INDIAN INSTITUTE OF INFORMATION TECHNOLOGY

Dharwad

ज्ञानेन विकासः

Optimization, Approximation, and Stochastic

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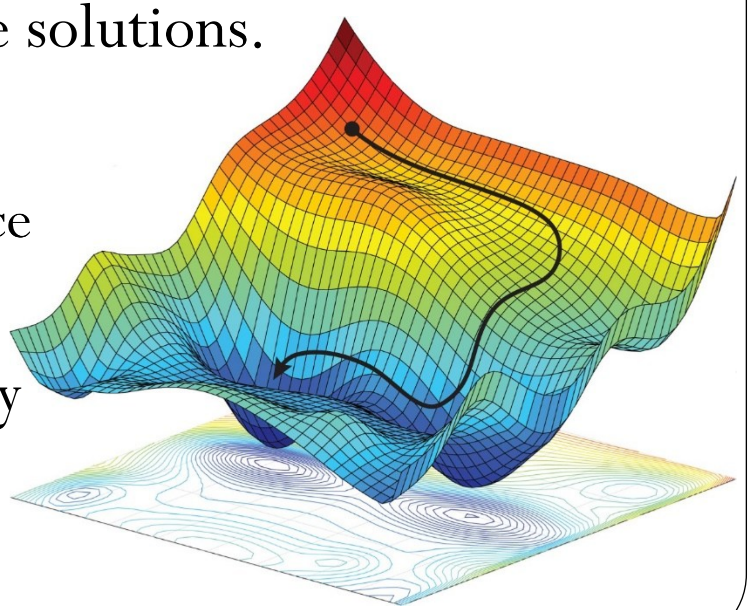
PDPM Indian Institute of Information Technology, Design and Manufacturing, Jabalpur

Optimization

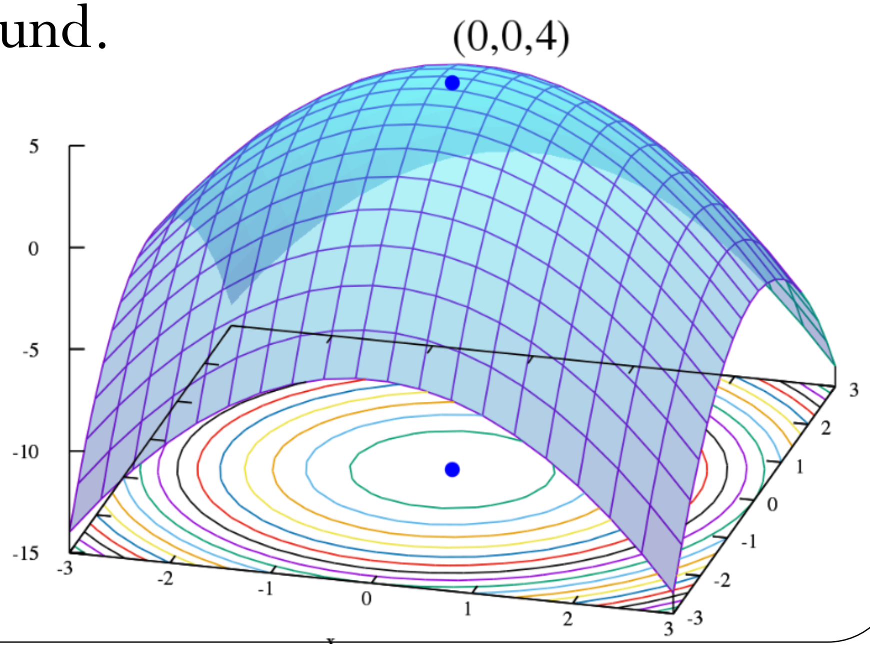
- An optimization problem can be represented in the following way:
  - Given: a function $f : A \to \mathbb{R}$ from some set A to the real numbers
  - Goal: an element $x_0 \in A$ such that $f(x_0) \leq f(x)$ for all $x \in A$ ("minimization") or such that $f(x_0) \geq f(x)$ for all $x \in A$ ("maximization").
- The domain A of f is called the search space or the choice set
- The elements of A are called candidate solutions or feasible solutions.
- A is some subset of the
  - Euclidean space $\mathbb{R}^n$, Geometry represent N-dimensional space
  - Specified by a set of constraints,
  - Equalities or inequalities that the members of A have to satisfy



