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MODULE 5

Rotation / circular motion

⇒ Motion of a body along a circular path.

⇒ centre of rotation remains constant

⇒ Displacement of particle in circular motion is measured in terms of angular displacement θ.

Angular velocity, $\omega = \frac{d\theta}{dt}$ ~~unit~~ rpm (revolution per sec)

$$\omega = \frac{2\pi N}{60} \quad N: \text{Angular velocity in rpm.}$$

Angular accelera^n, $\alpha = \frac{d\omega}{dt}$

Relation b/w linear & angular velocity ; $v = r\omega$

angular accelera^n ; $a = r\alpha$

tangential accelera^n, $a_t = r\frac{d^2\theta}{dt^2}$

normal accelera^n, $a_n = r\omega^2$

equations of motion

Angular motion	Rectilinear motion
$\omega_2 = \omega_1 + \alpha t$	$v = u + at$
$\omega_2^2 = \omega_1^2 + 2\alpha\theta$	$v^2 = u^2 + 2as$
$\theta = \omega_1 t + \frac{1}{2}\alpha t^2$	$s = ut + \frac{1}{2}at.$

Q. The armature of an electric motor, has angular speed of 1800 rpm at the instant when the power is cut off. If it comes to rest in 6 sec, calculate the angular deceleraⁿ assuming it is constant. How many revolutions does the armature make during this period.

$N = 1800\,rpm$. $\quad \omega_1 = \frac{2\pi N_1}{60} = 2\pi \times 30$ rad/s

$t = 6$ sec

$\omega_2 = \omega_1 + \alpha t$

$0 = 2\pi \times 30 + \alpha \times 6$

$\alpha = \underline{-31.4}$ rad/s²

Angle turned, $\theta = \omega_1 t - \frac{1}{2}\alpha t^2$

$= 2\pi \times 30 \times 6 - \frac{1}{2} \times 31.4 \times 6^2$

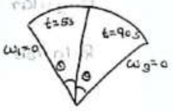
$= 565.77$ rad $= \frac{565.77}{2\pi}$

$= 90$ revolutions

Q. A grinding wheel is attached to the shaft of an electric motor of rated speed of 1800rpm. when the power is switched on the unit attains the rated speed in 5sec, & when the power is switched of the unit comes to rest in 90sec assuming uniformly accelerated motion, determine the no. of revolutions the unit turns.

i) to attain the rated speed

ii) to come to rest



$N = 1800$ rpm $\quad \omega = \frac{2\pi \times 1800}{60} = 2\pi \times 30$ rad/s.

$t = 5$ sec $\quad = 60\pi$ rad/s

$\omega_2 = \omega_1 + \alpha t$

$60\pi = 0 + 5\alpha$

$\alpha = 12\pi$ rad/s²

i) Angular displacement, $\theta = \omega_1 t + \frac{1}{2}\alpha t^2$

$= \frac{1}{2} \times 12\pi \times 5^2$

$= 150\pi$ rad

No. of revolutions to attain the rated speed $\Big\} = \frac{150\pi}{2\pi} = 75$

ii) $\omega_3 = 0$

$t = 90s$

$\omega_3 = \omega_2 + \alpha t$

$0 = 60\pi + 90\alpha$

$\alpha = -\frac{2}{3}\pi$ rad/s²

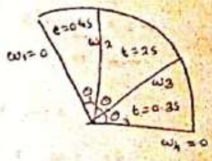
$\theta = \omega_2 t + \frac{1}{2}\alpha t^2$

$= 60\pi \times 90 + \frac{1}{2} \times \frac{-2}{3}\pi \times 90^2$

$= 2700\pi$ rad

$= 1350$ revolutions

Q. A wheel accelerates from rest to a speed of 180rpm uniformly in 0.4 sec. It then rotates at that speed for 2sec before decelerating to rest in 0.3 sec. Determine the total revolutions made by the wheel.



