INDIAN INSTITUTE OF INFORMATION TECHNOLOGY

Tree - Data Structures

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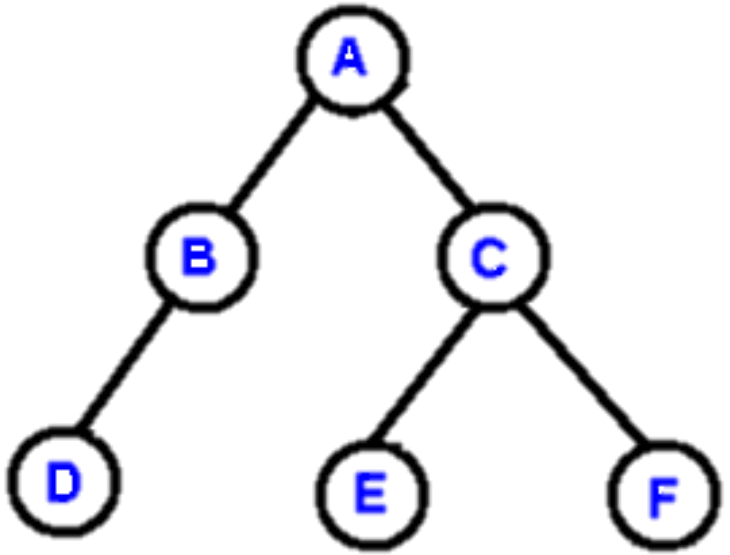
Trees

- Tree is special kind of Graph.
- Family Trees
- Organization Charts
- Classification trees
  - what kind of flower is this?
  - is this mushroom poisonous?
- File directory structure
  - folders, subfolders in Windows
  - directories, subdirectories in UNIX
- Non-recursive procedure call chains



Tree Terminology

root:

leaf:

child:

parent:

sibling:

ancestor:

descendent:

subtree:

depth:

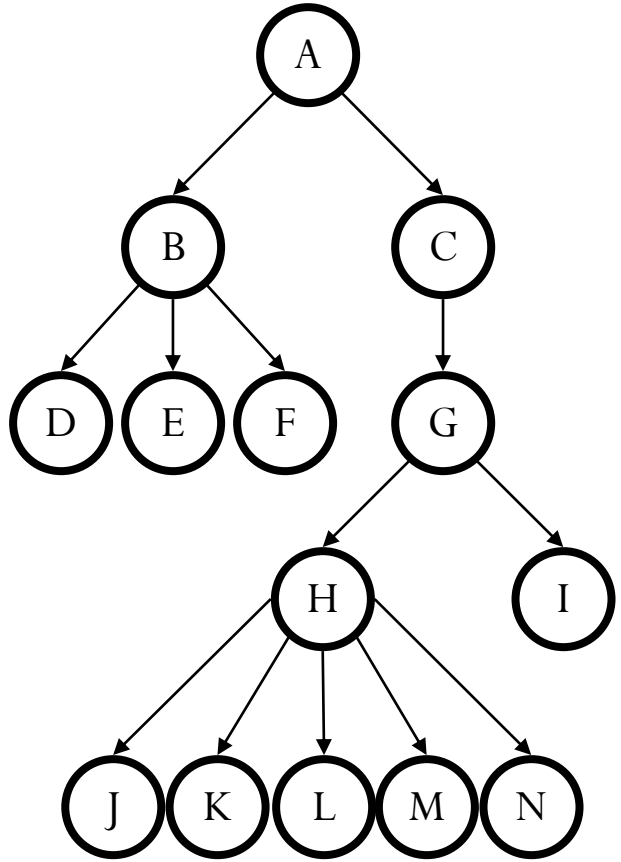
height:

degree:

branching factor:

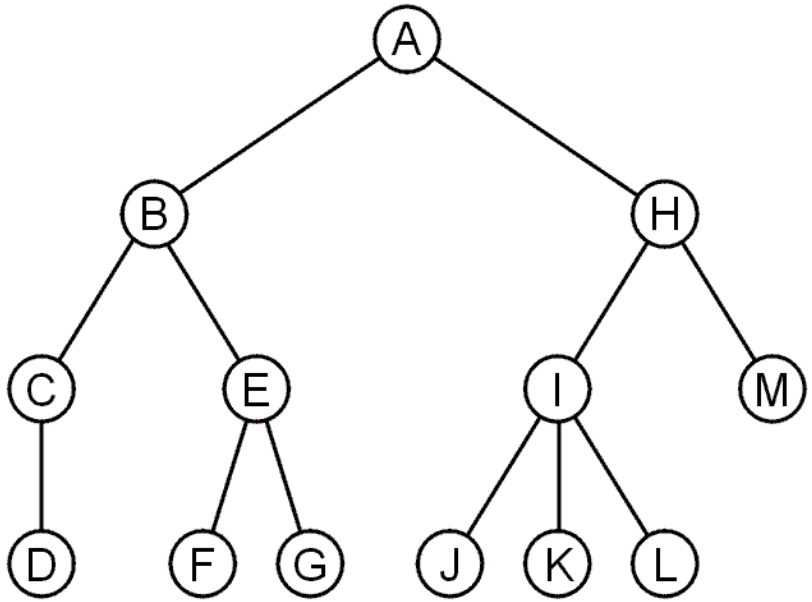
preorder traversal:

postorder traversal:



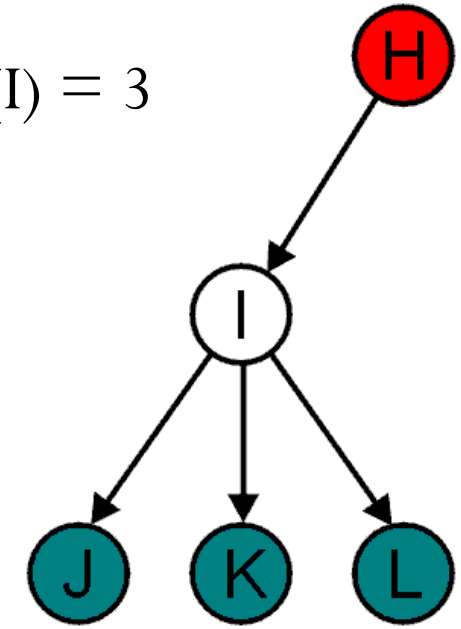
Trees

- A rooted tree data structure stores information in *nodes*
  - Similar to linked lists:
    - There is a first node, or *root*
    - Each node has variable number of references to successors
    - Each node, other than the root, has exactly one node pointing to it



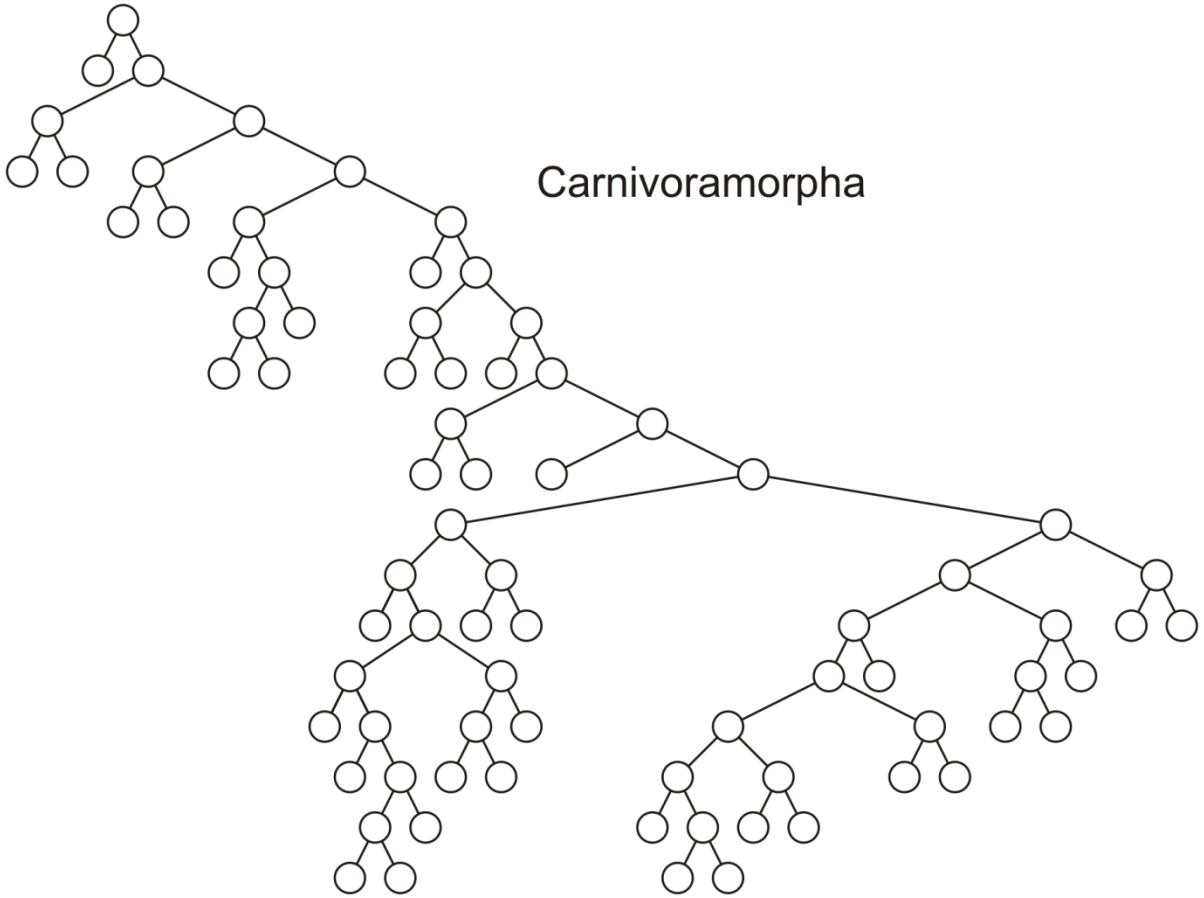
Terminology

- Each node can be connected to arbitrary number of nodes, called ***children***
- All nodes will have zero or more child nodes or *children*
  - I has three children: J, K and L
- For all nodes other than the root node, there is one parent node
  - H is the parent I
- The *degree* of a node is defined as the number of its children: deg(I) = 3
- Nodes with the same parent are *siblings*
  - J, K, and L are siblings



Terminology

- Phylogenetic trees have nodes with degree 2 or 0:

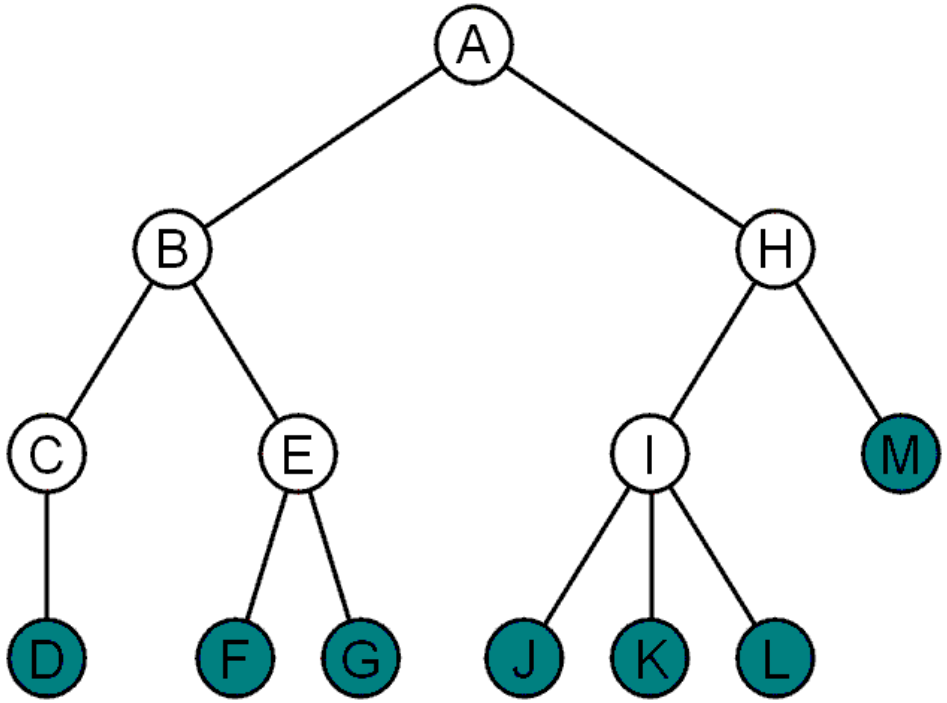


Wesley-Hunt, G. D.; Flynn, J. J. “Phylogeny of the Carnivora: basal relationships among the Carnivoramorphans, and assessment of the position of ‘Miacoidea’

Terminology

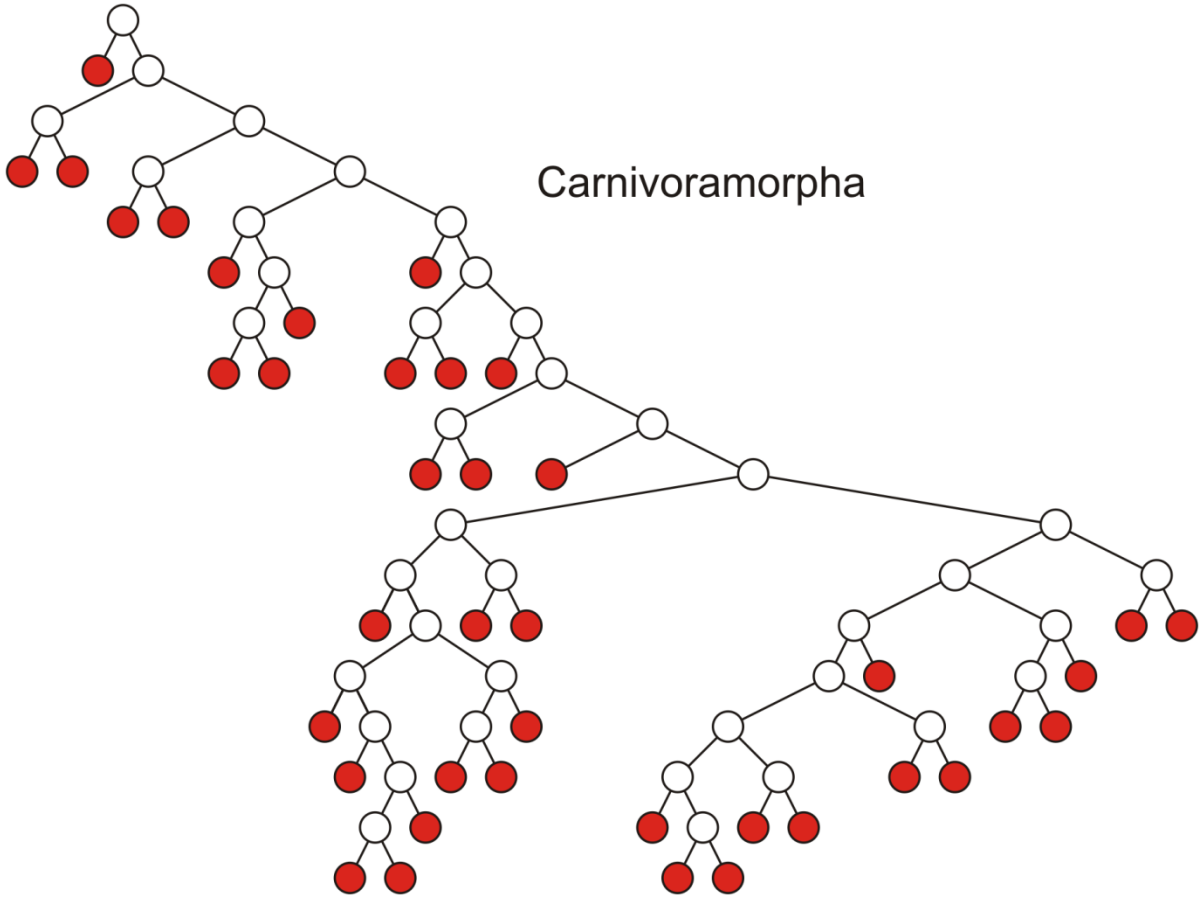
- Nodes with degree zero are also called *leaf nodes*

- All other nodes are said to be *internal nodes*, that is, they are internal to the tree



Terminology

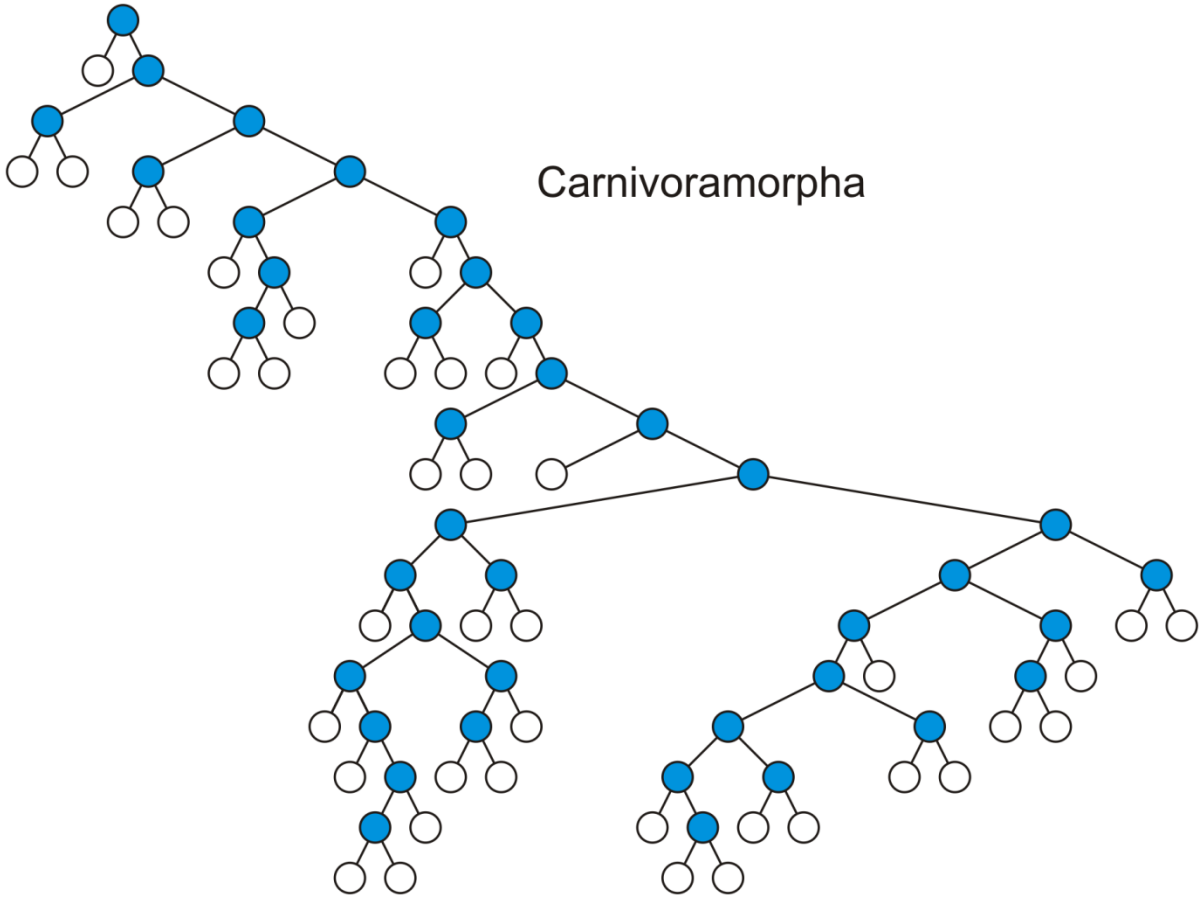
Leaf nodes:



Wesley-Hunt, G. D.; Flynn, J. J. “Phylogeny of the Carnivora: basal relationships among the Carnivoramorphans, and assessment of the position of ‘Miacoidea’

Terminology

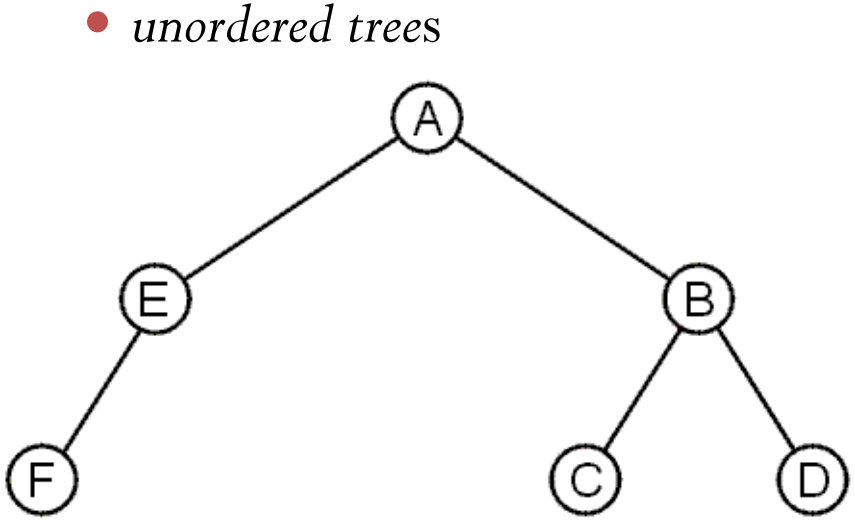
Internal nodes:

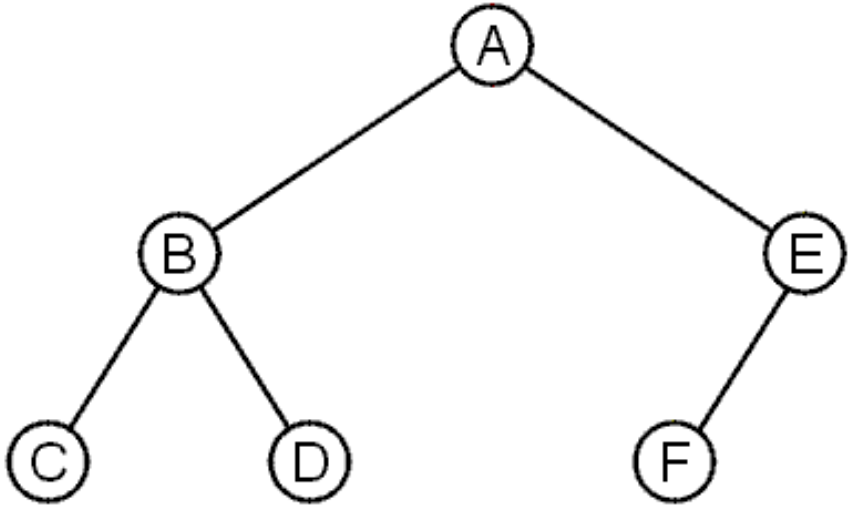


Wesley-Hunt, G. D.; Flynn, J. J. “Phylogeny of the Carnivora: basal relationships among the Carnivoramorphans, and assessment of the position of ‘Miacoidea’

Terminology

- These trees are equal if the order of the children is ignored
  - *unordered trees*

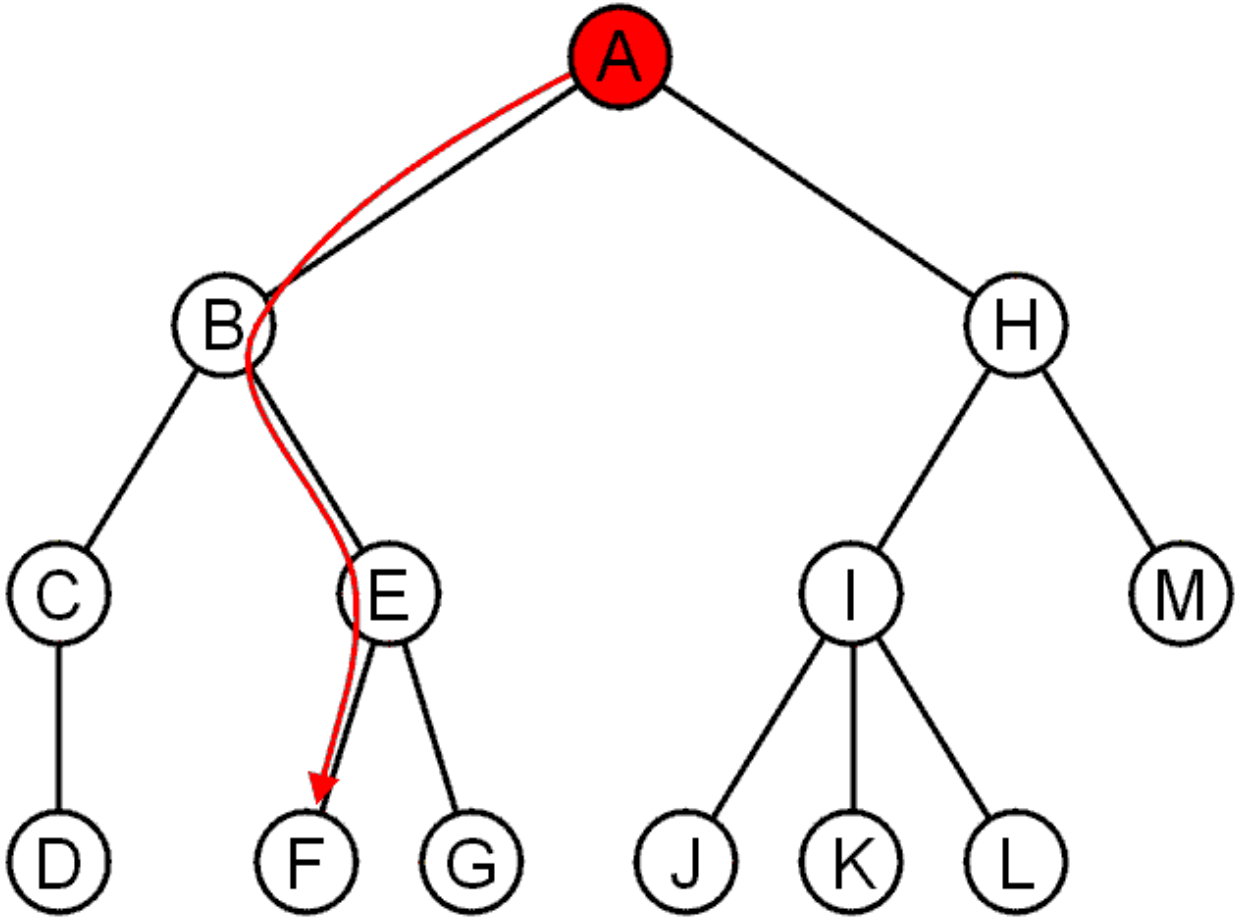




- They are different if order is relevant (*ordered trees*)
  - We will usually examine ordered trees (linear orders)
  - In a hierarchical ordering, order is not relevant

Terminology

- The shape of a rooted tree gives a natural flow from the *root node*, or just *root*



Terminology

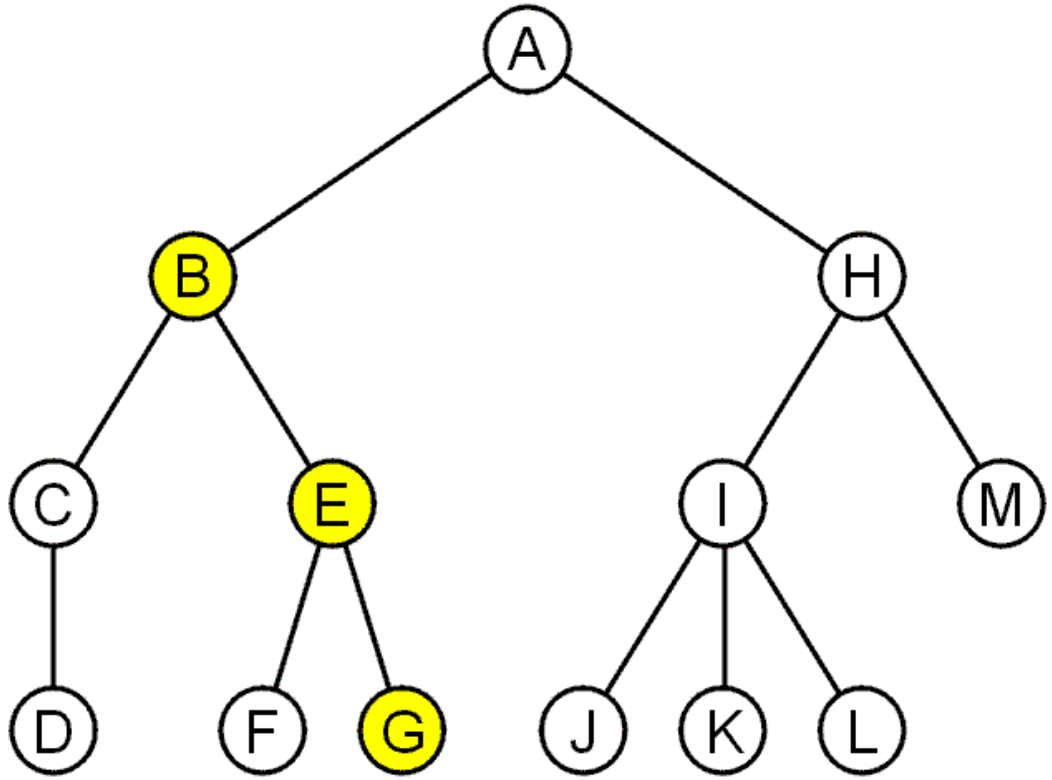
- A path is a sequence of nodes

$$(a_0, a_1, \ldots, a_n)$$

where a_{k+1} is a child of a_k is

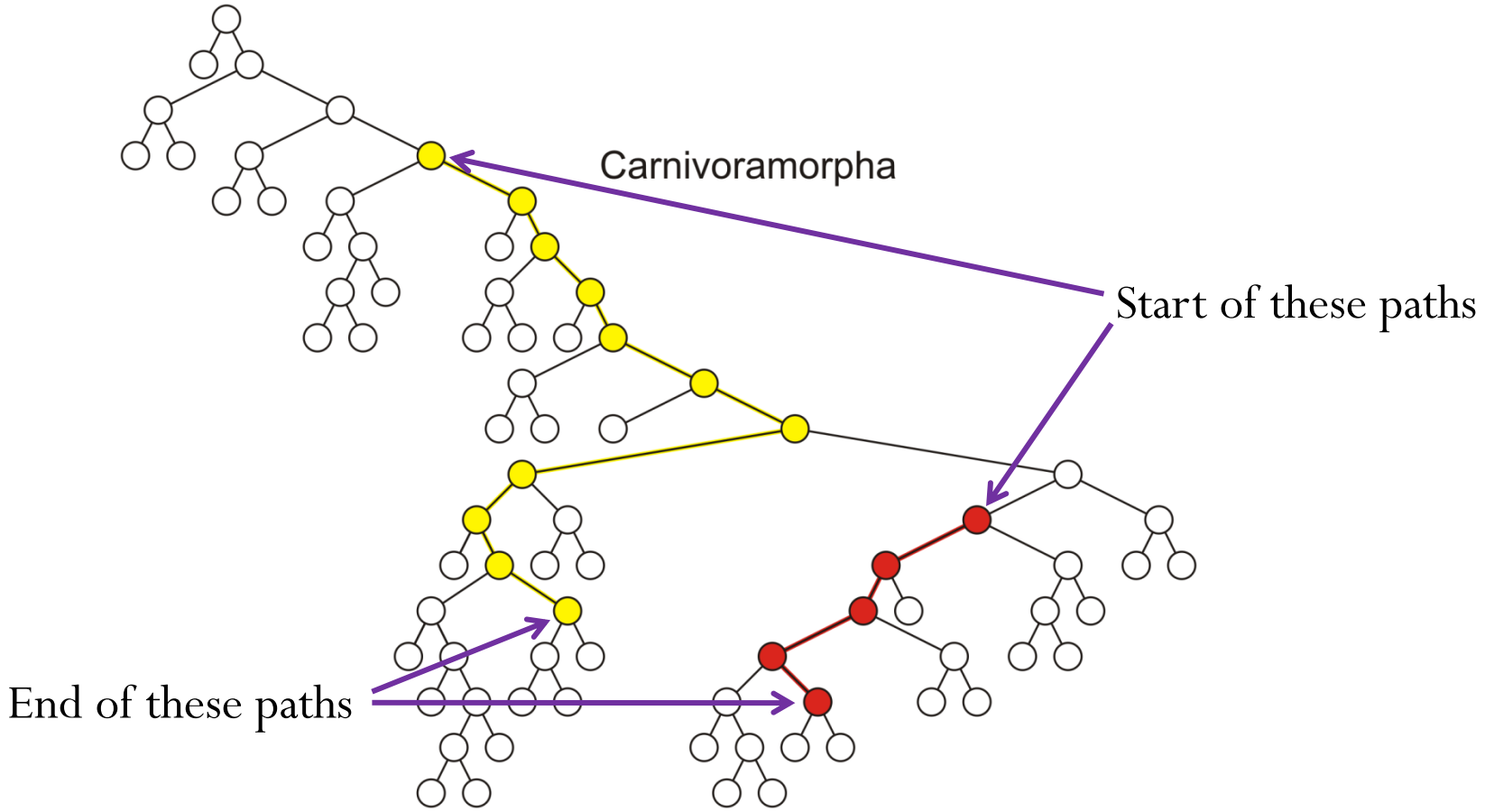
- The length of this path is n

- *E.g.*, the path (B, E, G) has length 2



Terminology

- Paths of length 10 (11 nodes) and 4 (5 nodes)



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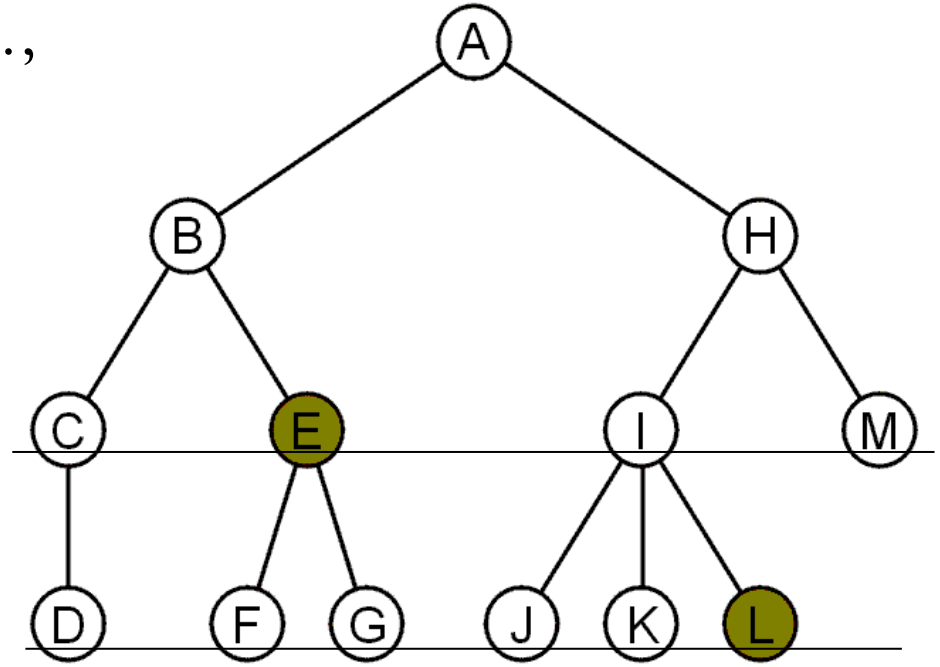
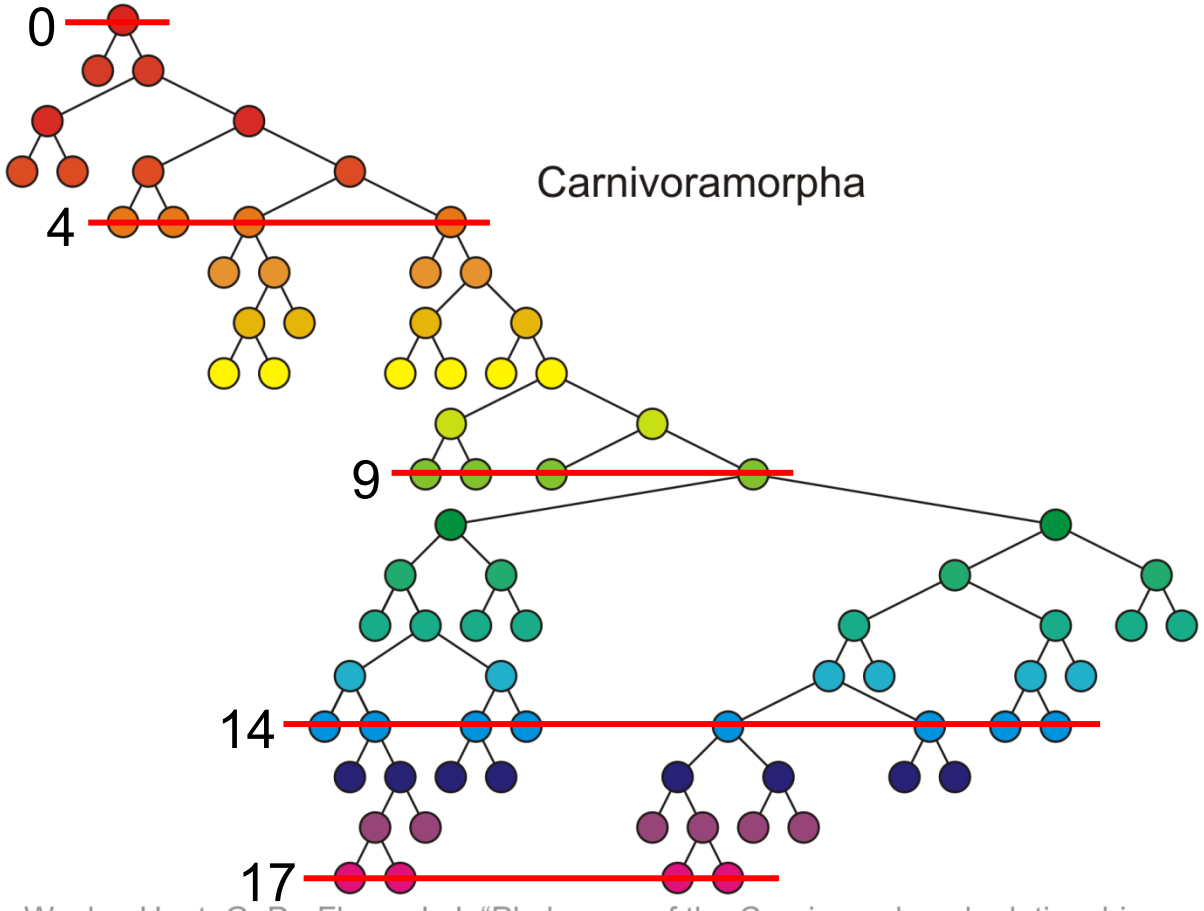
Tree Terminology