https://en.wikipedia.org/wiki/Mathematical_optimization

Optimization

- Problem of finding the best solution from all feasible solutions.
- **Discrete optimization:** A problem with discrete variables in which an object must be found from a countable set like integer, permutation or graph
  - **Combinatorial optimization**
- **Continuous optimization:** A problem with continuous variables in which an optimal value from a continuous function must be found.
  - **Constrained problems**
  - **Multimodal problems**



https://en.wikipedia.org/wiki/Optimization_problem

Combinatorial optimization

- Finds an optimal object from a finite set of objects, where the set of feasible solutions is discrete or can be reduced to a discrete set.
  - Exhaustive search uses algorithms that quickly rule out large parts of the search space,
  - Or use **Approximation or Probabilistic algorithms**.
- A combinatorial optimization problem A is a quadruple (I, f, m, g), where
  - I is a set of instances; given an instance $x \in I$, $f(x)$ is the set of feasible solutions;
  - Given an instance x and a feasible solution y of x, $m(x, y)$ denotes the measure of y, which is usually a positive real.
  - g is the goal function, and is either min or max.
  - Goal is to find for some instance x an optimal solution, that is, a feasible solution of y

$$m(x,y) = g\{m(x,y') \mid y' \in f(x)\}.$$

https://en.wikipedia.org/wiki/Combinatorial_optimization

Constrained optimization

- Process of optimizing an objective function with respect to some variables in the presence of constraints on those variables.
  - primarily equality constraints, inequality constraints, and integer constraints
  - set of candidate solutions that satisfy all constraints is called the feasible set
- A general constrained minimization problem may be written as follows:

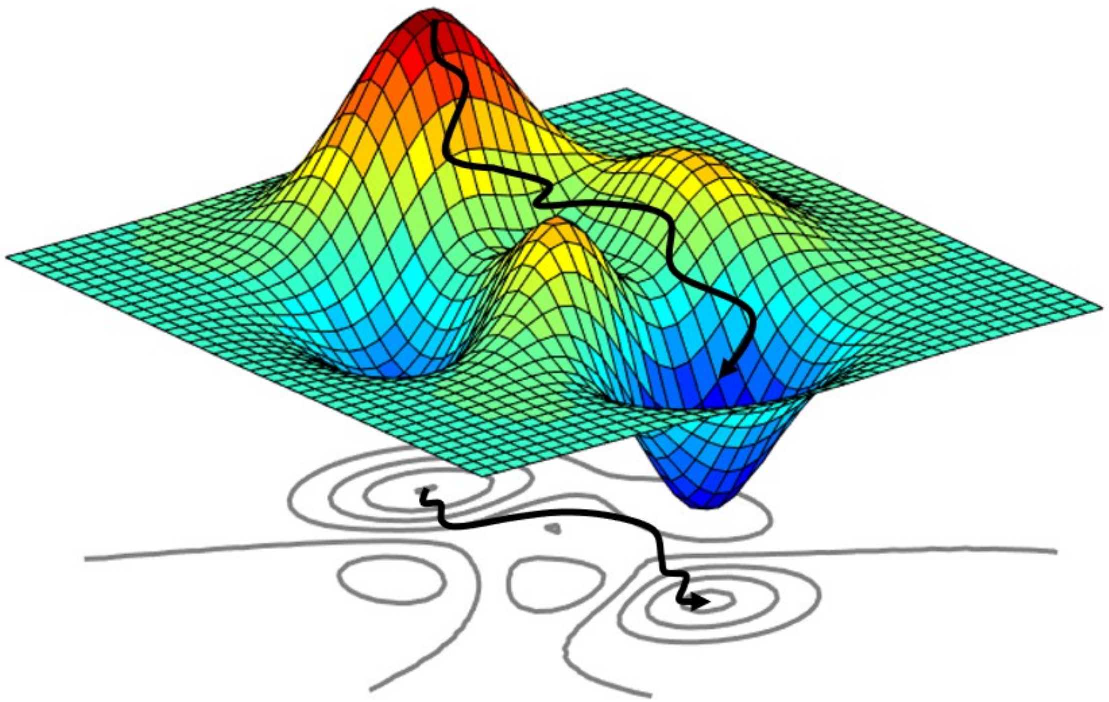
$$
\begin{array}{llll}
\min & f(\mathbf{x}) & & \\
\text{subject to} & g_i(\mathbf{x}) = c_i & \text{for } i = 1, \ldots, n & \text{Equality constraints} \\
& h_j(\mathbf{x}) \geqq d_j & \text{for } j = 1, \ldots, m & \text{Inequality constraints}
\end{array}
$$

