INDIAN INSTITUTE OF INFORMATION TECHNOLOGY

Dharwad
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Recursion and Tower of Hanoi

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Recursion

- A process by which a function calls itself repeatedly.
- Such a function is called a recursive function

- Recursion may be direct or cyclically in a chain
- Direct recursion.
- When a function f(…) calls f(…).

- Cyclically in a chain recursion.
  - f1(…) calls f2(…) , f2(…) calls f3(…) . . . fi(…) calls f1(…)

Some Examples of Recursion

- Example 1: Factorial calculation

  $n! = n \times (n-1) \times (n-2) \times \ldots \times 3 \times 2 \times 1$

  $n! = n \times (n-1)!$

  $factorial(n) = n \times factorial(n-1)$

- Example 2: Fibonacci number sequence

  1, 1, 2, 3, 5, 8, 13, 21, ...

  $n_i = n_{i-1} + n_{i-2}$

  $fibonacci(n) = fibonacci(n-1) + fibonacci(n-2)$

Some Examples of Recursion

- Example 3: GCD of two positive integers

  gcd(10, 15) = 5, gcd(11, 13) = 1

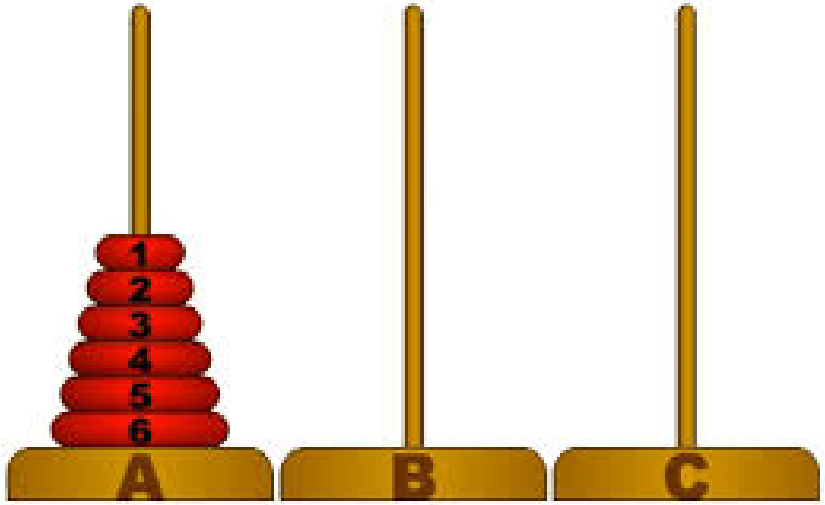
  gcd(m,n) = gcd(m-n,n), if m>n else gcd(m,n-m)

- Example 4: Recursion formula

  $T(n) = n + 2 \times T(n-1)$

  T(100) = ?

