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Functional Reactive Programming for Data Binding in $C\sharp$

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Original Aims of the Project

Aims of the project will go here.

Work Completed

Work completed will go here.

Special Difficulties

None

Declaration

I, David Barker of Jesus College, being a candidate for Part II of the Computer Science Tripos, hereby declare that this dissertation and the work described in it are my own work, unaided except as may be specified below, and that the dissertation does not contain material that has already been used to any substantial extent for a comparable purpose.

Signed [signature]

Date [date]

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Chapter 1

Introduction

It has become a firmly established practice when writing applications to separate the code managing the user interface from the business logic powering it. This is a key part of the principle of separation of concerns, and means that the interface can be safely changed without needing to modify the backend code (and vice versa, assuming the backend provides a consistent interface). Beginning with the traditional MVC architecture, this has led to the development of a family of system architectures such as MVPM and MVVM which enforce this principle.

1.1 Data binding

Data binding presents a mechanism for bridging the gap between the separated layers by allowing the developer to specify that some value in the user interface code should be bound to a property of the model. This usually simply means that whenever the value in the model changes, the data binding framework will ensure that the value in the user interface is also updated to the same value. However, bindings can often be more complex: for instance, there could be a text box in the user interface which reflects backend data but can also be used to change it (two-way binding). Alternatively, there might be a user interface component whose value is determined by some function of a variable in the model - consider a list view which displays a filtered and sorted version of a list stored in the model. There could even be values which depend on multiple sources, or more complex many-to-many bindings.

Many current languages and frameworks provide data binding features with varying levels of complexity. Java has a variety of extension libraries [examples] which allow the programmer to use data binding with Swing. Javascript too has a variety of possibilities, with a prominent example being the backbone.js MVC

framework. Unfortunately, many of these are quite limited and difficult to use - backbone.js, for example, only provides one-way binding and requires a lot of boilerplate code from the programmer.

1.2 Data binding in .NET

Microsofts .NET framework offers a particularly powerful example of data binding through Windows Presentation Foundation (WPF). Based on the MVVM architecture, it has many features to allow things like two-way binding, binding through functions and bindings based on list operations. One of its key advantages is that the user interface can be defined entirely in XAML ¹ with bindings being specified through XAML parameters. The view logic is then specified in the ViewModel which in turn communicates with the model. This means user interface designers can work purely in XAML without concern for the logic or binding mechanisms in place behind the scenes.

However, WPF suffers from a similar problem to many other data binding frameworks: advanced data bindings can be very complex and difficult to manage, and setting up bindings in the first place requires quite a lot of boilerplate code in the model and view. Furthermore, binding through functions requires special value converter classes to be written. This is essentially an application of the Template pattern, and the value converters are not type safe - they take objects as input and return objects as output (and bindings with multiple inputs will simply take arrays of objects with no guarantee that the right number of values has been passed). [Maybe more disadvantages?] Clearly a more simple and general binding framework would definitely make application development simpler.

1.3 Project inspirations

Many useful ideas for data binding come from the area of functional programming. Functional reactive programming (FRP), for example, is a paradigm which nicely models the concept of data binding in a general way. Another useful concept is that of the 'arrows' implemented in Haskell. An 'arrow from type A to type B' essentially represents a process taking an input of type A and returning something of type B, and these can be arbitrarily combined in interesting ways to build up complex functions. More detail on both will be given in the next chapter.

¹Extensible Application Markup Language, an XML-based markup language for defining user interfaces and simple behaviour

3

The FRP paradigm was the inspiration for the project, which sought to implement a general-purpose data binding framework in C‡ using concepts derived from functional programming. I successfully implemented a framework which provides data binding in both directions, through arbitrary functions with arbitrary numbers of inputs and outputs, all with a simple syntax based on lambda expressions. Minimal boilerplate code is required to set up bindable properties—the containing class should extend Bindable, and from there marking arbitrary member variables and properties with the [Bindable] tag will make it possible to create bindings to and from them with a single function call. I also implemented a large variety of arrow types and arrow operators along with a number of utility functions to make creating and combining them simple.

Chapter 2

Preparation

This chapter is empty still!

