State Space Search

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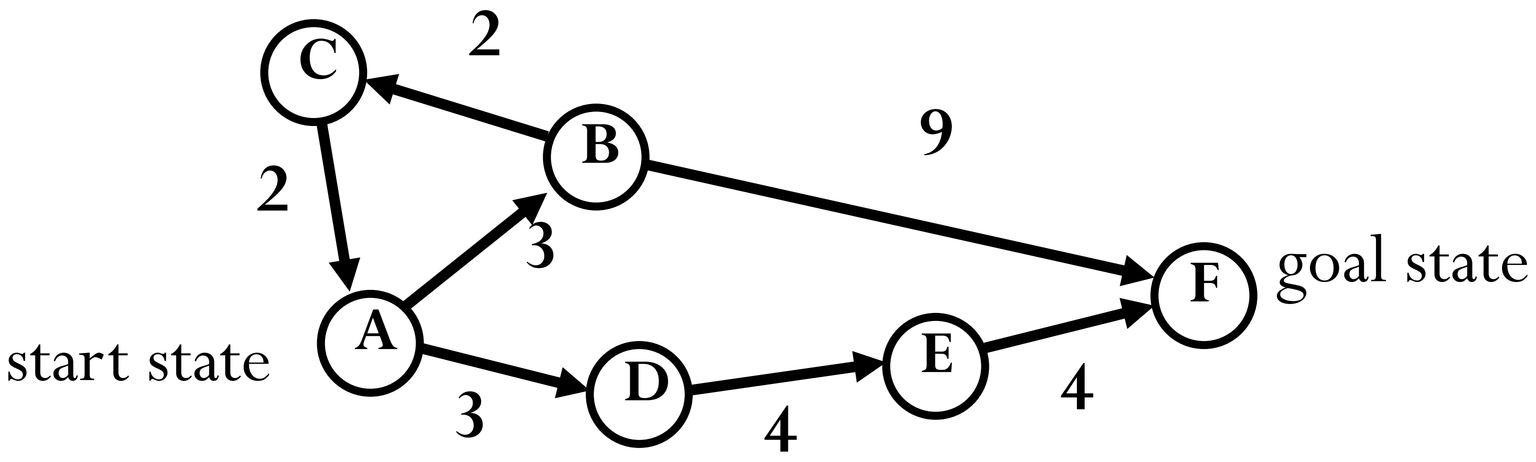
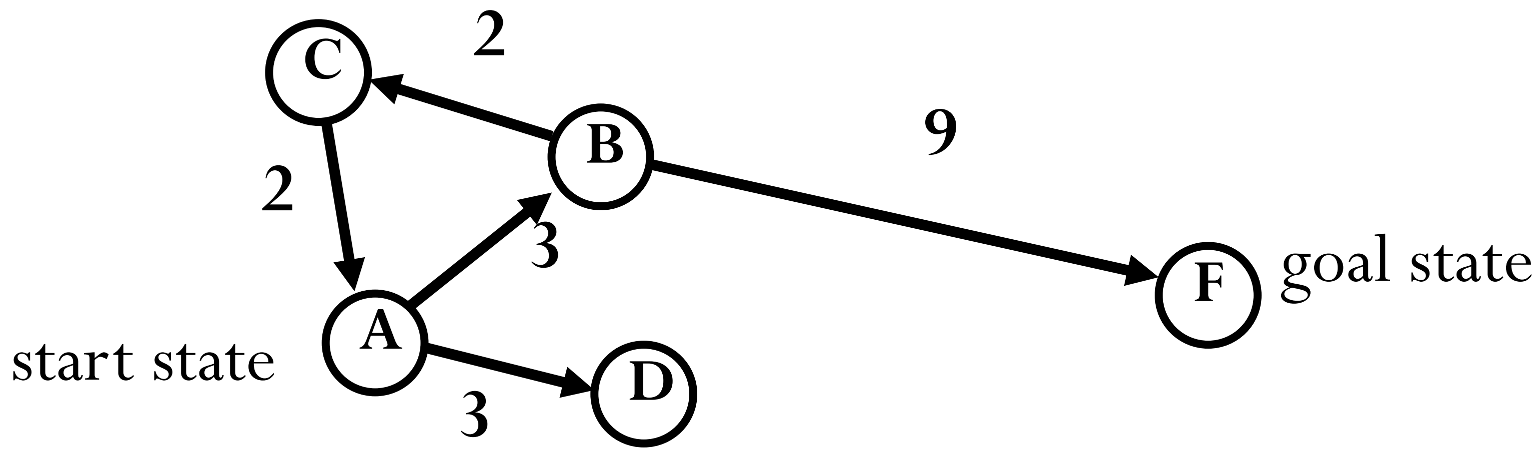
Search

- We have some actions that can change the state of the world
  - Change induced by an action perfectly predictable
- Try to come up with a sequence of actions that will lead us to a goal state
  - May want to minimize number of actions
  - More generally, may want to minimize total cost of actions
- Do not need to execute actions in real life while searching for solution!
  - Everything perfectly predictable anyway

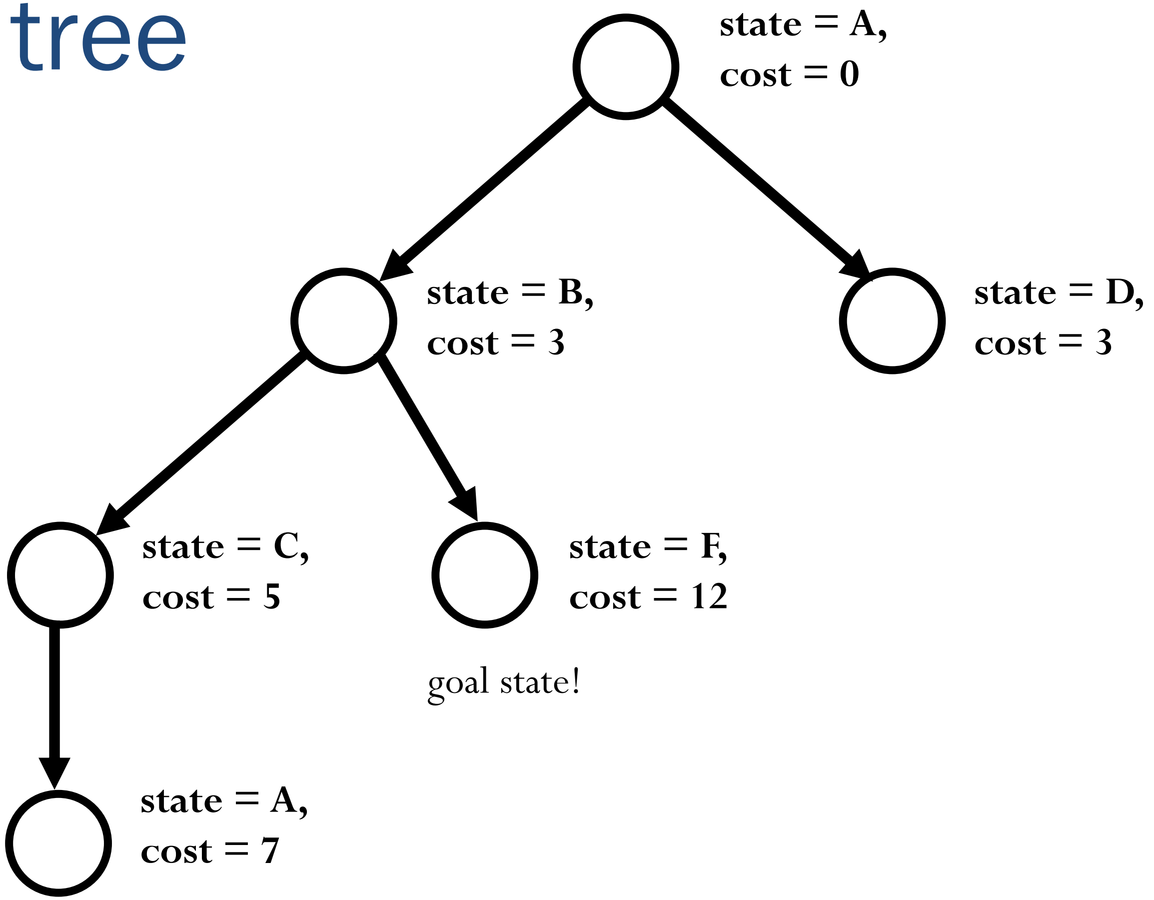
A simple example: traveling on a graph



Searching for a solution

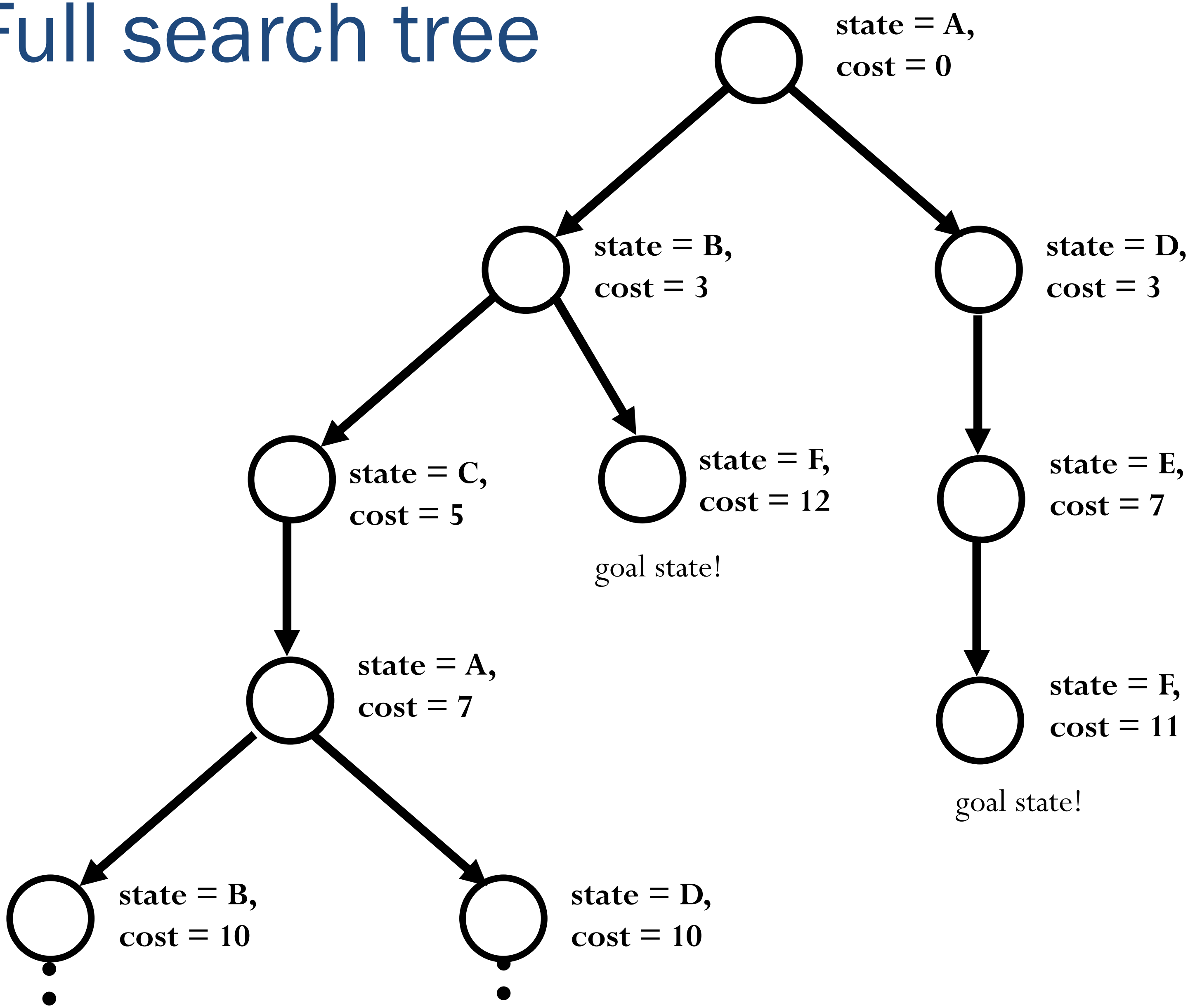


Search tree

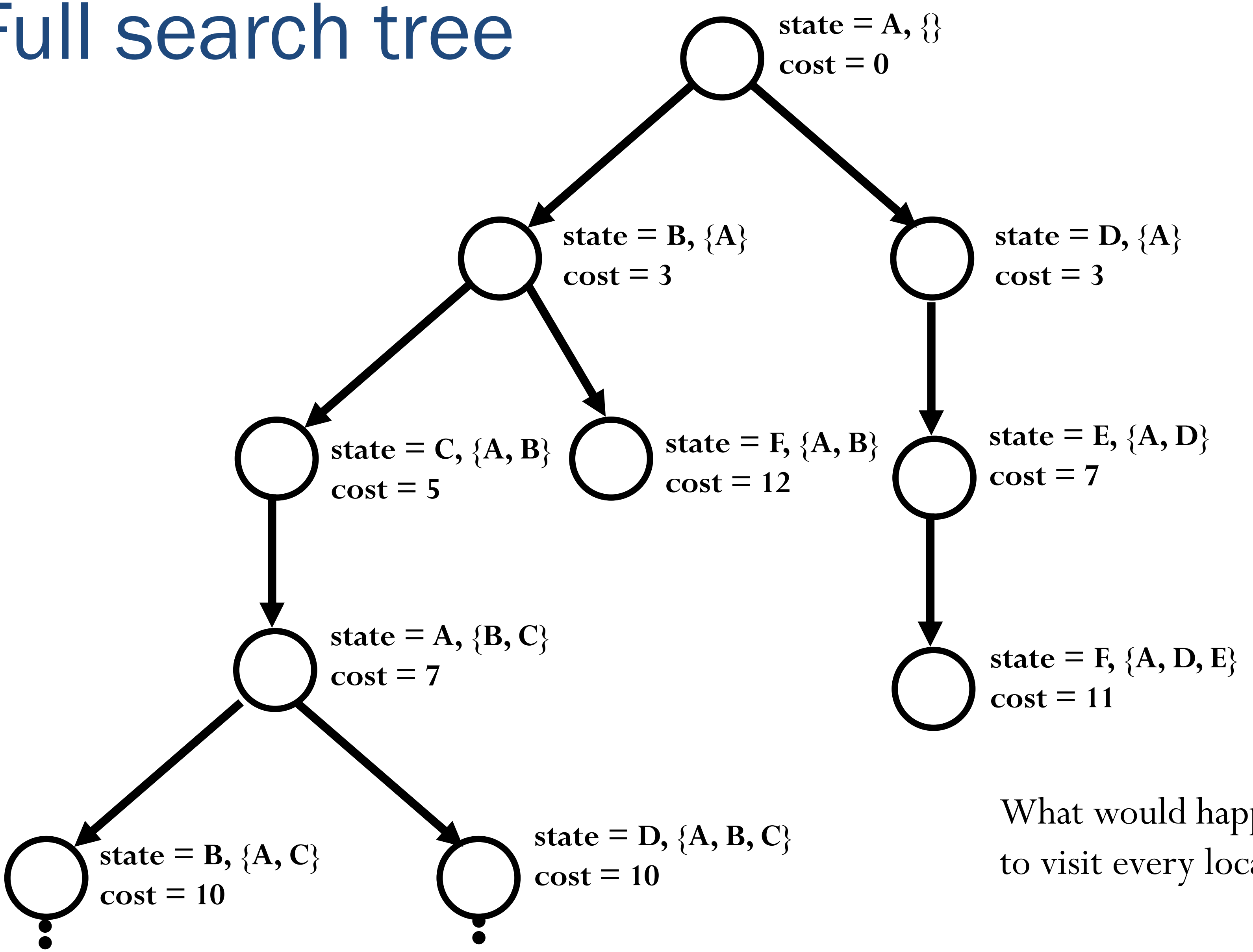


search tree nodes and states are not the same thing!

Full search tree



Full search tree



What would happen if the goal were to visit every location twice?

