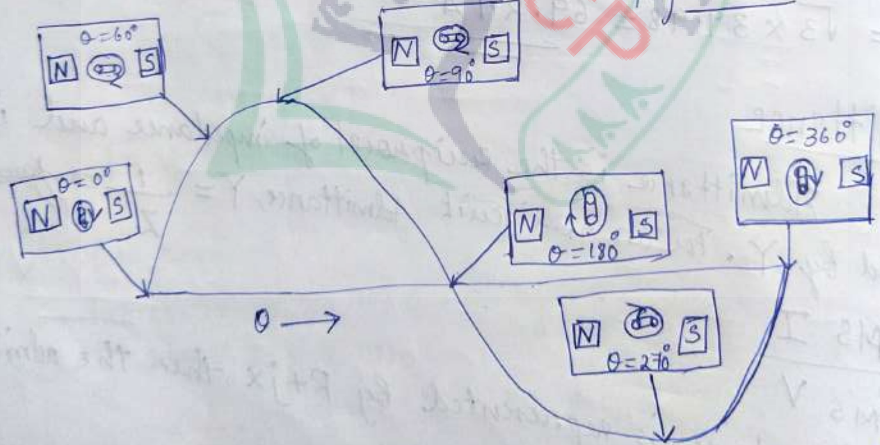
Admittance Triangle



→ Impedance Triangle

AC Fundamentals

Figure -1



Relation b/w the coil position and induced voltage.

Figure-2

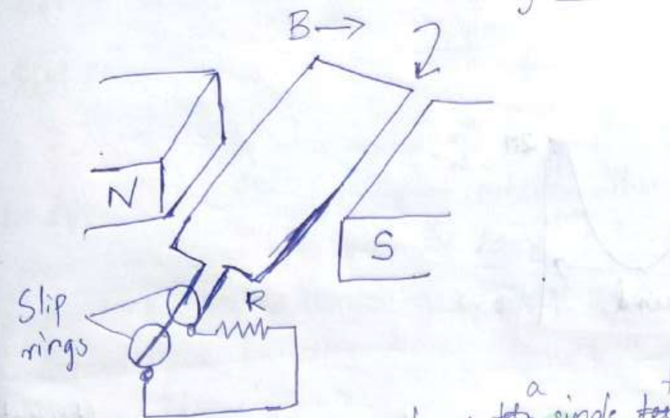
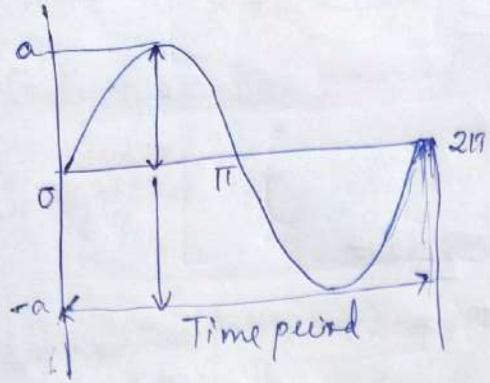


Figure 1 shows a single turn coil placed in a magnetic field, the coil is rotated in an axis ⊥ to the direction of magnetic field and the ends of the coil are connected to two slip rings when the coil is rotated an emf is induced in the coil with magnitude proportional to the magnetic field, length of the conductor and speed of cutting the magnetic field. As the coil rotates in the magnetic field the voltage induced ~~in the~~ changes as shown in figure-2. Here the voltage induced in the coil is given by $e = \cancel{VLB\sin\theta}\ Blv\sin\theta$. When the coil sides moves parallel to the field ($\theta = 0°$, the induced voltage zero). As θ increases from 0° to 90°, the induced voltage increases and reaches the maximum value when $\theta = 90°$. From 90° to 180°, the induced voltage reduces as shown in fig. From 180° to 360°, the induced voltage will be -ve and the negative peak occurs at 270°. At 360° the coil reaches in it's position and the cycle is repeated.

Considering the angular speed of the coil as ω radian/s, the induced voltage is given by the equation $V = V_m \sin\omega t$, where V_m peak value of induced emf.

Angular speed $\omega = 2\pi f$



Cycle

One complete set of +ve and -ve values of an alternating quantity is called a cycle

Periodic time

The time taken for one cycle is known as time period or periodic time.

$T = \frac{1}{f}$

Amplitude

It is the magnitude of the maximum +ve or negative value of alternating quantity.

Frequency

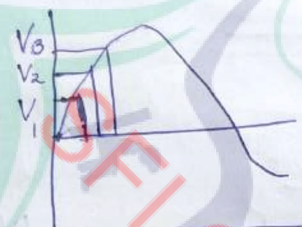
The no. of cycles completed in one second is called frequency of alternating quantity. It is expressed in cycles/second or Hertz.