- The depth of a node is the number of edges from the root to the node.
- The height of a node is the number of edges from the node to the deepest leaf.
- The height of a tree is a height of the root.
- For each node in a tree, there exists a unique path from the root node to that node
- The length of this path is the *depth* of the node, *e.g.*,
  - E has depth 2
  - L has depth 3



Terminology

- Nodes of depth up to 17



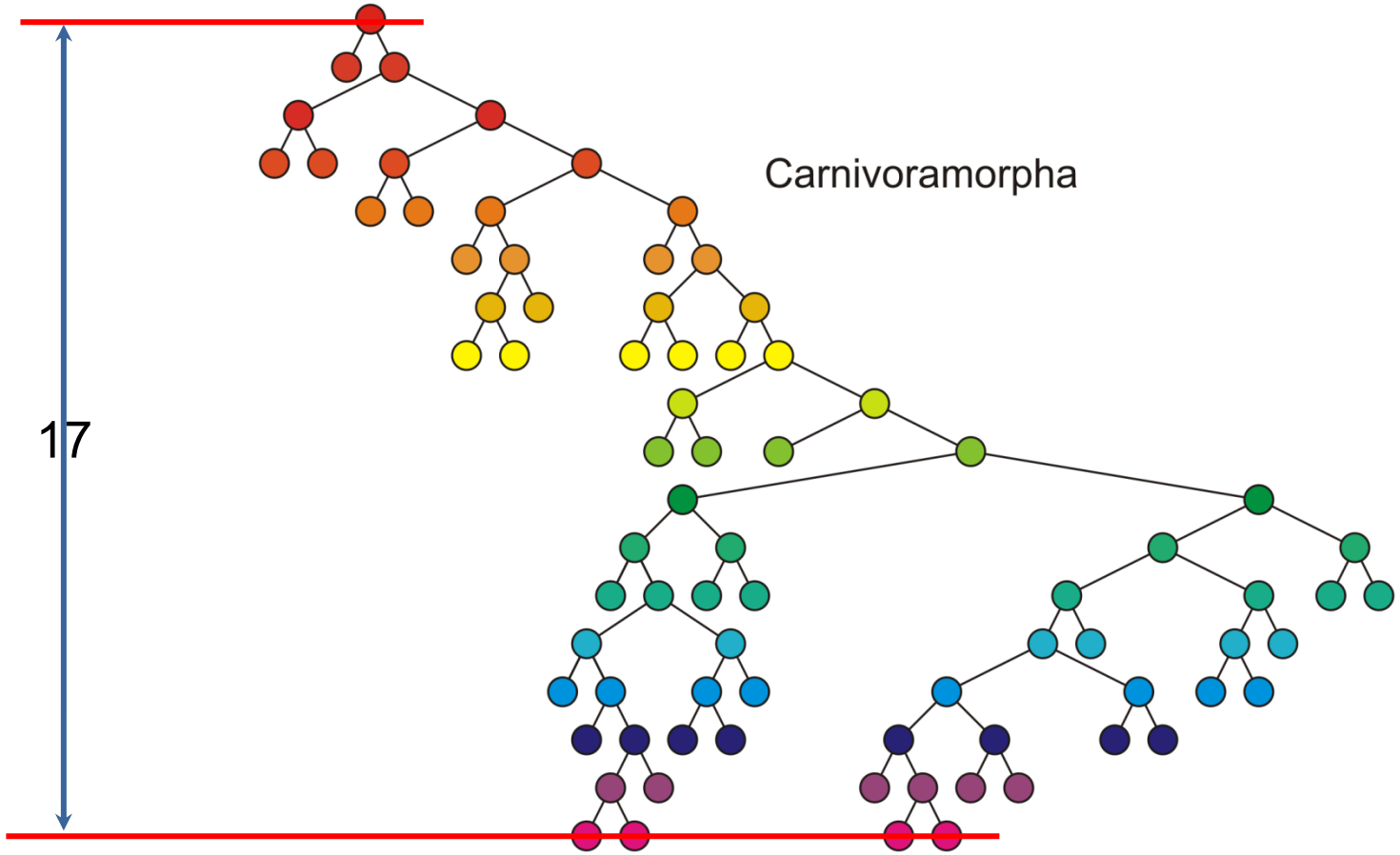
Wesley-Hunt, G. D.; Flynn, J. J. “Phylogeny of the Carnivora: basal relationships among the Carnivoramorphans, and assessment of the position of ‘Miacoidea’

Terminology

- The *height* of a tree is defined as the maximum depth of any node within the tree
- The height of a tree with one node is 0
  - Just the root node
- For convenience, we define the height of the empty tree to be –1

Terminology

- The *height* of this tree is 17



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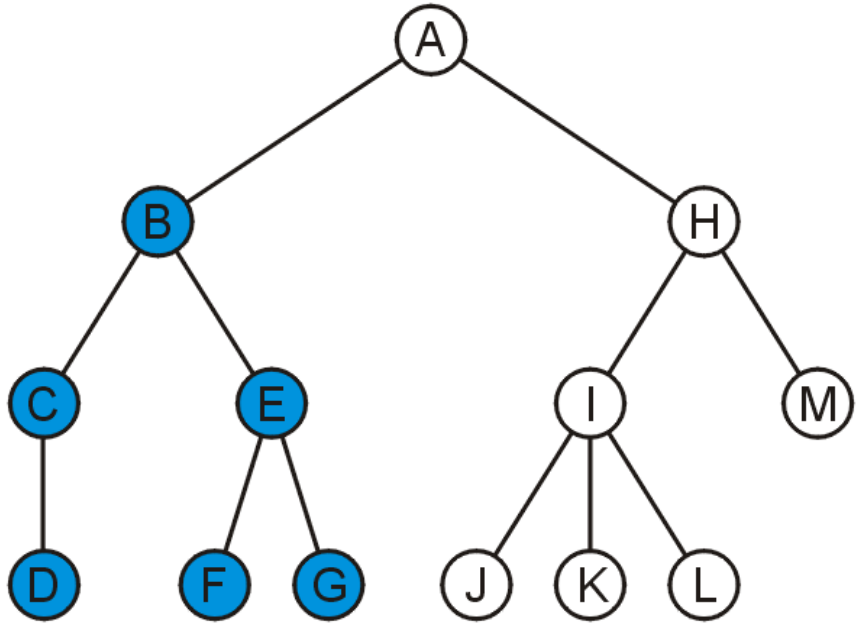
Terminology

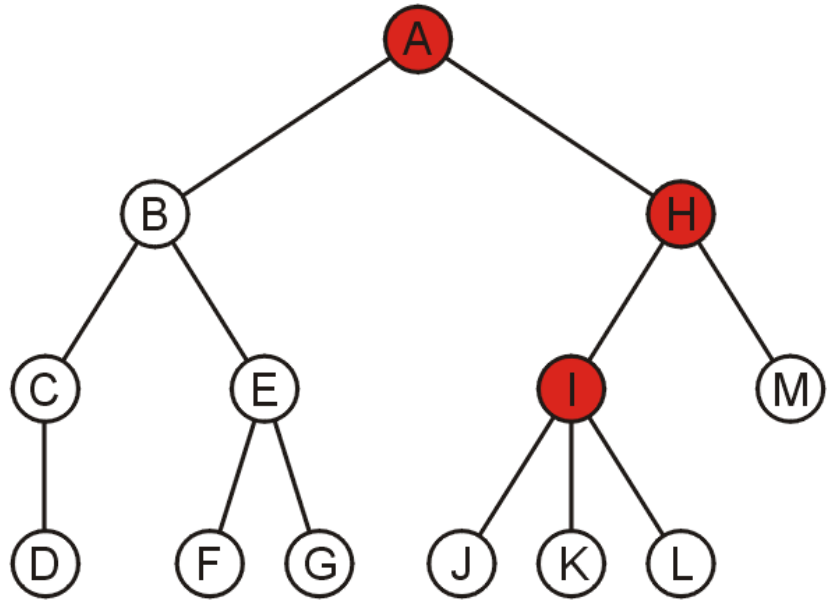
- If a path exists from node *a* to node *b*:
  - *a* is an *ancestor* of *b*
  - *b* is a *descendent* of *a*
- Thus, a node is both an ancestor and a descendant of itself
  - We can add the adjective *strict* to exclude equality: *a* is a *strict descendent* of *b* if *a* is a descendant of *b* but $a \neq b$
- The root node is an ancestor of all nodes

Terminology

- The descendants of node B are B, C, D, E, F, and G:

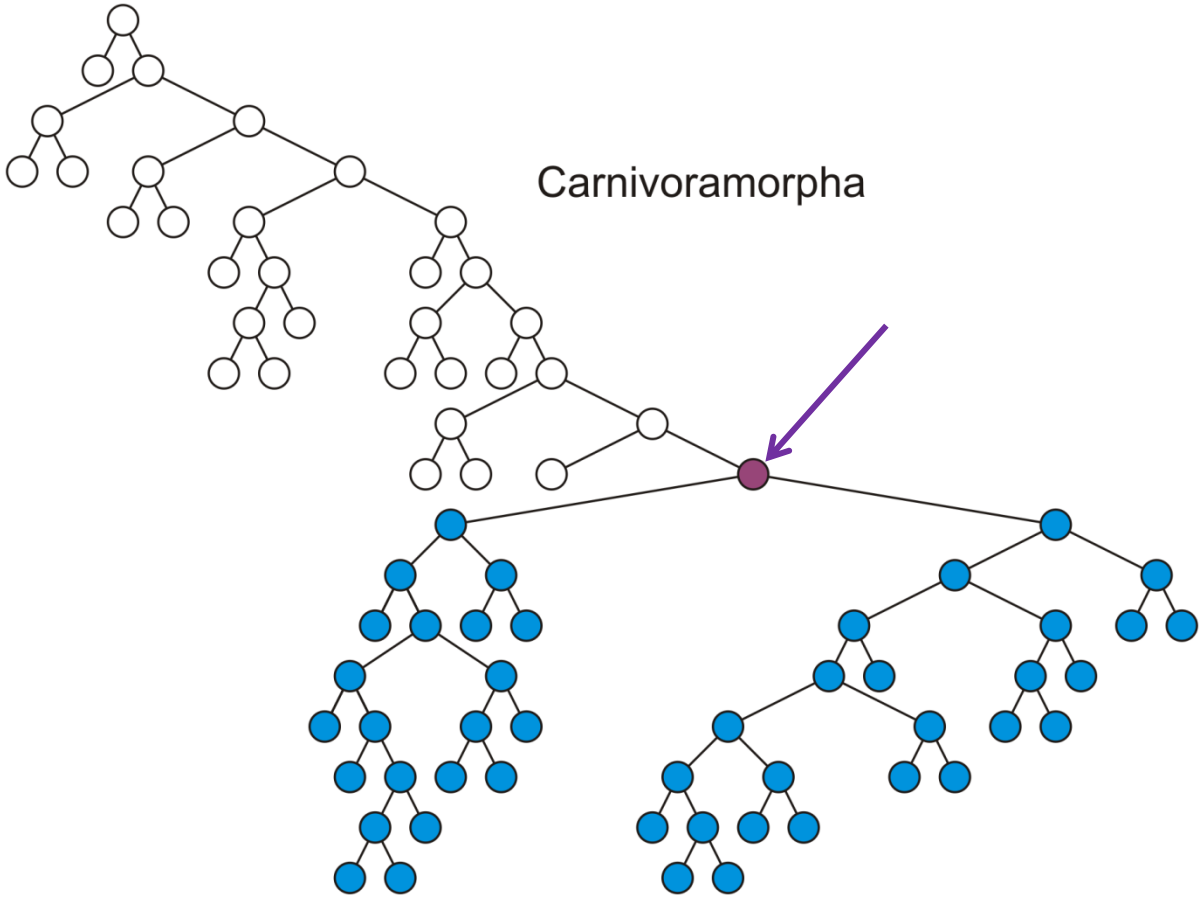


- The ancestors of node I are I, H, and A:



Terminology

All descendants (including itself) of the indicated node



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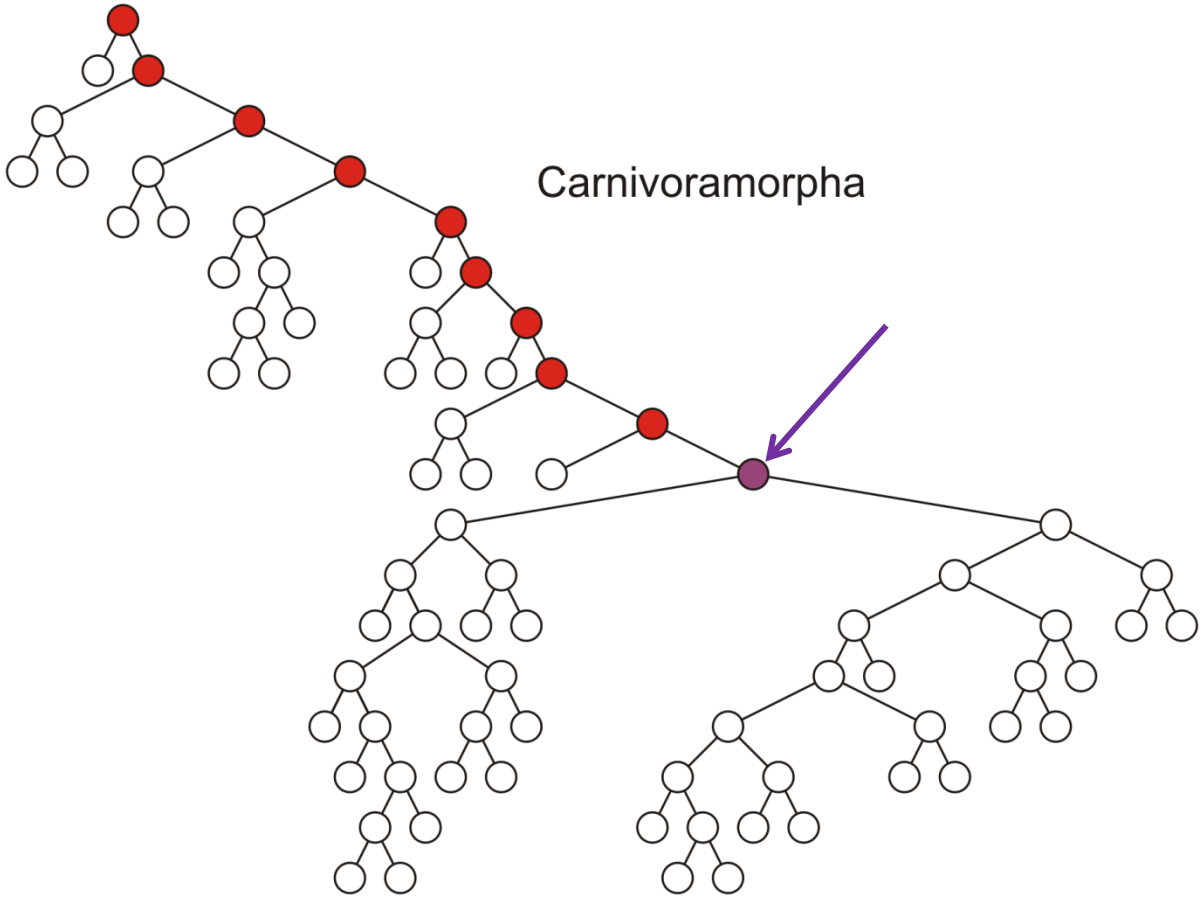
Terminology

All ancestors (including itself) of the indicated node



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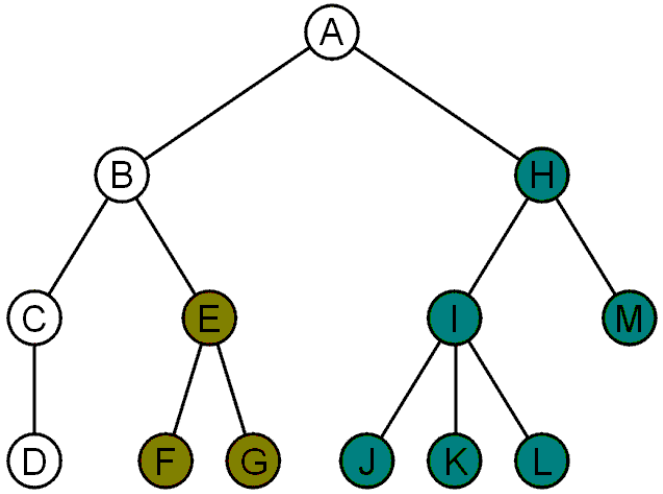
Terminology

- Another approach to a tree is to define the tree recursively:
  - A degree-0 node is a tree
  - A node with degree *n* is a tree if it has *n* children and all of its children are disjoint trees (*i.e.*, with no intersecting nodes)

- Given any node *a* within a tree with root *r*, the collection of *a* and all of its descendants is said to be a *subtree of the tree with root a*



Example: XHTML and CSS

- The XML of XHTML has a tree structure

- Cascading Style Sheets (CSS) use the tree structure to modify the display of HTML

Example: XHTML and CSS

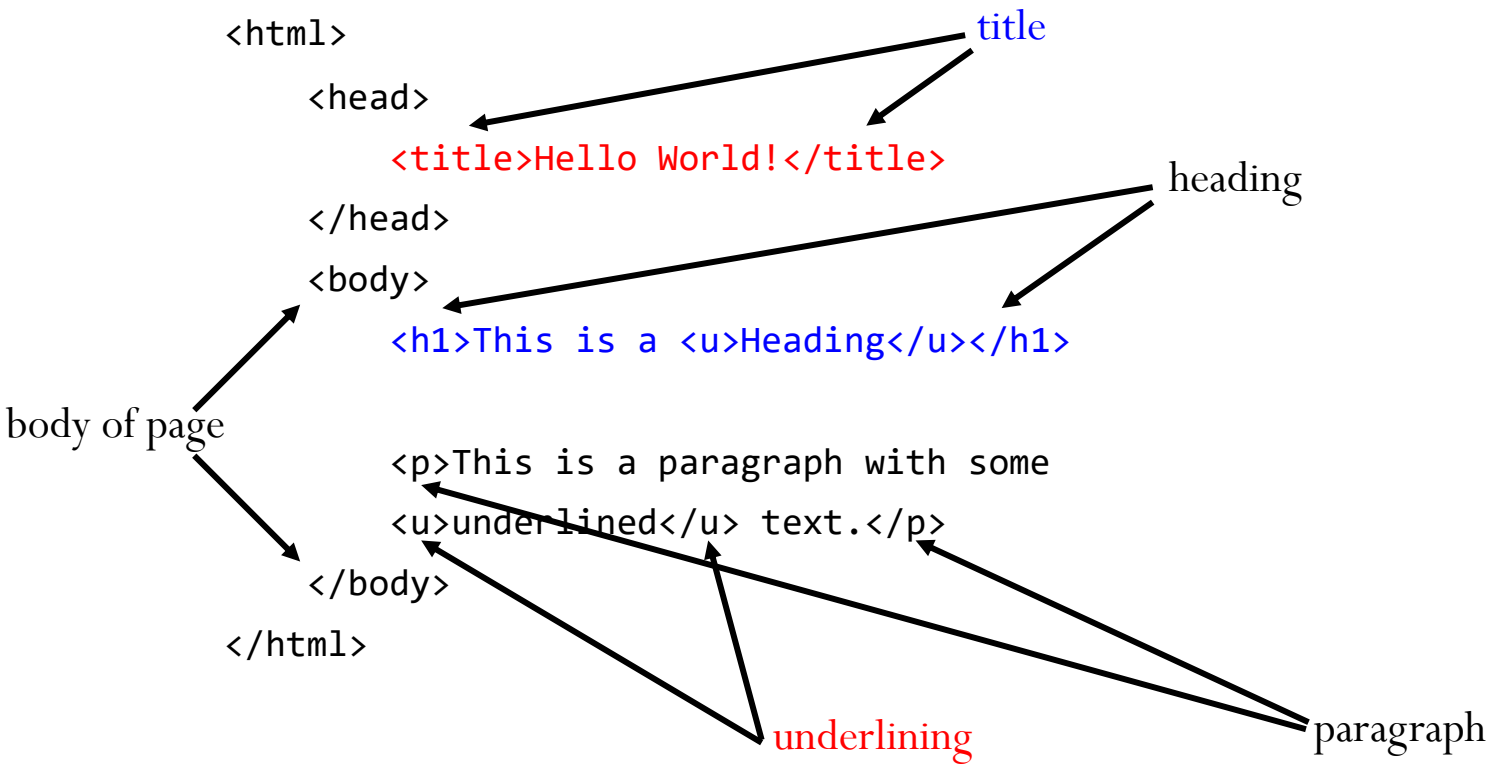
- Consider the following XHTML document

```
<html>
    <head>
        <title>Hello World!</title>
    </head>
    <body>
        <h1>This is a <u>Heading</u></h1>

        <p>This is a paragraph with some
        <u>underlined</u> text.</p>
    </body>
</html>
```

Example: XHTML and CSS

- Consider the following XHTML document

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    </body>
</html>
```

Example: XHTML and CSS

- The nested tags define a tree rooted at the HTML tag

```
<html>
    <head>
        <title>Hello World!</title>
    </head>
    <body>
        <h1>This is a <u>Heading</u></h1>

        <p>This is a paragraph with some
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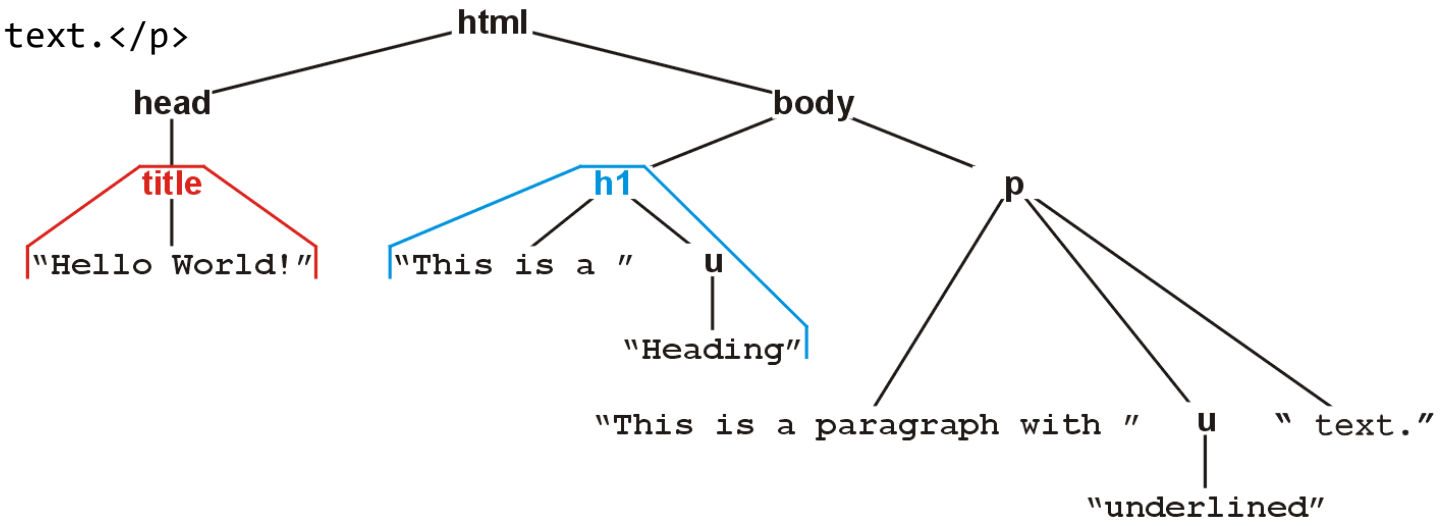
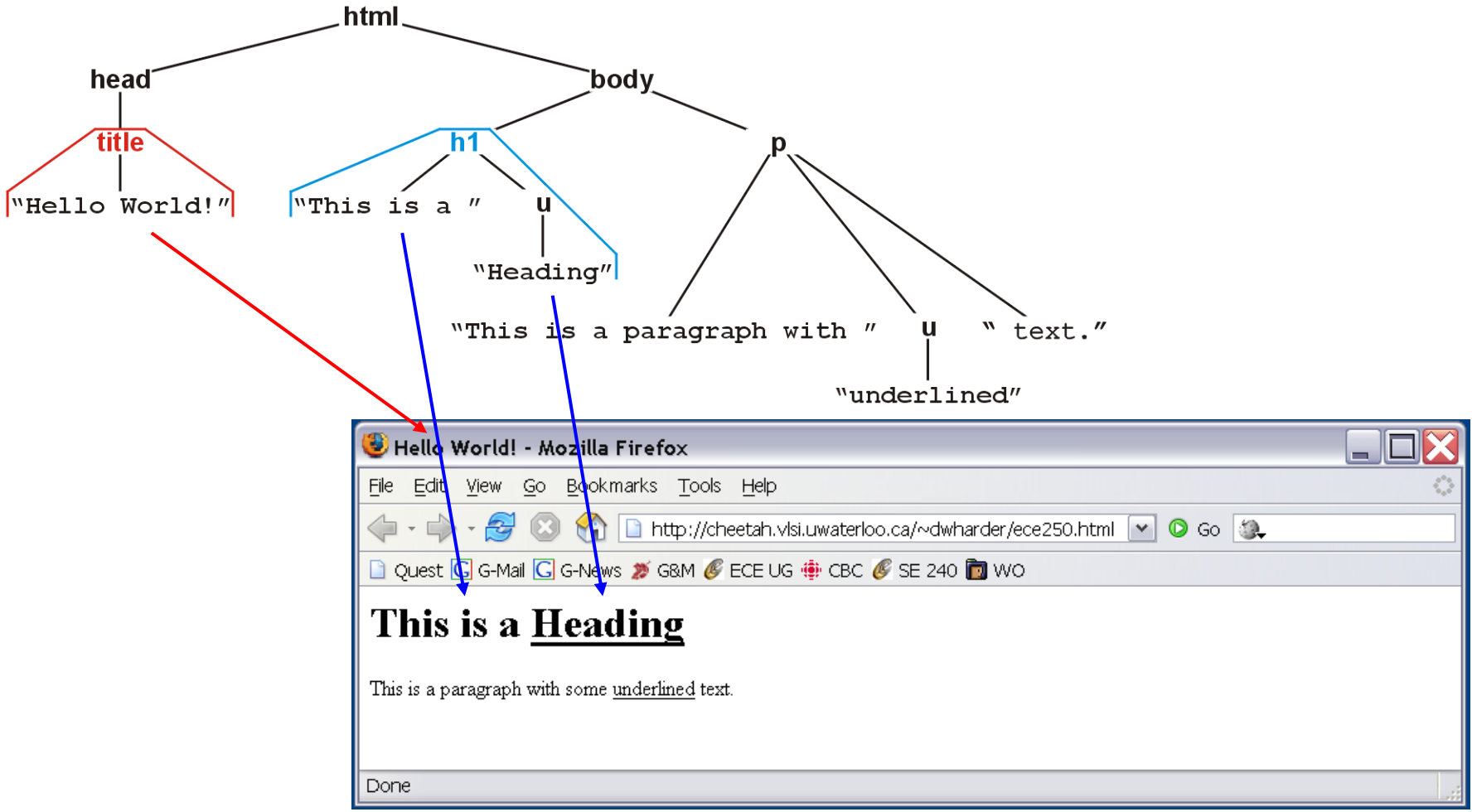
    </body>
</html>
```



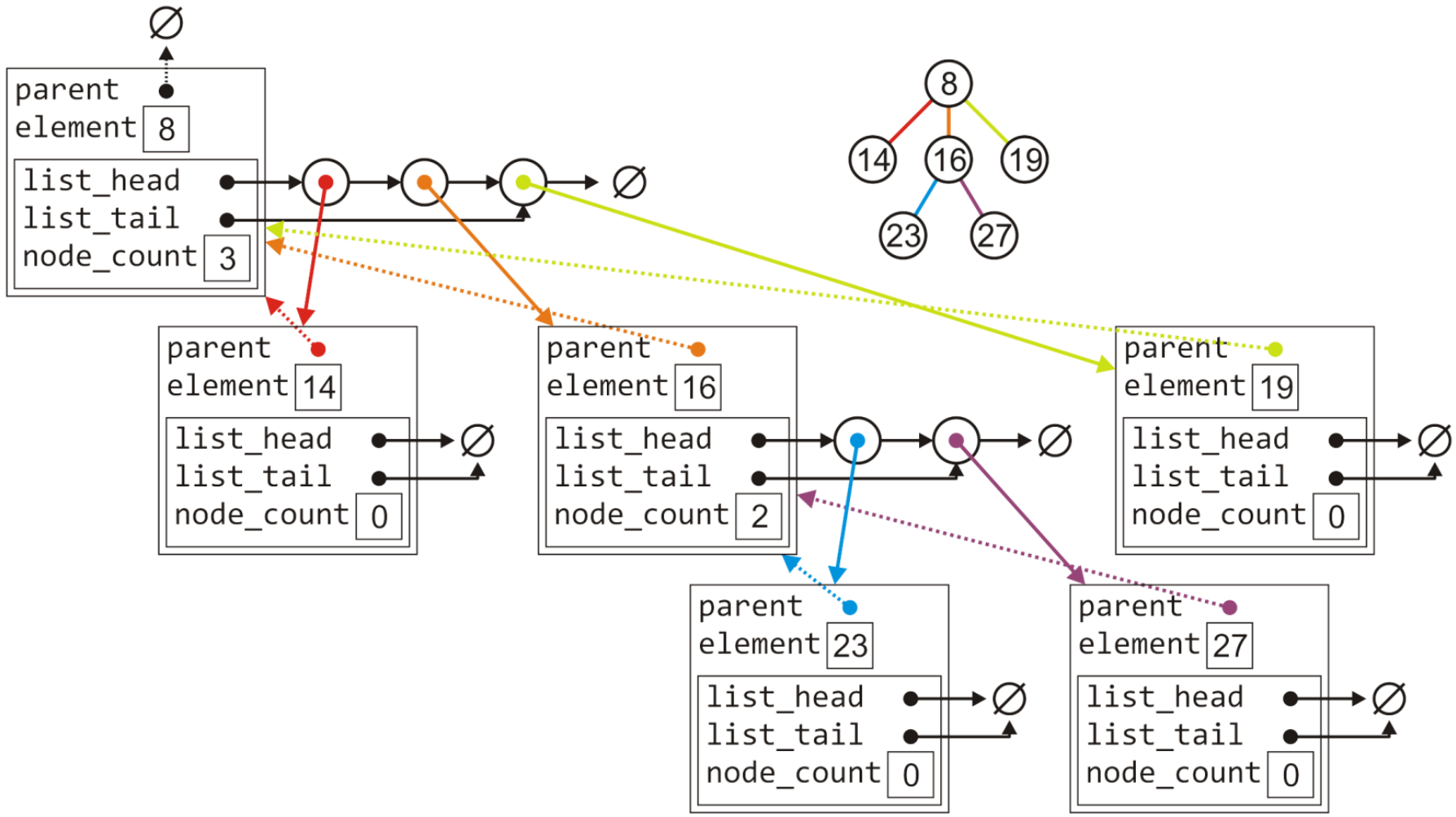
Example: XHTML and CSS

- Web browsers render this tree as a web page



Tree Structure

- The tree with six nodes would be stored as follows:



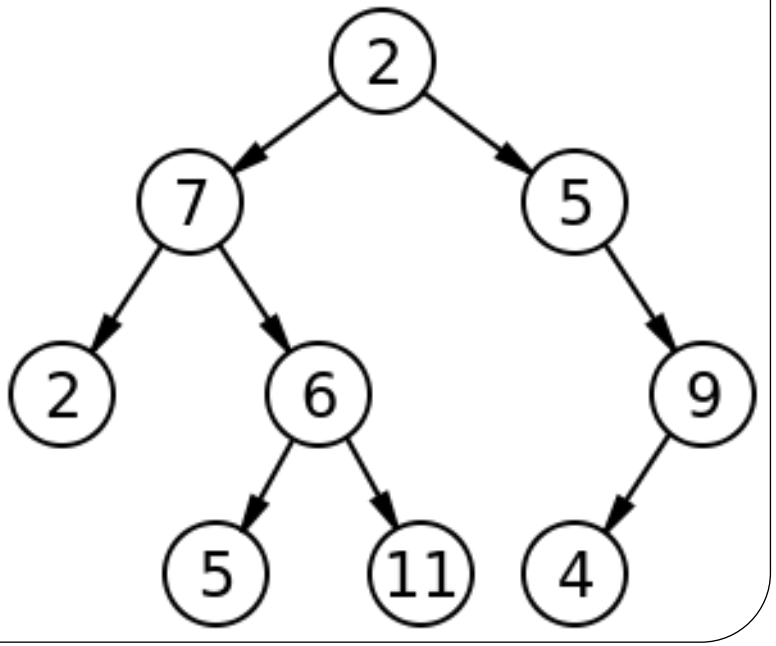
Advantages of Trees

- Trees are so useful and frequently used, because they have some very serious advantages:-
  - Trees reflect structural relationships in the data
  - Trees are used to represent hierarchies
  - Trees provide an efficient insertion and searching
  - Trees are very flexible data, allowing to move subtrees around with minimum effort

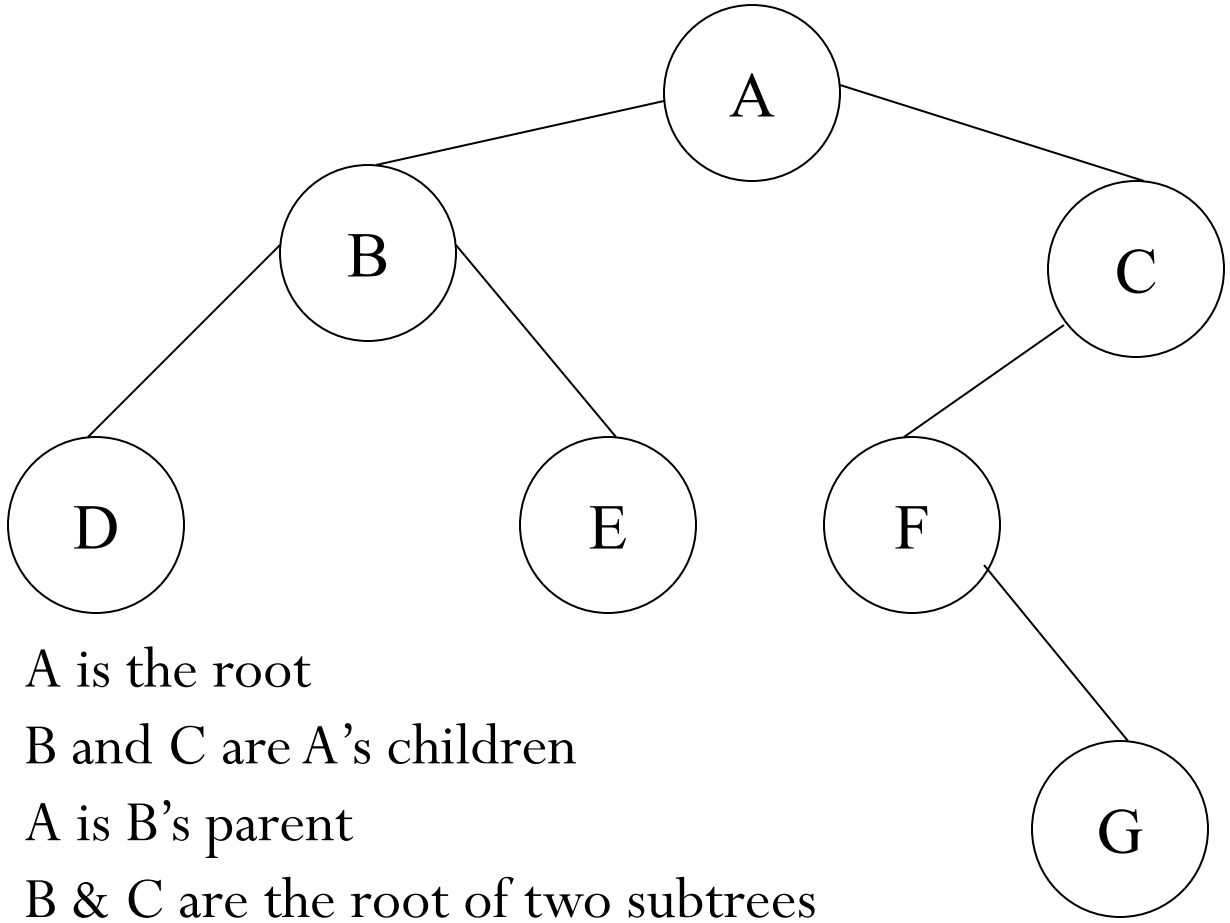
Binary Tree

Definitions and Terminologies

- A data structure in which a record is linked to two successor records, usually referred to as the left branch when greater and the right when less than the previous record.
- A binary tree is made of nodes, where each node contains a "left" reference, a "right" reference, and a data element. The topmost node in the tree is called the ***root***
- Every node (excluding a root) in a tree is connected by a directed edge from exactly one other node which is called a ***parent***
- Each node can be connected to two nodes, called ***children***
- Nodes with no children are called ***leaves*** or ***external nodes***
- Nodes which are not leaves are called ***internal nodes***
- Nodes with the same parent are called ***sibling***



Definitions and Terminologies



A is the root

B and C are A's children

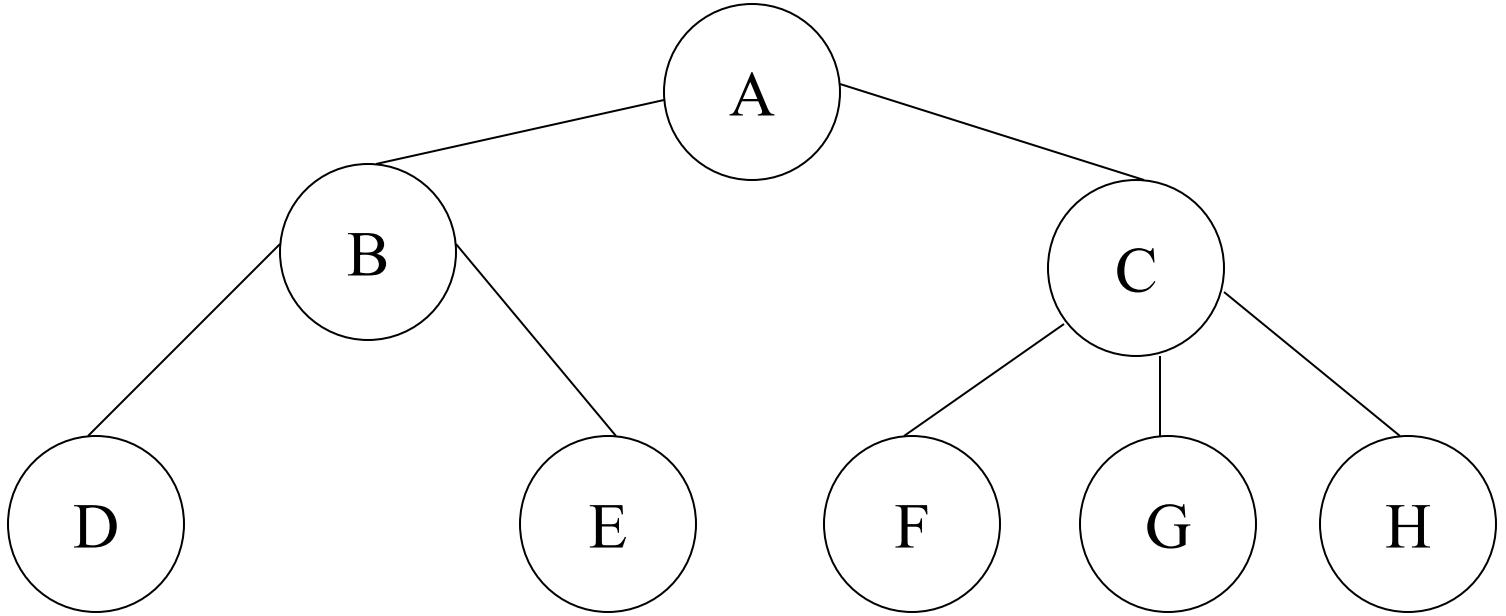
A is B's parent

B & C are the root of two subtrees

D, E, and G are leaves

Definitions and Terminologies

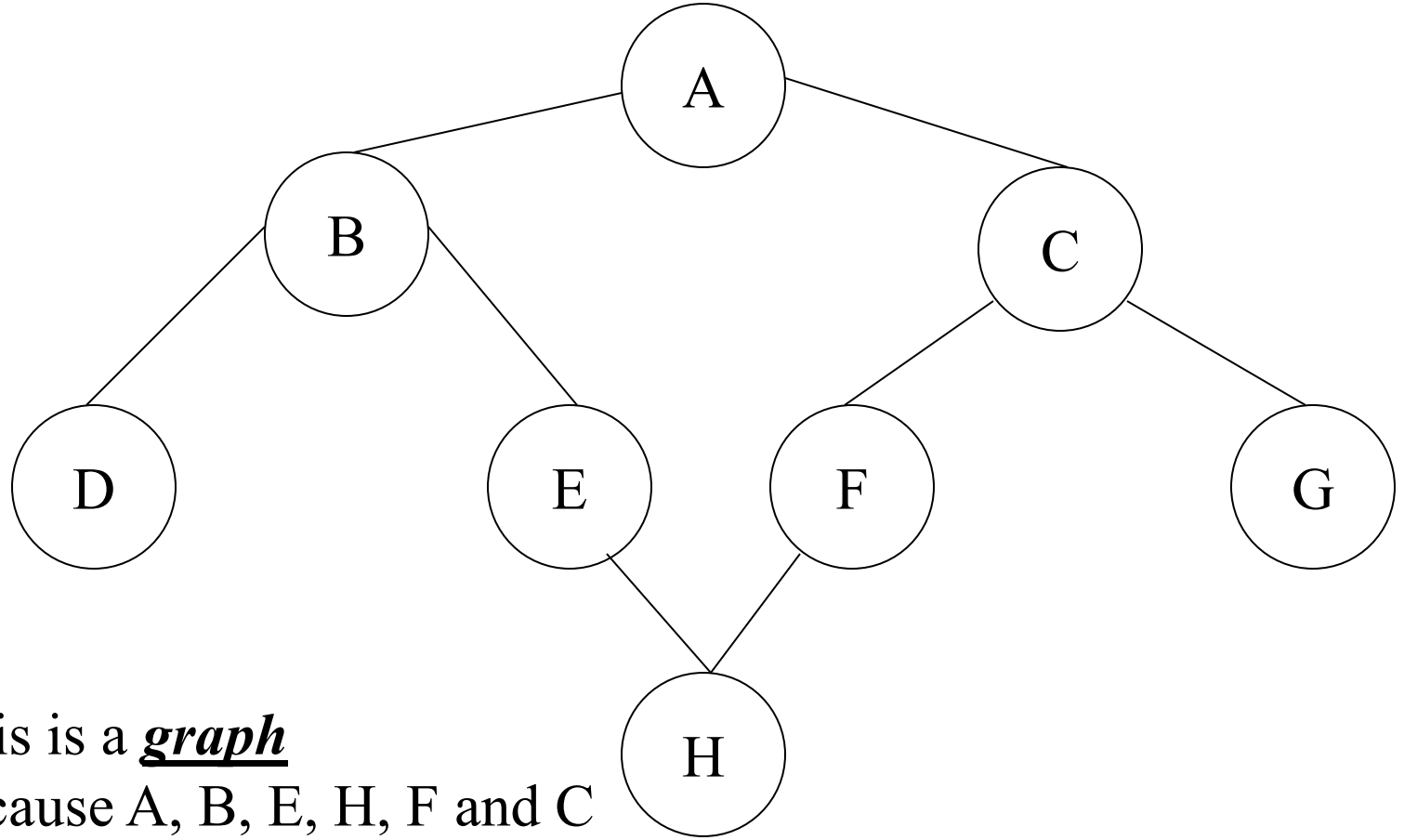
This is ***NOT*** A Binary Tree



This is a ***general tree*** because C has three child nodes

Definitions and Terminologies

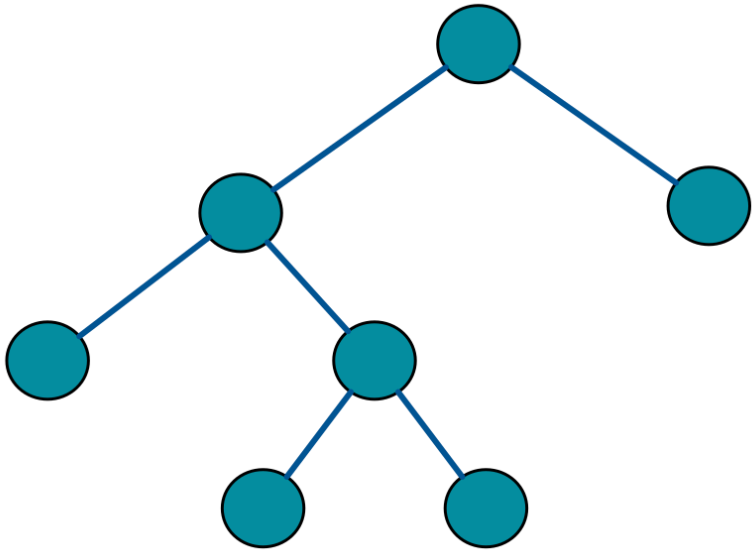
This is ***NOT*** A Binary Tree

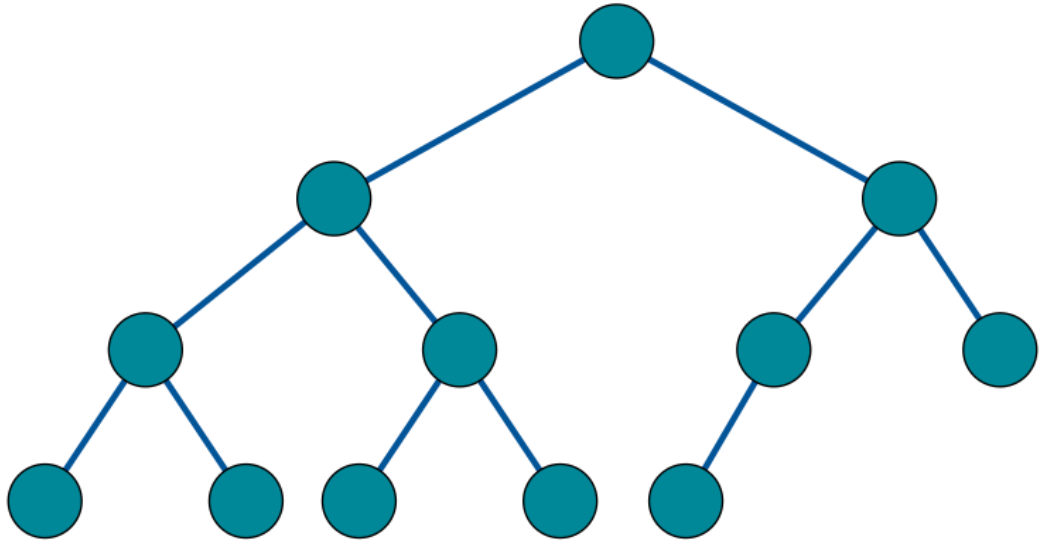


This is a ***graph***
because A, B, E, H, F and C
form a circuit

Full & Complete Binary Tree

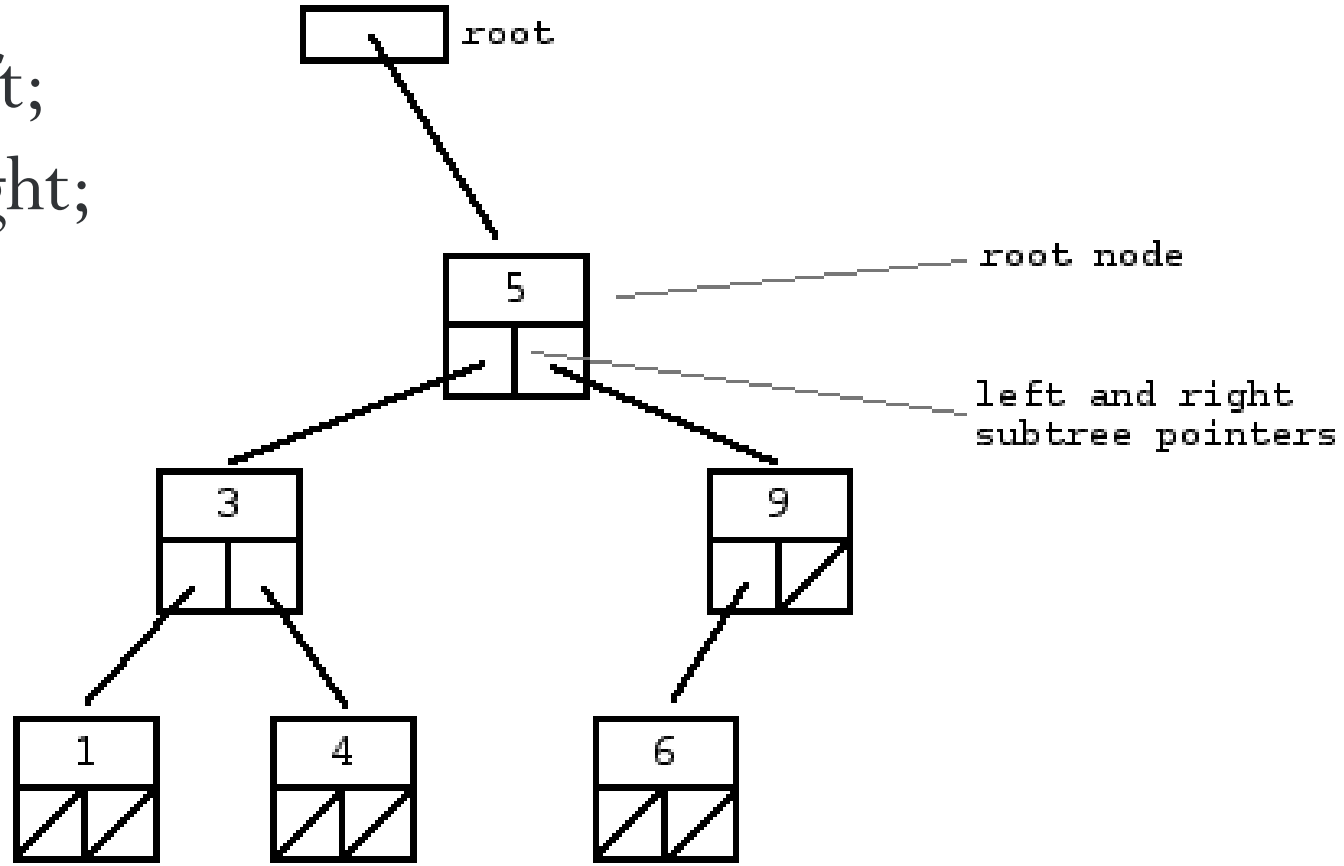
- A full binary tree is a binary tree in which each node has exactly zero or two children.
- A complete binary tree is a binary tree, which is completely filled, with the possible exception of the bottom level, which is filled from left to right.





Binary Tree in C