where $g_i(x)$ and $h_j(x)$ are constraints that are required to be satisfied, and $f(x)$ is the objective function that needs to be optimized subject to the constraints

https://en.wikipedia.org/wiki/Constrained_optimization

Multimodal optimization

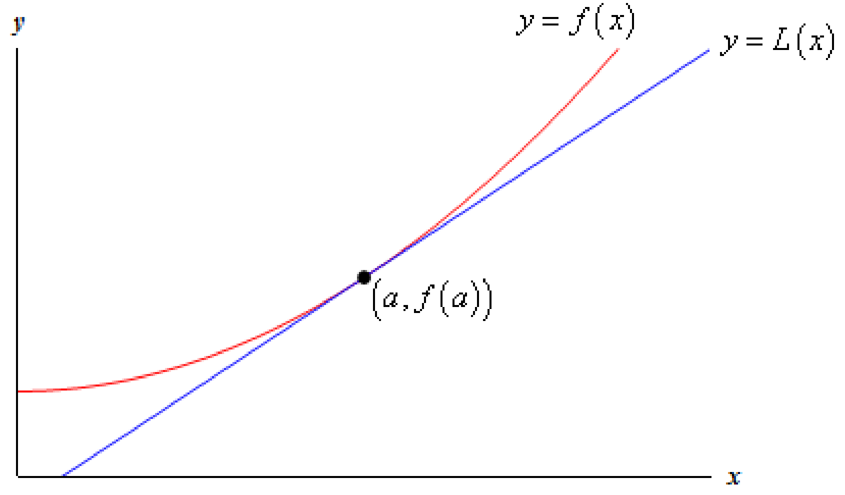
- Finds all or most of the multiple (at least locally optimal) solutions of a problem, as opposed to a single best solution.
  - Evolutionary Multimodal Optimization is a branch of **Evolutionary Computation**
- **Evolutionary algorithms:**
  - Genetic Algorithms (GAs),
  - Evolution Strategy (ES),
  - Differential Evolution (DE),
  - Particle Swarm Optimization (PSO) etc.



https://en.wikipedia.org/wiki/Evolutionary_multimodal_optimization

Approximation

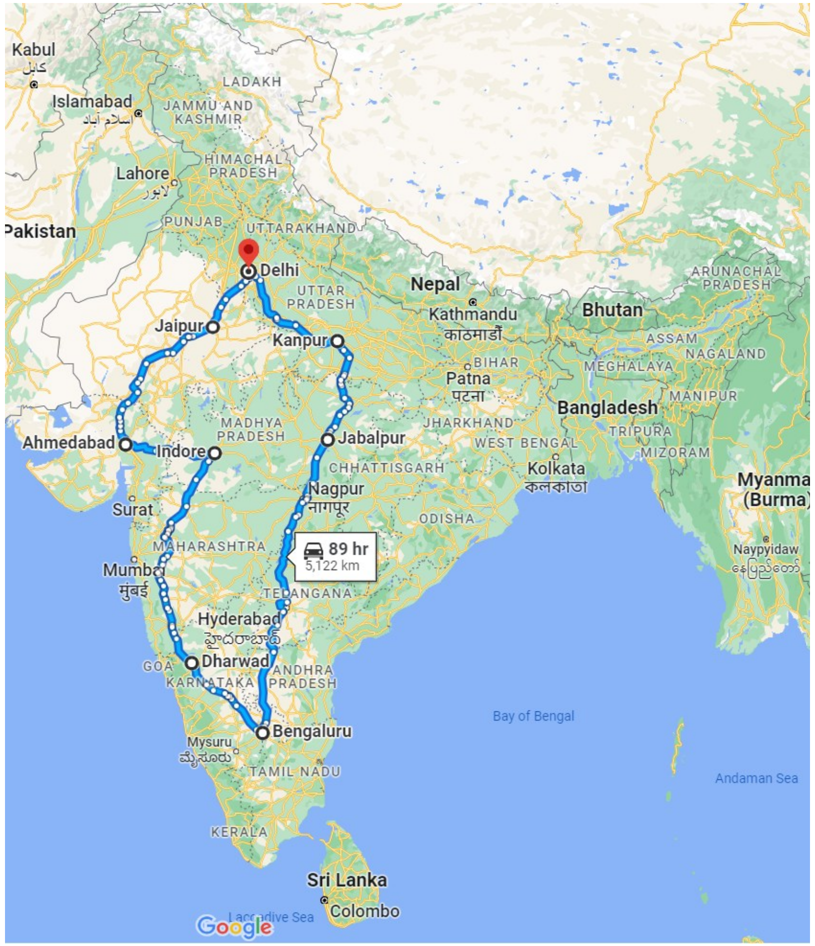
- Efficient algorithms that find **approximate solutions** to NP optimization problems.
- Solution with provable guarantees on the distance of the returned solution to the optimal one.
- Example:
  - Vertex Cover Problem,
  - Traveling Salesman Problem,
  - Set-covering problem (resource-selection problems)
  - Subset-sum problem



