Deterministic and Non-Deterministic

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Computational complexity theory

- Fields in
  - Theoretical Computer Science
  - Analysis of Algorithms
- An algorithm solves a computation problem by mathematical steps.
- A computational problem (such as an algorithm) is a task solved by a computer.
- Focuses on classifying computational problems according to the resource usage
- Resource usage: amount of resources needed to solve computational problem,
- Resources: such as time and storage.

https://en.wikipedia.org/wiki/Computational_complexity_theory

Single Source Shortest-Paths Implementation

algorithm	restriction	typical case	worst case	extra space
topological sort	no directed cycles	$E + V$	$E + V$	V
Dijkstra (binary heap)	no negative weights	$E \log V$	$E \log V$	V
Bellman–Ford	no negative cycles	$E\,V$	$E\,V$	V
Bellman–Ford (queue–based)		$E + V$	$E\,V$	V

Easy

Medium

Hard

Remark 1. Directed cycles make the problem harder.

Remark 2. Negative weights make the problem harder.

Remark 3. Negative cycles makes the problem intractable.

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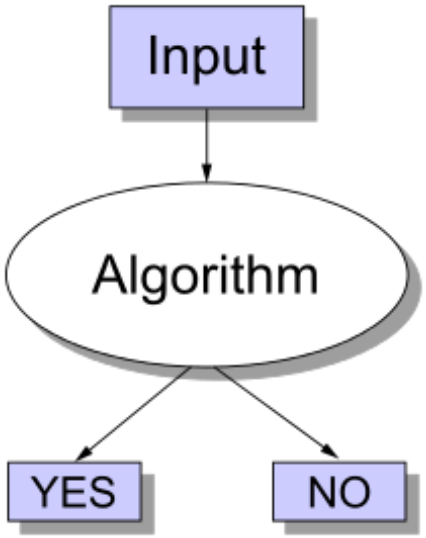
Deterministic algorithm

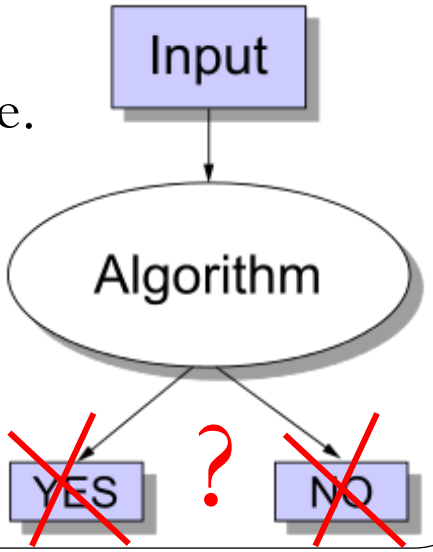
- Given a particular input, will always produce the same output, with the underlying machine always passing through the same sequence of states.
- State machine: a state describes what a machine is doing at a particular instant in time.
- State machines pass in a discrete manner from one state to another.
- Enter the input, initial state or start state.
- Current state determines what will be next state, the set of states is predetermined.