```
struct node {
        int value;
        struct node * left;
        struct node * right;
    }
```

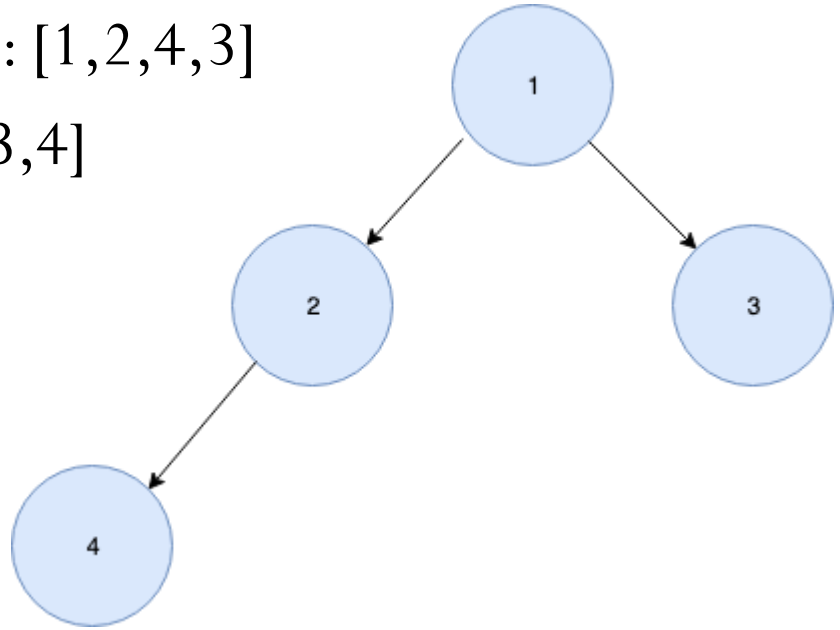


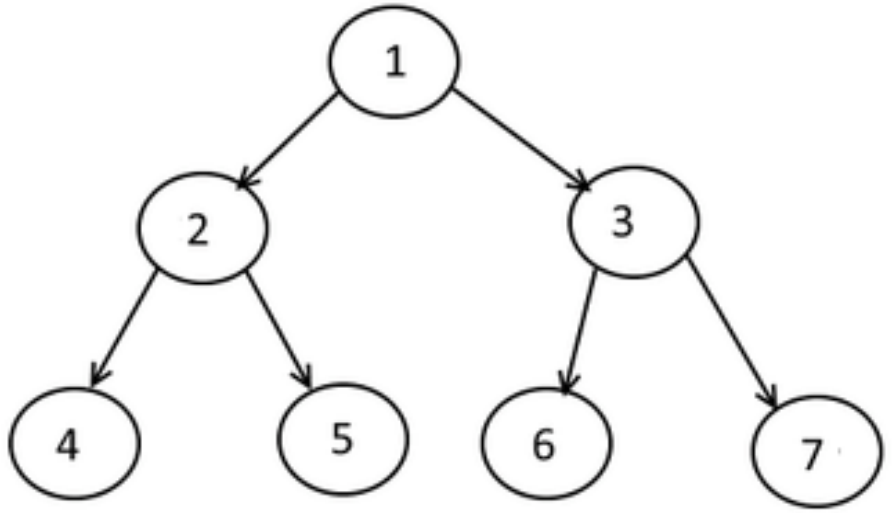
Traversals

- A traversal is a process that visits all the nodes in the tree. Since a tree is a nonlinear data structure, there is no unique traversal. We will consider several traversal algorithms with we group in the following two kinds:-
  - depth-first traversal
  - breadth-first traversal
- There are three different types of depth-first traversals:-
  - **PreOrder traversal** - visit the parent first and then left and right children;
  - **InOrder traversal** - visit the left child, then the parent and the right child;
  - **PostOrder traversal** - visit left child, then the right child and then the parent;

Traversals

- In-order traversal only in binary tree.
- The pre-order traversal is a form of the Depth-First-Search (DFS) traversal.
- The pre-order traversal is different from the Breadth First Search (BFS) traversal.
- Pre-Order: [1,2,4,3]
- BFS: [1,2,3,4]





Inorder Traversal: 4 2 5 1 6 3 7
Preorder Traversal: 1 2 4 5 3 6 7