https://en.wikipedia.org/wiki/Approximation_algorithm

Approximation

- Karp (1972) proved the TSP to be NP-hard, but effective heuristic approximation methods were developed (Lin and Kernighan, 1973).
- The traveling-salesperson problem (TSP) is a standard combinatorial problem in theoretical computer science (Lawler et al., 1992).
- Arora (1998) devised a fully polynomial approximation scheme for Euclidean TSPs.



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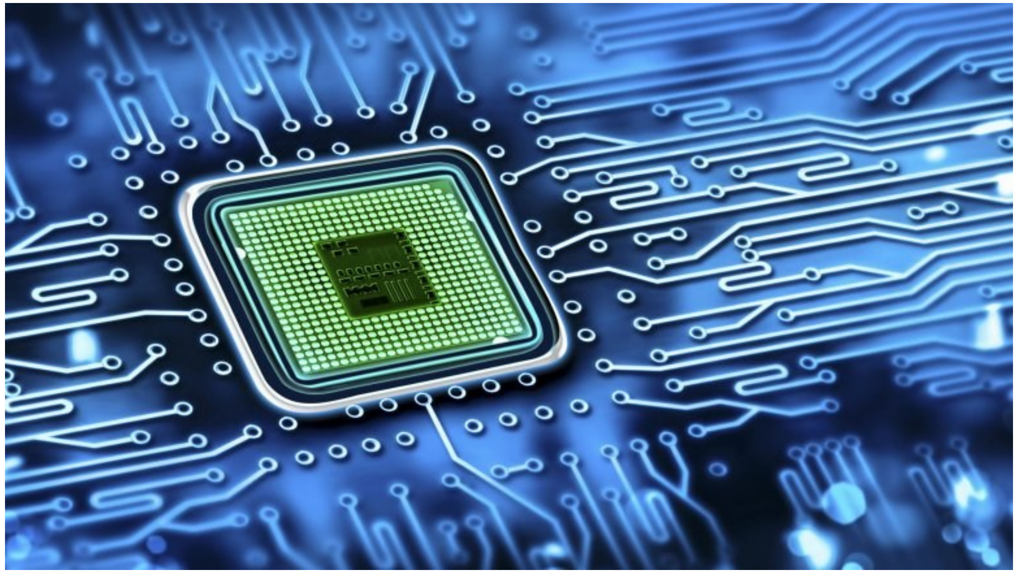
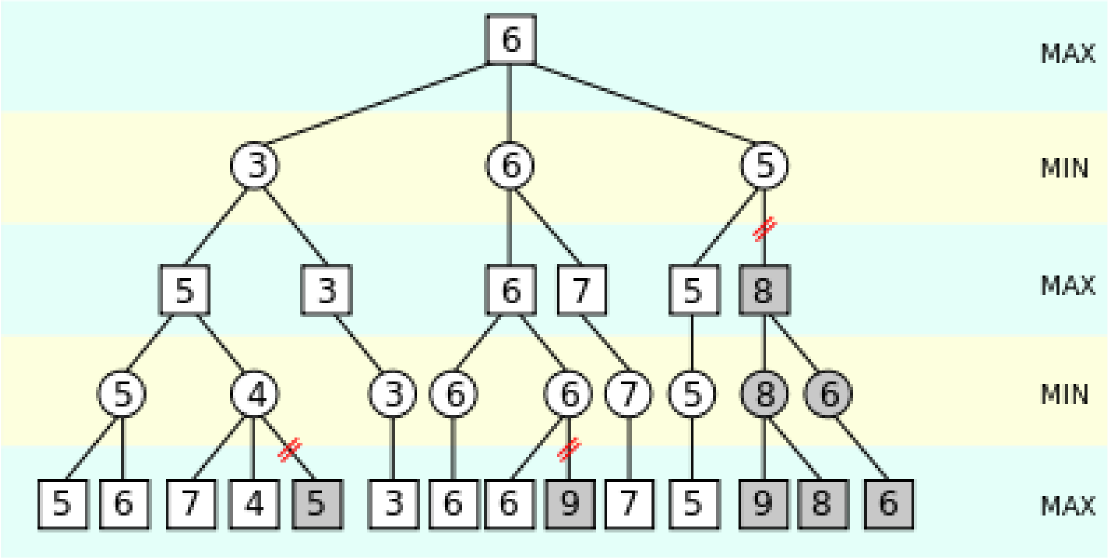
Stuart Russel, and Peter Norvig. "Artificial intelligence: A modern approach. Third edit." Upper Saddle River, New Jersey 7458 (2015). http://aima.cs.berkeley.edu/

