PRATUSH orbit

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Abstract

The Cosmic Dawn (CD) and Epoch of Reionization (EoR) are some of the most poorly understood eras of the evolution of our Universe. The rest-frame 21-cm signal of neutral Hydrogen (HI) emitted in Cosmic Dawn can be used as a tracer to detect and study the period of formation of the first stars in the Universe. This signal observed by us today is highly redshifted, and with a wavelength of about a meter, it falls in the meter wave region of the electromagnetic spectrum. Observing this redshifted 21-cm HI line from ground-based observatories has limitations due to man-made terrestrial Radio Frequency Interference (RFI) and ionospheric influences. Building scientific payloads in space is the next step to circumvent the restrictions of ground-based instruments. This project deals with the problem of determining a suitable orbit for a lunar orbiter carrying a payload purpose built to observe the 21-cm signal from the radio-quiet far side of the moon. The final aim of this project is to propose a low altitude orbit that shows long-term stability and ensures maximum time spent in radio-quiet regions on the lunar far side, taking into account the non-uniform gravitational field of the Moon.

Contents

1 Chapter1: The Epoch of Reionization and 21cm-HI line

The Universe started with the Big Bang 15 billion years ago and has been expanding ever since. Fig 1 depicts the various stages of Universe formation. This project deals with methods to study the Epoch Of the Reionization period which begun sometime after the formation of first stars.

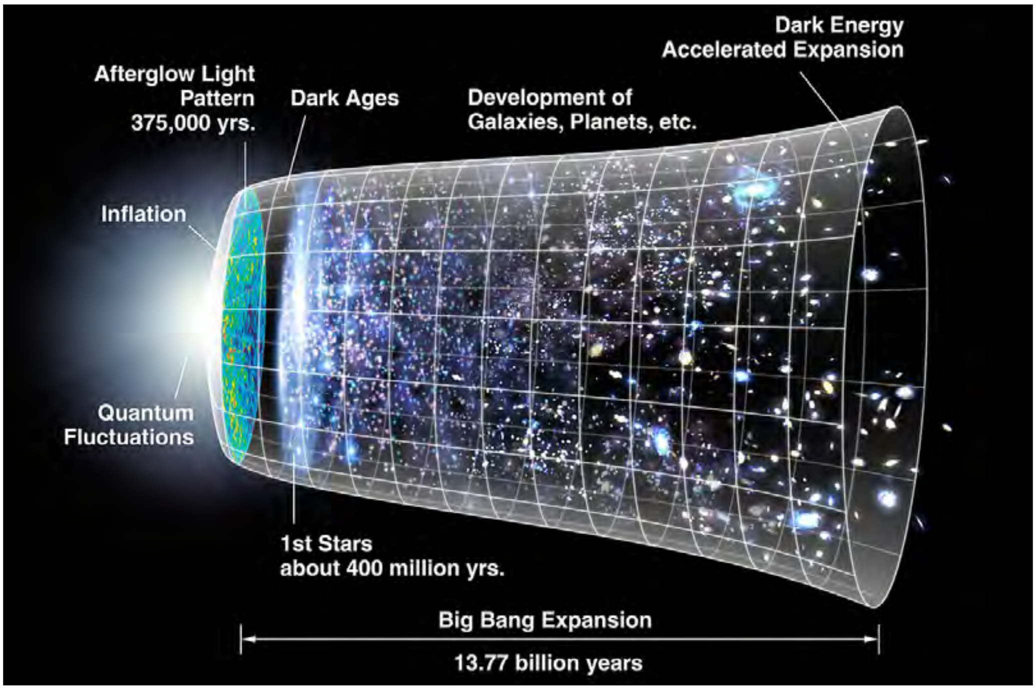


Figure 1: Evolution of Universe after the Big Bang. The right end of the image depicts the present universe while the left end depicts the earliest stage that we can probe. The expansion of the Universe is depicted by the conical shape of the graph. The expansion was rapid in the beginning and gradual for the next several billion years. The expansion in recent history has been accelerated possibly due to the Dark Energy. Picture credits: National Aeronautics and Space Administration (NASA)

Initially the Universe was a hot dense mix of baryons and photons. The mean free path of photons was very small due to their scattering off free electrons, making the Universe essentially opaque. The Universe was expanding and the temperature and density of this baryon-photon soup was decreasing. About 370,000 years after the Big Bang, the temperature of the Universe had fallen enough to allow the protons to capture free electrons and form the first Hydrogen (and Helium) atoms. This marked the onset of the Epoch of Recombination (of protons and electrons) and decoupling (of photons and baryons). The photons were no longer scattered by free electrons and their mean free path increased, turning the Universe transparent. All the photons left after decoupling are what we observe as the Cosmic Microwave Background (CMB) today. For the next several 100 million years the Universe entered the Cosmic Dark Ages and there were no new sources of light.

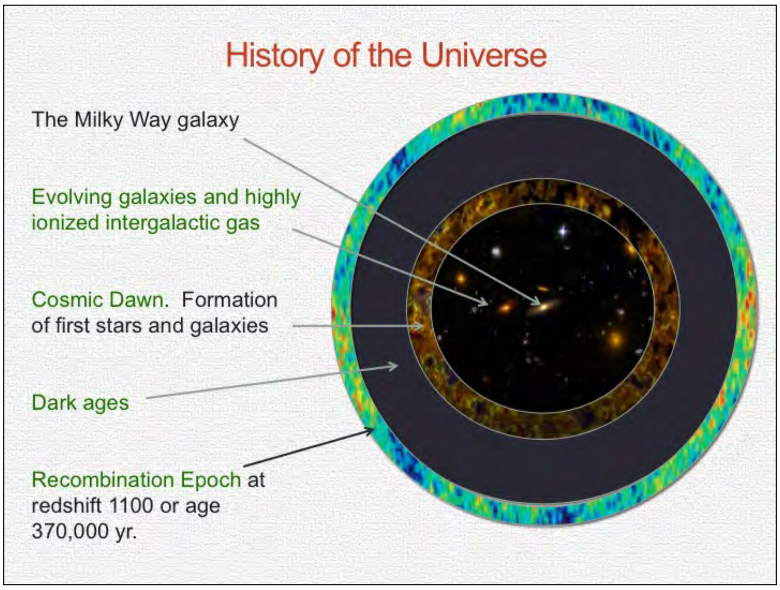


Figure 2: The farther we look, the more redshifted signal is observed.
Picture credits: Raman Research Institute, Bangalore

The expansion of the Universe stretches the wavelength of the original emitted signal and it is observed with a longer wavelength. The cosmological redshift z is a measure of the lengthening of signals given by $z = \dfrac{\lambda_{observed} - \lambda_{rest}}{\lambda_{rest}}$ due to expansion of space.

Sometime after Cosmic Dawn, new stars formed marking the beginning of Cosmic Dawn. The question of when these new stars formed remains unanswered due to lack of observational evidence from the Cosmic Dark Ages. These newly formed stars emitted ionising radiation which ionized the neutral Hydrogen gas in the surrounding intergalactic medium marking the beginning of The Epoch of Reionization (EOR). The exact boundary between Cosmic Dawn and EoR is still unknown. Approximately one billion years after the Big Bang (at redshifts $\approx$ 6) the EoR ended and much of the Universe was reionized. Observing the 21-cm line emitted from spin-flip transition in neutral Hydrogen atoms is one of the ways to probe this period. A Hydrogen atom in ground state consists of a proton and an electron. The hyperfine splitting of the ground state of neutral Hydrogen atom occurs due to interaction of the magnetic dipole moment of the electron and proton[1]. When the dipole moments of electron and proton are anti-parallel (singlet state) the atom has slightly lower energy than when they are parallel (triplet state) .The energy difference between the two states is ΔE= 5.9×10^{-6} eV. When a photon of wavelength 21 cm and frequency 1420 MHz is absorbed/emitted by an atom it goes from one state to another. Hydrogen atoms go from singlet to triplet state mainly due to collision (with other H atoms, protons, electrons) and absorption of photons of suitable wavelength.

The spin-flip transition from triplet state to singlet state in neutral H atom is a very rare occurrence with a mean lifetime of 11 million years[2] but the vast amount of neutral hydrogen during early phases of the universe makes 21-cm signal an abundant resource. The spin temperature of a Hydrogen cloud determines the strength of the 21-cm signal emitted. It is dependent on the ratio of the number of Hydrogen atoms in triplet and singlet states. The 21-cm signal observed on Earth today is redshifted due to the expansion of the Universe and falls in the Microwave part of the spectrum. The CMB acts as a back light and the spin temperature determines whether the signal is seen in absorption or emission against it by our receivers. The strength is very small (at max -100 mK) compared to the human-made RFI in the same frequency range. The next chapter discusses limitations of ground based radio telescopes and need for space based observatories.

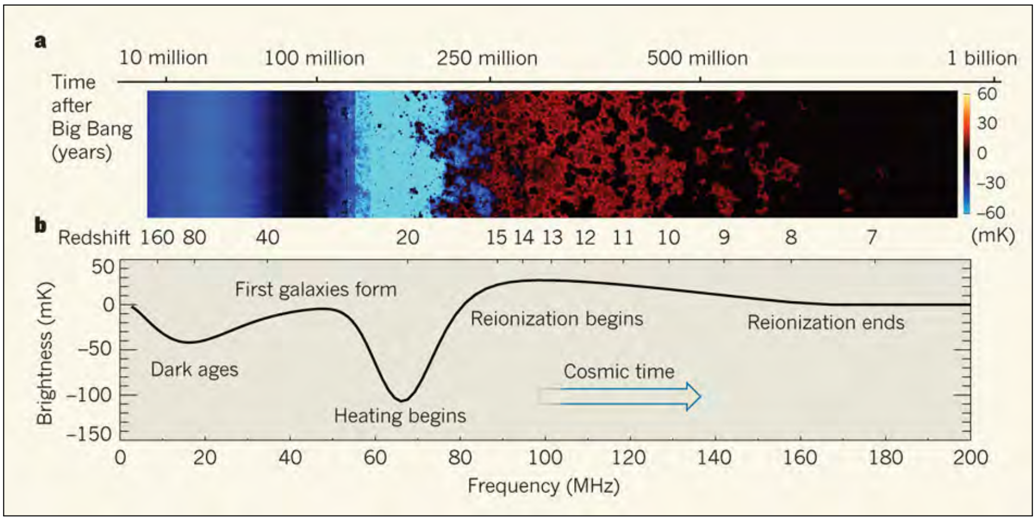


Figure 3: Predicted time evolution of 21-cm brightness from Dark ages to end of Epoch of Reionization
Picture credits: "21-cm cosmology in the 21st Century" by Jonathan R. Pritchard and Abraham Loeb

2 Chapter 2: Need for Lunar observatories

Any ground based measurements of the 21-cm signal will always be infested with human made RFI. Even specially designed radio quiet zones for Murchison Radio-astronomy Observatory or Green Bank observatory are not free from ionospheric influences. The Sun emits radiation throughout the electromagnetic spectrum and contributes to the solar RFI.

The nearest truly radio quiet region in space is the lunar far side[3] where the Moon acts as a natural blocker of terrestrial, ionospheric and solar RFI[4]. There is a Earth shadow and Sun shadow region behind the Moon and the intersection of these two radio quiet regions is the "Prime cone region", ideal for observing the 21-cm signal.

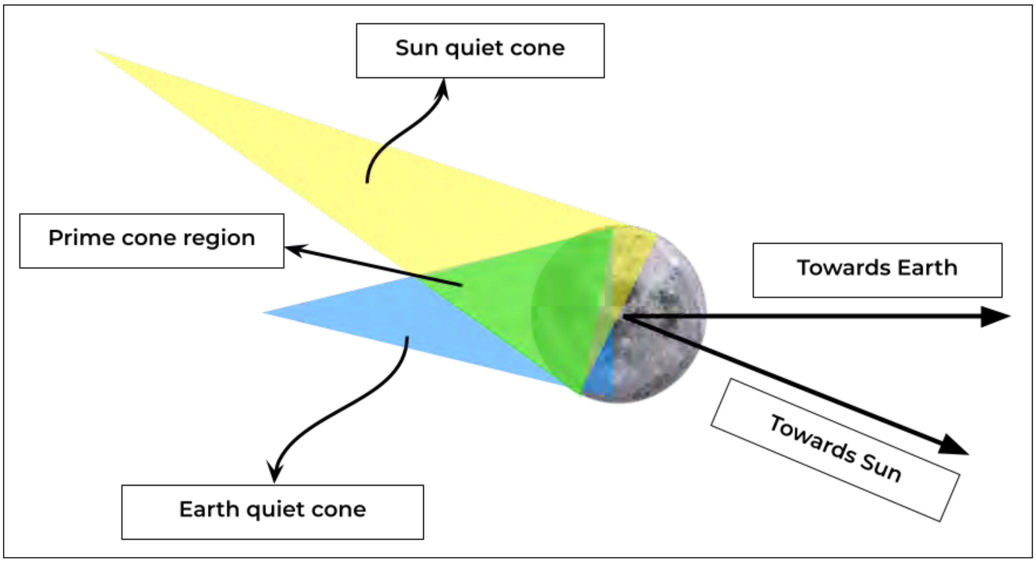


Figure 4: Blue cone represents the region free from terrestrial RFI
Yellow cone represents region free from Solar RFI
Green cone is the prime cone region formed by the intersection of the 2 radio quiet zones.

The lunar far side was unseen to human eyes until the Soviet spacecraft Luna 3 photographed it in 1959. Since then multiple satellites have captured the far side of the Moon with higher resolutions. All the moon landings (manned and unmanned) have occurred on the near side with an exception of Chang'e 4 in 2019. The Queqiao relay satellite was put into a halo orbit around the Earth-Moon L_2 point to facilitate communications between the Chang'e 4's Yutu rover on the lunar far side with Earth. Additionally this relay satellite also hosts the Netherlands-China Low-Frequency Explorer (NCLE)[5] to observe the 21-cm HI line. However since the Earth-Moon L2 lagrangian point is at a distance of approx 61 000 km from the Moon it is not in the true radio quiet regions. The NCLE is the first step of a four phase plan aimed at deploying a space-based ultra-low frequency radio telescope named Orbiting Low Frequency Antennas for Radio Astronomy (OLFAR), that will explore the Cosmic Dark ages.

Apart from this a surface based radio telescope located in a crater on the lunar far side is also being proposed[6]. Such a telescope will always be free from RFI from Earth based sources, ionosphere and artificial satellites around Earth. Furthermore during lunar nights, which last about 14 days, it will be free from Sun based radio interferences. The Daedalus crater located approximately at the center of the Lunar far side is an ideal location for such a telescope. Such an antenna (shown in Fig 5) would require a relay satellite similar to Chang'e 4's Queqiao to facilitate communication with the Earth based stations. Since only one successful landing has taken place on lunar farside, building an antenna in a crater is still uncharted territory in space exploration.

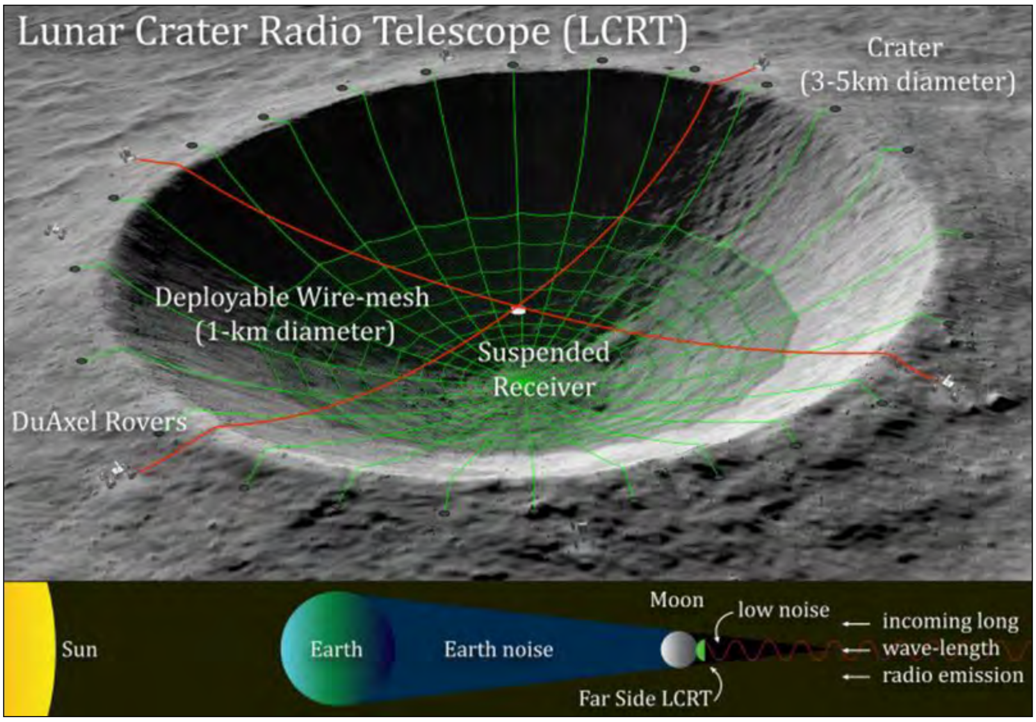


Figure 5: Proposed plan for Lunar Crater Radio Telescope
Picture credits Saptarshi Bandyopadhyay, NASA

The most cost efficient and plausible method based on humankind's current engineering capabilities is to observe the 21-cm line from a satellite in lunar orbit. The orbit for such a satellite will have to be planned in such a way as to maximise the time spent in radio quiet regions during the full mission sequence. At least two such missions are in works right now. These will be discussed below

2.1 PRATUSH (**P**robing **R**eioniz**AT**ion of the **U**niverse using **S**ignal from **H**ydrogen)

PRATUSH[7] is a future radiometer in lunar orbit proposed by Raman Research Institute (India), intended to study the 21-cm signal from the radio quiet region on the far side of the Moon. The nominal lifetime of the mission will be two years for achieving required sensitivity and sufficient sky coverage.

For a mission like PRATUSH a lunar orbit is required that fulfills the following criteria.

1. Frozen (stable) orbit that requires little to no orbital station keeping maneuvers for a mission duration of 2 years taking into account the non-uniform gravitational field of the Moon

2. Maximise time spent in the Earth and Sun shadow region behind the Moon

3. The orbit should be stable for a range of lunar orbital insertion parameters like semi-major axis (SMA) length, inclination and eccentricity.

It is obvious that setting the perilune at the lunar far side will promise maximum time spent in the Earth shadow region. However, with time the longitude of perilune may change due to the non uniform gravitational field of the Moon.

The following steps have been taken to analyze any set of initial Keplerian orbital parameters in this project.

1. Initially the spacecraft is positioned at nearside and the perilune of its orbit is set at the center of lunar farside.
   (a) Propagate the spacecraft to perilune then back to apolune repeatedly for two years or until it crashes
   (b) Store the altitude of the spacecraft at these two points
   (c) If the orbit shows long term stability it can be analysed further in step 2
2. With the same set of initial Keplerian parameters estimate the time spent in radio quiet region in one day, one lunar month and 2 years
   (a) Propagate the spacecraft till its longitude increases by 20° in a coordinate system fixed with respect to lunar surface. Repeat this step for the whole mission duration.
   (b) Store the number of days elapsed, altitude, longitude, right ascension (RA) and declination (DEC) values of the spacecraft at each step in a .csv file
   (c) From the position of the spacecraft at each step determine whether it lies in the radio quiet cone regions of Sun and Earth or not.
   (d) Calculate the cumulative time it spends in the radio quiet cone for a mission duration of 2 years

Estimations of ΔV budget, suitable launch dates and trajectory from the Earth to Moon are not discussed. It was observed that the spacecraft spends maximum time in the radio quiet zone near a Full Moon of each lunar month. The time spent in radio quiet zone for different lunar months are comparable. For this reason time spent in radio quiet region in one lunar month and on a full moon were studied first.

2.2 DARE (**D**ark **A**ges **R**adio **E**xplorer)

A mission similar to PRATUSH named DARE[8] (Dark Ages Radio Explorer), proposed by University of Colorado has been a great resource for this project. Its aim is to observe the redshifted 21-cm signal in the range of 40-120 MHz from lunar far side with a mission duration of two years. The proposed DARE mission orbit has been used as a case study in this project to verify the consistency of simulations and as a starting point in determining the orbital parameters for PRATUSH.

PRATUSH and DARE missions are proposed to observe the 21-cm signal from the radio quiet regions behind the Moon. This region of space free from RFI from the Sun and the Earth is not fixed. When viewed from the celestial north pole, the Earth revolves around the Sun in a counterclockwise direction. The Moon also revolves around the Earth in the same way. Additionally both the Earth and Moon have prograde rotation, meaning their direction of rotation is same as their direction of revolution around the primary body.

As seen in Fig 6 the Earth quiet cone in blue colour rotates by 360° with respect to distant stars in one lunar month. Similarly the Sun quiet cone in yellow colour will rotate by 360° with respect to distant stars in one solar year (not shown in figure). The intersection of these two regions shown in green colour is the true radio quiet region which will be ideal for the mission.

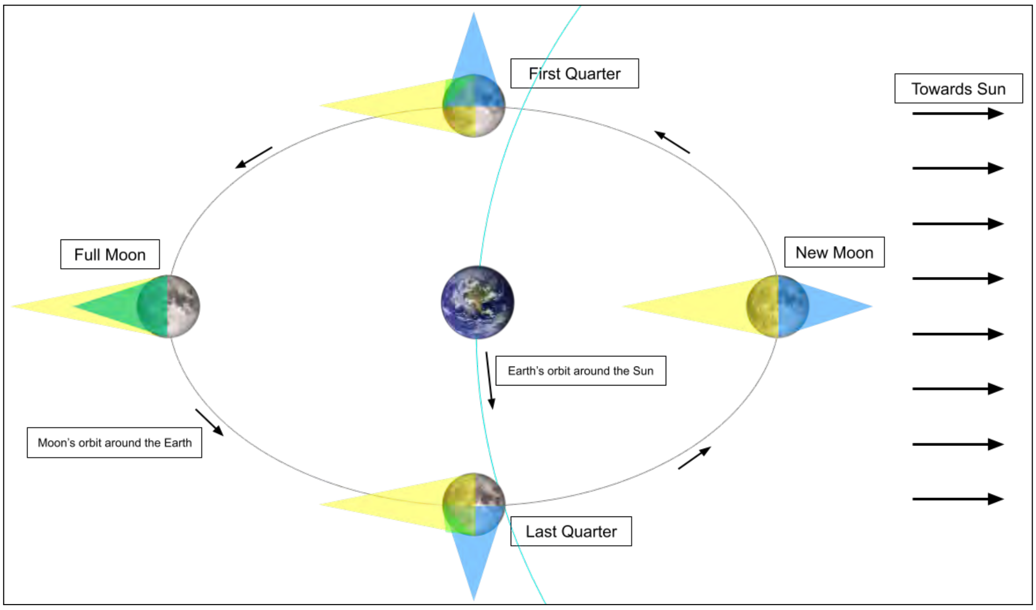


Figure 6: Yellow cone: Region of space free from solar RFI
Blue cone: Region of space free from terrestrial RFI
Green cone: Region of space free from both solar and terrestrial RFI

3 Chapter 3: Setting up the simulations

3.1 GMAT

Since the final aim of this project is to find a suitable orbit for PRATUSH, a tool for designing space based missions is required. There are many such tools available like System Tools Kit (STK) by AGI, FreeFlyer by a.i. Solutions and GMAT by NASA . This project utilises GMAT[9] (General Mission and Analysis Tool) an open source software developed by NASA and other public/private contributors which promises high fidelity in all its simulations. It is being used extensively in maneuver and operational planning for the ACE mission by NASA[10].

The missions in GMAT can be planned either by graphical (GUI) or script interface but both need to be synchronised regularly. It can easily generate 3D figures of a satellite's orbit, ground track plots, 2D plots during a mission. However for missions with longer mission durations where real time analysis is not required such plots become computationally complex and it is better to generate a report or ephemeris file which can later be used to generate the plots. This approach is used here and all the plots in this project have been generated using Python in Jupyter notebook.

Since all the simulations in this project were made using extensive use of the GUI, wherever possible a snapshot of the GMAT window is provided for anyone to verify it in any future works.

The GMAT scripts,report files and Python notebooks used to generate results presented in this report are available in the GitHub repository [11] The next sections discuss the setup required for the simulations in this project.

3.2 Coordinate system

Before discussing the coordinate systems used in the forthcoming simulations it is important to discuss certain notations used in this project. A "coordinate system" is defined by specifying an origin at a well defined point and three orthogonal xyz axes.

The origin can be the center of any celestial body, barycenter of the solar system or even a spacecraft. All the coordinate systems used in this project have their origin at the center of the Moon. The axes can be defined as inertial (fixed with respect to distant stars) or body fixed (fixed with respect to the celestial body where the origin lies). The direction and time dependence of the three axes is important when considering lunar missions.

