LINDAU NOBEL LAUREATE MEETINGS

INDIAN INSTITUTE OF INFORMATION TECHNOLOGY

HEIDELBERG LAUREATE FORUM

Deterministic, Non-Deterministic, Optimization, Approximation, and Probabilistic

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PhD: **IIT Indore** MTech: **IIITDM Jabalpur**

acm India Council

Pingala Interactions In Computing

Indian Institute of Technology Indore

भारतीय प्रौद्योगिकी संस्थान इंदौर

PDPM

Indian Institute of Information Technology, Design and Manufacturing, Jabalpur

Deterministic and Non-deterministic algorithms

Computational complexity theory

- Fields in
  - Theoretical Computer Science
  - Analysis of Algorithms
- An algorithm solves a computation problem by mathematical steps.
- A computational problem (such as an algorithm) is a task solved by a computer.
- Focuses on classifying computational problems according to the resource usage
- Resource usage: amount of resources needed to solve computational problem,
- Resources: such as time and storage.

https://en.wikipedia.org/wiki/Computational_complexity_theory

Single Source Shortest-Paths Implementation

algorithm	restriction	typical case	worst case	extra space
topological sort	no directed cycles	$E + V$	$E + V$	V
Dijkstra (binary heap)	no negative weights	$E \log V$	$E \log V$	V
Bellman–Ford	no negative cycles	$E\,V$	$E\,V$	V
Bellman–Ford (queue–based)		$E + V$	$E\,V$	V

Easy

Medium

Hard

Remark 1. Directed cycles make the problem harder.

Remark 2. Negative weights make the problem harder.

Remark 3. Negative cycles makes the problem intractable.

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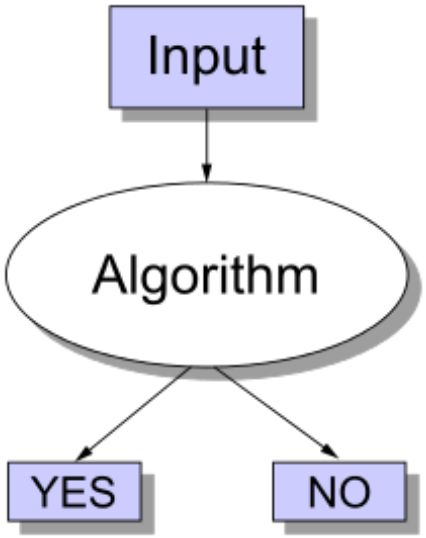
Remark 1. Directed cycles make the problem harder.

Remark 2. Negative weights make the problem harder.

Remark 3. Negative cycles makes the problem intractable.

Deterministic algorithm

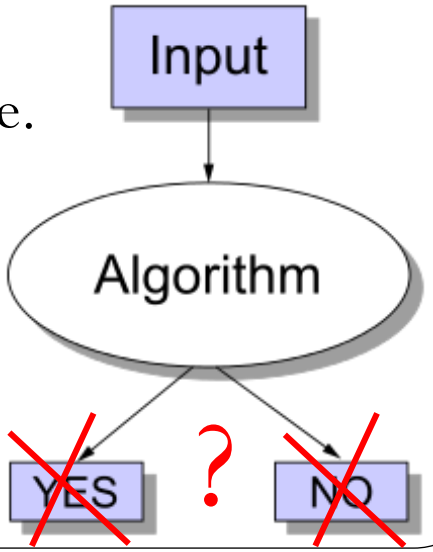
- Given a particular input, will always produce the same output, with the underlying machine always passing through the same sequence of states.
- State machine: a state describes what a machine is doing at a particular instant in time.
- State machines pass in a discrete manner from one state to another.
- Enter the input, initial state or start state.
- Current state determines what will be next state, the set of states is predetermined.



https://en.wikipedia.org/wiki/Deterministic_algorithm

Non-deterministic Algorithms

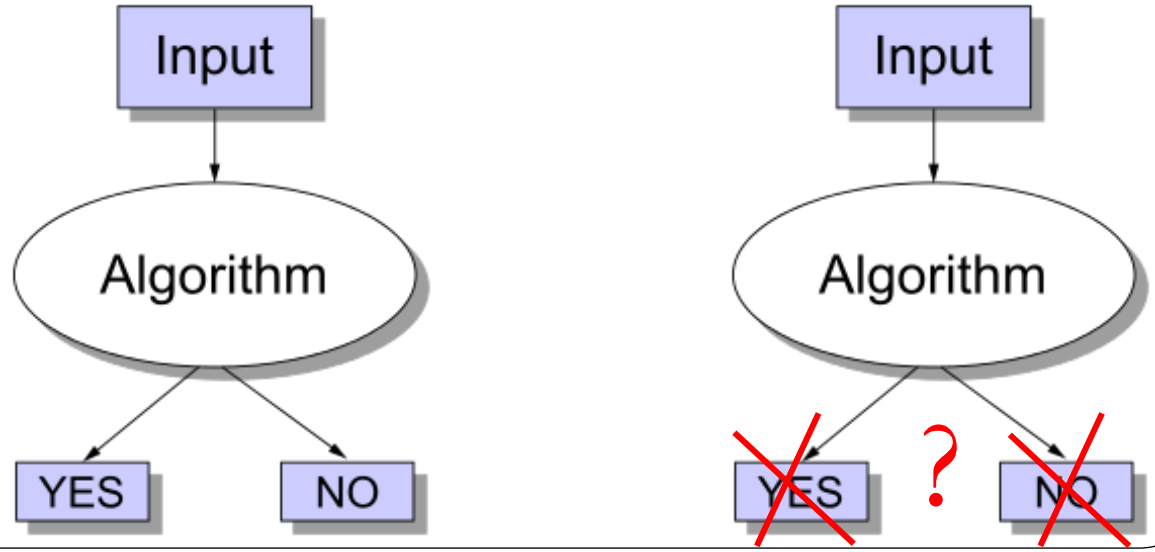
- If it uses external state other than the input, such as
  - user input,
  - a global variable,
  - a hardware timer value,
  - a random value, or
  - stored disk data.
- If it is timing-sensitive,
  - e.g. if it has multiple processors writing to the same data at the same time.
- If a hardware error causes its state to change in an unexpected way.
- The order each processor writes data will affect the result.



https://en.wikipedia.org/wiki/Deterministic_algorithm

Deterministic and Non-deterministic Algorithms

- Disadvantages of Determinism
  - predictable future by players or predictable security by hacker
  - e.g. predictable card shuffling program or security key
- Pseudorandom number generator is often not sufficient,
  - thus cryptographically secure pseudo-random number generator,
  - hardware random number generator.