-Explain FRP here -Explain arrows here -Software engineering approach - iterations at first, issue-driven Agile

Chapter 3

Implementation

3.1 Overview

Give a general overview

3.2 Arrows

3.2.1 Overview

Implementing the highly functional idea of an arrow in C# posed an interesting challenge. Fundamentally, there were two main obstacles to overcome: C#'s syntax, which is far clunkier than Haskell's; and the type system, which is far more restrictive and relies on static analysis at compile time. It was decided early on that arrows would be built on lambda expressions as these seemed the most natural way of expressing the desired functionality. C#'s generic Func<T1, T2> type provided a good way of handling arbitrary lambda expressions between arbitrary types, and so this became the basis for the arrow type.

Tackling the type system was particularly difficult. I initially took the approach of writing arrow combinator functions so that they required no type parameters and instead inferred all their types by reflection. This made the syntax far neater and, using some run-time type checking, could plausibly be made type safe (though the compiler would have no way of enforcing it). However, after some experimenting it became clear that writing new operators and arrow types would be incredibly difficult using this approach, and the resulting code was very complicated and difficult to understand.

It was therefore decided that arrows and combinators should all be statically typed, as this would allow the compiler to do type checking and lead to much cleaner code. However, this meant the issue of the programmer having to provide long lists of type parameters to every combinator was still there. I was eventually able to solve this problem by writing the combinators in such a way that the compiler could always infer types from the arguments without the programmer needing to supply them. Several more situations were fixed by including static extension methods which would use the source object to infer some parameters, and a series of static helper methods were included in the class Op to help fill in the gaps. This was easier said than done in several cases (which will be discussed later), but ultimately I managed to eliminate type parameters completely from the syntax.

A further problem was how best to allow arrows from multiple inputs to multiple outputs. Simply having a Func with multiple inputs would not have worked as the arrow would have had to be rewritten for every given number of inputs, and to make matters worse Func is limited to one output. For some time I debated writing a generic binary-tree-structured type¹ to handle this, but on discovering C‡'s built-in Tuple<T1, T2> type I felt this would be the simplest option. This ended up complicating the data binding (as I will discuss later), but was entirely sufficient for supporting the arrow implementation.

[Maybe mention arrow intercompatibility?]

3.2.2 Simple arrows

Summarise functionality, combinators and syntactic efforts made here, mention things like the parallelism of the And operator as well.

- Basic operators how to create, combine and first/second arrows
- Other operators like And (mention it's parallel)
- Advanced operators
- Table illustrating mapping between C# operators and Haskell equivalents

The first objective in implementing arrows was to get the simple function-based arrow working, as all the others derive from this. As mentioned earlier, it was implemented using the C# Func<A, B> class to hold its function. An Invoke method is then exposed for using the arrow.

Whilst arrows can be constructed using new Arrow<A, B>(function), a simpler syntax is provided by an extension method:

¹All arrows work on nested pairs

3.2. ARROWS 9

```
var increment = ((int x) \Rightarrow x + 1).Arr();
```

The Arr operator effectively lifts a function to arrow status, in much the same way as the arr operator in Haskell. Arrow composition is also possible:

```
var combinedArrow = arrow.Combine(anotherArrow);
```

This is equivalent to arrow >>> anotherArrow in the Haskell syntax. Implementing a >>> operator was unfortunately not possible in C# as it does not allow user-defined operators. The first operator was then implemented to complete the basic arrow:

```
var firstArrow = arrow.First(default(T));
Or, equally:2
var firstArrow = Op.First(arrow, default(T))
```

The above syntax may not immediately make sense. Essentially, the default(T) passes in null if T is a reference type and some sensible default if it is a null type. The only purpose for this parameter is to allow the compiler to infer the type of the second argument to the resulting arrow, and so any value of the appropriate type would work – default(T) is used for simplicity. This is the one syntax 'glitch' which I was unable to remove, and is explained in more detail at the end of this subsection.

Going beyond the basic operators, many composite operators were also implemented based on these simple combinators. For instance, Second was implemented in terms of First. The standard parallel combinator was also implemented as And:

```
var parallelArrow = leftArrow.And(rightArrow);
```

This is equivalent to the *** operator in Haskell, and executes the two arrows in parallel. This allows a number of complex operations to be conveniently parallelised in one arrow, which could likely be useful when working with data bindings between complex objects. The Fanout operator makes good use of this: given one input arrow of type Arrow<A, B> and two following arrows of types Arrow<B, C> and Arrow<B, D>3, it returns an arrow which executes the input arrow, splits the result and then passes it through the two following arrows in parallel yielding a pair of results.