Every celestial body needs a well defined surface mapping system similar to the latitude and longitude system defined on Earth. There are 2 types of widely used body fixed coordinate systems, planetocentric and planetographic. As seen in Fig 7 these two systems differ in describing the latitude and altitude of a point. For a perfectly spherical body both the systems are the same. The use of either system depends on mission objectives and the difference in latitude and altitude depends upon the celestial body. From Mars based missions it is observed that the latitude and altitude reported in the two systems differ by as much as 0.34° and 2 km[12]. All the coordinate system used in this project are planetocentric as defined by NASA for Lunar Reconnaissance Orbiter[13]. The longitude is measured from the prime meridian and it is the same in both the systems. The prime meridian for most of the celestial bodies in our solar system have been well defined.

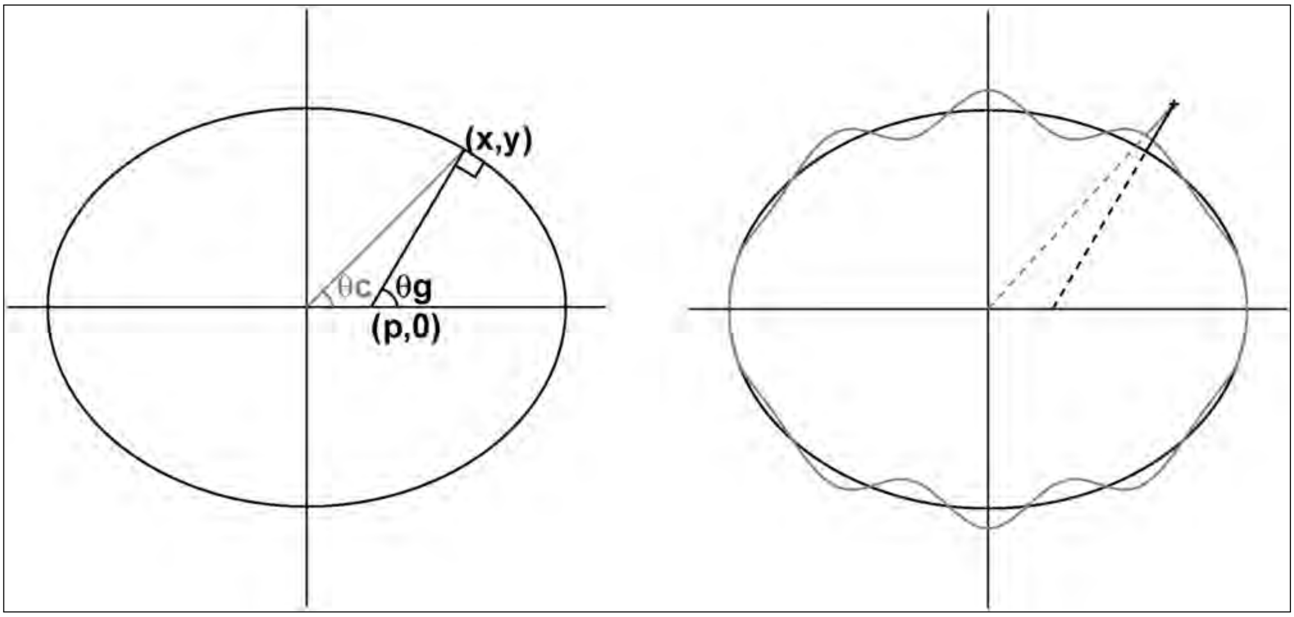


Figure 7: In the left figure θ_g is the latitude in planetographic system while θ_c is the latitude in planetocentric system.
The diagram on the right shows the difference in altitude in both systems
Picture credits: Withers, Paul, and B. M. Jakosky. "Implications of MAVEN's planetographic coordinate system for comparisons to other recent Mars orbital missions."

GMAT provides the freedom to set up a coordinate system in which the direction of the axes is defined by the position and velocity of one celestial body with respect to another. It is referred to as an "Object referenced coordinate system" in GMAT and defined using the following

1. Primary body
2. Secondary body
3. $\vec{r}$ relative position of secondary body with respect to primary body
4. $\vec{v}$ relative velocity of the secondary with respect to primary
5. $\vec{n}$ mutually perpendicular to $\vec{r}$ and $\vec{v}$

Fig 8 shows the three vectors along with the position of the primary and secondary body. The vectors $\vec{r}$ and $\vec{v}$ lie in the orbital plane of the secondary while $\vec{n}$ is perpendicular to it. Two out of the six vectors $\{\vec{r}, \vec{v}, \vec{n}, -\vec{r}, -\vec{v}, -\vec{n}\}$ are used to describe the xyz axes in the coordinate system centered at the secondary body. By describing two of the xyz axes the third one is computed internally such that $z = x \times y$ thus completing the orthogonal set of axes.

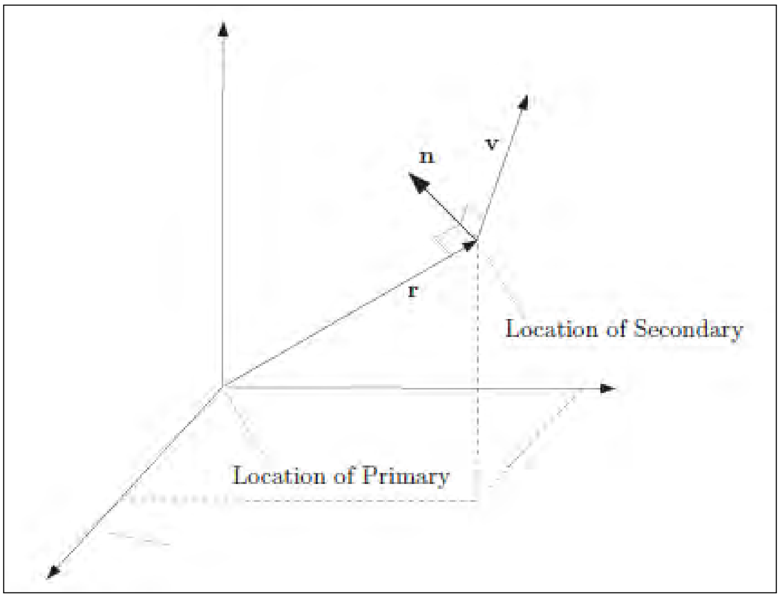


Figure 8: Picture credits: GMAT

3.3 Body Inertial

The three axes of a body inertial coordinate system are fixed with respect to distant stars . The origin is defined at the center of a certain celestial object. Many body inertial coordinate systems are predefined for most celestial bodies in GMAT. The Moon centered body inertial system set up in Fig 9 was used for some simulations in this project. This system is considered an industry standard when planning lunar missions since the direction of axes is time independent.

In the predefined body inertial system for the Moon, the x-axis points along the line formed by the intersection of the Moon's equatorial plane and Earth's mean equatorial plane at J2000 epoch. The z-axis points along the Moon's spin axis direction at the J2000 epoch. A suitable y-axis completes the right-angled system. The J2000 epoch refers to the Gregorian date January 1, 2000, at 12:00 TT (Terrestrial Time).

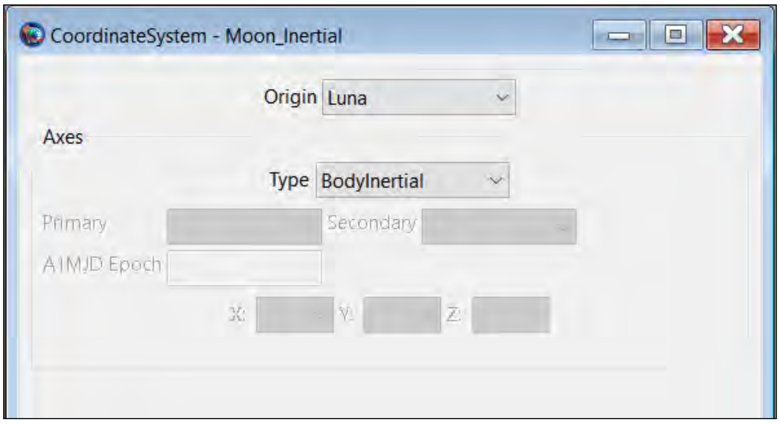


Figure 9: Setting up an inertial coordinate system centered at the Moon

Other inertial systems like MJ2000Eq set up in Fig 10 differ from "body inertial" only by the convention used to set up the axes. More information about the convention used to set up the coordinate system can be found in GMAT documentation. Infact the axes of all inertial systems centered at the Moon differ from each other due to the convention used to define them.

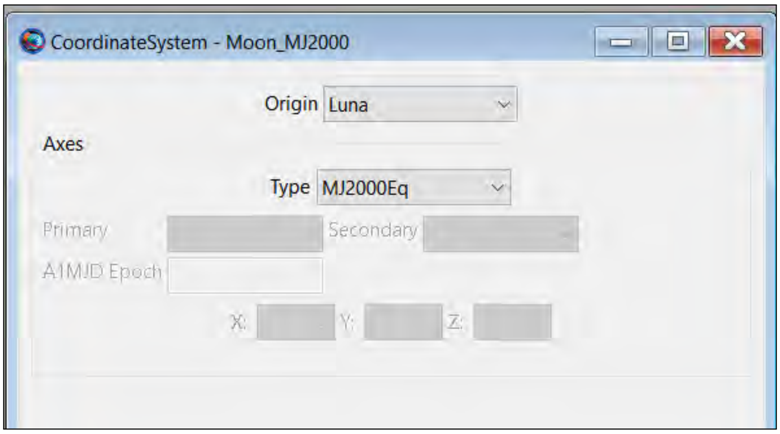


Figure 10: Setting up a MJ2000Eq coordinate system centred at the Moon

3.4 Body Fixed

There is an inbuilt planetocentric, principal axis coordinate system predefined in GMAT for all the planets in our Solar system and the Moon. The axes are fixed with respect to the surface of the body. The axes for a body fixed system for the Moon are defined by the DE file used to model the Moon[1]. Fig 11 shows the setting up of a body fixed coordinate system centered at the Moon. For the rest of the project this system will be referred to as the "LunaFixed" coordinate system.

In the forthcoming simulations of Chapter 4, the spacecraft is propagated in certain steps in longitude in this coordinate system and at each step its position in other coordinate systems is stored in an external file.

3.5 Earth_X

This system is defined in Fig 12 using the 'Object referenced coordinate system' option available in GMAT discussed before. The Earth is set as the 'primary body' and Moon as the 'secondary

[1]Refer to GMAT docs

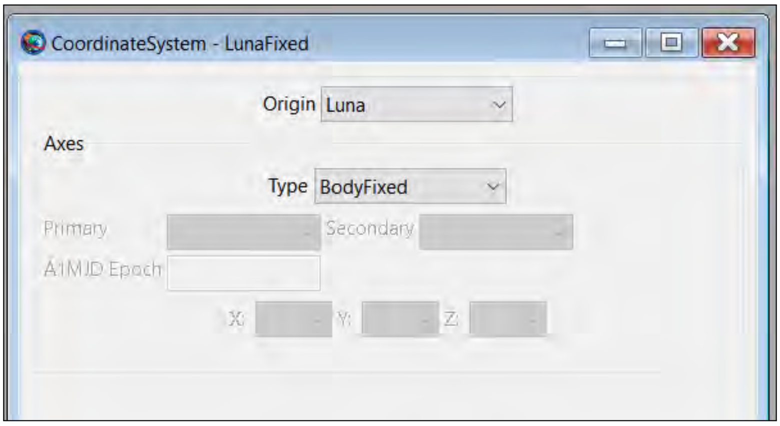


Figure 11: Setting up the LunaFixed coordinate system

body'. The origin is thus at the center of the Moon and the x-axis is in direction of $\vec{r}$ which points towards the centre of the Earth. The z-axis is in the direction of $\vec{n}$ and perpendicular to the orbital plane of the Moon. The y-axis is computed accordingly, completing the set of three orthogonal axes.

The eccentricity and inclination of the Moon's orbit around the Earth means that the three axes are not fixed with respect to the Lunar surface. The x-axis rotates 360° with respect to distant stars in one lunar month. Since the x-axis always points towards the Earth, the radio quiet region of Earth in this system is fixed in negative x direction. The coordinates of spacecraft in this system were used to determine whether it was in the radio quiet region of the Earth.

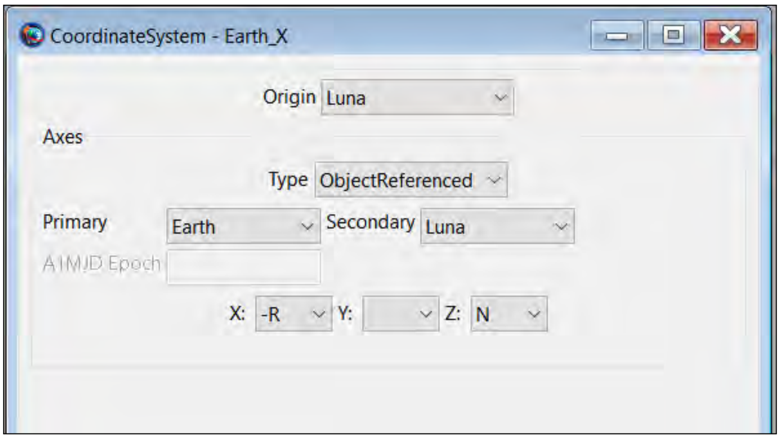


Figure 12: Setting up the Earth_X coordinate system

3.6 Sun_X

Similar to the Earth_X system discussed earlier, this was an 'Object referenced' coordinate system. However, unlike the Earth_X system here the Sun was set as the 'primary body' and Moon as the 'secondary body'. The setup is shown in Fig 13 in which the x-axis is set in direction of $\vec{r}$ which points from the Moon to the Sun. Thus the Sun shadow region is fixed in negative x direction. The x-axis rotates 360° with respect to distant stars in one year. The coordinates of the satellite in this system are used to determine whether it is in the radio quiet region of the Sun.

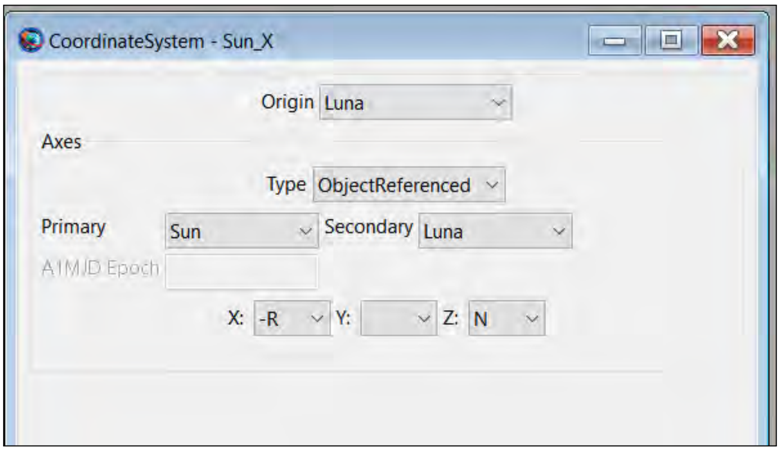


Figure 13: Setting up the Sun_X coordinate system

3.7 Orbital Elements of a satellite

After a coordinate system consisting of an origin and three axes has been set up, the position and velocity of the spacecraft at any point in space can be defined. Keplerian and state vector notation are the most common and discussed in next sections.

3.7.1 Keplerian orbital elements

The orbit of a satellite around any celestial body can be uniquely represented by 6 Keplerian orbital elements. The position and velocity of a satellite are defined at a certain isntance of time (epoch), so time is sometimes considered as the 7^{th} Keplerian element. The 7 elements together specify the position and velocity of the satellite at a certain time. Although the velocity of the spacecraft is not stated as a parameter, it can be calculated from suitable transformation to orbital state vectors (discussed in next section).

1. Epoch: Specific time at which the parameters are defined
2. Eccentricity (e): of the orbit around the celestial body. Based on its value the orbit can have different shapes
   (a) $e = 0$ circular orbit
   (b) $e > 0$ and $e < 1$ elliptical orbit
   (c) $e = 1$ parabolic orbit
   (d) $e > 1$ hyperbolic orbit
3. Inclination (i): Angle between orbital plane and some reference plane passing through the center of central body. Based on its value an orbit can be of the following type
   (a) $i \geq 0$ and $i < 90°$ prograde orbit. A body in prograde revolves and rotates in the same direction around its primary body.
   (b) $i = 90°$ polar orbit
   (c) $i > 90°$ and $i < 180°$ retrograde orbit. A body in retorgrade orbit rotates opposite to its direction of revolution around its primary.
4. Semi Major Axis Length (a): Half of the distance between the farthest point (Apogee) and nearest point (Perigee). These points are referred as Apolun and Perilune when the central body is the Moon
5. Right Ascension of Ascending Node (RAAN) (Ω): A node refers to the points where the orbital plane intersects the reference plane. There are 2 such points for inclinations $> 0°$ and undetermined for inclinations=$0°$ or $180°$. The ascending node is the node where the satellite goes from below the reference plane to above it (depending on the direction conventions used). The other is the descending node situated directly opposite to the ascending node. The RAAN is measured in reference plane as opposed to Argument of perigee
6. Argument of Perigee (AOP) (ω): Measured in the orbital plane and defines the position of perigee relative to the ascending node. For a circular orbit every point is at the same distance to the centre, so the argument of perigee is taken to be $0°$ or $90°$ by convention. The value AOP+ RAAN gives the longitude of perigee which describes the position of perigee relative to reference x direction

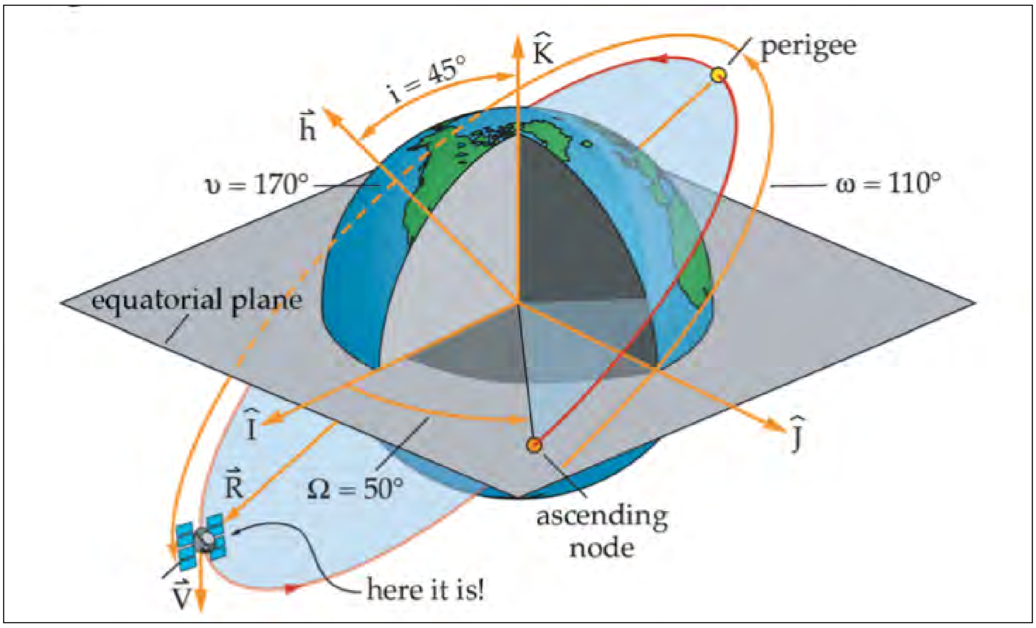


Figure 14: Pictorial representations of all the 6 Keplerian orbital elements
Picture credits: Federal Aviation Administration

7. True Anomaly (υ): This angle is measured in the orbital plane and describes the position of the satellite relative to the perigee.

A quick way to visualize all the Keplerian parameters in real time can be found at `https://orbitalmechanics.info/` The Keplerian system is useful in visualizing a satellite's orbit around a central body and predict its motion ignoring atmospheric drag and third body effects. However, when determining a satellite's orbit by numerical integration, taking into account all perturbations, the state vector notation is more useful.

3.7.2 State vector notation

Another way to specify the position of a satellite is to provide its position and velocity relative to the center of a central body at a certain time. These three quantities $\vec{r}$, $\vec{v}$ and time are the three orbital state vectors of a satellite at a specific time. Fig 15 shows the orbit and the corresponding state vectors of an orbiting body

A satellite's state vectors can be used to compute its Keplerian elements and vice versa. GMAT supports both these systems for providing the initial position and velocity to the spacecraft. It can also compute the corresponding elements in other systems. All the simulations in this project use the Keplerian system to provide the initial parameters for a satellite in lunar orbit.

3.8 Setting up the moon model and propagator

If the position and velocity of a spacecraft is known at some point, the position and velocity at some later time can be calculated by integrating the differential equations of motion. These equations take into account the force from all sources like celestial objects, solar radiation pressure (SRP), atmospheric drag etc. Alternatively GMAT allows modelling the trajectory of any object by providing an ephemeris file. This method is not suited for this project and the numerical integration method is used in all simulations

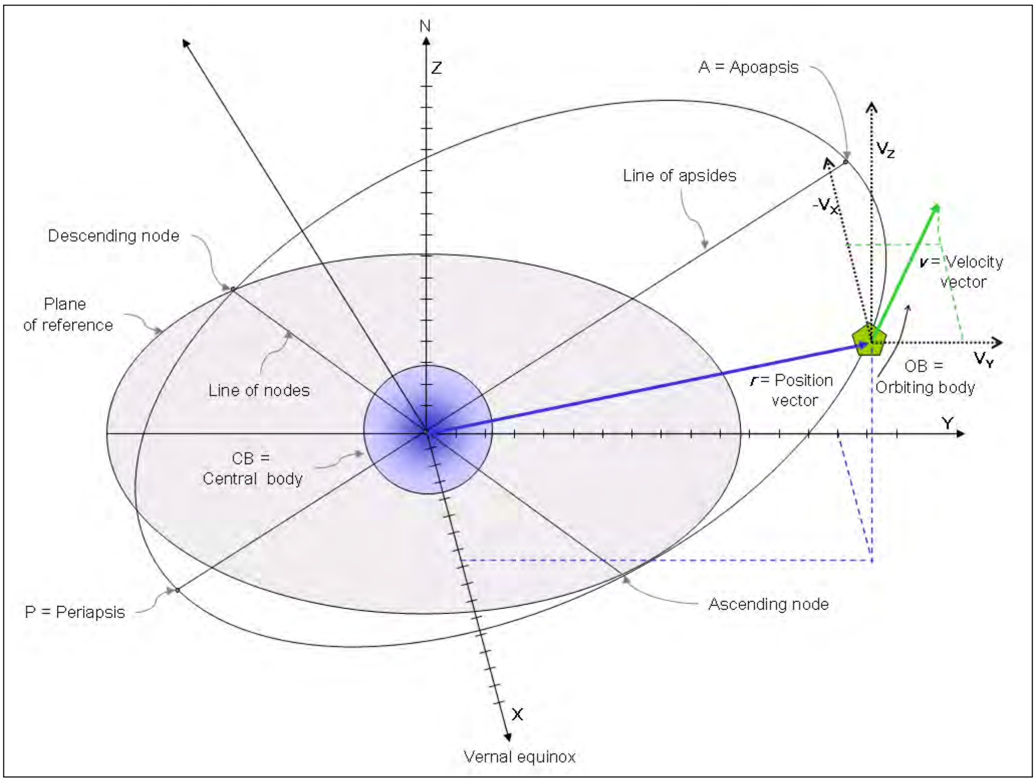


Figure 15: The two state vectors of a satellite, $\vec{r}$ shown in blue color and $\vec{v}$ shown in green color

3.8.1 Physical properties of the Moon

This project is concerned only with the orbit of the satellite in the vicinity of the Moon and as such the physical properties of the Moon should be rechecked beforehand. The moon has a spheroid shape with a mean equatorial radius of 1738.1 km and mean polar radius of 1736.0 km. The moon thus has a flattening factor of 0.0012. The standard Gravitational parameter μ $(= G \times M)$ value for the Moon is set to 4902.8005821478 km^3/s^2. All these values along with other physical parameters of the Moon can be accessed in fact sheet released by NASA[14]. Some of these values were different from the default ones set in GMAT for DE405 ephemeris and were changed form the Moon properties tab shown in Fig 18.

Every celestial body needs to have a well defined position ,orientation and trajectory for us to know its gravitational strength on a spacecraft during a mission design. This is where the SPICE kernel and ephemeris files come into picture. An ephemeris file along with the the SPICE[15] Planetary Constant Kernel[16] (PCK) determine the position and orientation of the models of every celestial object during a mission in GMAT. This project uses the default "SPICELunaCurrentKernel.bpc" file in the PCK field which comes preloaded for the Moon for the DE405 ephemeris. Knowing the position and orientation of celestial bodies is important when trying to determine when a satellite is occulted from other celestial bodies or ground stations.

