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# Constraint Satisfaction Problems (CSPs)

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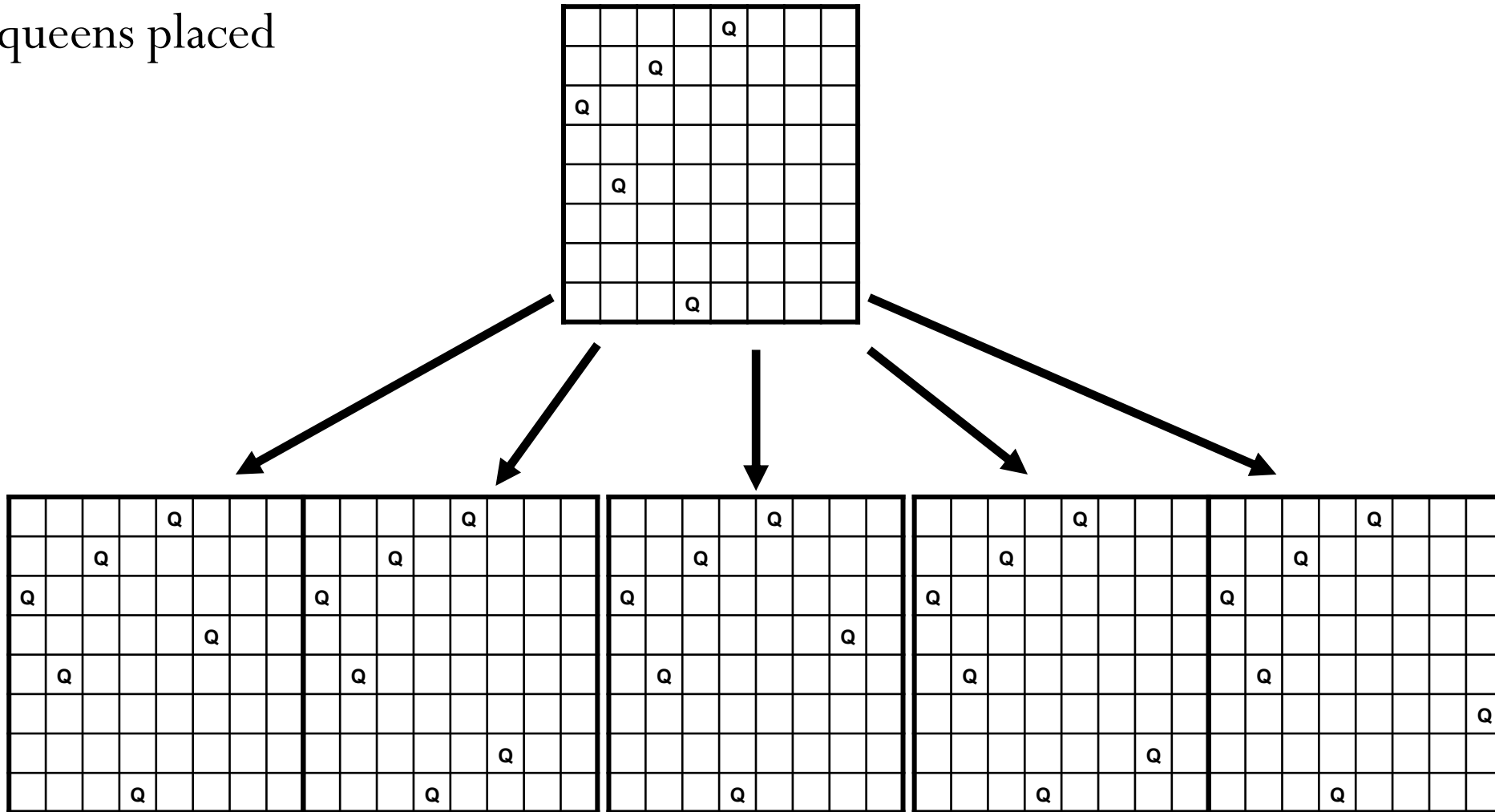
# Queens puzzle

- Place eight queens on a chessboard so that no two attack each other

				Q			
		Q					
Q							
						Q	
	Q						
							Q
					Q		
			Q				

# Search formulation of the queens puzzle

- Successors: all valid ways of placing additional queen on the board; goal: eight queens placed



# Keeping track of remaining possible values

- For every variable, keep track of which values are still possible
- General heuristic: branch on variable with fewest values remaining

				Q	X	X	X
		Q			X	X	X
Q					X	X	X
							X
	Q				X	X	X
					X	X	
							X
			Q		X	X	X

only one possibility for last  
column; might as well fill in

				Q	X	X	
		Q			X	X	
Q					X	X	
					X		
	Q				X	X	
					X	X	Q
						X	
			Q		X	X	

now only one left for other  
two columns

				Q			
		Q					
Q							
						Q	
	Q						
							Q
					Q		
			Q				

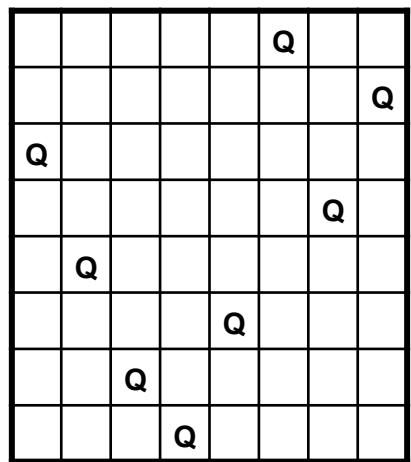
done!  
(no real branching needed!)