Some Examples of Recursion



- Example 5: Tower of Hanoi

  Move n disks from A C

  = move (n-1) disks from A to B

  + move the disk from A to C

  + move (n-1) disk from B to C

- Example 6: Sorting

  Merge Sort, Quick Sort,

- Example 7: Traversal of Binary Tree

  Pre-order, Inorder, Postorder Traversal of Binary Tree

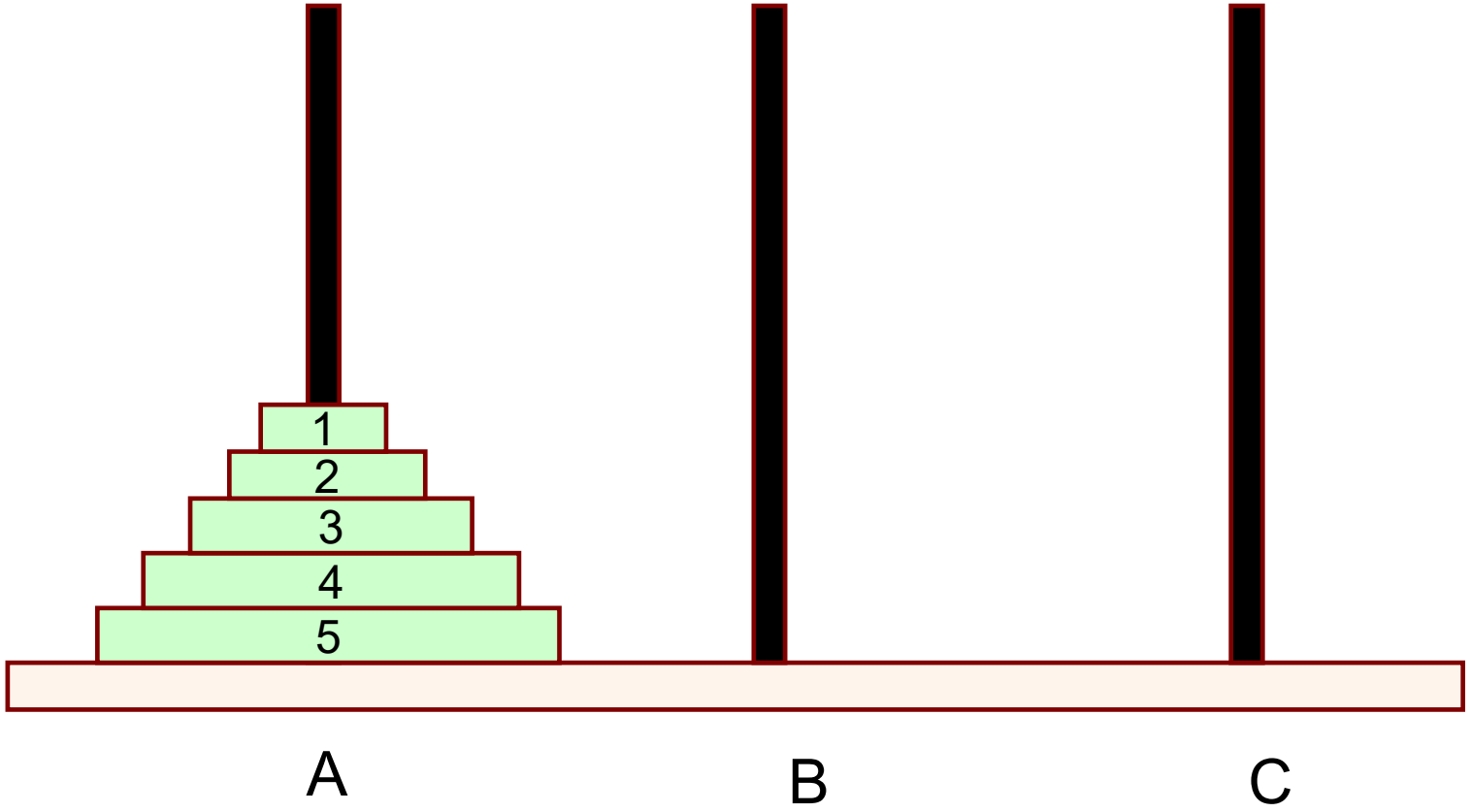
Base case and Recursive case

- For a function to be written in recursive form, two conditions are to be satisfied:

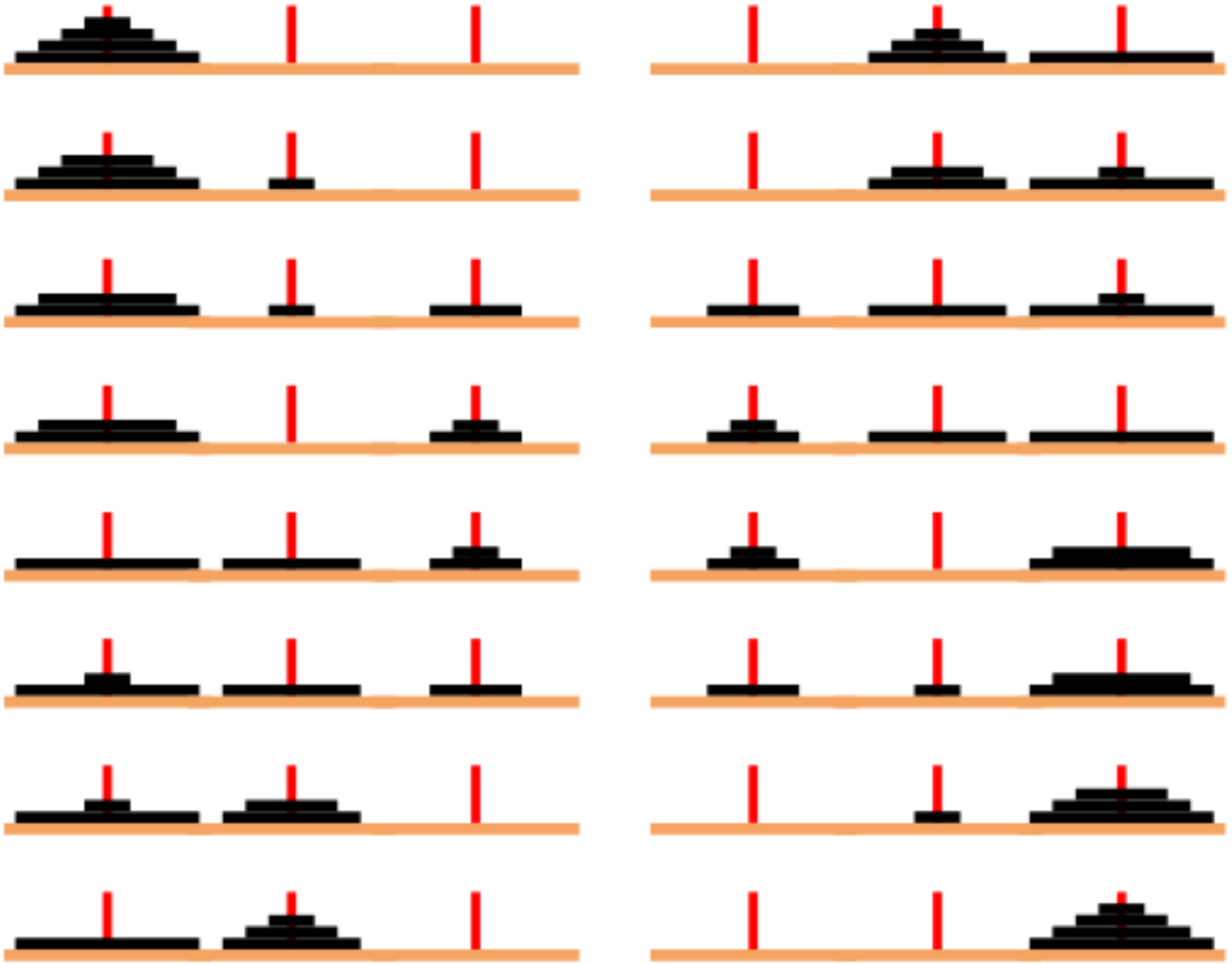
- Condition 1: **Base case**
  - The problem statement must include a stopping condition. A simple occurrence that can be answered directly.

- Condition 2: **Recursive case**
  - It should be possible to express the problem in recursive form.
  - A more complex occurrence of the problem that cannot be directly answered, but can instead be described in terms of smaller occurrences of the same problem.