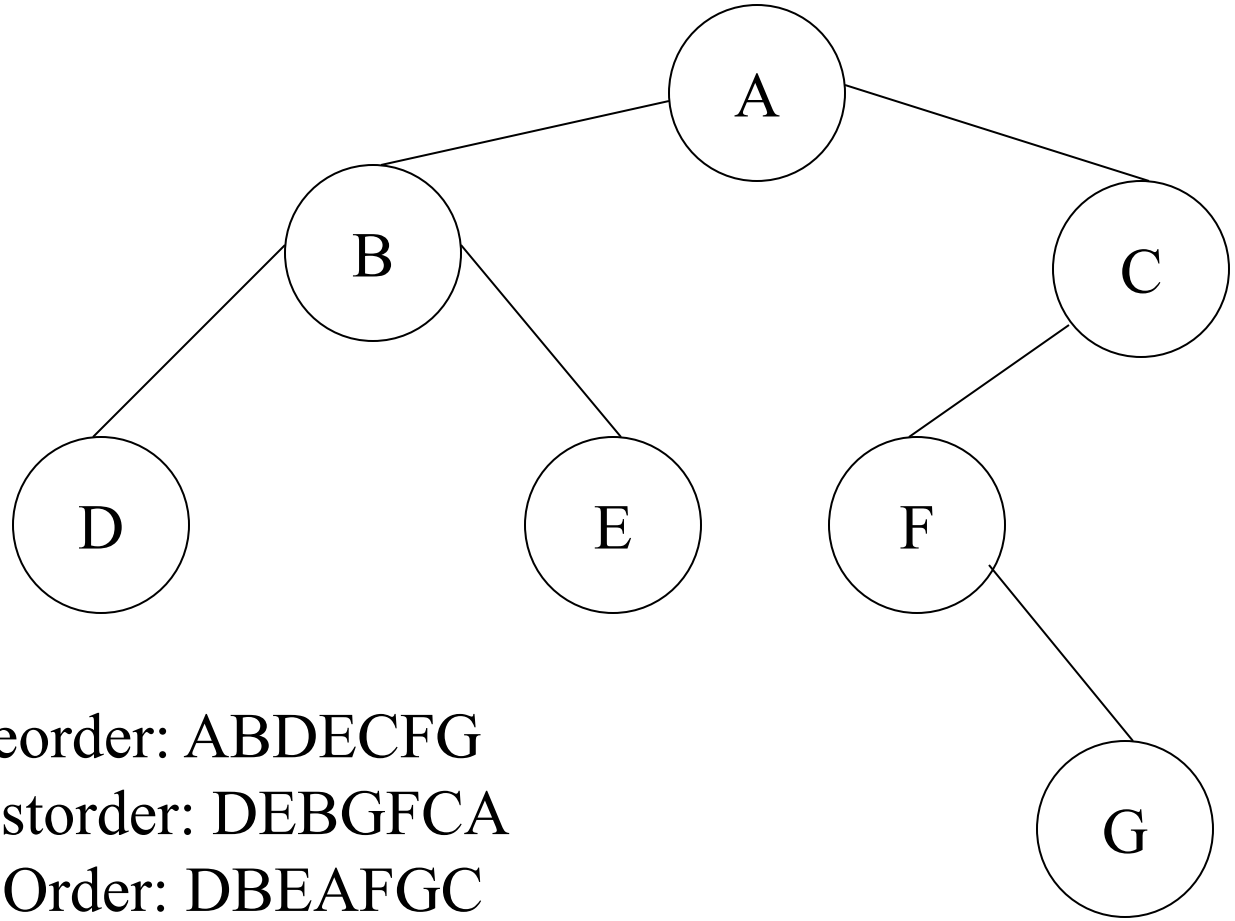
Breadth-First Search: 1 2 3 4 5 6 7
Depth-First Search: 1 2 4 5 3 6 7

Postorder Traversal: 4 5 2 6 7 3 1
(Left to Right)
Postorder Traversal: 7 6 3 5 4 2 1
(Right to Left) -- Reverse Order

Reverse order Traversals

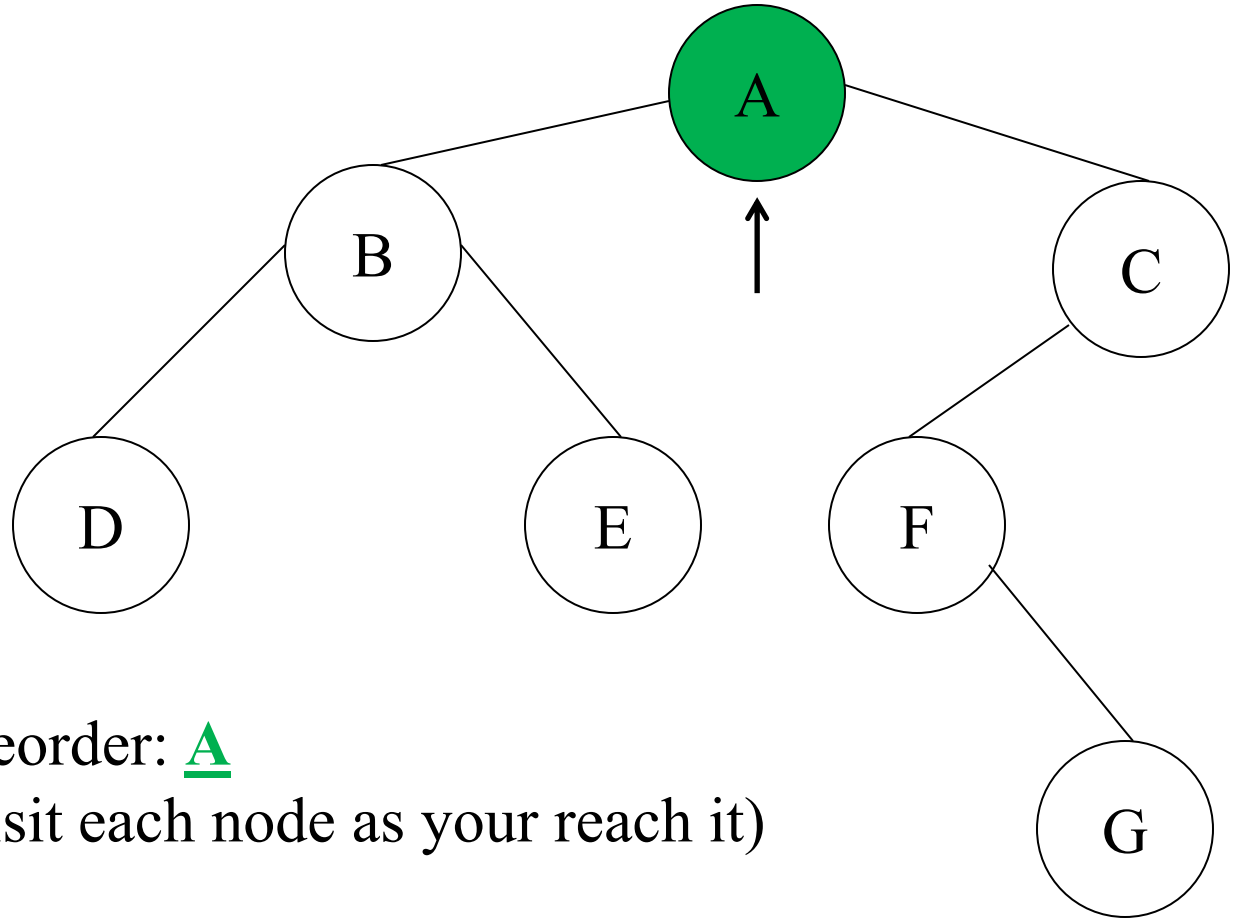
- The other traversals are the reverse of these three standard ones
  - the right subtree is traversed before the left subtree is traversed
- Reverse preorder: root, right subtree, left subtree
- Reverse inorder: right subtree, root, left subtree
- Reverse postorder: right subtree, left subtree, root

Tree Traversals: An Example



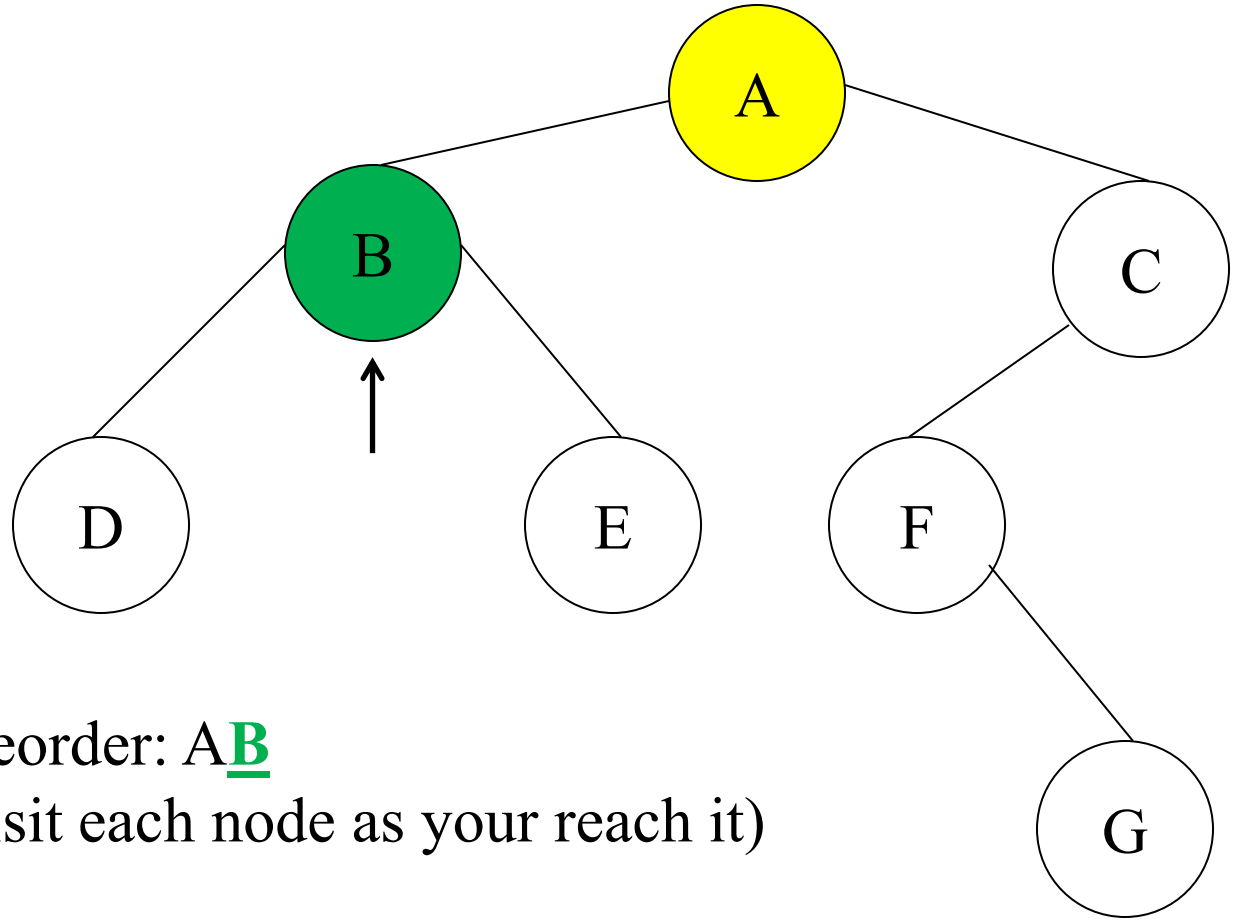
Preorder: ABDECFG
Postorder: DEBGFCA
In Order: DBEAFGC

Tree Traversals: An Example



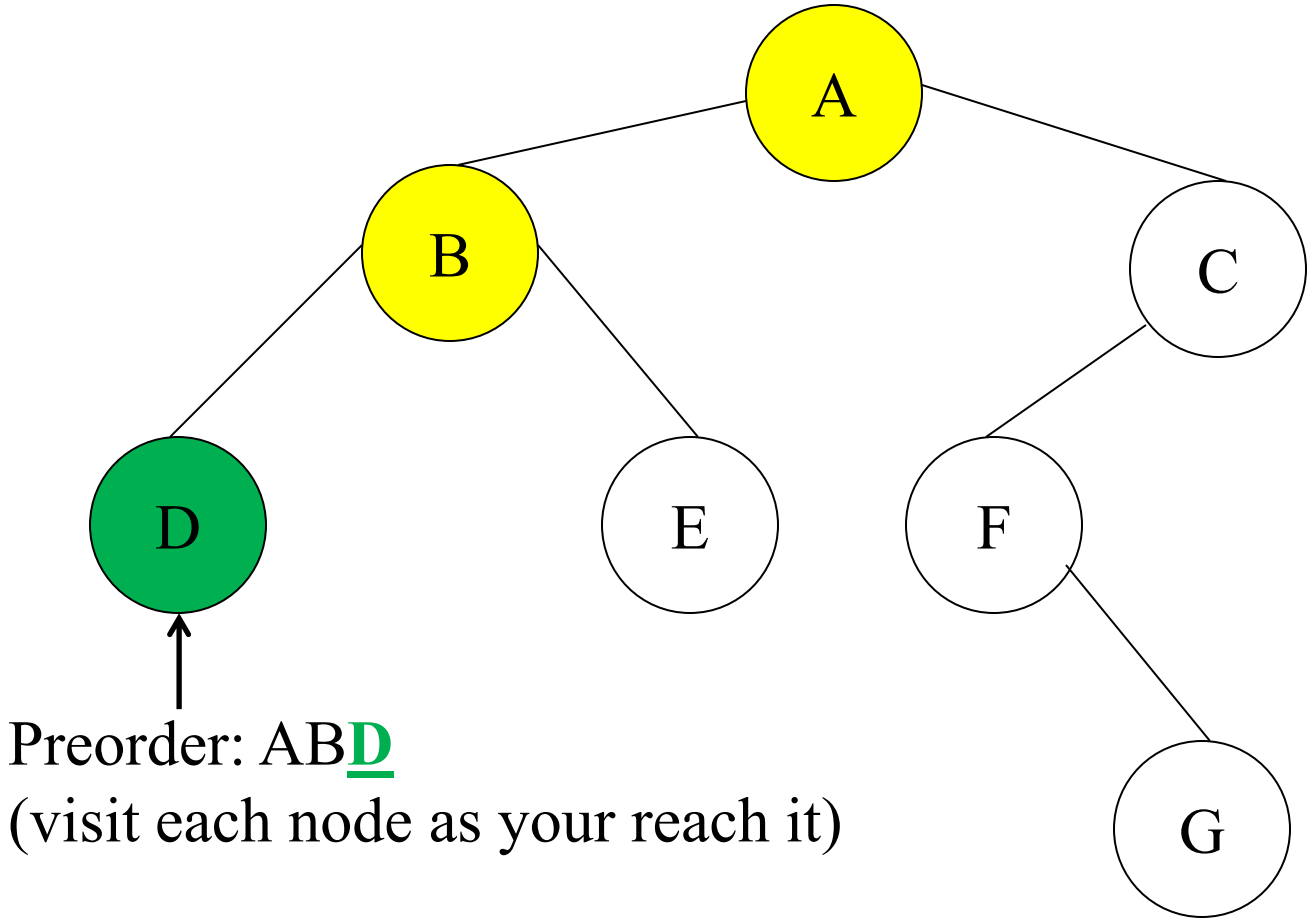
Preorder: A
(visit each node as your reach it)

Tree Traversals: An Example

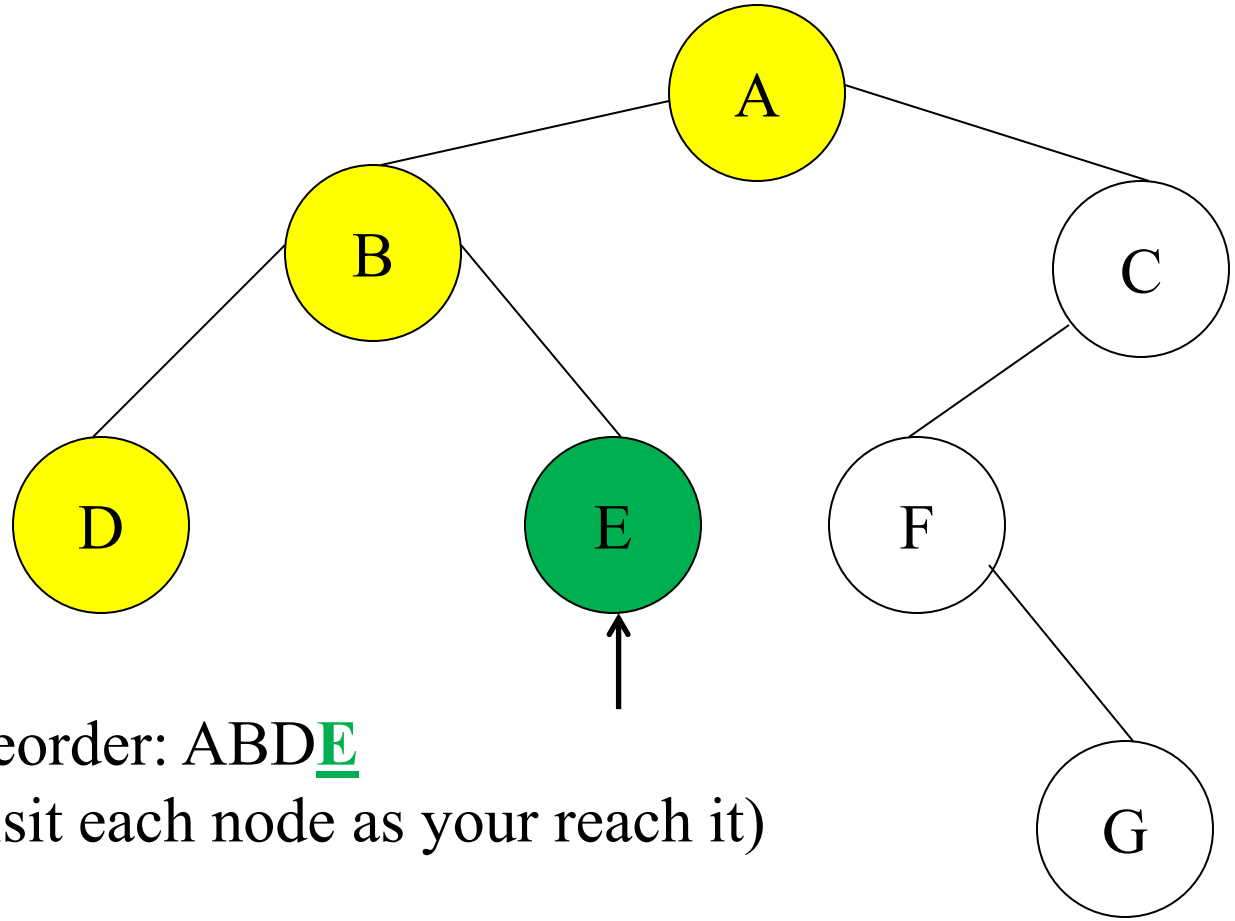


Preorder: A**B**
(visit each node as your reach it)

Tree Traversals: An Example

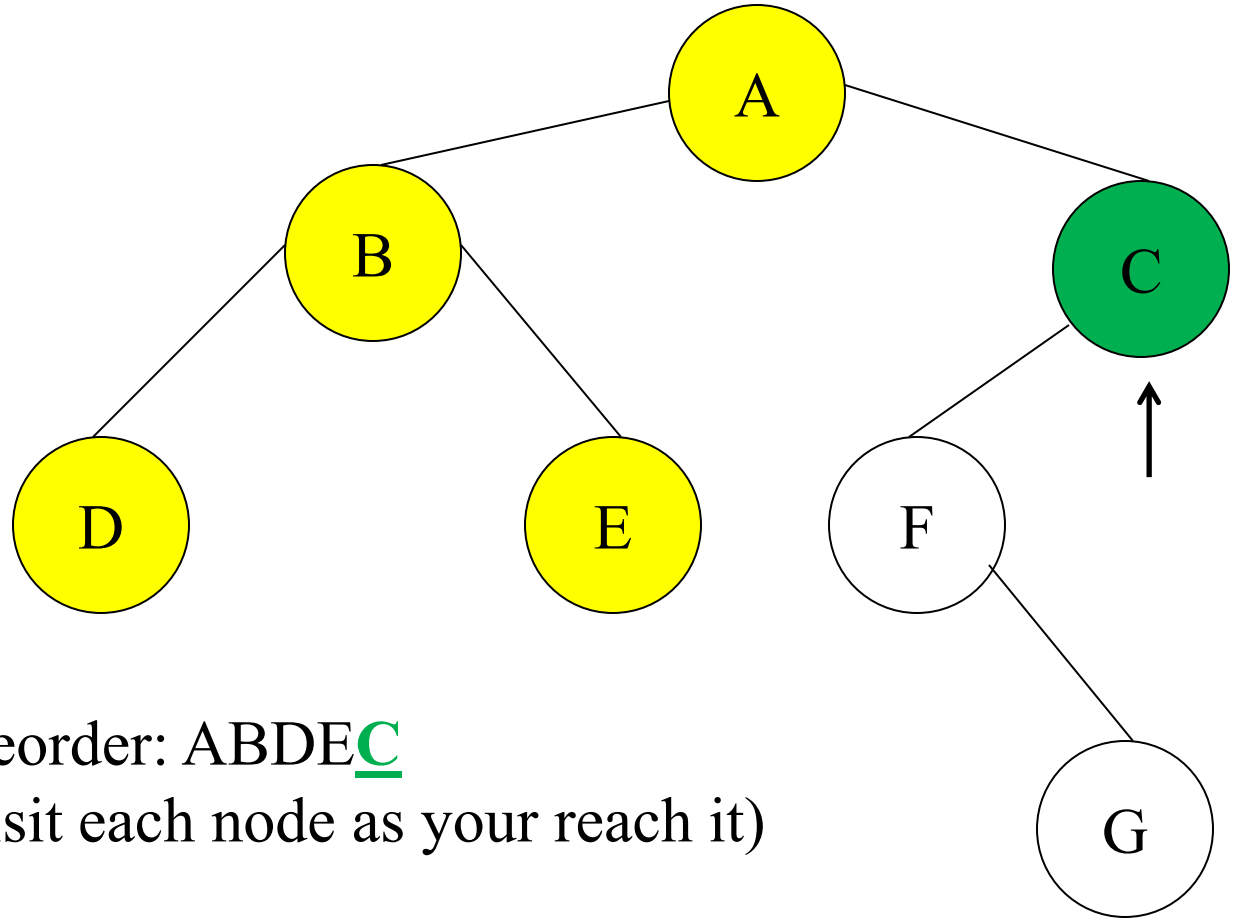


Tree Traversals: An Example



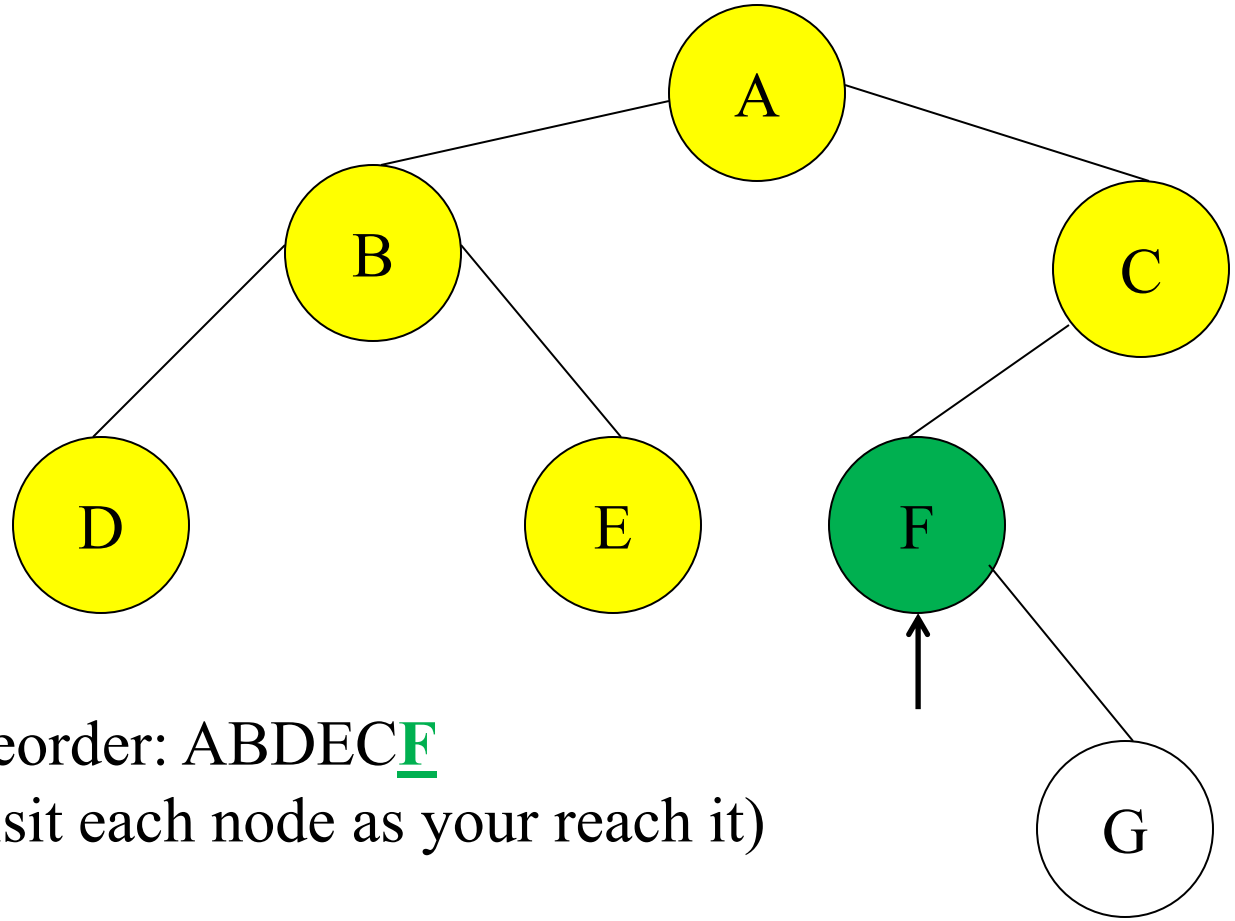
Preorder: ABDE