Approximation

- VLSI layout methods are surveyed by Shahookar and Mazumder (1991), and many layout optimization papers appear in VLSI journals.
- Approximation in Mini-Max is to cut the search off at some point





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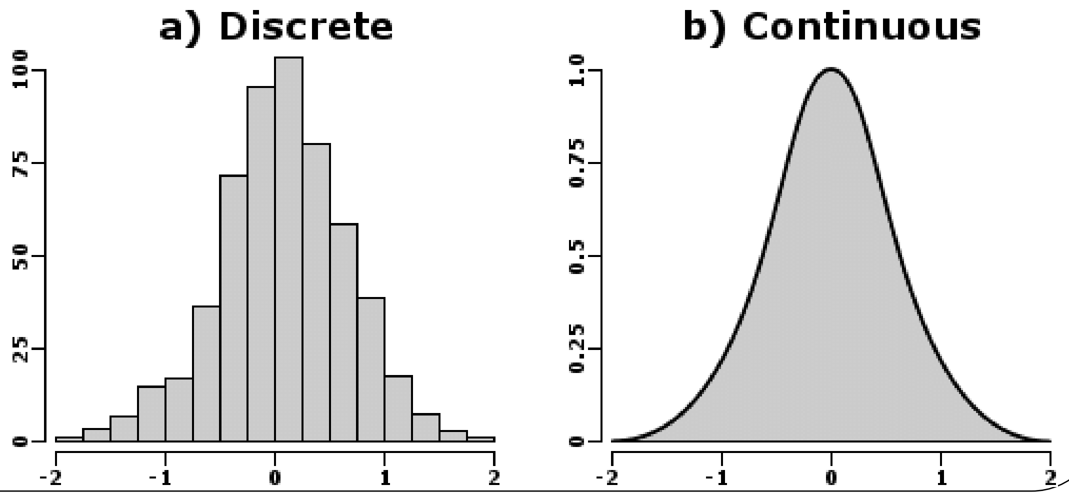
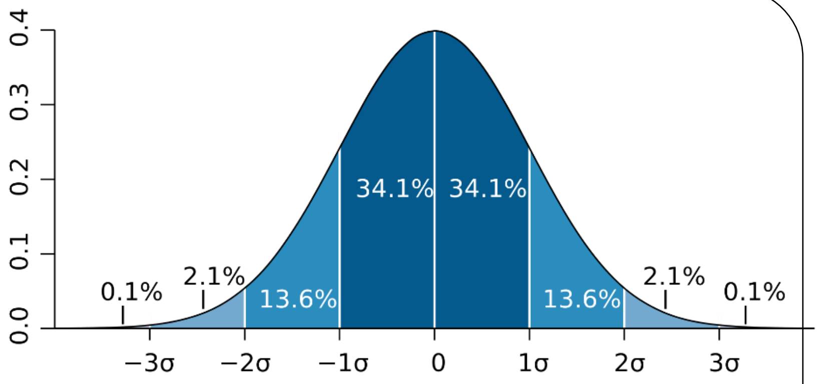
Probabilistic Vs Stochastic