https://en.wikipedia.org/wiki/Deterministic_algorithm

P (Polynomial) Time Problems

- Contains all decision problems that can be solved deterministically using a polynomial time (polynomial amount of computation time).
- A problem is in P-complete, if it is in P
- P is the class of computational problems that are "efficiently solvable" or "tractable".
- Class P, typically take all the "tractable" problems for a sequential algorithm,
- But, in reality, some problems not known to be solved in polynomial P time.

https://en.wikipedia.org/wiki/P_(complexity)
https://en.wikipedia.org/wiki/P-complete

P (Polynomial) Time Problems

- Programmable Function (or method) is polynomial-time
  - if completes in constant-time or polynomial time,
  - then the entire algorithm takes polynomial time.
- Polynomial-time algorithms:
  - Minimum Spanning Tree: Kruskal's $\mathbf{O(}E \lg V\mathbf{)}$ and Prim's $\mathbf{O(}E + V \lg V\mathbf{)}$ algorithm
  - Shortest Path Algorithms: Djikstra's **O(E lgV)** and Bellman-Ford's $\mathbf{O(}EV\mathbf{)}$ algorithm

https://en.wikipedia.org/wiki/P_(complexity)
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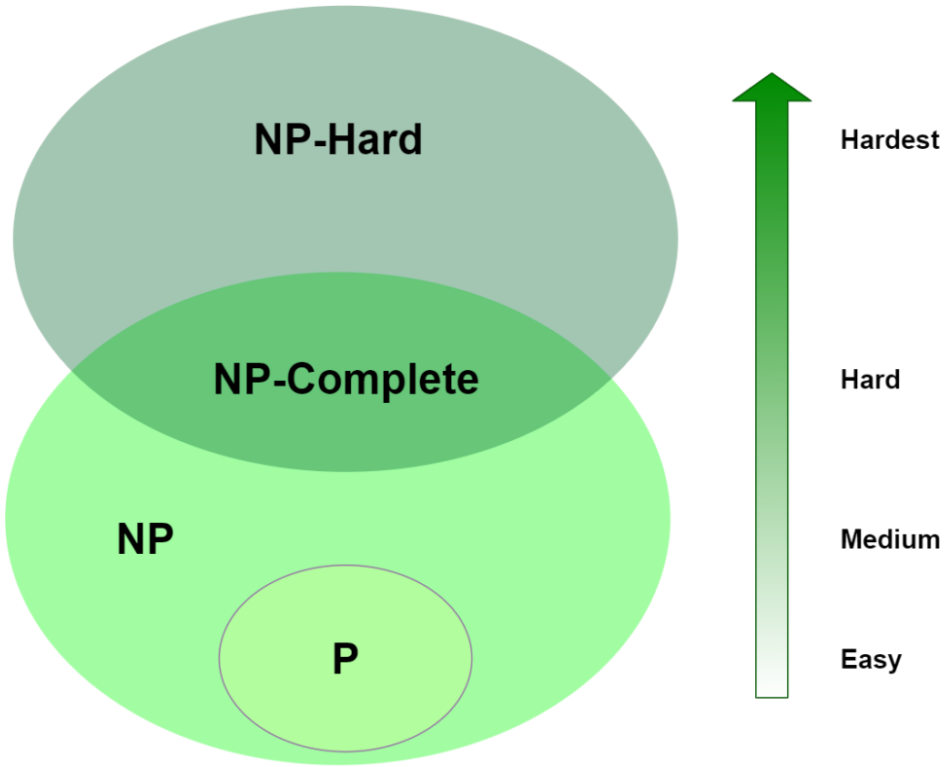
NP - Naming convention

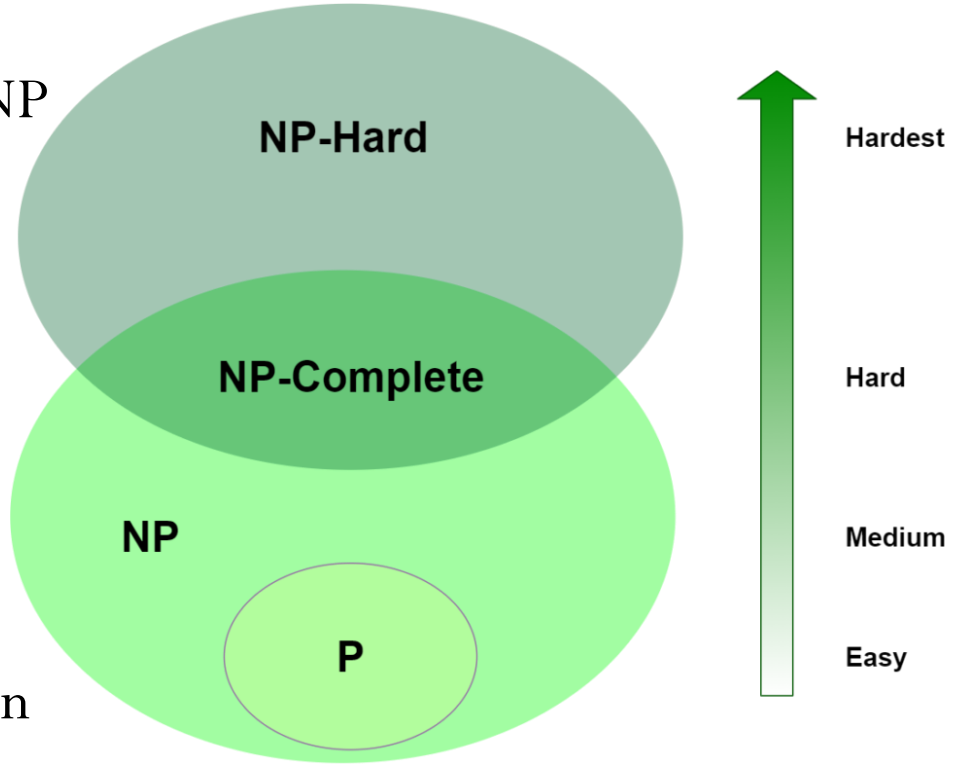
- Classification
  - Hardest → NP-Hard
  - Hard → NP-Complete
  - Medium → NP
  - Easy →P
- Order of N inputs
  - O(1) – constant-time
  - $O(\log_2(n))$ – logarithmic-time
  - O(n) – linear-time
  - $O(n^2)$ – quadratic-time
  - $O(n^k)$ – polynomial-time
  - $O(k^n)$ – exponential-time
  - O(n!) – factorial-time



