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Problem 1

The Hilbert matrix, known to be severely illconditioned, is defined as

$$H_n = \begin{bmatrix} 1 & 1/2 & 1/3 & .. & 1/N \\ 1/2 & 1/3 & .. & .. & 1/(N+1) \\ 1/3 & . & .. & .. & . \\ . & . & . & .. & . \\ 1/N & .. & .. & .. & 1/(2N-1) \end{bmatrix}$$

Condition Number$(H_n) = |||H_n||| \; |||{H_n}^{-1}|||$
The elements of H_n is given by

$$H_{n_{ij}} = \frac{1}{(i+j-1)}$$

The elements of H_n^{-1} is given by

$$H_{n_{ij}}^{-1} = \frac{(-1)^{i+j} c_i c_j}{(i+j-1)}$$

where

$$c_i = \frac{(n+i-1)!}{(n-i)!((i-1)!)^2}$$

(i)
We calculate the 1-norm of the hilbert matrix and its inverse using:

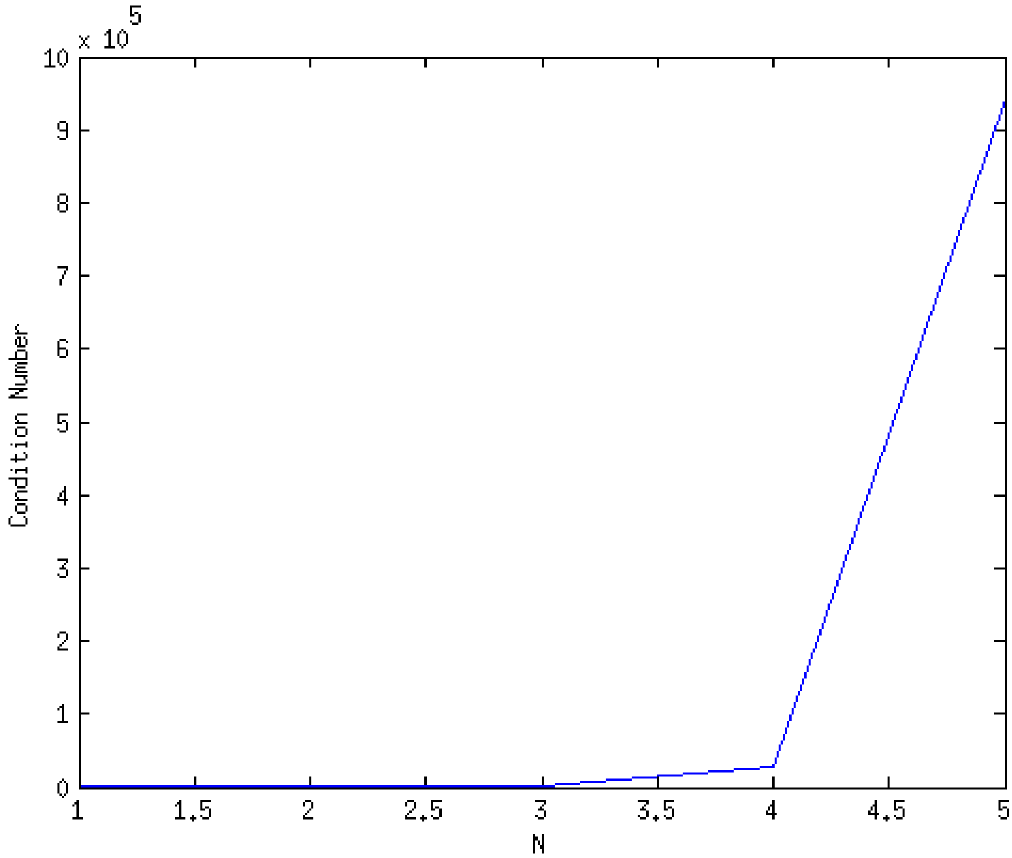
$$||A||_1 = max(\sum_{j=1}^{n} A_{1j}, \sum_{j=1}^{n} A_{2j}, .., \sum_{j=1}^{n} A_{nj})$$

After calculating using MATLAB code.