https://en.wikipedia.org/wiki/Deterministic_algorithm

Non-deterministic Algorithms

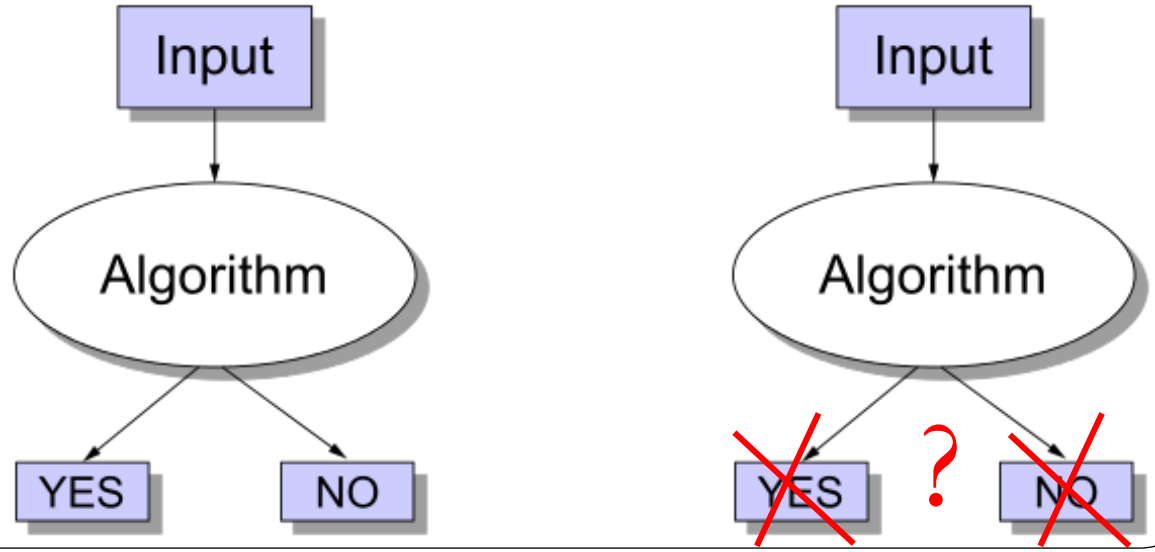
- If it uses external state other than the input, such as
  - user input,
  - a global variable,
  - a hardware timer value,
  - a random value, or
  - stored disk data.
- If it is timing-sensitive,
  - e.g. if it has multiple processors writing to the same data at the same time.
- If a hardware error causes its state to change in an unexpected way.
- The order each processor writes data will affect the result.



https://en.wikipedia.org/wiki/Deterministic_algorithm

Deterministic and Non-deterministic Algorithms

- Disadvantages of Determinism
  - predictable future by players or predictable security by hacker
  - e.g. predictable card shuffling program or security key
- Pseudorandom number generator is often not sufficient,
  - thus cryptographically secure pseudo-random number generator,
  - hardware random number generator.



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P (Polynomial) Time Problems

- Contains all decision problems that can be solved deterministically using a polynomial time (polynomial amount of computation time).
- A problem is in P-complete, if it is in P
- P is the class of computational problems that are "efficiently solvable" or "tractable".
- Class P, typically take all the "tractable" problems for a sequential algorithm,
- But, in reality, some problems not known to be solved in polynomial P time.

https://en.wikipedia.org/wiki/P_(complexity)
https://en.wikipedia.org/wiki/P-complete

P (Polynomial) Time Problems

- Programmable Function (or method) is polynomial-time
  - if completes in constant-time or polynomial time,
  - then the entire algorithm takes polynomial time.
- Polynomial-time algorithms:
  - Minimum Spanning Tree: Kruskal's $\mathbf{O(}E \lg V\mathbf{)}$ and Prim's $\mathbf{O(}E + V \lg V\mathbf{)}$ algorithm
  - Shortest Path Algorithms: Djikstra's **O(E lgV)** and Bellman-Ford's $\mathbf{O(}EV\mathbf{)}$ algorithm