https://www.baeldung.com/cs/p-np-np-complete-np-hard

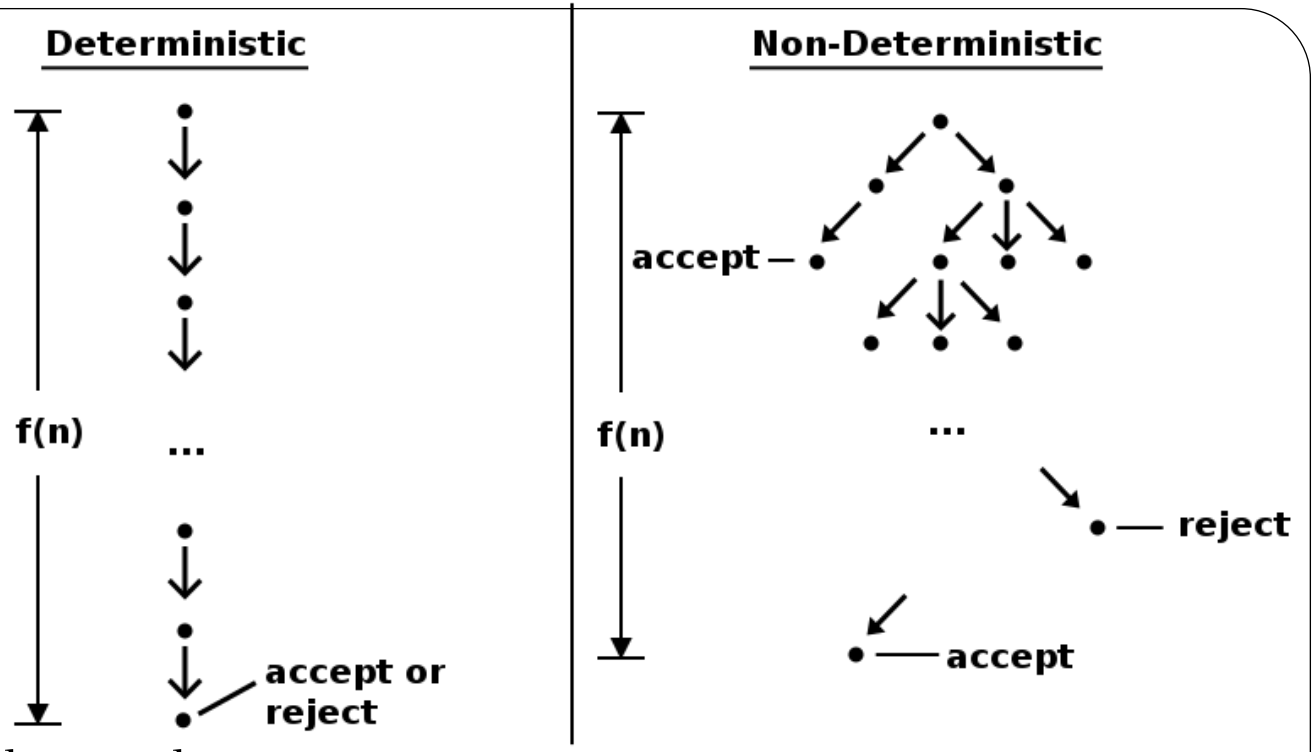
NP - Naming convention

- NP-hard: Class of problems are at least as hard as the hardest problems in NP.
- NP-hard problems do not have to be in NP; means NP hard problem may not even be decidable.
- NP-complete: Class of decision problems which contains the hardest problems in NP. Each NP-complete problem has to be in NP.
- NP: Class of computational decision problems for which any given yes-solution can be verified as a solution in polynomial time
- NP-easy: At most as hard as NP, but not necessarily in NP.



https://en.wikipedia.org/wiki/NP-hardness
https://www.baeldung.com/cs/p-np-np-complete-np-hard

P and NP Problems



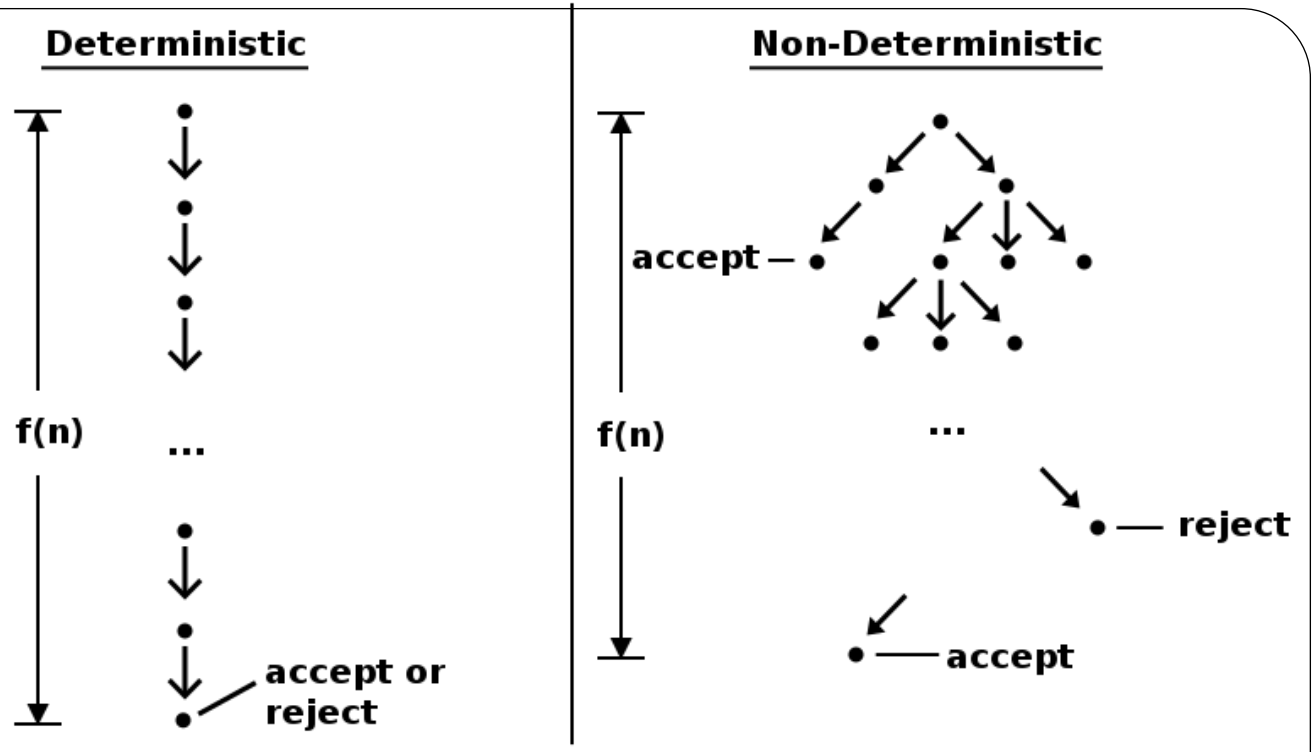
- Nondeterministic Polynomial-time
- “Nondeterministic” refers to
  - “luckiest possible guesser”
- "Complete" refers to
  - “in the same complexity class”
- **P** versus **NP** determine
  - whether a problem can be verified in polynomial time
  - whether the problem can also be solved in polynomial time.
- If it turned out that **P** ≠ **NP**, (widely accepted/believed),
- There are problems in **NP** that are harder to compute than to verify:
- NP problems could not be solved in polynomial time, but the answer could be verified in polynomial time.