$N_2 = 180$ rpm. $\quad \omega_2 = \frac{2\pi \times 180}{60}$

$t = 0.4$ sec

$= 6\pi$ rad/s

$\omega_1 = 0$

$\omega_2 = \omega_1 + \alpha t$

$6\pi = 0.4\alpha$

$\alpha_1 = 47.1$ rad/s²

$\theta_1 = \omega_1 t + \frac{1}{2}\alpha_1 t^2$

$= \frac{1}{2} \times 47.1 \times 0.4^2$

$= 3.77$ rad

$\omega_3 = \omega_2 \quad (\alpha = 0)$

$\theta_2 = \omega_2 t = 6\pi \times 2 = 12\pi$ rad

$\omega_4 = \omega_3 + \alpha t$

$\alpha_3 = \frac{0 - 6\pi}{0.3} = -62.8$ rad/s²

$\theta_3 = \omega_2 t + \frac{1}{2}\alpha_3 t^2$

$= 6\pi \times 0.3 - \frac{1}{2} \times 62.8 \times 0.3^2$

$= 2.83$ rad

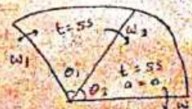
Total angular displacement, $\theta = \theta_1 + \theta_2 + \theta_3$

$= 3.77 + 12\pi + 2.83$

$= 44.28$ rad $= \frac{44.28}{2\pi}$

$= 7.05$ revolutions

Q. A wheel rotates for 5 sec with a const angular acceleration & describes during that time 100 rad. It then rotates with constant angular velocity & during the next 5 sec describes 80 rad. Find the initial angular velocity & the angular acceleration.



$\omega_2 = \omega_1 + \alpha_1 t_1$

$\omega_2 = \omega_1 + 5\alpha_1$ ——(1)

$\theta = \omega_1 t_1 + \frac{1}{2}\alpha_1 t_1^2$

$100 = 5\omega_1 + \frac{1}{2}\alpha_1 \times 5^2$

$20 = \omega_1 + 2.5\alpha_1$ —— (2)

$\theta_2 = \omega_2 t_2 + \frac{1}{2}\alpha_2 t_2^2$

$80 = 5\omega_2 + 0$

$\underline{\omega_2 = 16 \text{ rad/s}}$

$\therefore \omega_2 = \omega_1 + 5\alpha_{21}$ —— (I)

$16 = \omega_1 + 5\alpha_1$

$20 = \omega_1 + 2.5\alpha_1$ —— (II)

$-4 = 2.5\alpha_1$

$\alpha_1 = \underline{-1.6 \text{ rad/s}^2}$

$16 = \omega_1 + 5\alpha_1$

$= \omega_1 - 5 \times 1.6 = \underline{24 \text{ rad/s}}$

2. The rotation of a flywheel is defined by the eqn, $\omega = 3t^2 - 2t + 2$, where ω is in rad/s & t is in sec. After one second from the start, the angular displacement was 4 rad. Determine the angular displacement, angular velocity & angular acceleration of the flywheel when $t = 3s$.

$\omega = 3t^2 - 2t + 2$

at $t = 1s$, $\theta = 4$ rad

$\theta|_{t=3} = \int (3t^2 - 2t + 2)\,dt$

$4 = 1 - 1 + 2 \times 1 + c$

$c = \underline{2}$

$= \cancel{3}\frac{t^3}{\cancel{3}} - \cancel{2}\frac{t^2}{\cancel{2}} + 2t + c$

$= 3^3 - 3^2 + 2 \times 3 + 2$

$= \underline{26 \text{ rad}}$

$\omega|_{t=3} = 3 \times 3^2 - 2 \times 3 + 2$

$= \underline{23 \text{ rad/s}}$

$\alpha|_{t=3} = \frac{d\omega}{dt} = \frac{d}{dt}(3t^2 - 2t + 2)$

$= 6t - 2$

$= \underline{16 \text{ rad/s}^2}$

Kinetics of rotation

Turning moment / torque , $T = I\alpha$ — I = mass moment of inertia, α = angular accelraⁿ

⇒ It is analogues Newton's IIlaw

$$\text{torque}, T = F \times r.$$

Work done in rotation

$$\text{workdone} = T\theta = \frac{T \times 2\pi N}{60}$$

KE due to rotation

$$KE = \frac{1}{2} I\omega^2.$$ I - mass moment of inertia, ω - angular velocity.

work-energy equⁿ for rotation:

$$T\theta = \frac{1}{2} I\omega_2^2 - \frac{1}{2} I\omega_1^2.$$

SHM: Any motion which repeat after interval of time.

$$\text{Period}, \quad t_p = \frac{2\pi}{\omega}$$

Free vibration: If a disturbing force is applied just to start the motion & is then removed from the system is said to undergo free vibration.

