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SFI GEC PALAKKAD

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

First Semester B.Tech Degree Regular and Supplementary Examination December 2020 (2019 Scheme)

Course Code: MAT101
Course Name: LINEAR ALGEBRA AND CALCULUS
(2019 Scheme)

Max. Marks: 100 Duration: 3 Hours

PART A

Answer all questions, each carries 3 marks.

1	Determine the rank of the matrix $A = \begin{bmatrix} 2 & 1 & -1 \\ 0 & 3 & -2 \\ 2 & 4 & -3 \end{bmatrix}$	(3)
2	Show that the quadratic form $4x^2 + 12xy + 13y^2$ is positive definite.	(3)
3	If $z = \sin(y^2 - 4x)$ find the rate of change of z with respect to x at the point $(3,1)$ with y held fixed.	(3)
4	Find $\frac{dz}{dt}$ by chain rule ,where $z = 3x^2 y^2$, $x = t^4, y = t^3$	(3)
5	Find the mass of the lamina with density function x^2 which is bounded by $y = x$ and $y = x^2$.	(3)
6	Evaluate $\iint_R y^2 x \, dA$ over the region $R = \{(x,y), -3 \le x \le 2, 0 \le y \le 1\}$	(3)
7	Test the convergence of the series $\sum_{k=1}^{\infty} \left(\frac{k}{100}\right)^k$	(3)
8	Does the series $\sum_{k=1}^{\infty} \left(\frac{-3}{4}\right)^k$ converge? If so, find the sum.	(3)
9	Find the binomial series for $f(x) = (1+x)^{1/3}$ up to third degree term.	(3)
10	Find the Maclaurin's series of $f(x) = \log(1+x)$ up to third degree term.	(3)

PART B

Answer one full question from each module, each question carries 14 marks

Module-I

11 a) Solve the following linear system of equations using Gauss elimination method. $x + y + z = 6, \quad x + 2y - 3z = -4, \quad -x - 4y + 9z = 18$ (7)

b) Find eigen values and eigen vectors of the matrix

$$A = \begin{bmatrix} 1 & 1 & 2 \\ -1 & 2 & 1 \\ 0 & 1 & 3 \end{bmatrix}$$ (7)

12 a) Show that the equations

$x + y + z = a, \quad 3x + 4y + 5z = b, \quad 2x + 3y + 4z = c$

(i) have no solution if $a = b = c = 1$ (7)

(ii) have many solutions if $a = \frac{b}{2} = c = 1$

b) Find the matrix of transformation that diagonalize the matrix (7)

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$ Also, find the diagonal matrix.

Module-II

13 a) Find the local linear approximation of $\frac{4y}{x+z}$ at $(1,1,1)$ (7)

b) Find the absolute extrema of the function $f(x, y) = x^2 - 3y^2 - 2x + 6y$ over the square region with vertices $(0,0), (0,2)$ $(2,2)$ and $(2,0)$. (7)

14 a) If $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$ find the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}$. (7)

b) Locate all relative extrema of $f(x, y) = 2xy - x^3 - y^2$ (7)

Module-III

15 a) Use double integrals to find the area of the region enclosed between the parabola $2y = x^2$ and the line $y = 2x$ (7)

b) Find the volume of the solid in the first octant bounded by the coordinate planes and the plane $x + 2y + z = 6$. (7)

16 a) Change the order of integration and hence evaluate $\int_0^4 \int_{\frac{x^2}{4}}^{2\sqrt{x}} dy\,dx$ (7)

b) Evaluate $\iiint_G z\,dV$, where G is the wedge in the first octant cut off from the cylindrical solid $y^2 + z^2 \leq 1$ and the planes $y = x$ and $x = 0$. (7)

Module-IV

17 a) Test the convergence of the series

$$1+\frac{1.3}{3!}+\frac{1.3.5}{5!}+\frac{1.3.5.7}{7!}+\ldots\ldots\ldots$$ (7)

b) Find the sum of the series $\sum_{k=1}^{\infty}\frac{1}{k(k+1)}$ (7)