https://en.wikipedia.org/wiki/P_versus_NP_problem

NP Complete

- Nondeterministic Polynomial-time

Complete

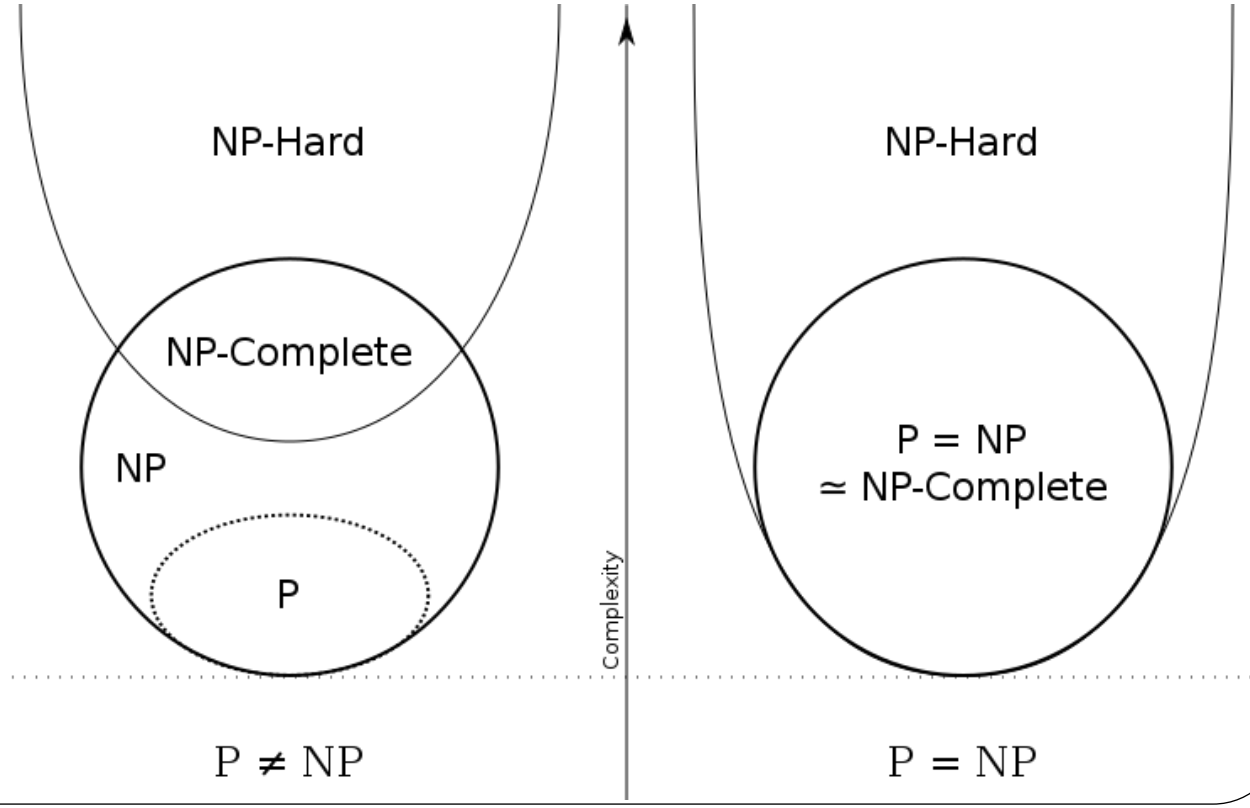


- A problem is NP-complete when:
  - a brute-force search algorithm can solve it, and the correctness of each solution can be verified quickly, and
  - the problem can be used to simulate any other problem with similar solvability.
- NP-complete problem can be *verified* "quickly",
- There is no known way to *find* a solution quickly.

https://en.wikipedia.org/wiki/NP-completeness

NP - Hard Problems

- Non-Deterministic Polynomial-time hardness
- At least as hard as the hardest problems in NP
- The might be some polynomial-time algorithms for NP-hard problems but might not been discovered yet
- NP-hard but not NP-complete
  - halting problem: "given a program and its input, will it run forever?"
  - traveling salesman problem