Force vibration: If the disturbing force act at periodic intervals on the system. The system is said to undergo forced vibraⁿ.

Degree of freedom: It is the no. of independent co-ordinates require to define the configuraⁿ of system

$$F = kx$$

$$\omega = \sqrt{\frac{k}{m}}$$

$$f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}}.$$

$$\frac{1}{K_e} = \frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{K_3}$$ spring in series.

$$K_e = K_1 + K_2 + K_3$$ parallel.

Q. A body, moving with SHM, has an amplitude of 1m & period of oscillations is 2sec. Find the velocity & acceleration of the body at $t = 0.4sec$, when time is measured from

i) the mean position

ii) the extreme position.

i) $r = 1m$

$t_p = 2s$

$t = 0.4s$

$\omega = \frac{2\pi}{t_p} = \frac{2\pi}{2} = \pi \ rad/s$

$\omega t = 0.4\pi \ rad$

$= \left(\frac{180}{\pi} \times 0.4\pi\right)^{\circ} = \underline{\underline{72^{\circ}}}$

$x = r\sin\omega t$

$x = 1\sin 72 = \underline{\underline{0.95m}}$

Velocity, $V = \omega\sqrt{r^2 - x^2} = \pi \times \sqrt{1^2 - (0.95)^2} = \underline{\underline{0.98 m/s}}$

Acceleration, $a = \omega^2 x = \pi^2 \times 0.95 = \underline{\underline{9.3 m/s^2}}$

ii) $x = r\cos\omega t = 1 \times \cos 72$

$= 0.31m$

$V = \omega\sqrt{r^2 - x^2} = \pi\sqrt{1^2 - 0.31^2} = \underline{\underline{2.99 m/s}}$

$a = \omega^2 x = \pi^2 \times 0.31 = \underline{\underline{3.06 m/s^2}}$

Q. A body moving with SHM has velocities of 10m/s & 4m/s at 2 & 4 m distance from the mean position. Find the amplitude & time period of the body.

$x = 2m$, $V = 10 m/s$

$x = 4m$, $v = 4m/s$

$V = \omega\sqrt{r^2 - x^2}$

$10 = \omega\sqrt{r^2 - 4}$

$4 = \omega\sqrt{r^2 - 16}$

$\frac{10}{4} = \frac{\sqrt{r^2 - 4}}{\sqrt{r^2 - 16}}$

$6.25 = \frac{r^2 - 4}{r^2 - 16}$

$6.25r^2 - 100 = r^2 - 4$

$5.25r^2 = 96$

$r = \underline{\underline{4.28m}}$

$10 = \omega\sqrt{r^2 - 4}$

$10 = \omega\sqrt{4.28^2 - 4}$

$= 3.78\omega$

$\omega = \underline{\underline{2.64 \ rad/s}}$

$t_p = \frac{2\pi}{\omega} = \frac{2\pi}{2.64}$

$= \underline{\underline{2.38s}}$

Q. A flywheel weighing 50KN & having radius of gyration 1m loses its speed from 400rpm in 120sec. Calculate

i) The retarding torque acting on it.

ii) Change in the KE during the above period.

$N_1 = 400 rpm$. $N_2 = 300 rpm$

$t = 120 s$ $W = 50 \times 10^3 N$

$k = 1 m$.

$\omega_1 = \frac{2\pi N}{60} = \frac{2\pi \times 400}{60} = 41.89 \text{ rad/s}$.