- Deterministic system are predictable.
  - The state of the system can be forecasted for given input, constraints, and mathematical model.
  - States of a deterministic system are pre-determined.
- Non-Deterministic system are unpredictable
  - Stochastic and Probabilistic are usually used interchangeably.
  - Both represent the randomness present in the system.
  - Probabilistic is superset concept of stochastic.



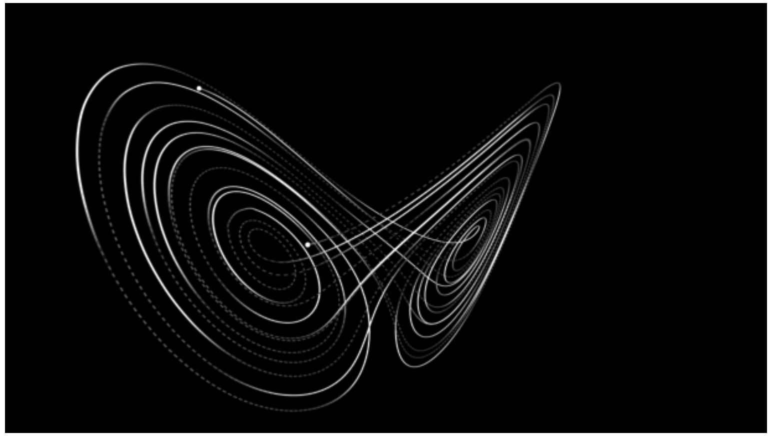
Probabilistic Vs Stochastic



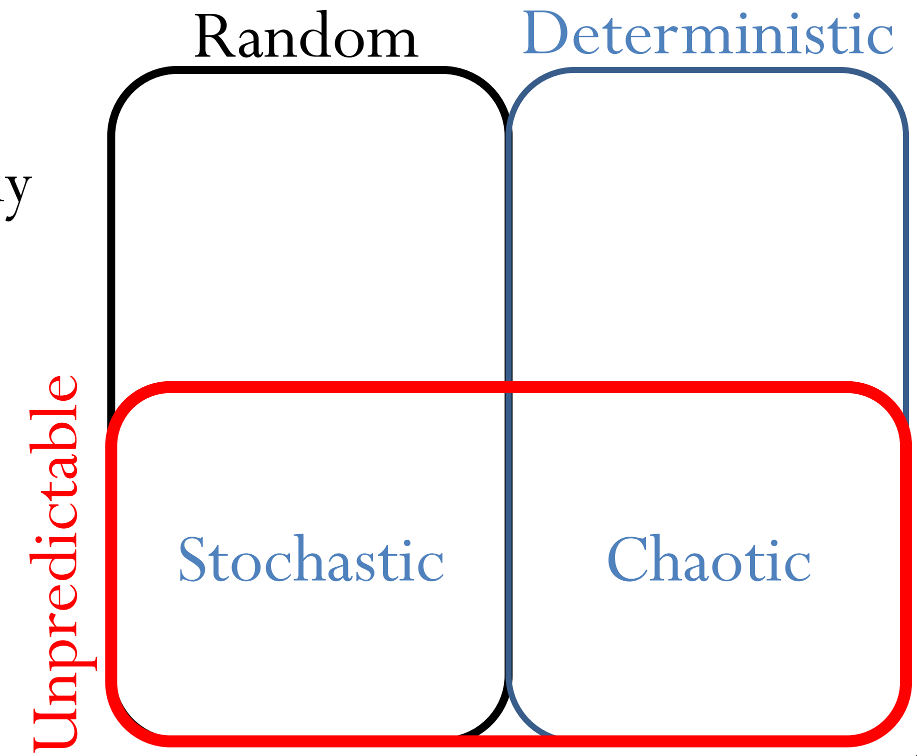
- Probabilistic models are independent of time,
  - which describes system with numerical chances or likelihood of an event to occur.
  - E.g., Lottery numbers are independent of each other.
  - Each instance is determined by the same probability distributions, but with no memory of older instance.
- Stochastic models are time dependent systems,
  - whose changes are described by its past and probabilities for successive changes.
  - E.g., Price of a stock is its old price and an uncertain change.
  - The uncertainty are small, which is semi-predictable.
  - If the stock was closed at 100,
    - then its opening value is predictable around 90 or 110.