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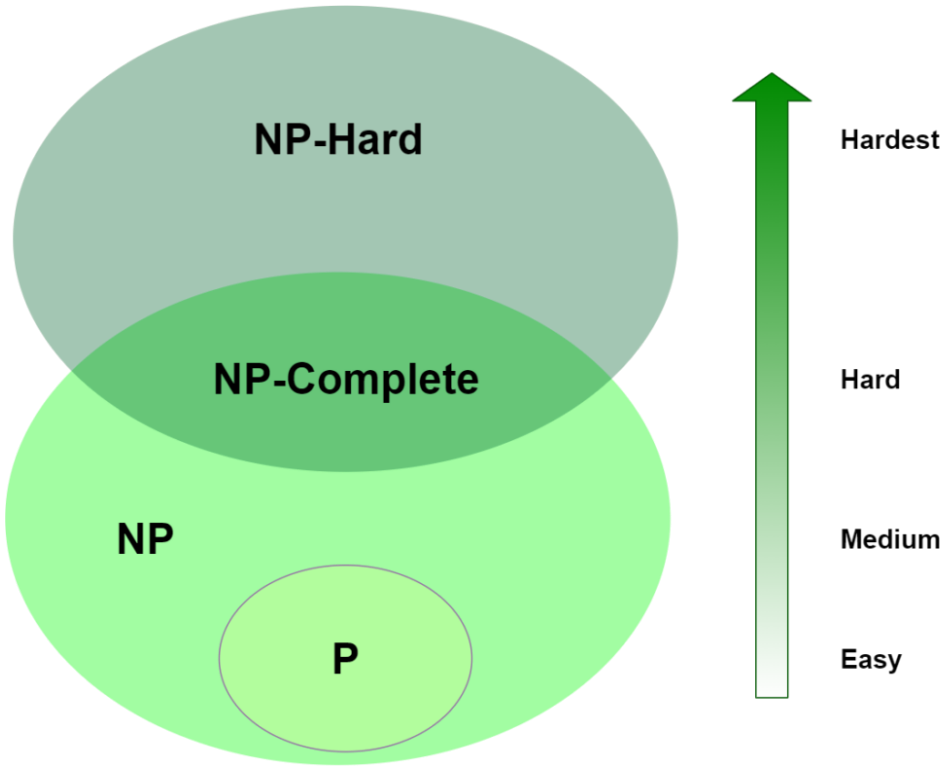
NP - Naming convention

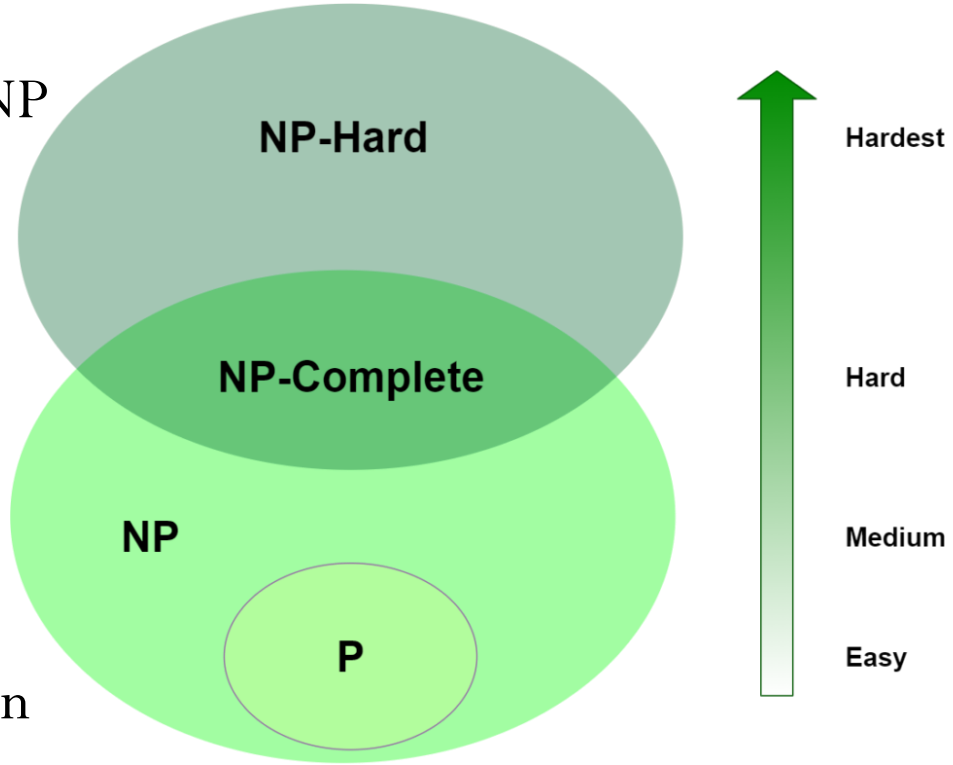
- Classification
  - Hardest → NP-Hard
  - Hard → NP-Complete
  - Medium → NP
  - Easy →P
- Order of N inputs
  - O(1) – constant-time
  - $O(\log_2(n))$ – logarithmic-time
  - O(n) – linear-time
  - $O(n^2)$ – quadratic-time
  - $O(n^k)$ – polynomial-time
  - $O(k^n)$ – exponential-time
  - O(n!) – factorial-time