$\omega_2 = \frac{2\pi \times 300}{60} = \underline{31.42 \text{ rad/s}}$

i) Retarding torque, $T = I\alpha$

$= mk^2\alpha$

$= \frac{50\times10^3}{9.81} \times 1^2 \times \alpha$

$\omega_2 = \omega_1 + \alpha t$ $= \frac{50\times10^3}{9.81} \times 1 \times 0.087$

$31.42 = 41.891 \alpha \times 120$

$\alpha = \underline{\underline{-0.087 \text{ rad/s}^2}}$ $= \underline{443.43 \text{ Nm}}$

ii) change in KE = initial KE - final KE

$= \frac{1}{2} I\omega_1^2 - \frac{1}{2} I\omega_2^2$

$= \frac{1}{2} I (\omega_1^2 - \omega_2^2)$

$= \frac{1}{2} mk^2 (\omega_1^2 - \omega_2^2)$

$= \frac{1}{2} \times \frac{50\times10^3}{9.81} \times 1 \; (41.89^3 - 31.42^2)$

$= \underline{\underline{1956.05 \text{ kNm}}}$

Q. A right circular disc of weight 1500 N & 750 mm diameter is free to rotate about its geometric axis & is constantly accelerated from rest to 300 rpm in 20s. Determine the constant torque required to produce this accelerat^n.

$W = 1500 N$

$\omega_1 = 0$, $t = 20 s$

$\omega_2 = \frac{2\pi N}{60} = \frac{2\pi \times 300}{60} = 31.41 m/s$

$T = I\alpha$

moment of inertia, $I = \frac{mr^2}{2}$

$$= \frac{\omega}{g} \frac{r^2}{2}$$

$$= \frac{1500}{9.81} \times \left(\frac{\frac{0.75}{2}}{2}\right)^2$$

$$= \underline{\underline{10.75 \, kgm^2}}$$

$\omega_2 = \omega_1 + \alpha t$

$31.41 = 20\alpha$

$\alpha = \underline{\underline{1.57 \, rad/s^2}}$

torque, $T = I\alpha$

$= 10.75 \times 1.57$

$= \underline{\underline{16.88 \, Nm}}$

? A shaft of radius r rotates with constant angular speed ω in bearenges for which the co-efficient of friction is μ. Through what angle θ will it rotate after the driving torque is removed.

frictional force $= \mu R_N = \mu \omega$

$F_r = \mu \omega_r$

$\omega_1 = \omega$

$\omega_2 = 0$

$T = I\alpha$

$\mu \omega_r = I\alpha$

$$\alpha = \frac{\mu \omega_r}{I} = \frac{\mu \omega_r}{\frac{mr^2}{2}} = \frac{\mu mg r}{mr^2/2}$$

$$= \frac{2\mu g}{r}$$

$\omega_2^2 = \omega_1^2 - 2\alpha \times \theta$

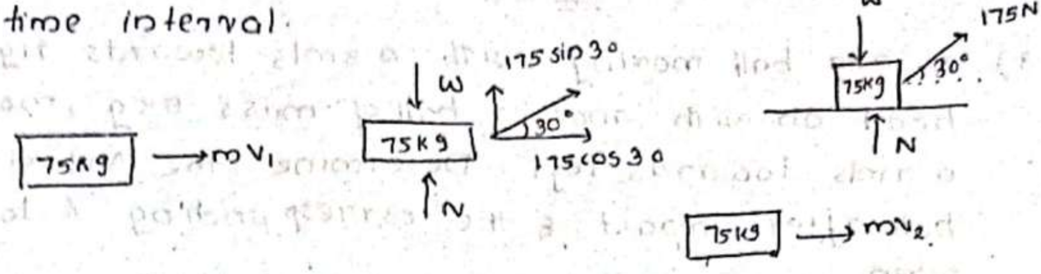
$0 = \omega^2 - 2\alpha\theta$

$$\theta = \frac{\omega^2}{2\alpha}$$

$$= \frac{\omega^2}{2\left(\frac{2\mu g}{r}\right)}$$

$$\theta = \underline{\frac{\omega^2 r}{4\mu g} \text{ rad}}$$

1) The 75 kg crate is originally at rest on the smooth horizontal surface. If a towing force of 175 N, is acting at an angle of 30° is applied for 12 s, determine final velocity & normal force which the surface exerts on the crate during this time interval.