Table

N	$\|\|H_n\|\|$	$\|\|H_n^{-1}\|\|$	Condition Number(K)
1	1.000	1	1
2	1.500	18	27
3	1.833	408	748
4	2.083	13620	28375
5	2.283	413280	943656
6	2.450	11865420	29070279

After N=5 CN exceeds 10^6.
CN as a function of N upto N=5 is given by the plot below



(ii)
We calculate the 2-norm of the hilbert matrix and its inverse using:

$$||A||_2 = \sqrt{\lambda_{max}(A \times A)}$$

where λ_max is the maximum among the eigen values.
After calculating using MATLAB code.

Table

N	$\|\|H_n\|\|$	$\|\|H_n^{-1}\|\|$	Condition Number(K)
1	1.000	1	1
2	1.267	15.211	19.281
3	1.408	372.115	524.057
4	1.500	10341.015	15513.739
5	1.567	304142.842	476607.250
6	1.619	9235320.244	14951058.640

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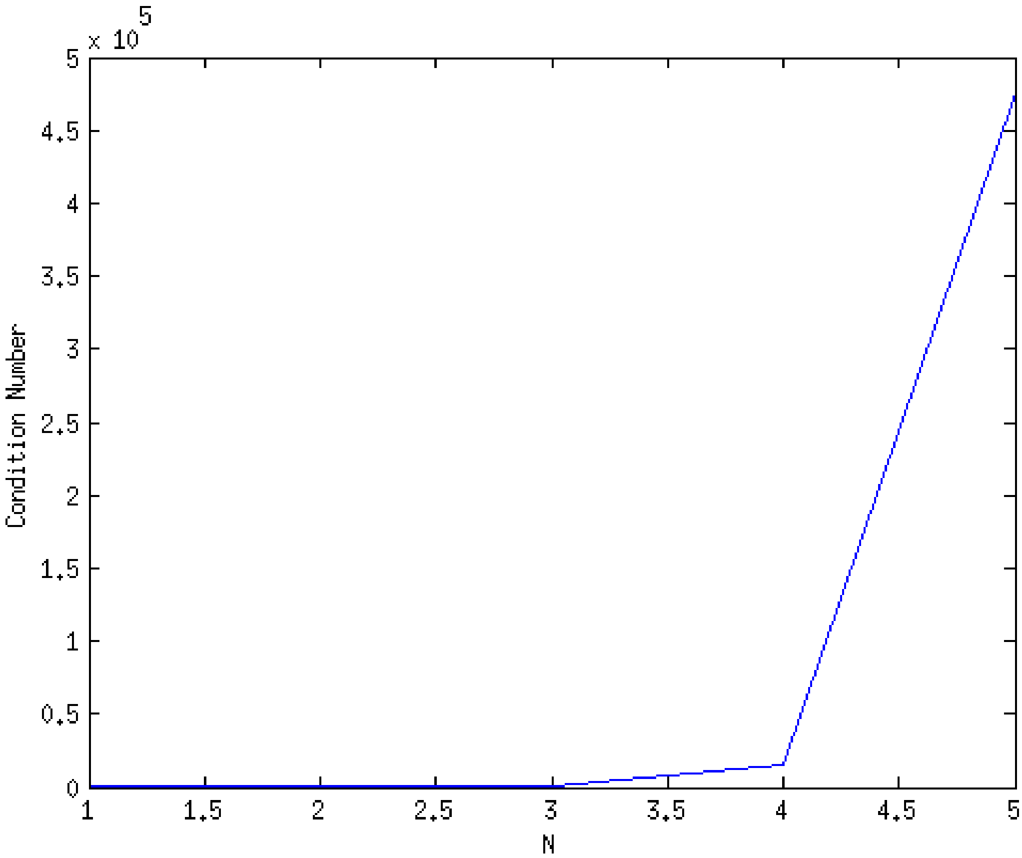
CN as a function of N upto N=5 is given by the plot below