# Constraint Satisfaction Problems (CSPs)

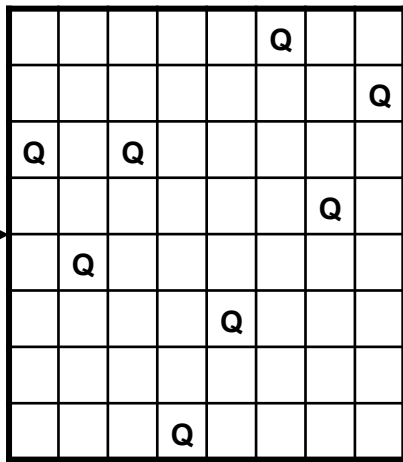
- Defined by:
  - A set of **variables**  $x_1, x_2, \dots, x_n$
  - A **domain**  $D_i$  for each variable  $x_i$
  - **Constraints**  $c_1, c_2, \dots, c_m$
- A constraint is specified by
  - A subset (often, two) of the variables
  - All the allowable joint assignments to those variables
- Goal: find a **complete, consistent** assignment
- Queens problem: (other examples in next slides)
  - $x_i$  in  $\{1, \dots, 8\}$  indicates in which row in the  $i^{\text{th}}$  column to place a queen
  - For example, constraint on  $x_1$  and  $x_2$ :  $\{(1,3), (1,4), (1,5), (1,6), (1,7), (1,8), (2,4), (2,5), \dots, (3,1), (3,5), \dots\}$

# Local search: hill climbing

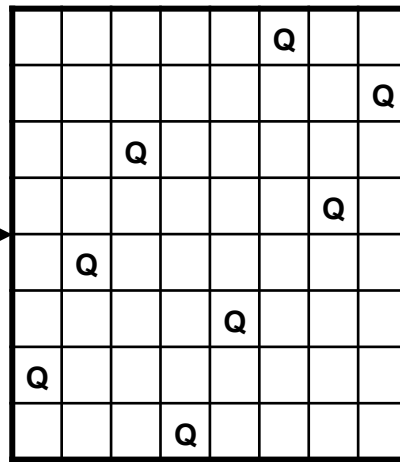
- Start with a complete state
- Move to successor with best (or at least better) objective value
  - Successor: move one queen within its column
- Local search can get stuck in a **local optimum**