In x-direcⁿ

$$(mv_1)_x + (\text{impulse}_{1\text{-}2})_x = (mv_2)_x$$

$$mv_1 + \Sigma F_x \times t = (mv_2)_x$$

$$0 + 175\cos 30 \times 12 = 75 V_2$$

$$V_2 = \underline{\underline{24.24\,m/s}}$$

In y-direcⁿ

$$(m_1 v_1)_y + (\text{impulse}_{1\text{-}2})_y = (mv_2)_y$$

$$(mv_1)_y + \Sigma F_y \times t = (mv_2)_y$$

$$0 + N(12) - 75 \times 9.81 \times 12 + 175\sin 30 \times 12 = 0$$

$$N = \underline{\underline{648.25\,N}}$$

2) A man of weight 700N is standing on one end of a boat of weight 2200 N & 3m long. He then walks to the other end of the boat. what is the corresponding displacement of the boat? (Neglect water resistance)

Apply conservaⁿ of momentum in the x-direcⁿ

initial momentum = Final momentum.

$$0 = (mv)_{man} + (mv)_{boat}$$

$$0 = \frac{700}{g} \times V_{man} + \frac{2200}{g} V_{boat}$$

$$= 700\left(\frac{dx}{dt}\right)_{man} + 2200\left(\frac{dx}{dt}\right)_{boat}$$

$$= 700\, dx_{man} + 2200\, dx_{boat}$$

$$= 700 \int_0^{3+x} dx_{man} + 2200 \int_0^{x} dx_{boat}$$

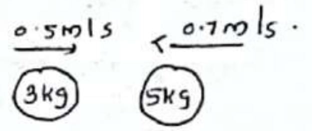
$$0 = 700\left[x_{man}\right]_0^{3+x} + 2200\left[x_{boat}\right]_0^{x}$$

$$0 = 700\left[3+x\right] + 2200x$$

$$x = -0.724\text{ m} \quad \text{(backwards)}.$$

3) A 3 kg ball moving with 0.5 m/s towards right collides head on with another ball of mass 5 kg, moving with 0.7 m/s towards left. Determine the velocities of the ball after impact & the corresponding % loss of KE when

1) The impact is perfectly elastic $e = 1$
2) The impact is perfectly plastic $e = 0$
3) The impact is such that $e = 0.7$.



1) Using conserva^n of momentum

$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B'$$

$$3 \times 0.5 + 5 \times -0.7 = 3v_A' + 5v_B'$$

$$-2 = 3v_A' + 5v_B' \quad \text{---(1)}$$

using co-efficient of restitution equ

$$v_B' - v_A' = e\left[v_A - v_B\right]$$

$$v_B' - v_A' = 1\left[0.5 - (-0.7)\right]$$

$$v_B' = 1.2 + v_A' \quad \text{---(2)}$$

From (1) & (2) $\Rightarrow$ $v_A' = -1\text{ m/s} = 1\text{ m/s}$

$$v_B' = 0.2\text{ m/s}.$$

Since the impact is perfectly elastic there will be no loss of KE.

2) impact is perfectly plastic ie $e = 0$

$$v_A' = v_B' = v'$$

using conserva^n of momentum.

$$3 \times 0.5 + 5 \times -0.7 = 3v' + 5v'$$

$$v' = -0.25\text{ m/s}.$$

KE of the system before impact

$$KE = \frac{1}{2} m v_A^2 + \frac{1}{2} m v_B^2$$

$= \frac{1}{2} \times 3 \times (0.5)^2 + \frac{1}{2} \times 5 \times (0.7)^2$

$= \underline{1.6\ J}$

KE of the system after impact

$= \frac{1}{2} \times 3 \times (0.25)^2 + \frac{1}{2} \times 5 \times (0.25)^2$

$= \underline{0.25\ J}$

% of KE $= \frac{1.6 - 0.25}{1.6} \times 100 = \underline{84.375\ \%}$

3) impact when $e = 0.7$

$m_A v_A + m_B v_B = m_A v_A' + m_B v_B'$

$3 \times 0.5 + 5 \times -0.7 = 3 v_A' + 5 v_B'$ —— (3)

using co-efficient of restitution

$v_B' - v_A' = e [v_A - v_B]$

$v_B' - v_A' = 0.7 [0.5 - (-0.7)]$

$v_B' = 0.84 + v_A'$ —— (4)

from (3) & (4) $\Rightarrow$ $v_A' = -0.775\ m/s = \underline{0.775\ m/s}$.