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a) Test the convergence of (i) $\sum_{k=1}^{\infty}\frac{k!}{3!\,(k-1)!\,3^k}$ (ii) $\sum_{k=1}^{\infty}\left(\frac{4k-5}{2k+1}\right)^k$ (7)

b) Test the absolute or conditional convergence of $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}\,k^2}{k^3+1}$ (7)

Module-V

19 a) Expand into a Fourier series, $f(x) = e^{-x}$, $0< x < 2\pi$ (7)

b) Find the half range cosine series for $f(x)=(x-1)^2$ in $0 \le x \le 1$. (7)

20 a) Find the Fourier series of the function $f(x) = |x|$ in $-1 \le x \le 1$ (7)

b) Find the Fourier sine series of $f(x) = x\cos x$ in $0 < x < \pi$ (7)

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1) $A = \begin{bmatrix} 2 & 1 & -1 \\ 0 & 3 & -2 \\ 2 & 4 & -3 \end{bmatrix}$

$R_3 \rightarrow R_3 - R_1$

$$\begin{bmatrix} 2 & 1 & -1 \\ 0 & 3 & -2 \\ 0 & 3 & -2 \end{bmatrix}$$

$R_3 \rightarrow R_3 - R_2$

$$\begin{bmatrix} 2 & 1 & -1 \\ 0 & 3 & -2 \\ 0 & 0 & 0 \end{bmatrix} \quad \therefore \rho(A) = 2$$

$\therefore$ Rank of $A = 2$

2) $4x^2 + 12xy + 13y^2$

Coefficient Matrix A =

$$\begin{array}{c|cc} & x & y \\ \hline x & 4 & 6 \\ y & 6 & 13 \end{array}$$

$$A = \begin{bmatrix} 4 & 6 \\ 6 & 13 \end{bmatrix}$$

Characteristic equation is $|A - \lambda I| = 0$

$$\begin{vmatrix} 4-\lambda & 6 \\ 6 & 13-\lambda \end{vmatrix} = 0$$

$(4-\lambda)(13-\lambda) - 36 = 0$

$\lambda^2 - 17\lambda + 16 = 0$

$(\lambda-16)(\lambda-1) = 0$

$\therefore \lambda = 1, 16$

The eigen values are 16, 1

The quadratic form is said to be positive definite if all eigen values are +ve ($\lambda_i > 0$)

$\therefore$ The quadratic form $4x^2 + 12xy + 15y^2$ is +ve definite.

3) $z = \sin(y^2 - 4x)$

$$\frac{dz}{dx} = \cos(y^2-4x)(-4)$$

$$= -4\cos(y^2-4x)$$

$$\left.\frac{dz}{dx}\right|_{(3,1)} = -4\cos(1-4\times3)$$

$$= -4\cos(-11)$$

$$= -3.926$$

4) $z = 3x^2y^2 \qquad x = t^4 \qquad y = t^3$

$$\frac{dz}{dt} = \frac{\partial z}{\partial x} \times \frac{dx}{dt} + \frac{\partial z}{\partial y} \times \frac{dy}{dt}$$

$$= 3y^2 \times 2x \times 4t^3 + 3x^2 \times 2y \times 3t^2$$

$$= 6xy^2 \times 4t^3 + 6x^2y \times 3t^2$$

$= 2 + t^3 \times t^4 \times t^6 + 18\, t^{8} \times t^3 \times t^2$

$= 42 t^3$

7) $\sum_{K=1}^{\infty} a_K = \sum_{K=1}^{\infty} \left(\frac{K}{100}\right)^K$

$a_K = \left(\frac{K}{100}\right)^K$

$a_K^{1/K} = \frac{K}{100}$

$\lim_{K \to \infty} a_K^{1/K} = \lim_{K \to \infty} \frac{K}{100} > 1$

$\Rightarrow$ The series is divergent.