Key concepts in search

- Set of states that we can be in
  - Including an initial state…
  - … and goal states (equivalently, a goal test)
- For every state, a set of actions that we can take
  - Each action results in a new state
  - Typically defined by successor function
    - Given a state, produces all states that can be reached from it
- Cost function that determines the cost of each action (or path = sequence of actions)
- Solution: path from initial state to a goal state
  - Optimal solution: solution with minimal cost

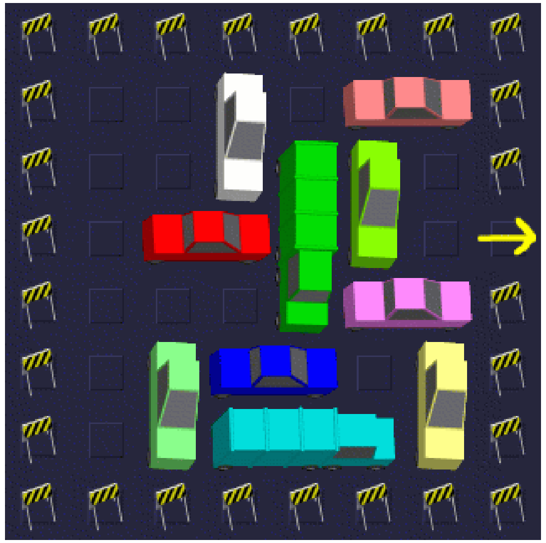
Uninformed search

Uninformed search

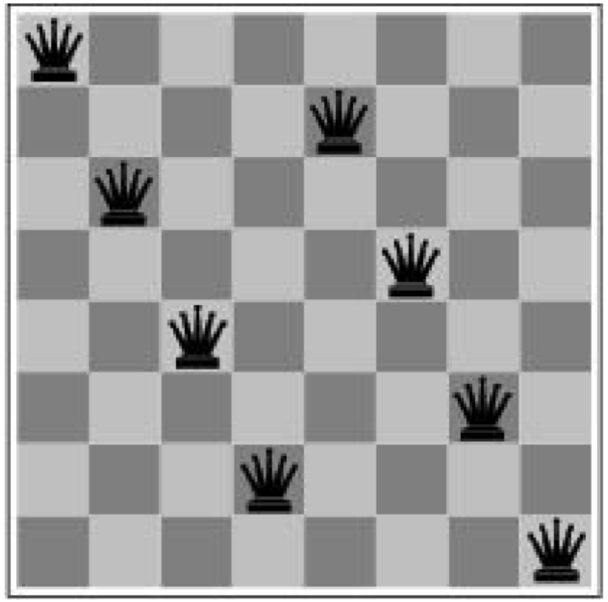
- Given a state, we only know whether it is a goal state or not
- Cannot say one nongoal state looks better than another nongoal state
- Can only traverse state space blindly in hope of somehow hitting a goal state at some point
  - Also called blind search
  - Blind does **not** imply unsystematic!

Searching Examples

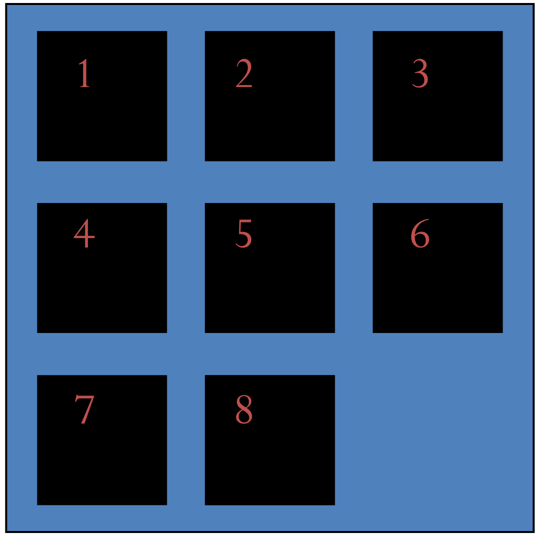
Rush Hour: Move cars forward and backward to “escape”



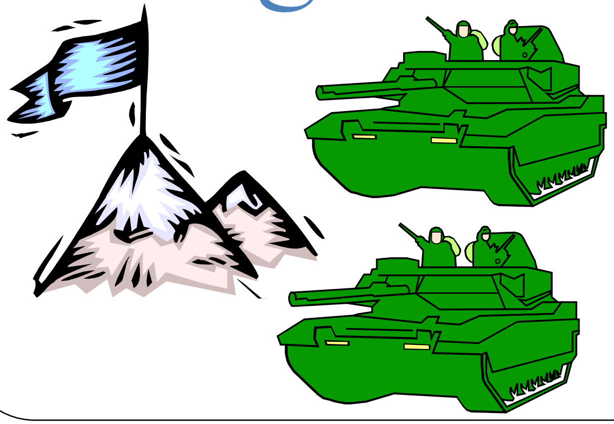
8-queens problem



8-puzzle



Logistics



Generic search algorithm

- Fringe = set of nodes generated but not expanded
- fringe := {initial state}
- loop:
  - if fringe empty, declare failure
  - choose and remove a node v from fringe
  - check if v's state s is a goal state; if so, declare success
  - if not, expand v, insert resulting nodes into fringe
- Key question in search: Which of the generated nodes do we expand next?

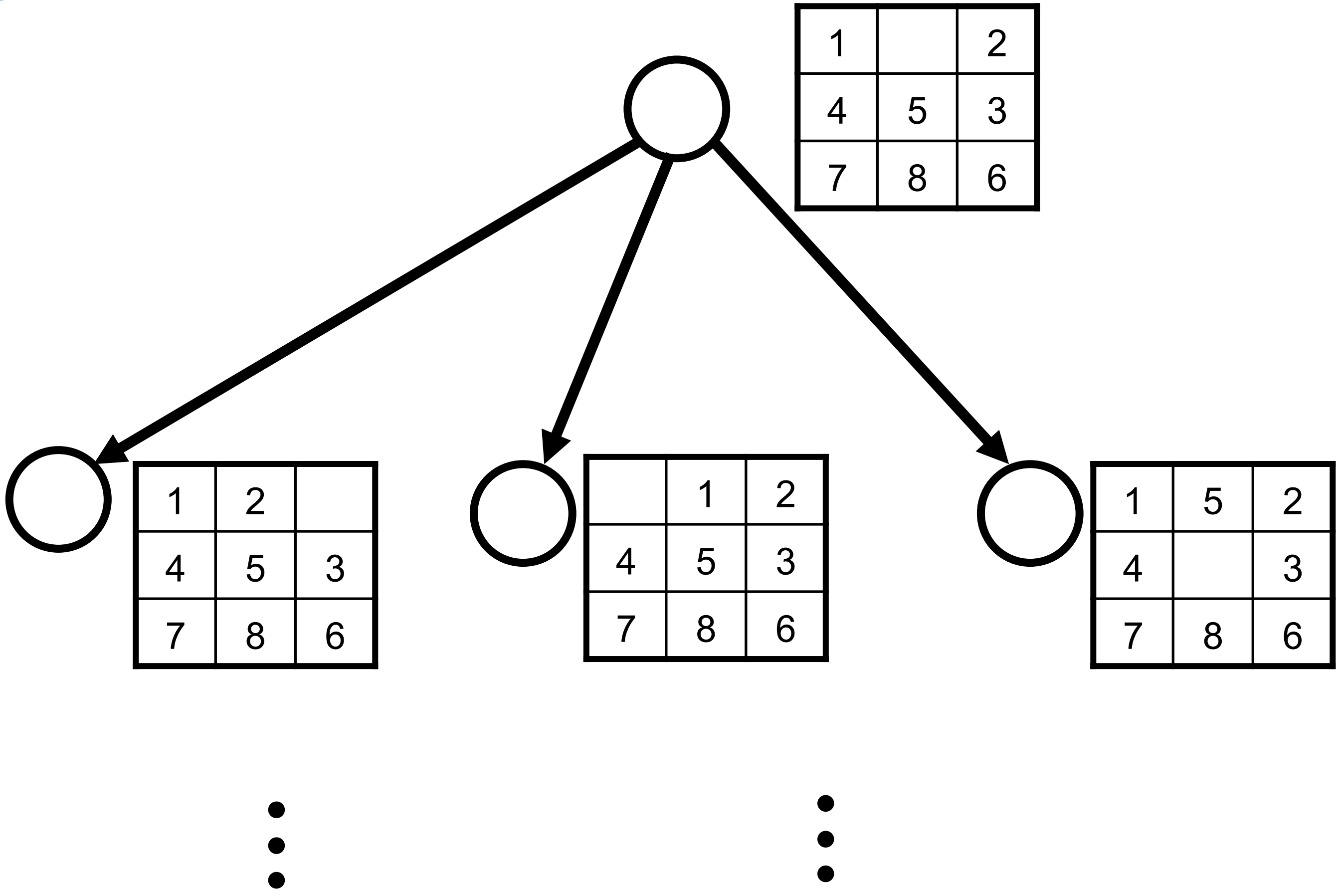
8-puzzle

1		2
4	5	3
7	8	6

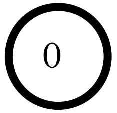
1	2	3
4	5	6
7	8	

goal state

8-puzzle



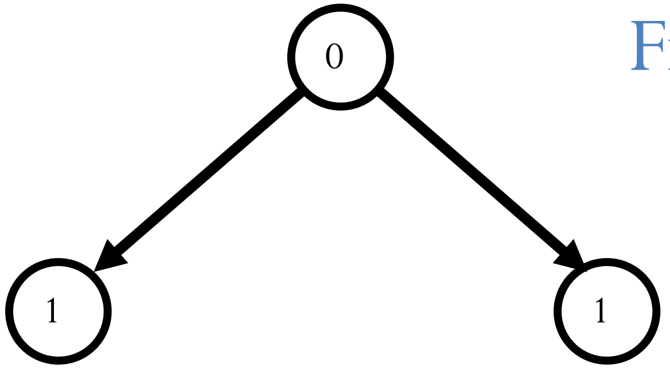
Breadth-First Search



First-In-First-Out (FIFO) Queue

Animated PPT

Breadth-First Search



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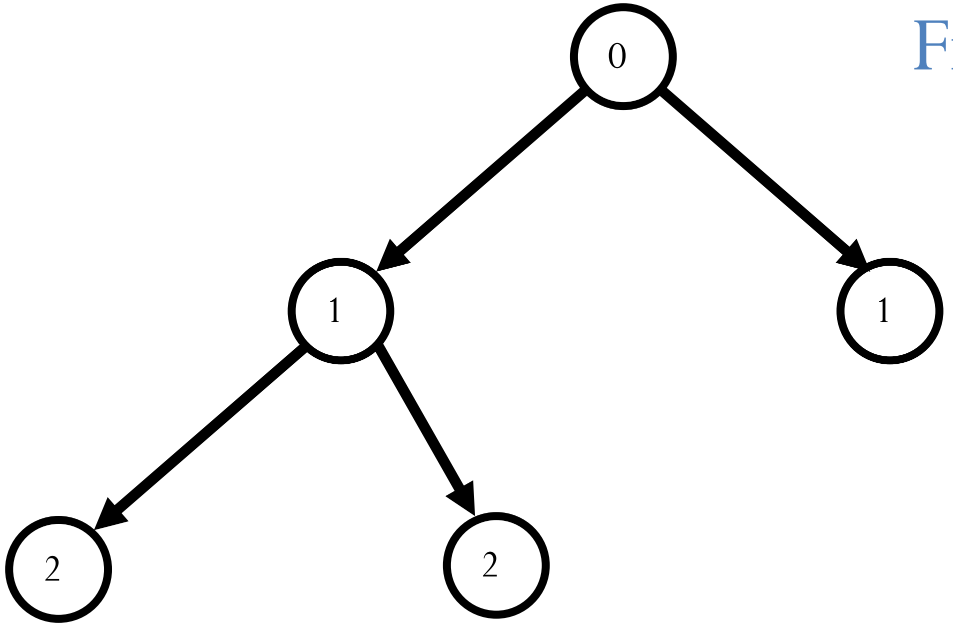

First-In-First-Out (FIFO)
Queue

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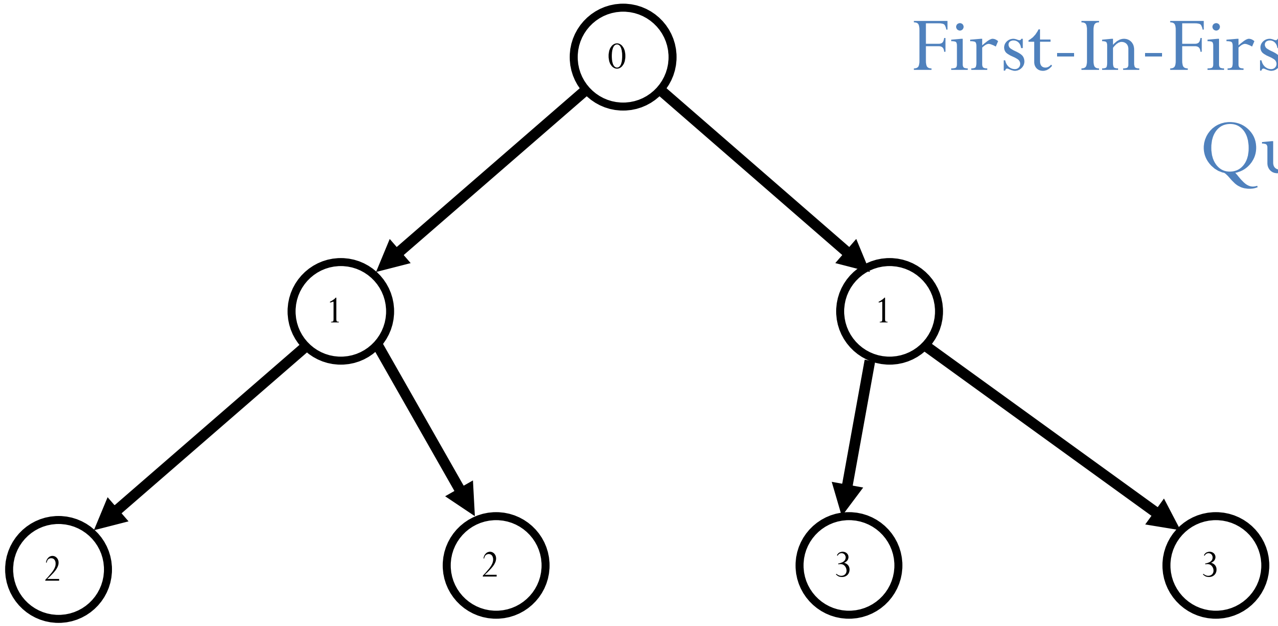
Breadth-First Search