$v_B' = \underline{0.065\ m/s}$

KE of the system after impact

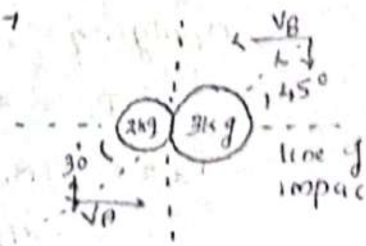
$\frac{1}{2} \times 3 \times (0.775)^2 + \frac{1}{2} \times 5 \times (0.065)^2$

$= \underline{0.9115\ J}$

% loss in KE $= \frac{1.6 - 0.9115}{1.6} \times 100$

$= \underline{43.03\ \%}$

4) 2 smooth balls collide as shown. find the velocities after impact. $m_A = 2kg$, $m_B = 3\ kg$ & $e = 0.7$



$V_{AN} = 3.46\ m/s \Rightarrow V_{At} = 2\ m/s$

$V_{BN} = 1.732\ m/s \Rightarrow V_{Bt} = 1\ m/s$

$1.724 = 2 V_{An}' + 3 V_{Bn}'$ —— (1)

$V_{Bn}' - V_{An}' = 3.63$ —— (2)

$V_{An}' = -1.83\ m/s = 1.83\ m/s$

$V_{Bn}' = 1.79\ m/s$

$V_{At} = V_{At}' = 2 m/s \uparrow \qquad V_{Bt} = V_{Bt}' = 1 m/s \downarrow$

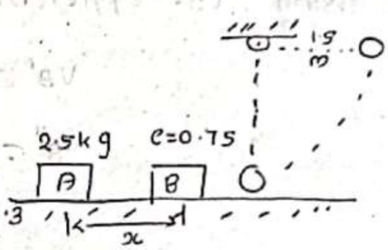
$$V_A' = \sqrt{(V_{An}')^2 + (V_{At}')^2}$$

$$= \sqrt{(1.83)^2 + 2^2} = \underline{\underline{2.71 m/s}}$$

$$\theta_A' = \tan^{-1}\left(\frac{V_{At}'}{V_{An}'}\right) = \tan^{-1}\left(\frac{2}{1.83}\right) = \underline{\underline{47.54^\circ}}$$

$$V_A' = \underline{\underline{2.71 m/s}} \qquad \theta_A' = \underline{\underline{47.54^\circ}}$$

5. A sphere of mass 3kg is released from rest. It swings as a pendulum & strikes a block B of mass 2.5kg resting on a horizontal surface. Determine how far the block until move after impact. Take $\mu = 0.3$ b/w the block B & horizontal surface & $e = 0.75$.



$T_1 + \Sigma U_{1-2} = T_2$ ——(1)

$T_1 = 0$ since it starts from rest.

$T_2 = \frac{1}{2} m v^2 = \frac{1}{2} \times 3 v^2.$

U_{1-2} = only weight force is acting

$= mgh = 3 \times 9.81 \times 1.5 = \underline{\underline{44.145 J}}.$

(1) $\Rightarrow \quad 0 + 44.145 = \frac{1}{2} 3 v^2$

$v = \underline{\underline{5.42 m/s}}$

$m_A V_A + m_B V_B = m_A V_A' + m_B V_B'$

$3 \times 5.425 + 2.5 \times 0 = 3 V_A' + 2.5 V_B'$

$3 V_A' + 2.5 V_B' = 16.275$ ——(2)

$V_B' - V_A' = 0.75(5.425 - 0) = 4.068$ ——(3)

From (2) & (3) $\quad 3 V_A' = \underline{\underline{1.11 m/s}} \qquad V_B' = \underline{\underline{5.178 m/s}}$

Applying work-energy principle to block B.