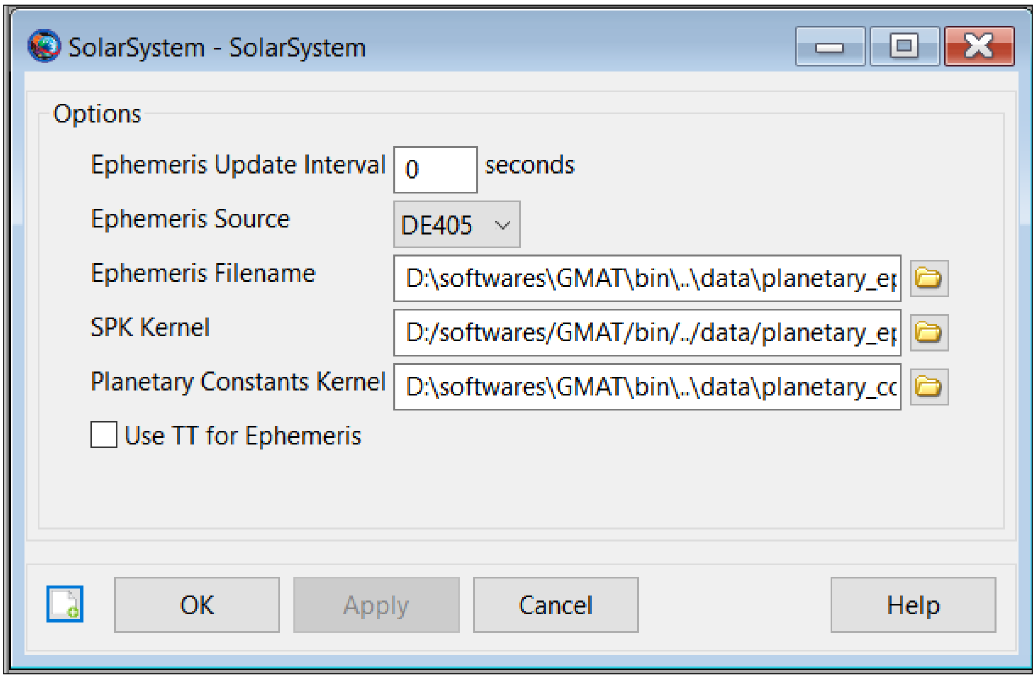


Figure 16: The solar system tab gives an option to select the Ephemeris source, SPK kernel and Planetary constants kernel for all the major celestial bodies. The default values of these fields are used in this project.

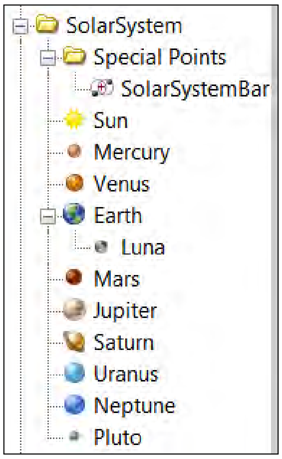


Figure 17: Solar system tree showing the list of all bodies modelled in a simulation

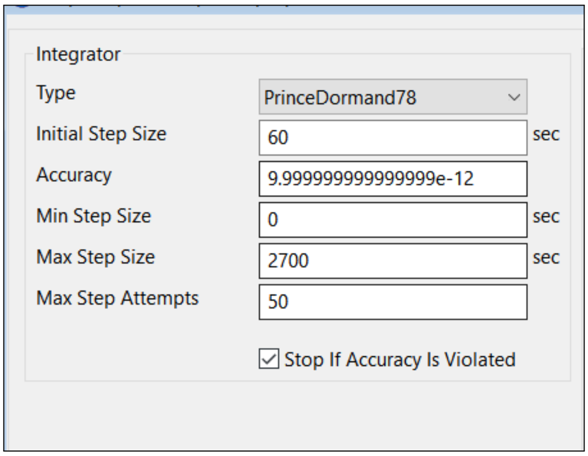


Figure 19: Setting up the PrinceDormand78 numerical integrator

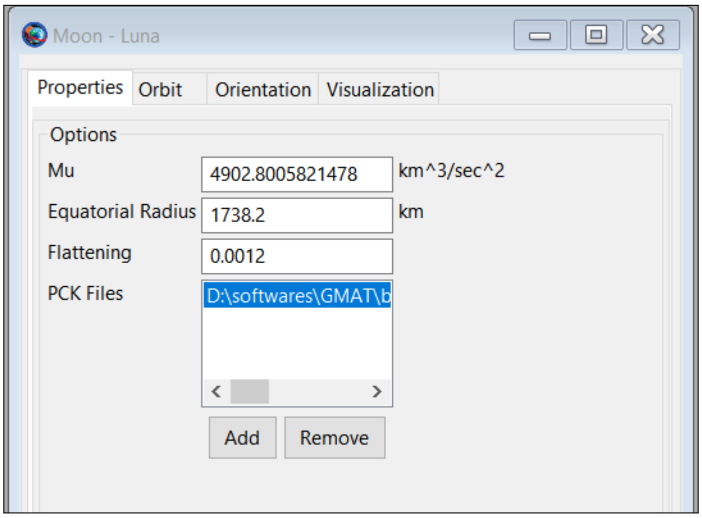


Figure 18: The Moon properties window is used to set the physical parameters of the Moon

3.8.2 Propagator and Force model

The "propagator" is a GMAT component which models the motion of spacecraft by using an ephemeris file or by numerical integration method. This project will use the numerical integration method to solve the differential equations of motion of the spacecraft. From a range of inbuilt options, the PrinceDormand78, an adaptive step, eighth order Runge-Kutta integrator with seventh order error control is used to model the spacecraft in all the simulations. By setting up the "Min step size" field (see Fig 19) to zero the integrator's adaptive step algorithm decides the optimum minimum integration step size.

More information about different numerical integrators and their limitations can be accessed in GMAT guide.

The "Force Model" is a GMAT component comprising of all the forces considered during planning a mission and is essentially a model of all the forces which are to be considered during a mission. The propagator integrates the equations of motions by using the force model.

Currently GMAT supports only one central body in the force model for which a detailed non uniform gravity model can be provided. Due to the nature of this project all simulations in this project consider the Moon as the central body and the rest of the planets in the Solar system and Sun as point objects. The atmospheric drag due to the Moon's exosphere and solar radiation pressure has been excluded from the force model.

A detailed gravity model of the central body can be provided in the force model. The gravity model of any celestial body is described in terms of spherical harmonics and the degree and order of the harmonics essentially dictates its spatial resolution. Higher degree and order of gravity models provide more accuracy but at the same time increase the computational load.

The gravity model used for this project was GRGM 900c[17] with a degree and order 100X100. The GRCGM 900c is a gravity field solution in spherical harmonics to degree and order 900 from the tracking data of the Gravity Recovery and Interior Laboratory (GRAIL) Primary (1 March to 29 May 2012) and Extended Missions (30 August to 14 December 2012). Fig 20 shows the setting up of the force model in GMAT.

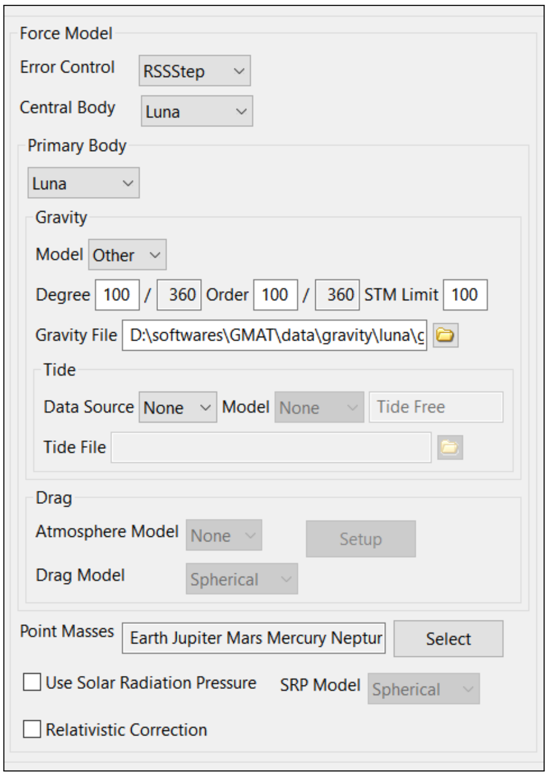


Figure 20: Setting up central body, point masses and spherical harmonics gravity model in the force model

3.9 Setting up initial parameters of the spacecraft

Up until now the coordinate systems, propagator and force model have been set up and a spacecraft's trajectory is ready to be simulated. However before starting any mission sequence in GMAT the physical attributes of the satellite such as mass, orientation, drag area etc along with its initial position and velocity need to be defined. Since this project does not deal with the complete trajectory of the spacecraft all the way from Earth, a certain starting date is required for the simulations. It is assumed that at this starting day the satellite has already been inserted into a lunar orbit and will not perform any maneuvers using its thrusters. During the mission sequence its mass will remain constant and the only forces acting on it will be the ones defined in the force model.

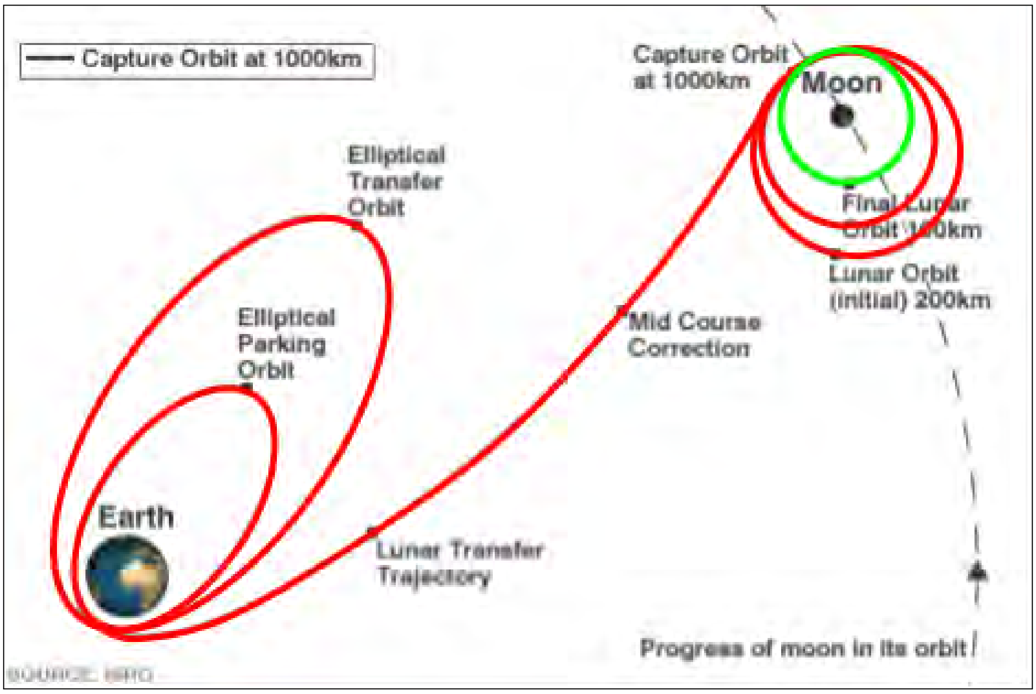


Figure 21: Trajectory of the Chandrayaan I mission. This project only deals with the part of the trajectory shown in green while all the part in red is not considered.

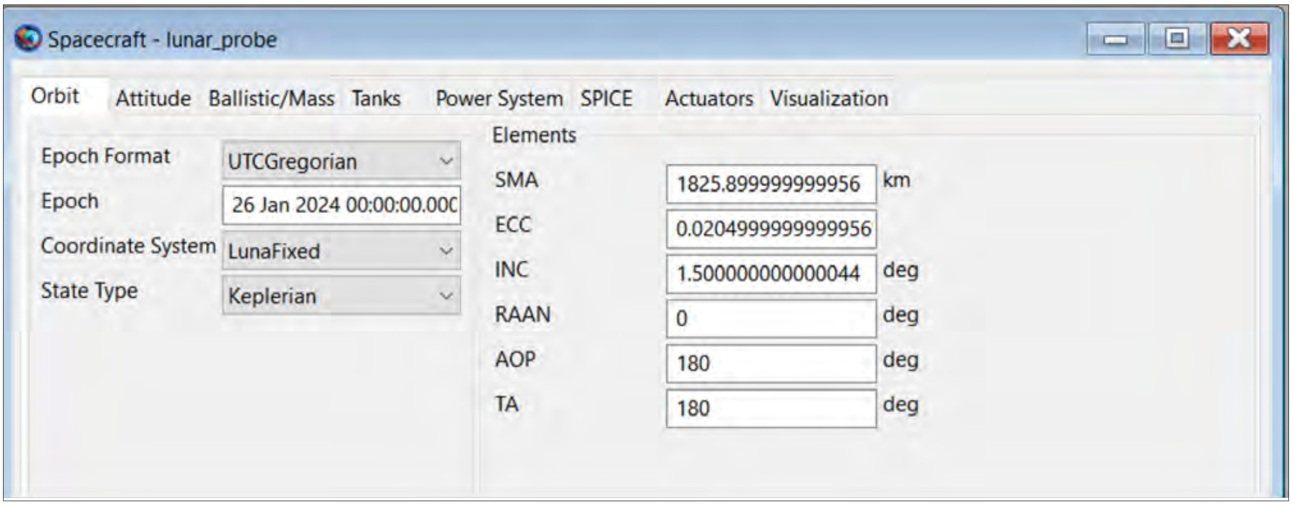


Figure 22: Setting up the initial parameters of the spacecraft in Keplerian state type. The "coordinate system" field and the "state type" field defines the system and notation in which the initial position and velocity for the satellite are provided.

Setting up the initial parameters of the spacecraft in Keplerian state type The "coordinate system" field and the "state type" field defines the system and notation in which the initial position and velocity for the satellite are provided.

After setting the 6 Keplerian parameters, if the "state type" (or "coordinate system") field is changed, the corresponding elements in other notations are computed internally as seen in Fig 23. This inbuilt feature of GMAT to calculate Keplerian elements in multiple coordinate systems will be utilised in the simulations later.

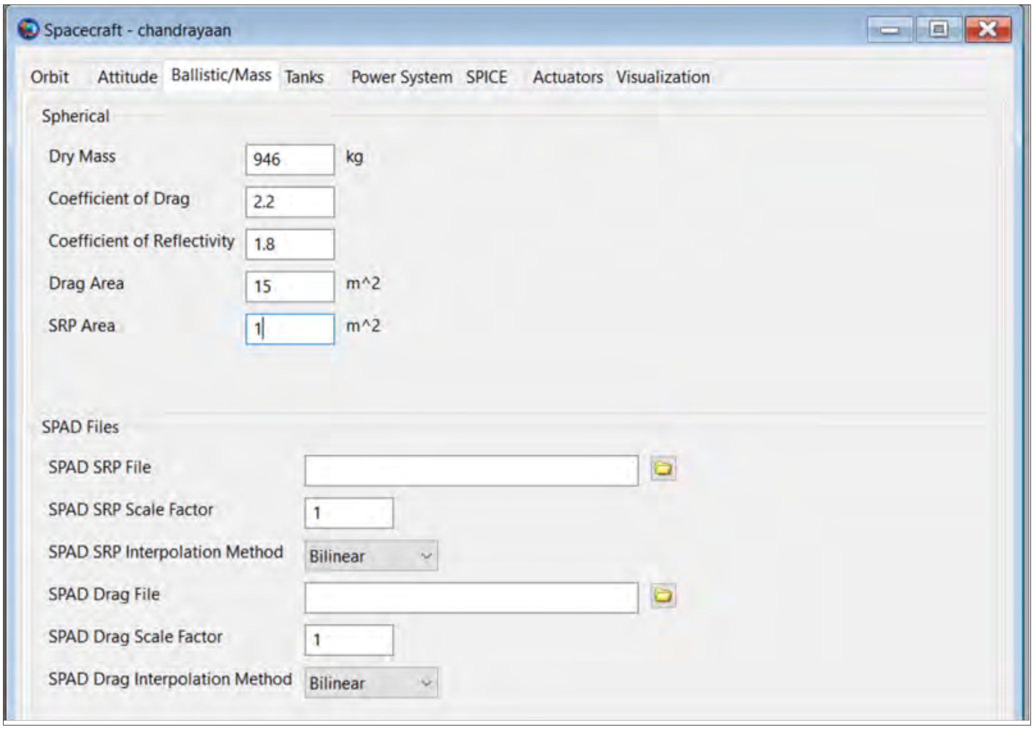


Figure 24: Only the mass field is relevant to this project. Other fields such as drag area and solar radiation pressure (SRP) must be used in any future works where the effects of drag and pressure forces are considered

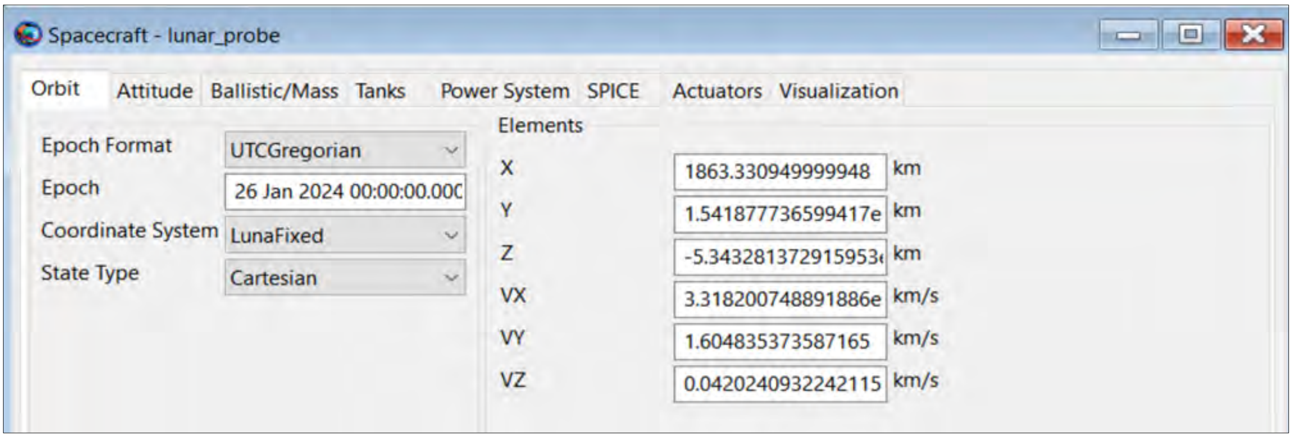


Figure 23: State vector notation for the same initial position and velocity set in Fig 22

The Ballistic/Mass tab shown in Fig 24 can be used to set the mass and drag area of the spacecraft. This project is concerned only with the "dry mass" field and other fields such as "drag area", "SRP area" can be used if atmospheric drag and solar radiation pressure are considered in force models.

3.10 Calculation of the Radio quiet regions on Lunar far side

There exists a certain region of space behind the Moon where the RFI from the Earth is attenuated to such a high extent that this region can be considered virtually free from terrestrial RFI. Given the spherical shape of Earth and Moon it is reasonable to predict that this region of space will be of conical shape with its axis along the line joining the Earth and the Moon. A similar region of space will be formed for the Sun Moon system which will be free from solar RFI. These radio quiet regions of space will be referred to as "Earth quiet cone" and "Sun quiet cone" respectively. Before proceeding any further it is important to characterize these quiet cones formed behind the Moon for the Earth-Moon, Sun-Moon and Earth-Moon-Sun systems.

3.10.1 Earth quiet cone

The geometrical shadow behind the Moon can be imagined as the region of space not in direct line of sight of Earth or its satellites. The geosynchronous orbit is the highest orbit with a significant amount of satellites that can contribute to radio noise. There are a few satellites with higher altitudes like NASA's Aqua and Magnetospheric Multiscale Mission but they should not contribute much radio noise on the Lunar far side. The region of space from where the Moon occults the geosynchronous satellites is defined as the 'quiet cone' in Fig 25 below. As more and more satellites are launched into orbits higher than geosynchronous orbit the radio quiet region behind the Moon shrinks.

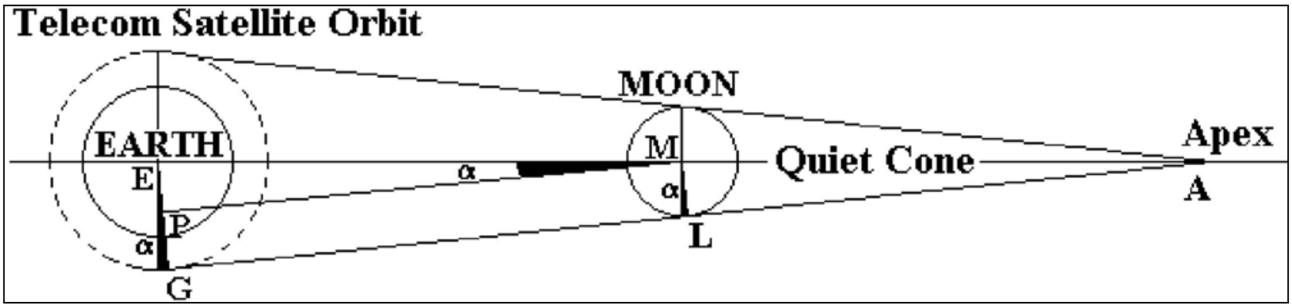


Figure 25: E :center of the Earth
G :point in geosynchronous orbit
L :point of starting of quiet cone on lunar surface
Picture credits : Maccone, Claudio. "The quiet cone above the far side of the moon."

The height of this quiet cone (MA) can be estimated geometrically. Taking altitude of geosynchronous orbits (EG) as 36,000 km and distance between the Earth and Moon (EM) as 384,400 km, radius of the Moon (ML) as 1,734 km the , the value of α (in Fig 3.15) turns out to be approximately 5°. The value of MA turns out to be approximately 19,900 km. So the apex of the quiet cone lies approximately at a height of 18,200 km above the surface of the Moon on lunar far side.

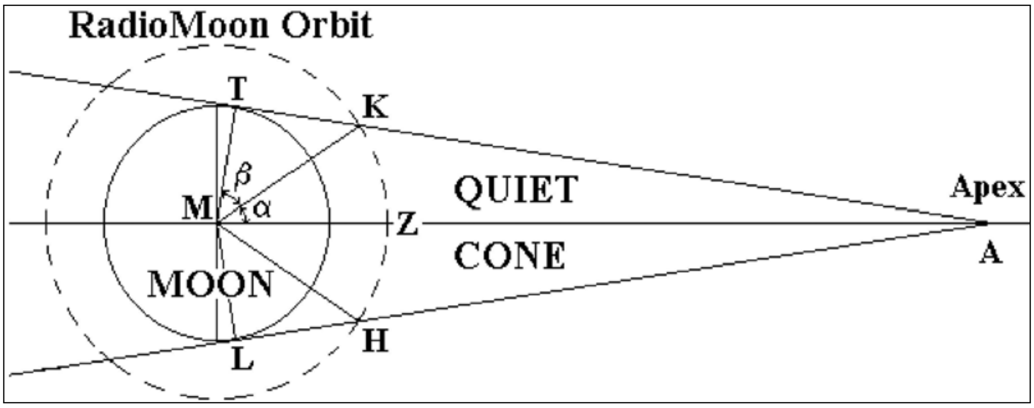


Figure 26: A :Apex of the quiet cone
M :center of the Moon
K and H :points on a satellites orbit when spacecraft enters/leaves the quiet cone
Z :point in satellite's orbit directly opposite to the Earth and in center of the radio quiet cone
T and L :points on Lunar surface where quiet cone starts
α :half of the total angle spacecrafts subtends in quiet cone
β : angle between H and L or T and K
Image credits : Maccone, Claudio. "The quiet cone above the far side of the moon."

From Fig 3.16 it is clear that as the altitude of the spacecraft increases the value of α decreases. This means the satellite spends a lesser amount of time in quiet cone.

This method of estimating the Earth quiet cone did not take into account the diffraction of radio waves around the lunar surface. If we consider the density profile, topography of the Moon and diffraction of radio waves around edges, the radio quiet region will be different from the geometrical shadow region. For this reason only an electrodynamic simulation[18] of radio waves (using finite-difference time-domain method[19]) as it propagates through and around the Moon can give the true extent of the radio quiet region behind the Moon. It is known that radio waves of lower frequencies diffract more than higher frequencies. Since PRATUSH mission will be observing in the frequency band 35-200 MHz, frequencies at 35MHz will give an estimate of the maximum permissible width of the cone relevant to this mission. The extent of Earth quiet cone estimated from this method is used in this project.

Depending upon the frequency and attenuation required a longitude vs height graph is plotted in Earth_X coordinate system. The Earth quiet cone is always centered around the 180° longitude in Earth_X system. Between the two red/blue lines in Fig 3.17 terrestrial RFI is attenuated by at least -90dB.