8) $\sum_{K=1}^{\infty} a_K = \sum_{K=1}^{\infty} \left(\frac{-3}{4}\right)^K$

$a_K = \left(\frac{-3}{4}\right)^K$

$\lim_{K \to 2} a_K^{1/K} = \lim_{K \to \infty} \left(\frac{-3}{4}\right)$

$= \frac{-3}{4} < 1$

$\Rightarrow$ The series is convergent

(c) $f(x) = f(0) + \frac{x}{1!} f'(0) + \frac{x^2}{2!} \cdot f''(0) + \frac{x^3}{3!} f'''(0)$

$f(x) = \log(1+x)$

$f(0) = \log 1 = 0$

$f'(x) = \frac{1}{1+x}$, $f'(0) = 1$

$f''(x) = \frac{-1}{(1+x)^2}$; $f''(0) = -1$

$f'''(x) = \frac{-1}{(1+x)^3}$; $f'''(0) = 2$

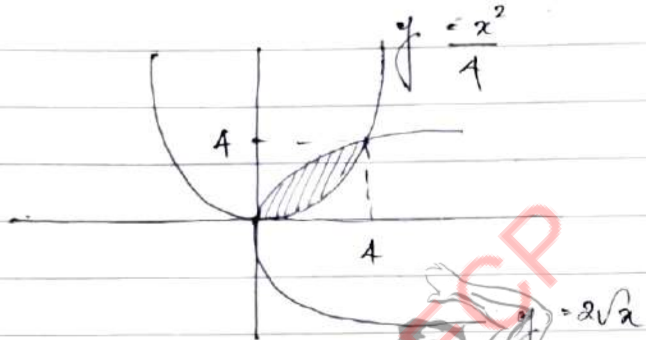
$f(x) = 0 + \frac{x}{1!} \times 1 + \frac{x^2}{2!} \times -1 + \frac{x^3}{\cancel{3!}\,3} \times \cancel{2}$

$f(x) = x + \frac{x^2}{2} + \frac{x^3}{3}$

16) (a) $\displaystyle\int_0^4 \int_{\frac{x^2}{4}}^{2\sqrt{x}} dy\,dx$

$y = \frac{x^2}{4} \to 2\sqrt{x}$

$x : 0 \longrightarrow 4$



$y = \frac{x^2}{4}$ & $y = 2\sqrt{x}$

$2\sqrt{x} = \frac{x^2}{4}$

$4x = \frac{x^4}{16}$

$64x = x^4$

$x^4 - 64x = 0$

$x(x^3 - 64) = 0$

$x = 0, \quad x = 4$

$y = 0, \quad y = 4$

On changing the order of integration we have,

$\displaystyle\int_0^4 \int_{\frac{y^2}{4}}^{2\sqrt{y}} dx\,dy.$

$$\int_0^4 [x]_{\frac{y^2}{4}}^{2\sqrt{y}} \, dy$$

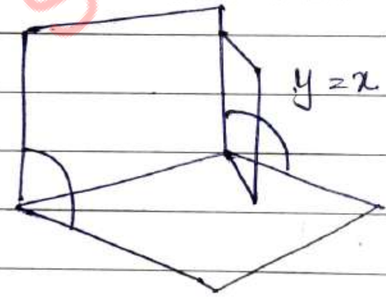
$$\int_0^4 \left(2\sqrt{y} - \frac{y^2}{4}\right) dy$$

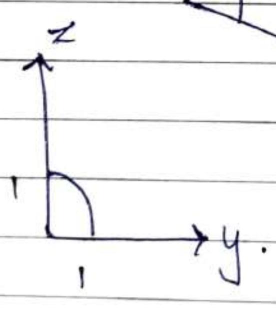
$$= \left[\frac{2y^{3/2}}{3/2} - \frac{y^3}{12}\right]_0^4$$

$$= \frac{4}{3} \times 8 - \frac{\cancel{64}\ 16}{\cancel{12}\ 3}$$

$$= \frac{32}{3} - \frac{16}{3} = \frac{16}{3}$$

16 b) $\iiint_G z \, dV$





$z : 0 \rightarrow \sqrt{1-y^2}$

$y : 0 \rightarrow 1$

$x = 0 \rightarrow y$.