Given a pair result, it is then possible to recombine the two elements using a Func<A, B, C> operator using Unsplit. The following example recombines the tuple by adding its elements together:

²Op is the static class containing all arrow operators and extension methods

³Note that the input types of the two arrows have to match the output type of the first arrow

```
var unsplitArrow = arrowOnPairs.Unsplit((x, y) => x + y);
```

Challenges encountered

A few - mention type inference challenges, overcoming type system whilst maintaining type safety and difficulties with the First operator (also alternatives considered for this – the three iterations used, and why the last one makes sense). Also implementing something like a general identity arrow or swap arrow – ultimately needed to make it a special arrow subclass as having a polymorphic arrow is impossible in C#.

3.2.3 Invertible arrows

Discuss the alternative ways of implementing this - a set of preset arrows designed to form a Turing-complete basis, forcing the user to specify an arrow for each direction etc. Final decision to simply construct using a function for each direction then allow arbitrary combination, as inspired by the paper on invertible arrows.

3.2.4 List arrows

- Wrapping Linq
- SQL-style syntax helped by various extension methods
- Functional-style operations foldl and foldr

3.2.5 Choice arrows

- Not deeply implemented or tested but could be handy for some things, most likely convenient exception handling
- This wouldn't be much good for bindings though

3.2.6 Further utility arrows

To simplify various common tasks, a number of simple utility arrows were written. These simply inherited from an arrow on the appropriate types, and initialised the arrow to some particular function in the constructor.

Identity arrows The identity arrow, though very rarely used in practice, plays a key part in several of the arrow laws. Creating it from a Func every time made the tests a lot messier, so a simple IDArrow<T> class was written which takes a type parameter and returns an identity arrow on that type. An invertible version of this was also written for the invertible arrow laws.

Swap arrow This fulfilled a need which came up surprisingly often: given two types A and B, create an arrow on a Tuple<A, B> which swaps them around and outputs a Tuple<B, A>.

Tuple reassociation arrows The tuple operations assoc and cossa⁴ turn up reasonably frequently in functional programming, and feature in several of the arrow laws. As these are fairly fiddly to implement I decided it would make sense to create utility arrows for both these functions.

3.2.7 Feedback in arrows

Discuss whether this would be useful or not, reasons for it not being there (lack of real-world use cases?) and how one might implement it.

3.3 Data binding

3.3.1 Overall architecture

Use diagrams and stuff to explain it.

3.3.2 Creating bindable sources and destinations

One of the main problems with WPF data binding is the complexity of making sources 'bindable'. This usually requires that the programmer manually implement the INotifyPropertyChanged interface, creating appropriate events and overriding the set methods of all their properties such that they throw the right events. This often becomes a case of copying and pasting the code used for other data sources as it is almost always identical and includes boilerplate tasks like ensuring a variable's new value is different from its old one before throwing the event, and checking that the event isn't null.

For this project I decided to abstract these details away into a base class, called Bindable. This provides events for binding and a set of methods used by

⁴assoc takes a tuple ((a, b), c) and returns (a, (b, c)), whilst cossa does the opposite

binding classes to manage data binding, many of which will be discussed later in this section. The most important are those which allow getting and setting of arbitrary variables by name using reflection, as these are the main interface used by the bindings manager. These have been designed to be dynamically type safe (that is, throwing exceptions at run time where type errors are encountered), to ensure the properties being accessed exist and to abstract away the differences between properties and simple public member variables⁵.

The result of this is that instead of writing all the boilerplate code one would usually write, the programmer need only make their sources and destinations extend Bindable and all the usual things will be handled elsewhere leading to considerably less code clutter. A representative example of the final syntax is given in Listing 3.1.

Another addition that can be seen in the listing is the [Bindable] attribute which has been used above the value property. This uses a PostSharp aspect, explained in the next section, to intercept the setter for throwing events and so relieve the programmer of having to do this themselves.

Listing 3.1: Creating a bindable source class public class DataSource : Bindable {
 [Bindable]
 public int value { get; set; }

PostSharp

}

PostSharp is a C# extension which provides aspect-oriented programming (AOP) functionality. Essentially, AOP is a means of separating cross-cutting concerns from the code they affect, allowing 'aspects' to be written providing this common functionality. Markers can then be placed in the appropriate points, and the aspects will generate and inject their code into these locations at compile time.

In this case, the Bindable aspect will intercept the setter of any property or member variable it is placed above and insert code to check whether the value has changed and throw appropriate events if it has. This does not interfere with any setters the programmer may already have specified, as it defers to the programmer's setter before throwing the event for the value changing (thus ensuring the event is only thrown once all values are updated and any side-effects have occurred).