4 attacking pairs

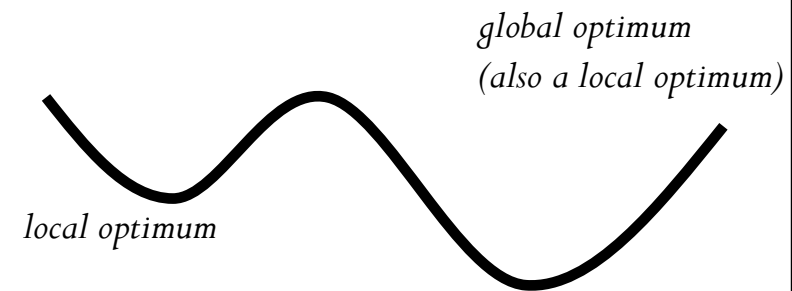


3 attacking pairs



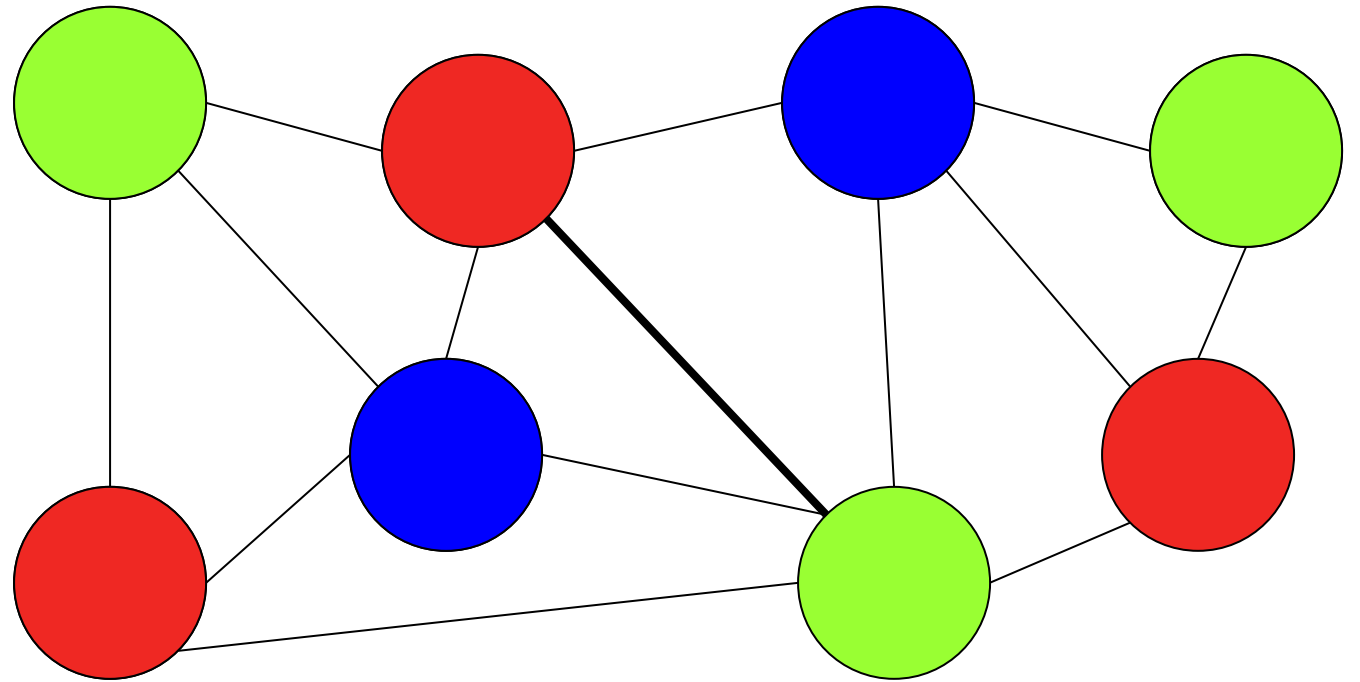
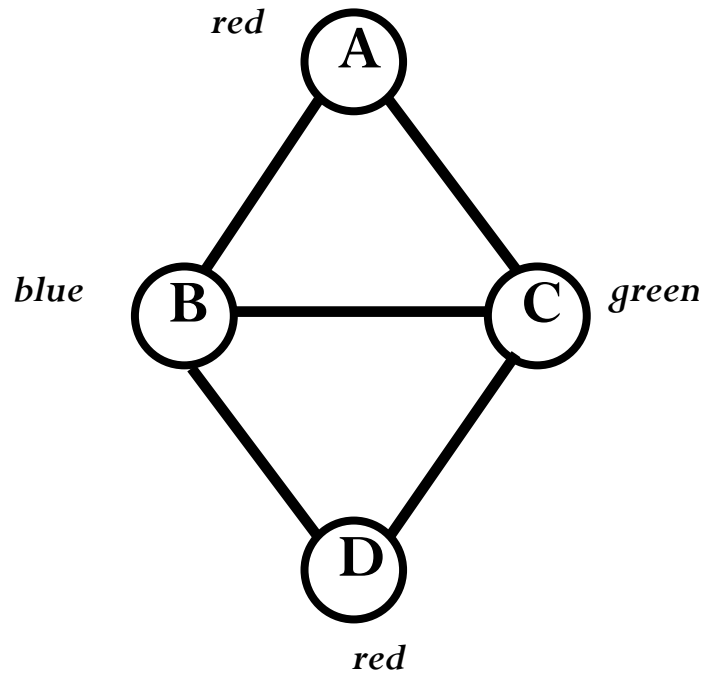
2 attacking pairs

no more  
improvements



# Graph coloring

- Fixed number of colors; no two adjacent nodes can share a color



# Constrained optimization

- Process of optimizing an objective function with respect to some variables in the presence of constraints on those variables.
  - primarily equality constraints, inequality constraints, and integer constraints
  - set of candidate solutions that satisfy all constraints is called the feasible set
- A general constrained minimization problem may be written as follows:

$$\begin{array}{ll} \min & f(\mathbf{x}) \\ \text{subject to} & g_i(\mathbf{x}) = c_i \quad \text{for } i = 1, \dots, n \quad \text{Equality constraints} \\ & h_j(\mathbf{x}) \geq d_j \quad \text{for } j = 1, \dots, m \quad \text{Inequality constraints} \end{array}$$

where  $g_i(\mathbf{x})$  and  $h_j(\mathbf{x})$  are constraints that are required to be satisfied, and  $f(\mathbf{x})$  is the objective function that needs to be optimized subject to the constraints



# References

- Vincent Conitzer, CPS 270: Artificial Intelligence  
<http://www.cs.duke.edu/courses/fall08/cps270/>

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