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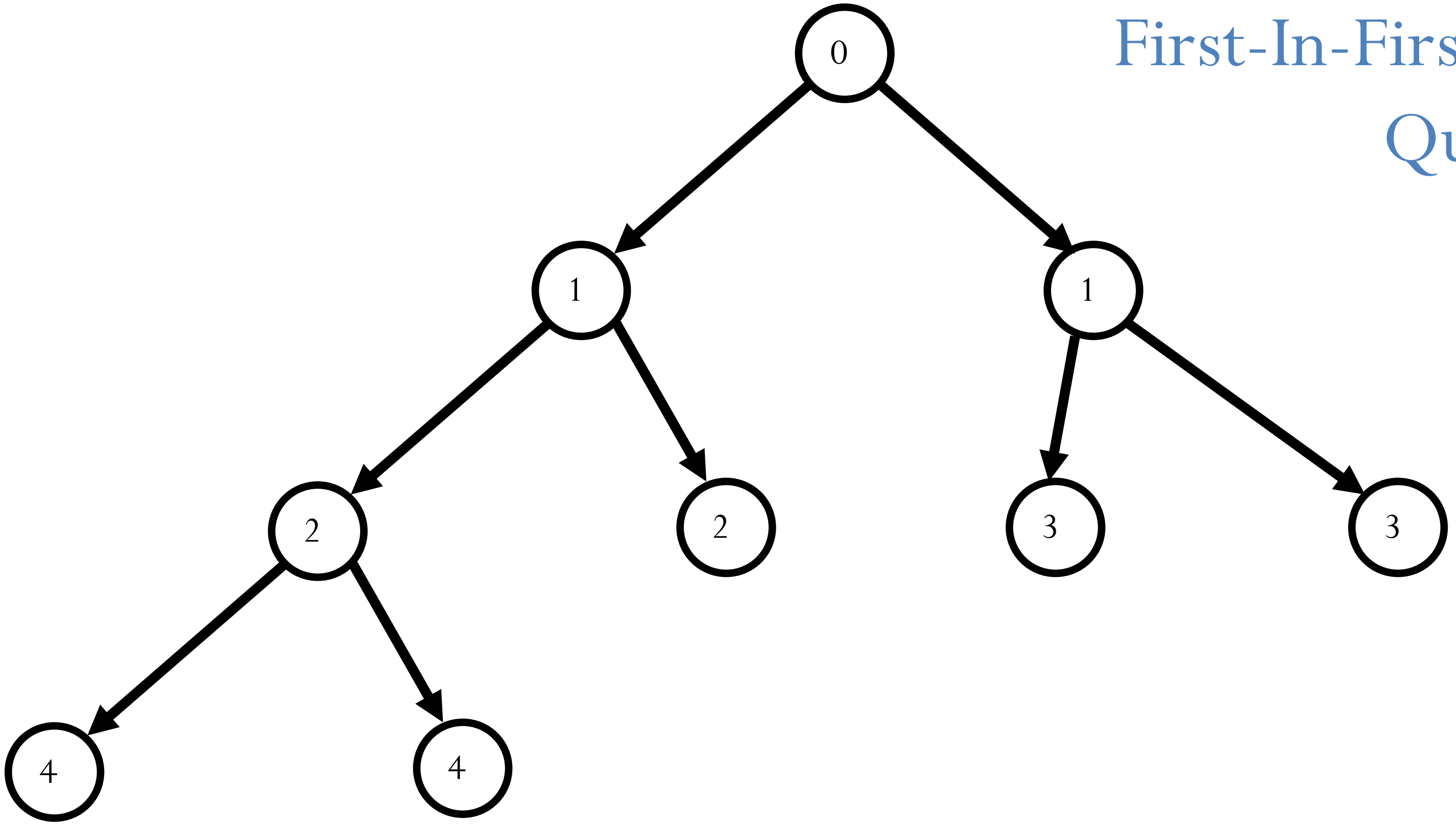

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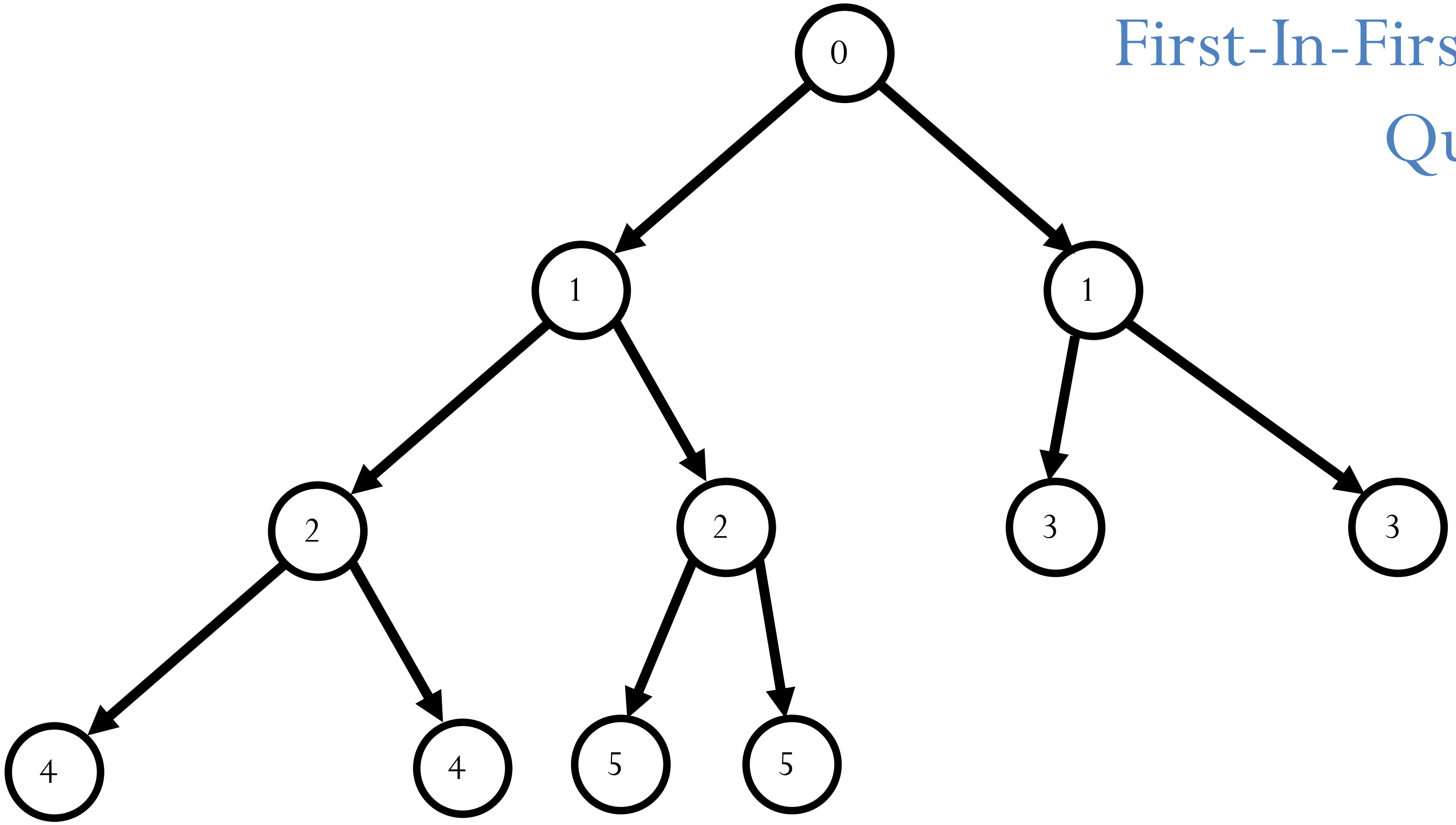
Breadth-First Search



First-In-First-Out (FIFO) Queue

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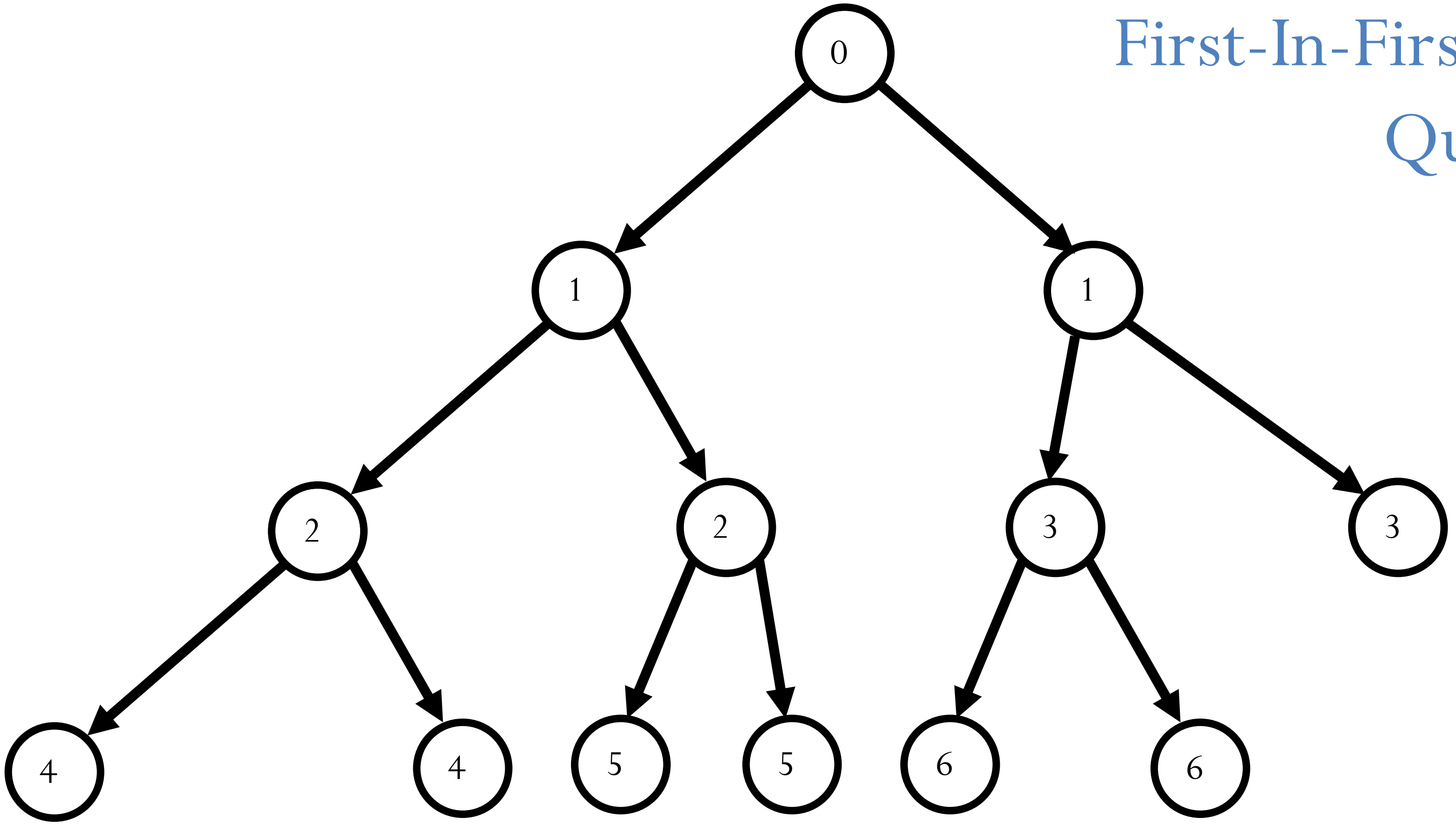
Breadth-First Search



First-In-First-Out (FIFO)
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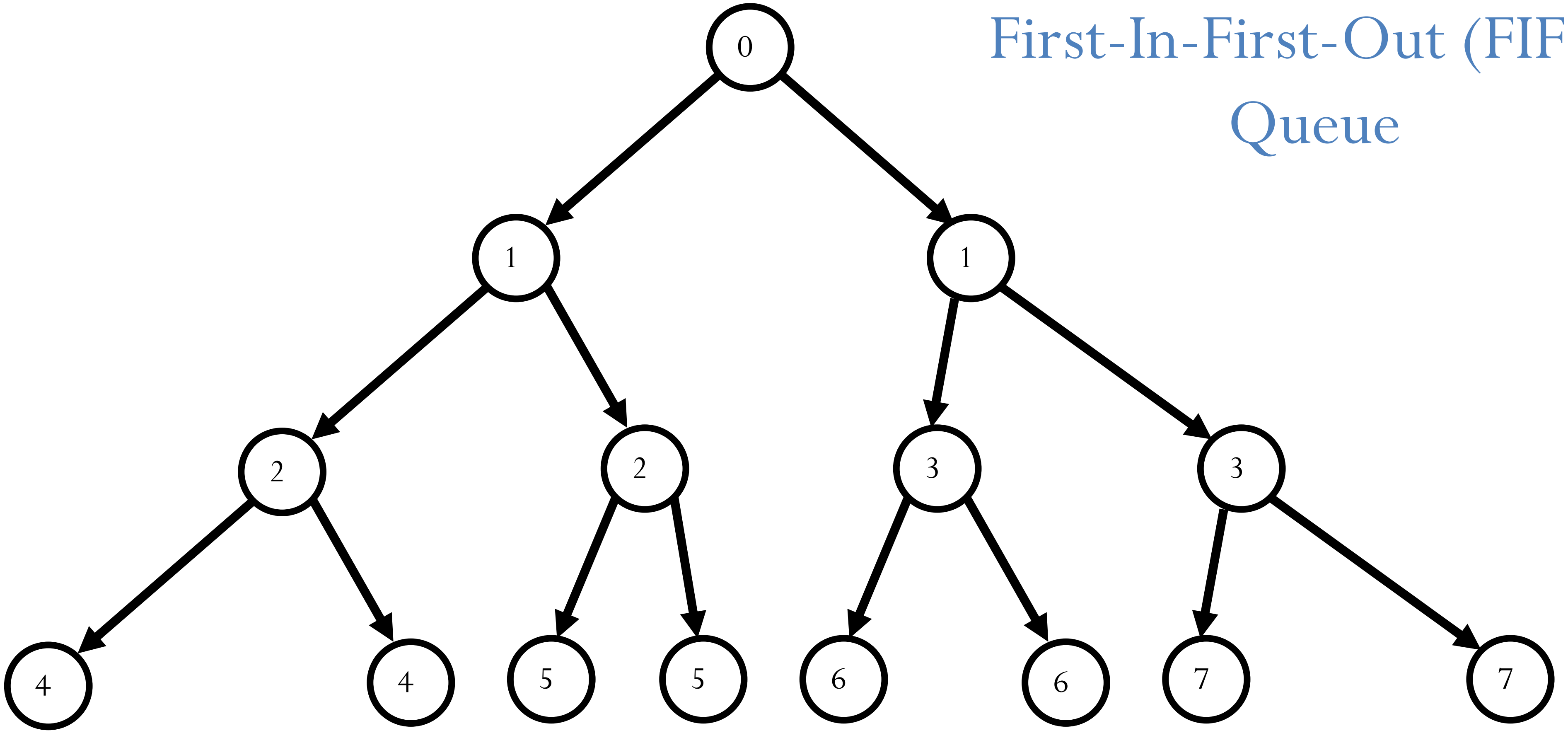
Breadth-First Search

First-In-First-Out (FIFO)
Queue



Animated PPT

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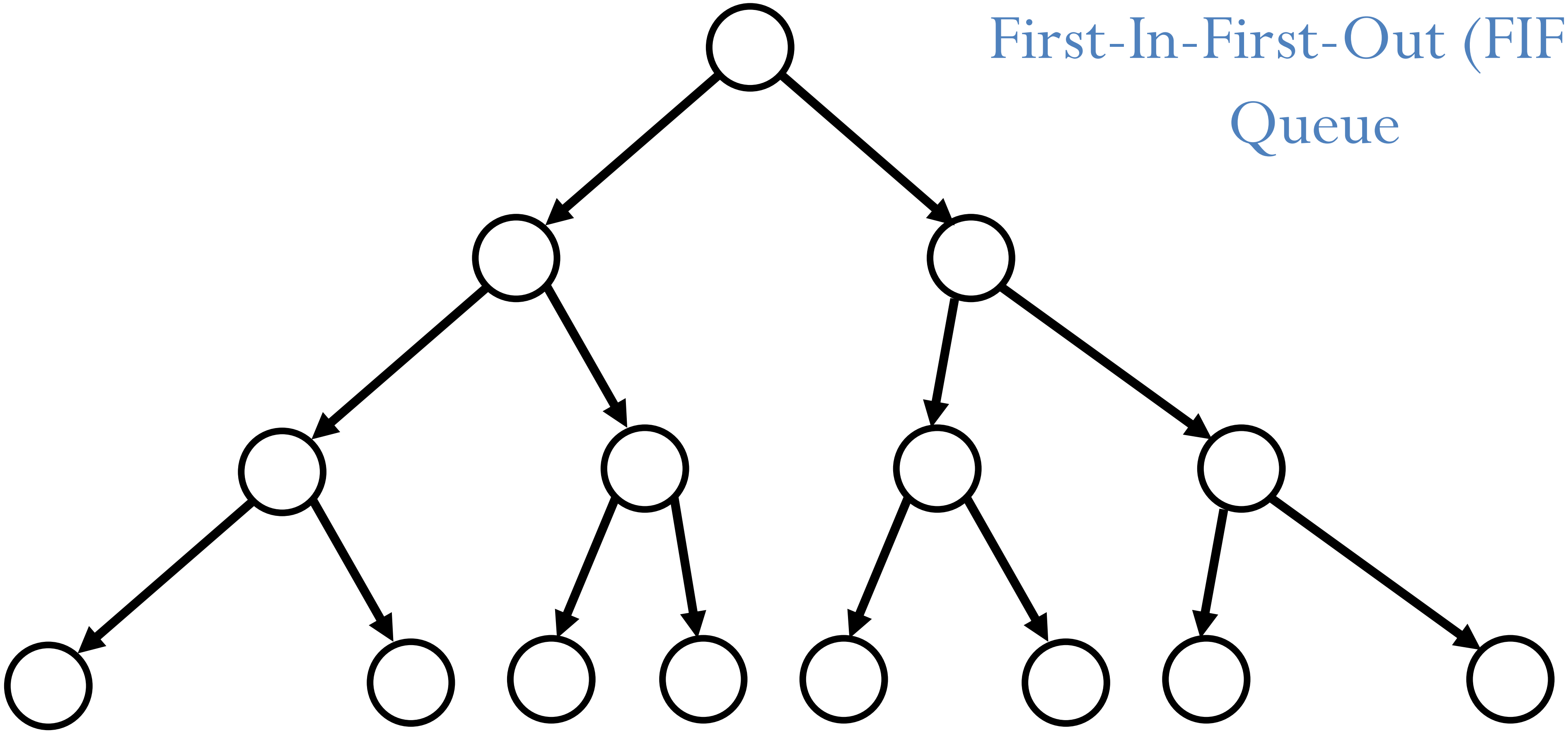