Instantaneous value

The value of V or I at a particular instant is wave is the arithmetic mean of the ordinates at equal intervals over a half cycle of the wave.

Root Mean Square value (r.m.s value)

rms value of an alternating quantity may be defined as the d.c. current which when flows through a given resistance produces the same amount of heat as that produced by the alternating current passing through the same resistance for the same time. RMS value is also called the effective value.



$$V_{r.m.s} = \sqrt{\frac{V_1^2 + V_2^2 \cdots V_n^2}{n}}$$

$$= \sqrt{\frac{1}{2\pi}\int_0^{2\pi} (V_m \sin\omega t)^2 \, d\omega t}$$

$$= \sqrt{\frac{1}{2\pi}\int_0^{2\pi} V_m^2 \sin^2\omega t \, d\omega t}$$

$$= \sqrt{\frac{V_m^2}{2\pi}\int_0^{2\pi}\left(\frac{1-\cos 2\omega t}{2}\right) d\omega t}$$

$$= \sqrt{\frac{V_m^2}{4\pi}\left[\int_0^{2\pi}\left[d\omega t - \cos 2\omega t \, d\omega t\right]\right]}$$

$$= \sqrt{\frac{V_m^2}{4\pi}\left[\omega t - \frac{\sin 2\omega t}{2}\right]_0^{2\pi}}$$

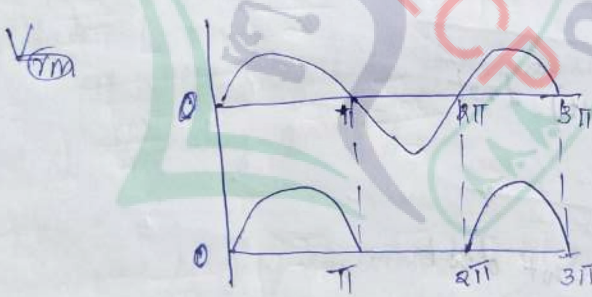
$$= \sqrt{\frac{V_m^2}{4\pi}\left[2\pi - \frac{\sin 2\times 2\pi}{2} - (0-0)\right]}$$

$$V_{rms} = \sqrt{\frac{V_m^2 \times \cancel{2\pi}}{2\,\cancel{4\pi}}}$$

$$= \sqrt{\frac{V_m^2}{2}}$$

$$\boxed{V_{rms} = \frac{V_m}{\sqrt{2}}}$$

1) Find the r.m.s value for a half wave rectifier circuit.



$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T V^2\, d\omega t}$$

$$V_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} (V_m^2 \sin^2 \omega t)\, d\omega t}$$

$$= \sqrt{\frac{1}{2\pi}\left[\int_0^{\pi} V_m^2 \sin^2 \omega t\, d\omega t + \int_{\pi}^{2\pi} 0\right]}$$

$$= \sqrt{\frac{1}{2\pi}\left[\int_0^{\pi} V_m^2 \left(\frac{1-\cos 2\omega t}{2}\right) d\omega t\right.}$$

$$= \sqrt{\frac{V_m^2}{2\pi}\int_0^{\pi} \frac{d\omega t - \cos 2\omega t \, d\omega t}{2}}$$

$$= \sqrt{\frac{V_m^2}{4\pi}\left[\omega t - \frac{\sin 2\omega t}{2}\right]_0^{\pi}}$$

$$= \sqrt{\frac{V_m^2}{4\pi}\left[\pi - \frac{\sin 2\pi}{2} - \left(0 - \frac{\sin 2 0}{2}\right)\right]}$$

$$= \sqrt{\frac{V_m^2}{4\pi} \times \pi}$$

$$= \sqrt{\frac{V_m^2}{4\cancel{\pi}} \times \cancel{\pi}}$$

$$\boxed{V_{r.m.s} = \frac{V_m}{2}}$$

$$V_{average} = \frac{1}{T}\int_0^T$$

i) For a half wave rectified sinusoidal alternating current find the following.

(i) r.m.s value

(ii) Average value.

Ans: (i) $I_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} I_m^2 \sin^2 \omega t \, dt}$

$$= \sqrt{\frac{1}{2\pi}\left[\int_0^{\pi} I_m^2 \sin^2 \omega t \, d\omega t + \int_{\pi}^{2\pi} 0\right.}$$

$$I_{avg} = \frac{1}{2\pi}\int_0$$

$$= \sqrt{\frac{1}{2\pi} \times I_m^2 \int_0^\pi \sin^2\omega t \, d\omega t}$$

$$= \sqrt{\frac{I_m^2}{2\pi}\int_0^\pi \left(\frac{1-\cos 2\omega t}{2}\right) d\omega t}$$

$$= \sqrt{\frac{I_m^2}{2\pi}\int_0^\pi d\omega t - \frac{\cos 2\omega t \, d\omega t}{2}}$$