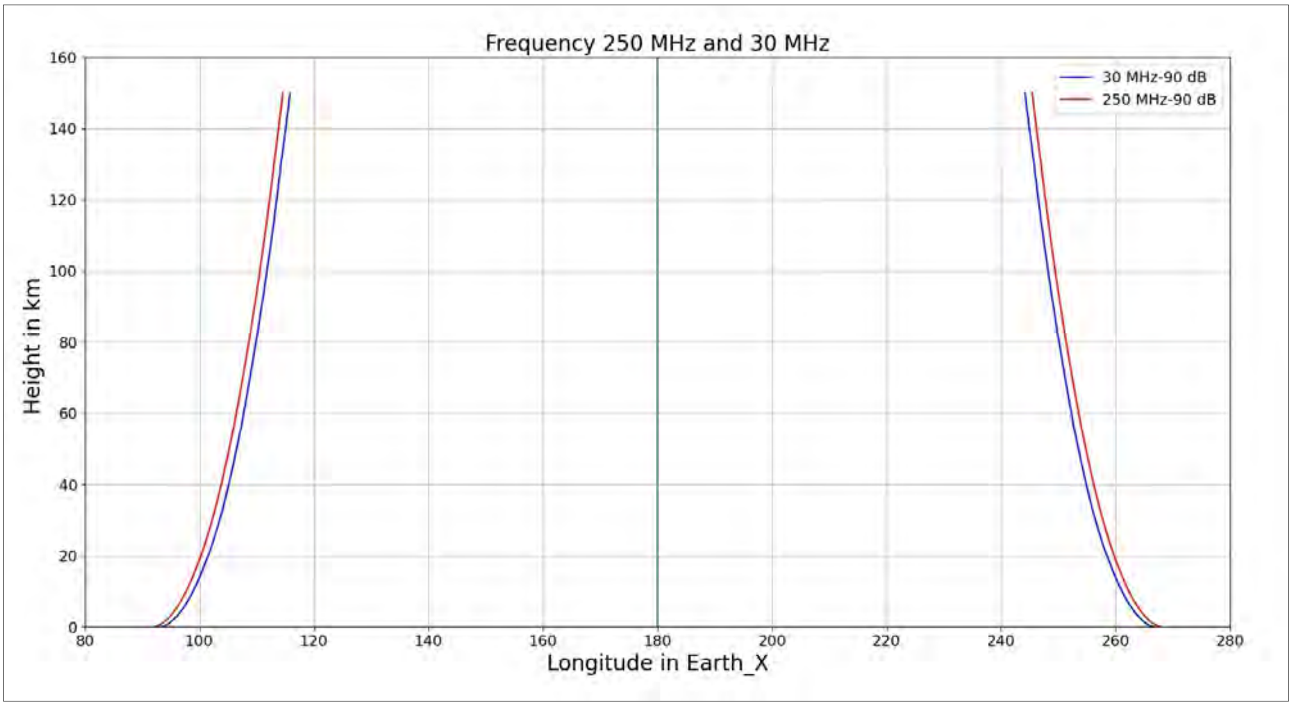


Figure 27: X-axis shows Longitude in Earth_X coordinate system
Y-axis shows height above Lunar surface in km
The red line marks the boundary of radio quiet cone for 250 Mhz
Blue line marks boundary of radio quiet cone for 30 MHz

3.10.2 Sun quiet cone

Since the Sun is approximately 400 times farther from the Moon compared to Earth, the sun quiet cone is expected to extend farther into space. The geometrical Sun shadow region behind the Moon can be estimated similar to the last section using Fig 25 with Sun instead of Earth. The distance of the apex of the quiet cone from the centre of the Moon comes out to be 371,960 km. This is almost equal to the distance between Earth and Moon which is evident during the total Solar eclipses when Sun quiet cone reaches the Earth (at some places).

The height of the apex of quiet cone varies throughout the year as distance and orientation of the Moon with respect to the Sun changes but it is always greater than the apex of Earth shadow region. This cone rotates 360° relative to distant stars in a span of one year.

This project sets 110° as the limit of the Sun quiet cone. Whenever the right ascension (RA) value of the spacecraft is more than 110° (or less than −110°) in the Sun_X coordinate system, it is assumed to be inside the Sun quiet cone.

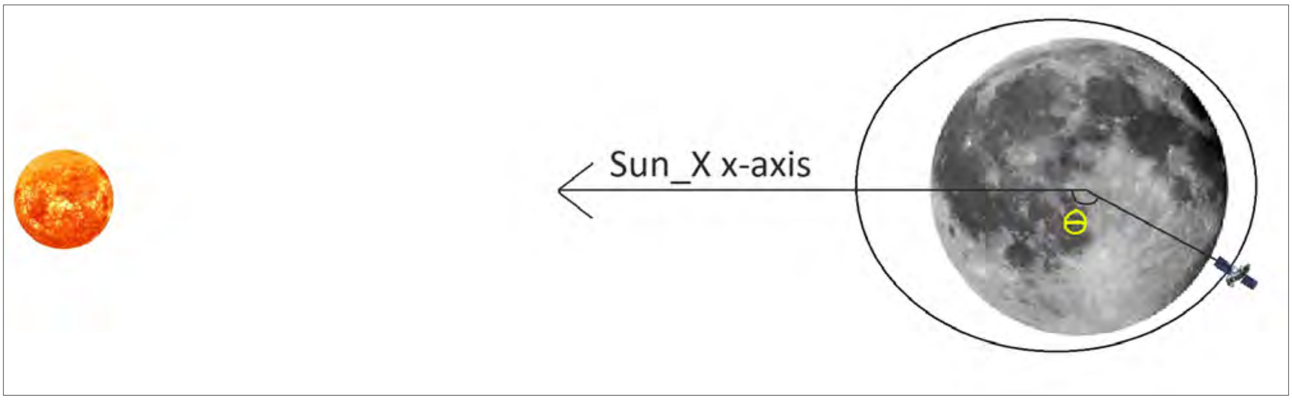


Figure 28: The RA (θ) value of the spacecraft in Sun_X system determines whether it inside the Sun quiet cone or not. For all values of $|\theta| > 110°$ the spacecraft is inside the sun quiet cone

3.10.3 Prime cone region

The intersection of the Earth and Sun shadow regions discussed so far is what will be referred to as the "prime cone region" in this project. It is a region of space free from both terrestrial and solar RFI, ideal for making observations by a mission like PRATUSH or DARE. Since the Earth quiet cone is fixed at lunar far side the prime cone region also forms on lunar far side only. The width of this region is expected to vary depending on the relative position of the Earth-Sun-Moon system. Fig 29 is a 2d representation of the prime cone region along with the Earth and Sun quiet cones.

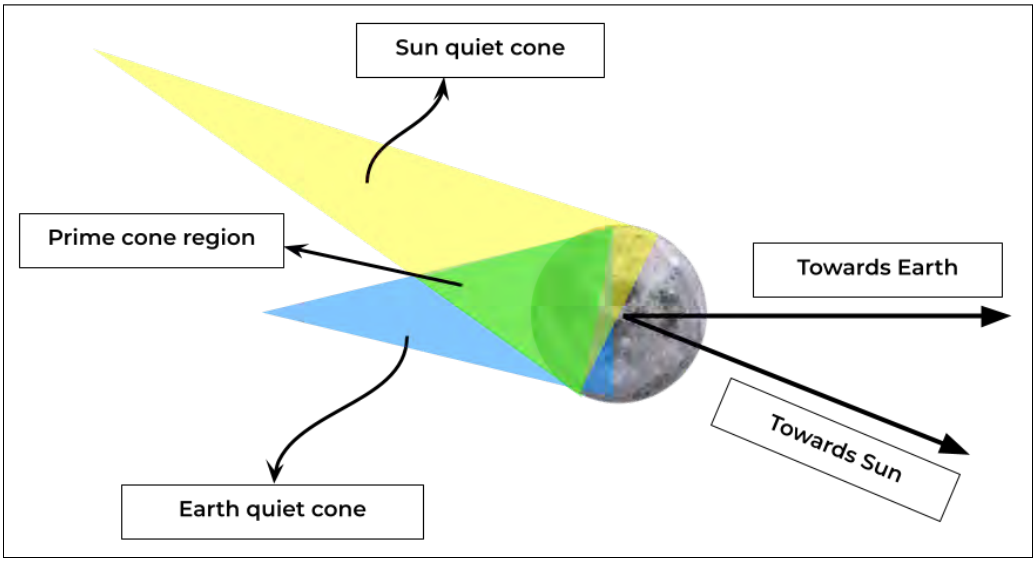


Figure 29: Blue cone represents the Earth quiet cone
Yellow cone represents the Sun quiet cone
Green cone is the prime cone region

3.11 Simulating a satellite's orbit in GMAT

To verify the setup of GMAT and its components discussed above a certain reference case must be simulated and checked for consistency with actual mission. Two such cases reference cases will be studied

1. Chandrayaan 2 mission and

2. DARE (proposed) mission

3.11.1 Chandrayaan 2 mission

India's most recent lunar mission Chandrayaan 2 is an ideal case for getting started with simulating an orbit. A series of maneuvers were performed after the launch on 22 July 2019 to put the spacecraft in the intended lunar orbit. The last and final lunar orbit maneuver was performed at 12:51 UTC, 1 september 2019 while the spacecraft was in lunar orbit. This maneuver lasted 52 seconds and put the spacecraft (orbiter and Vikram lander) in a 119.5 km $\times$ 127.1 km polar orbit[20]. The Vikram lander separated from the orbiter at 07:45 UTC and performed 2 de-orbiting maneuvers in the next two days in preparation for its landing on the lunar south pole.

This project will only replicate the orbiter's trajectory post final orbiter maneuver and take only the mass of the orbiter into consideration. The spacecraft simulated for 690 days is expected to show long term stability just like the real life mission. The inbuilt GMAT feature of generating XY plot is not used since it increases computational load on the computer. Instead a (.csv) report file is generated which can be accessed after the simulation is complete. After the simulation is complete, a python script is used to plot values from this report file.

A body fixed inertial coordinate named Moon_MJ200 system is set up (Fig 9) to provide the initial Keplerian parameters to the spacecraft. The origin of this system is at the Moon and the axes are set according to the Earth direction and orbital plane of the Moon at J2000. epoch[2]

The initial parameters of the spacecraft are set in Fig 30 where the "epoch" field is set at 12:52 UTC, 1 Sep 2019 and the Keplerian parameters set as discussed above. The true anomaly is kept 0° since its value in the real mission could not be found.

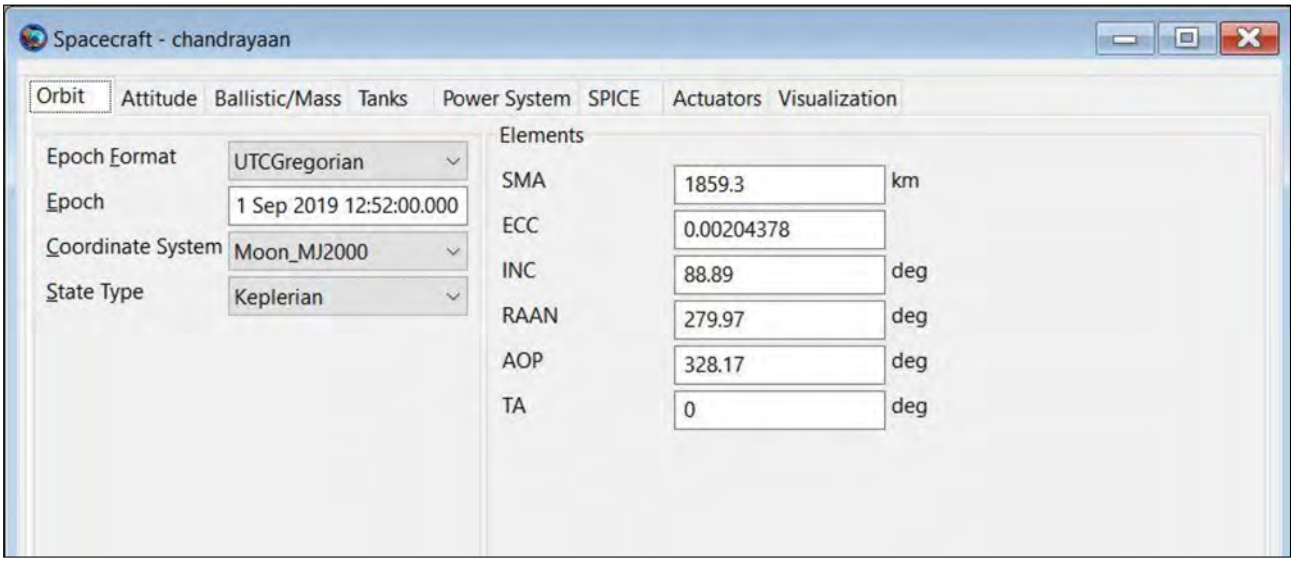
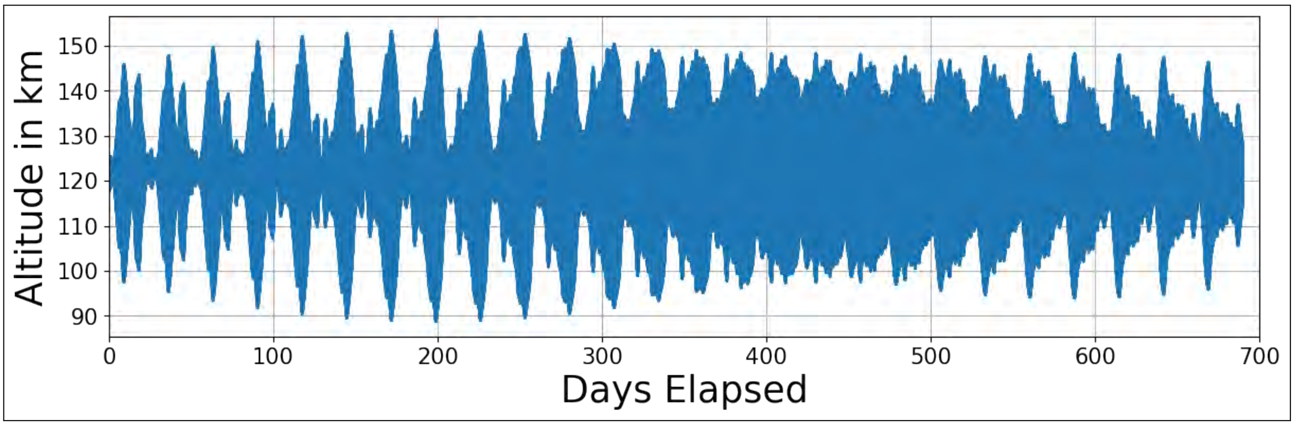


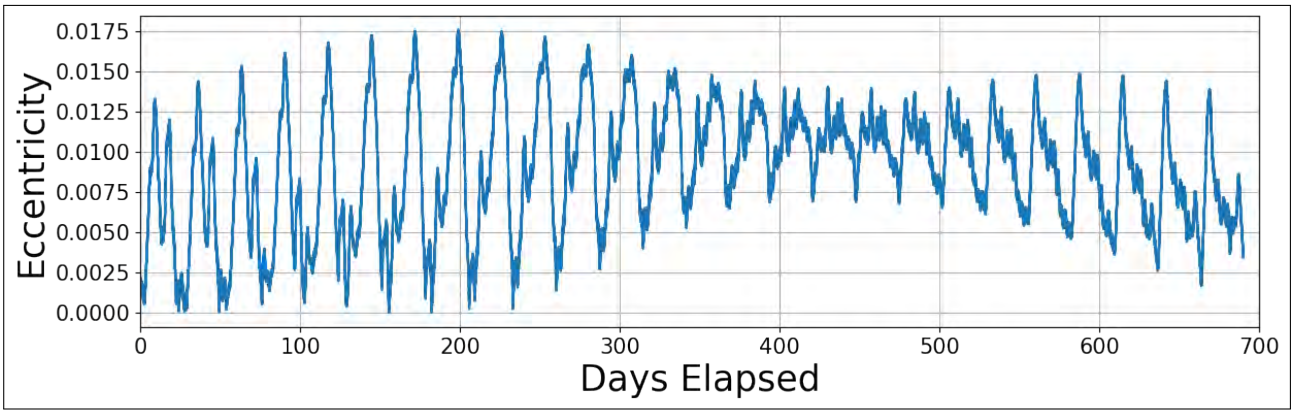
Figure 30: Setting up the initial parameters of the spacecraft similar to real mission

From the report file generated, plots of the altitude, eccentricity, Semi-major axis length and RAAN were plotted. It is seen that all these parameters showed regular pattern suggesting long term stability of the orbit. The altitude of perilune falls to approximately 90 km near 200 days but increases afterwards. Near the end it again starts to decrease suggesting a periodic trend.

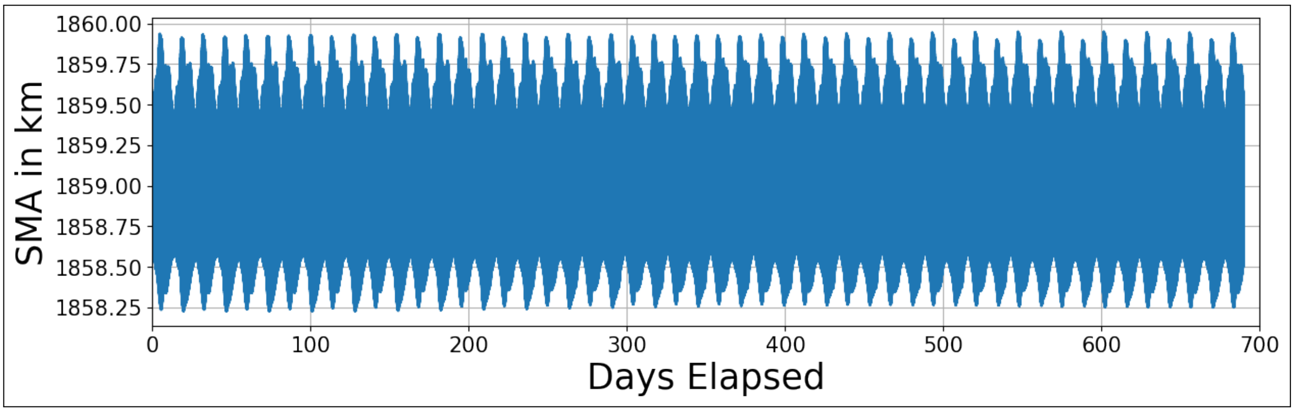
[2]More details can be found in GMAT documentations.



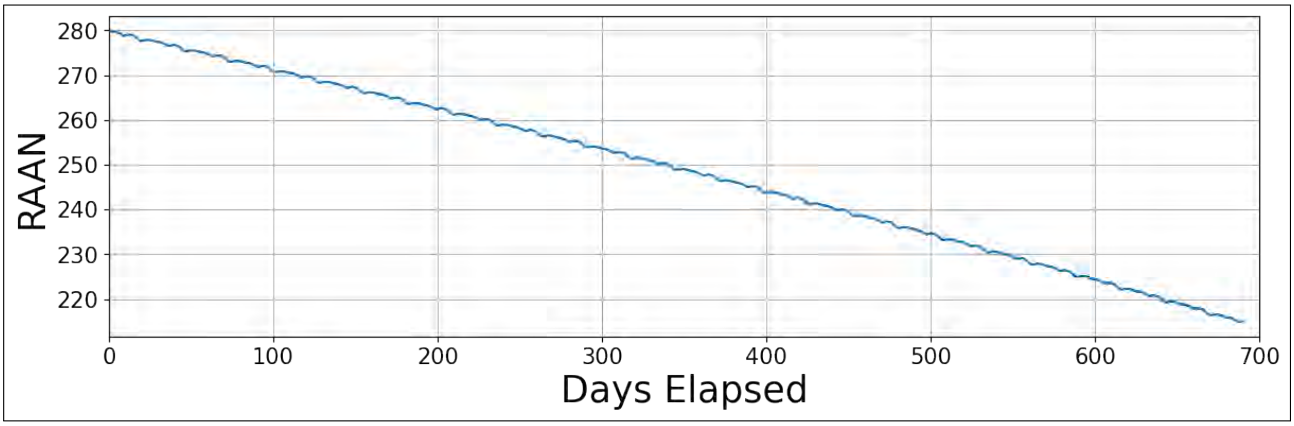
(a) Altitude vs Days



(b) Eccentricity vs Days

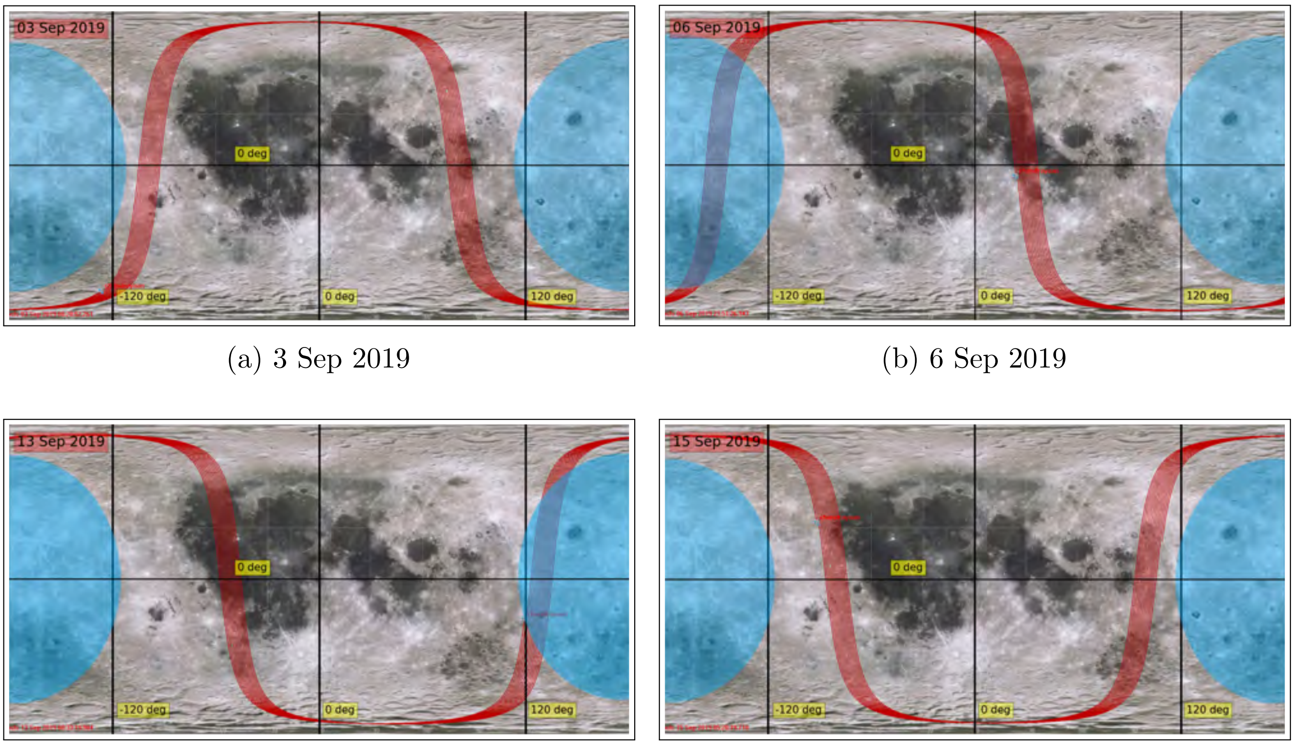


(c) SMA vs Days



(d) RAAN vs Days

The ground track plots centered at 0° longitude in Fig 32 reveal that the orbit appears to shift with respect to the lunar surface. The blue area marks the approximate boundary of the Earth shadow region on the lunar far side. The width of this region is maximum near the equator and decreases as one moves towards the lunar poles. On many days like 03 Sep 2019 (Fig 32a) and 15 Sep 2019 (Fig 32d) the satellite will be outside the Earth quiet cone throughout its orbit. This makes polar orbits unsuitable for a mission like PRATUSH.



(a) 3 Sep 2019

(b) 6 Sep 2019

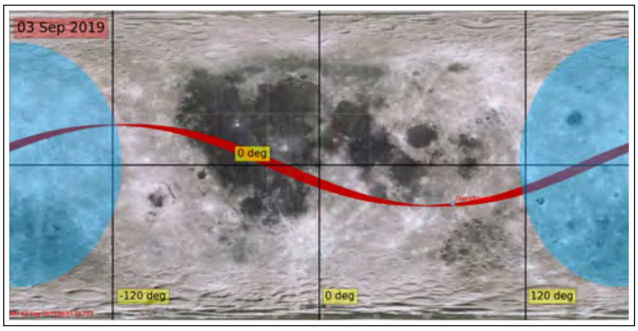
(c) 13 Sep 2019

(d) 15 Sep 2019

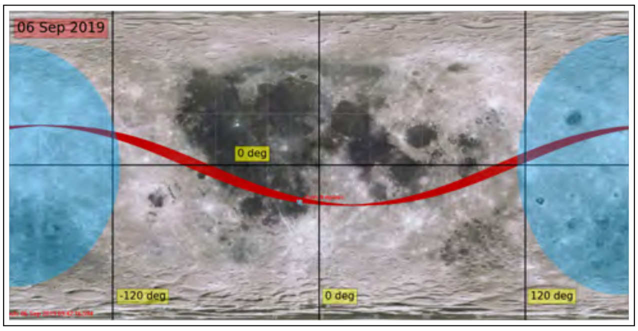
Figure 32: Groundtrack plots of the spacecraft

3.11.2 Chandrayaan 2 mission with near equatorial inclination

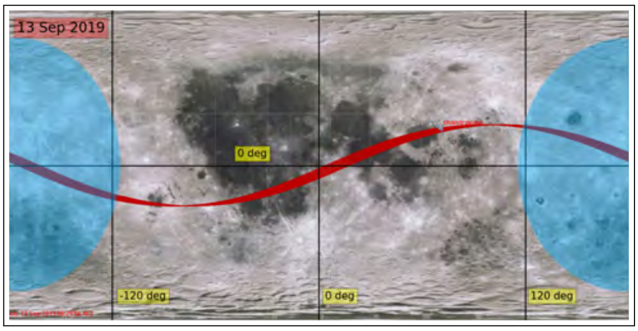
A spacecraft with all the initial parameters same as Fig 30 but with inclination 0° is propagated. The ground track plot reveals an ideal orbit in which a satellite passes through the Earth quiet region during every orbit. Such an orbit is ideal for a mission like PRATUSH. The axes of the Moon_MJ2000 system are by definition different from the body fixed system and time independent. This is the reason that orbits with inclination $i = 0°$ in Moon_MJ2000 do not coincide with the 0° latitude in Fig 33



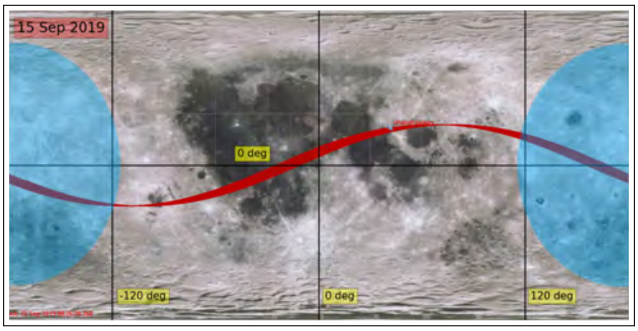
(a) 3 Sep 2019



(b) 6 Sep 2019



(c) 13 Sep 2019



(d) 15 Sep 2019

Figure 33: Groundtrack plots of the spacecraft if inclination was 0°

3.11.3 Stable orbits

Not all low altitude lunar orbits show long term stability due to the presence of mass concentrations (mascons) present mainly in lunar basins. Every time a low altitude satellite passes over these mascons it experiences a force which tugs it away from its ideal path. Over time these perturbations accumulate and without station keeping maneuvers the satellite impacts the surface. Satellites in low altitude orbits around the Moon require frequent orbital station keeping maneuvers to prevent them from crashing. NASA's Lunar prospector mission in 1998 had to perform monthly station keeping maneuvers during its mission to map the entire terrain of the moon.