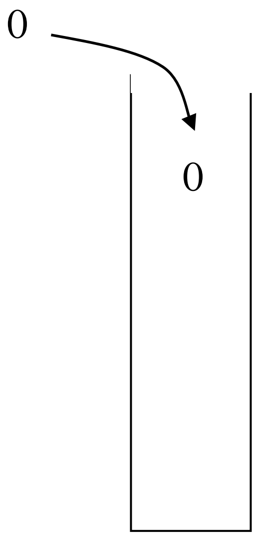
First-In-First-Out (FIFO)
Queue

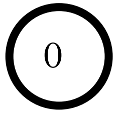
Animated PPT

Properties of Breadth-First Search

- Nodes are expanded in the same order in which they are generated
  - Fringe can be maintained as a First-In-First-Out (FIFO) queue
- BFS is complete: if a solution exists, one will be found
- BFS finds a shallowest solution
  - Not necessarily an optimal solution
- If every node has b successors (the branching factor), first solution is at depth d, then fringe size will be at least b^d at some point
  - This much space (and time) required ☹

Depth-First Search

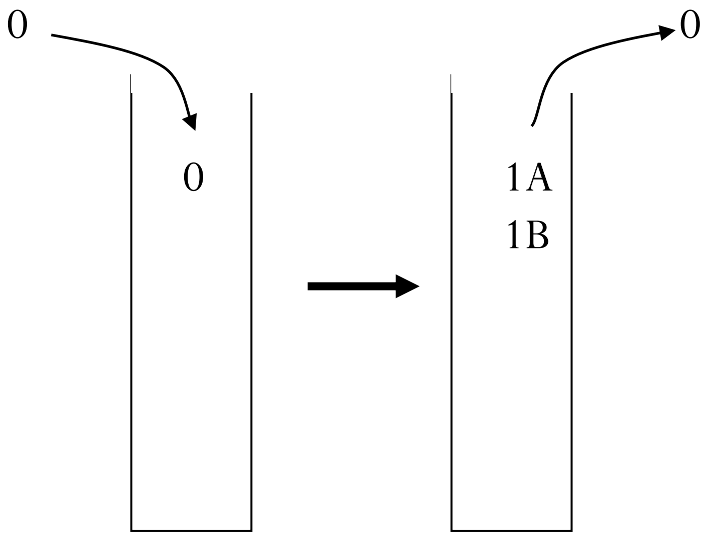


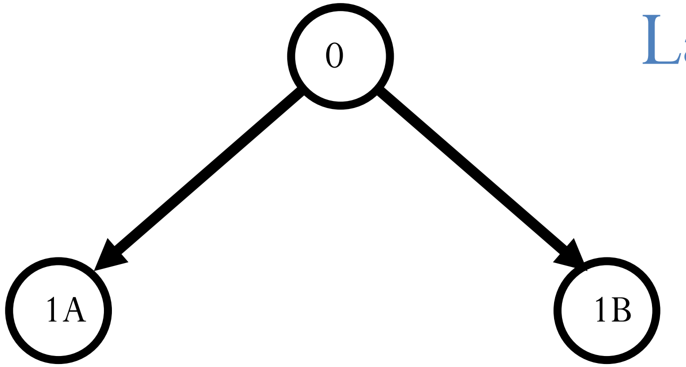


Last-In-First-Out (LIFO)
Stack

Animated PPT

Depth-First Search

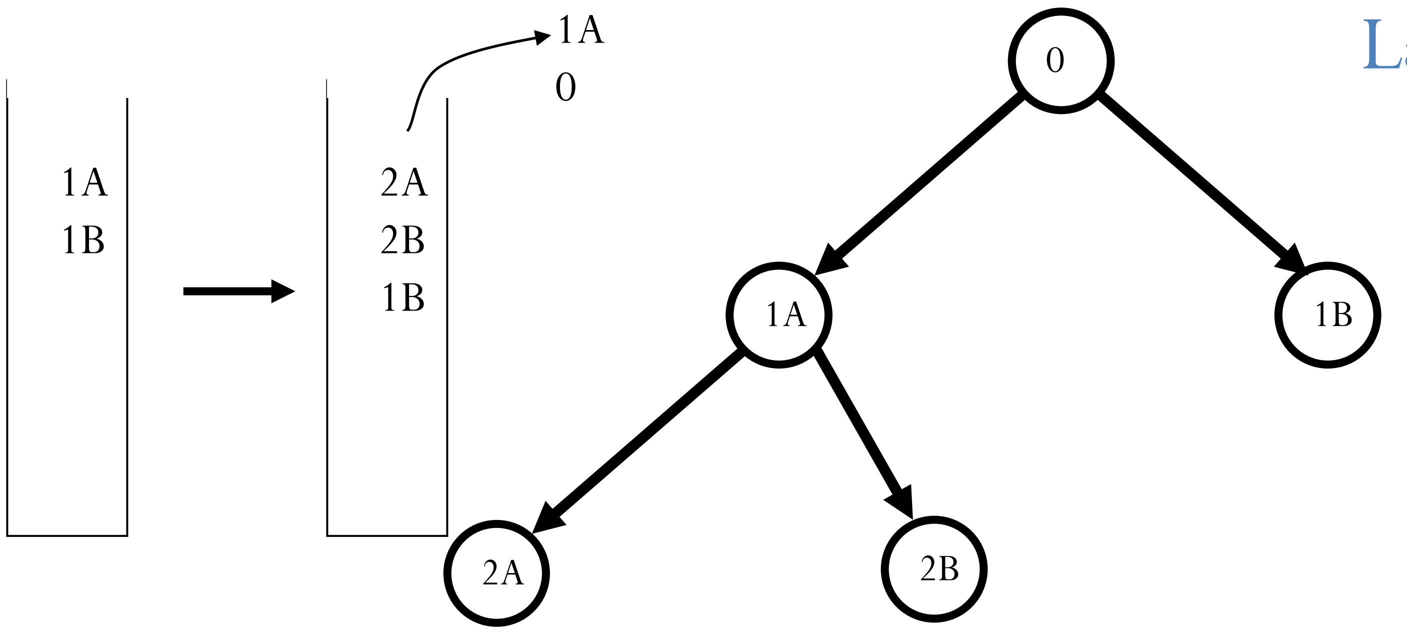




Last-In-First-Out (LIFO)
Stack

Animated PPT

Depth-First Search

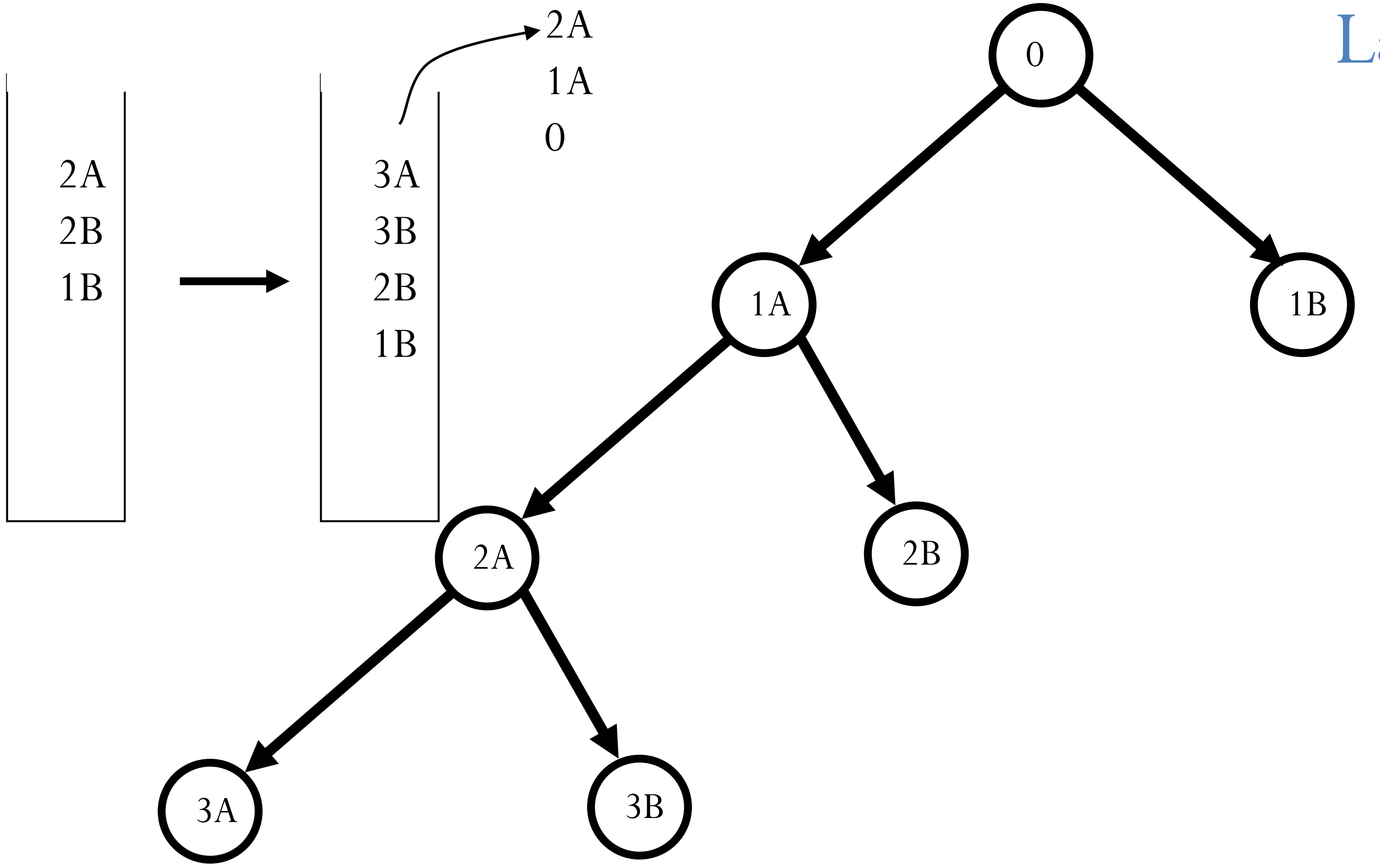


Last-In-First-Out (LIFO)

Stack

Animated PPT

Depth-First Search



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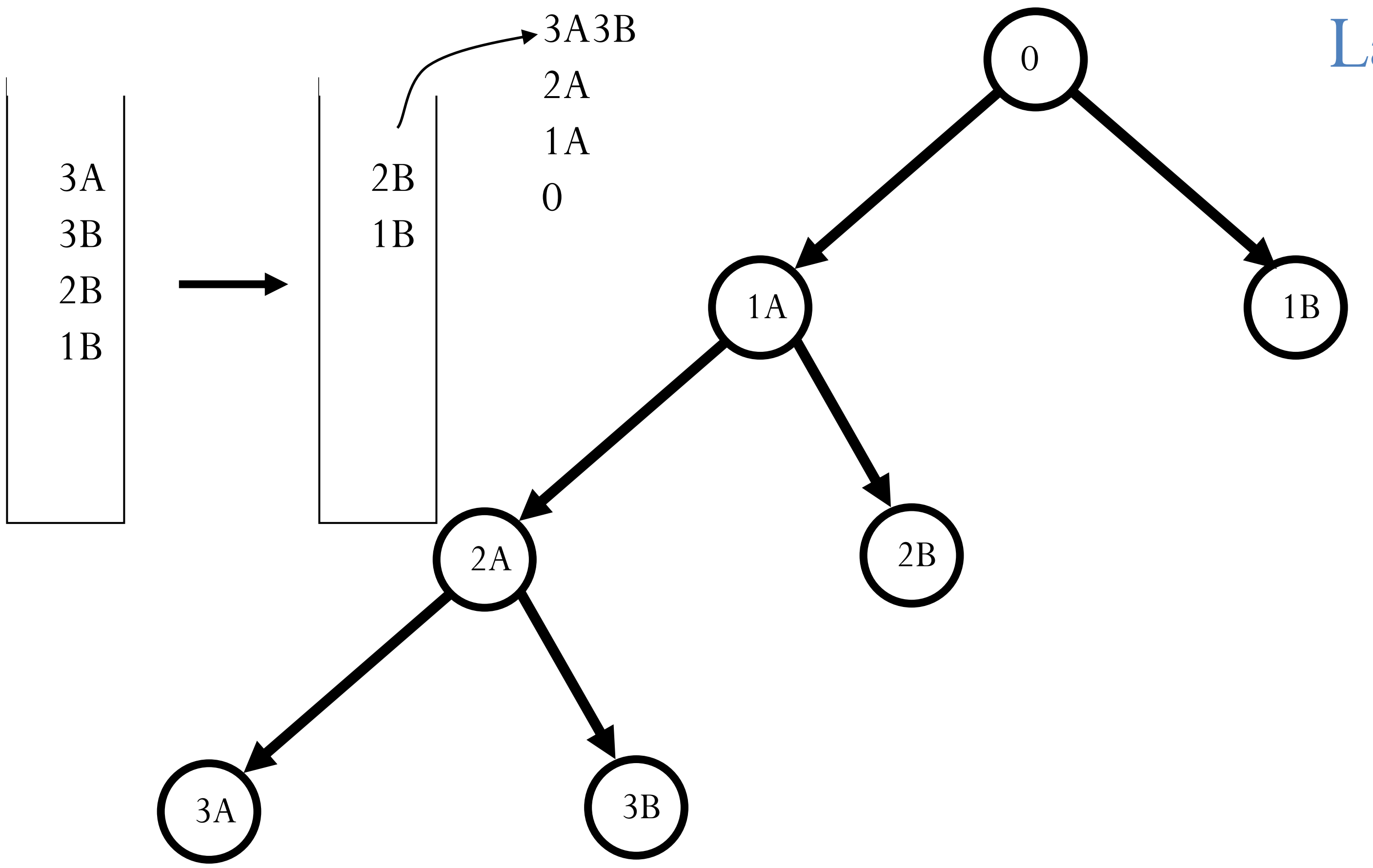
Depth-First Search