$$\iiint_G z\,dV = \int_0^1 \int_0^y \int_0^{\sqrt{1-y^2}} z\,dz\,dx\,dy.$$

$$= \int_0^1 \int_0^y \left[\frac{z^2}{2}\right]_0^{\sqrt{1-y^2}} dx\,dy$$

$$= \int_0^1 \int_0^y \frac{1-y^2}{2}\,dx\,dy$$

$$= \int_0^1 \left[\frac{x}{2} - \frac{xy^2}{2}\right]_0^y dy$$

$$= \int_0^1 \left(y/_2 - y^3/_2\right) dy$$

$$= \frac{1}{2}\left[\frac{y^2}{2} - \frac{y^4}{4}\right]_0^1$$

$$= \frac{1}{2}\left[1/_2 - 1/_4\right)$$

$$= 1/_2 \times 1/_4$$

$$= 1/_8 //.$$

12 (a) $x + y + z = a$

$3x + 4y + 5z = b$

$2x + 3y + 4z = c$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 4 & 5 \\ 2 & 3 & 4 \end{bmatrix} \quad B = \begin{bmatrix} a \\ b \\ c \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 3 & 4 & 5 \\ 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$$[AB] = \begin{bmatrix} 1 & 1 & 1 & a \\ 3 & 4 & 5 & b \\ 2 & 3 & 4 & c \end{bmatrix}$$

i) $a = b = c = 1$

$$[AB] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 3 & 4 & 5 & 1 \\ 2 & 3 & 4 & 1 \end{bmatrix} \quad \begin{matrix} R_2 \to R_2 - 3R_1 \\ R_3 \to R_3 - 2R_1 \end{matrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & -2 \\ 0 & 1 & 2 & -1 \end{bmatrix} \quad R_3 \to R_3 - R_2$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$R(AB) \neq R(A)$

$\therefore$ The system has no solution.

ii)

$a = \frac{b}{2} = c = 1$

$\Rightarrow a = 1,\ b = 2,\ c = 1$

$$[AB] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 3 & 4 & 5 & 2 \\ 2 & 3 & 4 & 1 \end{bmatrix} \quad \begin{array}{l} R_2 \to R_2 - 3R_1 \\ R_3 \to R_3 - 2R_1 \end{array}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & -1 \\ 0 & 1 & 2 & -1 \end{bmatrix} \quad R_3 \to R_3 - R_2$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R(AB) = R(A) \neq$ no: of variable.

$\therefore$ The system has infinite no: of solution.

b) $$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

The characteristic eqn is

$\lambda^3 - (6+3+3)\lambda^2 + (8+14+14)\lambda - 32 = 0$

$\lambda^3 - 12\lambda^2 + 36\lambda - 32 = 0$

$\lambda = 8, 2, 2.$

To find the eigen vectors:

$[A - \lambda I]X = 0$

$$\begin{bmatrix} 6-\lambda & -2 & 2 \\ -2 & 3-\lambda & -1 \\ 2 & -1 & 3-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

When $\lambda = 8$

$$\begin{bmatrix} -2 & -2 & 2 \\ -2 & -5 & -1 \\ 2 & -1 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad \begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 + R_1 \end{matrix}$$

$$\begin{bmatrix} -2 & -2 & 2 \\ 0 & -3 & -3 \\ 0 & -3 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad R_3 \to R_3 - R_2$$

$$\begin{bmatrix} -2 & -2 & 2 \\ 0 & -3 & -3 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-3x_2 - 3x_3 = 0 \Rightarrow x_2 + x_3 = 0$

$-2x_1 - 2x_2 + 2x_3 = 0 \Rightarrow x_1 + x_2 - x_3 = 0$

$$X_1 = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

When $\lambda = 2$

$$\begin{bmatrix} 4 & -2 & 2 \\ -2 & 1 & -1 \\ 2 & -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad \begin{matrix} R_2 \to R_2 + \frac{1}{2} R_1 \\ R_3 \to R_3 - \frac{1}{2} R_1 \end{matrix}$$

$$\begin{bmatrix} 4 & -2 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

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$4x_1 - 2x_2 + 2x_3 = 0 \Rightarrow 2x_1 - x_2 + x_3 = 0$

$$x_2 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} \qquad x_3 = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$$

$$P = \begin{bmatrix} 2 & 1 & 1 \\ -1 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$