$T_1 = \frac{1}{2} m v^2 = \frac{1}{2} \times 2.5 \times 5.178^2 = \underline{\underline{33.51 J}}.$

$T_3 = 0$ since the block comes to rest

U_{2-3} = only friction force will act

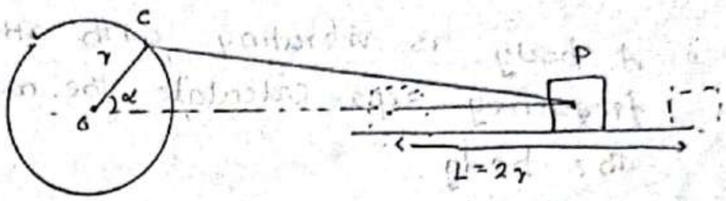
$= -\mu_k N \cdot S = 0.3 \times (2.5 \times 9.81)$

$= \underline{-7.36 J}$

$33.51 + (-7.36 x) = 0$

$x = \underline{\underline{4.55 m}}$

Q. The piston of a IC engine moves with SHM. The crank rotates at 420 rpm & the stroke length is 40cm. Find the velocity & accelera^n of the piston when it is at a distance of 10 cm from the mean position.



Speed of crank = 420 rpm

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 420}{60} = 43.98 \text{ rad/s}$$

Stroke length, L = 2 × crank radius

$$r = \frac{L}{2} = 20\text{cm} = 0.2\text{ m}$$

$$x = 10\text{cm} = 0.1\text{ m}$$

$$v = \omega\sqrt{r^2 - x^2}$$

$$= 43.98\sqrt{0.2^2 - 0.1^2}$$

$$= 7.62\text{ m/s}$$

$$a = \omega^2 x = 43.98^2 \times 0.1 = 193.42\text{ m/s}^2$$

Q. A particle moving with SHM has an amplitude of 4.5m & period of oscillation is 3.5 sec. Find the time required by the particle to pass 2 points which are at a distance of 3.5m & 1.5m from the centre & on the same side of mean position.

Amplitude, $r = 4.5$ m

$$t_p = 3.5\text{s}$$

$$\omega = \frac{2\pi}{t_p} = \frac{2\pi}{3.5} = 1.8 \text{ rad/s}$$

Let x_1 & x_2 be the distance of the 1^{st} & 2^{nd} point from mean position

$$x = r\cos\omega t$$

$$x_1 = r\cos\omega t_1$$

$$3.5 = 4.5\cos\left(1.8 \times t_1 \times \frac{180}{\pi}\right)^\circ$$

$$t_1 = 0.385$$

$$x_2 = r\cos\omega t_2$$

$$1.5 = 4.5\cos\left(1.8 \times t_2 \times \frac{180}{\pi}\right)$$

$$t_2 = 0.68\text{ s}.$$

time required to pass the 2 points

$$t = t_2 - t_1$$
$$= 0.68 - 0.38$$
$$= 0.35$$

Q. A body is vibrating with SHM of amplitude 150mm & frequency 3cps. Calculate the max. velocity & acceleration of the body.

$r = 150mm = 0.15 m$

$f = 3$ cps

$\omega = 2\pi f = 2\pi \times 3 = 6\pi$ rad/s

$V_{max} = r\omega = 0.15 \times 6\pi = 2.83$ m/s

$a_{max} = \omega^2 r = (6\pi)^2 \times 0.15 = 53.3$ m/s^2

Q. A 80N weight is hung on the end of a helical spring & is set vibrating vertically. The weight makes 4 oscillations per sec. Determine the stiffness of the spring.

$W = 80N$

$m = \frac{80}{g}$

$F = 4$ cps.

$f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

$f_n^2 = \frac{1}{4\pi^2} \frac{k}{m}$

$k = f_n^2 \times m (4\pi)^2$

$= 4 \times \frac{80}{9.81} \times 16 \times \pi^2$

$= 5151 N/m$

Q. If a helical spring having a stiffness of 90 N/cm is available what weight should be hug on it so that will oscillate with a periodic time of 1 sec.