https://www.baeldung.com/cs/p-np-np-complete-np-hard

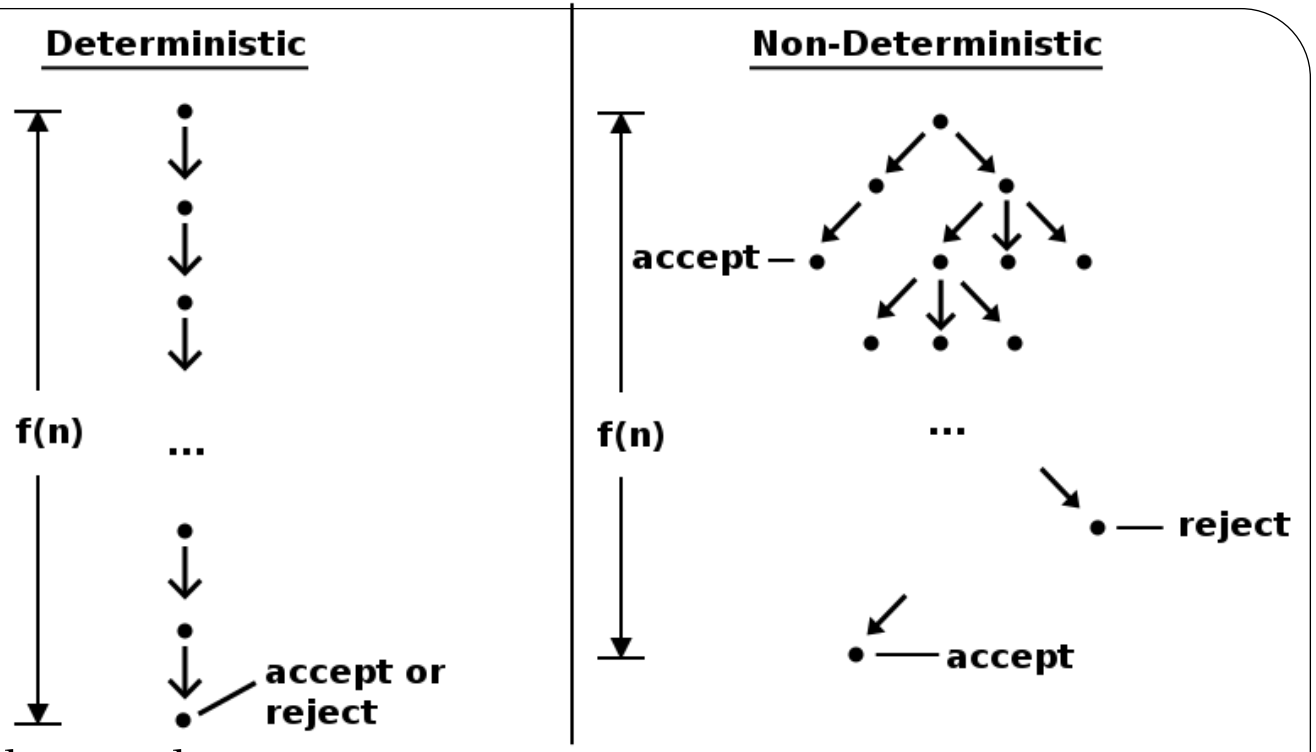
NP - Naming convention

- NP-hard: Class of problems are at least as hard as the hardest problems in NP.
- NP-hard problems do not have to be in NP; means NP hard problem may not even be decidable.
- NP-complete: Class of decision problems which contains the hardest problems in NP. Each NP-complete problem has to be in NP.
- NP: Class of computational decision problems for which any given yes-solution can be verified as a solution in polynomial time
- NP-easy: At most as hard as NP, but not necessarily in NP.



https://en.wikipedia.org/wiki/NP-hardness
https://www.baeldung.com/cs/p-np-np-complete-np-hard