$$D = P^{-1}AP$$

$$= \begin{bmatrix} 2 & 1 & 1 \\ -1 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}^{-1} \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} \begin{bmatrix} 2 & 1 & 1 \\ -1 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$

$$= \frac{1}{8} \begin{bmatrix} 4 & -2 & -2 \\ -2 & 3 & -1 \\ -2 & 1 & 5 \end{bmatrix} \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} \begin{bmatrix} 2 & 1 & 1 \\ -1 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$

$$= \frac{1}{8} \begin{bmatrix} 4 & -2 & -2 \\ -2 & 3 & -1 \\ -2 & 1 & 5 \end{bmatrix} \begin{bmatrix} 16 & 2 & 10 \\ -8 & 4 & -4 \\ 8 & 0 & 8 \end{bmatrix}$$

$$= \frac{1}{8} \begin{bmatrix} 64 & 0 & 0 \\ 0 & 16 & 0 \\ 0 & 0 & 16 \end{bmatrix} = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

13) a) $L(x,y,z) = f(x_0,y_0,z_0) + f_x(x_0,y_0,z_0)(x-x_0)$
$+ f_y(x_0,y_0,z_0)(y-y_0)$
$+ f_z(x_0,y_0,z_0)(z-z_0)$

$f(x,y,z) = \frac{4y}{x+z}$

$(x_0, y_0, z_0) \to (1,1,1)$

$f(x_0, y_0, z_0) = \frac{4}{2} = 2$

$f_x(x,y,z) = \frac{-4y}{(xz)^2}$

$f_x(x_0, y_0, z_0) = \frac{-4}{4} = -1$

$f_y(x,y,z) = \frac{4}{x+z}$.

$f_y(x_0, y_0, z_0) = \frac{4}{2} = 2$.

$f_z(x,y,z) = \frac{-4y}{(x+z)^2}$

$f_z(x_0, y_0, z_0) = \frac{-4}{4} = -1$ //

13b) $f(x,y)$

$f(x,y) = 0$

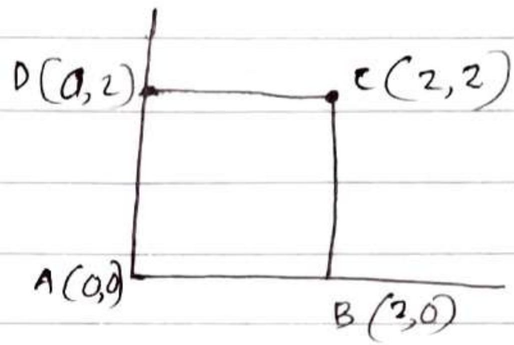
$2x - 2 = 0$

$x = 1$

$f(x,y) = 0$

$6y + 6 = 0$

$y = -1$



$(1,1)$ is a critical point

Along AB

$y = 0$

$f(x,y) = x^2 \; 3y^2 - 2x + 6y$

$f(x,0) = x^2 - 2x$

$f'(x,0) = 0$

$2x - 2 = 0$

$x = 1$

$(1,0)$ is the critical point on line AB

Along BC

$x = 2$

$f(2,y) = \cancel{4} - 3y^2 - \cancel{4} + 6y$

$= -3y^2 + 6y$

$f'(2,y) = 0$

$(-6y + 6) = 0$

$y = 1$ $(2,1)$ is the critical point on BC

Along CD

$y = 2$

$f(x,2) = x^2 - 12 - 2x + 12$

$= x^2 - 2x$

$f'(x,2) = 0$

$2x - 2 = 0$

$x = 1$

$(1,2)$ is the critical point on CD.