$K = 90 N/cm = 90 \times 10^2$ N/m

$t_p = 1$ sec

$f_n = \frac{1}{t_p} = 1$ cps

$f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

$m = \frac{k}{(2\pi)^2 f_n^2} = \frac{90 \times 10^2}{4 \times \pi^2 \times 1^2}$

$= 227.97$ kg

Q A weight of 50N suspended from a spring vibrates vertically with an amplitude of 8 cm & a frequency of 1 oscillation/sec. Find

a) the stiffness of the spring

b) the max. tension included in the spring.

c) Max. velocity of weight.

$$m = \frac{\omega}{g} = \frac{50}{9.81}$$

$x = 8\,cm = 0.08\,m$.

$F = 1\,cps$.

(a) $$f_n = \frac{1}{2\pi}\sqrt{\frac{K}{m}}$$

$$K = f_n^2 \times 4\pi^2 \times m = 1 \times 4 \times \pi^2 \times \frac{50}{9.81} = 201.22\,N/m$$

(b) max. tension in the spring $= Kx$

$= 201.22 \times 0.08$

$= 16.1\,N$

(c) max. velocity, $V_{max} = \omega x$

$= (2\pi f) \times x$

$= 2\pi \times 1 \times 0.08$

$= 0.5\,m/s$

Q A body of mass 50kg is suspended by 2 springs of stiffness 4 KN/m & 6KN/m as shown in fig. The body is pulled 50mm down from its equilibrium position & then released. Calculate.

a) frequency of oscillation.

b) max. Velocity

c) max. acceleraⁿ.

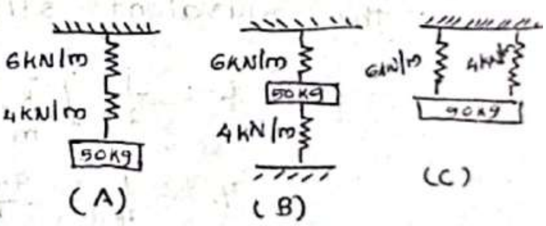


Fig A.

$$\frac{1}{K_e} = \frac{1}{K_1} + \frac{1}{K_2} = \frac{1}{6} + \frac{1}{4} = \frac{10}{24}$$

$K_e = 2.4\,KN/m = 2.4 \times 10^3\,N/m$

a) $$F = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{2.4 \times 10^3}{50}} = 1.10\,cps.$$

$\omega = 2\pi f = 6.93\,rad/s$.

b) $V_{max} = \omega x = 6.93 \times 0.05 = 0.35\,m/s$

c) $a_{max} = \omega^2 x = (6.93)^2 \times 0.05 = 2.4\,m/s^2$

Fig B.

$$K_e = k_1 + k_2 = 4 + 6 = 10\,kN/m$$

a) $f = \frac{1}{2\pi}\sqrt{\frac{K}{m}} = \frac{1}{2\pi}\sqrt{\frac{10\times10^3}{50}} = 2.25\,cps$

b) $V_{max} = \omega x = 2\pi \times 2.25 \times 0.05 = 0.71\,m/s.$

c) $a_{max} = \omega^2 x = (2\pi \times 2.25)^2 \times 0.05 = 10\,m/s^2$

Fig c

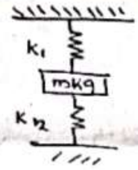
$K_e = 10\,kN/m.$

a) $f = 2.25\,cps$

b) $V_{max} = 0.71\,m/s$

c) $a_{max} = 10\,m/s^2$

Q. A spring of stiffness 6kN/m is cut into 2 halves & fixed to a mass m. If the system vibrates with frequency 3Hz, determine the mass m.



$f = 3Hz = 3cps.$

$k = 6\,kN/m = 6\times10^3\,N/m$

Stiffness of spring $\propto \frac{1}{\text{no. of coils}}$ $\therefore$ when this spring is cut into 2 halves, the stiffness of each half is doubles. $K_1 = K_2 = 2\times6 = 12\,kN/m$. Since the spring are in parallel, the equvalent stiffness, $ke = k_1 + k_2 = 24\,kN/m = 24\times10^3\,N/m$

$$f = \frac{1}{2\pi}\sqrt{\frac{K}{m}} = \frac{1}{2\pi}\sqrt{\frac{24\times10^3}{m}}$$

$$9 = \frac{1}{4\pi^2}\,\frac{24\times10^3}{m}$$

$$m = \frac{24\times10^3}{4\pi^2\times9} = 67.62\,kg$$