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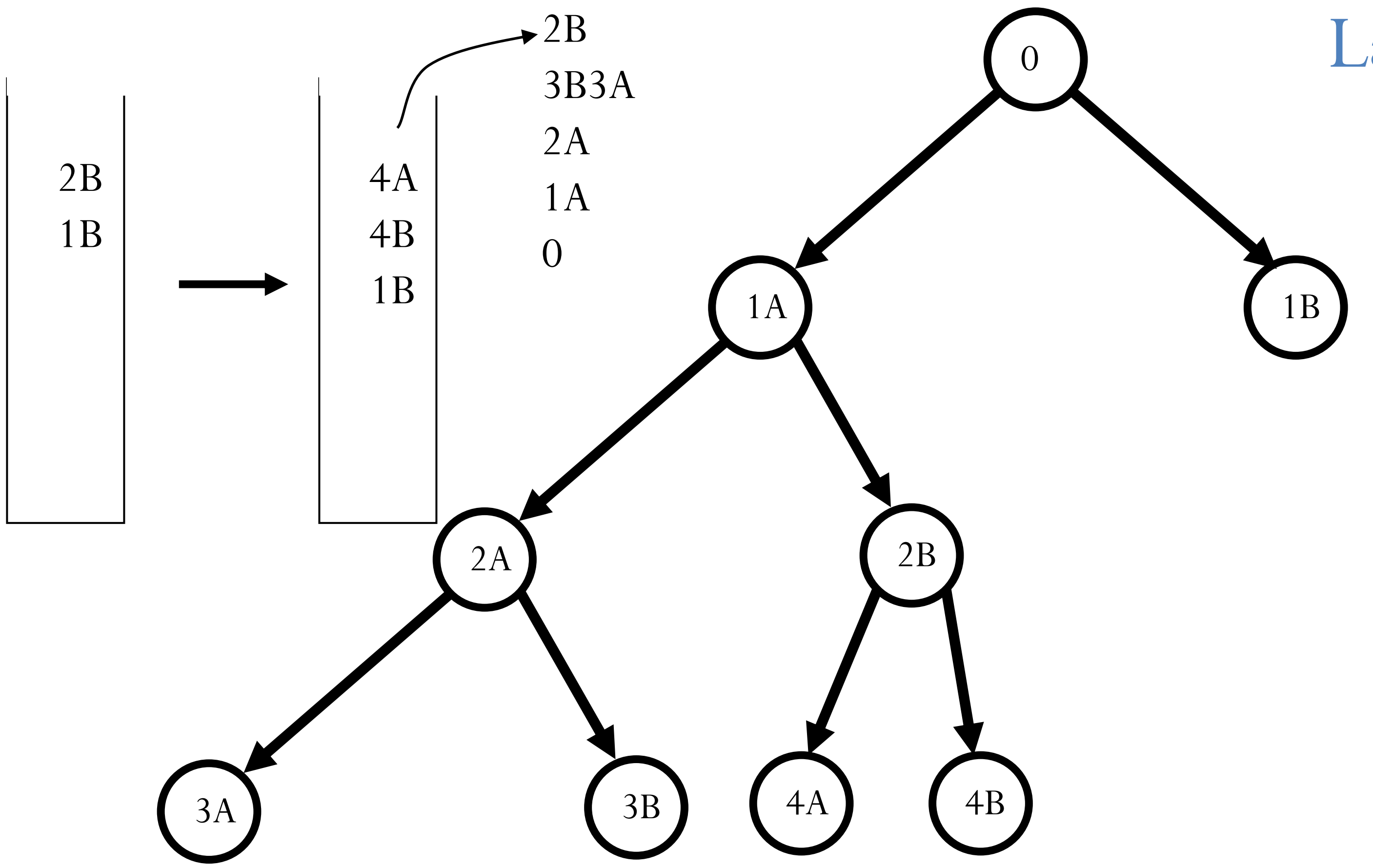

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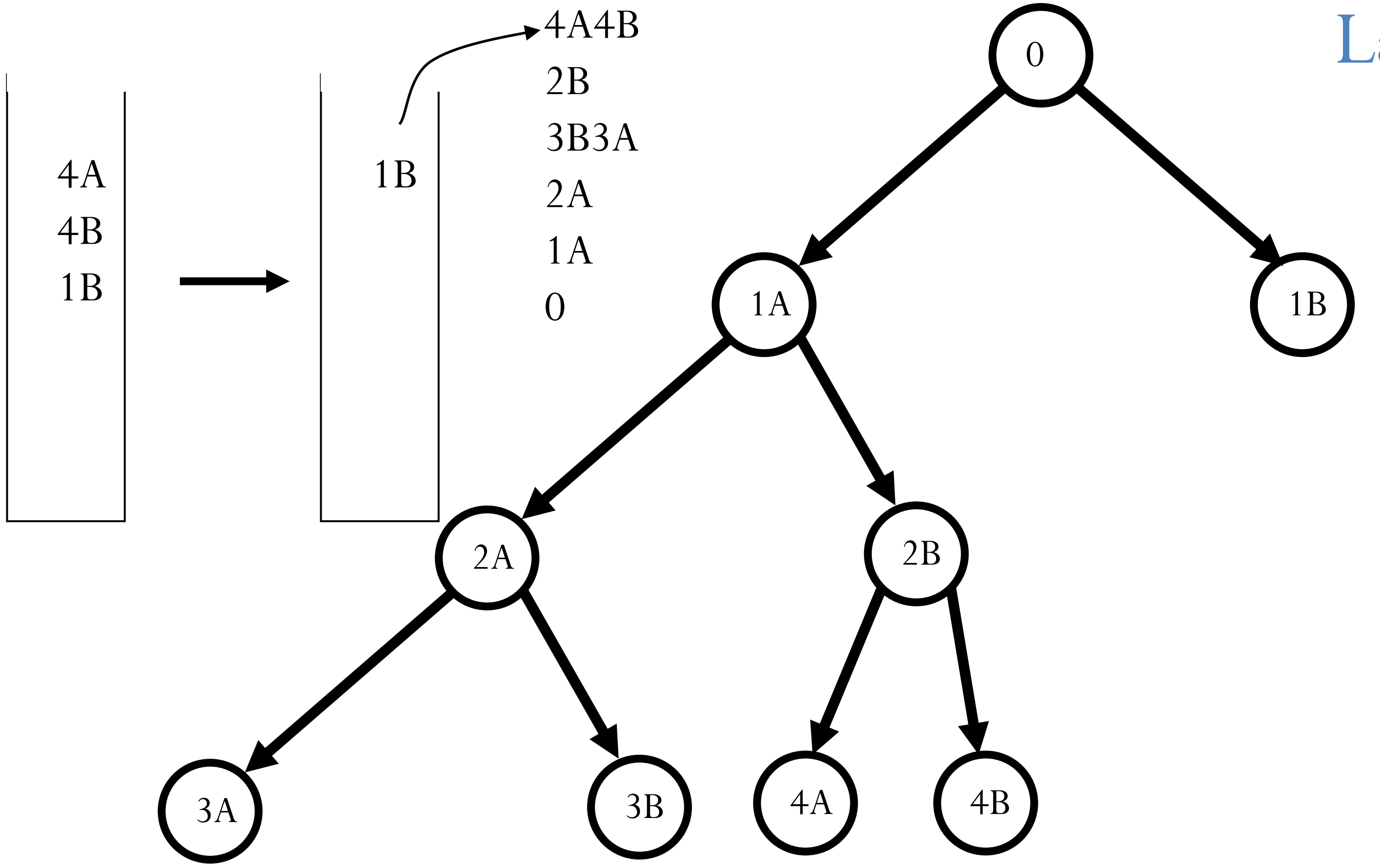

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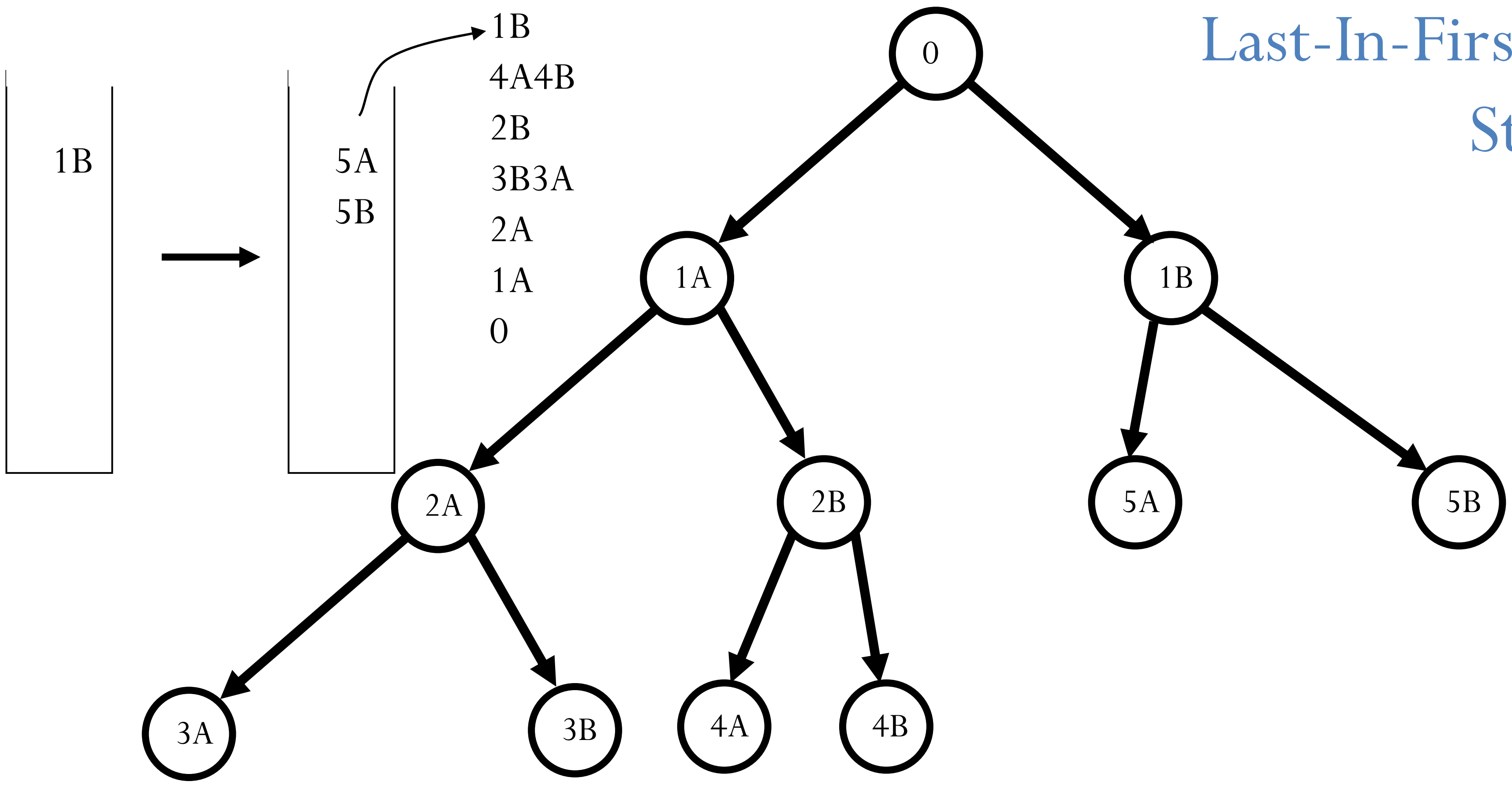
Animated PPT