During its primary mission period of one year it stayed at an altitude of 100km and performed maneuvers once a month to stay at the intended altitude. After this the spacecraft's altitude was lowered to extended mission periods in low altitude (100km and 30km) orbits. In the end it was deliberately crashed on lunar south pole to search for evidence of water. The aim of the Lunar Prospector Doppler Gravity Experiment (DGE) was to learn about the mass distribution of the Moon. The data from the primary and extended mission sequence was used to generate gravity models of the moon such as the LP165.

An orbit which shows long term stability with minimal or no orbital keeping maneuvers is termed as frozen orbit. In such an orbit the initial parameters are chosen in such a way that changes in eccentricity, inclinations and longitude of perigee are minimized. Polar orbits are generally stable and utilised in missions such as Chandrayaan 1 and 2. As the altitude of a spacecraft decreases the effect of drag forces due to Moon's exosphere, non-uniform gravitational forces of mascons increases and below a certain altitude it is bound to crash in absence of station keeping maneuvers. This minimum altitude depends on a lot a factors and this project sets 20 km as the minimum altitude criteria for an orbit to be termed "stable".

The near equatorial orbit discussed in the previous section guarantees more time in earth quiet cone compared to polar orbits. However whether such an equatorial orbit will show long

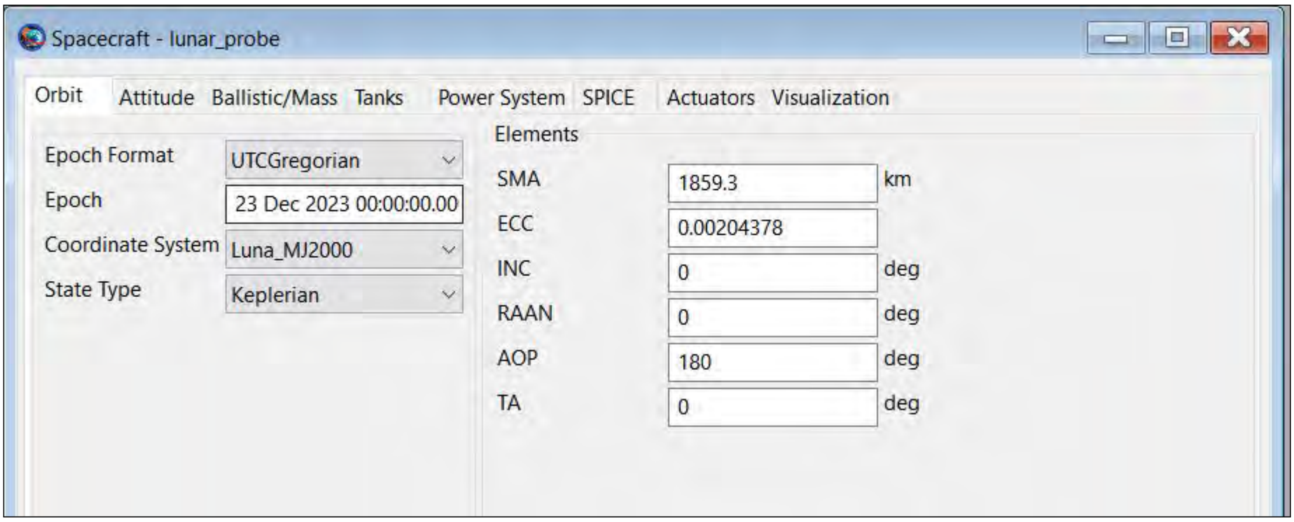


Figure 35: Initial parameters for the spacecraft during first run.
In subsequent runs only the inclination value is changed by 2.5°
RAAN and AOP values set such that initially the perilune is at the lunar far side

term stability like polar orbits is unknown.

The altitude vs time plot for this orbit in Fig 34 shows that the altitude of the perilune falls to as much as 37 km but never below 20 km. So the equatorial orbit discussed above showed promising results both in terms of time spent in quiet cones as well as long term stability. However since the forces of drag and solar radiation pressure were not considered, the perilune may fall below 37 km in a real life mission.

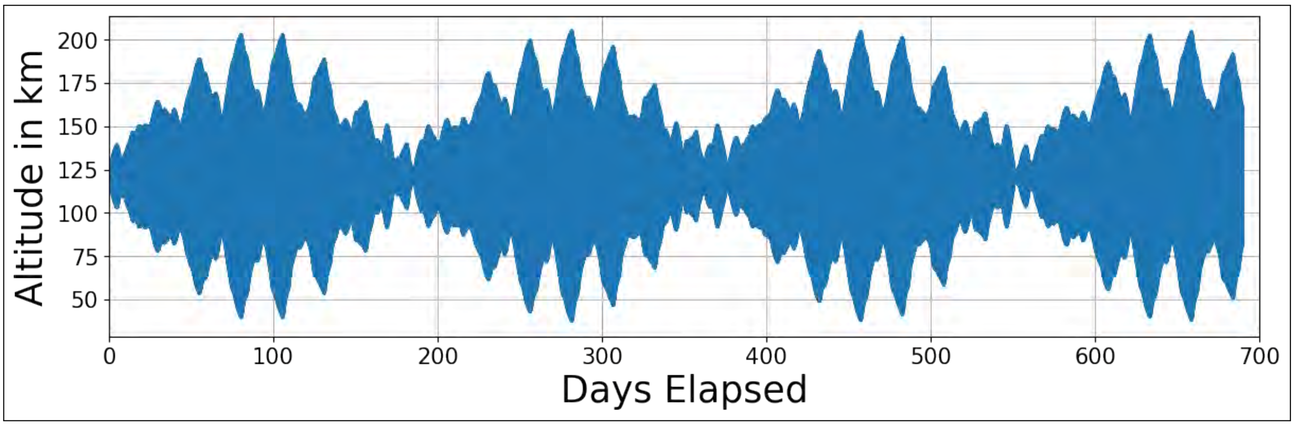


Figure 34: Altitude vs number of days elapsed

3.11.4 Varying the initial inclination

For the next step the effect of initial inclination on orbital stability was investigated by simulating a satellite with different initial inclinations. From real life missions like Chandrayaan 2 it is expected for near circular, near polar orbits to show long term stability.

The initial parameters set in simulations in Fig 35 are similar to the Chandrayaan 2 mission with only the inclination, RAAN and AOP values changed.

The satellite was set with an initial set of Keplerian parameters at a certain epoch and allowed to propagate for 60 days or until the altitude of the perilune dropped below 20 km. The initial inclination values varied from 0° to 90° in steps of 2.5° keeping other 6 parameters fixed.

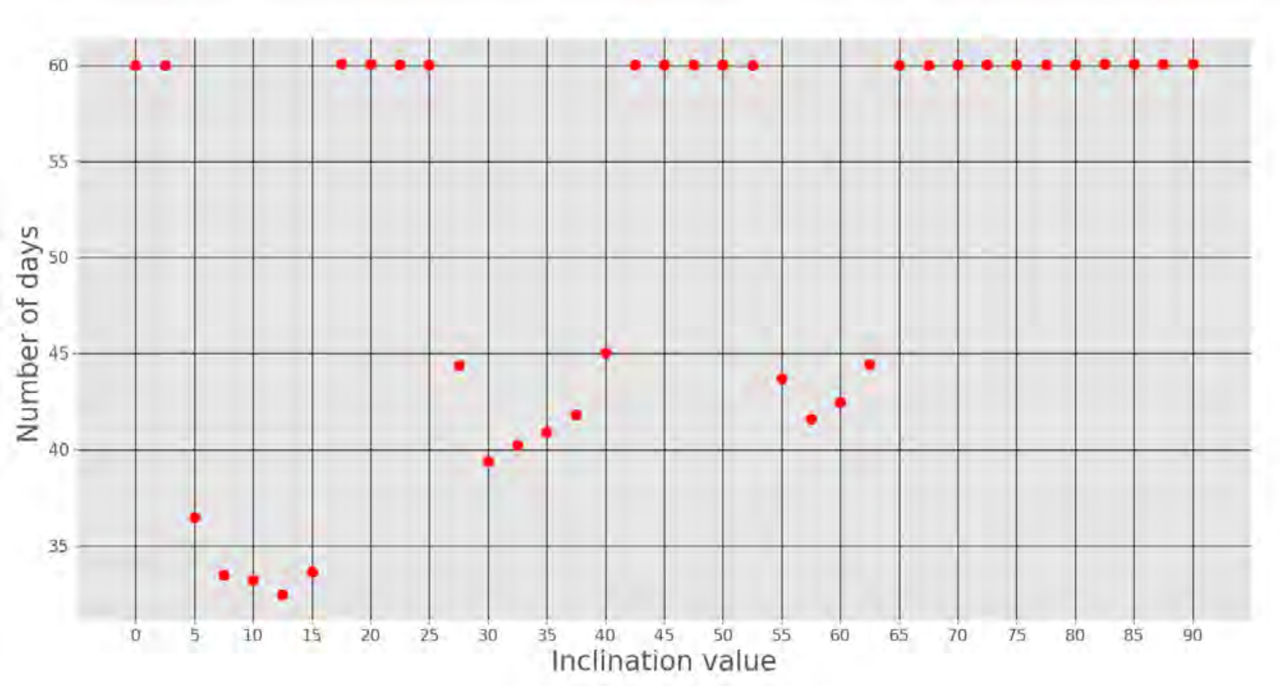


Figure 36: A stable band of inclinations seen at $0° - 2.5°$, $17.5° - 25°$, $42.5° - 52.5°$ and $65° - 90°$. For other inclinations the altitude of the spacecraft decayed below 20 km in less than 45 days.

Even at inclinations that do not show long term stability in Fig 36, a stable orbit can be achieved by varying the SMA and eccentricity values. The task of finding a low altitude stable lunar orbit is all about finding the right combination of these 6 (7 if epoch is considered) Keplerian orbital parameters. Clearly, varying all the 6 parameters through all possible values is not computationally possible and a reference case is required. Since the DARE mission is the only mission with mission objectives similar to PRATUSH. The work done in proposing an orbit for the DARE mission acts as a cornerstone for simulations in this project.

The following papers discuss the DARE orbit in detail and will frequently be referred to in this project.

1. "Genova, Anthony L., et al. "Trajectory design from GTO to lunar equatorial orbit for the dark ages radio explorer (DARE) spacecraft." arXiv preprint arXiv:1504.00410 (2015)" which will henceforth be referred to as "DARE 2015"[21].

2. "Plice, Laura, Ken Galal, and Jack O. Burns. "DARE Mission Design: Low RFI Observations from a Low-Altitude Frozen Lunar Orbit." arXiv preprint arXiv:1702.00286 (2017)" will henceforth be referred to as "DARE 2017"[22].

3.12 Choosing initial parameters for simulating DARE mission

Since the trajectory for the DARE mission has been planned all the way from Earth in DARE 2017, simulating its whole trajectory is not necessary for this project. This project intends to study the DARE orbit only after it has been inserted in a lunar orbit. The following approach is taken to simulate the DARE orbit in this project

1. Select a date after all the maneuvers (Hohman transfer, Lunar orbit acquisition, inclination change maneuvers) in DARE mission have been performed and the probe has entered a lunar orbit.

2. At this date (22 December 2023 in my case) set the initial parameters as defined in the table below with spacecraft at near side of the moon

The table 1 summarises the findings reported in DARE 2017 in terms of suitable orbits around the Moon. This project replicates this set of parameters in all of the simulations. The DARE baseline columns state the ideal initial parameters which have been used in the forthcoming simulations. The other two columns give an estimate of the error margin allowed in achieving the initial parameters during real life lunar orbital insertion maneuvers.

Parameters	**DARE baseline**	**Wider constraints**	**Tighter constraints**
Semi-major axis length (km)	1825.9	1805.4 - 1835.9	1820.9 - 1835.9
Eccentricity	0.0205	0.0151 - 0.0342	0.0178 - 0.0205
Inclination	1.5°	0.5° - 3.5°	0.5° - 1.5°
Longitude of Periapsis	180°	150° - 210°	180°
Initial Longitude of Ascending Node	180°	0° - 360°	210°

Table 1: Inital parameters taken from DARE 2017

3.13 Verification of the parameters stated in Table 1

Since all the simulations in this project utilise the initial parameters stated in "DARE baseline" column in Table 1 it is important to independently verify the results stated in DARE 2017 with the GMAT setup used for Chandrayaan 2 mission in previous section.

This project inspects only the inclination and RAAN values while other parameters from DARE 2017 can be verified in any future works.

3.13.1 Verification the initial RAAN and inclination values of DARE mission

Since changing the inclination of a satellite's orbit is one of the most expensive maneuvers in orbital design, only the inclination value was inspected in this project. A range of initial inclination values were checked for stability with a full range of initial RAAN values. The RAAN value of the orbit keeps on changing with time due to regression of nodes.

In Fig 37 the x-axis shows initial RAAN values chosen, from 0° - 360° in steps of 100°.

Y-axis initial inclination values chosen from 0°-5° in steps of 0.5°. Z-axis shows number of days s/c propagates before its altitude falls below 20km (60 days chosen as upper limit due to computational restraints)

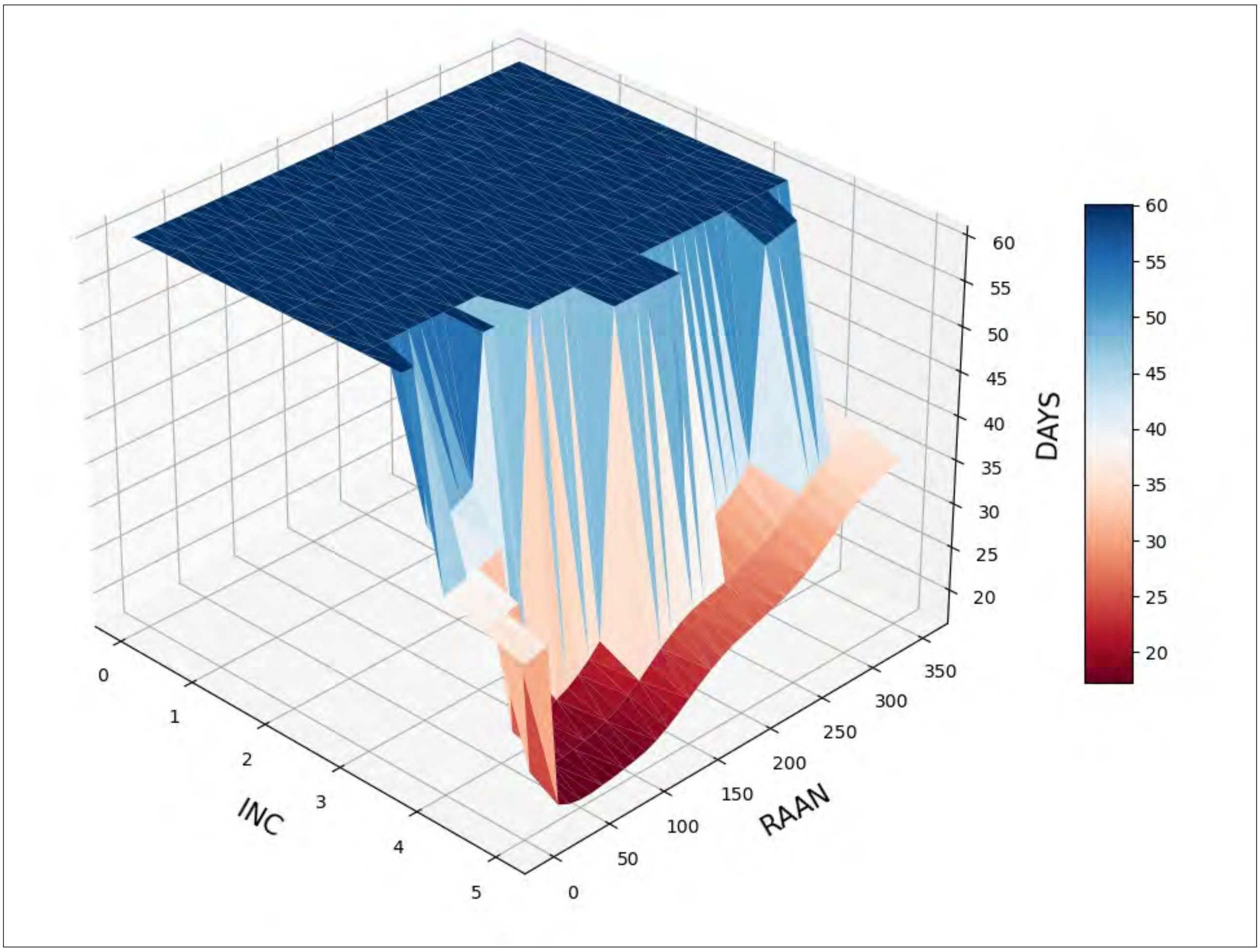


Figure 37: Orbital stabilty in terms of initial inclination and RAAN

In conclusion, the near circular, near equatorial orbits with any value of initial RAAN value show promising results in terms of orbital stability and can be studied further. Satellites in orbits with inclinations higher than 3.5° quickly fall below 20 km and are hence rejected.

3.13.2 Verification of orbital stability of DARE mission

A spacecraft is set with initial parameters (Fig 38) as stated in "DARE baseline" in table 1. Since the DARE orbit is expected to show stability for all possible RAAN values, initial RAAN was set to be 0°.

The satellite is initially set at perilune situated at the far side and propagated in steps of 10° (in LunaFixed system) for 730 days. After every step the coordinates (RA and DEC value) of the satellite in other systems (Eart_hX and Sun_X), altitude, number of days elapsed and other relevant information are stored in a .csv file. After the simulation is complete the time evolution of different parameters are plotted using a Python in jupyter notebook.

Fig 39 shows the time evolution of a satellite's altitude for a duration of two years. It is clear that the orbit shows long term stability without the need for station keeping maneuvering. In the zoomed in section of the plot in Fig 40 we see the altitude of the spacecraft goes from a maximum at apolune to a minimum at perilune during each orbit and the the altitude of the spacecraft never falls below 40km.

The longitude of perilune is defined as the sum of a spacecraft's RAAN and AOP. The spacecraft makes closest approach to the Moon at perilune so the longitude of perilune should ideally be at the center of lunar far side (180° in LunaFixed system) to ensure that it spends

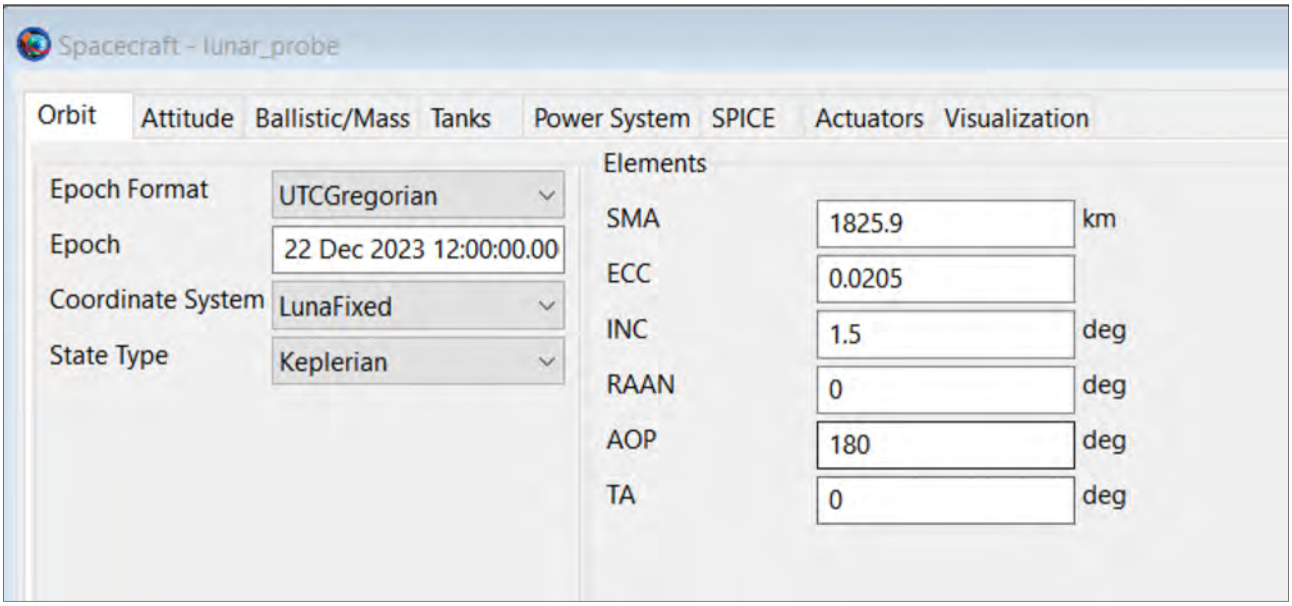


Figure 38: Setting up the initial parameters of the spacecraft similar to DARE.