(visit each node as your reach it)

Tree Traversals: An Example



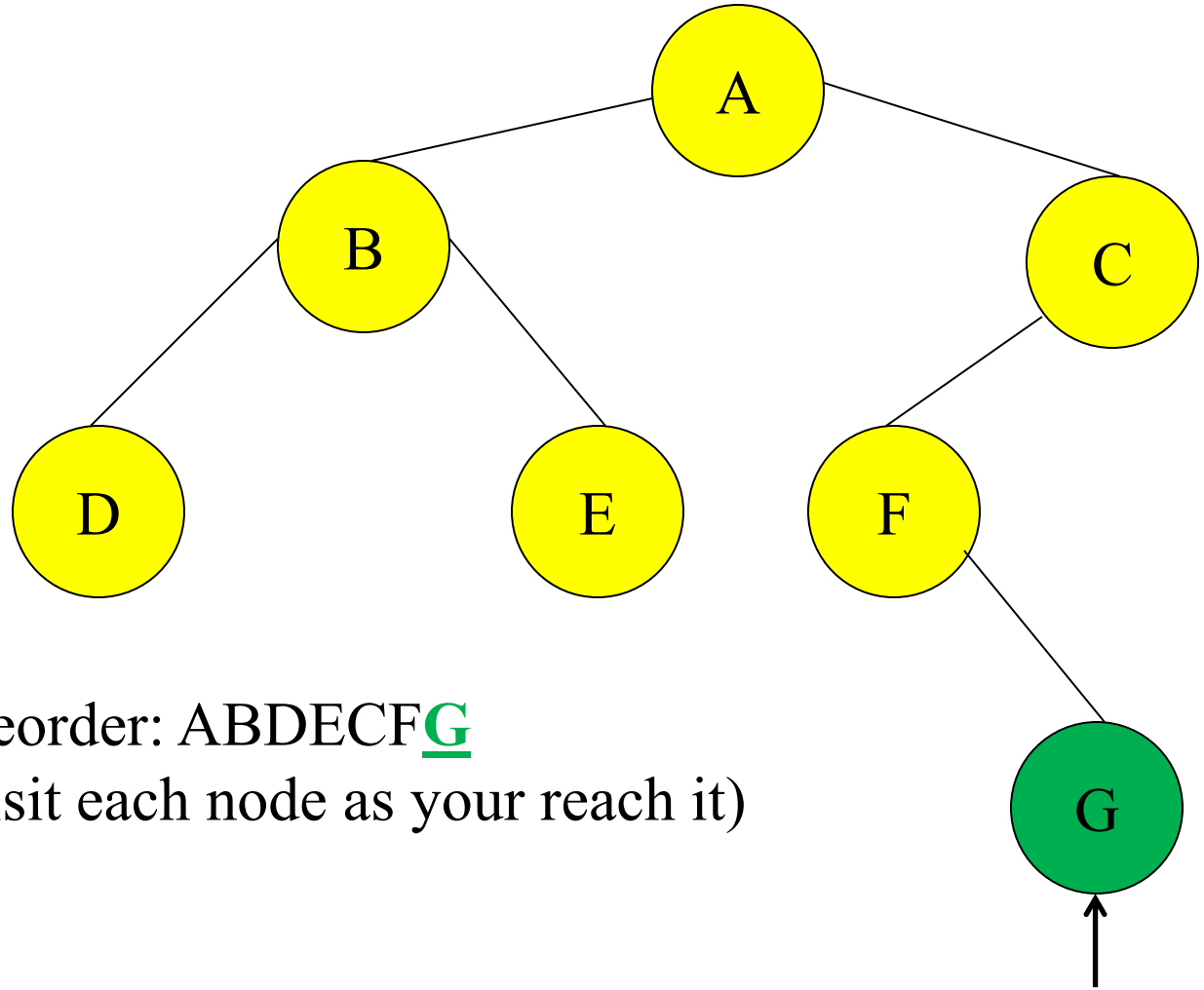
Preorder: ABDEC
(visit each node as your reach it)

Tree Traversals: An Example



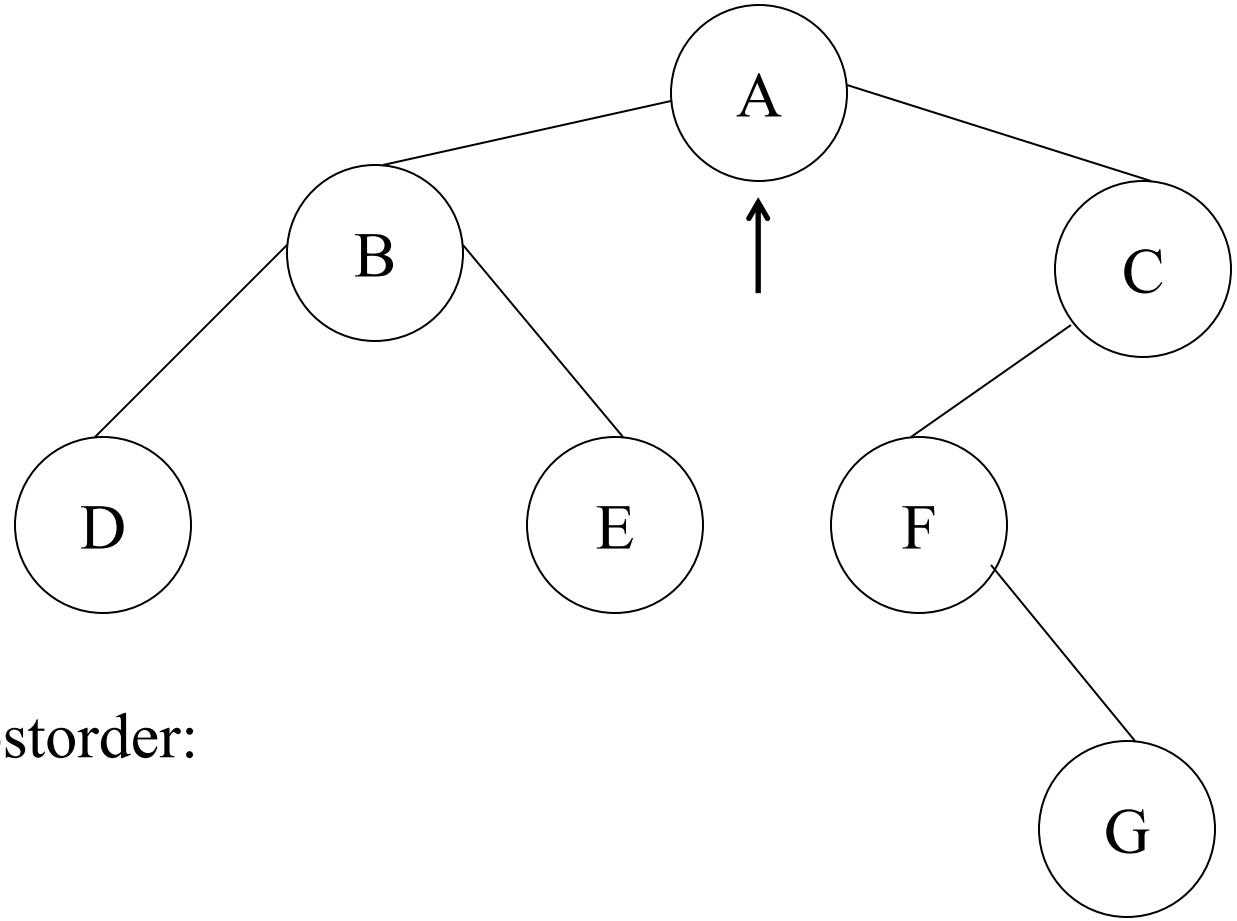
Preorder: ABDECF
(visit each node as your reach it)

Tree Traversals: An Example



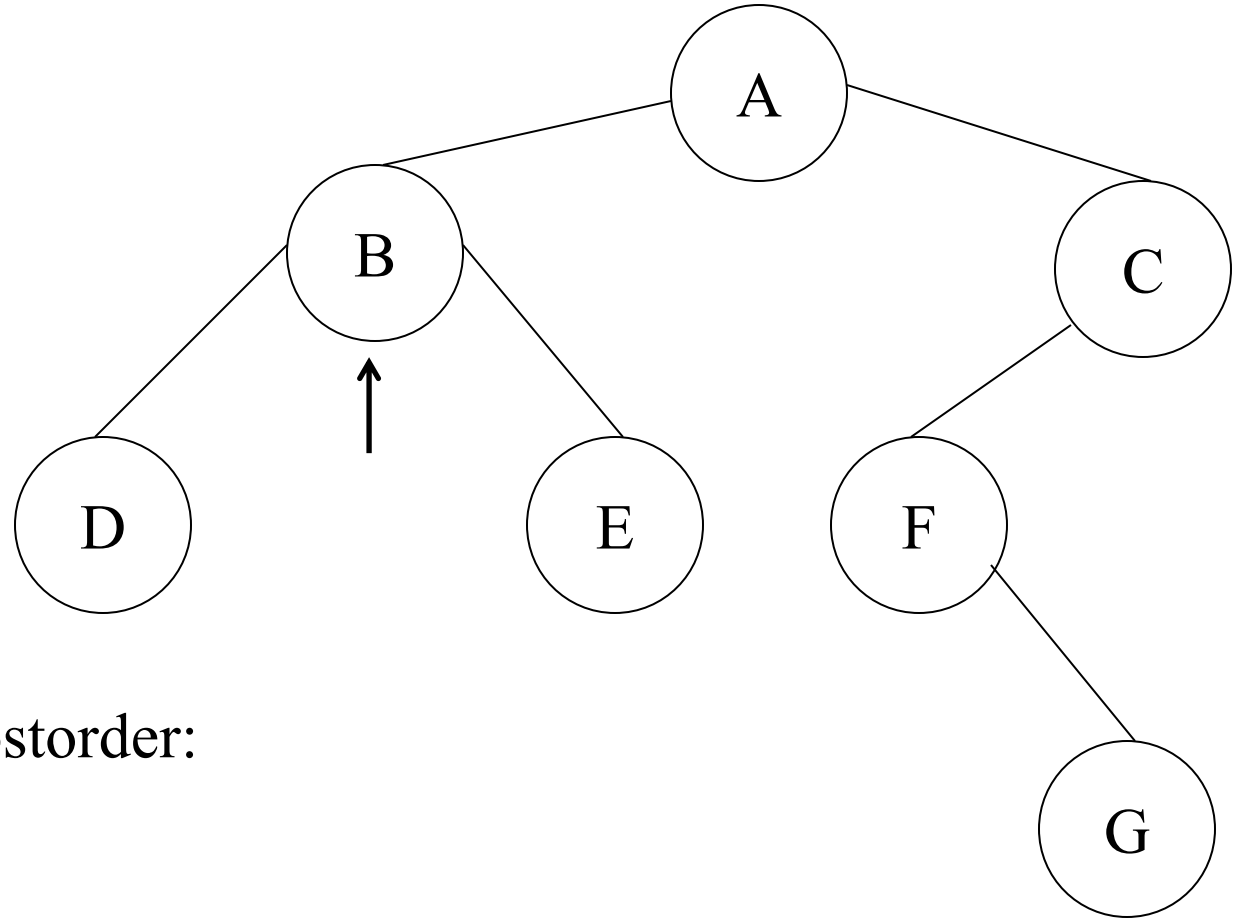
Preorder: ABDECFG
(visit each node as your reach it)

Tree Traversals: An Example



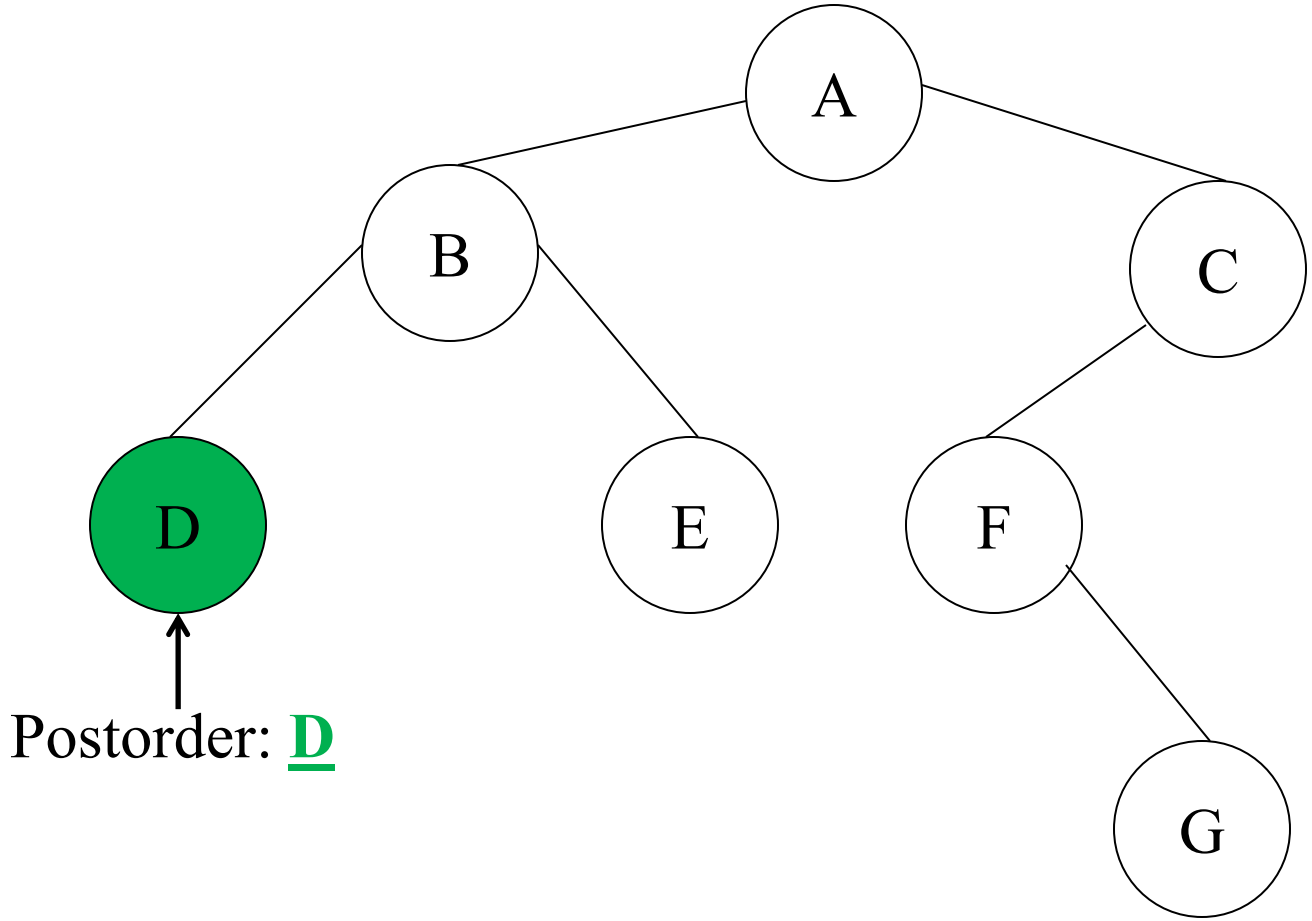
Postorder:

Tree Traversals: An Example

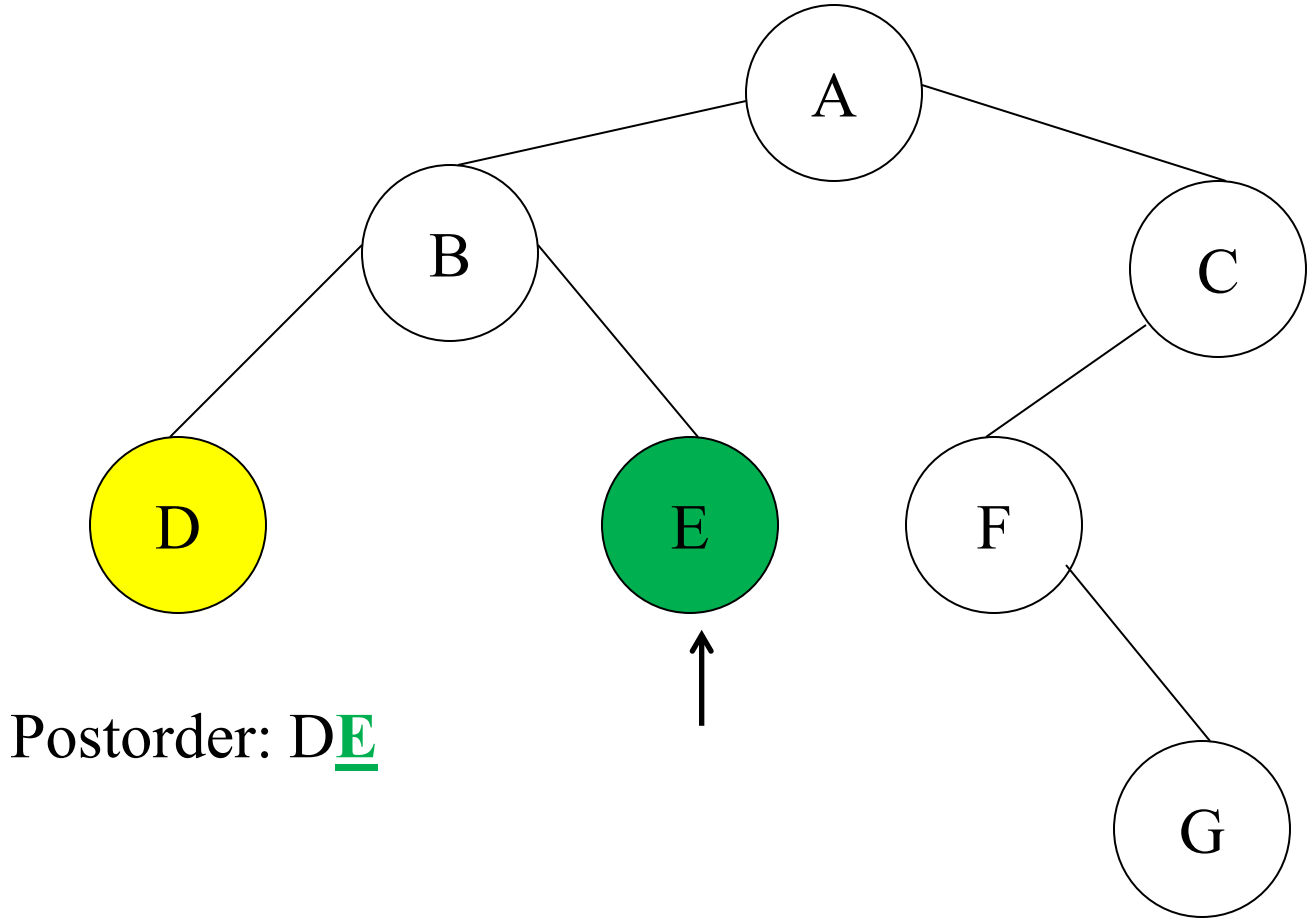


Postorder:

Tree Traversals: An Example

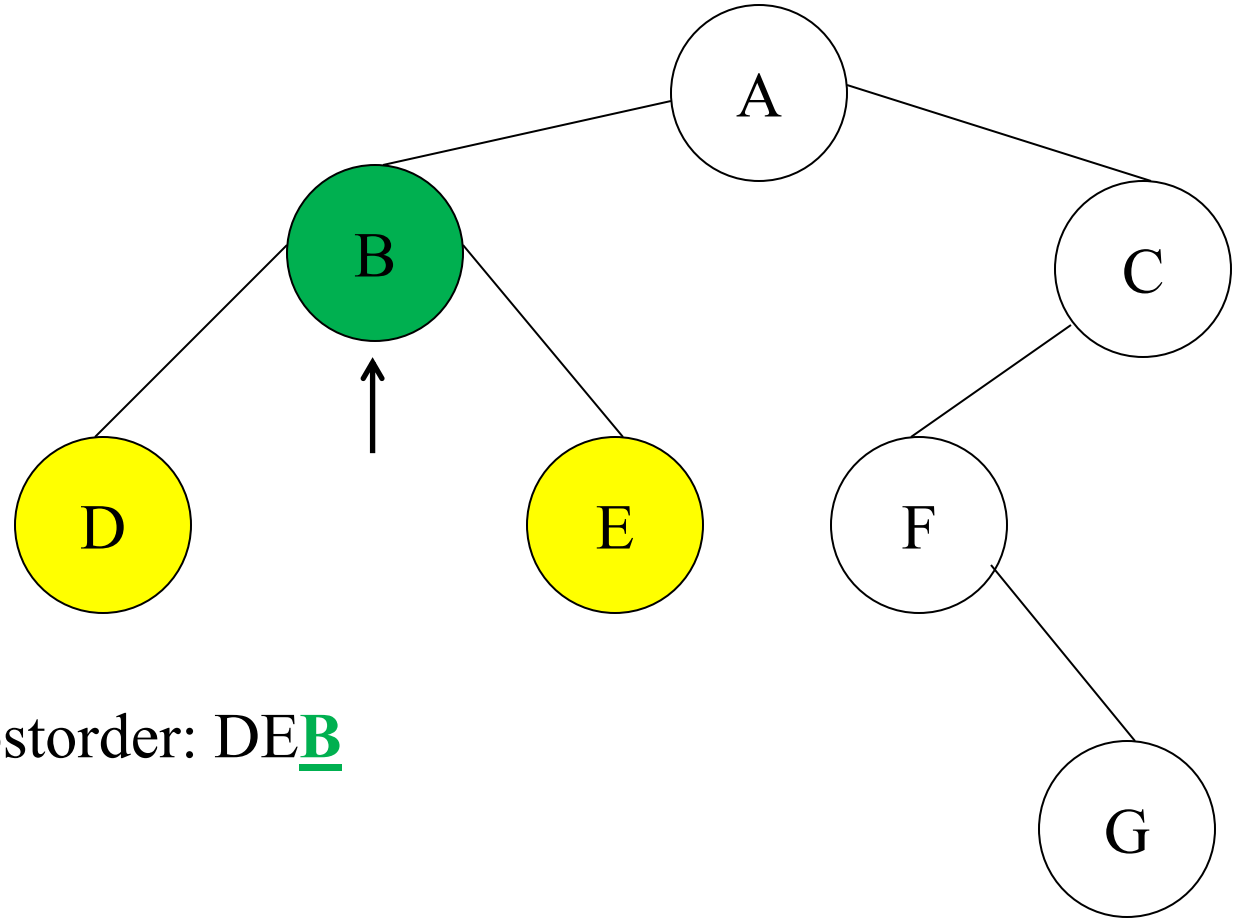


Tree Traversals: An Example



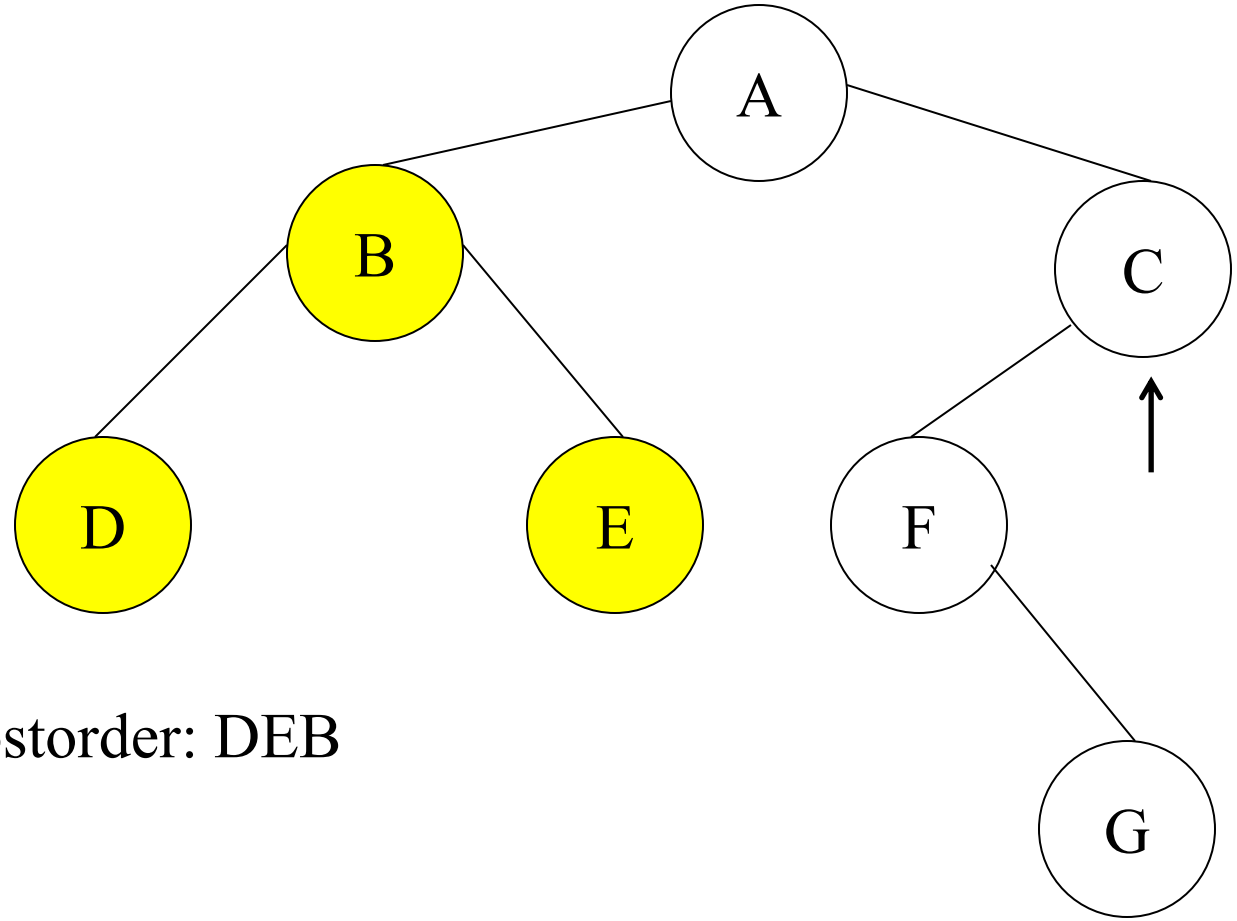
Postorder: D**E**

Tree Traversals: An Example



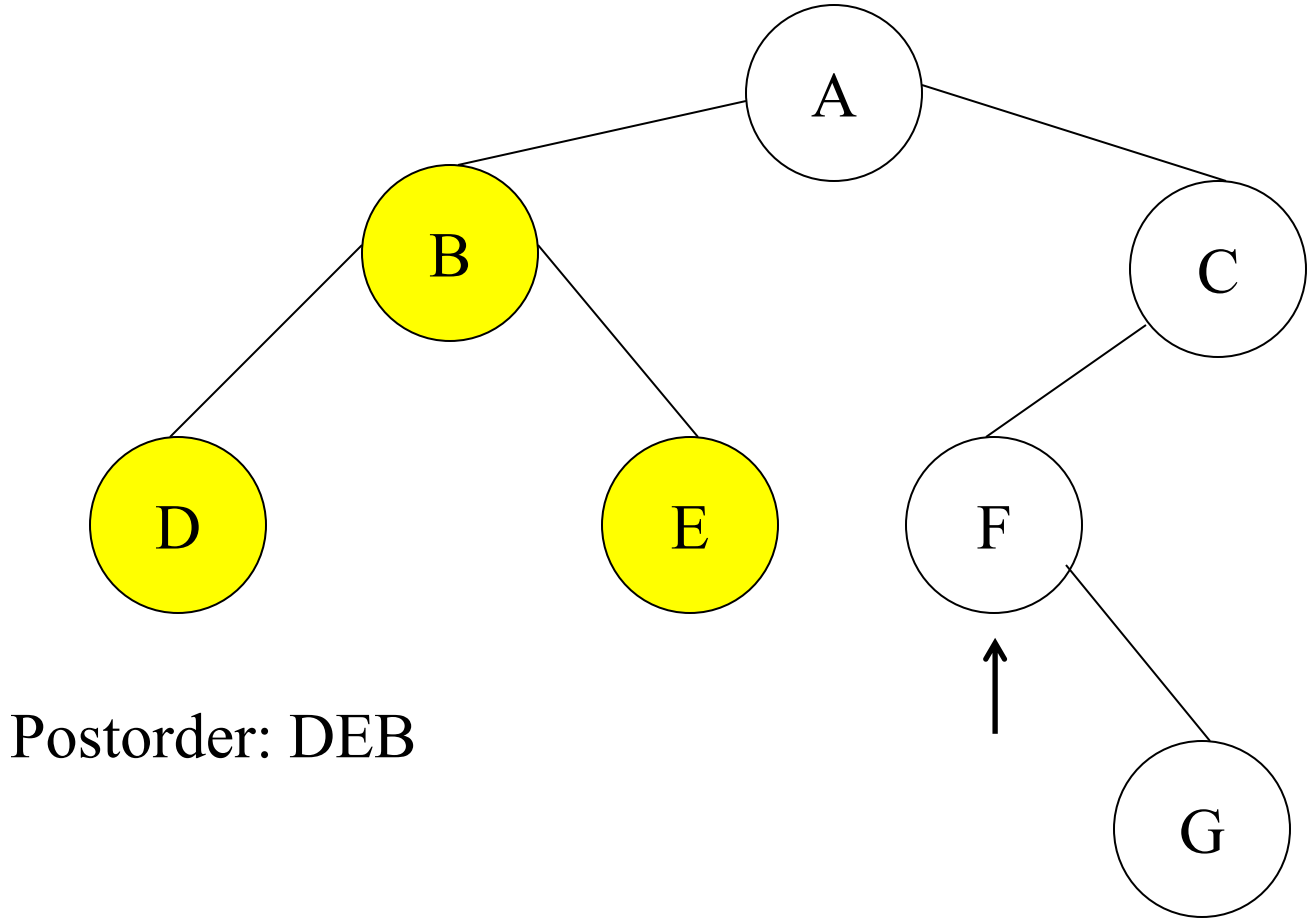
Postorder: DE**B**

Tree Traversals: An Example



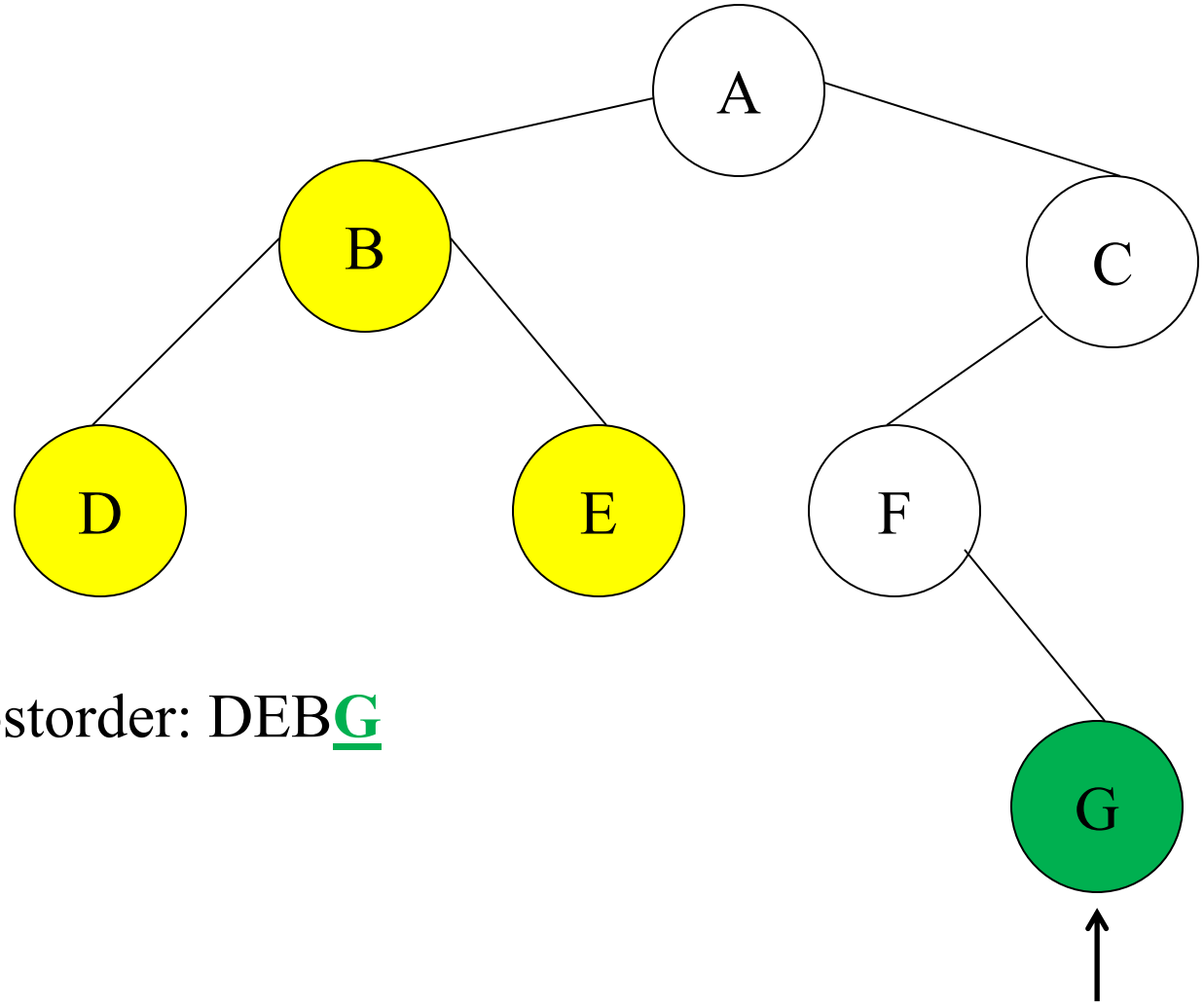
Postorder: DEB

Tree Traversals: An Example



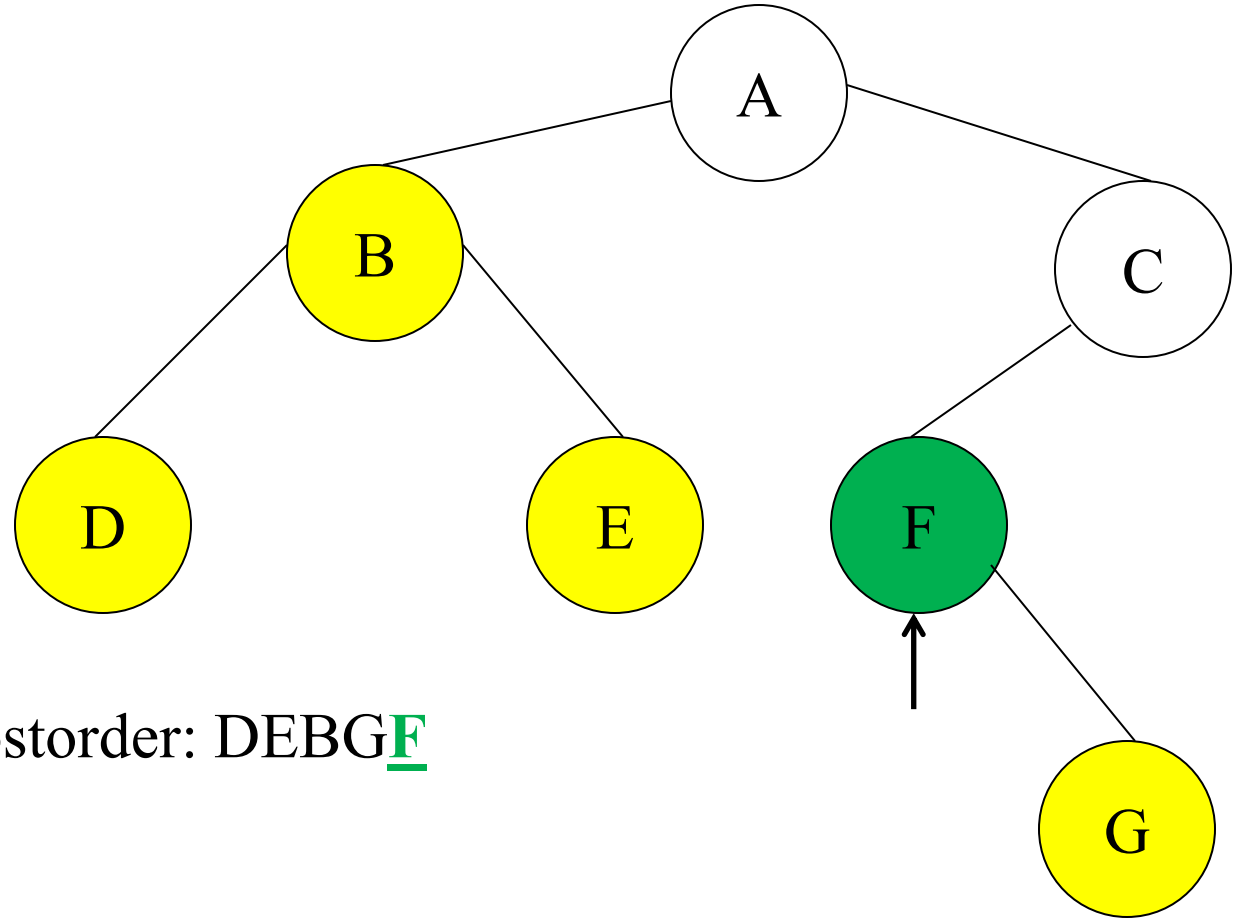
Postorder: DEB

Tree Traversals: An Example



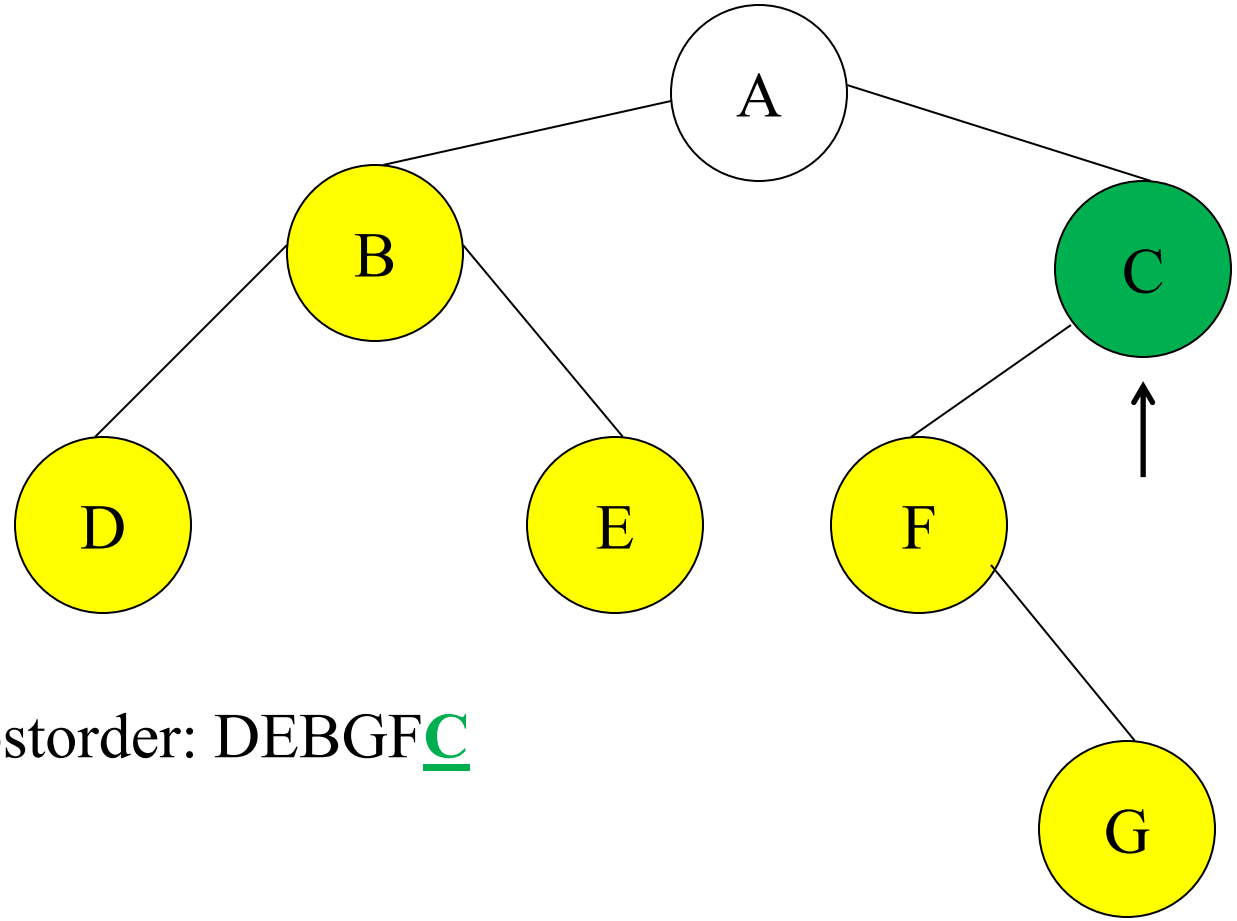
Postorder: DEBG

Tree Traversals: An Example



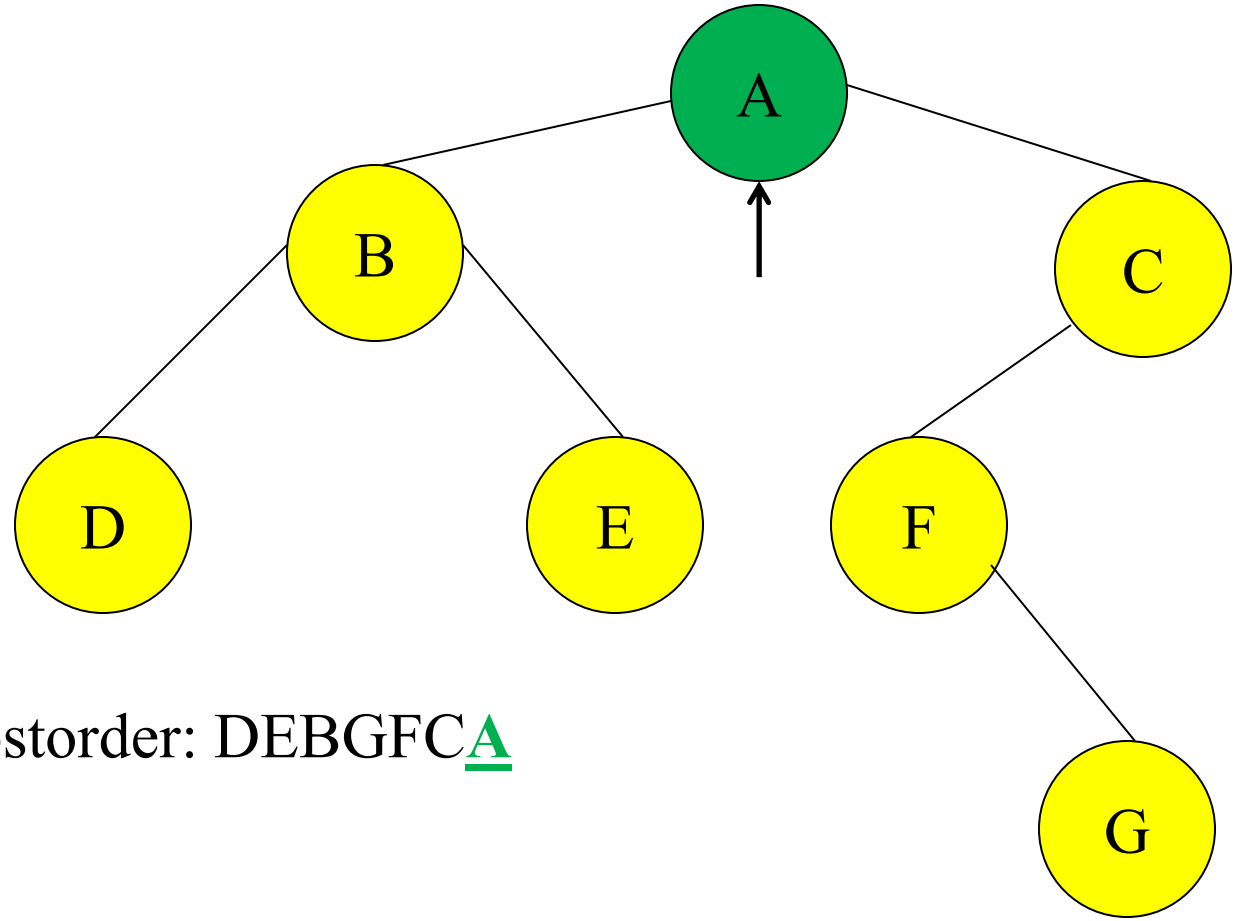