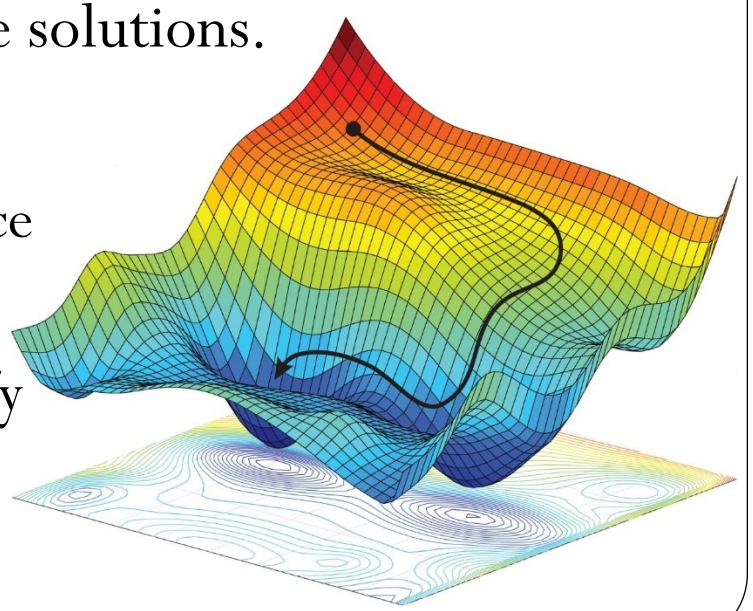


https://en.wikipedia.org/wiki/NP-hardness

Optimization, Approximation, and Probabilistic

Optimization

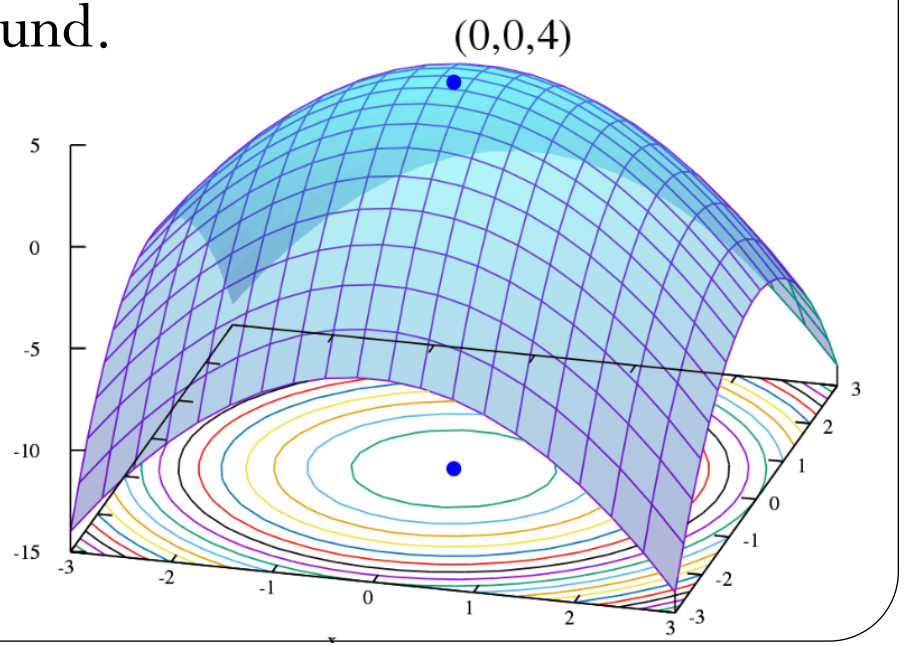
- An optimization problem can be represented in the following way:
  - Given: a function $f : A \to \mathbb{R}$ from some set A to the real numbers
  - Goal: an element $x_0 \in A$ such that $f(x_0) \leq f(x)$ for all $x \in A$ ("minimization") or such that $f(x_0) \geq f(x)$ for all $x \in A$ ("maximization").
- The domain A of f is called the search space or the choice set
- The elements of A are called candidate solutions or feasible solutions.
- A is some subset of the
  - Euclidean space $\mathbb{R}^n$, Geometry represent N-dimensional space
  - Specified by a set of constraints,
  - Equalities or inequalities that the members of A have to satisfy



https://en.wikipedia.org/wiki/Mathematical_optimization

Optimization

- Problem of finding the best solution from all feasible solutions.
- **Discrete optimization:** A problem with discrete variables in which an object must be found from a countable set like integer, permutation or graph
  - **Combinatorial optimization**
- **Continuous optimization:** A problem with continuous variables in which an optimal value from a continuous function must be found.
  - **Constrained problems**
  - **Multimodal problems**



https://en.wikipedia.org/wiki/Optimization_problem

Combinatorial optimization

- Finds an optimal object from a finite set of objects, where the set of feasible solutions is discrete or can be reduced to a discrete set.
  - Exhaustive search uses algorithms that quickly rule out large parts of the search space,
  - Or use **Approximation or Probabilistic algorithms**.
- A combinatorial optimization problem A is a quadruple (I, f, m, g), where
  - I is a set of instances; given an instance $x \in I$, $f(x)$ is the set of feasible solutions;
  - Given an instance x and a feasible solution y of x, $m(x, y)$ denotes the measure of y, which is usually a positive real.
  - g is the goal function, and is either min or max.
  - Goal is to find for some instance x an optimal solution, that is, a feasible solution of y

$$m(x, y) = g\{m(x, y') \mid y' \in f(x)\}.$$

https://en.wikipedia.org/wiki/Combinatorial_optimization

Constrained optimization

- Process of optimizing an objective function with respect to some variables in the presence of constraints on those variables.
  - primarily equality constraints, inequality constraints, and integer constraints
  - set of candidate solutions that satisfy all constraints is called the feasible set
- A general constrained minimization problem may be written as follows:

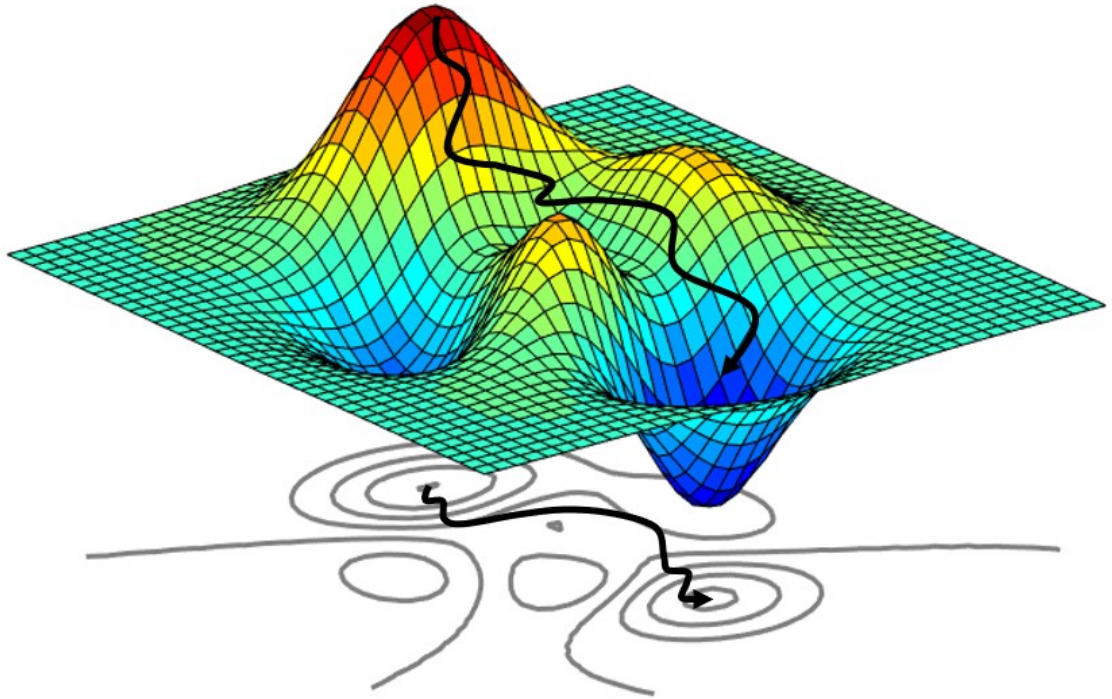
$$\begin{aligned} \min \quad & f(\mathbf{x}) \\ \text{subject to} \quad & g_i(\mathbf{x}) = c_i \quad \text{for } i = 1, \ldots, n \quad \text{Equality constraints} \\ & h_j(\mathbf{x}) \geqq d_j \quad \text{for } j = 1, \ldots, m \quad \text{Inequality constraints} \end{aligned}$$

where $g_i(x)$ and $h_j(x)$ are constraints that are required to be satisfied, and $f(x)$ is the objective function that needs to be optimized subject to the constraints