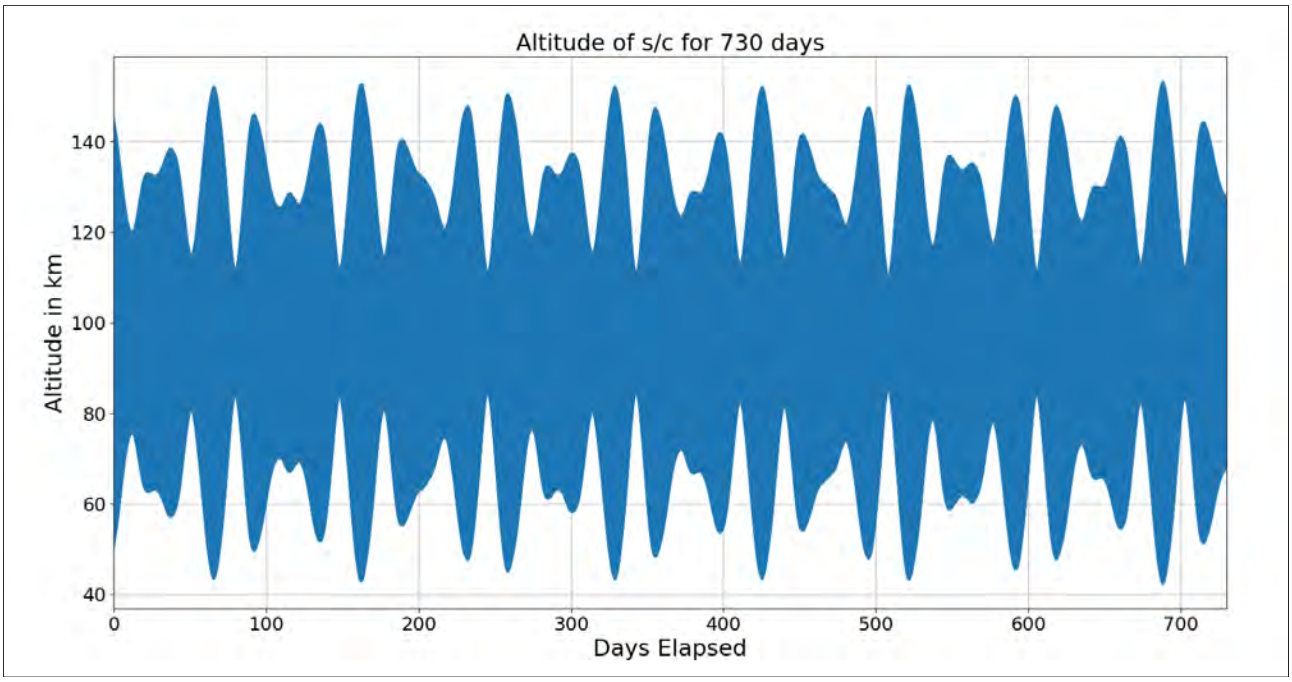


Figure 39: Altitude of the spacecraft for 730 days

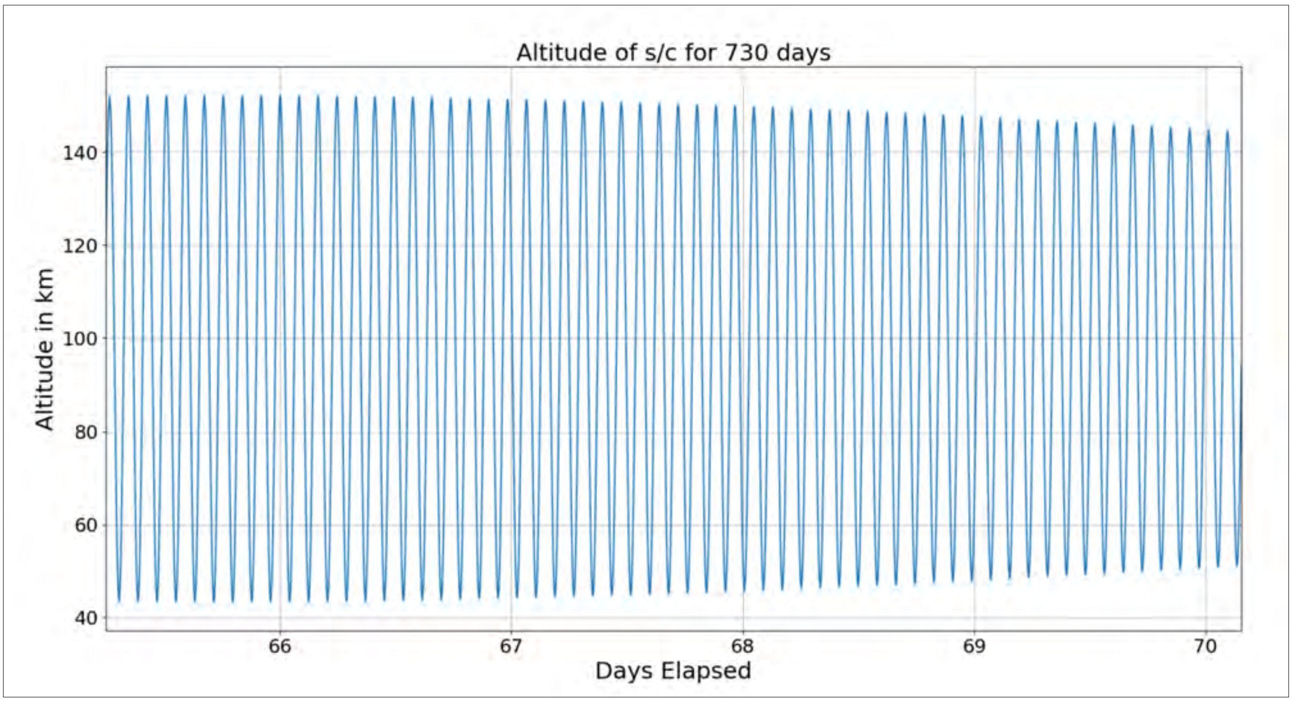


Figure 40: A zoomed in section of Fig 39 showing the altitude of the spacecraft as it goes from apolune to perilune to apolune again during each orbit

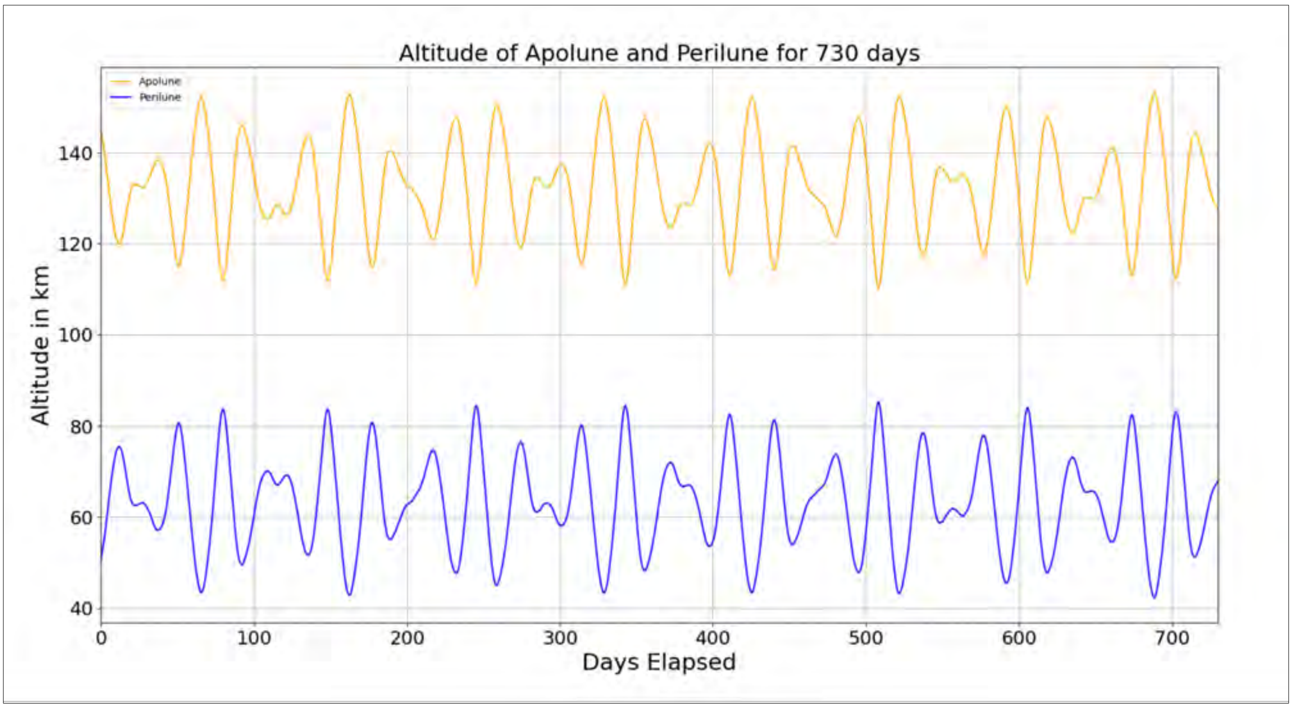


Figure 41: Altitude of the apogee and perigee of the spacecraft for 730 days

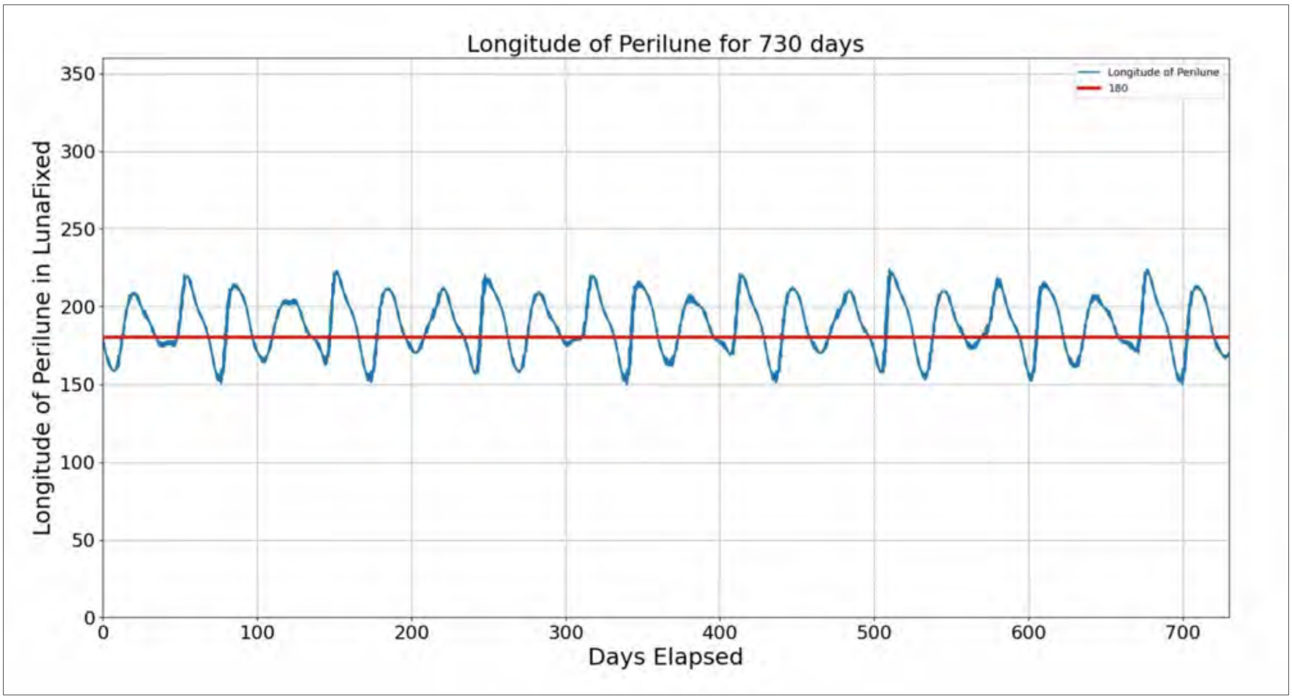


Figure 42: Longitude of the perilune of the spacecraft for 730 days

maximum time in radio quiet region. The longitude of perilune initially set at 180° in Fig 38 and during the course of 730 days it fluctuates to a maximum of 224° and a minimum of 149°.

Time evolution of altitude of spacecraft in LunaFixed system shows the following features of the orbit

1. Long term stability and the altitude of spacecraft never goes below 40 km
2. The spacecraft makes the closest approach to the Moon at lunar far side in all of the orbits. This will ensure maximum time spent in radio quiet regions.

3.14 Conclusions from simulating DARE mission orbit

In the last section the DARE mission was used as a reference case to verify the GMAT setup. The results achieved from these simulations were similar to ones stated in DARE 2017. The following similarities were observed

1. The orbit showed long term stability without requiring any station keeping maneuvers.
2. The longitude of apolune remains fixed at lunar far side near 180°
3. The orbit showed stability even when initial inclination and RAAN values were varied

However any future works trying to simulate the DARE mission orbit should be aware of the following

1. DARE 2017 utilised the GRAIL660 gravity model for its simulations while this project used a much coarser GRAIL100 model. The choice of lunar gravity model plays a significant role since the altitude of the spacecraft evolves during every orbit. Furthermore, this project did not account for the the solar radiation pressure and lunar atmosphere in any of its simulations.

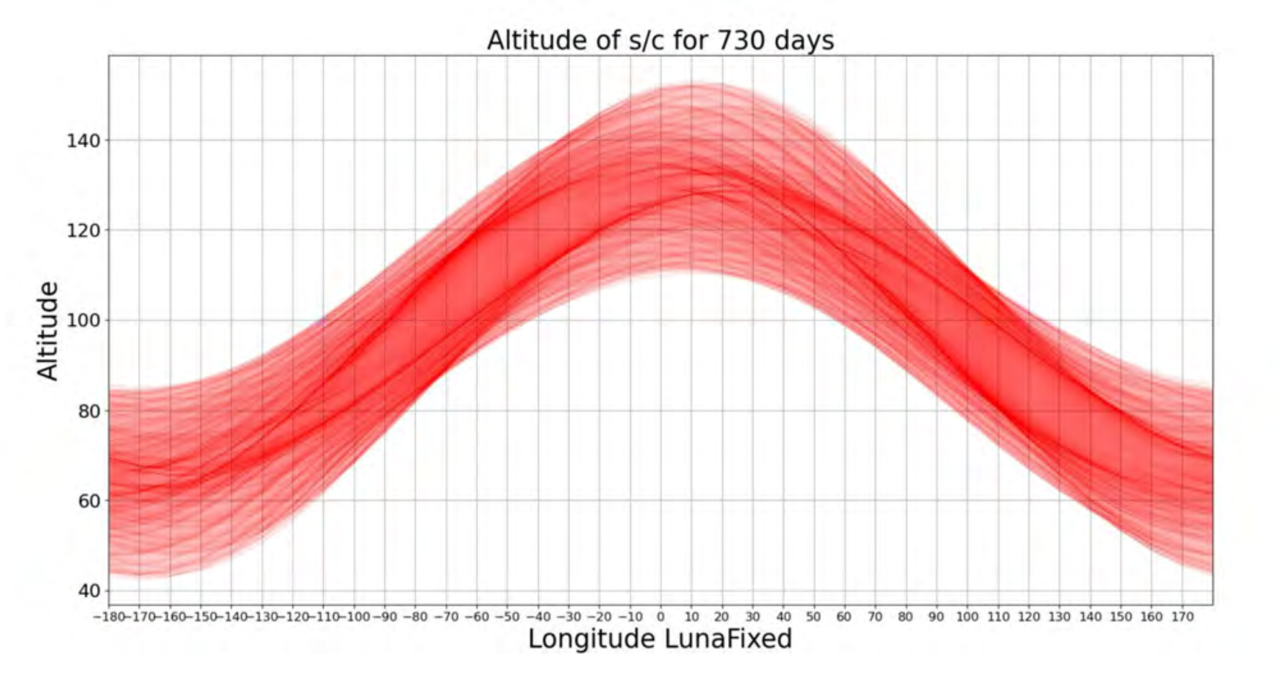


Figure 43: Altitude of the spacecraft in LunaFixed system

2. A non-inertial, moon fixed coordinate system was used in setting up the initial parameters due to the ease of setting the perilune of the spacecraft at far side. The coordinates of the far side in an inertial coordinate system are time dependent. Since lunar missions are generally planned in inertial coordinates systems, any future works should use the MJ2000 coordinate system or other suitable inertial system only.

The next chapter will be all about estimating the time spent in Earth cone, Sun cone and prime cone regions by a spacecraft.

4 Chapter 4: Time spent in radio quiet regions

All the simulations henceforth will use the following initial Keplerian parameters set in the LunaFixed coordinate system with a varying epoch.

Eccentricity	SMA	Inclination	RAAN	AOP	TA
0.0205	1825.9 km	1.5°	0°	180°	180°

Table 2: Initial parameters for simulations in LunaFixed system

4.1 Methodology for finding time spent in radio quiet regions

After the spacecraft has been set with the initial parameters stated in table 2 at a suitable epoch, it is propagated 2° at a time in the LunaFixed coordinate system. At each step the RA and DEC value of the spacecraft in all three coordinate systems are stored in a .csv file. The extent of the Earth quiet cone as a function of height and longitude in the Earth_X system is known. From the RA (in Earth_X system) and altitude value of the spacecraft it can estimated whether a spacecraft lies inside the earth quiet cone or not and the amount of time spent it spends in it. A similar approach is utilised for estimating whether the spacecraft is in sun quiet cone and the time it spends in it.

During each orbit around the Moon the spacecraft spends anywhere between 0 to around 44 minutes inside the prime cone region. However, each such pass through the prime cone region may not be suitable for observations since some time is required for readying the antenna. If this time is greater than time spent in the prime cone region during that orbit then no observation can be made. Since this time is dependent on hardware specification of the payload and is not yet defined for PRATUSH mission, a rough estimate (30 minutes) was used. It may be due to this approximation that the maximum observation time in prime cone region suitable for observation estimated from simulation in this project (around 44 minutes) is different from the one quoted in DARE 2017 (23 minutes). The time spent in the prime cone region is expected to be maximum on a full moon and due to the repetition in orientation of the Earth-Sun-Moon system during each lunar month it also expected to show a repetitive trend.

In all the calculations, the DEC value of the spacecraft in Earth_X or Sun_X system has been neglected when determining whether the spacecraft is radio quiet regions of Earth and Sun. This is because for a low altitude, near equatorial lunar orbit the DEC value in either system does not increase by a significant amount and can be ignored. However when working with higher inclinations both RA and DEC values should be considered.

The altitude vs longitude of spacecraft plots in the proceeding sections use the following notations to distinguish when the spacecraft is in the Earth/Sun quiet cone and prime cone regions.

1. The x-axis shows the RA value of the spacecraft in Earth_X system. The y-axis shows the altitude of the spacecraft in km.

2. Green line marks the boundary of Earth quiet cone region estimated from simulations discussed earlier in "Chapter 3: Earth quiet cone"

3. Bigger marker size indicates positions where the spacecraft is in Sun quiet cone.

4. Blue markers indicate the positions where the spacecraft is in the prime cone region.

In the plots which show the time spent by the spacecraft in the prime cone region, the following notations have been used.

1. The x-axis shows the index number of the orbit (approximately 12 orbits in a day). The y-axis shows the time spent inside the prime cone region in minutes.

2. The time spent in minutes is written in blue above every data point.

3. Similarly the width of the prime cone region is written in red below every data point. The width of prime cone is the difference of the longitude of the spacecraft in LunaFixed system when it enters and exits the prime cone region during any pass.

4.2 Simulating the spacecraft on a Full Moon for one day

A full moon which occurs once during every lunar month presents the ideal conditions for observation as the width of the prime cone region and the time spent in it by the spacecraft is maximum. Assuming that the spacecraft has a mission duration of two years and it performs all the lunar orbit insertion and lunar bound maneuvers prior to 22 December 2023, its trajectory for the next two years can be simulated. Since the orbital period of the spacecraft is approximately 2 hours it will orbit the moon 12 times on a full Moon. Five cases of lunar eclipses (2 total, 1 partial and 2 almost lunar eclipses) were investigated further.

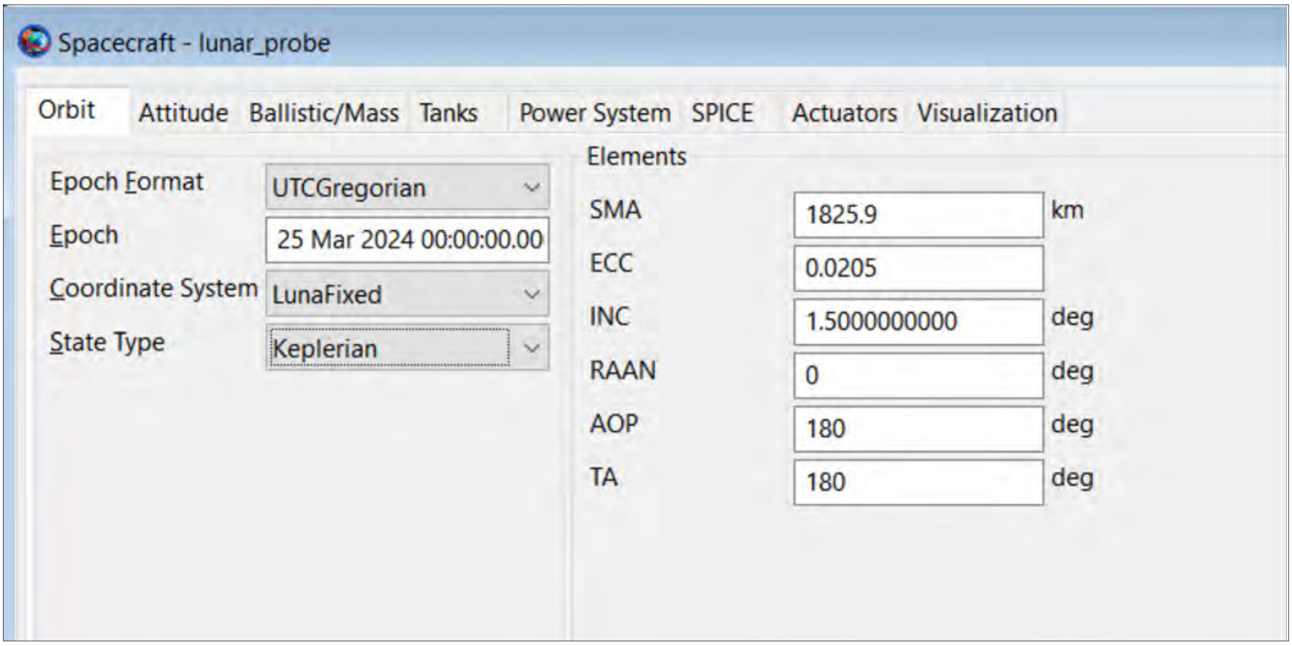


Figure 44: Initial parameters set for the first case. In next cases only the date of epoch is changed

Fig 44 shows the initial Keplerian parameters set for simulating the spacecraft on 25 March 2024. The spacecraft is set initially at the near side as opposed to previous simulations so that the time spent in prime cone during all 12 orbits made around the Moon on 25th March can be estimated. For other full moons/eclipses, similar initial parameters are used and only the date in the epoch field is changed.

4.2.1 25 January 2024

Since the full Moon occurs on 25 Jan 2024 17:54 UTC, the time spent in prime cone is expected to be maximum towards the end of the simulation. From Fig 45 it is seen that the spacecraft spends a maximum of approximately 43 minutes towards the end of 25 January 2024.

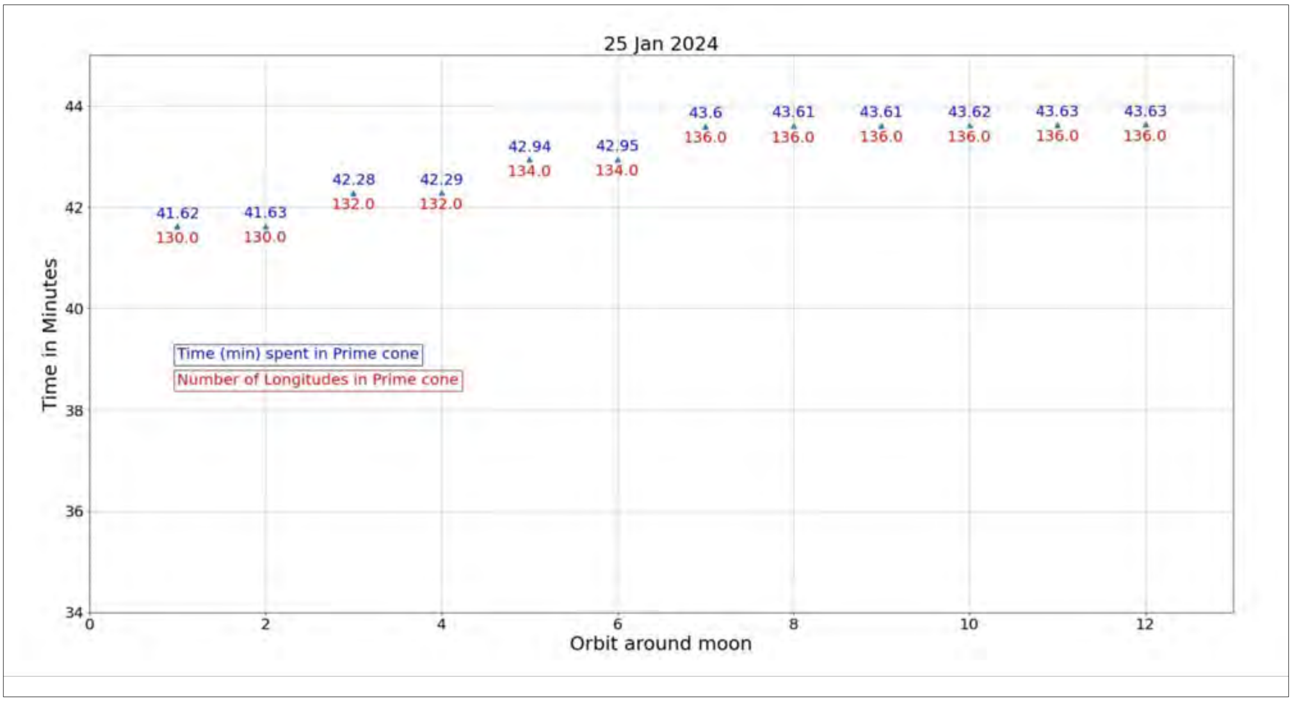


Figure 45: Time spent in prime cone for all 12 orbits

In Fig 45 and all such plots in proceeding sections the time spent in the prime cone decreases in steps of approximately 0.6 minutes (36 seconds). The number of longitudes the spacecraft crosses in the prime cone during each orbit decreases by 2° (in LunaFixed system) which is equal to the step size of the simulation set in GMAT. A smaller step size in simulation will result in more accurate values at the expense of computation simplicity.