Postorder: DEBGF

Tree Traversals: An Example



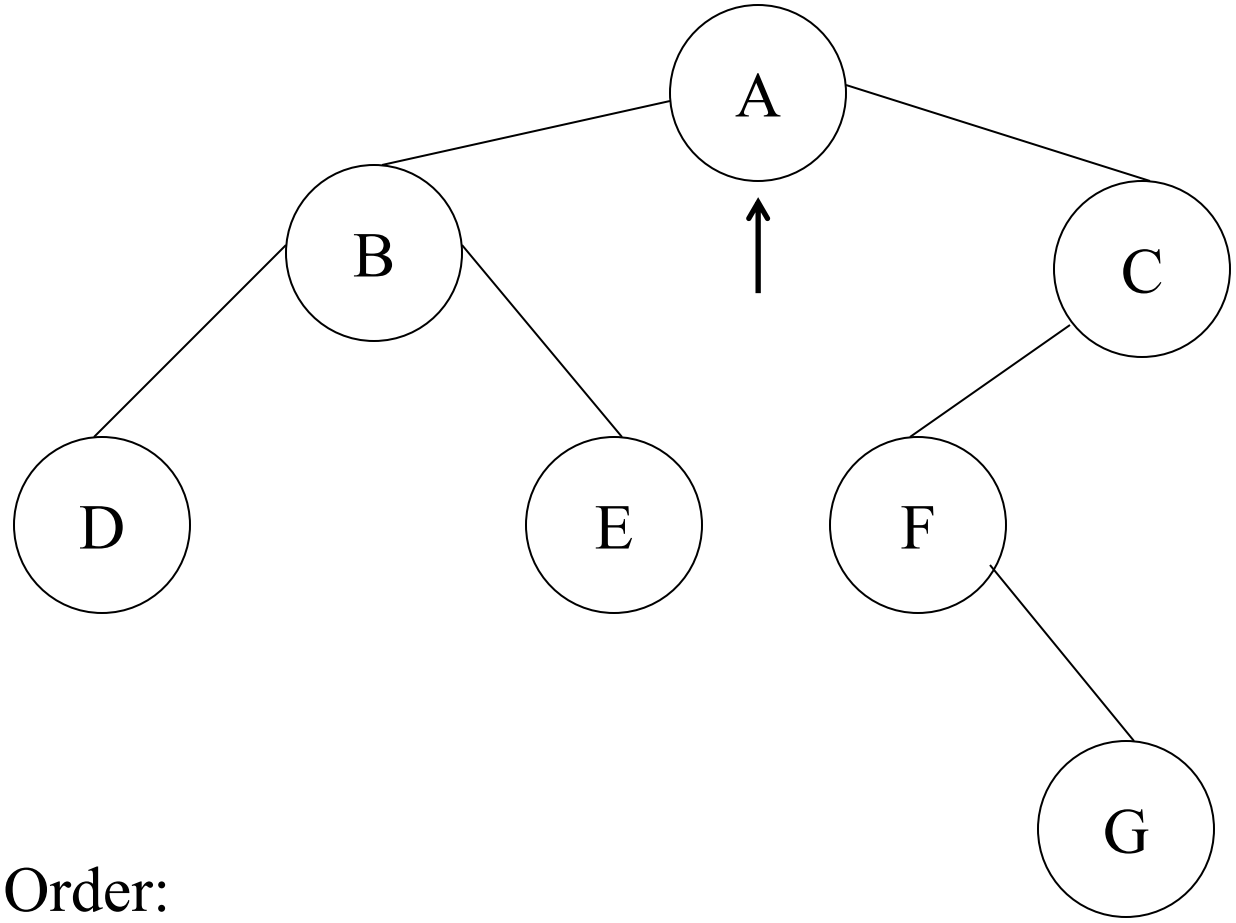
Postorder: DEBGFC

Tree Traversals: An Example



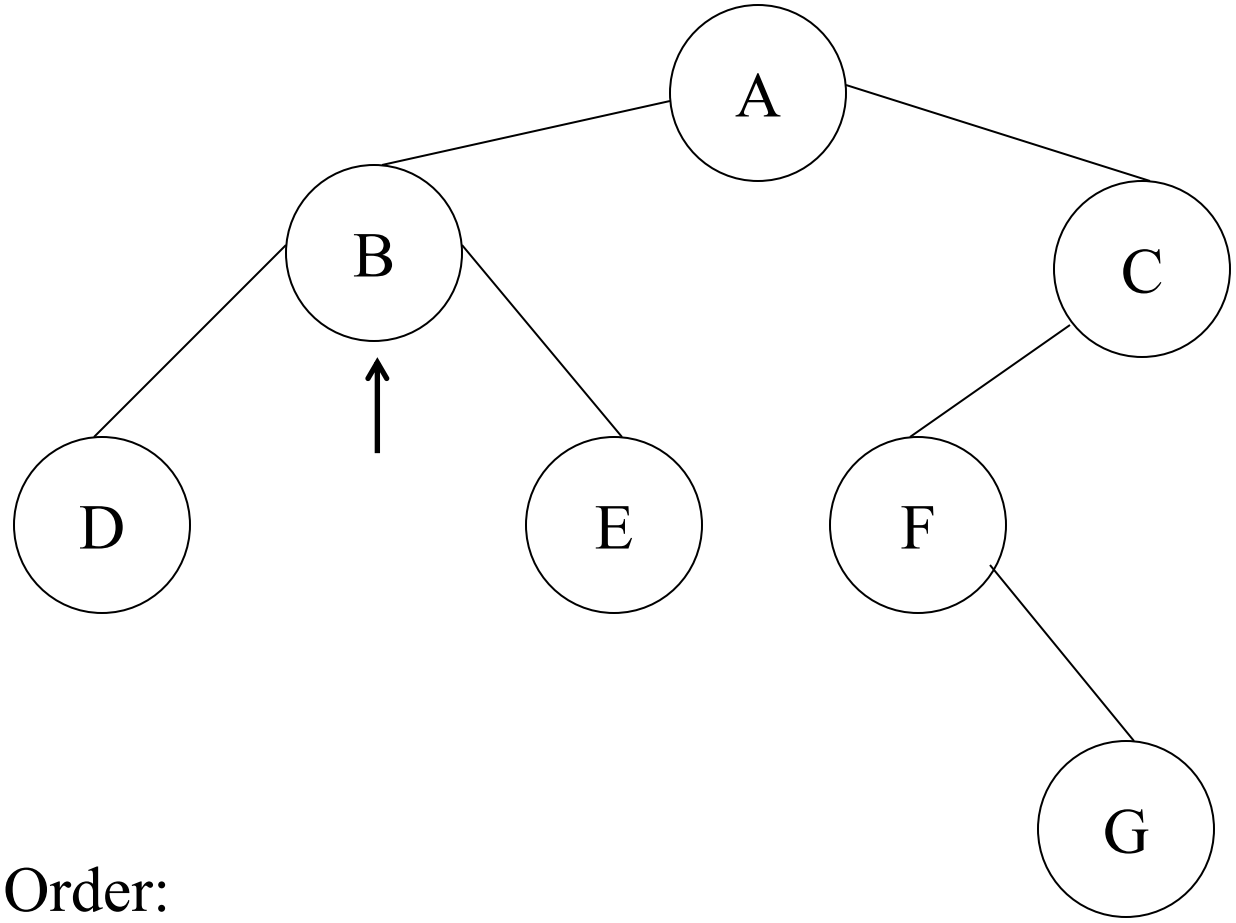
Postorder: DEBGFCA

Tree Traversals: An Example



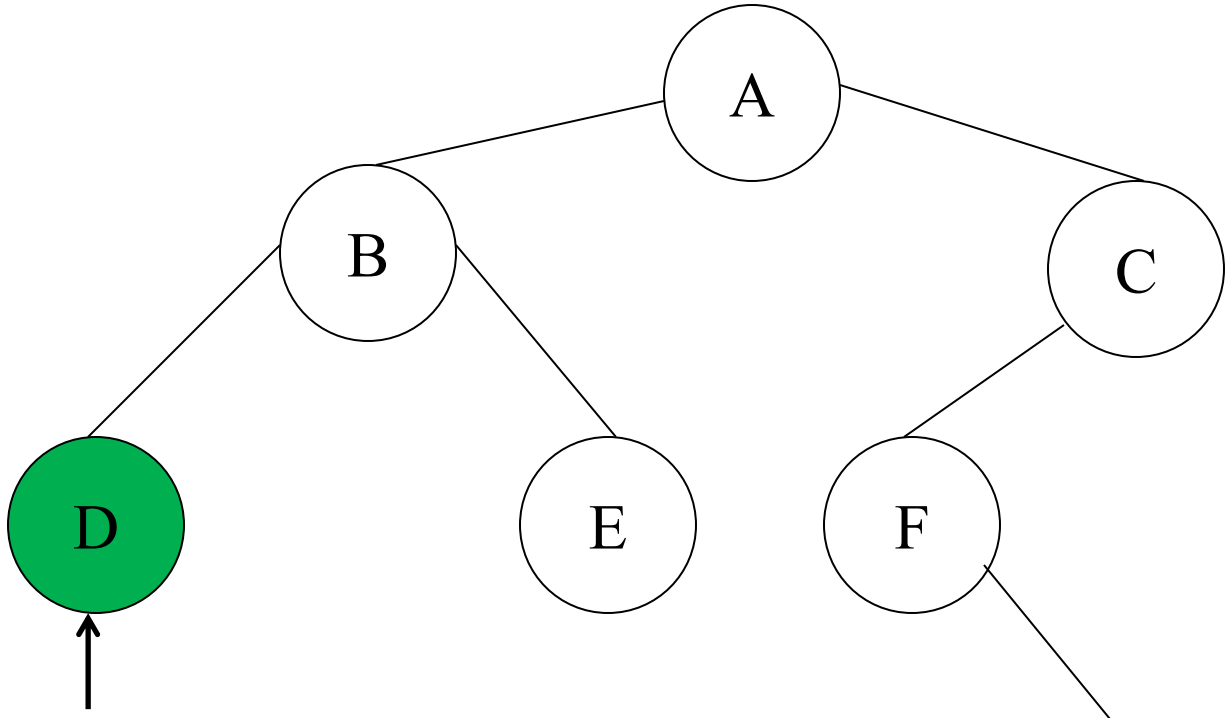
In Order:

Tree Traversals: An Example



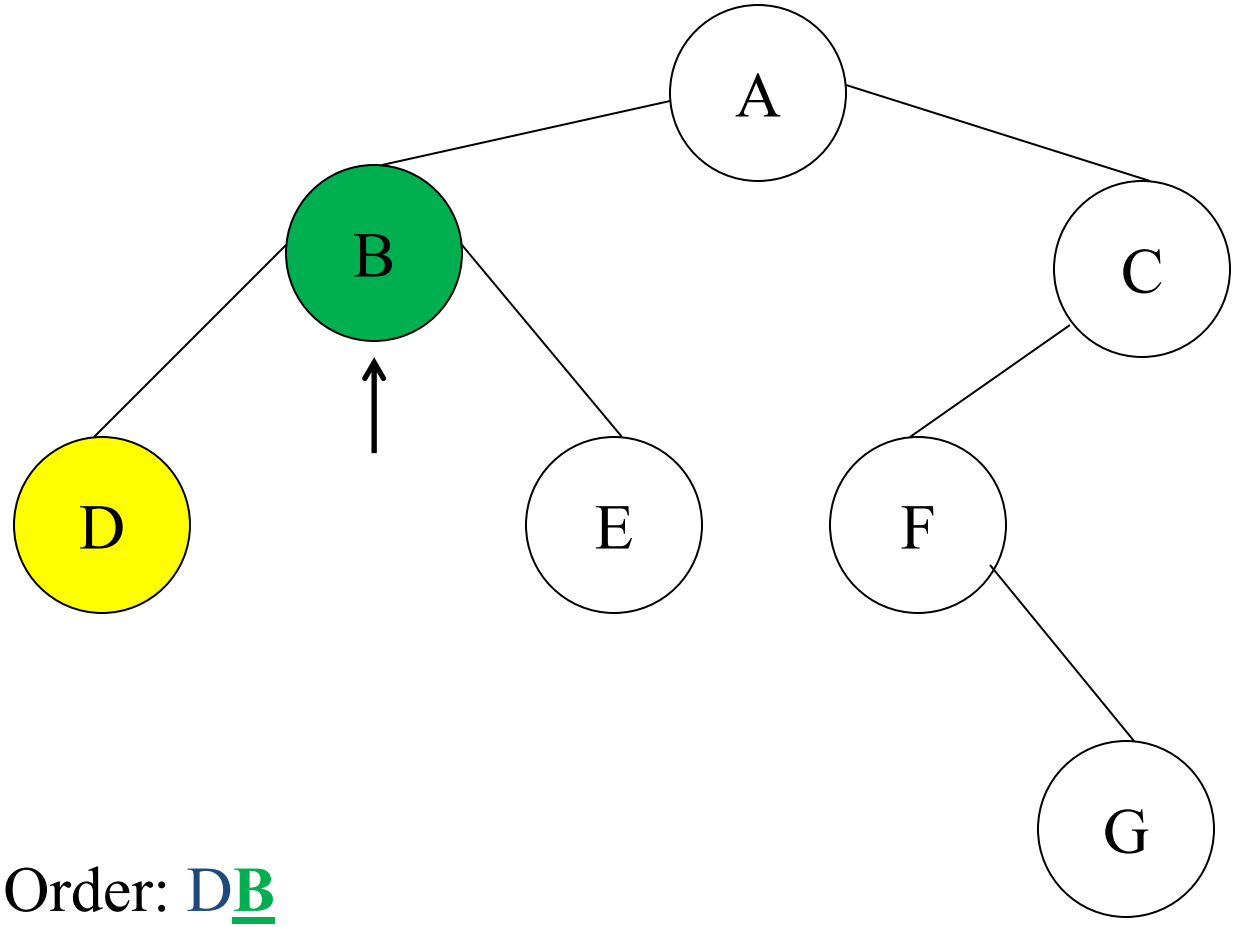
In Order:

Tree Traversals: An Example



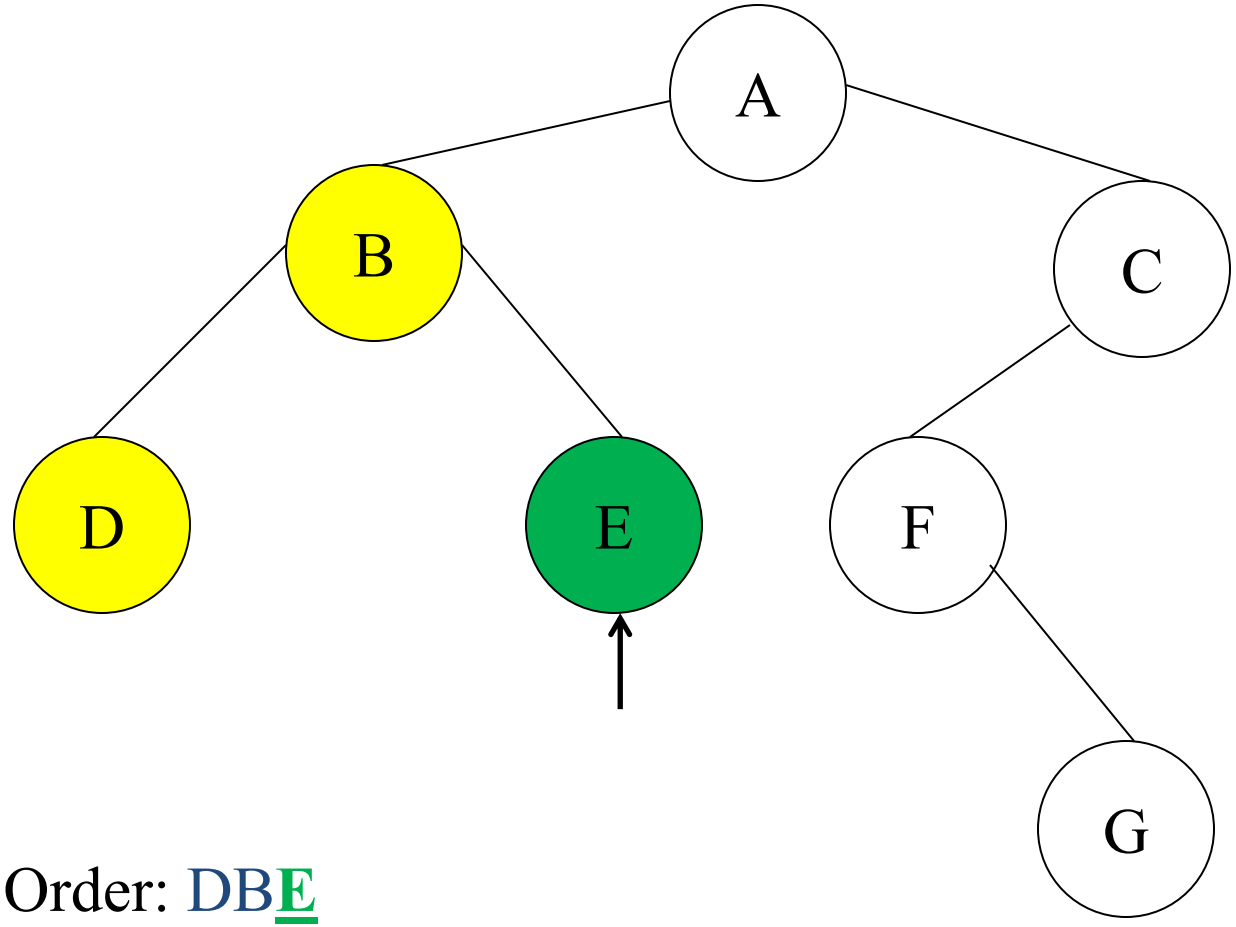
In Order: **D**

Tree Traversals: An Example



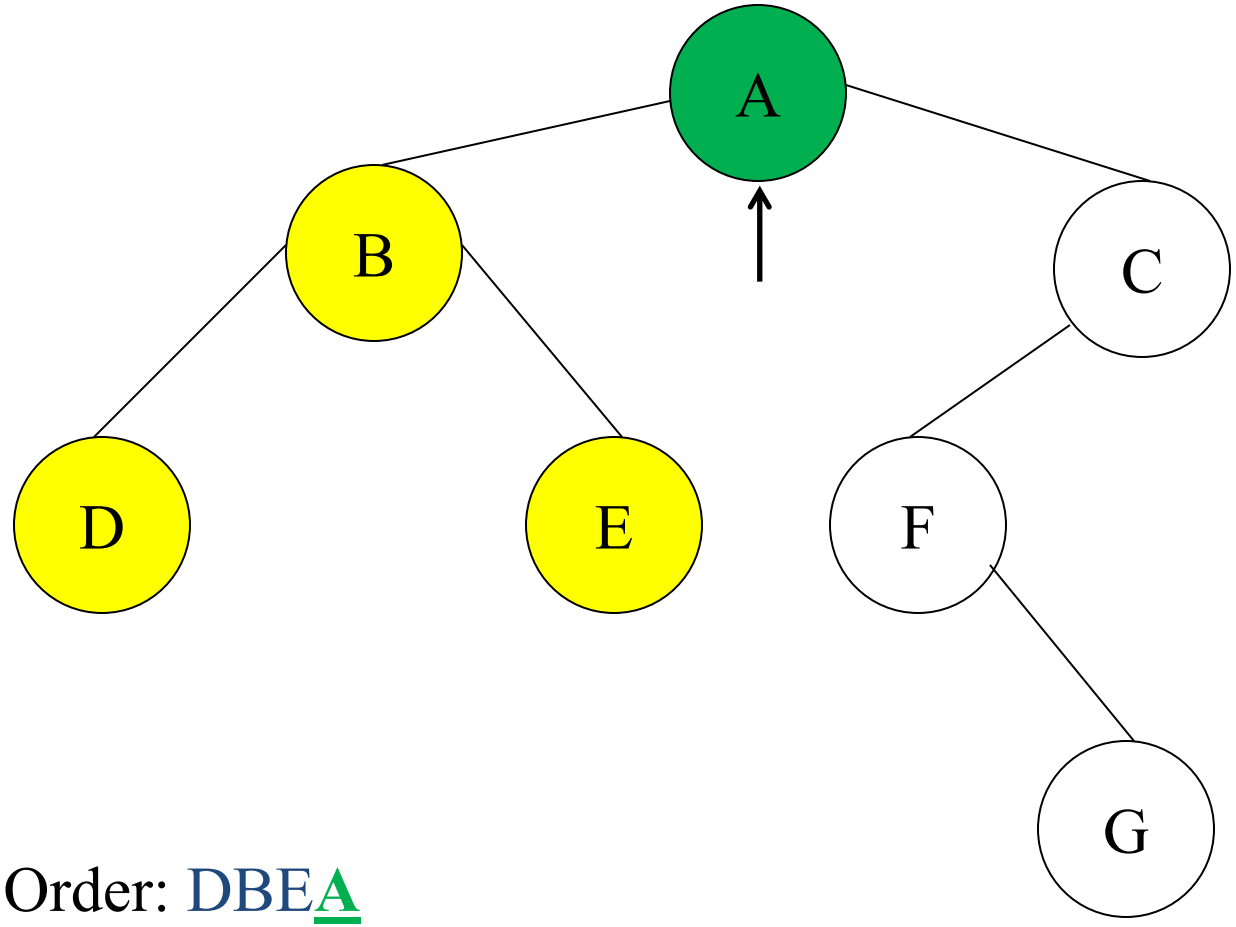
In Order: DB

Tree Traversals: An Example



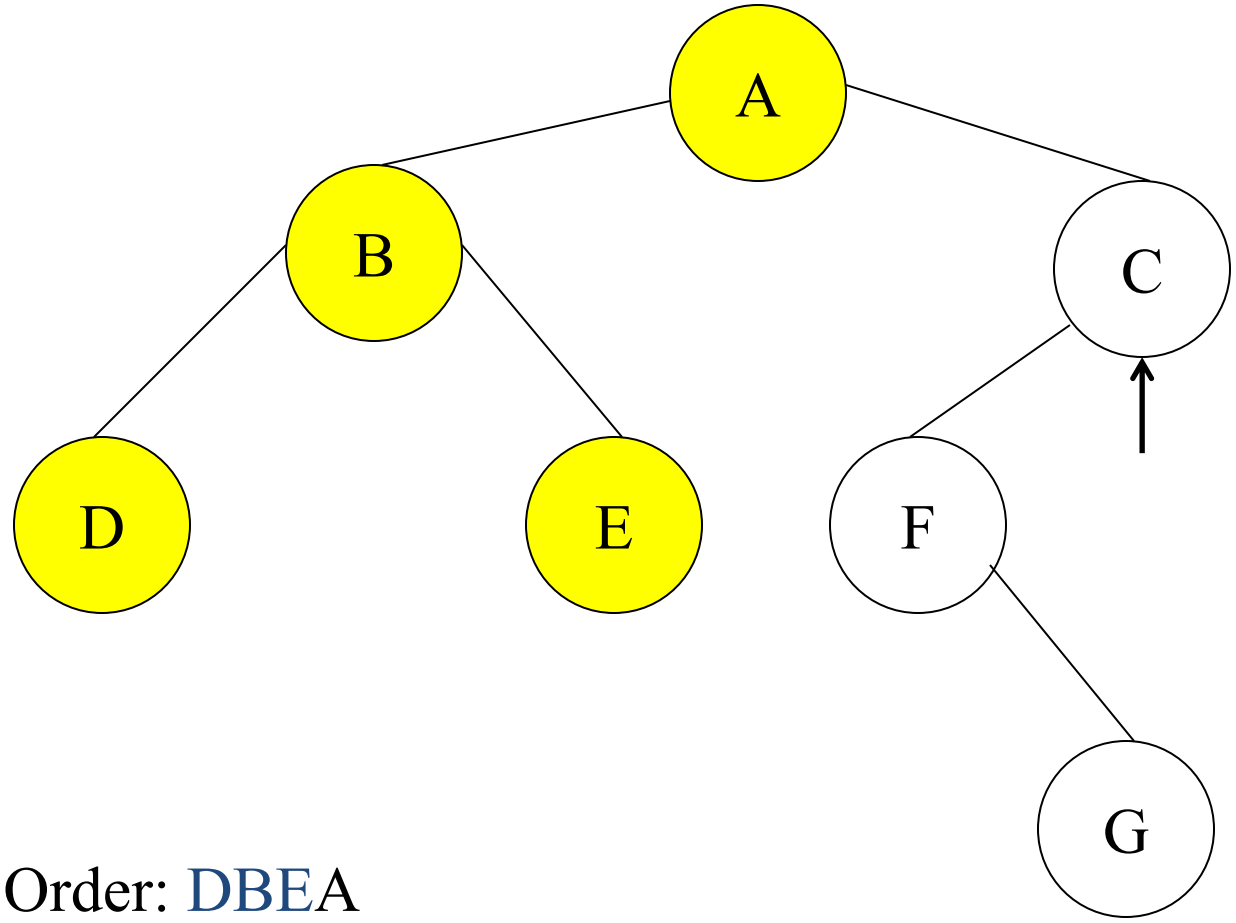
In Order: DBE

Tree Traversals: An Example



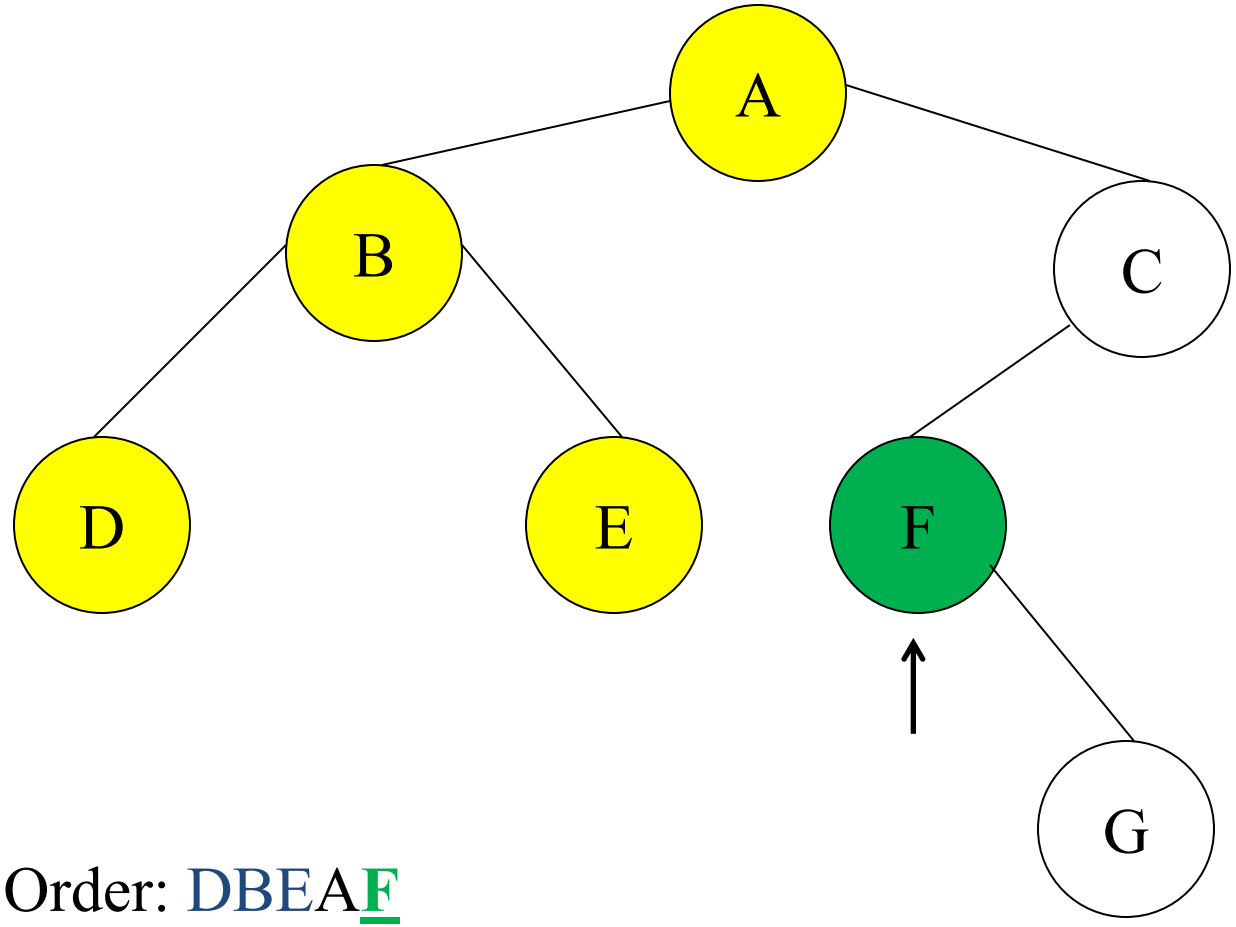
In Order: DBEA

Tree Traversals: An Example



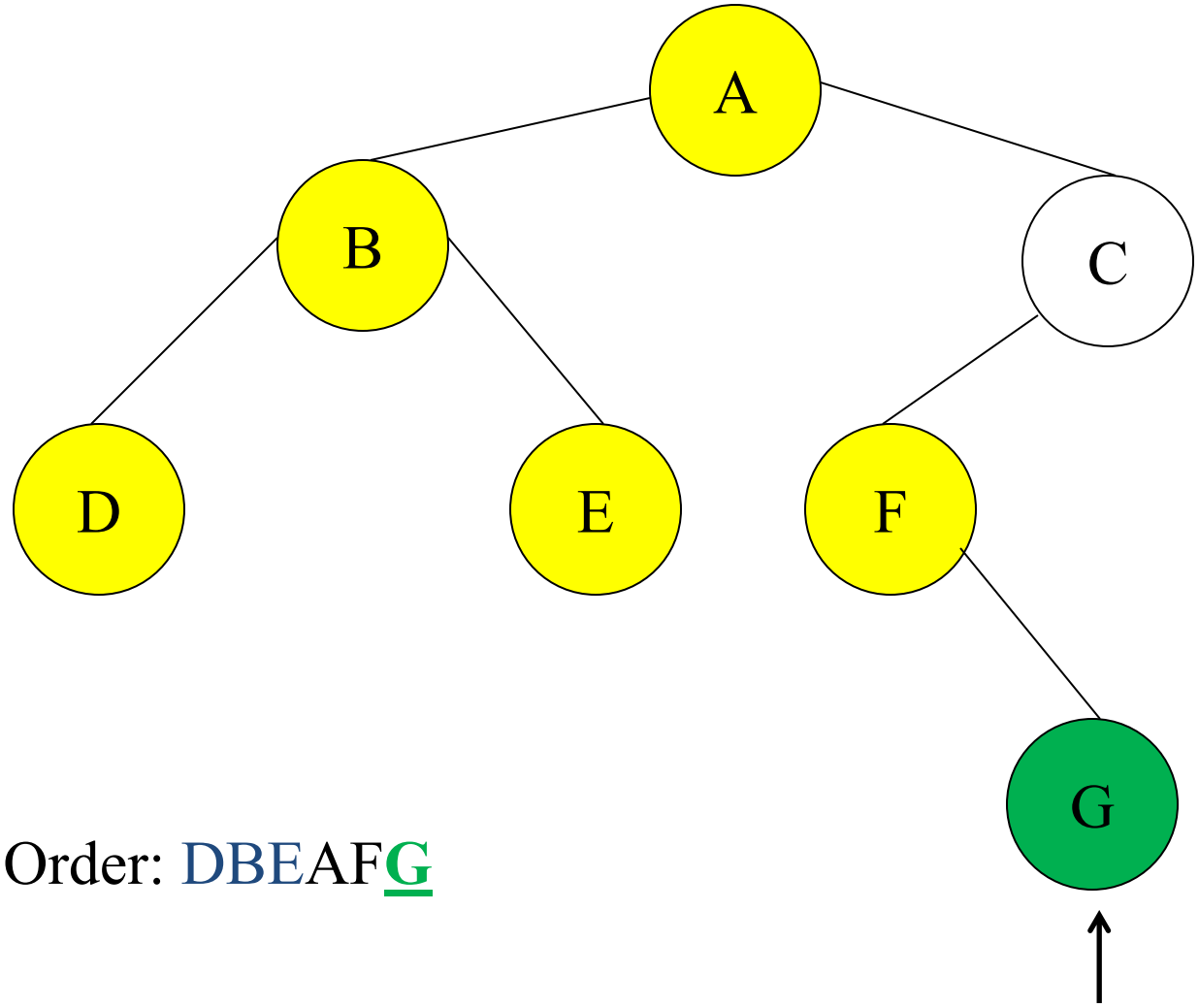
In Order: DBEA

Tree Traversals: An Example



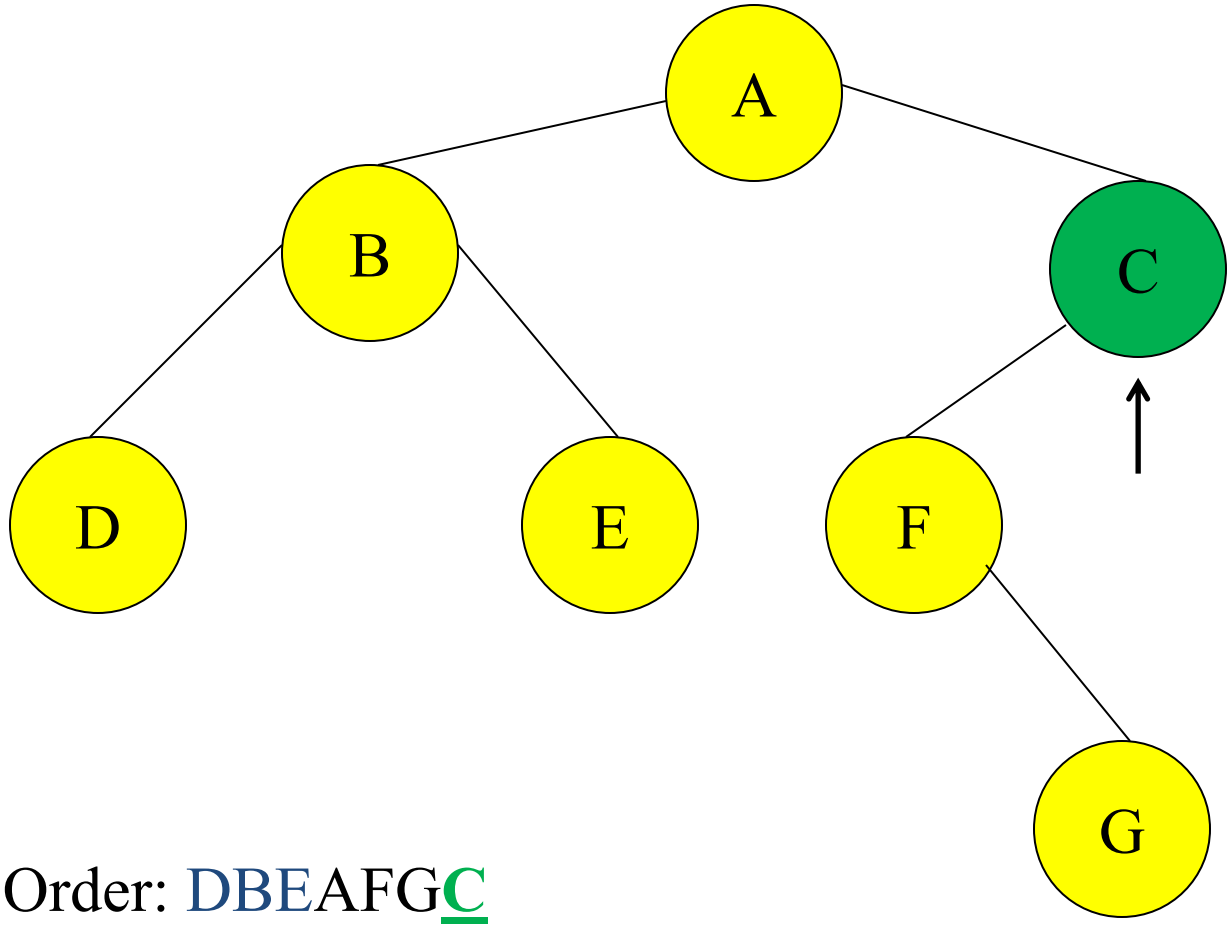
In Order: DBEAF

Tree Traversals: An Example



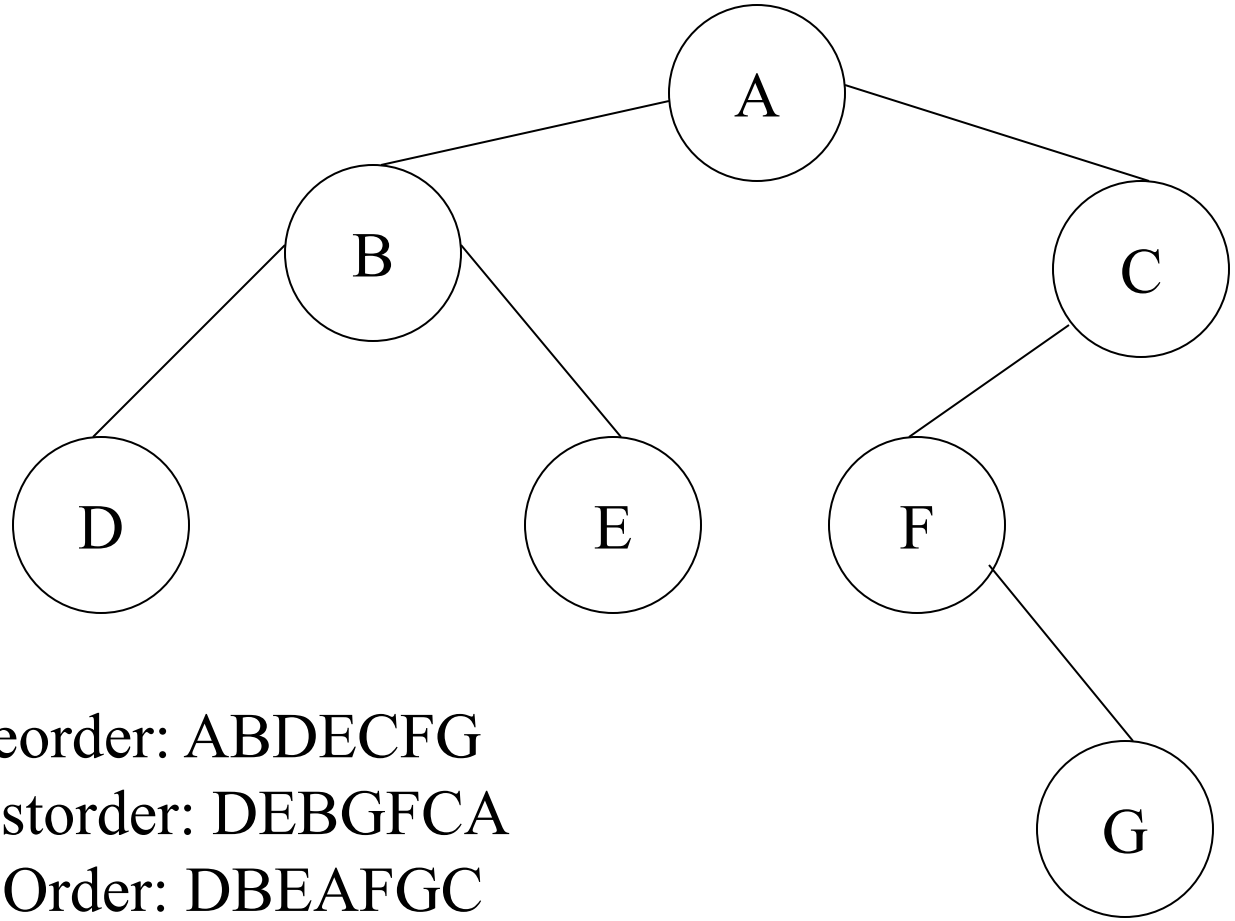
In Order: DBEAFG

Tree Traversals: An Example



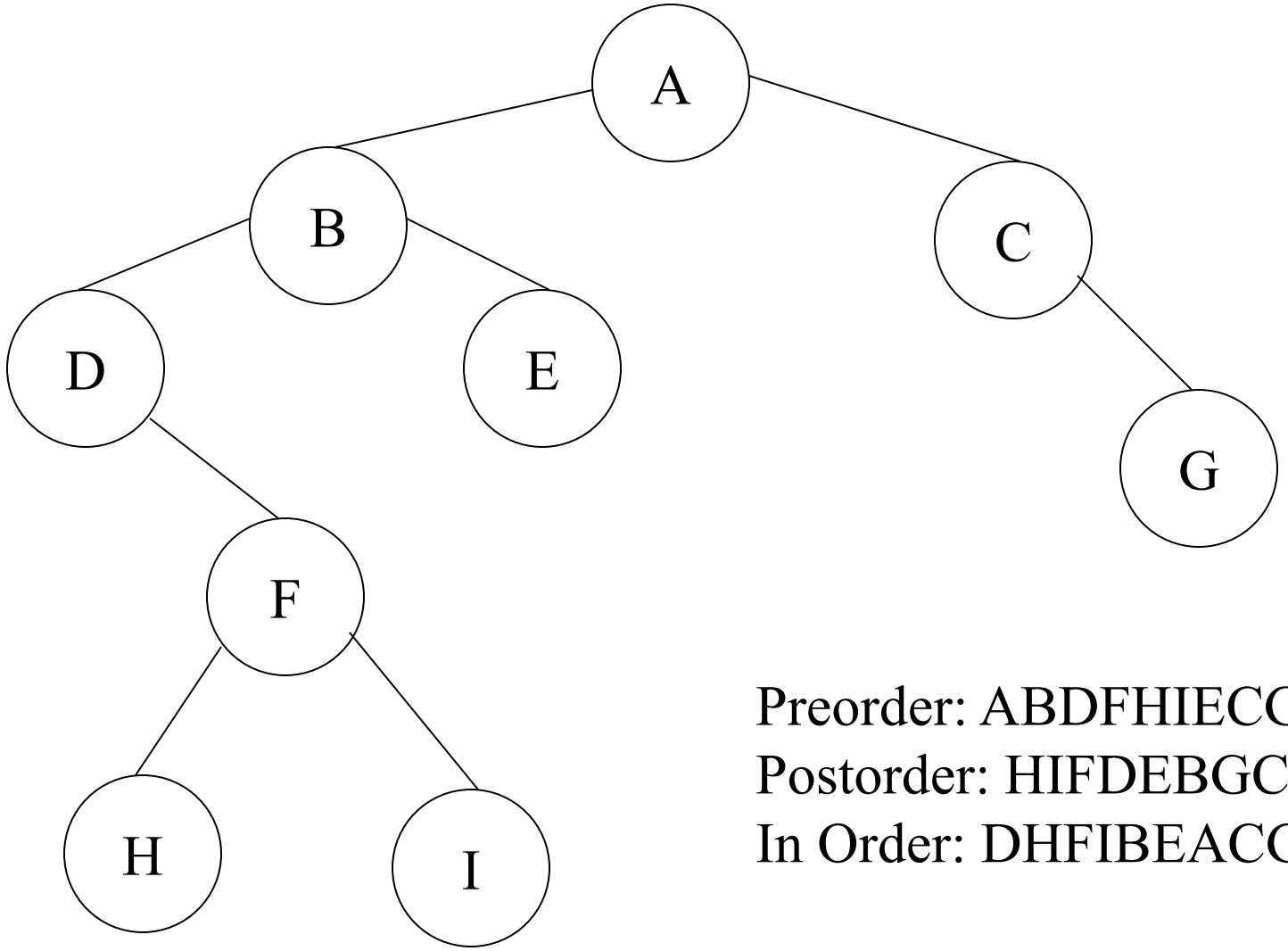
In Order: DBEAFGC

Tree Traversals: An Example



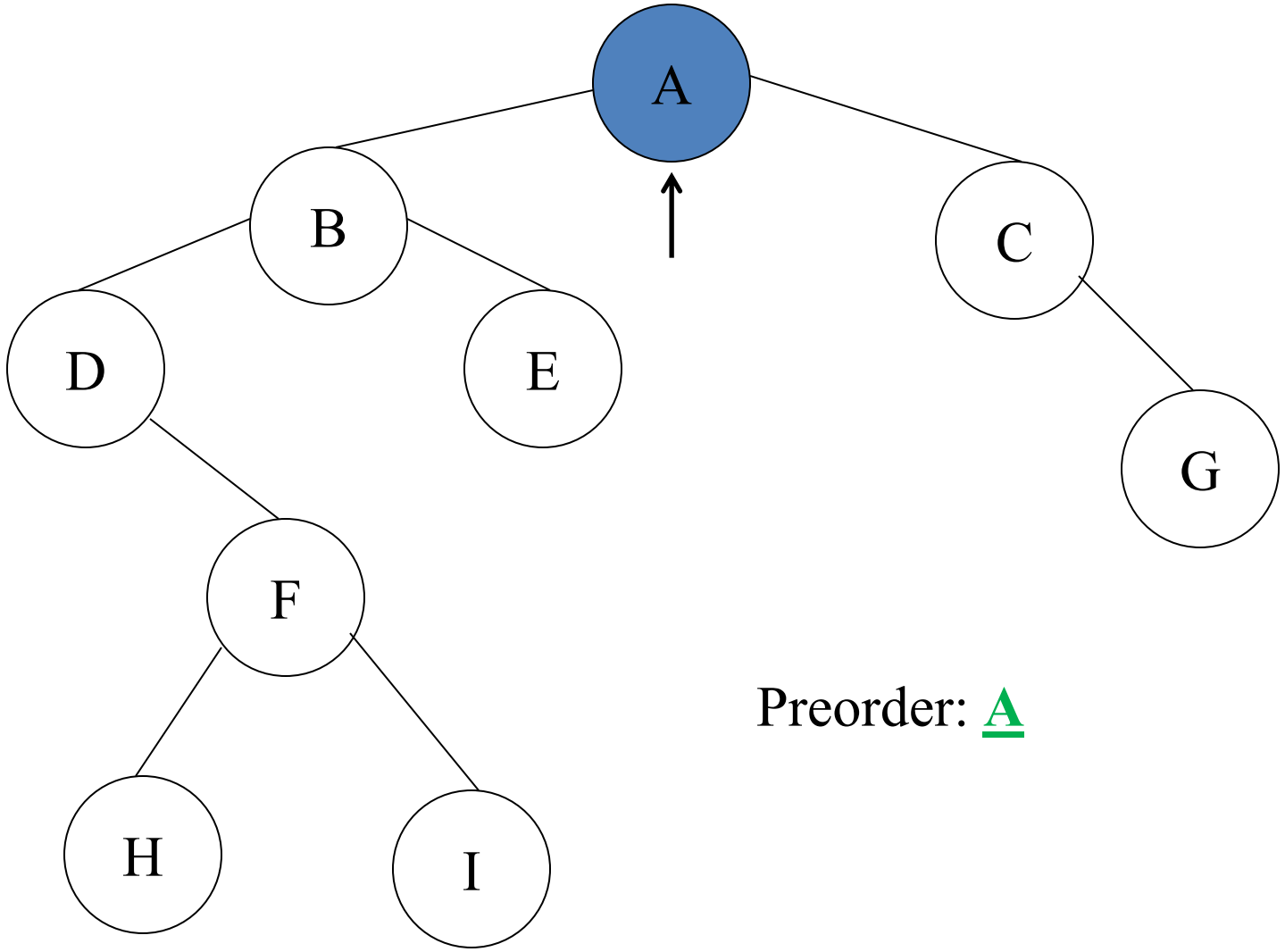