https://en.wikipedia.org/wiki/Constrained_optimization

Multimodal optimization

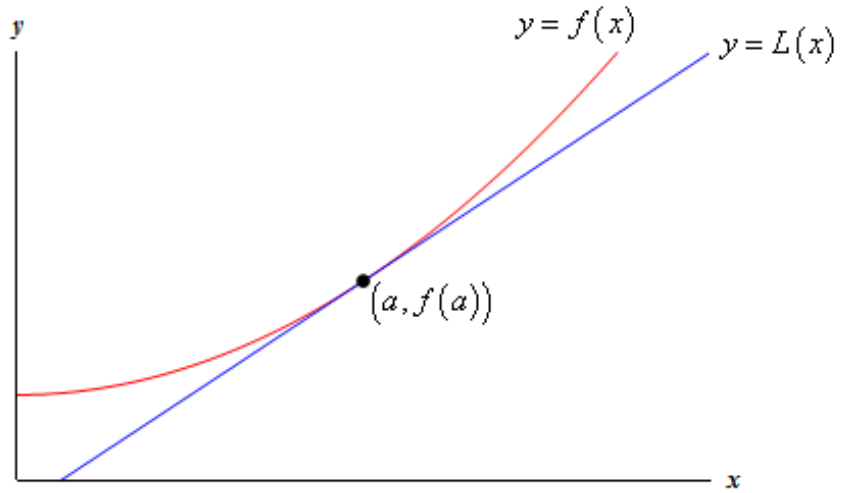
- Finds all or most of the multiple (at least locally optimal) solutions of a problem, as opposed to a single best solution.
  - Evolutionary Multimodal Optimization is a branch of **Evolutionary Computation**
- **Evolutionary algorithms:**
  - Genetic Algorithms (GAs),
  - Evolution Strategy (ES),
  - Differential Evolution (DE),
  - Particle Swarm Optimization (PSO) etc.



https://en.wikipedia.org/wiki/Evolutionary_multimodal_optimization

Approximation

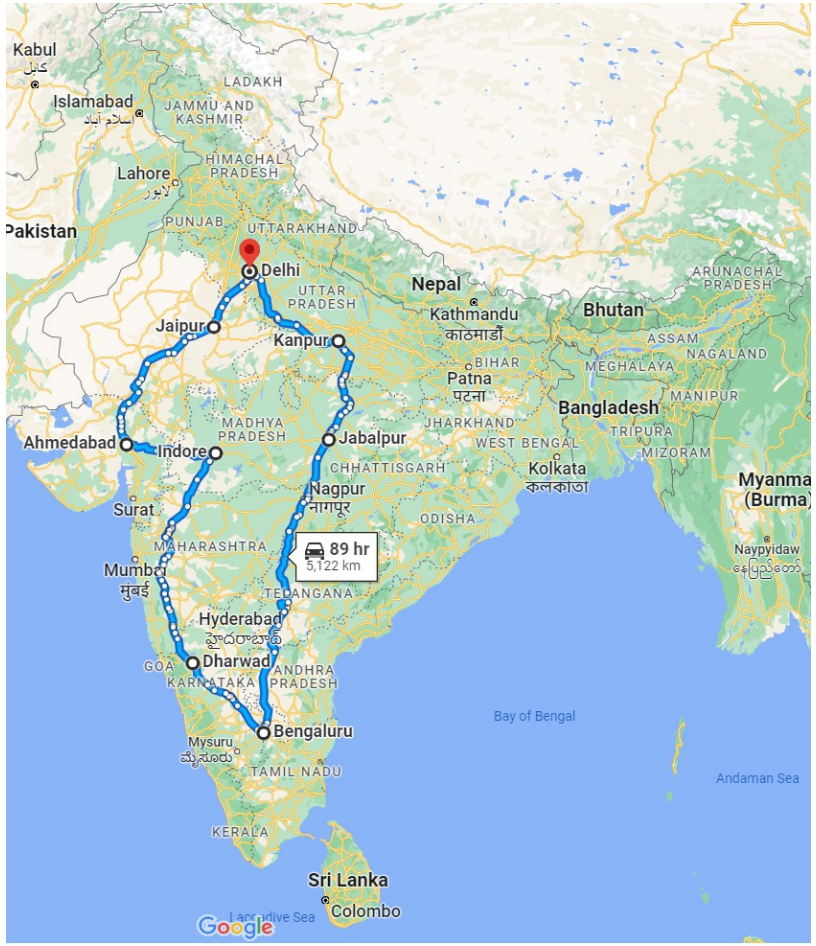
- Efficient algorithms that find **approximate solutions** to NP optimization problems.
- Solution with provable guarantees on the distance of the returned solution to the optimal one.
- Example:
  - Vertex Cover Problem,
  - Traveling Salesman Problem,
  - Set-covering problem (resource-selection problems)
  - Subset-sum problem



https://en.wikipedia.org/wiki/Approximation_algorithm

Approximation

- Karp (1972) proved the TSP to be NP-hard, but effective heuristic approximation methods were developed (Lin and Kernighan, 1973).
- The traveling-salesperson problem (TSP) is a standard combinatorial problem in theoretical computer science (Lawler et al., 1992).
- Arora (1998) devised a fully polynomial approximation scheme for Euclidean TSPs.



