Interview for the position of Lecturer in Computer Science

Pierre Pradic (University of Oxford)
Swansea University, July 7th 2021

Part I: Research

My research interests in a nutshell

I am active in a few areas connected to **logic in computer science**

Some salient keywords

Linear logic, automata theory, proof theory

My research

- connects rather distinct traditions in "logic in computer science"
- is thematically linked to topics in software verification

Common thread: applications of ideas coming from proof theory

In this presentation

Overview in three parts of some of my work on:

- constructiveness of Monadic Second Order logic — PhD topics
- proof-theoretic approaches to implicit definability — postdoc topic
- connections betweem λ-calculi and automata-theoretic transducer models

PhD: Monadic Second Order logic and constructivity

Monadic Second-Order logic (MSO)

Definition

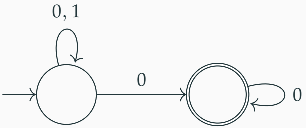
Restriction of second-order logic with only *unary second-order variables*.

Decidability [Büchi (1962)]

MSO over $(\mathbb{N}, <)$ is decidable.

$$\begin{tikzcd}
\text{MSO formulas over } \Sigma \arrow[rr, bend left, "\varphi \mapsto \mathcal{A}_\varphi"] \arrow[dr, "\varphi \mapsto \mathcal{L}(\varphi)"'] & & \text{automata over } \Sigma \arrow[dl, "\mathcal{A} \mapsto \mathcal{L}(\mathcal{A})"] \\
& \text{Semantics} \subseteq \mathcal{P}(\Sigma^\omega) &
\end{tikzcd}$$

For finite word automata: via easy complementation for *deterministic* automata.



...but Büchi automata are hard to determinize.

- non-constructive proofs of soundness!

  usual proofs: infinite Ramsey theorem, weak König's lemma

- $\mathsf{MSO}(\mathbb{N}, <)$ inherently classical

How (non)-constructive is MSO?

What axiomatic strength characterizes a given MSO *theory?*

- With H. Michalewski, L. Kołodziejczyk and M. Skrzypczak in Warsaw.

When can we extract computational content from MSO *proofs?*

- With C. Riba in Lyon.

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$\rightsquigarrow$ Metatheoretical analysis of Büchi's decidability theorem.

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$\rightsquigarrow$ Refinement of MSO($\mathbb{N}$) with witness extraction.

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$\leadsto$ Metatheoretical analysis of Büchi's decidability theorem.

Tools: Reverse Mathematics, descriptive set theoretic complexity

When can we extract computational content from MSO *proofs?*

- With C. Riba in Lyon.

$\leadsto$ Refinement of $\mathsf{MSO}(\mathbb{N})$ with witness extraction.

Tools: Intuitionistic/linear logic, categorical semantics

Weak König's lemma

Infinite Ramsey theorem

$\Downarrow$

(Over RCA_0)

Σ^0_2-induction $\Longrightarrow$ Additive Ramsey

$\Uparrow$ $\Downarrow$

$\mathsf{MSO}(\omega)$ $\Longleftarrow$ Compl. of NBA

$\Downarrow$

$\Swarrow$ $\Downarrow$

Bounded weak König's lemma

Determinization of NBA

The Logical Strength of Büchi's Decidability Theorem (CSL,LMCS) [Kołodziejczyk, Michalewski, P., Skrzypczak, 2016]

PhD, topic 1/2: Reverse Mathematics of Büchi's theorem

$$\begin{tikzcd}
\text{Weak König's lemma} \arrow[ddd, Rightarrow] & & \text{Infinite Ramsey theorem} \arrow[d, Rightarrow] \\
& \Sigma^0_2\text{-induction} \arrow[r, Rightarrow] & \text{Additive Ramsey} \arrow[d, Rightarrow] \\
& \mathsf{MSO}(\omega) \arrow[u, Rightarrow] \arrow[dl, Rightarrow] & \text{Compl. of NBA} \arrow[l, Rightarrow] \arrow[d, Rightarrow] \\
\text{Bounded weak König's lemma} & & \text{Determinization of NBA}
\end{tikzcd}$$

(Over RCA_0)

The Logical Strength of Büchi's Decidability Theorem (CSL,LMCS) [Kołodziejczyk, Michalewski, P., Skrzypczak, 2016]

Further work (in progress)

Analysis of the topological complexity of $\mathsf{MSO}(\mathbb{Q}, <)$-definable sets.

Rough idea: strictly intermediate between $(\mathbb{N}, <)$ and the infinite tree, multiple compelling challenges to overcome

PhD, topic 2/2: constructivity and MSO($\mathbb{N}$,<)

Goal: a refinement of MSO($\mathbb{N}$) with extraction for **causal** functions.

- Approach inspired by realizability. [Kleene (1945), ...]