P and NP Problems



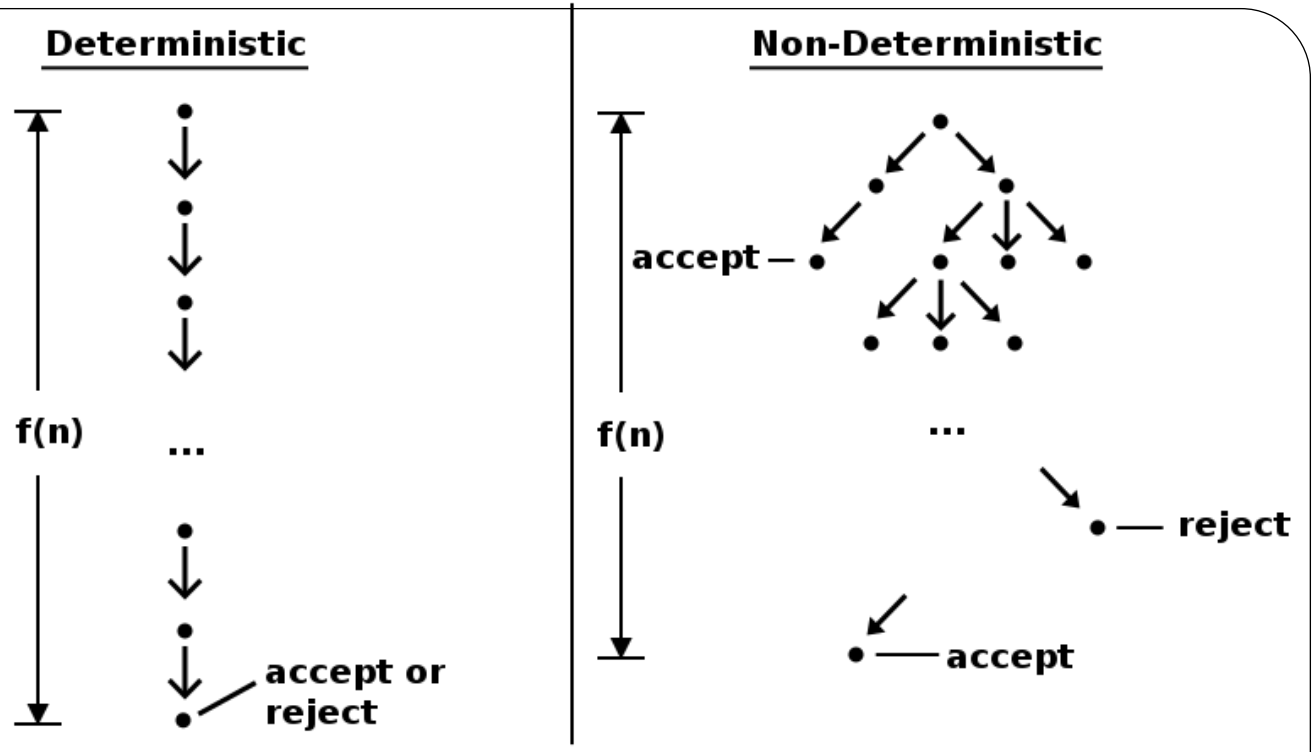
- Nondeterministic Polynomial-time
- “Nondeterministic” refers to
  - “luckiest possible guesser”
- "Complete" refers to
  - “in the same complexity class”
- **P** versus **NP** determine
  - whether a problem can be verified in polynomial time
  - whether the problem can also be solved in polynomial time.
- If it turned out that **P** ≠ **NP**, (widely accepted/believed),
- There are problems in **NP** that are harder to compute than to verify:
- NP problems could not be solved in polynomial time, but the answer could be verified in polynomial time.