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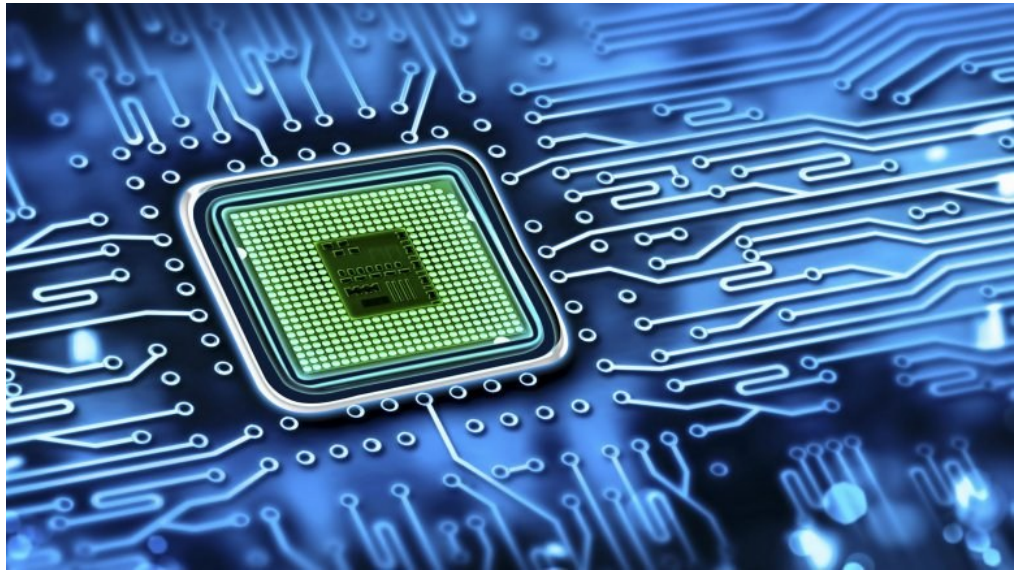
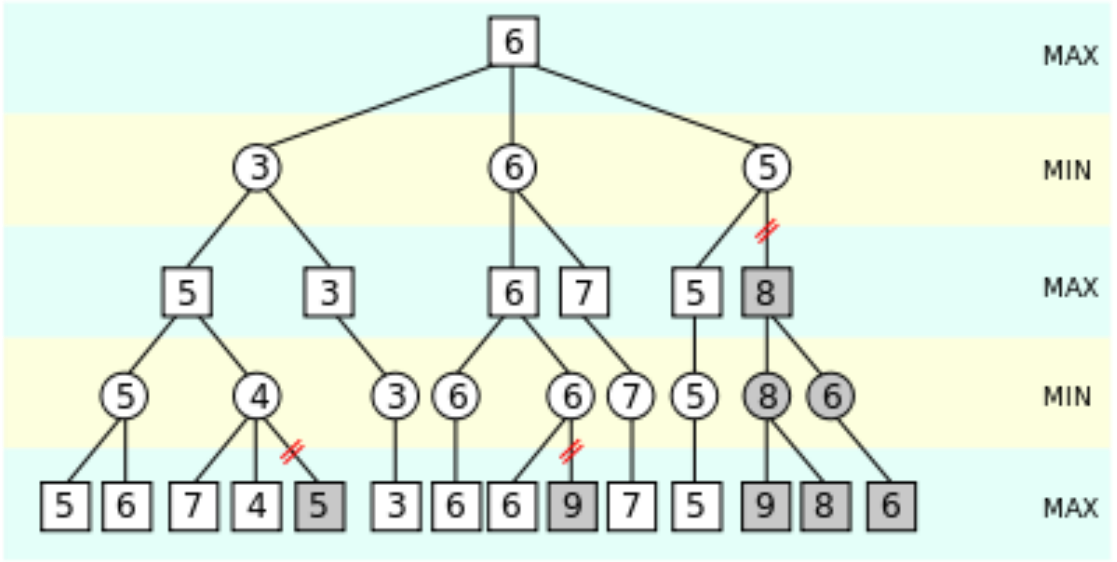
http://aima.cs.berkeley.edu/

Approximation

- VLSI layout methods are surveyed by Shahookar and Mazumder (1991), and many layout optimization papers appear in VLSI journals.
- Approximation in Mini-Max is to cut the search off at some point





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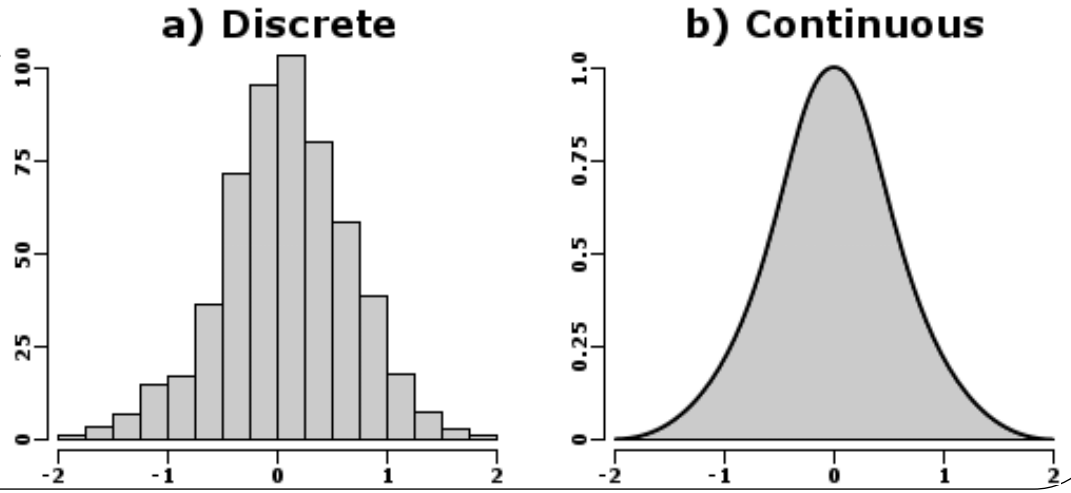
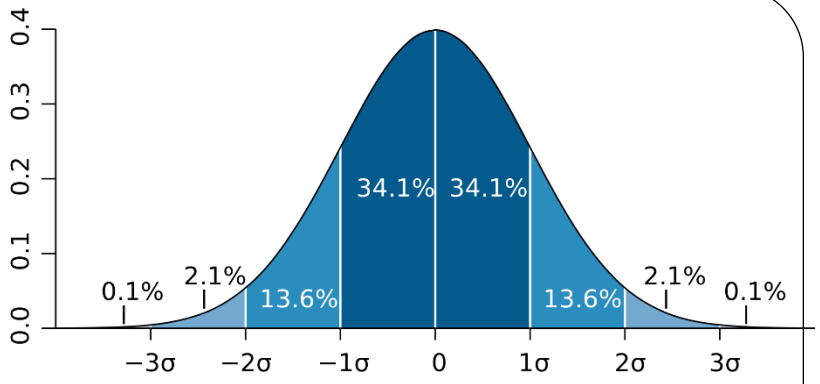
Probabilistic Vs Stochastic

- Deterministic system are predictable.
  - The state of the system can be forecasted for given input, constraints, and mathematical model.
  - States of a deterministic system are pre-determined.
- Non-Deterministic system are unpredictable
  - Stochastic and Probabilistic are usually used interchangeably.
  - Both represent the randomness present in the system.
  - Probabilistic is superset concept of stochastic.