ki) i) $l = x/y$

$m = y/z$

$n = z/x$

$$\frac{\partial v}{\partial x} = \frac{\partial v}{\partial l} \cdot \frac{\partial l}{\partial x} + \frac{\partial v}{\partial n} \frac{\partial n}{\partial x}$$

$$= \frac{l}{y} \frac{\partial v}{\partial l} + \left(\frac{-z}{x^2}\right) \frac{\partial v}{\partial n}$$

$$= l/y \frac{\partial v}{\partial l} - \frac{z}{x^2} \frac{\partial v}{\partial n}$$

$$\frac{\partial v}{\partial y} = \frac{\partial v}{\partial l} \cdot \frac{\partial l}{\partial y} + \frac{\partial v}{\partial m} \cdot \frac{\partial m}{\partial y}$$

$$= \frac{-x}{y^2} \frac{\partial v}{\partial l} + \frac{l}{z} \frac{\partial v}{\partial m}$$

$$\frac{\partial v}{\partial z} = \frac{\partial v}{\partial m} \cdot \frac{\partial m}{\partial z} + \frac{\partial v}{\partial n} \frac{\partial n}{\partial z}$$

$$= \frac{-y}{z^2} \frac{\partial v}{\partial m} + \frac{l}{x} \frac{\partial v}{\partial n}$$

$$x \frac{\partial v}{\partial x} + y \frac{\partial v}{\partial y} + z \frac{\partial v}{\partial z}$$

$$= \frac{x}{y} \frac{\partial v}{\partial l} - \frac{z}{x} \frac{\partial v}{\partial n} - \frac{x}{y} \frac{\partial v}{\partial l} + \frac{y}{z} \frac{\partial v}{\partial m} - \frac{y}{z} \frac{\partial v}{\partial m} + \frac{z}{x} \frac{\partial v}{\partial n}$$

$= 0$ //.

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b) $f(x,y) = 2xy - x^3 - y^2$

$$p = \frac{\partial f}{\partial x} = 2y - 3x^2$$

$$q = \frac{\partial f}{\partial y} = 2x - 2y$$

$$r = \frac{\partial^2 f}{\partial x^2} = -6x$$

$$s = \frac{\partial^2 f}{\partial x \partial y} = 2$$

$$t = \frac{\partial^2 f}{\partial y^2} = -2$$

$\frac{\partial f}{\partial x} = 0$ $\qquad$ $\frac{\partial f}{\partial y} = 0$

$2y - 3x^2 = 0 \rightarrow ①$ $\qquad$ $2x - 2y = 0$

$1 + 2 \Rightarrow -3x^2 + 2x = 0$

$3x^2 - 2x = 0$

$x(3x - 2) = 0$

$x = 0, \frac{2}{3}$

$y = 0, \frac{2}{3}$

Critical points are $(0,0)$ and $\left(\frac{2}{3}, \frac{2}{3}\right)$

$rt - s^2 = 12x - 4$

At $(0,0)$

$rt - s^2 = -4 < 0$

$(0,0)$ is a saddle point

At $(2/3, 2/3)$

$$rt^2 - s^2 = 4 > 0$$

$$r = -6 \times 2/3 = -4 < 0$$

17) a) Test the convergence of the series

$$1 + \frac{1.3}{3!} + \frac{1.3.5}{5!} + \frac{1.3.5.7}{7!} + \dots$$

$$\Rightarrow \sum_{n=0}^{\infty} \frac{2n+1}{(2n+1)!}.$$

By Ratio test,

$$v_k = \frac{2n+1}{(2n+1)!}$$

$$v_{k+1} = \frac{2(n+1)+1}{(2(n+1)+1)!} = \frac{2n+3}{(2n+3)!}$$

$$\therefore \lim_{k\to\infty} \frac{v_{k+1}}{v_k} = \lim_{k\to\infty} \frac{2n+3}{(2n+3)!} \times \frac{(2n+1)!}{2n+1}$$

$$= \lim_{k\to\infty} \frac{2n+3}{(2n+3)(2n+2)(2n+1)!} \times \frac{(2n+1)!}{(2n+1)}$$

$$= \lim_{k\to\infty} \frac{1}{(2n+2)(2n+1)}$$

$$= \lim_{k \to \infty} \frac{1}{2(n+1)(n+\frac{1}{2})} = \infty > 1$$

2) Find the sum of series $\sum_{k=1}^{\infty} \frac{1}{k(k+1)}$

ans) Converting to partial fraction

$$\frac{1}{k(k+1)} = \frac{A}{k} + \frac{B}{k+1}$$

$$1 = A(k+1) + Bk$$

when $k = 0$, $A = 1$

when $k = -1$, $B = -1$

$$\therefore \sum_{k=1}^{\infty} \frac{1}{k(k+1)} = \sum_{k=1}^{\infty} \frac{1}{k} - \sum_{k=1}^{\infty} \frac{1}{k+1}$$