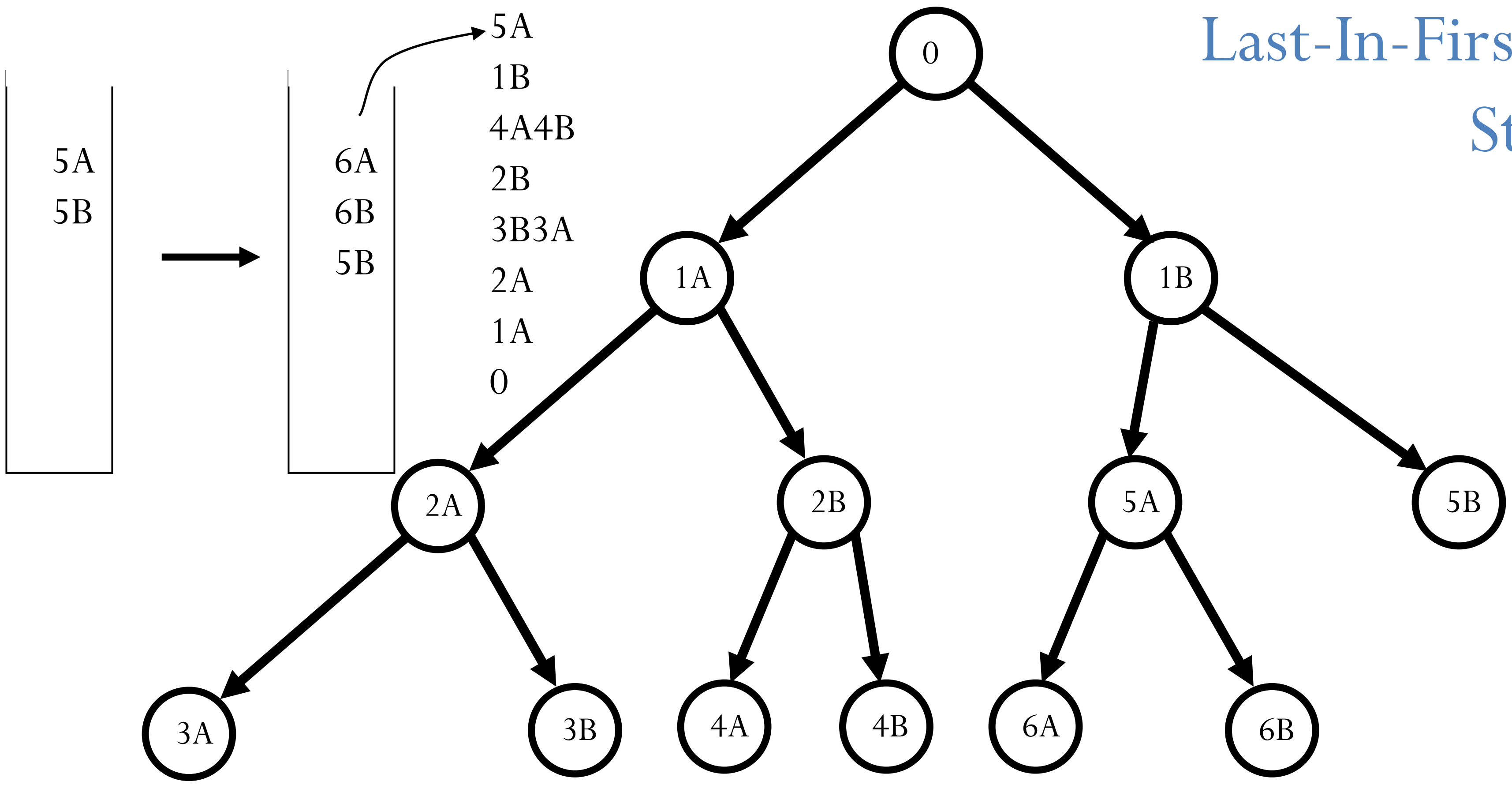
Depth-First Search



Last-In-First-Out (LIFO)
Stack

Animated PPT

Depth-First Search



Last-In-First-Out (LIFO)
Stack

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Depth-First Search

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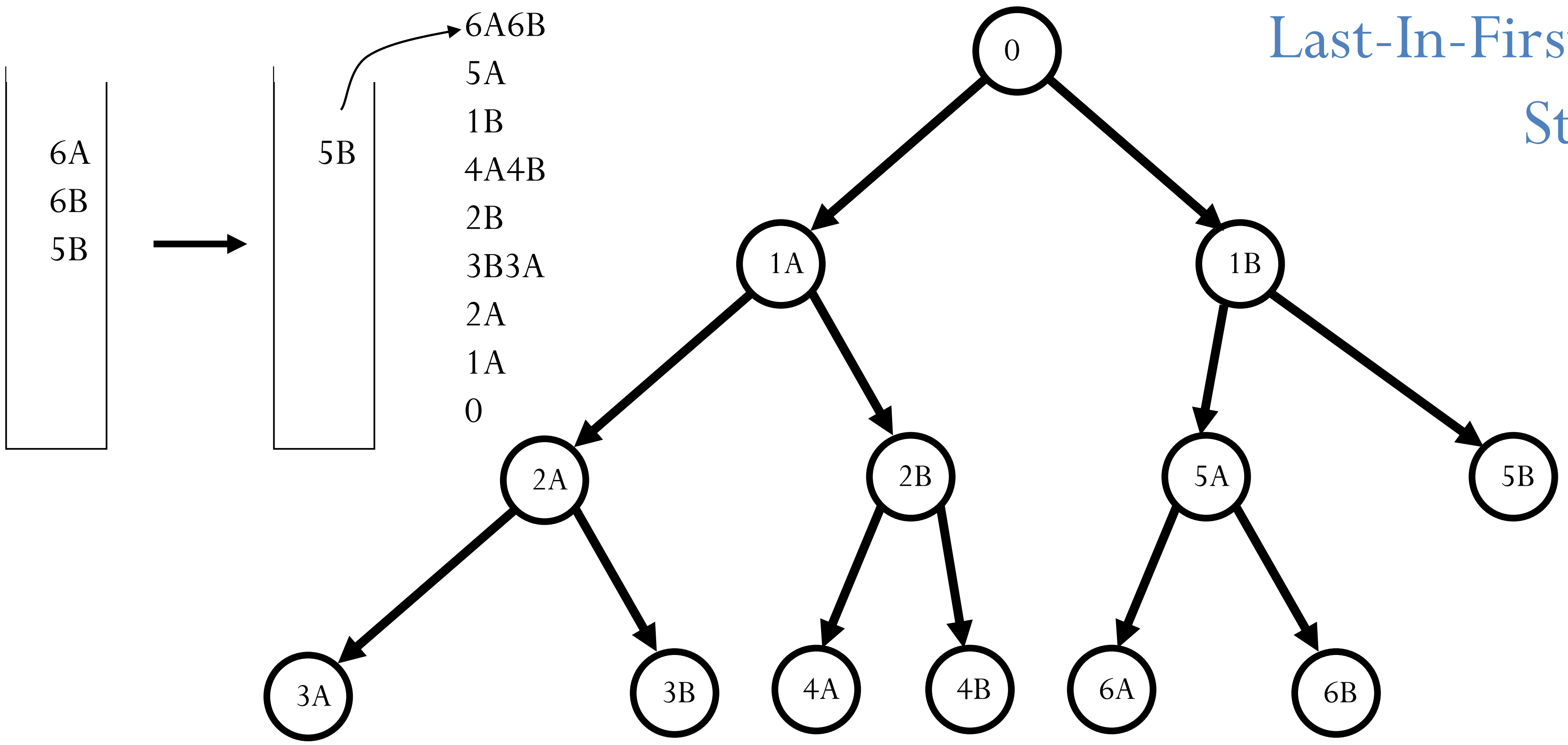
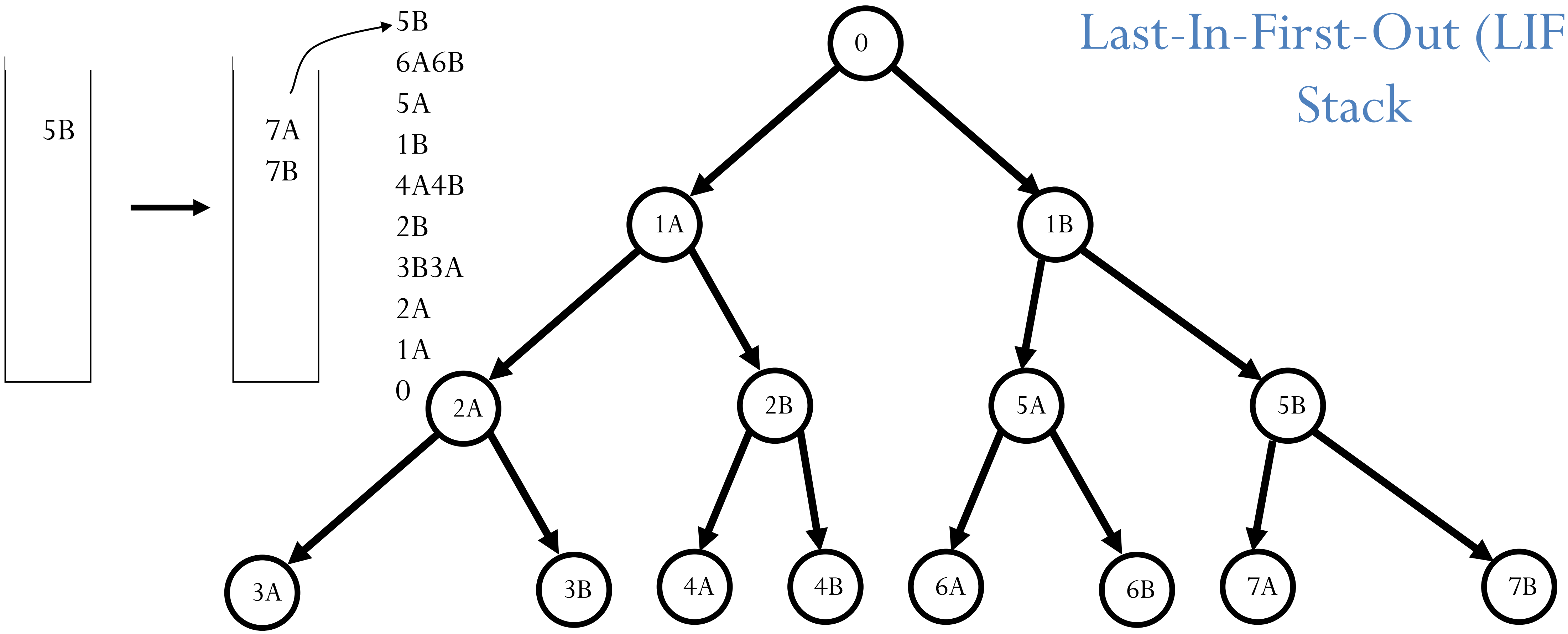


Animated PPT

Depth-First Search



Last-In-First-Out (LIFO)
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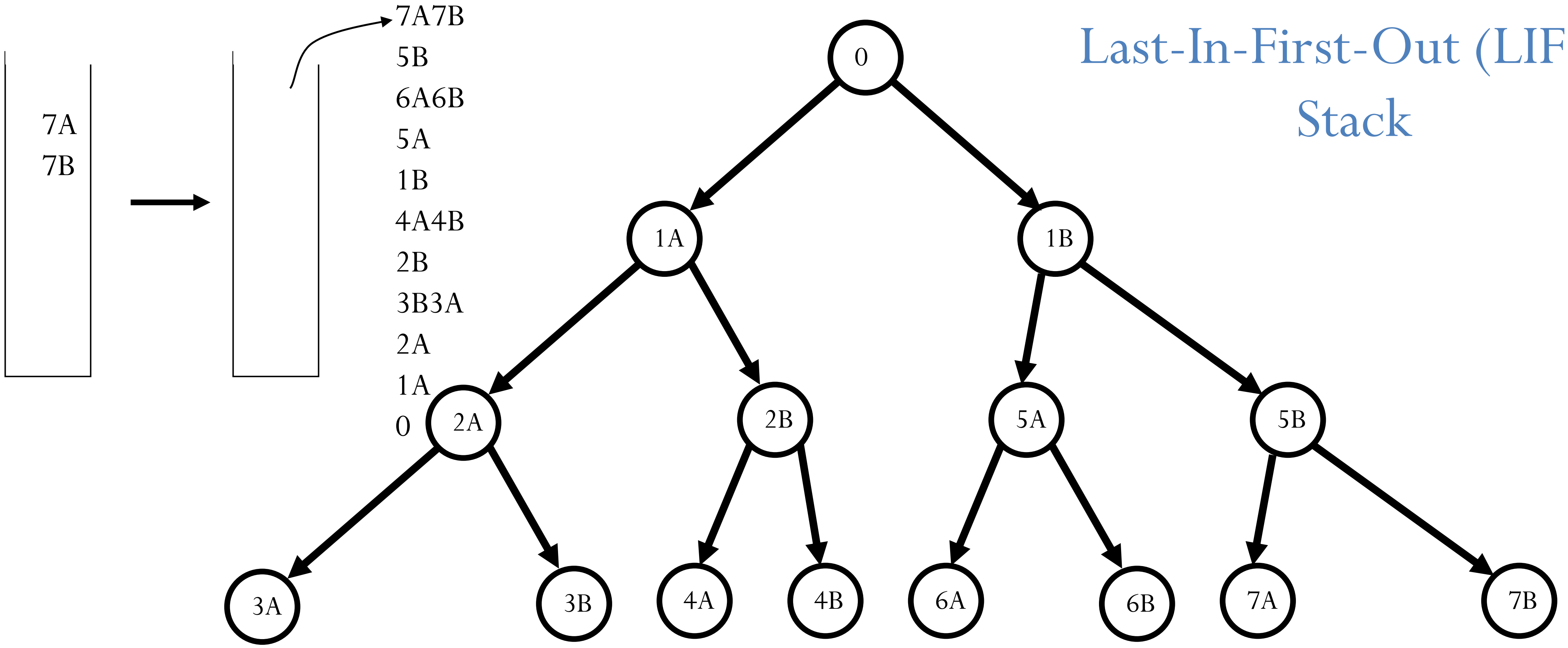
Animated PPT

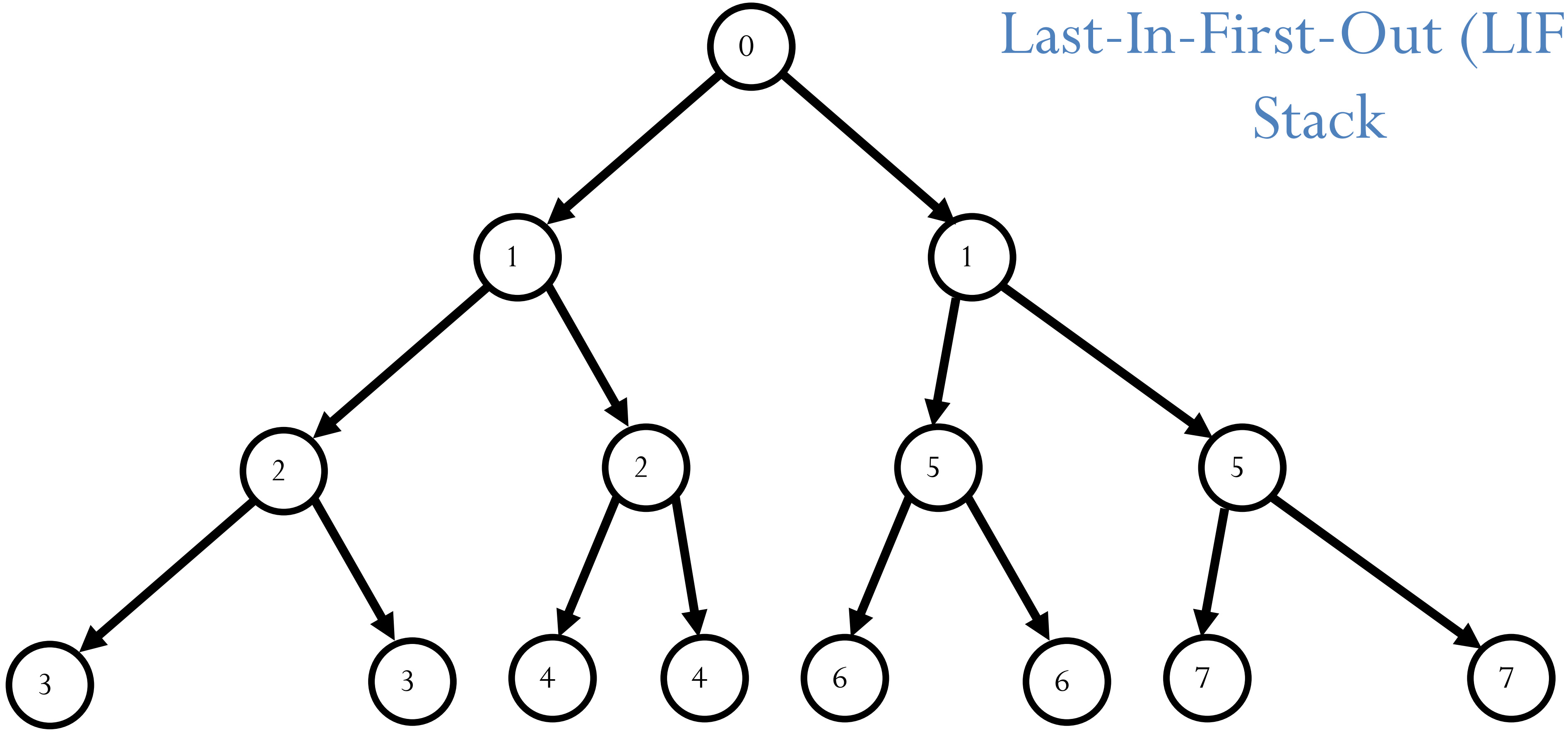
Depth-First Search

Last-In-First-Out (LIFO)
Stack



Animated PPT

Depth-First Search



Last-In-First-Out (LIFO)
Stack

Animated PPT

Implementing Depth-First Search

- Fringe can be maintained as a Last-In-First-Out (LIFO) queue (aka. a stack)
- Also easy to implement recursively:
- DFS(node)
  - If goal(node) return solution(node);
  - For each successor of node
    - Return DFS(successor) unless it is *failure*;
  - Return *failure;*

Properties of depth-first search

- Not complete (might cycle through nongoal states)
- If solution found, generally not optimal/shallowest
- If every node has b successors (the branching factor), and we search to at most depth m, fringe is at most bm
  - Much better space requirement ☺
  - Actually, generally don't even need to store all of fringe
- Time: still need to look at every node
  - $b^m + b^{m-1} + \ldots + 1$ (for $b>1$, $O(b^m)$)
  - Inevitable for uninformed search methods…

Combining good properties of BFS and DFS

- Limited depth DFS: just like DFS, except never go deeper than some depth d
- Iterative deepening DFS:
  - Call limited depth DFS with depth 0;
  - If unsuccessful, call with depth 1;
  - If unsuccessful, call with depth 2;
  - Etc.
- Complete, finds shallowest solution
- Space requirements of DFS
- May seem wasteful timewise because replicating effort
  - Really not that wasteful because **almost all effort at deepest level**
  - $db + (d-1)b^2 + (d-2)b^3 + \ldots + 1b^d$ is $O(b^d)$ for $b > 1$

Searching solution evaluation

- Comparing multiple searching algorithm based on
  - Completeness: does it always find a solution if one exist?
  - Time complexity: How long depends on number of nodes
  - Space complexity: Memory depends on number of nodes
  - Optimality: Find shortest path (or least cost solution)?
  - Systematicity: does it visit each state at most once?

Depth vs. Breadth-first

Let $|T(s)| \leq b$ (branching factor), goal at depth d

- How implement priority queue?
- Completeness?
- Time complexity?
- Space complexity?
- Optimality?

Breath First Search

- Completeness?
  - Yes
- Time complexity?
  - $O(b^d)$
- Space complexity?
  - $O(b^d)$ ☹
- Optimality?
  - yes

Depth First Search

- Completeness?
  - Yes, assuming state space finite
- Time complexity?
  - O(|V+E|), can do well if lots of goals
- Space complexity?
  - O(n), n deepest point of search
- Optimality?
  - No ☹

Let b as branching factor with goal at depth d

Depth-limited Search

DFS, only expand nodes depth $\leq l$.

- Completeness?
  - No, if $l \leq d$. ☹
- Time complexity?
  - $O(b^l)$
- Space complexity?
  - $O(l)$
- Optimality?
  - No ☹

Iterative Deepening

Depth limited, increasing l.

- Completeness?
  - Yes. ☺
- Time complexity?
  - $O(b^d)$, even with repeated work! ☺
- Space complexity?
  - $O(d)$ ☺
- Optimality?
  - Yes ☺

Bidirectional search

- Even better: search from both the start and the goal, in parallel!
- If the shallowest solution has depth d and branching factor is b on both sides, requires only $O(b^{d/2})$ nodes to be explored!

