VERY LITTLE EVOLUTIONARY GAME THEORY 2022
HOMEWORK SOLUTIONS

Week 1

It doesn't make sense that Dove's display has no fitness cost. If nothing else, it costs time and energy. Let d be the cost of display. Assume that Dove pays this cost whenever it meets another Dove, whether it wins the resource or not, but not when it retreats from a Hawk. Analyze this new version of the game.

Solution. Hawk can be evolutionarily stable when:

$$\frac{v-c}{2} > 0$$

So nothing has changed for that end of the dynamics. When Dove is common, it is evolutionarily stable when:

$$\frac{v-d}{2} > v$$

This is always satisfied. So again no change. What does change is the location of the mixed equilibrium. Let p be the frequency of Hawk. Assuming $v < c$, a stable mixed equilibrium is located where:

$$p\frac{v-c}{2} + (1-p)v = p(0) + (1-p)\frac{v-d}{2}$$

Solving for p yields $p = (v+d)/(c+d)$. As d increases, this is closer to $p = 1$, so more Hawks at the mixed equilibrium. The game is not qualitatively changed by adding display costs. But we should expect more fighting as a result of display costs.

Week 2

Analyze the evolutionary dynamics of the coordinate game from lecture, using the statistical assortment model. The coordination game is where "Safe" earns b when it meets itself, zero otherwise. "Risky" earns B when it meets itself, zero otherwise. Let p be the proportion of Risky in the population. Let r be the probability of assortment. Determine when each strategy is evolutionary stable and the location of any unstable equilibria.

Solution. This will be more transparent if we write general expected fitness expressions first. Then we can simplify them for the different stability scenarios. The expected fitness of Safe is:

$$\begin{aligned} V_S &= \Pr(\text{Safe-meets-Safe})b + \Pr(\text{Safe-meets-Risky})(0) \\ &= (r + (1-r)(1-p))b \end{aligned}$$

Now for Risky:

$$\begin{aligned} V_R &= \Pr(\text{Risky-meets-Risky})B + \Pr(\text{Risky-meets-Safe})0 \\ &= (r + (1-r)p)B \end{aligned}$$

Now when Safe is common, $p \approx 0$, so Safe is stable when:

$$b > rB$$

So if $r > b/B$, Risky can invade when rare. When Risky is common, $p \approx 1$, and Risky is stable when:

$$B > rb$$

So Risky is stable when $r < B/b$, which is always true, because $B > b$.

The domain of attraction moves too. If both strategies are stable, then there is an unstable equilibrium at where:

$$(r + (1-r)(1-p))b = (r + (1-r)p)B$$

Solving for p:

$$p = \frac{b - rB}{(1-r)(b+B)}$$

As r increases, this approaches $p \to 0$, meaning it is easier for Risky to invade.

Week 3

The iterated prisoner's dilemma is often criticized for presenting a too pessimistic view of the potential for cooperation, because many real contexts are not prisoner's dilemmas. Reanalyze the Tit-for-Tat strategy from lecture, but use a Stag Hunt payoff structure instead. This means that when both individuals cooperate, they both earn B. If one cooperates and the other does not, the cooperator earns zero (0). Non-cooperation always earns $b < B$. Consider when TFT is stable and can invade against ALLC and NO-C. Are there any qualitative differences from the prisoner's dilemma?

Solution. When TFT is common, it earns $B/(1-w)$, because the duration of the relationship is $1/(1-w)$ and the focal earns B on each turn. A rare ALLC earns the same (because there are no errors). So TFT is not stable against ALLC. A rare NO-C earns b. So TFT is stable against NO-C when $B/(1-w) > b$, which is always true, because $B > b$. Reciprocity (w) makes no difference here.

What about invasion? A rare TFT in a population of ALLC earns the same as ALLC. So it can invade. A rare TFT in a population of NO-C earns 0, so it cannot invade. Again, reciprocity makes no difference.

The Stag Hunt game makes reciprocity irrelevant to the maintenance of TFT when common. Cooperation can still not invade when rare, just like in a prisoner's dilemma.

Week 4

Add kin assortment to the repeated prisoner's dilemma from Week 3. Rederive the invasion and stability conditions for Tit-for-Tat, assuming pairs of individuals are relatives with coefficient of relatedness r. How much relatedness is needed for TFT to invade when rare? How do relatedness r and relationship duration w interact?

Solution. When TFT is common, it earns $(b-c)/(1-w)$. A rare NO-C earns:

$$(1-r)b + r(0)$$

So TFT is stable against NO-C when:

$$\frac{b-c}{1-w} > (1-r)b$$

This can be rearranged in several ways. Let's consider this form:

$$\big(r(1-w) + w\big)b > c$$

If we set $r = 0$, we get back $wb > c$, the condition in the absence of assortment. If we set $w = 0$, we get back $rb > c$, Hamilton's rule. Now notice that when w is large, assortment doesn't matter much. For stability in this scenario, reciprocity replaces assortment.

It's different for invasion though. When rare, TFT expects:

$$r\frac{b-c}{1-w} + (1-r)(-c)$$

A common NO-C earns zero. So TFT can invade when:

$$r\frac{b-c}{1-w} > (1-r)c$$

Again this can be arranged many ways. Let's consider:

$$r(b - wc) > c(1 - w)$$

If we set $w = 0$, we get $rb > c$, which is Hamilton's rule again. But for $w \to 1$, this goes to $r(b - c) > 0$, which is always true when $b > c$ and $r > 0$. For intermediate values of r and w, an increase in either makes the other more effective. For invasion, reciprocity and assortment are synergistic.

Week 5

No problem was assigned for week 5.