Preorder: ABDECFG
Postorder: DEBGFCA
In Order: DBEAFGC

Tree Traversals: Another Example



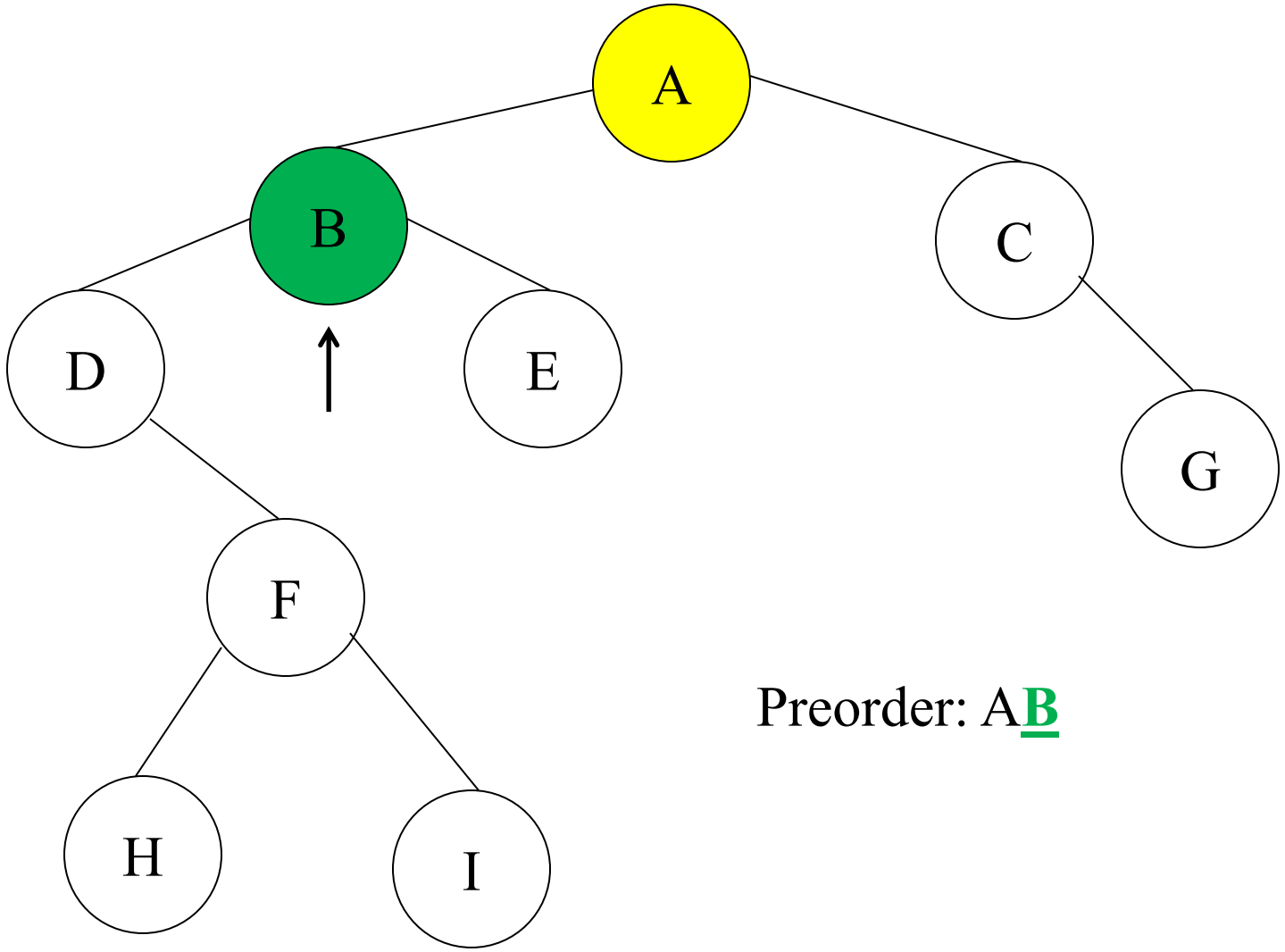
Preorder: ABDFHIECG
Postorder: HIFDEBGCA
In Order: DHFIBEACG

Tree Traversals: Another Example



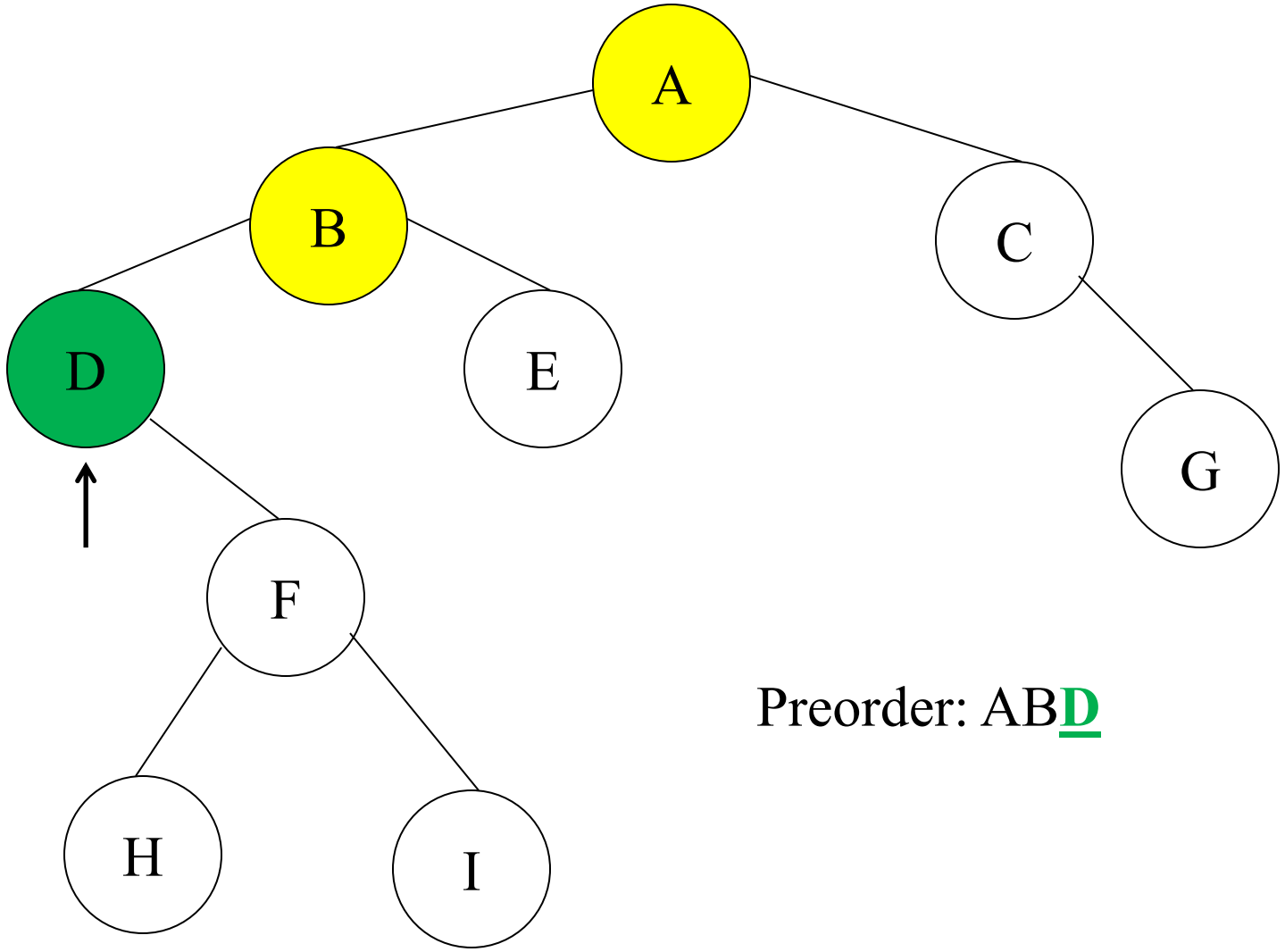
Preorder: A

Tree Traversals: Another Example



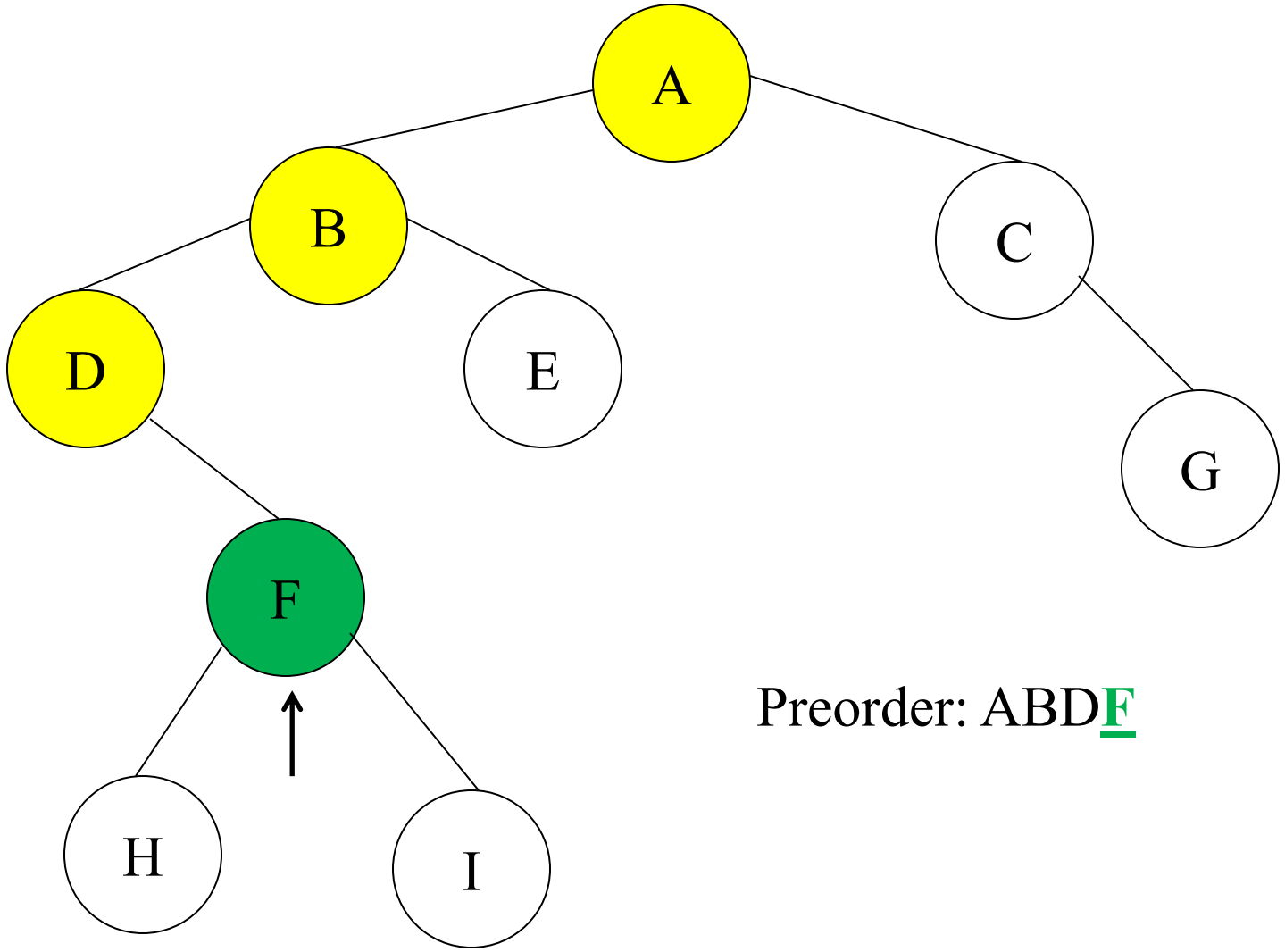
Preorder: AB

Tree Traversals: Another Example



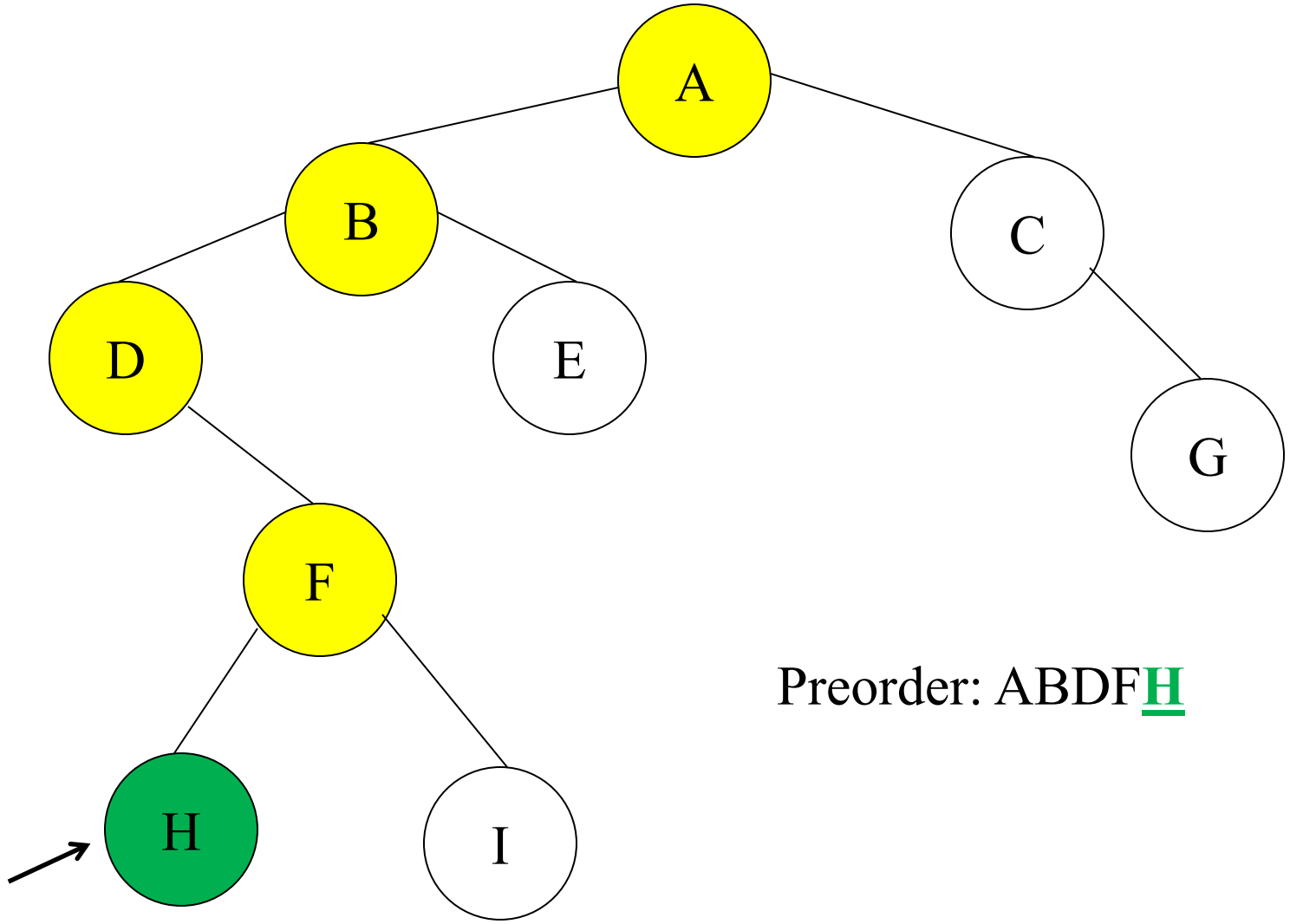
Preorder: ABD

Tree Traversals: Another Example



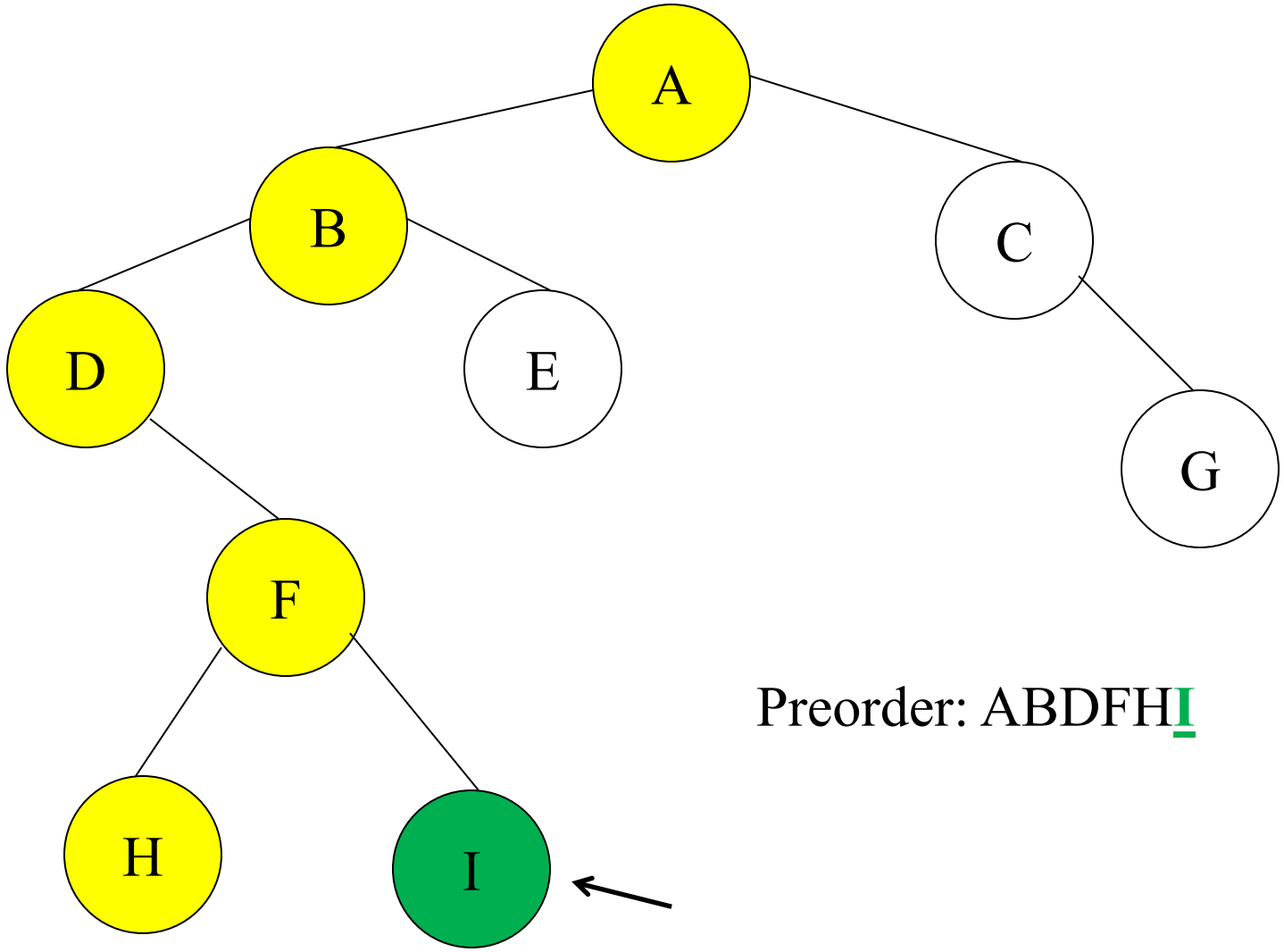
Preorder: ABD**F**

Tree Traversals: Another Example



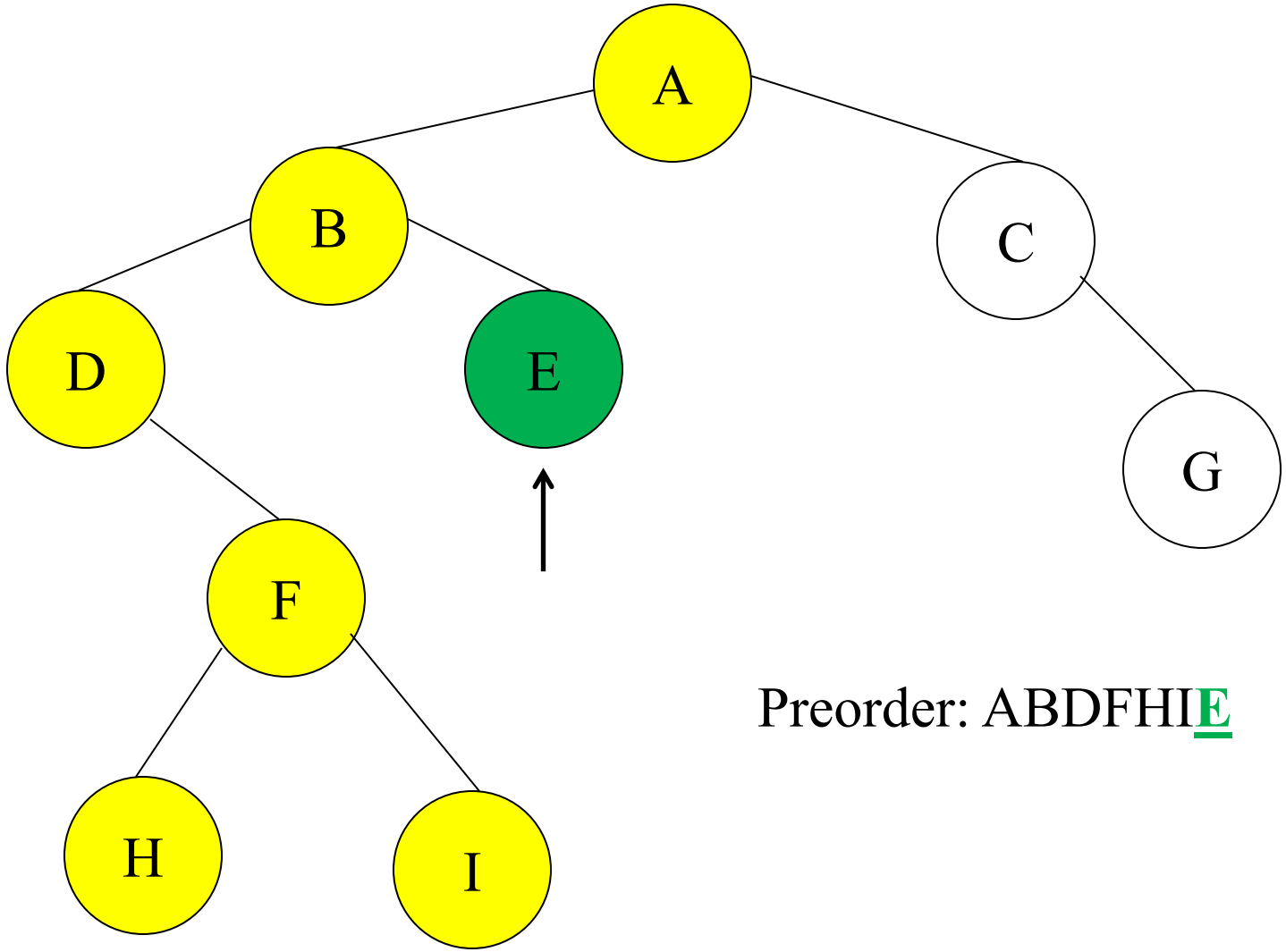
Preorder: ABDF**H**

Tree Traversals: Another Example



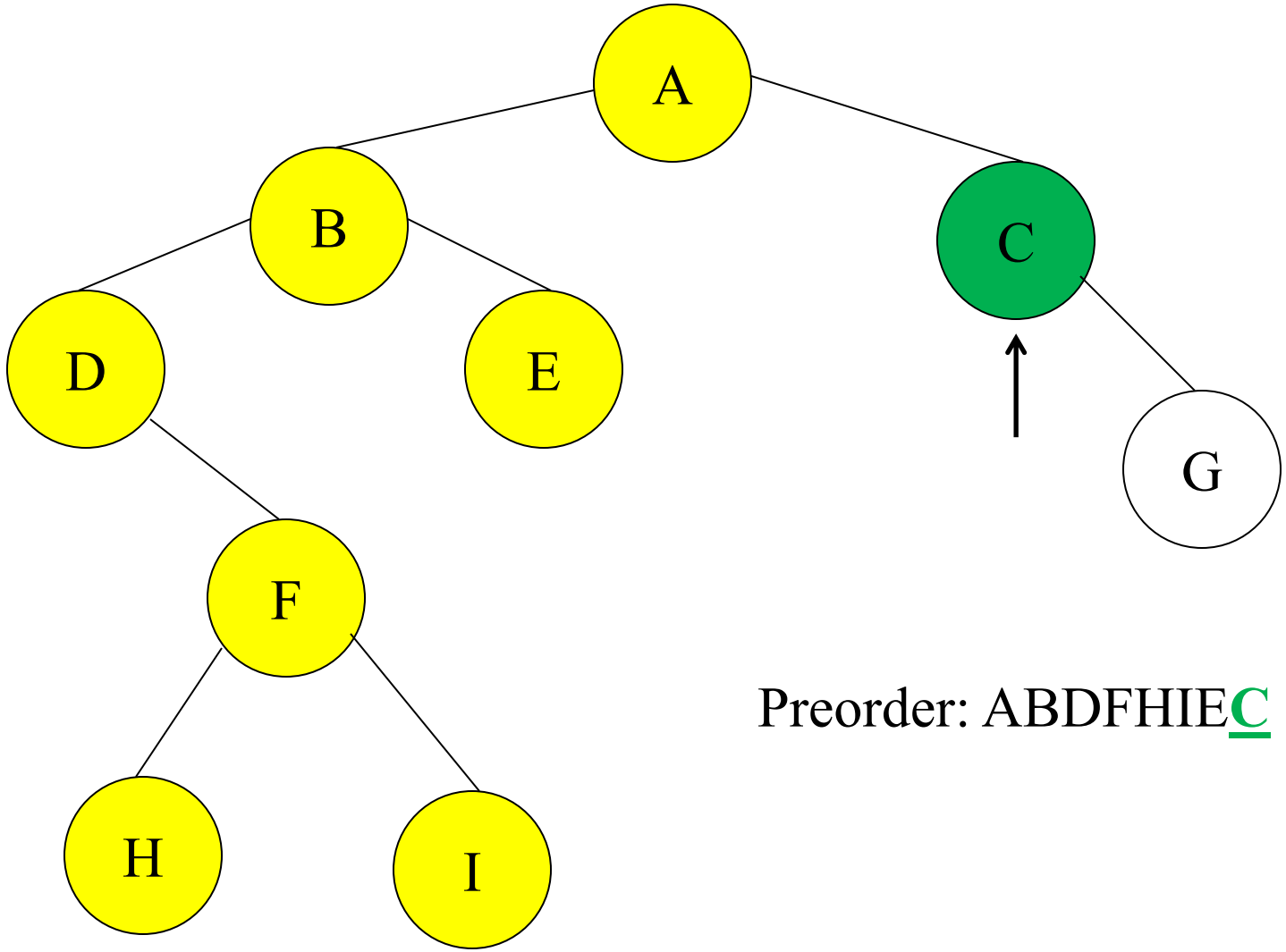
Preorder: ABDFHI

Tree Traversals: Another Example



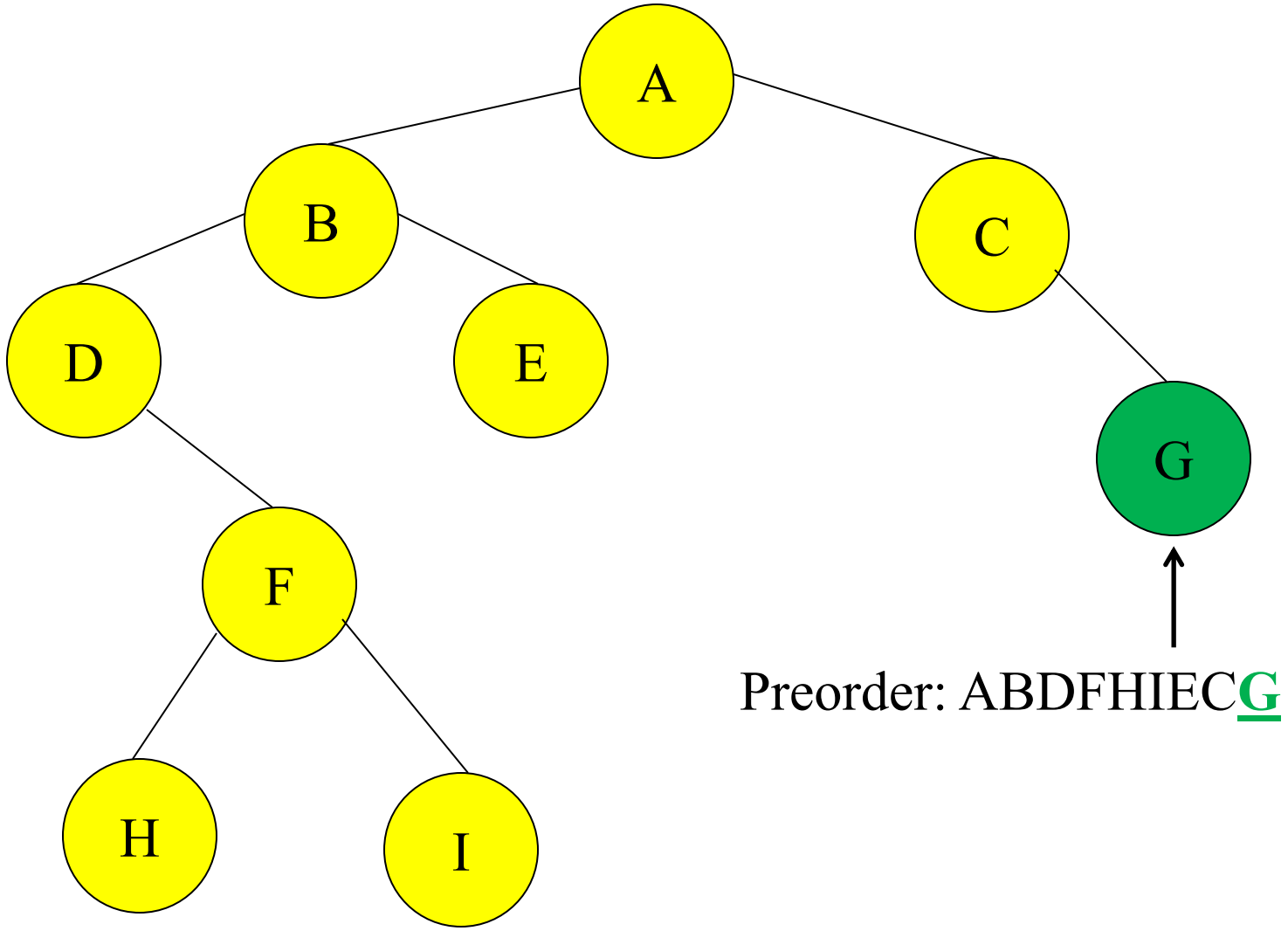
Preorder: ABDFHIE

Tree Traversals: Another Example

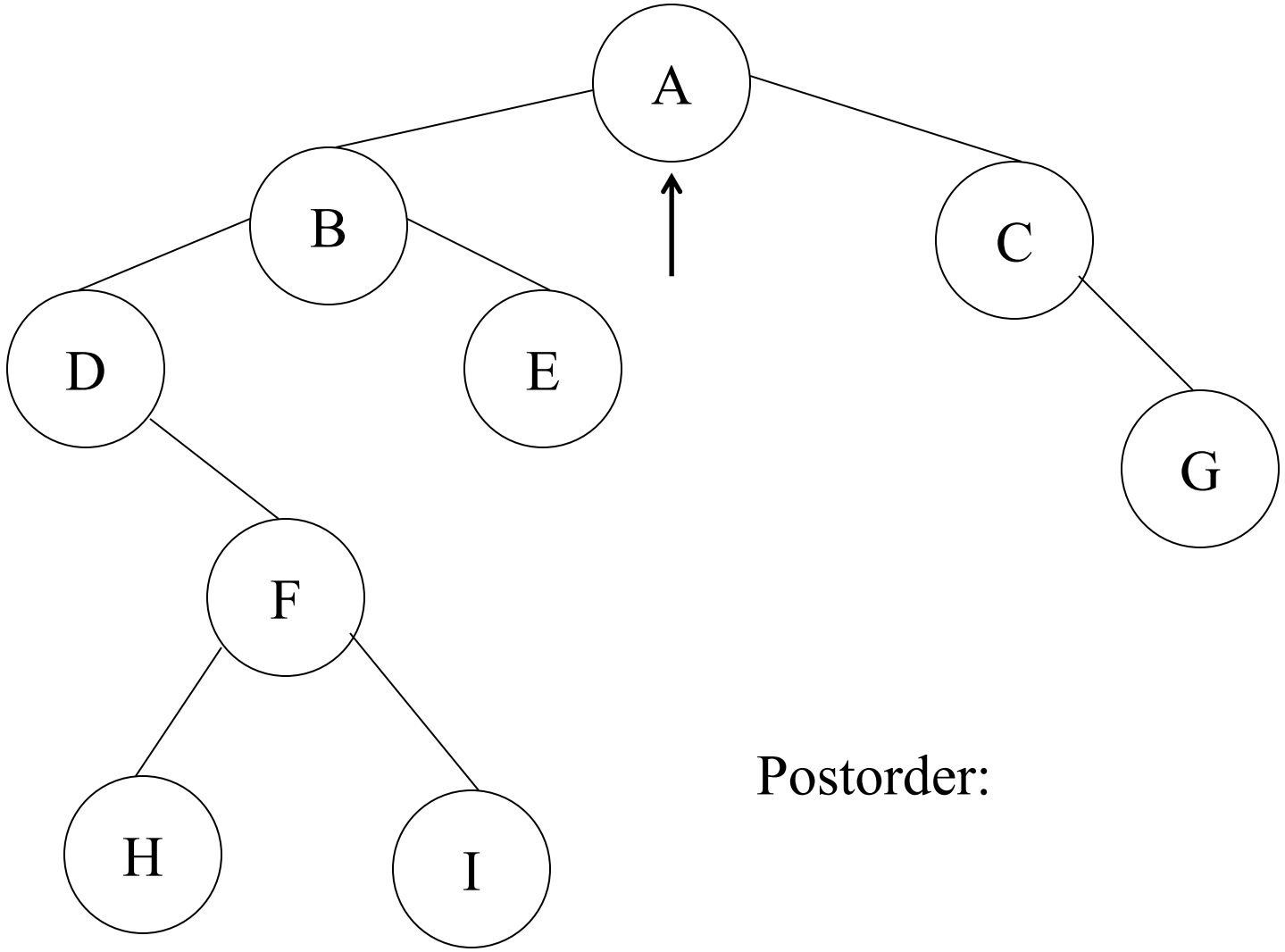


Preorder: ABDFHIE**C**

Tree Traversals: Another Example

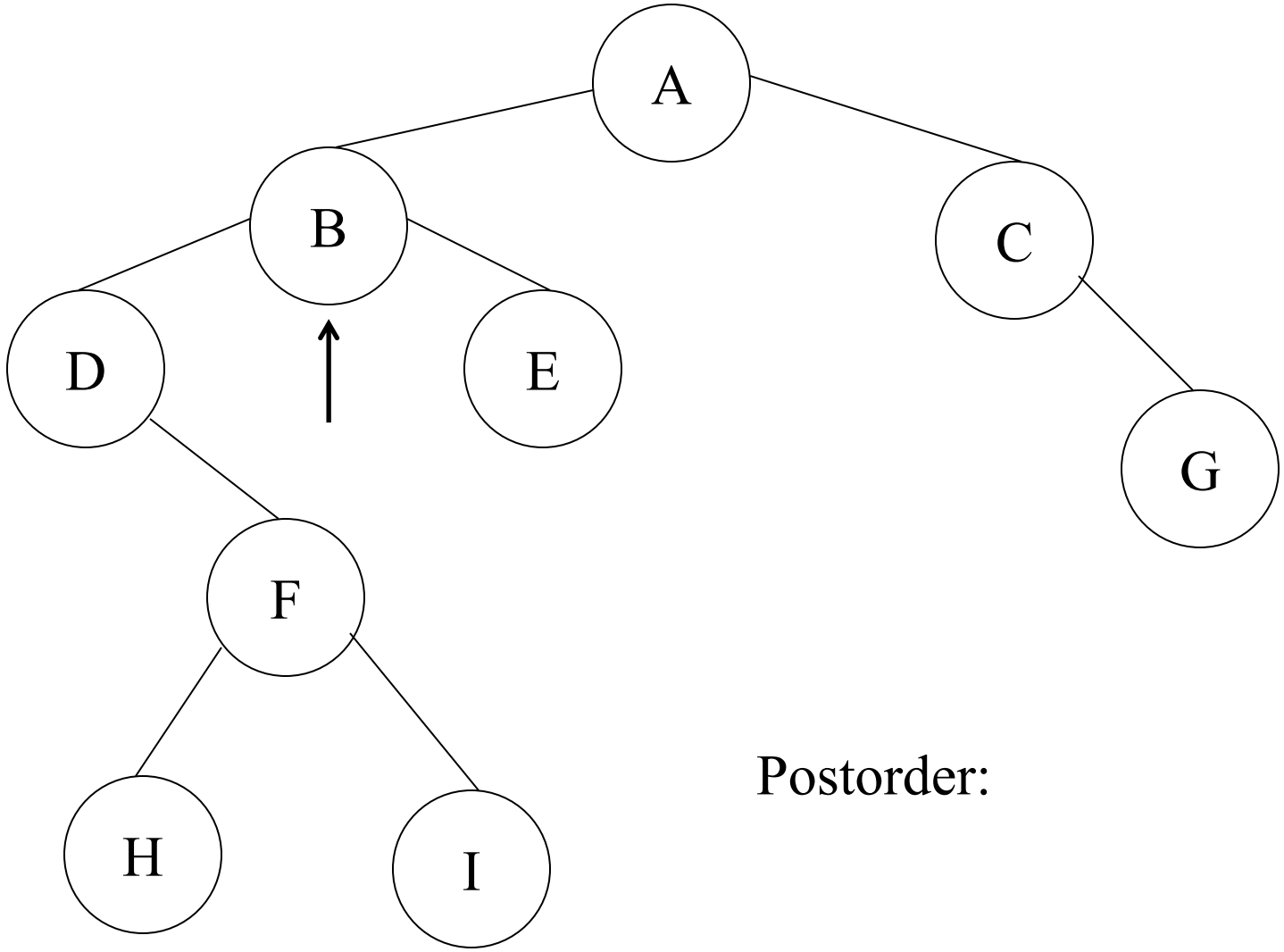


Tree Traversals: Another Example



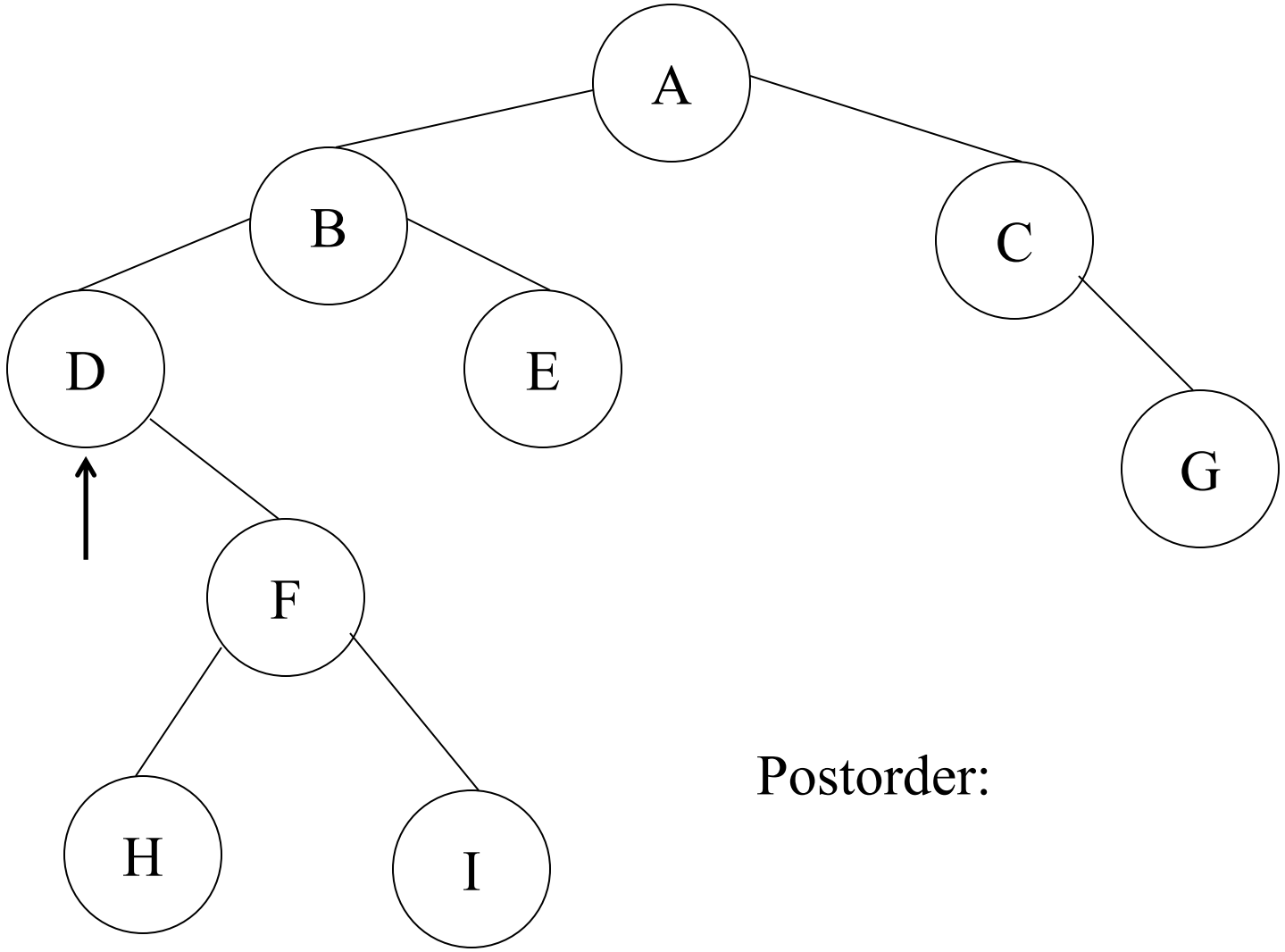
Postorder:

Tree Traversals: Another Example