 $^{^5{}m WPF}$ does not allow binding to member variables, but for this project I decided there was no obvious need to distinguish between the two

Unfortunately, the project budget only extended as far as the free version of PostSharp. Syntax for bindings could likely have been improved even further using the method and property introduction features provided by the full version, as this would have removed the need to extend Bindable. The unfortunate side-effect of the Bindable base class is that the class can no longer extend any other base class, as C‡ does not allow multiple class inheritance. However, in working with the binding framework it was found that this rarely caused problems that couldn't be worked around, and in any case patterns exist to overcome the lack of multiple inheritance in the general case⁶.

3.3.3 Creating bindings

It was decided that it would make sense to manage all bindings in a given project centrally, so that constraints on cycles and conflicts can be enforced and bindings can be dynamically added and removed. Therefore, the BindingsManager class was implemented to create and maintain bindings. This is a static class handling all bindings for the project it is used in and exposing methods for creating and removing particular bindings. The programmer can call it passing in the sources, destinations and arrow they wish to use, and it will return a binding handle which they can use to remove the binding later if needed. The bindings manager also handles cycle and conflict detection, automatic creation of two-way bindings based on the arrow type and support for many-to-many bindings, all of which will be explained in this section.

Syntax and usage

In order to abstract away details like types and specific object references, the bindings manager operates on 'bind points' — essentially, structures containing a reference to the Bindable source and a string representation of the property or member variable being bound. An extension method was written to make creating these simpler, as calling the constructor with the object and variable name turned out to make the overall syntax fairly cluttered. The standard way of making a bind point for an object 'obj' with property 'value' is thus obj.GetBindPoint("value"), which is a lot simpler to write.

Basic bindings (that is, from one source to one destination in one direction) can be set up by passing in a source bind point, and arrow to use for the binding and a destination bind point. An example of this is given in Listing 3.2.

⁶The 'composition over inheritance' pattern is one such technique which uses a shared base interface and a proxy object to 'inherit' from a class without using actual inheritance

With the binding created, the bindings manager subscribes to the update event exposed by the Bindable base class and will thus be notified of any updates. The event is set up to contain the name of the property or variable which has changed so that multiple values from the same object can be bound to without any additional complexity, and as mentioned the Bindable class provides an interface allowing the bindings manager to freely access and modify those variables which have been marked as bindable. Although this largely precludes static type safety, dynamic type safety is provided wherever possible by using reflection to check that types match up and throwing explanatory exceptions where something has gone wrong.

Listing 3.2: Creating a binding between two properties

Two-way binding

Mention inference of two-way bindings by reflecting on the type of the arrow passed in. Also mention the problems encountered - need for variable locking system to avoid infinite loop

Cycle and conflict detection

When setting up a large collection of arbitrary data bindings in a complex application, there is often the risk that some of the bindings could interfere with each other. This generally occurs in one of two ways:

- A collection of bindings ends up forming a cycle, with every update to the first source property ultimately leading to an update of that same property
- One property is directly or indirectly bound to more than one source through functions which will not necessarily yield the same result

The first scenario is especially problematic for this binding framework as binding updates are performed in the same thread, so a binding cycle will lead to an infinite loop of updates causing the application to freeze. The second scenario doesn't necessarily break the application, but the resulting behaviour will be en-

tirely unpredictable⁷. Naturally, a useful data binding framework would need to handle these situations properly.

To tackle this problem, I implemented a BindingGraph class which is used by the BindingsManager and keeps track of the topology of all the existing bindings (leaving the details of actual bind points to the bindings manager). Whenever a new binding is added or an old binding is removed, the binding graph is notified and it updates its map of the bindings appropriately. It then provides methods which allow the bindings manager to query whether a new binding will introduce a cycle or conflict, and if so the bindings manager will know to abort the binding with an exception.

Both cycles and conflicts can be reduced to the same question: for each node in the binding graph, is there another node which is reachable from it via more than one path? Both situations are therefore handled by the same algorithm, which proceeds by depth-first searching from each node and maintaining a set of nodes which have been seen along the way. Rough pseudocode for this method is given in Listing 3.3.