https://qr.ae/pvU18E
https://qr.ae/pvU11n

Stochastic and Chaotic



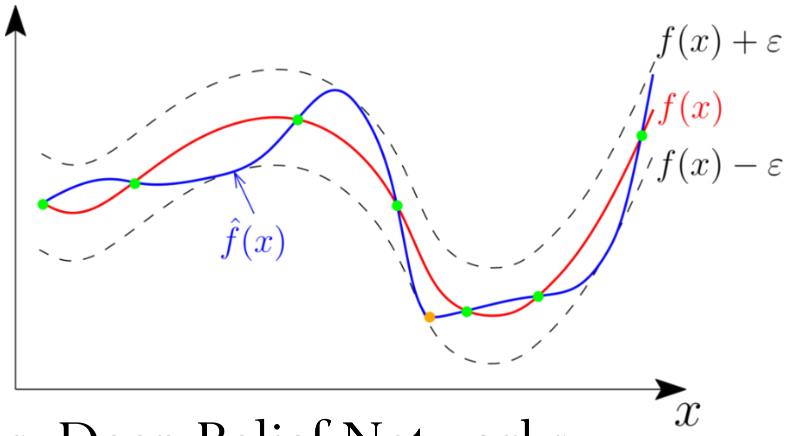
- A **chaotic** system is deterministic in theory.
  - It responds drastically to infinitesimal changes in initial and boundary conditions, making it in practice unpredictable and unstable.
- Deterministic chaos, or simply Chaos.
- **Chaos:** “When the present determines the future, but the approximate present does not approximately determine the future.”
- A **stochastic** system is a random phenomenon.
- Stochastic and Chaotic two terms interchangeably.
- It is hard to distinguish between chaotic and stochastic systems.

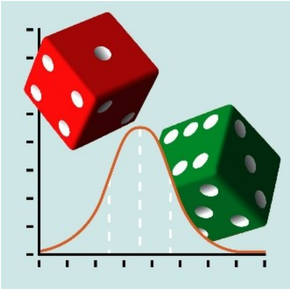


https://en.wikipedia.org/wiki/Chaos_theory
https://qr.ae/pvU11A

Optimization, Approximation, and Stochastic

- Deterministic nature of a system does not make it predictable.
- Exhaustive search is intractable for NP-hard problems
- Used Optimization, Approximations, and Probabilistic algorithms in
  - Travelling Salesman Problem (TSP), Minimum Spanning Tree (MST), and Knapsack
  - Minimax, Alpha–Beta pruning,
  - Monte Carlo, Markov chain Monte Carlo,
  - Constraint Satisfaction Problems,
  - Likelihood weighting, Maximum-Likelihood,
  - Stochastic Differential Equations (SDEs),
  - Belief states, Belief propagation, Bayesian Networks, Deep Belief Networks,
  - Various Machine Learning, Deep Learning, Reinforcement Learning algorithms,
  - and several other problems





https://mattiheino.com/2016/12/27/deterministic-not-predictable/

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- Images are from several sources e.g. movies, TV serials, internet, miscellaneous links, slides, blogs, etc.
- Quora
  - https://qr.ae/pvU18E
  - https://qr.ae/pvU11n
  - https://en.wikipedia.org/wiki/Chaos_theory
  - https://qr.ae/pvU11A

תודה רבה
Hebrew

Ευχαριστώ
Greek

Спасибо
Russian

Danke
German

धन्यवादः
Sanskrit

Merci
French

ধন্যবাদ
Bangla

நன்றி
Tamil

شكرا
Arabic

ಧನ್ಯವಾದಗಳು
Kannada

Thank You
English

നന്ദി
Malayalam

Grazie
Italian

多謝
Traditional Chinese

ధన్యవాదాలు
Telugu

આભાર
Gujarati

ਧੰਨਵਾਦ
Punjabi

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