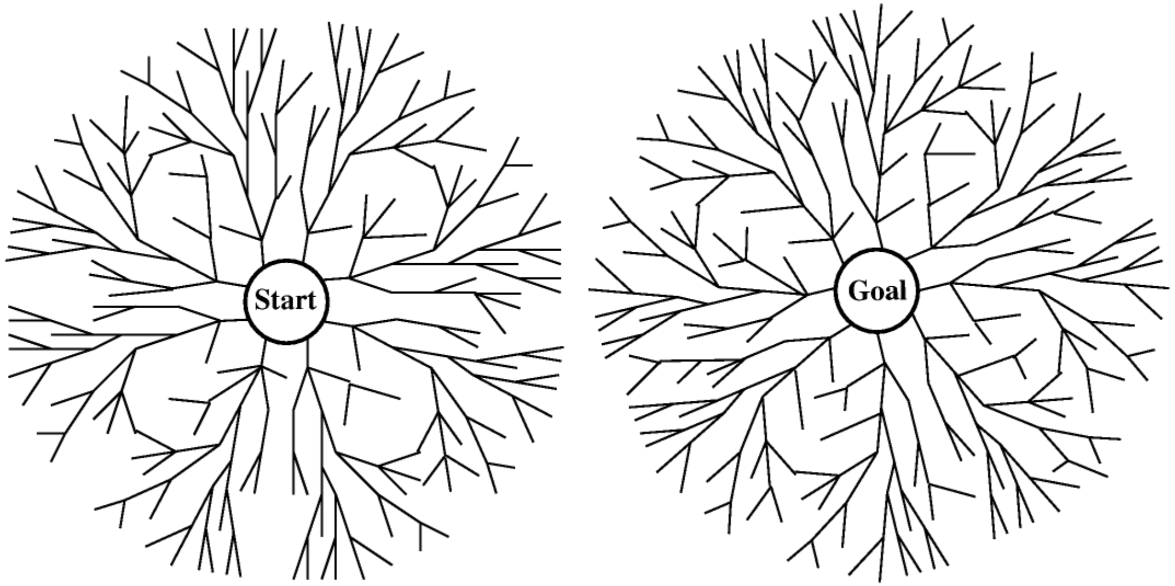


image from cs-alb-pc3.massey.ac.nz/notes/59302/fig03.17.gif

Bidirectional Search

BFS in both directions

Need N^{-1}

How could this help?

- b^l vs $2b^{l/2}$

What makes this hard to implement?

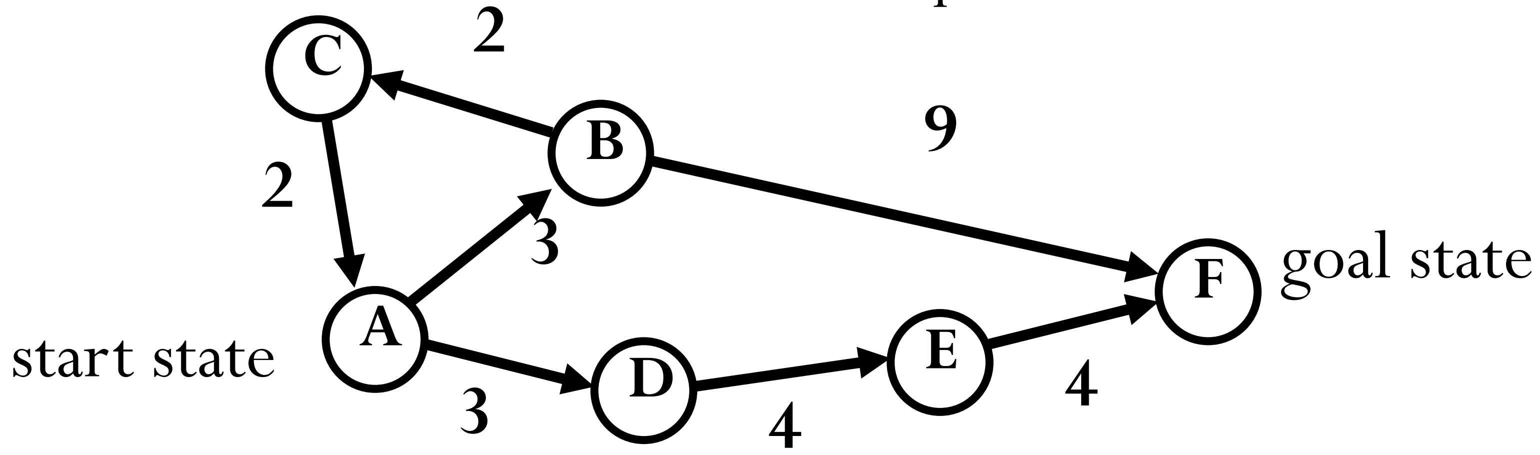
Informed Search

Informed search

- So far, have assumed that no nongoal state looks better than another
- Unrealistic
  - Even without knowing the road structure, some locations seem closer to the goal than others
  - Some states of the 8s puzzle seem closer to the goal than others
- Makes sense to expand closer-seeming nodes first

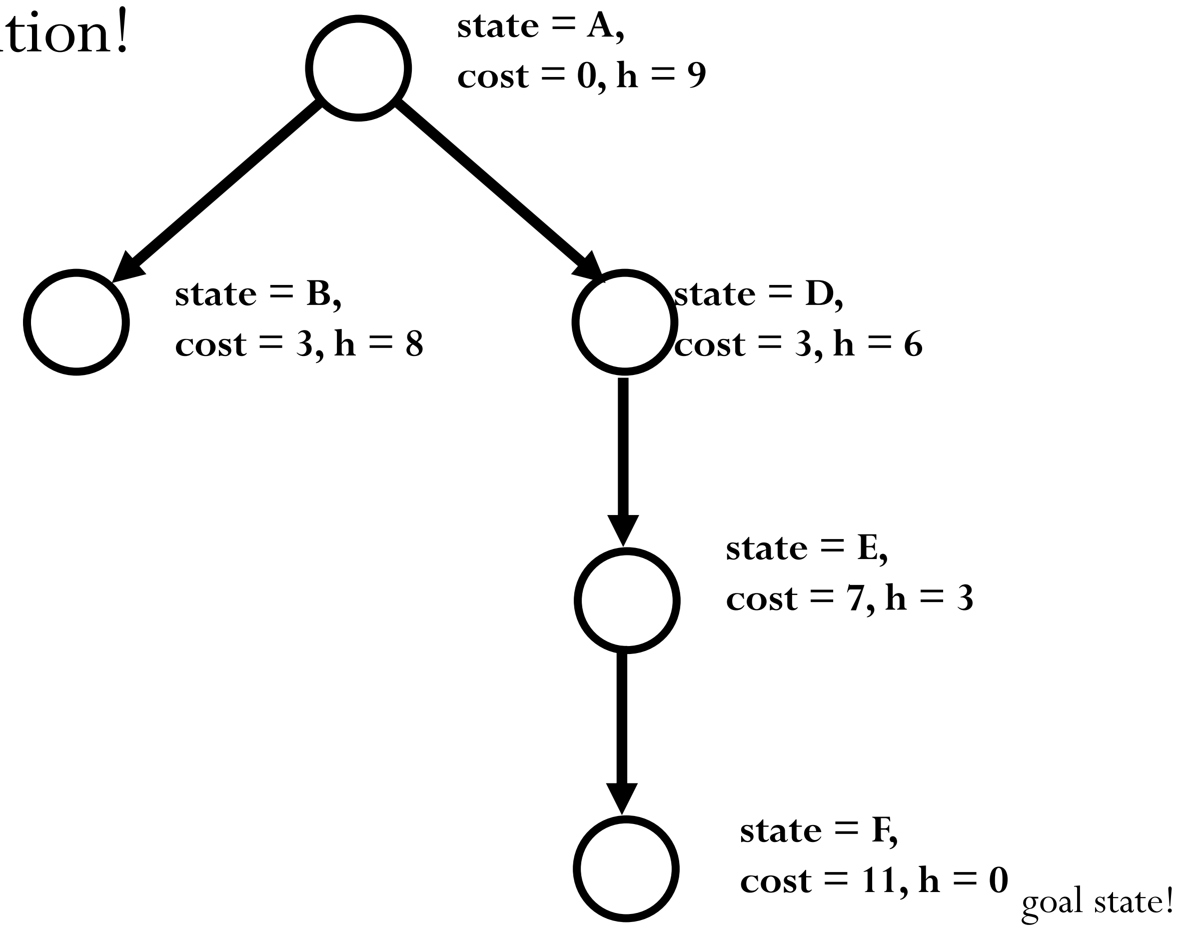
Heuristics

- Key notion: heuristic function h(n) gives an estimate of the distance from n to the goal
  - h(n)=0 for goal nodes
- E.g. straight-line distance for traveling problem
- Say: h(A) = 9, h(B) = 8, h(C) = 9, h(D) = 6, h(E) = 3, h(F) = 0
- We're adding something new to the problem!
- Can use heuristic to decide which nodes to expand first



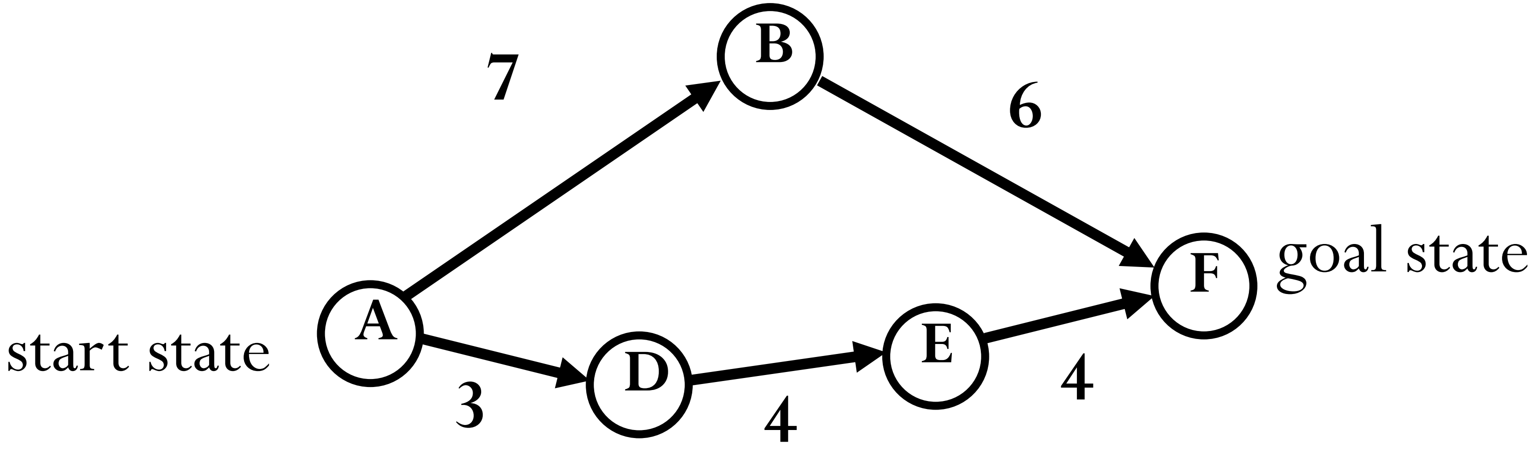
Greedy best-first search

- Greedy best-first search: expand nodes with lowest h values first
- Rapidly finds the optimal solution!
- Does it always?



A bad example for greedy

- Say: h(A) = 9, h(B) = 5, h(D) = 6, h(E) = 3, h(F) = 0
- Problem: greedy evaluates the promise of a node only by how far is left to go, does not take cost occurred already into account



IDA*

Memory Bounded

Just as iterative deepening gives a more memory efficient version of BFS, can define IDA* as a more memory efficient version of A*.

Just use DFS with a cutoff on f values. Repeat with larger cutoff until solution found.

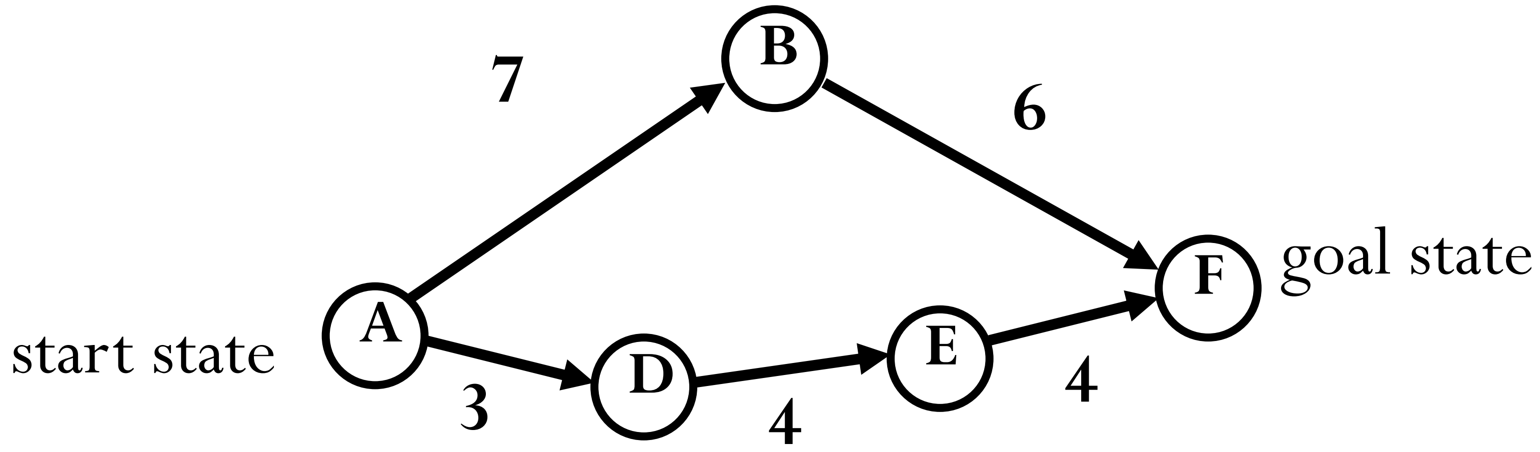
What to Learn?

The A* algorithm: its definition and behavior (finds optimal).

How to create admissible heuristics via relaxation.

A*

- Let g(n) be cost incurred already on path to n
- Expand nodes with lowest g(n) + h(n) first
- Say: h(A) = 9, h(B) = 5, h(D) = 6, h(E) = 3, h(F) = 0
- Note: if h=0 everywhere, then just uniform cost search



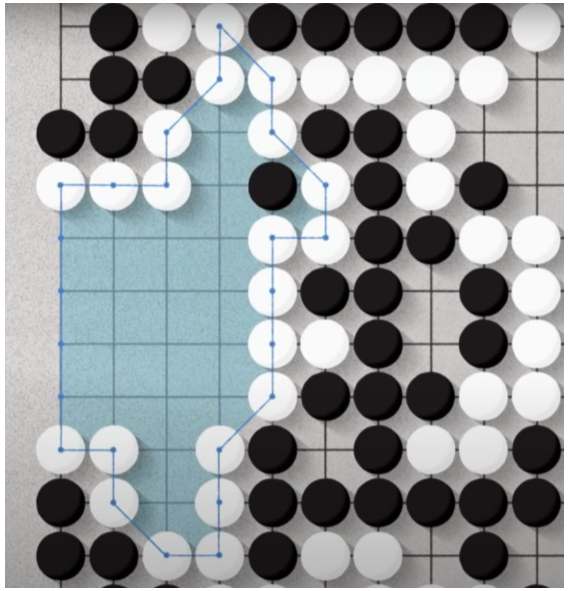
Admissibility

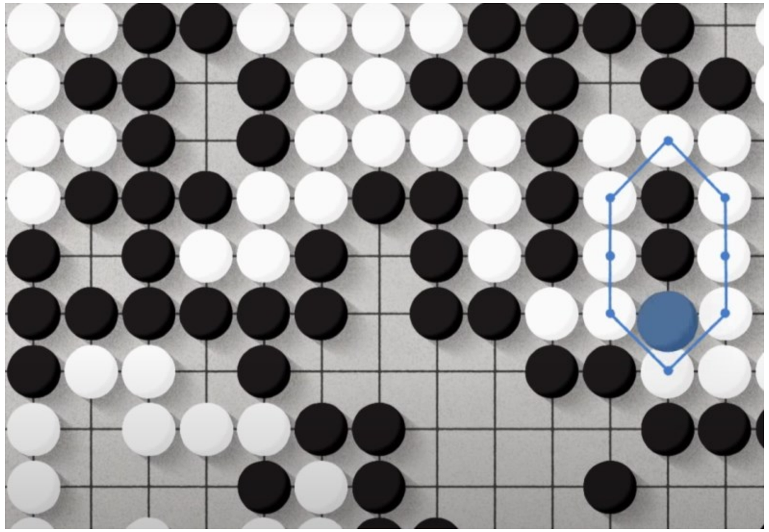
- A heuristic is admissible
  - if it never overestimates the distance to get the goal
  - If n is the optimal solution reachable from n', then $g(n) \geq g(n') + h(n')$
- Straight-line distance is admissible: can't hope for anything better than a straight road to the goal
- Admissible heuristic means that A* is always optimistic
- A* is guaranteed to return a least-cost path from start to goal.