Towers of Hanoi Problem

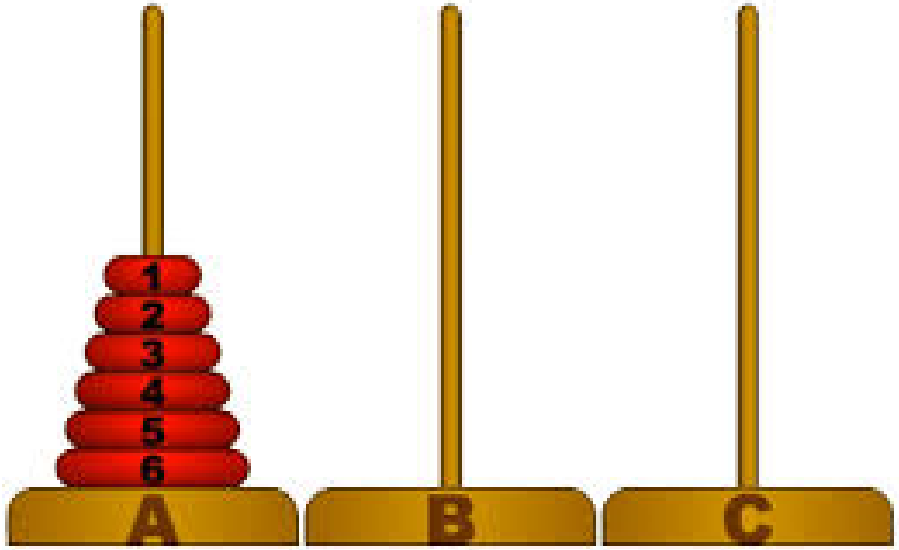


Towers of Hanoi Problem



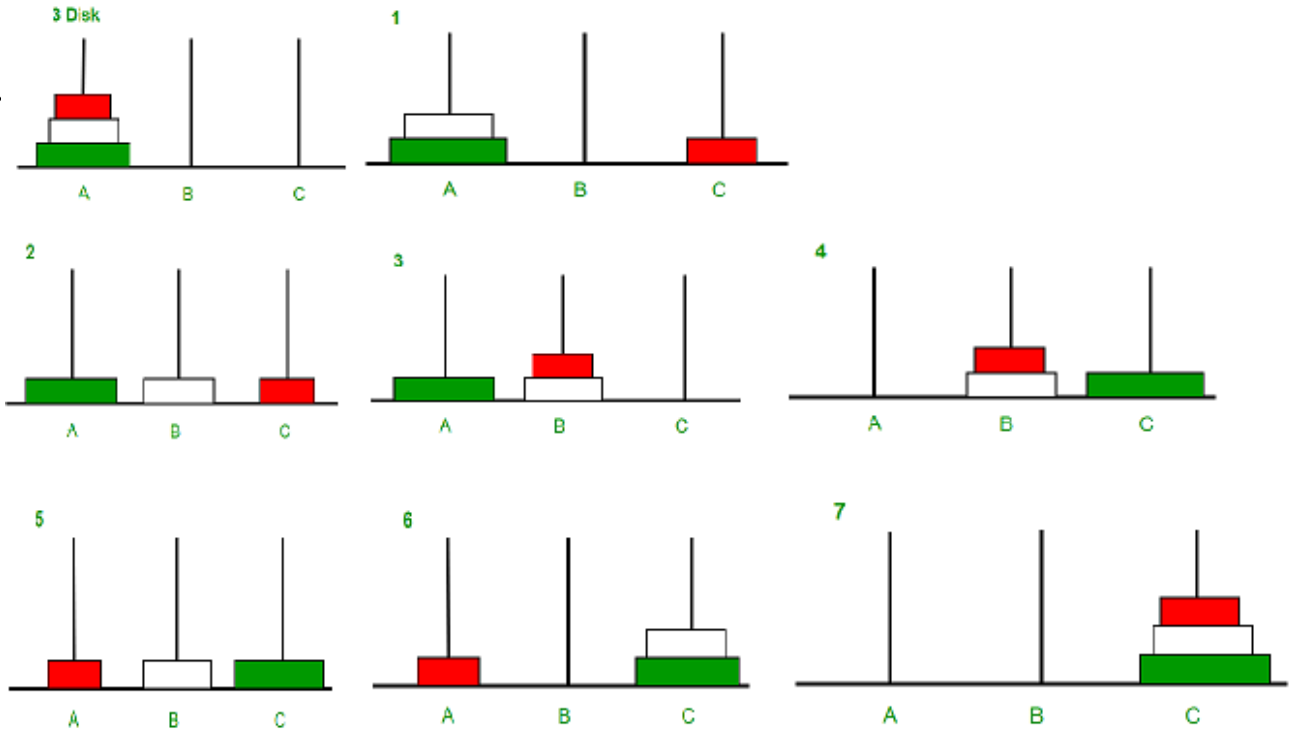
Towers of Hanoi Problem

- The problem statement
- Initially all the disks are stacked on the A pole.
- Required to transfer all the disks to the C pole.
  - Only one disk can be moved at a time.
  - A larger disk cannot be placed on a smaller disk.
- C pole is used for temporary storage of disks.



Towers of Hanoi Problem

- Recursive statement of the general problem of n disks
- Step 1:
  - Move the top (n-1) disks from A to B
- Step 2:
  - Move the largest disk from A to C.
- Step 3:
  - Move the (n-1) disks from B to C.



Towers of Hanoi Problem

```
#include <stdio.h>

void move(int n, char A, char B, char C);

int main()
{   int n;          /* Number of disks */
    scanf ("%d", &n);
    move (n, 'A', 'B', 'C');
    return 0;
}

void move (int n, char A, char B, char C)
{         if (n > 0) {
            move (n-1, A, C, B);
            printf ("Move disk %d from %c to %c \n", n, A, C);
            move (n-1, B, C, A);
            }
return;
}
```

Towers of Hanoi – Execution

n = 3
Move disk 1 from A to C
Move disk 2 from A to B
Move disk 1 from C to B
Move disk 3 from A to C
Move disk 1 from B to A
Move disk 2 from B to C
Move disk 1 from A to C

n = 4
Move disk 1 from Tower A to Tower C
Move disk 2 from Tower A to Tower B
Move disk 1 from Tower C to Tower B
Move disk 3 from Tower A to Tower C
Move disk 1 from Tower B to Tower A
Move disk 2 from Tower B to Tower C
Move disk 1 from Tower A to Tower C
Move disk 4 from Tower A to Tower B
Move disk 1 from Tower C to Tower B