Listing 3.3: Pseudocode for cycle and conflict detection

```
for each node in the graph:
    seen = {}
    checkCycles(node, seen)

function checkCycles(node, seen)
    if node in seen:
        return 'cycle'
    else:
        add node to seen
        for each child node:
        checkCycles(child, seen)
```

Further to detecting these situations, another issue which had to be resolved was what action to take on finding a problem. Were the programmer to create a new binding between two objects which were already bound, a possible response would be to simply overwrite the old binding. This would prevent errors from the framework disrupting the user, and is also a fairly reasonable behaviour to expect. However, it was decided that this made the binding process too opaque and could lead to confusing behaviour if the programmer had accidentally overwritten a binding – for instance, by mistyping the name of one of the objects they intended to bind. Therefore, whenever the programmer attempts to create a binding which would lead to a cycle or conflict, the framework throws an exception explaining

⁷This is because the order in which bindings are updated is governed by the order in which $C\sharp$ delivers events, and this is outwith the programmer's control

the error. The behaviour of 'overwriting' an old binding is still possible as the BindingsManager provides a method for removing specific bindings, but now the programmer has to do the overwriting explicitly.

Many-to-many bindings

Many-to-many bindings presented an additional challenge for the binding system. It was decided that the best approach in terms of syntax would be to have the user pass in a list of source bind points, an arrow and a list of destination bind points, so as to mirror the syntax used in simple bindings. However, this simple approach is out of step with the way parameters are passed to arrows: by construction, all arrows taking multiple inputs will take them in the form of a binary-tree-structured Tuple⁸. As there didn't appear to be any C\pmu syntax allowing the user to pass in binding sources in a similar fashion (such that the structure could be understood by the binding), it was decided that an argument marshaller/unmarshaller would be needed to handle the translation between lists of bind points and tree-structured tuples.

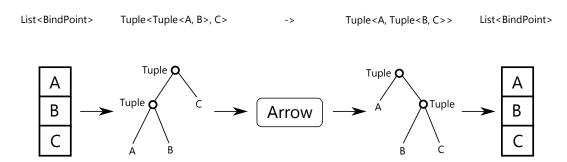


Figure 3.1: Marshalling a list of BindPoint sources to pass to an arrow and unmarshalling the result

By using reflection on the type of the arrow, I was able to convert a list of bind points to a binary-tree-structured Tuple. The code essentially does a recursive depth-first search through the input type of the arrow (obtained via reflection), simultaneously constructing a Tuple of the same structure and inserting values from the bind points wherever a 'leaf' is encountered in the type being searched. At the other end, a similar process is used to extract the values from the returned tuple and put them into a list which is then used to assign to the destination bind

⁸This can be verified by looking at the implementations of the operators; for instance, And converts two arrows (which may be on tuples) into an arrow on two-tuples containing the types of the original arrows, thus building up a binary tree structure

points. The process is illustrated in Figure 3.1, where a set of inputs of types A, B and C are being passed into an arrow from type Tuple<A, B>, C> to type Tuple<A, Tuple<B, C>>.

Problems encountered

Type safety All sort of problems here; difficulties of type checking arrows should also be mentioned.

Binding to lists They don't throw events! Binding to lists needs the list to be assigned to for it to update. Possible solutions like ObservableEnumerable (or whatever it was called), potentially more boilerplate, future work n shit.

3.3.4 Integration work with WPF

Mention difficulties with the value converter and inability to properly integrate it into XAML and stuff.

Chapter 4

Evaluation

4.1 Correctness of arrow implementations

Mention here that the main device for determining their correctness has been the arrow laws, and possibly list them with explanations? Also mention that 'special' arrows like ListArrows are correct because they are based on simple arrows and invertible arrows. Also, maybe say that ArrowChoice was ignored here as it's slightly extraneous and its correctness probably follows from the correctness of simple arrows anyway.

4.1.1 Automated testing

Simple arrows

Give tests and results for arrows.

Invertible arrows

Same for invertible arrows.

4.1.2 Correctness proof by decomposing into lambda calculus

Explain the technique used to translate into lambda calculus and the likely correctness of this, and give a few sample proofs for some non-trivial arrow laws.

4.2 Syntax evaluation

4.2.1 Arrow syntax

Arrow syntaxy stuff.

Comparison with Haskell

Might not be great, but why not?

4.2.2 Binding syntax

Pretty good. Say something about the demo apps and stuff:

Username two-way binding

Todo

List binding from a mock database

Todo

Some other demo

Todo

4.3 Performance testing

4.3.1 Arrow performance

Tests indicating how arrows perform vs. functions and funcs.

Measuring technique

Explain how the results were obtained

Simple function results

The simple function results can be seen in the graph.