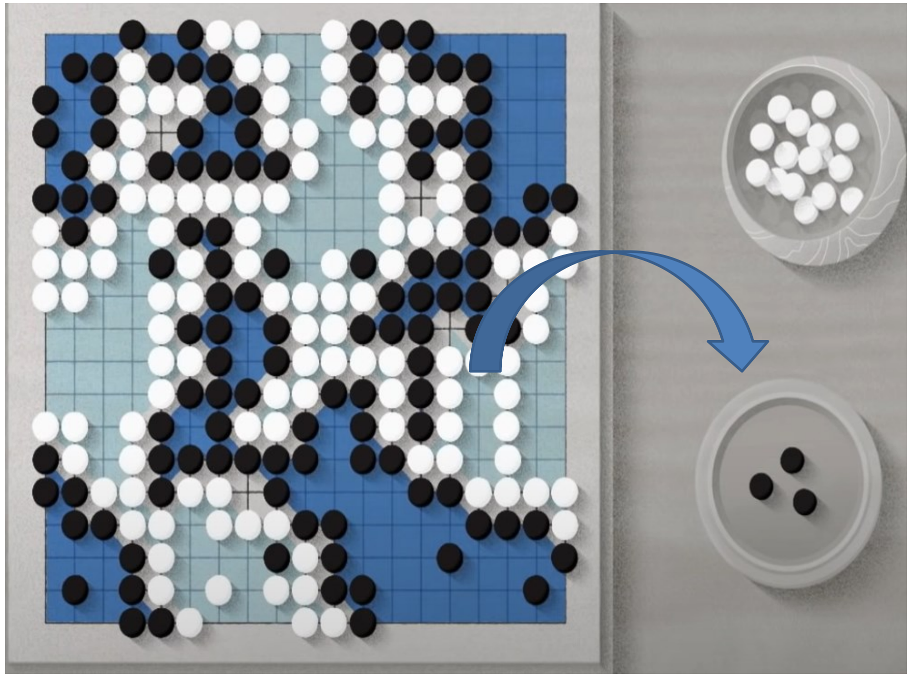
Alpha Go Searching Example

2016: Alpha Go

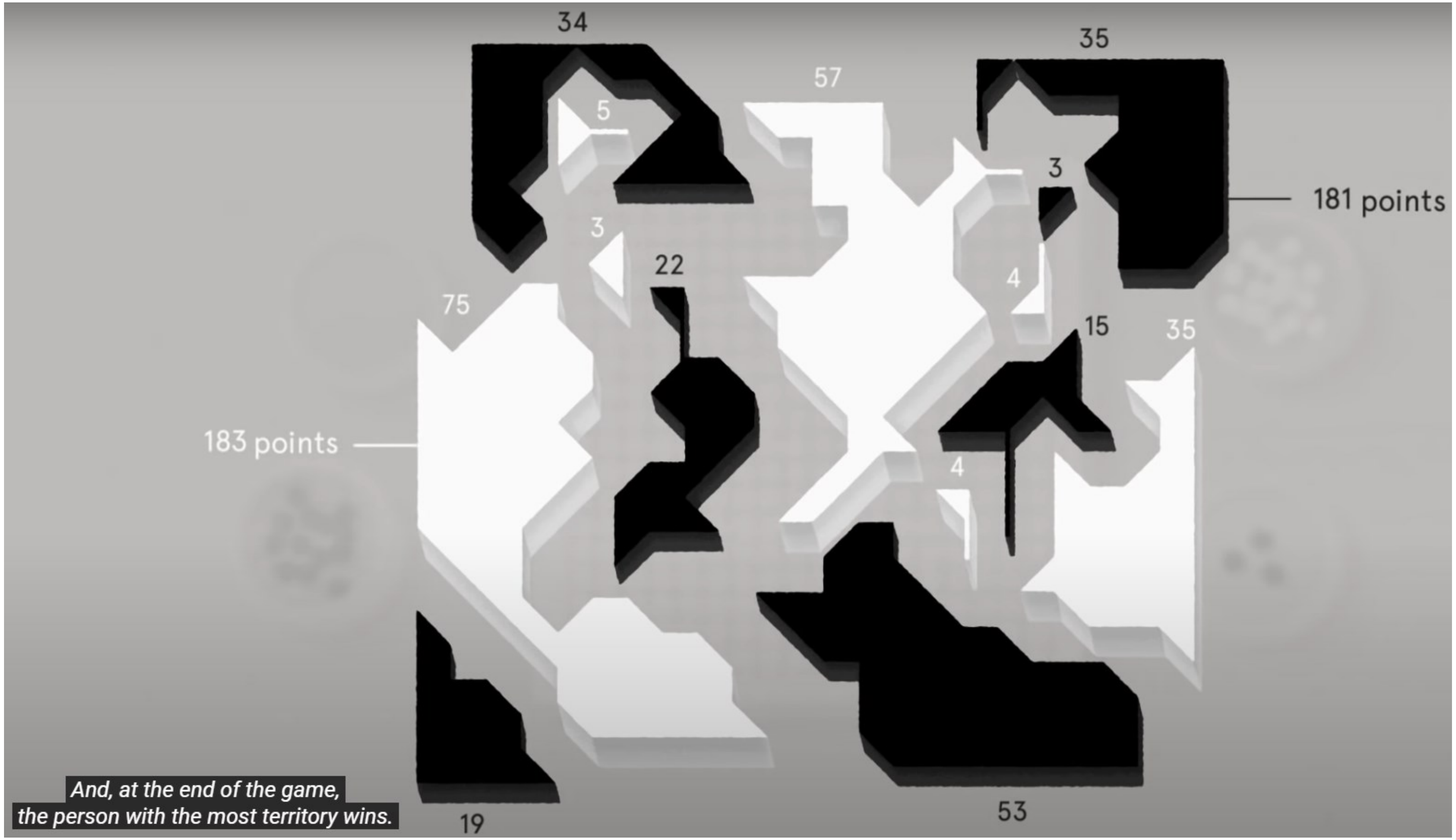
- Make territories
- Capture opponent pieces
- Largest territory







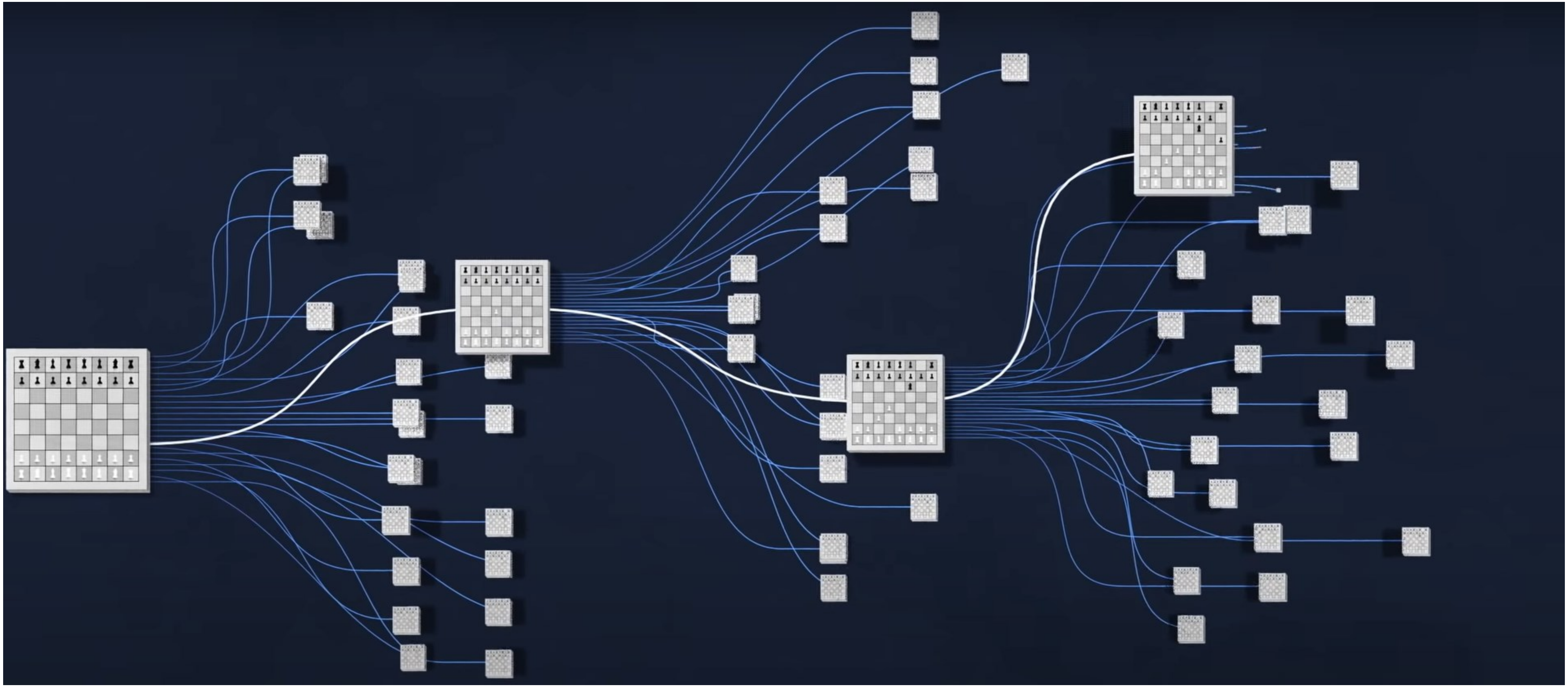
2016: Alpha Go



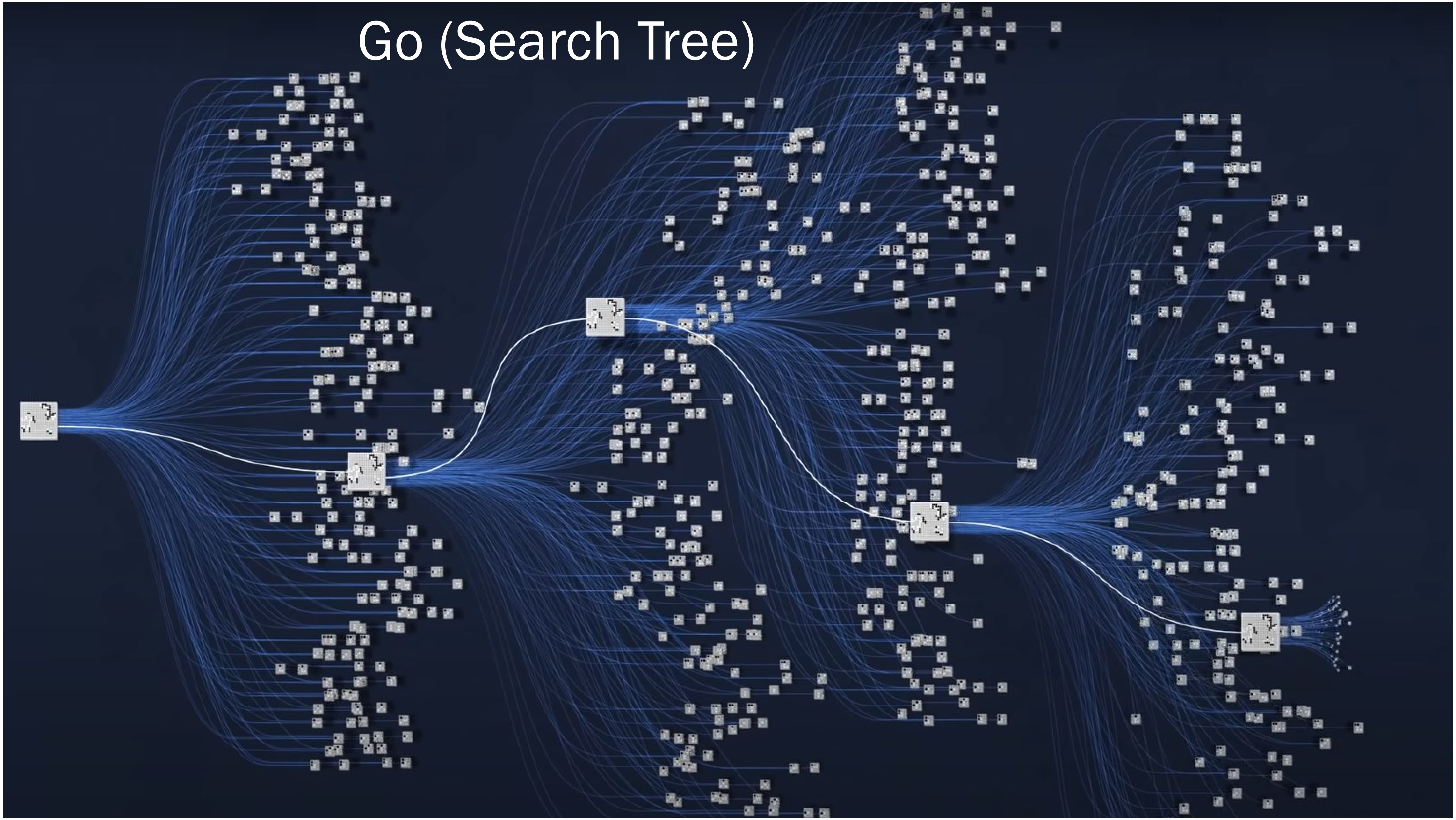
2016: Alpha Go

- https://www.buzzfeednews.com/article/alexkantrowitz/were-in-an-artificial-intelligence-hype-cycle

Chess (Search Tree)



Go (Search Tree)



Chess v/s Go

- At the opening move in Chess there are 20 possible moves.
- Go is simpler than Chess and yet more complex.
- In large games $\mathbf{O(b^d)}$, such as
  - chess (**b≈35**, **d≈80**)
  - Go (**b≈250**, **d≈150**),
- Exhaustive search is infeasible, but the effective search space can be reduced by two general principles.
  - the depth of the search may be reduced by position evaluation: truncating (cut) the search tree at state s and replacing the subtree below s by an approximate value function $\mathbf{v(s) \approx v^*(s)}$ that predicts the outcome from state s.
  - the breadth of the search may be reduced by sampling actions from a policy $\mathbf{p(a|s)}$ that is a probability distribution over possible moves **a** in position **s**.

Silver, David, et al. "Mastering the game of Go with deep neural networks and tree search." Nature 529.7587 (2016): 484-489.

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ขอบคุณ
Thai

Grazie
Italian

תודה רבה
Hebrew

धन्यवादः
Sanskrit

ಧನ್ಯವಾದಗಳು
Kannada

Ευχαριστώ
Greek

Thank You
English

Gracias
Spanish

Спасибо
Russian

Obrigado
Portuguese

شكراً
Arabic

https://sites.google.com/site/animeshchaturvedi07

Merci
French

多謝
Traditional Chinese

धन्यवाद
Hindi

Danke
German

多谢
Simplified Chinese

நன்றி
Tamil
Tamil

ありがとうございました
Japanese

감사합니다
Korean