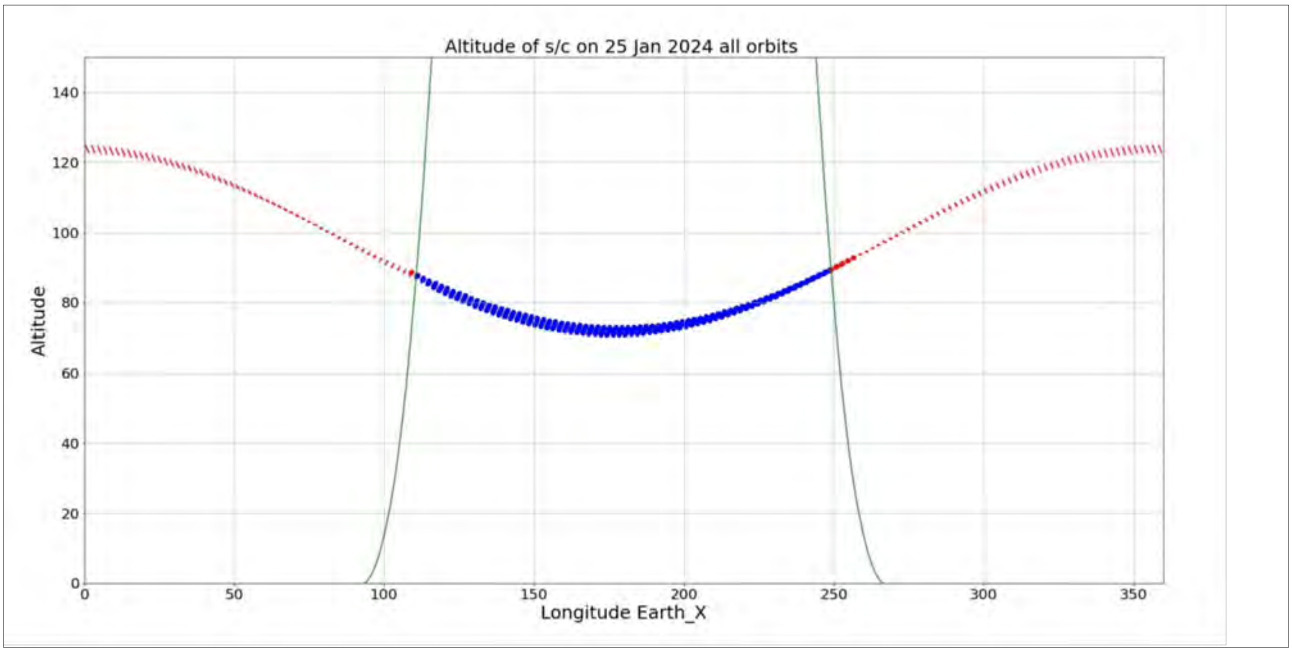


Figure 46: Altitude of the spacecraft for all 12 orbits

In Fig 46 Most of the positions where the spacecraft is in the Sun quiet cone (thick marker points) also correspond to the positions where it is inside the Earth quiet cone (between green lines). This is the reason why the spacecraft spends maximum time in the prime cone region on this day.

4.2.2 23 May 2024

The full Moon occurs on 13:55 UTC at which time the time spent in prime cone region is expected to be maximum. The plots from the simulations show expected results and the time spent in the prime cone is maximum (with slight variation) for 5th to 8th orbits around the Moon.

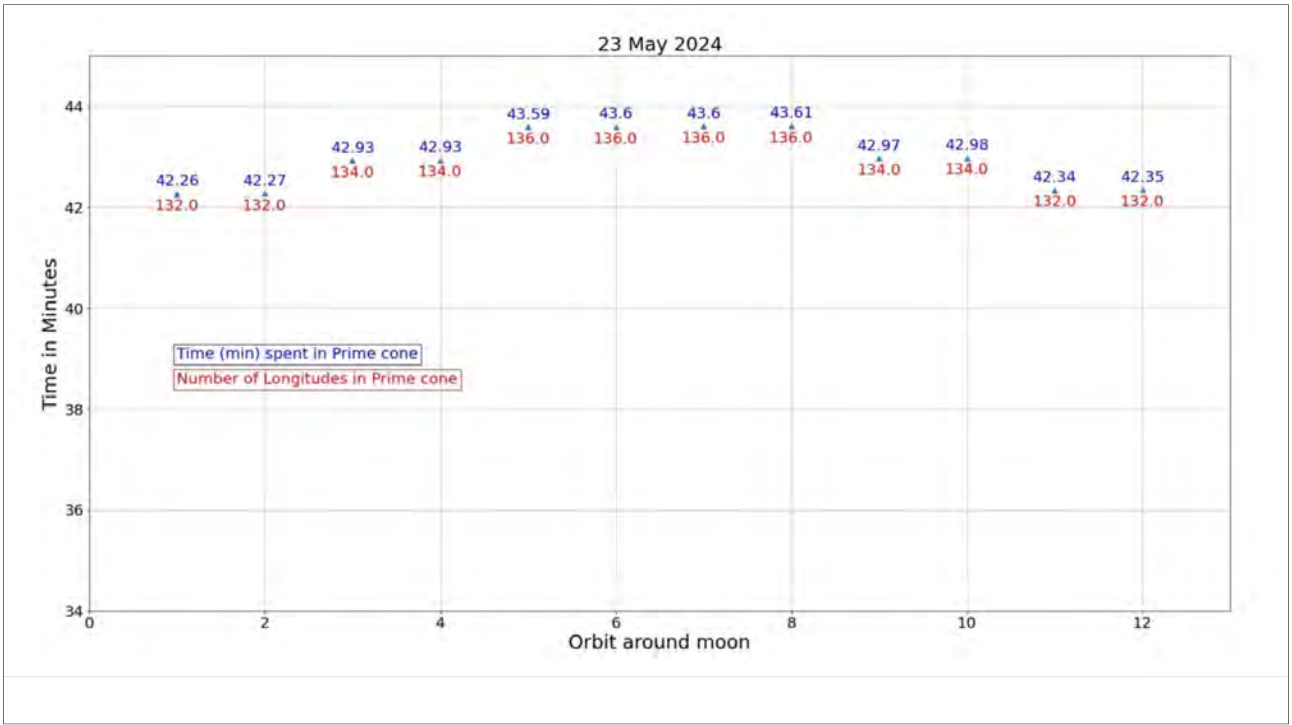


Figure 47: Time spent in prime cone for all 12 orbits

4.2.3 12 February 2025

The full Moon occurs at 13:54 UTC and as expected the time spent in prime cone is maximum around that time in Fig 48. The maximum time spent in prime cone is 43.6 minutes which is similar to previous simulations.

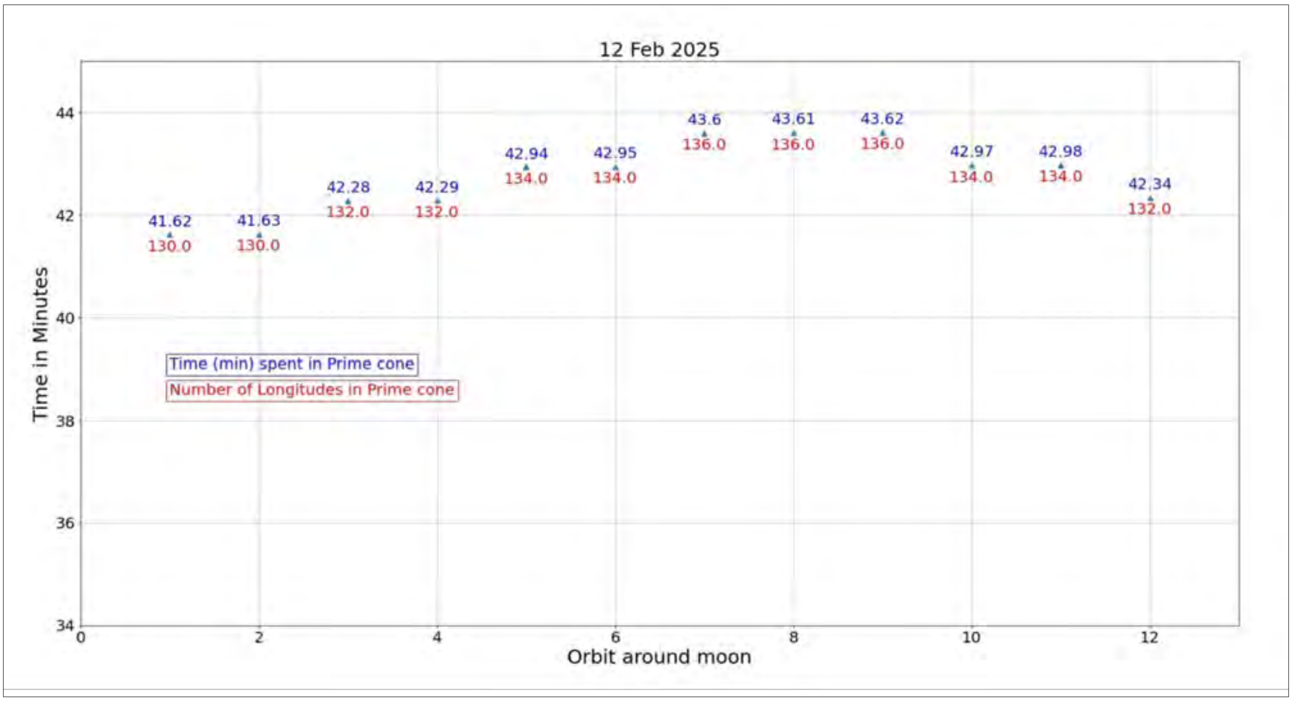


Figure 48: Time spent in prime cone for all 12 orbits

4.3 Simulating the spacecraft on a Lunar Eclipse

During Lunar eclipses the Moon moves either fully or partially into the Earth shadow region. The Sun, Moon and Earth lie in a straight line configuration and the width of the prime cone and time spent in it by a spacecraft is expected to be maximum. Five such cases were investigated in this project.

4.3.1 25 March 2024 Penumbral lunar eclipse

Maximum eclipse will occur at 07:14 UTC on 25 March 2024 and the spacecraft will spend a maximum of 43.59 minutes in prime cone region as seen in Fig 49.

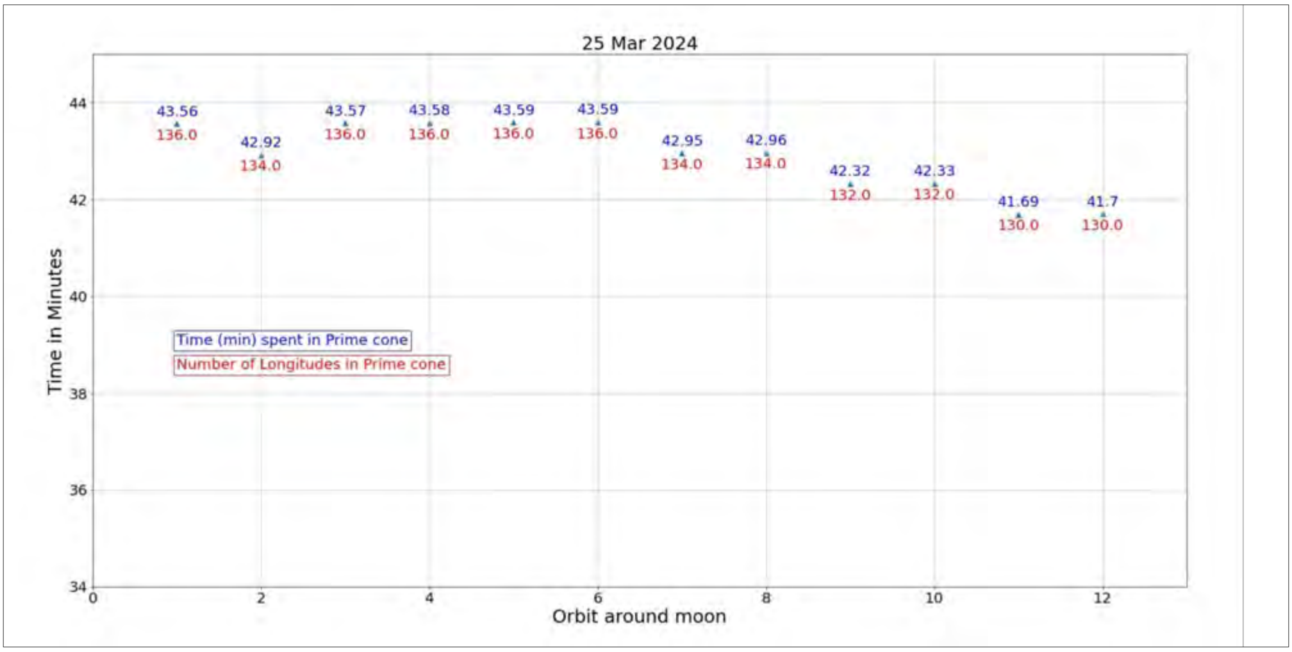


Figure 49: Time spent in prime cone for all 12 orbits

The reason the 2^{nd} orbit shows a slightly lesser amount of time spent in prime cone compared to 1^{st} and 3^{rd} orbits can be attributed to the step size of the simulation.

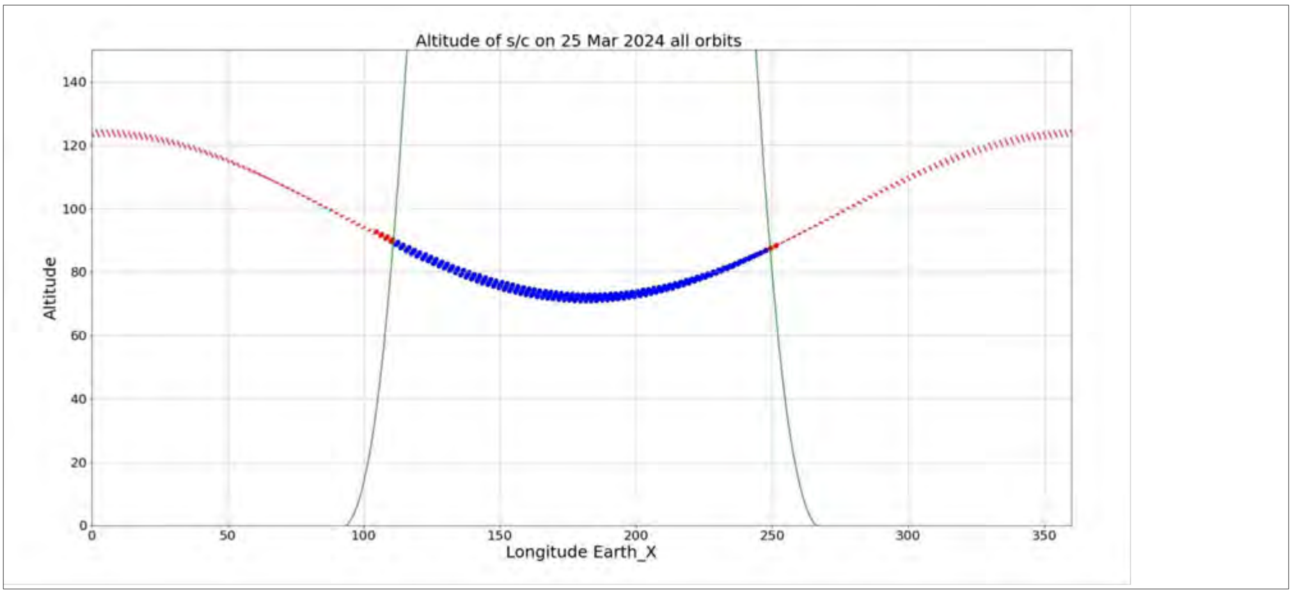


Figure 50: Altitude of the spacecraft for all 12 orbits

4.3.2 7 September 2025 Total lunar eclipse

Total lunar eclipse will start at 15:33 UTC lasting till 18:53 UTC when the Moon will move completely out of Earth's umbra. At 20:54 UTC the Moon will be completely outside the Earth's penumbra marking the end of partial lunar eclipse. The time spent in prime cone follows the expected trend, reaching a maximum value of 43.63 minutes during the 8^{th} to 11^{th} orbits of the day.

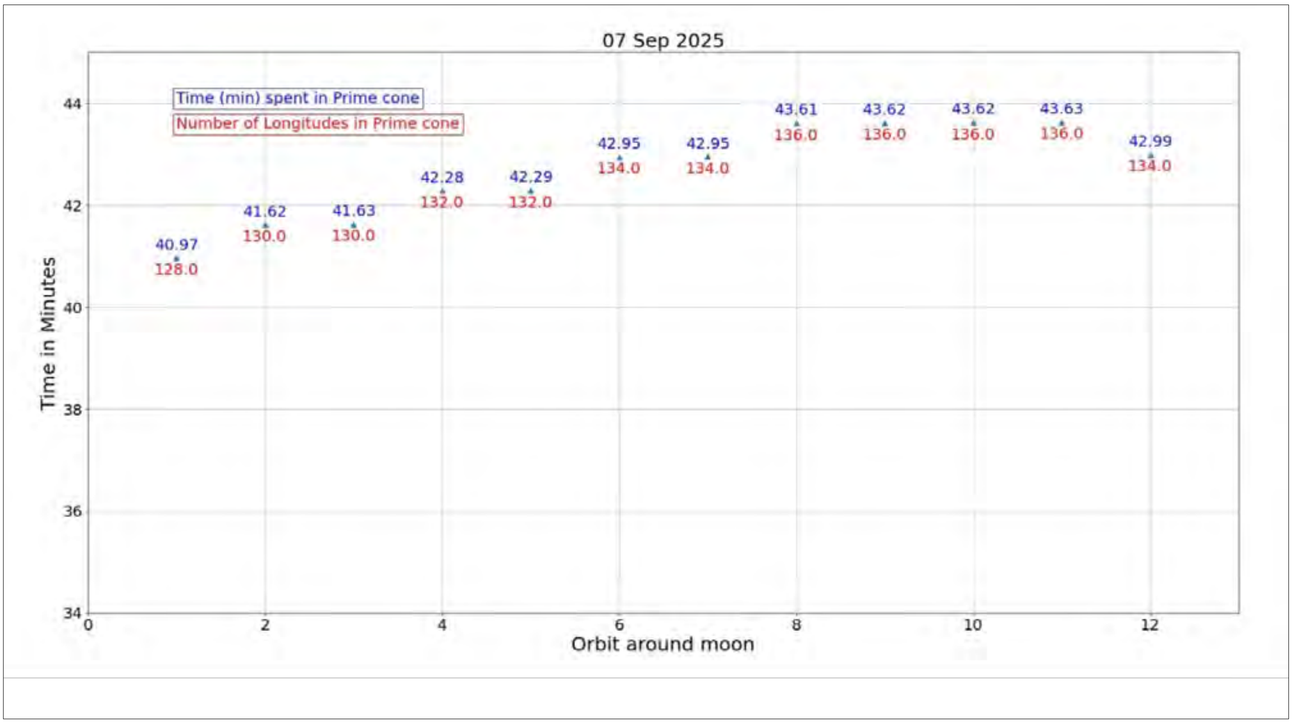


Figure 51: Time spent in prime cone for all 12 orbits

Few conclusions can be drawn from these plots

1. This method of finding time spent in prime cone is not well suited for this project since the orbital parameters of the spacecraft have not been allowed to evolve with time under the non uniform gravitational forces.
2. When the spacecraft is simulated with certain initial parameters for one day only, its altitude vs longitude graph of any day will be similar to any other day. Fig 46 and Fig 50.
3. The difference in time spent in prime cone region on full moons will be solely because of the relative positions of the Sun, Moon and Earth. For any full moon/ lunar eclipse the maximum time spent in prime cone was similar as seen in all figures showing time spent in prime cone Fig 45, 47, 48, 49, 51.
4. Time spent in prime cone region during luanr eclipses was comparable (or less) to that of full moons. If the DEC value of the spacecraft is also considered for determining whether the spacecraft lies in the Earth/ Sun quiet cone, then the time during lunar eclipses is expected to be more than full moons.

The next section deals with propagation of spacecraft for one lunar month. A simulation with longer duration allows the altitude of the spacecraft to evolve under the influence of non uniform forces of gravity.

4.4 Simulating the spacecraft for one lunar month

A suitable lunar month was chosen whose results can be compared with full moons/ eclipses discussed in previous section. The starting date of the simulation was set as the previous new moon and the duration of the simulation was kept as 30 days.

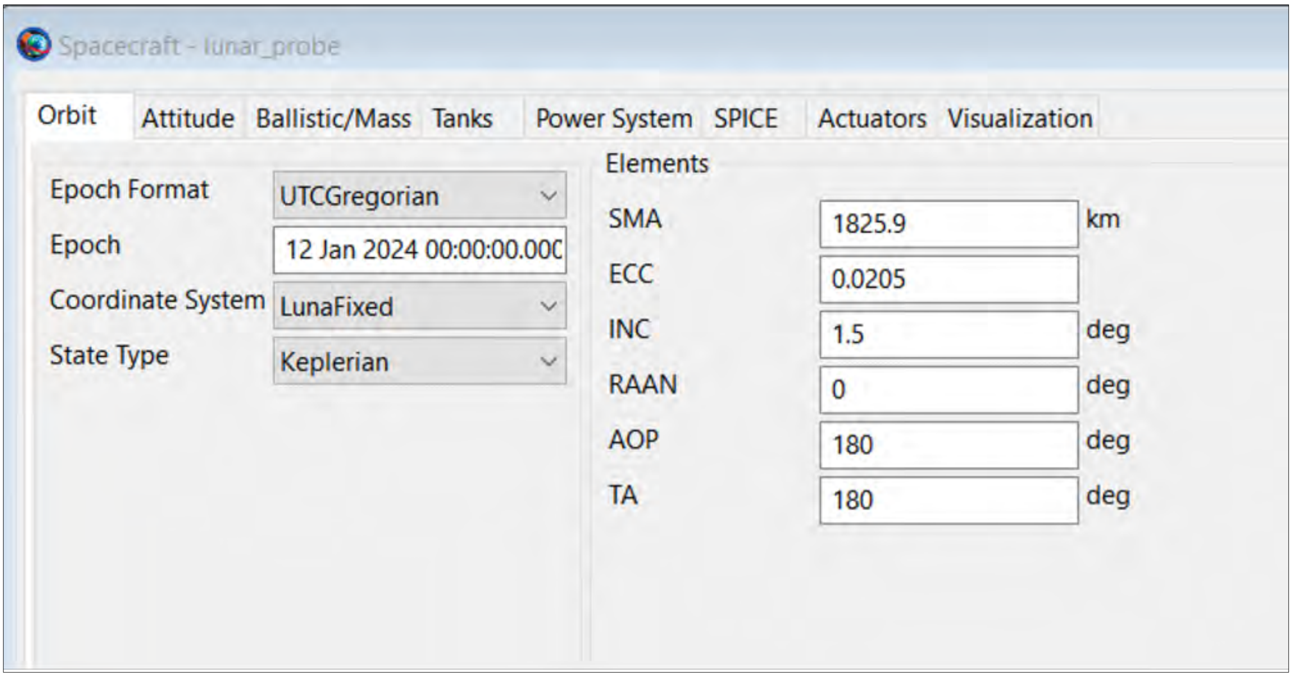


Figure 52: Initial parameters set for simulating the spacecraft for 30 days around 25 January 2024

4.4.1 12 January to 11 February 2024

The spacecraft is simulated for 30 days beginning from 12 January (new moon) so that the maximum time spent in prime cone on 25 January 2024 can be compared with the results of previous section.

Fig 53 shows the time spent in prime cone for one lunar month which is at a maximum 13.5 days into the simulation (25 January at noon) with a value of 43.67 minutes. This value was very close to the value discussed in the last section.