Problem 2

x	y
0.1	10.57
0.2	19.03
0.5	36.61
0.7	44.43
1.0	52.92
2.0	68.16
5.0	82.65
7.0	86.30
10.0	89.46

The saturation curve is given by

$$y = \frac{\alpha x}{\beta + x}$$

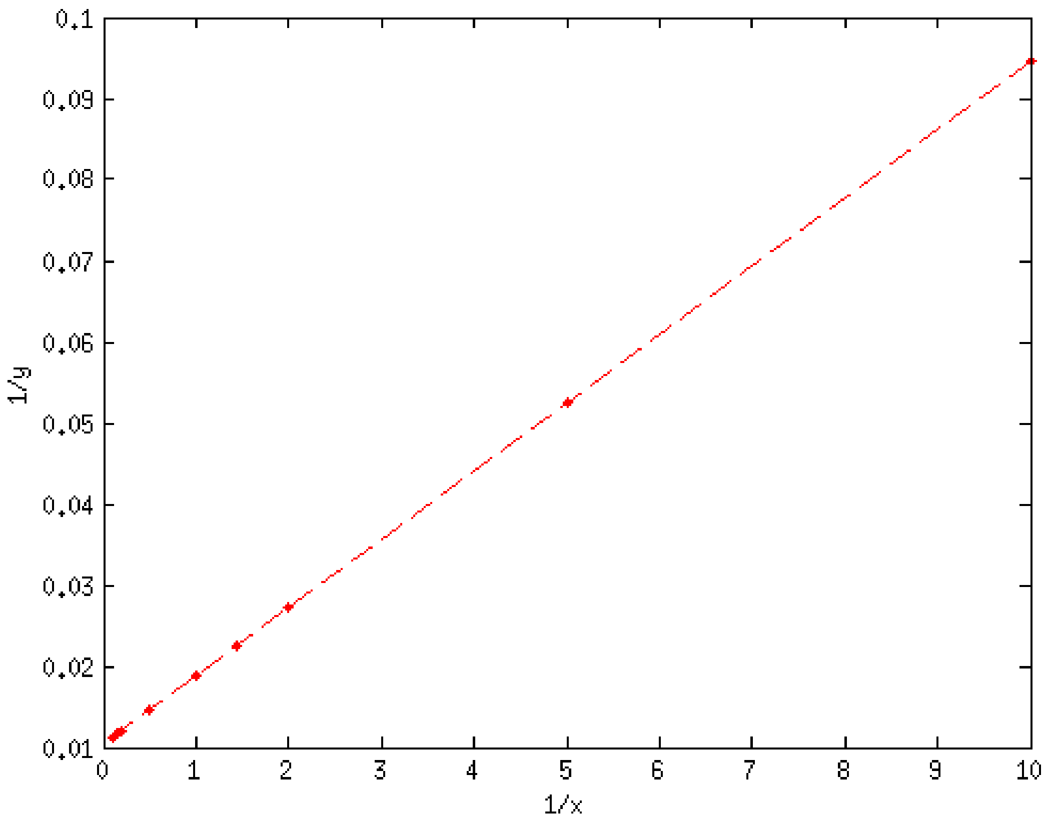
$$\frac{1}{y} = \frac{\beta}{\alpha}\frac{1}{x} + \frac{1}{\alpha}$$

Now we can perform linear regression and determine the constants β and α using the matlab code
Their values are

$$\alpha = 95.8729$$

$$\beta = 0.8073$$

The correlation coefficient is equal to 1 which suggests perfect positive fit as shown by the plot of 1/y vs 1/x below:



The y vs x curve generated using the new model is shown below