Move disk 2 from Tower C to Tower A
Move disk 1 from Tower B to Tower A
Move disk 3 from Tower C to Tower B
Move disk 1 from Tower A to Tower C
Move disk 2 from Tower A to Tower B
Move disk 1 from Tower C to Tower B

n = 5
Move disk 1 from A to C
Move disk 2 from A to B
Move disk 1 from C to B
Move disk 3 from A to C
Move disk 1 from B to A
Move disk 2 from B to C
Move disk 1 from A to C
Move disk 4 from A to B
Move disk 1 from C to B
Move disk 2 from C to A
Move disk 1 from B to A
Move disk 3 from C to B
Move disk 1 from A to C
Move disk 2 from A to B
Move disk 1 from C to B
Move disk 5 from A to C
Move disk 1 from B to A
Move disk 2 from B to C
Move disk 1 from A to C
Move disk 3 from B to A
Move disk 1 from C to B
Move disk 2 from C to A
Move disk 1 from B to A
Move disk 4 from B to C
Move disk 1 from A to C
Move disk 2 from A to B
Move disk 1 from C to B
Move disk 3 from A to C
Move disk 1 from B to A
Move disk 2 from B to C
Move disk 1 from A to C

How many moves are required for *n* disks?

תודה רבה
Hebrew

Ευχαριστώ
Greek

Danke
German

Спасибо
Russian

धन्यवादः
Sanskrit

Merci
French

ধন্যবাদ
Bangla

நன்றி
Tamil

شكراً
Arabic

ಧನ್ಯವಾದಗಳು
Kannada

Thank You
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നന്ദി
Malayalam

Grazie
Italian

多謝
Traditional Chinese

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Gujarati

ਧੰਨਵਾਦ
Punjabi

धन्यवाद
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Obrigado
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Japanese

ขอบคุณ
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감사합니다
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