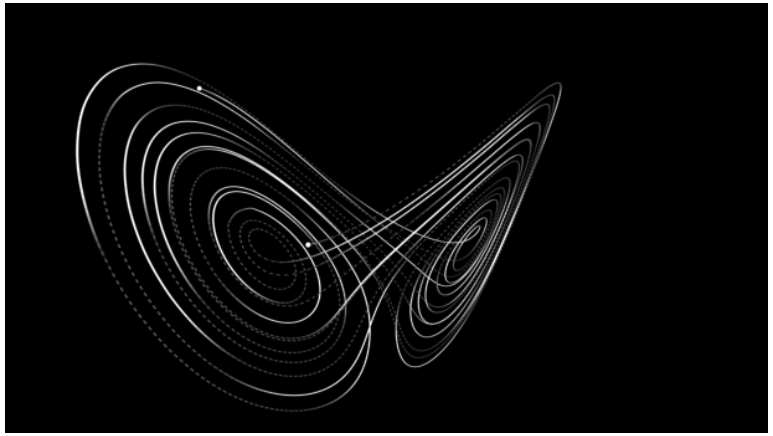
Probabilistic Vs Stochastic



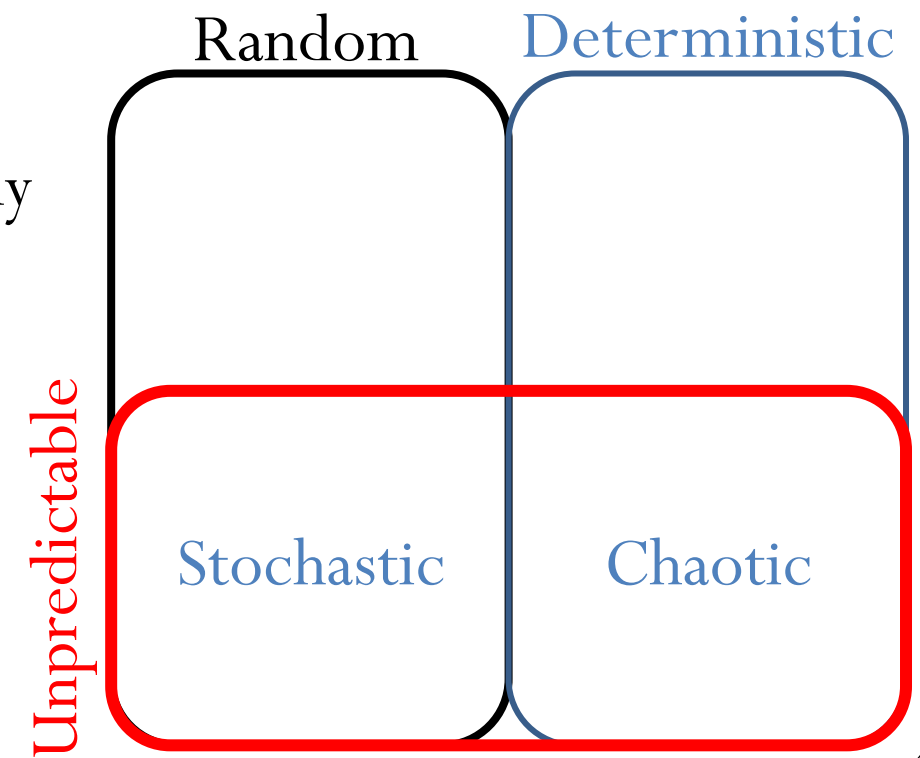
- Probabilistic models are independent of time,
  - which describes system with numerical chances or likelihood of an event to occur.
  - E.g., Lottery numbers are independent of each other.
  - Each instance is determined by the same probability distributions, but with no memory of older instance.
- Stochastic models are time dependent systems,
  - whose changes are described by its past and probabilities for successive changes.
  - E.g., Price of a stock is its old price and an uncertain change.
  - The uncertainty are small, which is semi-predictable.
  - If the stock was closed at 100,
    - then its opening value is predictable around 90 or 110.

https://qr.ae/pvU18E
https://qr.ae/pvU11n

Stochastic and Chaotic



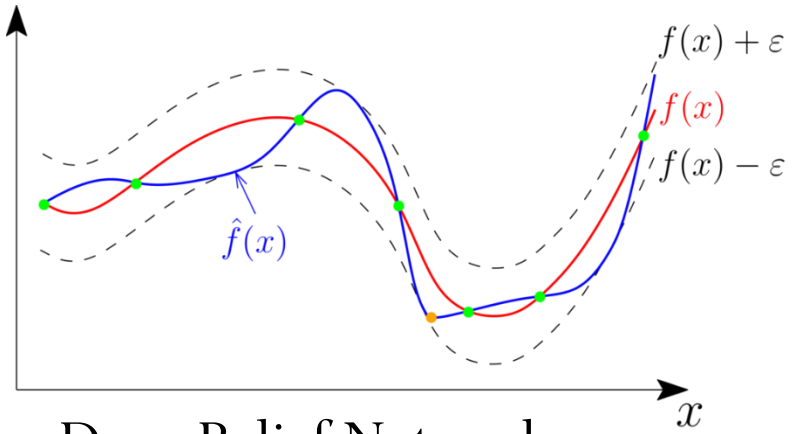
- A **chaotic** system is deterministic in theory.
  - It responds drastically to infinitesimal changes in initial and boundary conditions, making it in practice unpredictable and unstable.
- Deterministic chaos, or simply Chaos.
- **Chaos:** "When the present determines the future, but the approximate present does not approximately determine the future."
- A **stochastic** system is a random phenomenon.
- Stochastic and Chaotic two terms interchangeably.
- It is hard to distinguish between chaotic and stochastic systems.

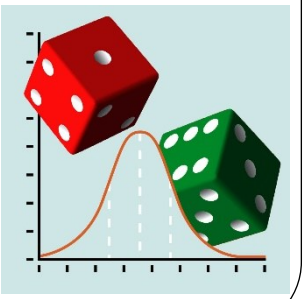


https://en.wikipedia.org/wiki/Chaos_theory
https://qr.ae/pvU11A

Optimization, Approximation, and Probabilistic

- Deterministic nature of a system does not make it predictable.
- Exhaustive search is intractable for NP-hard problems
- Used Optimization, Approximations, and Probabilistic algorithms in
  - Travelling Salesman Problem (TSP), Minimum Spanning Tree (MST), and Knapsack
  - Minimax, Alpha–Beta pruning,
  - Monte Carlo, Markov chain Monte Carlo,
  - Constraint Satisfaction Problems,
  - Likelihood weighting, Maximum-Likelihood,
  - Stochastic Differential Equations (SDEs),
  - Belief states, Belief propagation, Bayesian Networks, Deep Belief Networks,
  - Various Machine Learning, Deep Learning, Reinforcement Learning algorithms,
  - and several other problems





https://mattiheino.com/2016/12/27/deterministic-not-predictable/

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  - https://qr.ae/pvU11n
  - https://en.wikipedia.org/wiki/Chaos_theory
  - https://qr.ae/pvU11A

תודה רבה
Hebrew

Ευχαριστώ
Greek

Danke
German

Спасибо
Russian

धन्यवादः
Sanskrit

Merci
French

ধন্যবাদ
Bangla

நன்றி
Tamil

شكراً
Arabic

ಧನ್ಯವಾದಗಳು
Kannada

Thank You
English

നന്ദി
Malayalam

Grazie
Italian

多謝
Traditional Chinese

ధన్యవాదాలు
Telugu

આભાર
Gujarati

ਧੰਨਵਾਦ
Punjabi

धन्यवाद
Hindi & Marathi

Gracias
Spanish

多谢
Simplified Chinese

https://sites.google.com/site/animeshchaturvedi07

Obrigado
Portuguese

ありがとうございました
Japanese

ขอบคุณ
Thai

감사합니다
Korean