https://en.wikipedia.org/wiki/P_versus_NP_problem

NP Complete

- Nondeterministic Polynomial-time

Complete

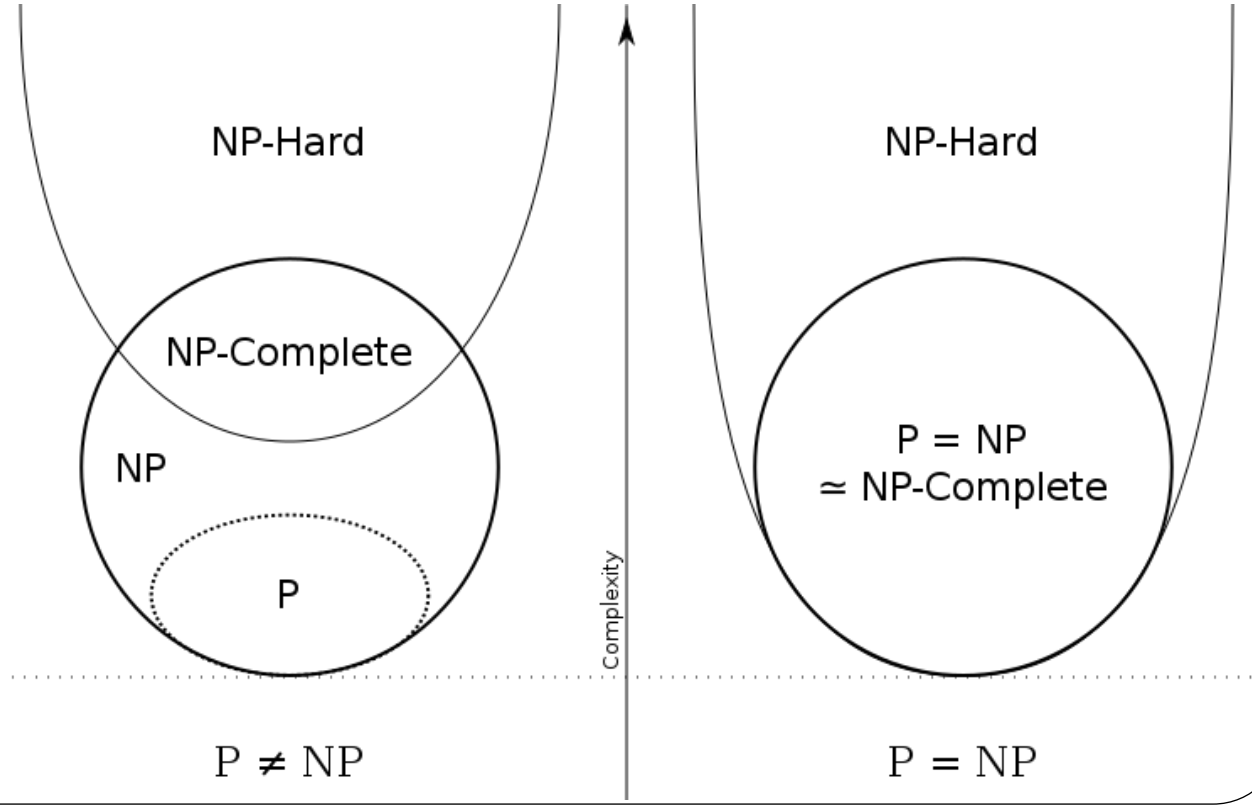


- A problem is NP-complete when:
  - a brute-force search algorithm can solve it, and the correctness of each solution can be verified quickly, and
  - the problem can be used to simulate any other problem with similar solvability.
- NP-complete problem can be *verified* "quickly",
- There is no known way to *find* a solution quickly.

https://en.wikipedia.org/wiki/NP-completeness

NP - Hard Problems

- Non-Deterministic Polynomial-time hardness
- At least as hard as the hardest problems in NP
- The might be some polynomial-time algorithms for NP-hard problems but might not been discovered yet
- NP-hard but not NP-complete
  - halting problem: "given a program and its input, will it run forever?"
  - traveling salesman problem



https://en.wikipedia.org/wiki/NP-hardness

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- Wikipedia pages https://www.wikipedia.org/
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Ευχαριστώ
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Danke
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French

ধন্যবাদ
Bangla

நன்றி
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Arabic

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Thank You
English

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多謝
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