Analogous example: extraction for intuitionistic arithmetic (HA)

If $HA \vdash \forall x \exists y \varphi(x, y)$, there is an algorithm computing

$$f : \mathbb{N} \to \mathbb{N} \text{ recursive} \qquad \text{such that} \qquad \forall x \;\; \varphi(x, f(x))$$

Analogy

Classical system	MSO($\mathbb{N}, <$)	PA
Realizers	**Causal functions**	System T
Intuitionistic system	???	HA

PhD topic 2/2 (cont.): subsystems of MSO and game models

Two restricted subsystems allowing extraction and able to interpret the classical system

- LMSO based on linear logic
- SMSO, a positive fragment allowing intuitionist reasoning

A refined automata/logic correspondence

$$
\begin{tikzcd}
\mathsf{LMSO} \arrow[rr, bend left, "\varphi \mapsto \mathcal{A}_\varphi"] \arrow[dr, "\varphi \mapsto [\![\varphi]\!]"'] & & \text{Alternating automata} \arrow[dl, "\text{Acceptance game}"] \\
& \text{Simulation games} &
\end{tikzcd}
$$

- **Further developments:** polarity system, a complete axiomatization...
- **Some tools:** Dialectica categories, categorical semantics, Church synthesis

A Curry-Howard Approach to Church's Synthesis (FSCD,LMCS) [P., Riba, 2017]

LMSO: A Curry-Howard Approach to Church's Synthesis via Linear Logic (LiCS) [P., Riba, 2018]

A Dialectica-Like Interpretation of a Linear MSO on Infinite Words (FoSSaCS) [P., Riba, 2019]

Nested relational queries and interpolation algorithms
j.w.w. M. Benedikt

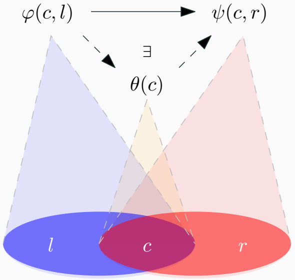
Beth definability and Craig interpolation

Beth definability

Let $\varphi(R)$ be a first-order formula.

If $\varphi(R) \wedge \varphi(R') \Rightarrow R \equiv R'$, then there is a FO $\psi(\vec{x})$ such that $\varphi(R) \Rightarrow \varphi(\psi)$. i.e., R is first-order definable

- Model-theoretic proof using amalgamation
- Proof-theoretic effective proof using interpolation



Craig interpolation

If $\varphi \Rightarrow \psi$, there exists θ such that

$$\varphi \Rightarrow \theta \qquad \text{and} \qquad \theta \Rightarrow \psi$$

and θ mentions *only* variables/relation symbols common to φ and ψ.

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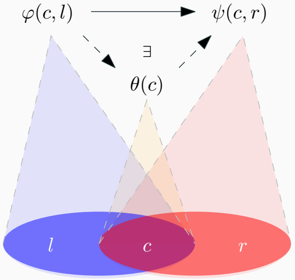

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Contribution

New definability and interpolation result in this vein.

The nested setting

Generalize to the *Nested Relational Calculus* (term language for set expressions)

$$\frac{\Gamma \vdash e : T \qquad \Gamma \vdash e' : T}{\Gamma \vdash \{e, e'\} : \mathsf{Set}(T)} \qquad \frac{\Gamma \vdash e_1 : \mathsf{Set}(T_1) \qquad \Gamma,\ x : T_1 \vdash e_2 : \mathsf{Set}(T_2)}{\Gamma \vdash \bigcup \{e_2 \mid x \in e_1\} : \mathsf{Set}(T_2)} \qquad \ldots$$

Definition

Call $\varphi(i, o)$ an **implicit definition** when it is functional: $\varphi(i, o) \wedge \varphi(i, o') \implies o = o'$

Extraction from Δ_0 intuitionistic implicit definitions **[Benedikt, P., 2021]**

For every Δ_0 implicit definition $\varphi(i, o)$, there is a compatible NRC term $e(i)$ such that

$$\varphi(i, o) \implies o = e(i)$$

Further, $e(i)$ may be efficiently computed from a cut-free **intuitionistic** functionality proof

- Intuitionistic case: a generalization of interpolation
- Current work: proof-theoretic approach to obtain an efficient algorithm for the classical case

Related questions: effective model-theoretic definability theorems via proof theory?

Implicit automata in λ-calculi
j.w.w. L.T.D. Nguyễn

Simply-typed λ-calculus with an anonymous base type

$$t, u \quad ::= \quad x \mid t\ u \mid \lambda x.t \qquad\qquad A, B \quad ::= \quad o \mid A \to B$$

Church encodings of *strings* over alphabet $\Sigma = \{a, b\}$:

- $\mathsf{Str}_{\{a,b\}} = (o \to o) \to (o \to o) \to o \to o$
- $abb \in \{a, b\}^* \rightsquigarrow \overline{abb} = \lambda f_a.\ \lambda f_b.\ \lambda x.\ f_a\ (f_b\ (f_b\ x)) : \mathsf{Str}_{\{a,b\}}$

Theorem [Hillebrand & Kanellakis, 1996]

For any type A and any simply typed λ-term $t : \mathsf{Str}_\Sigma[A/o] \to \mathsf{Bool}$, the corresponding language $L_t \subseteq \Sigma^*$ is *regular*.