$$= \sum_{k=1}^{\infty} \left[\frac{1}{k} - \frac{1}{k+1} \right]$$

$$= \Sigma\left[1 - \frac{1}{2}\right] + \left[\frac{1}{2} - \frac{1}{3}\right] + \cdots + \left[\frac{1}{k} - \frac{1}{k+1}\right]$$

$$= 1 - \frac{1}{k+1} = \frac{k+1-1}{k+1}$$

$$= \frac{k}{k+1}$$

3) Test the convergence of (i) $\sum_{k=1}^{\infty} \frac{k!}{3!(k-1)!3^k}$

i) By ratio test

$$u_k = \frac{k!}{3!(k-1)!3^k}$$

$$u_{k+1} = \frac{(k+1)!}{3!(k+1-1)!\,3^{k+1}} = \frac{(k+1)!}{3!\,k!\,3^{k+1}}$$

$$\lim_{k\to\infty} \frac{u_{k+1}}{u_k} = \lim_{k\to\infty} \frac{(k+1)!}{\cancel{3}!\,k!\,3^{k+1}} \times \frac{\cancel{3}!\,(k-1)!\,3^k}{k!}$$

$$= \underset{k \to \alpha}{Lt} \frac{k!\,(k+1)}{(k-1)k!\;3^k\,3} \times \frac{(k-1)!\,3^k}{k!}$$

$$= \underset{k \to \alpha}{Lt} \frac{k+1}{3k} \qquad \underset{k \to \alpha}{Lt}$$

$$= \underset{k \to \alpha}{Lt} \frac{k\left(1+\frac{1}{k}\right)}{3k}$$

$$= \frac{1}{3} < 1 \Rightarrow \text{series converges}$$

ii) $\sum_{k=1}^{\alpha} \left[\frac{4k-5}{2k+1}\right]^k$.

By root test

$$v_k = \left[\frac{4k-5}{2k+1}\right]^k$$

$$\underset{k \to \alpha}{Lt} \left[v_k\right]^{\frac{1}{k}} = \underset{k \to \alpha}{Lt} \left[\left[\frac{4k-5}{2k+1}\right]^k\right]^{\frac{1}{k}}$$

$$= \underset{k \to \alpha}{Lt} \frac{4k-5}{2k+1}$$

$$= \lim_{k \to \infty} \frac{k\left(4 - \frac{5}{k}\right)}{k\left(2 + \frac{1}{k}\right)} = \lim_{k \to \infty} \left[\frac{4 - \frac{1}{k}}{2 - \frac{1}{k}}\right]$$

$$= \frac{4}{2} = 2 > 1 \text{ diverges}$$

Q) Test the absolute or conditional convergence of $\sum_{k=1}^{\infty} \frac{(-1)^{k+1} k^2}{k^3+1}$

$$\star \quad \sum_{k=1}^{\infty} (-1)^{k+1} \frac{k^2}{k^3+1}$$

$$\sum_{k=1}^{\infty} \frac{k^2}{k^3+1} \quad \therefore |U_k| = \frac{k^2}{k^3+1}$$

$$a_k = \frac{k^2}{k^3+1}$$

$$b_k = \frac{k^2}{k^3} = \frac{1}{k}$$

By limit comparison test

$$\lim_{k \to \infty} \frac{a_k}{b_k} = \lim_{k \to \infty} \frac{k^2}{k^3+1} \times k$$

$$= \lim_{k \to \infty} \frac{k^3}{k^3+1}$$

$$= \lim_{k \to \infty} \frac{k^3}{k^3\left(1+\frac{1}{k^3}\right)} = 1$$

$\therefore \rho > 0 \Rightarrow$ both Σa_k and Σb_k converges or diverges together

$\Sigma b_k = \frac{1}{k} \Rightarrow$ harmonic series

$\therefore \Sigma b_k$ diverges

$\therefore$ Given series diverges absolutely