List function results

Results for list-based functions

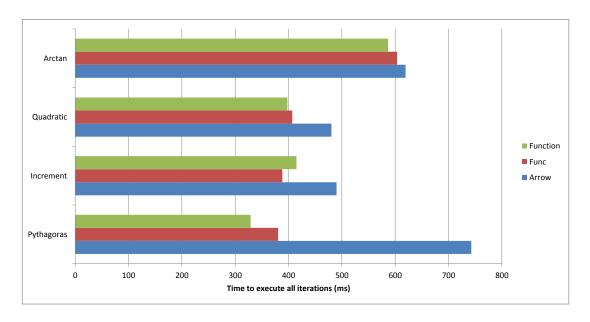


Figure 4.1: Performance of arrows, Funcs and normal functions in implementing simple functionality

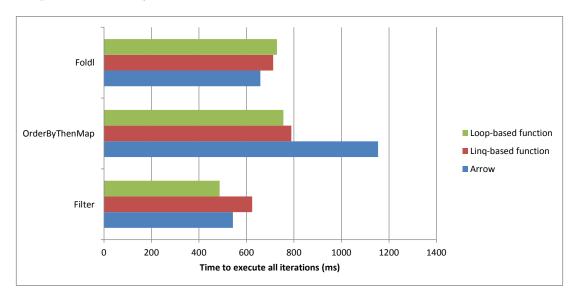


Figure 4.2: Performance of arrows, Linq queries and normal (loop-based) functions in implementing simple list functionality

Overhead due to arrow chaining

Explanation of the problem (which will have been highlighted in earlier results) along with test results for increasingly long chains of identity functions. Suggest reasons for this and potential solutions (or maybe the solutions should come in

later?)

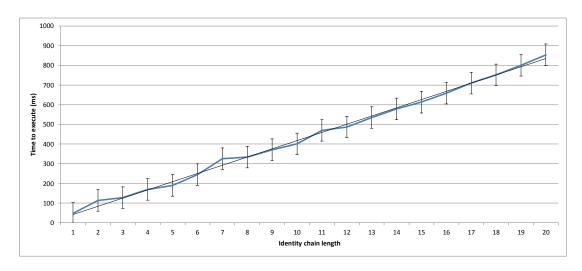


Figure 4.3: Performance of arrows, Linq queries and normal (loop-based) functions in implementing simple list functionality

4.3.2 Binding performance

Todo...

Chapter 5

Conclusion

Conclusion goes here!

5.1 Future work

- Performance issues optimisations possible with dynamic expression trees?
- Messiness caused by use of Tuples would be better with a custom tree type?
- Feedback arrows
- Better integration with WPF
- etc...

Appendix A

Arrow laws

Here the arrow laws used in testing are presented both in Haskell syntax and as they were translated into C‡. For convenience, the following definitions are used:

```
id = new Func<T, T>(x => x)
That is, id represents a C# identity function.
first_f = new Func<Tuple<T, T>, Tuple<T, T>>(
   tuple => Tuple.Create(f(tuple.Item1), tuple.Item2))
```

This defines a Func which invokes a function 'f' on the first element of a tuple.

A.1 Normal arrow laws

```
\begin{samepage}

Identity
arr id = id
new IDArrow<T>() ≈ id

Arr operator distributivity
arr (f >>> g) = arr f >>> arr g
Op.Arr(x => g(f(x))) ≈ Op.Arr(f) .Combine( Op.Arr(g) )

First operator distributivity
first (f >>> g) = first f >>> first g
f .Combine(g) .First<T>() ≈ f .First<T>() .Combine( g.First<T>()
)

Associativity of arr and first operators
first (arr f) = arr (first f)
Op.Arr(f).First<T>() ≈ Op.Arr ( first_f )
```

```
Piping commutativity
first f >>> arr (id *** g) = arr (id *** g) >>> first f
f.First<T>() .Combine( id.And(g) ) ≈ id.And(g) .Combine( f.First<
        T>() )

Piping simplification
first f >>> arr fst = arr fst >>> f
f.First<T>() .Combine( Op.Arr(tuple => tuple.Item1) ) ≈ Op.Arr(
        tuple => Tuple.Item1) .Combine( f )

Piping reassociation
first (first f) >>> arr assoc = arr assoc >>> first f
f .First<T>() .First<T>() .Combine( new AssocArrow<T>() ) ≈ new
        AssocArrow<T>() .Combine( f .First<T>() )
```