Proof: a nice and easy semantic evaluation argument, and a converse holds

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Project: generalize along several dimensions

- Considering linear/affine/planar type systems $\qquad A, B \ ::= \ o \mid A \multimap B \mid A \to B \mid A \& B$

  (with finer-grained types for Church encodings)
- Consider richer input and output types: Str instead of Bool, Tree instead of Str

A partial landscape

Implicit automata in typed λ-calculi I: aperiodicity in a non-commutative logic (ICALP) [P., Nguyễn, 2020]

Implicit automata in typed λ-calculi II: streaming transducers vs categorical semantics (preprint) [P., Nguyễn, 2020]

Comparison-free polyregular functions (ICALP) [P., Nguyễn, 2021]

Broader picture

$\mathsf{Str}_\Sigma[A] \multimap \mathsf{Bool}$ with A linear (adapted as needed):

λ-calculus	languages	status
simply typed	regular	✓ [Hillebrand & Kanellakis 1996]
linear or affine	regular	✓
non-commutative linear or affine	star-free	✓

$\mathsf{Str}_\Gamma[A] \multimap \mathsf{Str}_\Sigma$ with A affine (adapted as needed):

λ-calculus	transducers	status
affine	regular functions	✓
non-commutative affine	first-order regular fn.	✓?
linear/affine with additives	regular functions	✓
parsimonious	polyregular	??
simply typed	variant of CPDA???	???

Tools: semantic evaluation, SSTs, categorical semantics, monoid theory (Krohn-Rhodes), GoI...

Much remains to be done! (with some promising directions)

Other publications/collaborations

Publications that do not fit the three themes:

(still in the broad spirit of "application of proof-theoretic ideas")

Integrating Linear and Dependent Types (PoPL) [Benton, Krishnaswami, P., 2015]

(Dependent type theory, linear logic)

Cantor-Bernstein implies excluded middle (preprint) [Brown, P., 2019]

(Application of a theorem of Escardó, related to searchability of $2^{\mathbb{N}}$ [Berger, 1990])

Kleene Algebra with Hypotheses (FoSSaCS) [Doumane, Kuperberg, Pous, P., 2019]

(Complexity, verification)

From normal functors to logarithmic space queries (ICALP) [Nguyễn, P., 2019]

(Implicit complexity, categorical semantics of linear logic)

Part II: Integration

Research-wise

Common interests with faculty members of the theory group at Swansea

(e.g. with A. Beckmann, J. Blanck, U. Berger, A. Pauly, M. Seisenberger, A. Setzer)

- Proof theory and applications
- Constructivism
- Semantics
- Descriptive complexity

Among complementary interests:

- Linear logic
- Algebraic approaches to language theory
- Tree/string transductions

Project for First Grant

Implicit automata in λ-calculi and realizability

- Ongoing collaborations with L.T.D. Nguyễn and others
- Raises enough questions to sustain a project

Teaching experience

Three settings, mostly as teaching assistant:

At École Normale Supérieure de Lyon (ENSL) during my PhD

- Third and fourth year students
- Tutorials/labs: parallel programming and algorithmics, algorithmics, semantics of programming languages, models of computation (C, MPI, Coq)
- Small cohorts, a lot of freedom with e.g. teaching material/homework/exam design

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At Oxford University since then (as departmental lecturer and postdoc)

- Small groups
- Tutorials/labs for artificial intelligence, computer architecture and model checking (remote)

Other responsibilities

At École Normale Supérieure de Lyon

Evaluation of research internships (third and fourth year students)

- Mostly topics related to my research: formal verification, automata theory, type theory, ...

At Oxford University (2020)

Helping with selecting incoming master students

- Assessing applications and conducting interviews

Assessing projects and dissertation (third to fifth year)

- Varied topics: algorithmics, parity games, ZX-calculus, genetic programming, GUI design...

In a nutshell

Teaching/supervision

- I have TAed up to fourth year level for lectures in Lyon and Oxford; on occasion I
  - designed teaching material
  - marked midterms
- I feel I can *readily* help with a number of modules...

  e.g., CSF105, CS-205, CS-270, CSCM75...
- ...and I am eager to increase my range!

Research

- Compatible with the theory group
- Some vision for a project around λ-calculus, automata and realizability

Quick vitae

Some research interests

Linear logic, automata theory, proof theory

Doctoral studies (ÉNS Lyon/University of Warsaw, defended on 23/06/20)

- Joint supervision of C. Riba and H. Michalewski
- Topic: **Some proof-theoretical approaches to Monadic Second-Order logic**

Post-doctoral positions (Oxford)

January 2020–October 2020:	Departmental Lecturer
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10 international conference papers, 2 journal papers (extended versions), 2 preprints

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