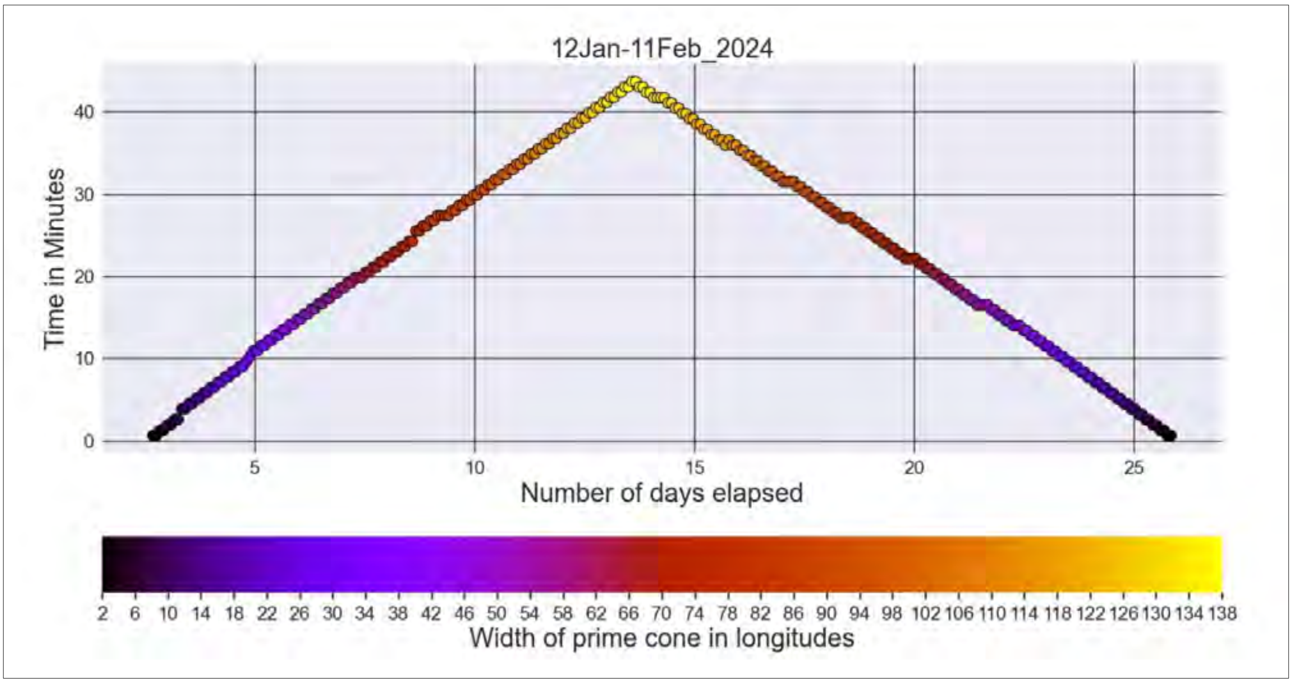


Figure 53: Time spent in prime cone for all orbits during one lunar month

The data points lie on discrete time steps due to the step size of 2° used in the simulations as mentioned earlier. The difference in these steps is also 0.6 minutes. The cumulative sum of time in prime cone is 106.73 hours and sum of all times greater than 30 minutes is approximately 56 hours

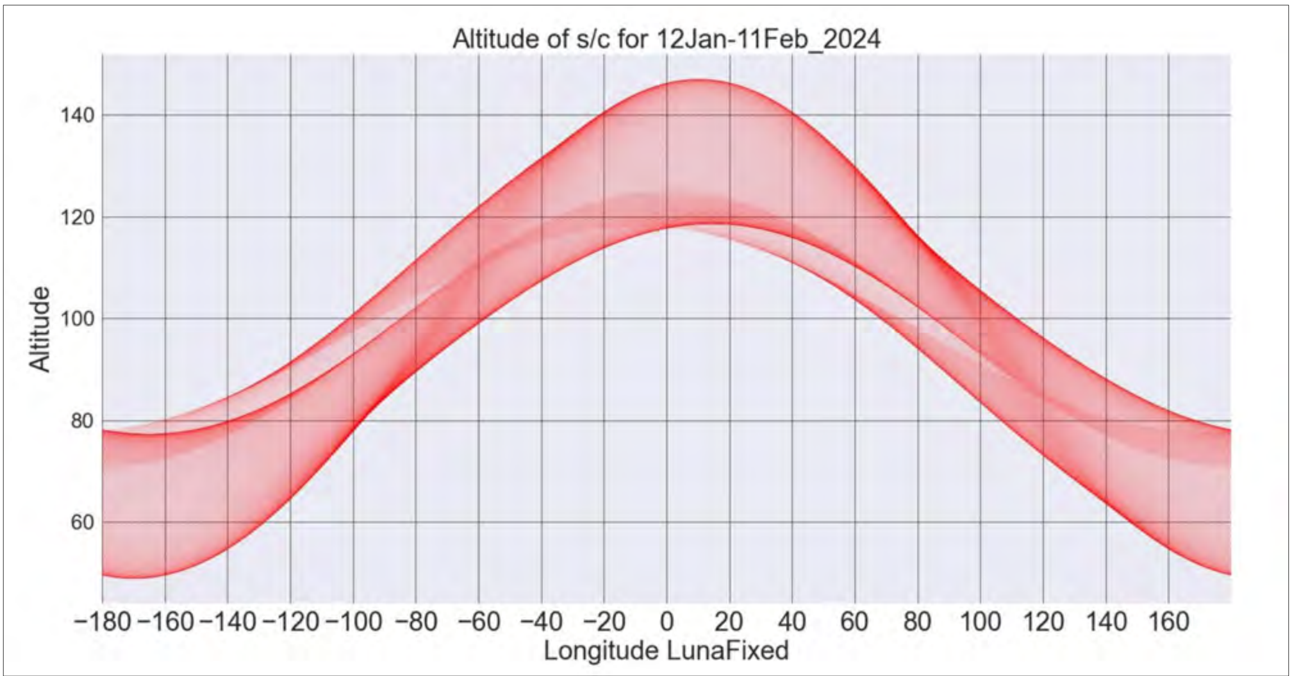
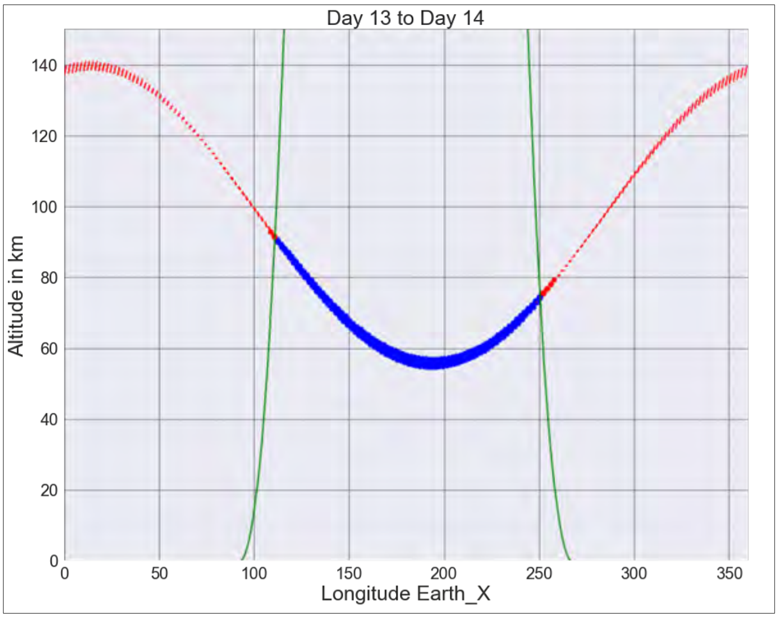


Figure 54: Altitude of the spacecraft for all orbits during one lunar month

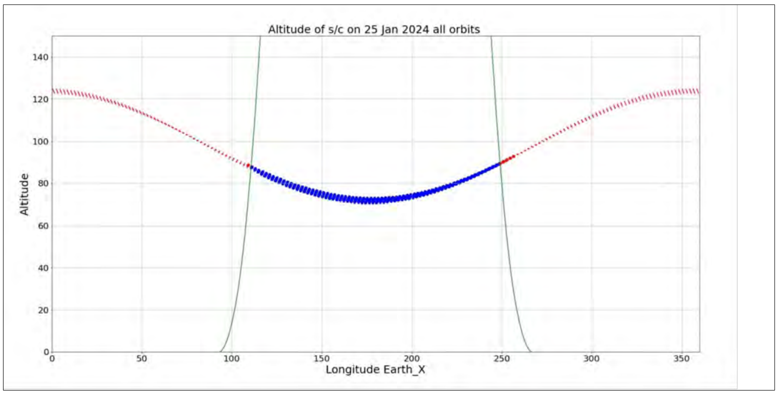
The evolution of altitude plays an important role in determining the time spent in prime cone region. Lower altitudes of the perilune on the lunar far side ensures that the spacecraft remains in radio quiet region for a longer time. Fig 55 compares the altitude of all orbits for the same day (25 January 2025) when simulated for 30 days Fig 55a and when simulated for one day Fig 55b. This results in a slight difference in time spent inside prime cone region for same day in both simulations

4.4.2 23 August to 22 September 2025

By simulating the spacecraft in this period the time spent in prime cone on a total lunar eclipse (7 September 2025) can be compared with the last section. On 7 September (15 days into the simulation) time spent in prime cone is maximum with a value of 43.54 minutes. Again, this value was close to the one calculated in the previous section.



(a) Simulation of 30 days



(b) Simulation of 1 day

Figure 55: Altitude of all orbits of the spacecraft for 25 Jan 2024 with different simulation durations

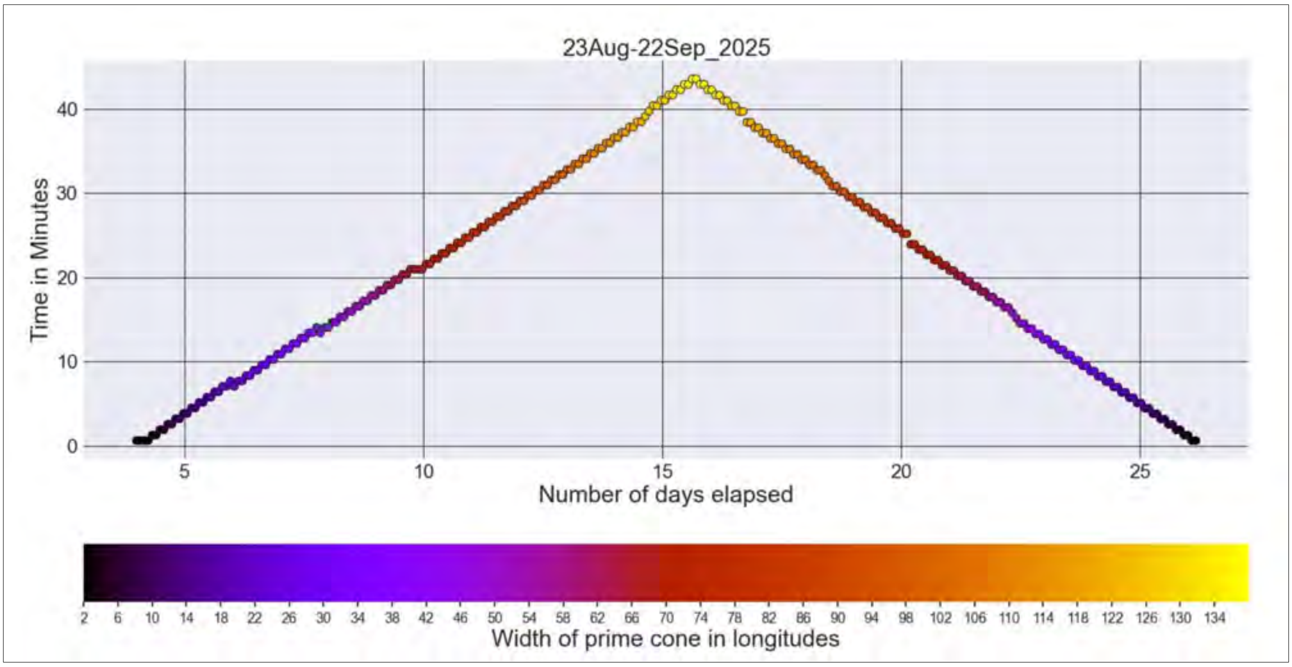


Figure 56: Altitude of the spacecraft for all orbits during one lunar month

The cumulative time spent in prime cone is 96.66 hours and the sum of all times greater than 30 minutes is approximately 49 hours.

These two cases discussed convey how important it is to let the simulation run for longer duration. In the next section a simulation of two years is discussed.

4.5 Simulating the spacecraft for two years

Simulating the spacecraft for 2 years beginning from 22 Dec 2023 allowed the altitude enough time to evolve under a non-uniform gravitational field. As expected, the time spent in prime cone follows a similar trend for each lunar month attaining a maximum value at full moon. This maximum value for each lunar month varies slightly around 44 minutes. For some full moons it is greater than 44 minutes which is more than the maximum time observed for any individual full moon simulation. This shows the importance of longer simulations in which time evolution of spacecraft's altitude changes the time spent in radio quiet zones.

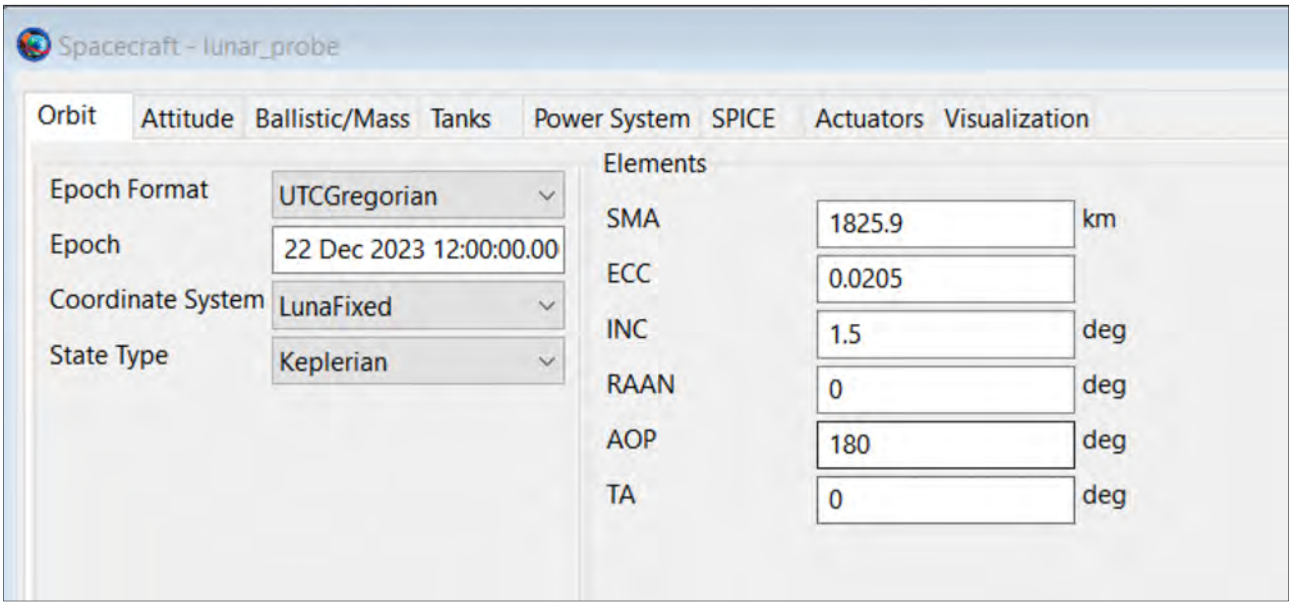


Figure 57: Setting up the initial parameters of the spacecraft

The spacecraft spends a total of 2506 hours inside the prime cone and a total of 1317 hours where time spent in prime cone is more than 30 minutes

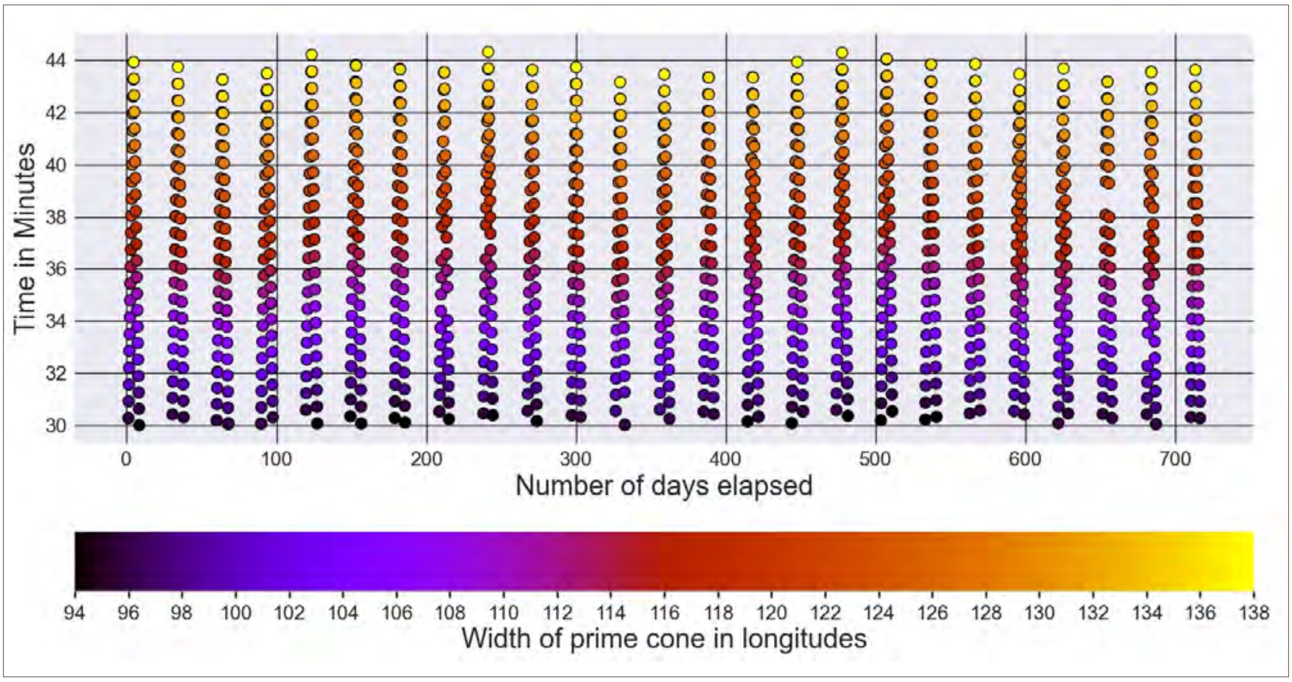
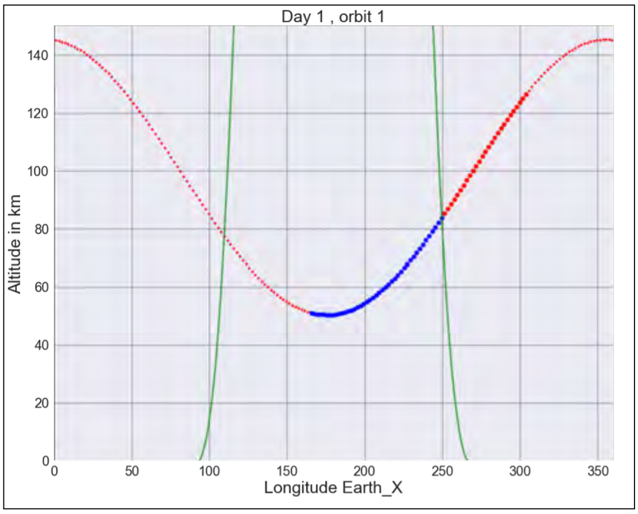
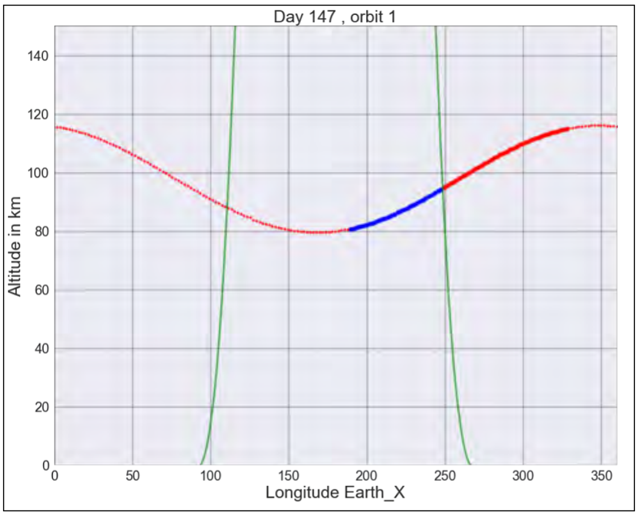


Figure 58: All the time spent in prime cone region which was greater than 30 minutes

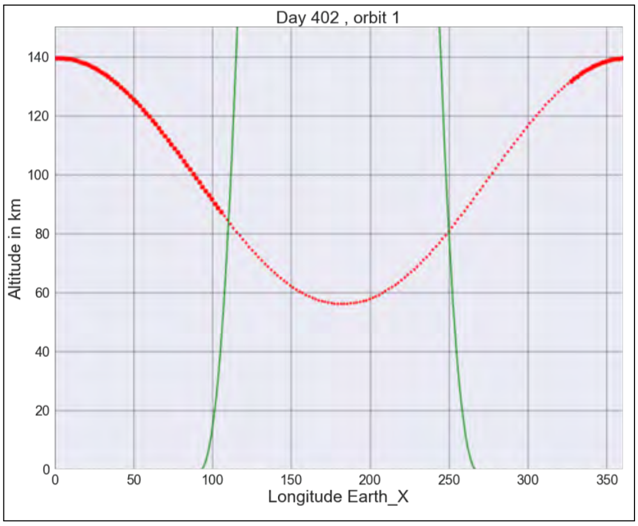
Altitude vs longitude/RA in Earth_X system for a few randomly selected days below shows the time evolution of spacecraft's altitude.



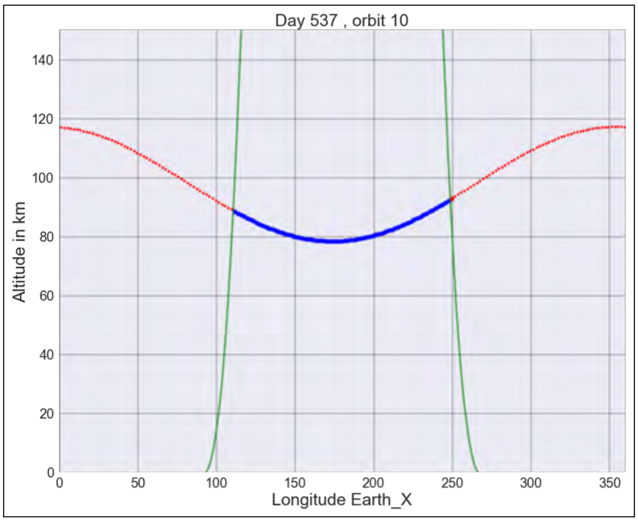
(a) Day 1, 1^{st} orbit



(b) Day 147, 1^{st} orbit



(c) Day 402, 1^{st} orbit



(d) Day 537, 10^{th} orbit

Figure 59: Altitude vs RA in Earth_X system for one orbit on some days

5 Chapter 5 : Conclusion and future works

The last chapter discussed in detail the orbit of a spacecraft simulated with the initial Keplerian parameters stated in Table 2 in Chapter 4 .These parameters are restated in Table 3.

Eccentricity	SMA	Inclination	RAAN	AOP	TA
0.0205	1825.9 km	1.5°	0°	180°	180°

Table 3: Initial parameters for simulations in LunaFixed system

Table 4 reiterates the criteria for an orbit to be considered for PRATUSH mission. The result column summarises the findings of last chapters.

S no.	Criteria	Findings
1	Frozen (stable) orbit that requires little to no orbital station keeping maneuvers for a mission duration of 2 years taking into account the non-uniform gravitational field of the Moon	Orbit was stable (did not decay below 20 km altitude) for two years. **Remark**: Solar Radiation Pressure and drag from lunar exosphere were not accounted for.
2	Maximise time spent in the Earth and Sun shadow region behind the Moon	Satellite spent a maximum of about 44 minutes during one orbit inside the prime cone region **Remark**: a lower threshold for observation time of one orbit should be decided to filter out orbits unusable for science observations
3	The orbit should be stable for a range of lunar orbital insertion parameters like semi-major axis (SMA) length, inclination and eccentricity.	Orbit was stable for a range of initial inclination and RAAN values **Remark**: other parameters were not investigated

Table 4: Criteria for an orbit to be considered suitable for PRATUSH along with the findings of this project

The simulations undertaken can be extended by considering the following higher level optimization for improved accuracy.

1. An inertial coordinate system should be used to set the initial Keplerian parameters of the spacecraft. This would ensure better comparison of results between simulations with different starting dates. The axes of the LunaFixed system are time dependent, so the orbit of a spacecraft initialized with same Keplerian parameters on different epochs will be different when viewed in inertial frame. A non inertial, moon fixed frame was used in this project for setting the satellite initially at the lunar nearside/farside easily.

2. A more refined set of criteria needs to be defined for PRATUSH mission. Only a portion of time spent in prime cone region during every pass can be utilised for observation and the remaining time is required for thermal stabilisation of antenna and other preparations. A certain threshold limit of cumulative observation hours during mission lifetime should be proposed to eliminate orbits that do not provide enough time in prime cone region. This project did not study the beamwidth of the on board antenna, the direction it should be pointing towards inside the prime cone region etc.

3. The region of space behind the moon free from radio frequency interference were characterized in the following way

    (a) **Earth quiet cone** Using finite difference time domain algorithm the propagation of electromagnetic waves around a spherical Moon with a constant density was simulated. The region of space where the radio waves were attenuated to levels below 90dB was considered free from terrestrial RFI (Earth quiet cone)

    (b) **Sun quiet cone** The geometrical shadow region behind the Moon for the Sun-Moon system was calculated and described as the Sun quiet cone throughout this project. This rudimentary definition should be improved upon to account for the solar radiation that diffracts around the edges of the Moon. Similar to Earth quiet cone a simulation of electromagnetic waves can help in this case.

4. When simulating higher inclinations, both the DEC and RA value along with the altitude of the satellite will determine whether it is inside radio quiet region. This method should be cross verified with the inbuilt GMAT event locator subsystem that uses SPICE kernels to determine when a spacecraft is occulted by a celestial body. It generates a report file stating all the eclipses experienced by the spacecraft during the full mission sequence.

5. A higher order and degree gravity model for the moon should be used. Additionally, the forces of drag from lunar exosphere and solar radiation pressure should not be neglected as they can amount to significant perturbations over time. A detailed physical model of the spacecraft is needed from which the drag area and SRP area can be estimated. This area should be specified in the spacecraft hardware properties window in GMAT

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A GitHub repository

All the GMAT scripts, Python notebooks used to generate the plots are available in a Github repository at `https://github.com/distortions21cm/PRATUSH_orbit.git`

The GMAT scripts in the repository should be changed before execution on a new machine depending on the location where the GMAT software is installed. All the gravity files, terrain models and sample scripts are located at this location. The following features of the script should be changed by selecting the appropriate location of the required files.

1. The grgm900c lunar gravity model file is located in the "data/gravity/luna" folder. In the force model tab for the propagator ,the correct location should be used.

2. Some of the scripts generate ground track plots of the satellite around the Moon. The location of the image file used to map the lunar surface should be changed before running the scripts.

3. The location of the report file generated in .csv format should be changed according to the local machine.