$$= \sqrt{\frac{I_m^2}{4\pi}\left[\omega t - \frac{\sin 2\omega t}{2}\right]_0^\pi}$$

$$= \sqrt{\frac{I_m^2}{4\pi}\left[\pi - \frac{\sin 2\pi}{2} - (0)\right]}$$

$$= \sqrt{\frac{I_m^2}{4\pi} \times \pi} = \frac{I_m}{2}$$

(ii) $I_{avg} = \frac{1}{2\pi}\int_0^\pi I_m \sin \omega t \, d\omega t$

$$= \frac{I_m}{2\pi}\int_0^\pi \sin\omega t \, d\omega t$$

$$= \frac{I_m}{2\pi}\left[-\cos\omega t\right]_0^\pi = \frac{I_m}{2\pi}\left[-\cos\pi + \cos 0\right]$$

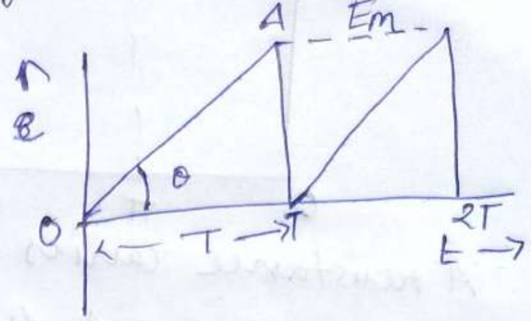
$-1+1$

$$= \frac{I_m}{2\pi} \times 2 = \frac{I_m}{\pi}$$

(2) Determine the form factor for the saw tooth wave form ~~for the sa~~

Let K be the slope of the portion OA. The instantaneous value of the wave form can be written as

$V = Kt$. $K = \frac{E_m}{T}$



$V = \frac{E_m}{T} t$

$V_{r.m.s} = \sqrt{\frac{1}{T}\int_0^T V^2 \cdot d\omega t}$

Form factor = $\frac{\text{RMS value}}{\text{Average value}}$

$E_{rms} = \sqrt{\frac{1}{T}\int_0^T \left(\frac{E_m t}{T}\right)^2 dt}$

$= \sqrt{\frac{1}{T^3} \times \int_0^T E_m^2 t^2 dt} = \sqrt{\frac{E_m^2}{T^3}\left[\frac{t^3}{3}\right]_0^T}$

$= \sqrt{\frac{E_m^2}{T^3}\left[\frac{T^3}{3}\right]}$

$= \sqrt{\frac{E_m^2}{\cancel{T^3}} \times \frac{\cancel{T^3}}{3}} = \frac{E_m}{\sqrt{3}}$ //

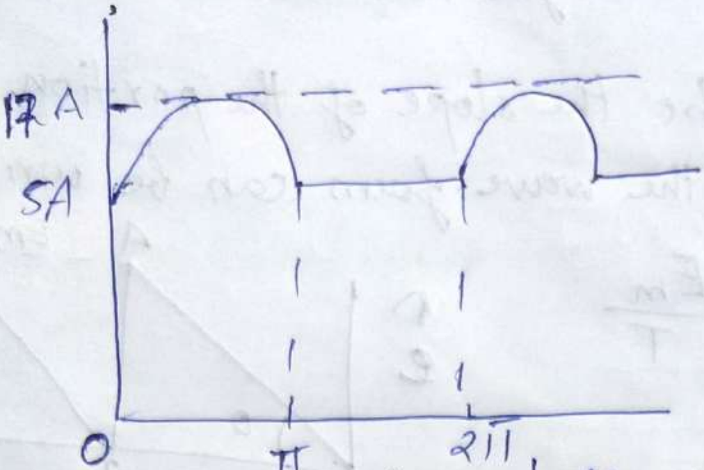
$E_{avg} = \frac{1}{T}\int_0^T \frac{E_m}{T} t\, dt$

$= \frac{E_m}{T^2}\left[\frac{t^2}{2}\right]_0^T = \frac{E_m}{2}$ //

Form factor = $\frac{\cancel{E_m}}{\sqrt{3}} \times \frac{2}{\cancel{E_m}} = \frac{2}{\sqrt{3}}$ //

$= \frac{I_m}{\pi}$

(3)



A resistance carries a half rectified sinusoidal current of peak value 12 A super impose on a dc of 5A. Determine the average and r.m.s values of the total current

$$I_{r.m.s} = \sqrt{\frac{1}{2\pi}\int_0^{\pi}(5+12\sin\omega t)^2\,d\omega t + \int_{\pi}^{2\pi}5^2\,d\omega t}$$

$$I_{avg} = \frac{1}{2\pi}\left(\int_0^{\pi}(5+12\sin\omega t)\,d\omega t + \int_{\pi}^{2\pi}5\,d\omega t\right)$$