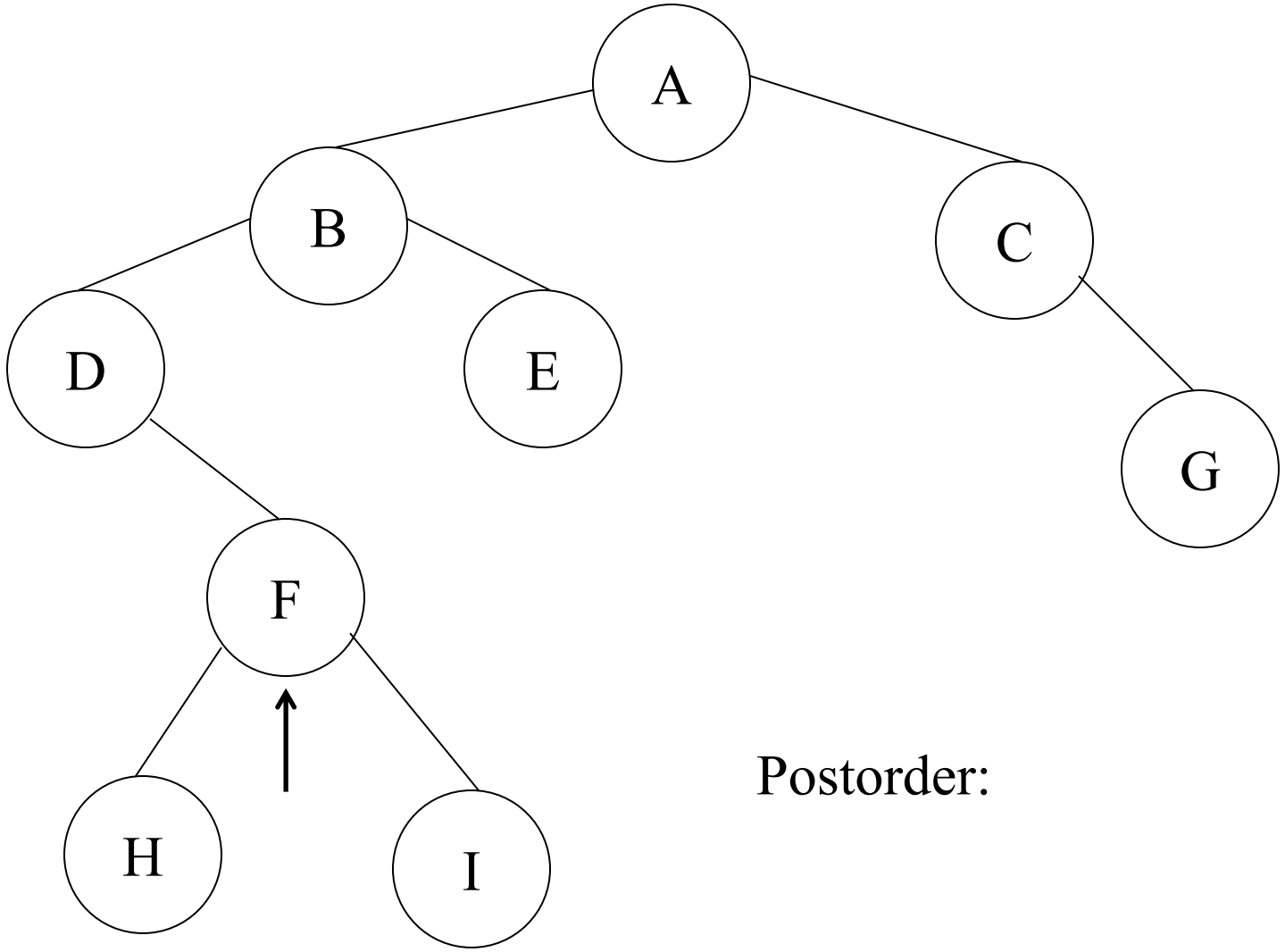
Postorder:

Tree Traversals: Another Example



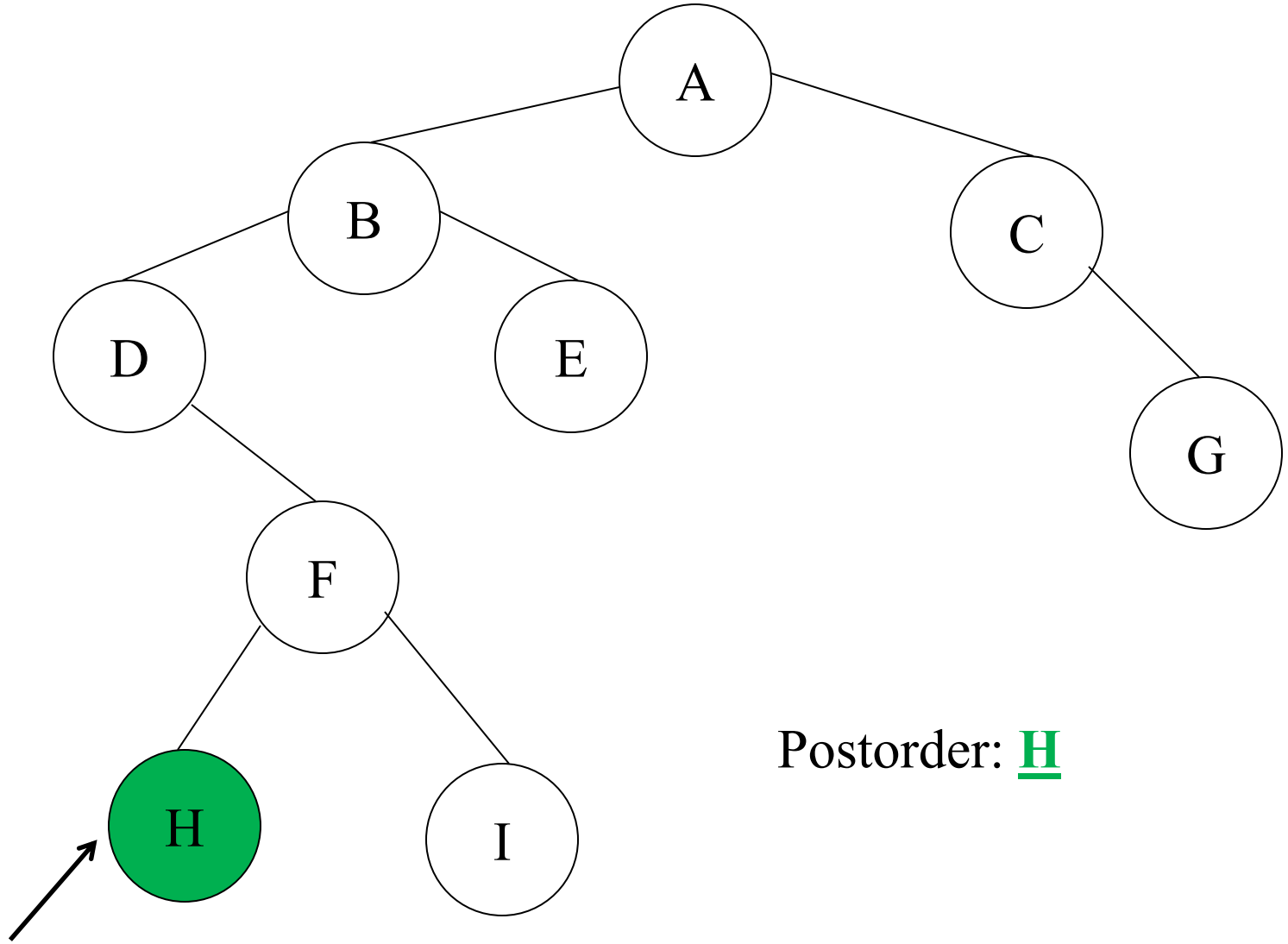
Postorder:

Tree Traversals: Another Example



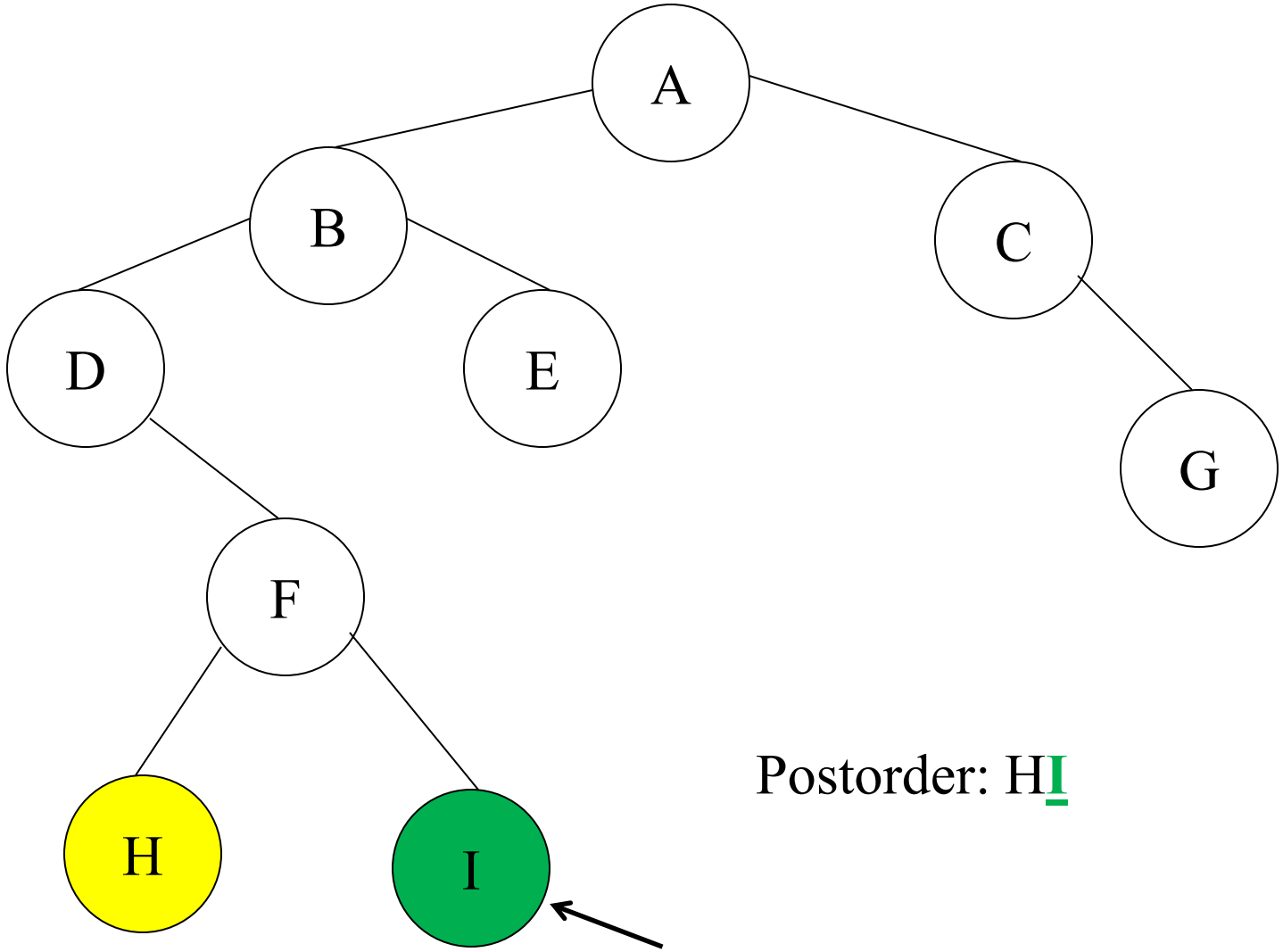
Postorder:

Tree Traversals: Another Example



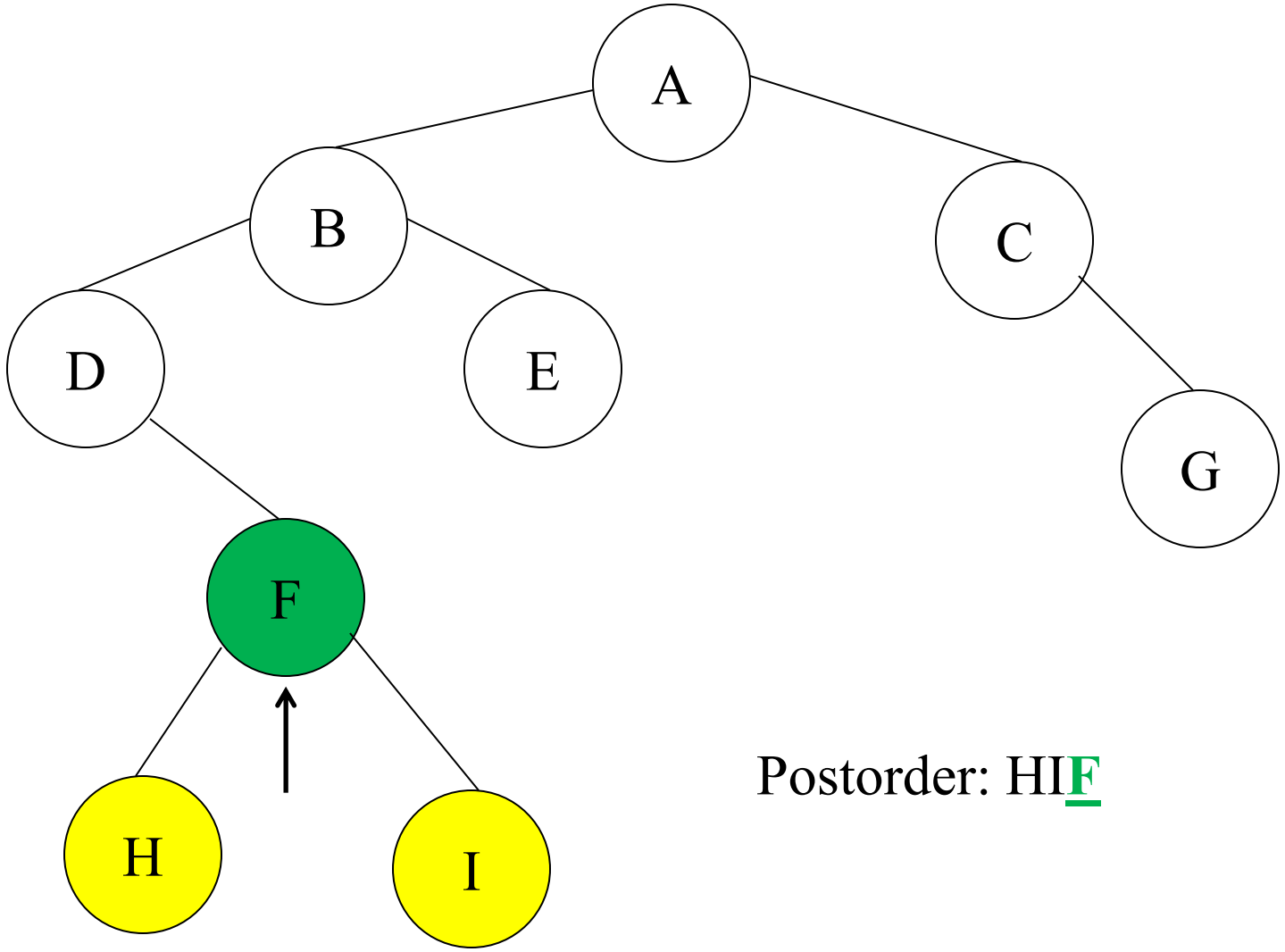
Postorder: H

Tree Traversals: Another Example



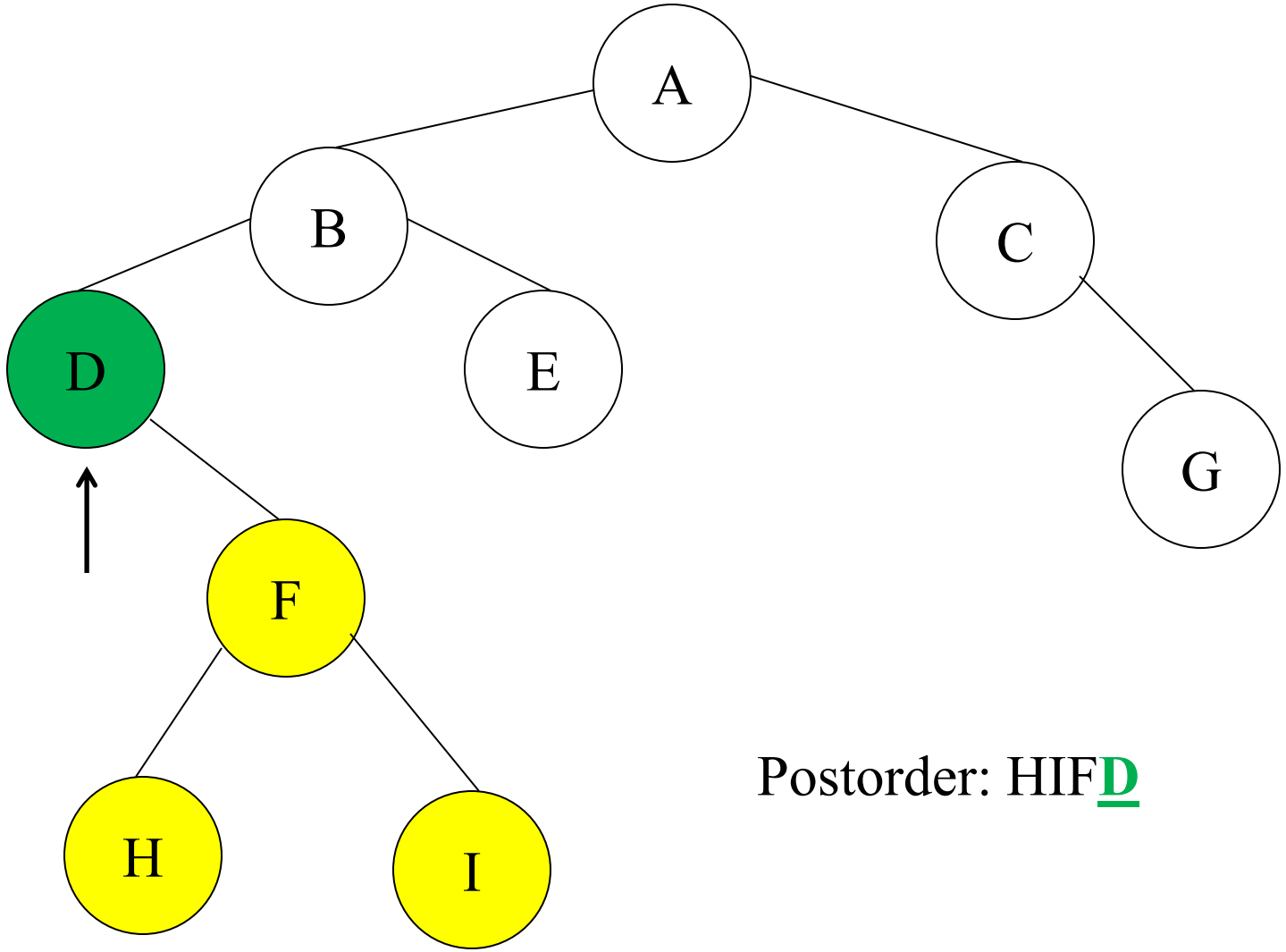
Postorder: HI

Tree Traversals: Another Example



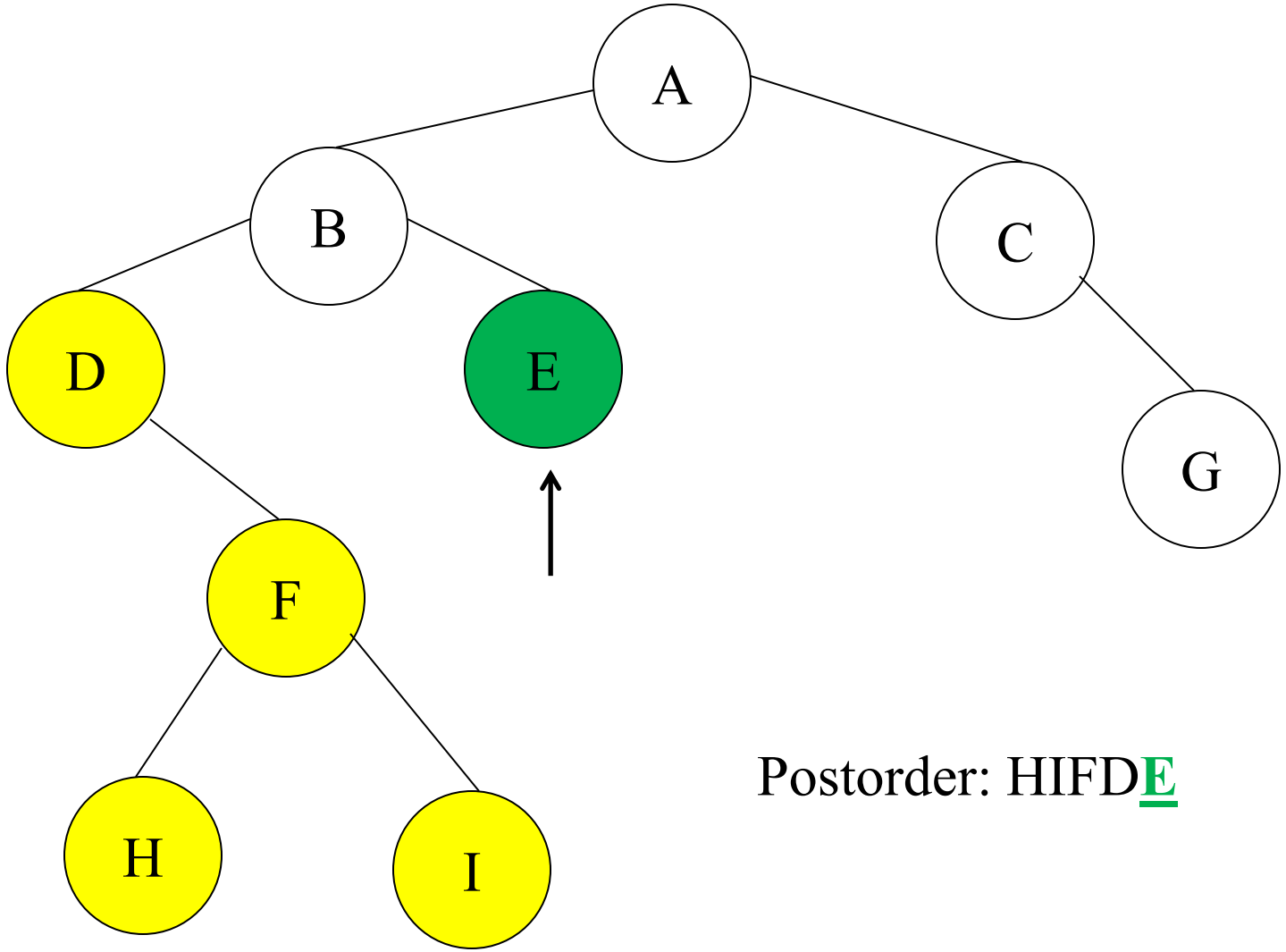
Postorder: HI**F**

Tree Traversals: Another Example



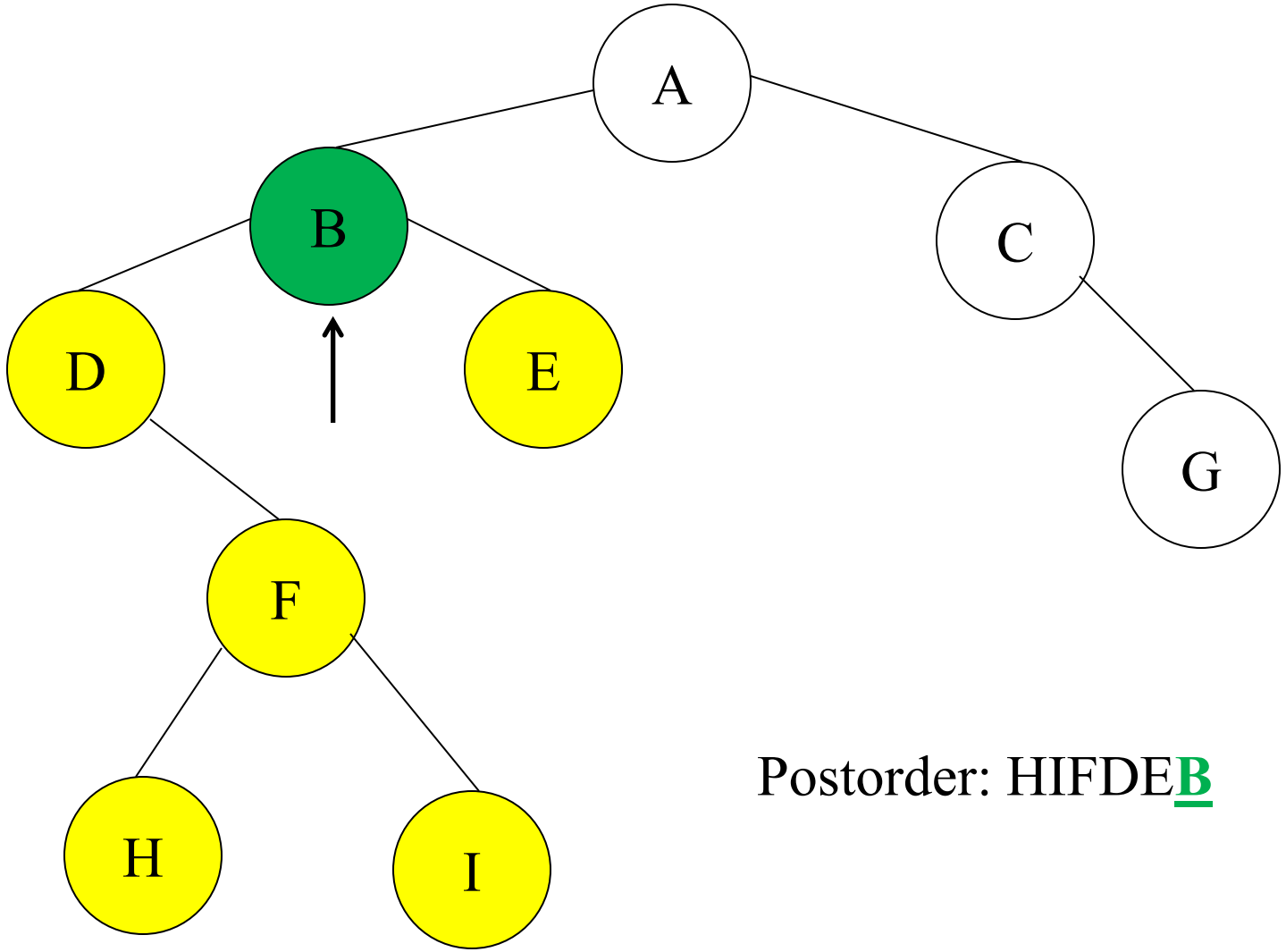
Postorder: HIF**D**

Tree Traversals: Another Example



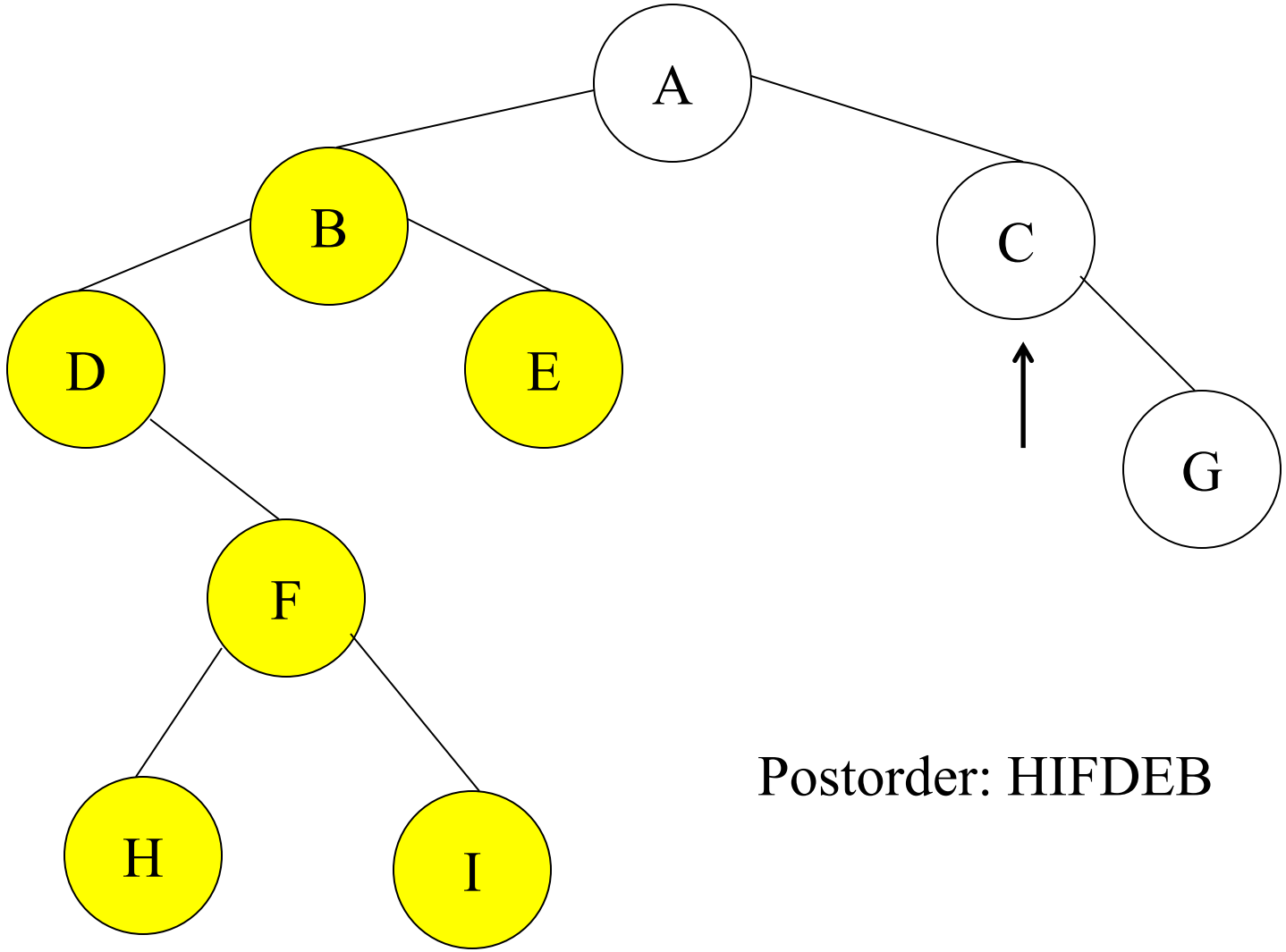
Postorder: HIFD**E**

Tree Traversals: Another Example



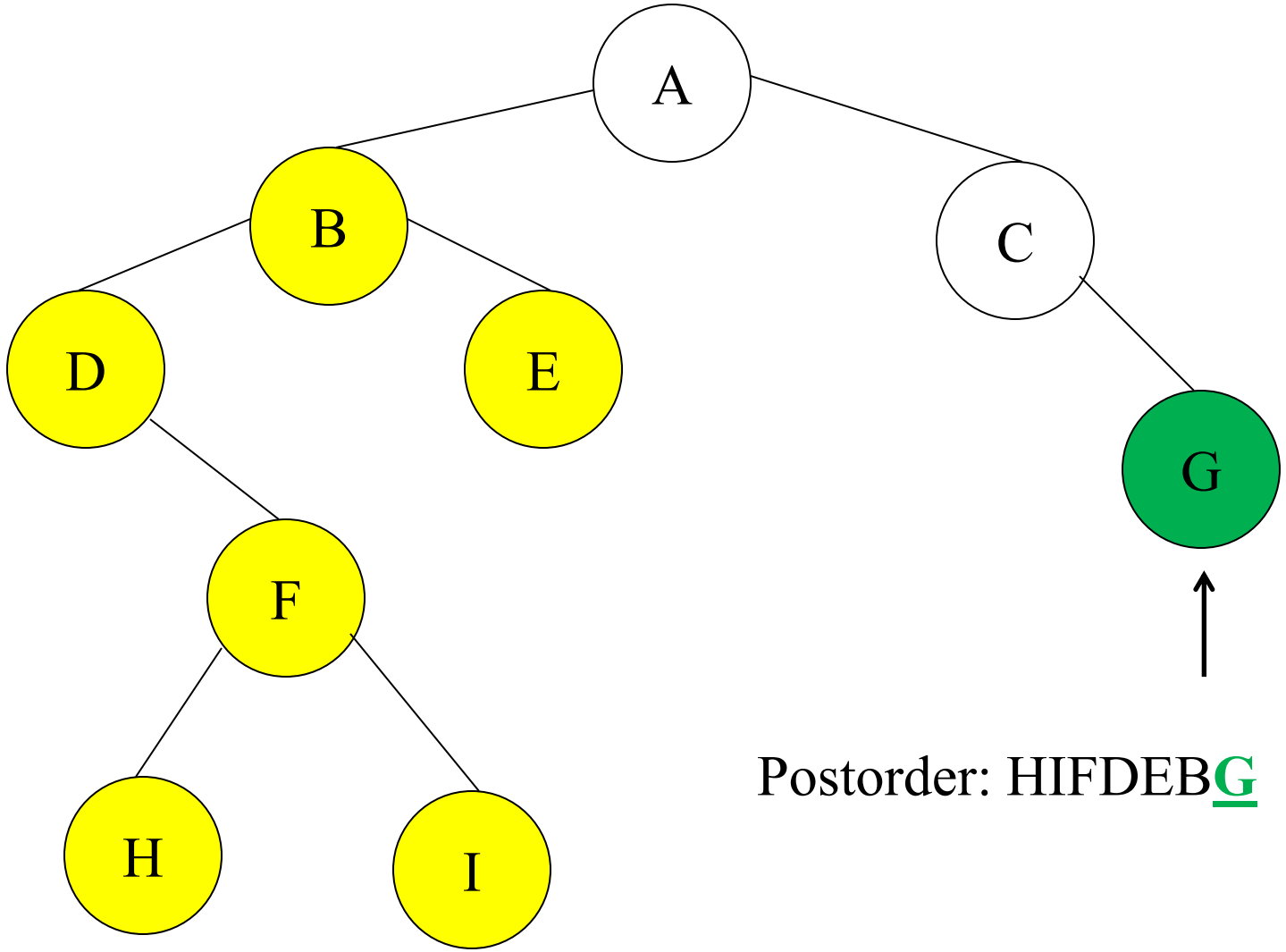
Postorder: HIFDEB

Tree Traversals: Another Example



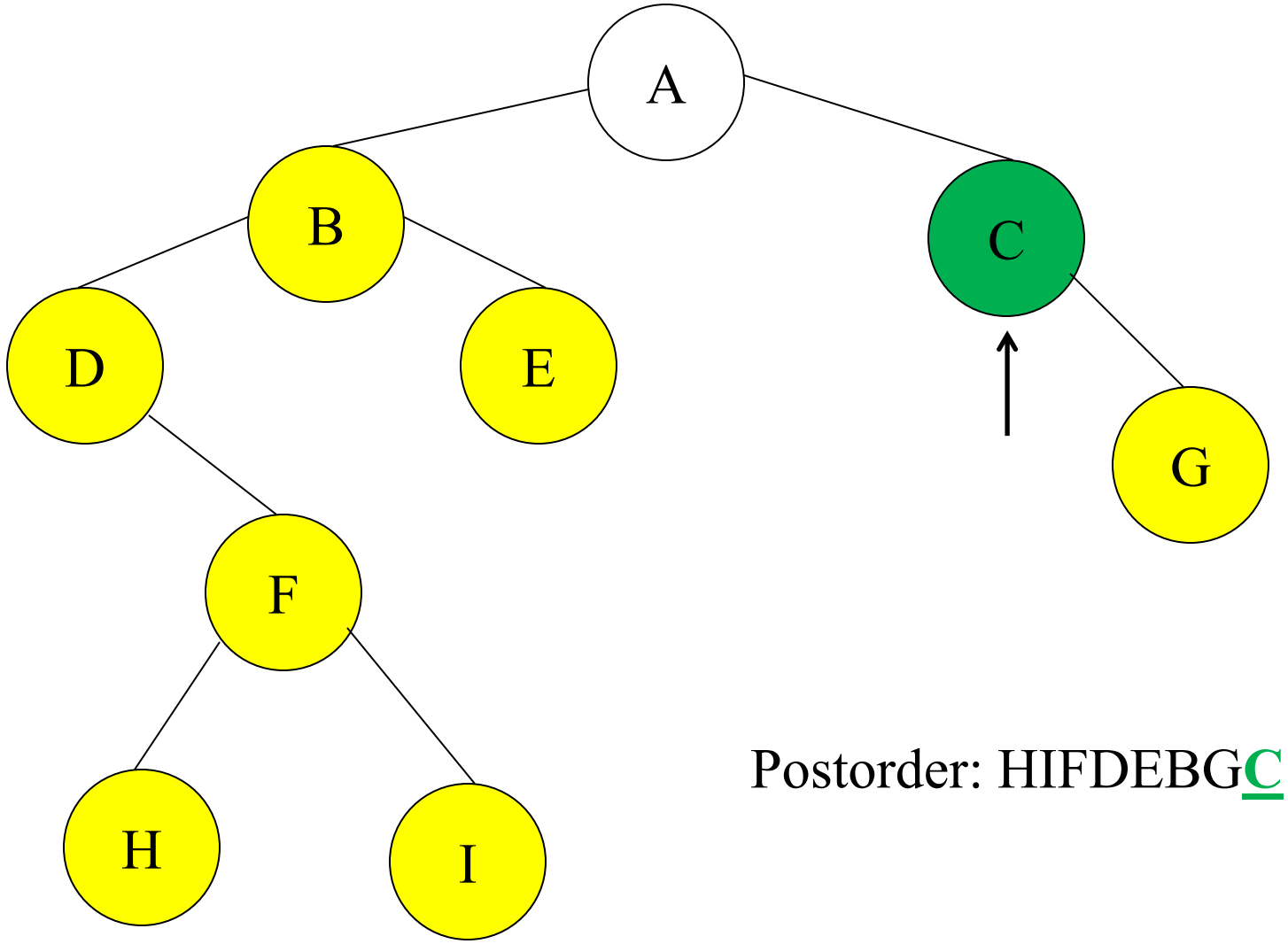
Postorder: HIFDEB

Tree Traversals: Another Example



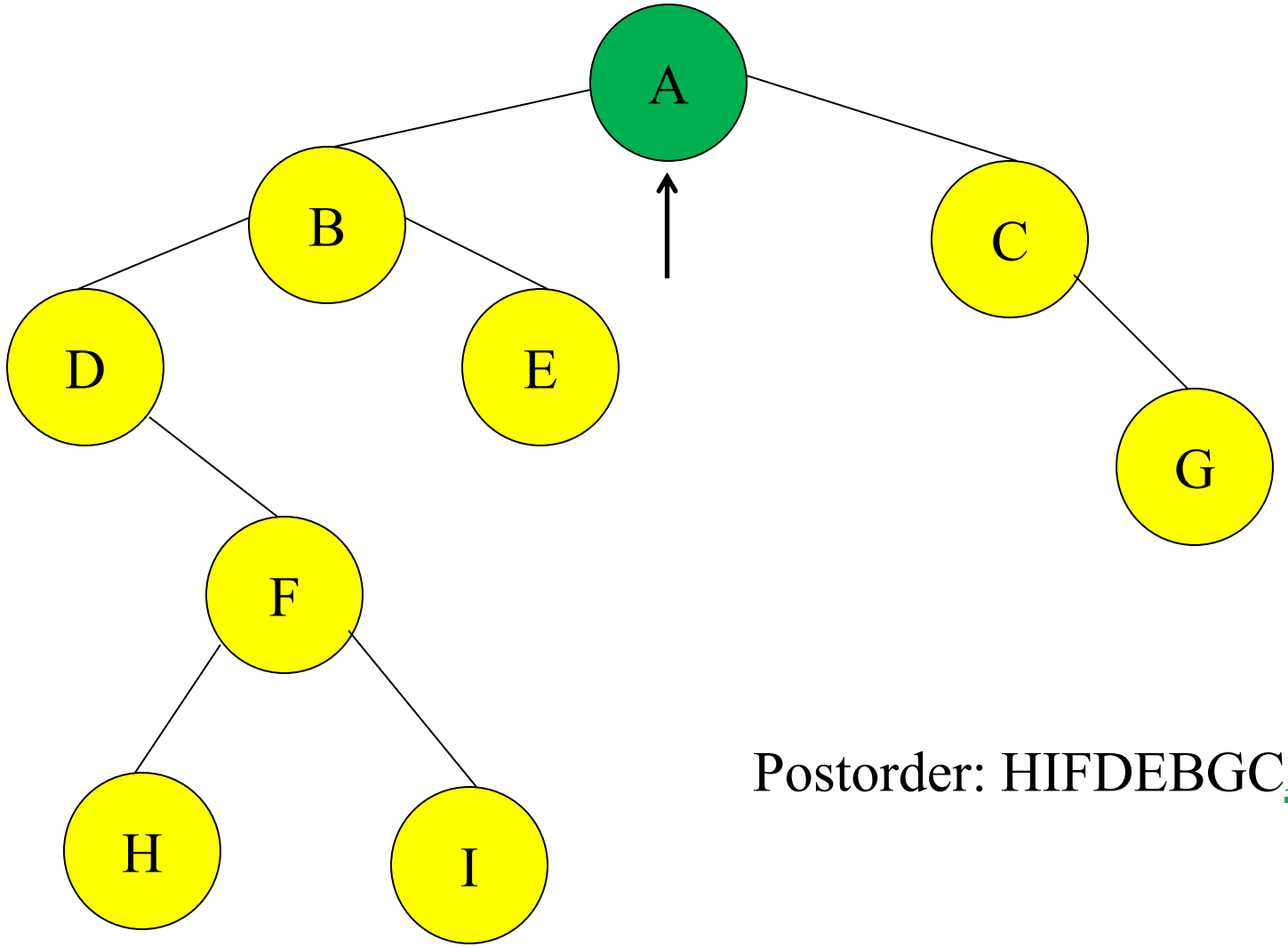
Postorder: HIFDEBG

Tree Traversals: Another Example



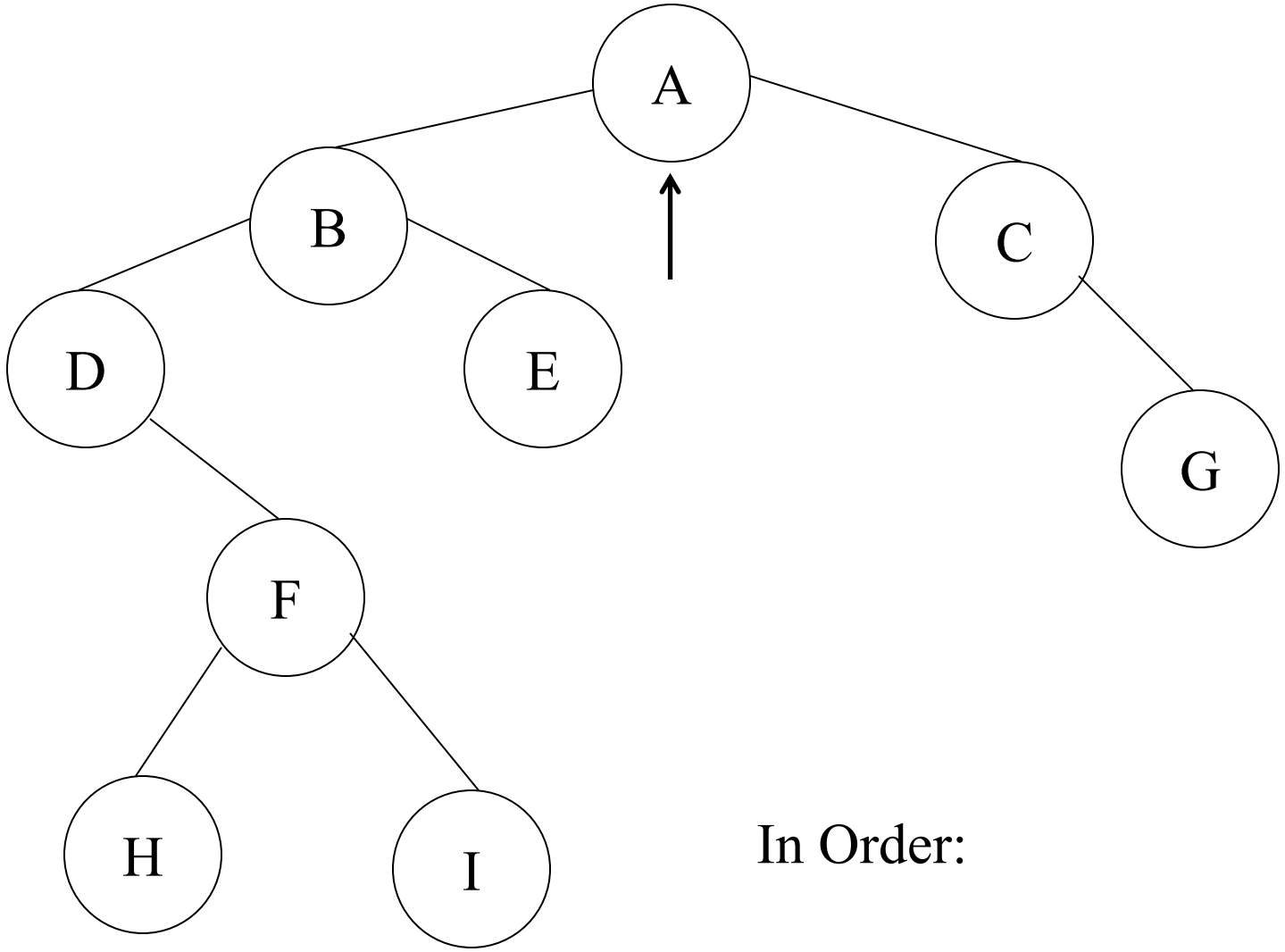
Postorder: HIFDEBGC

Tree Traversals: Another Example



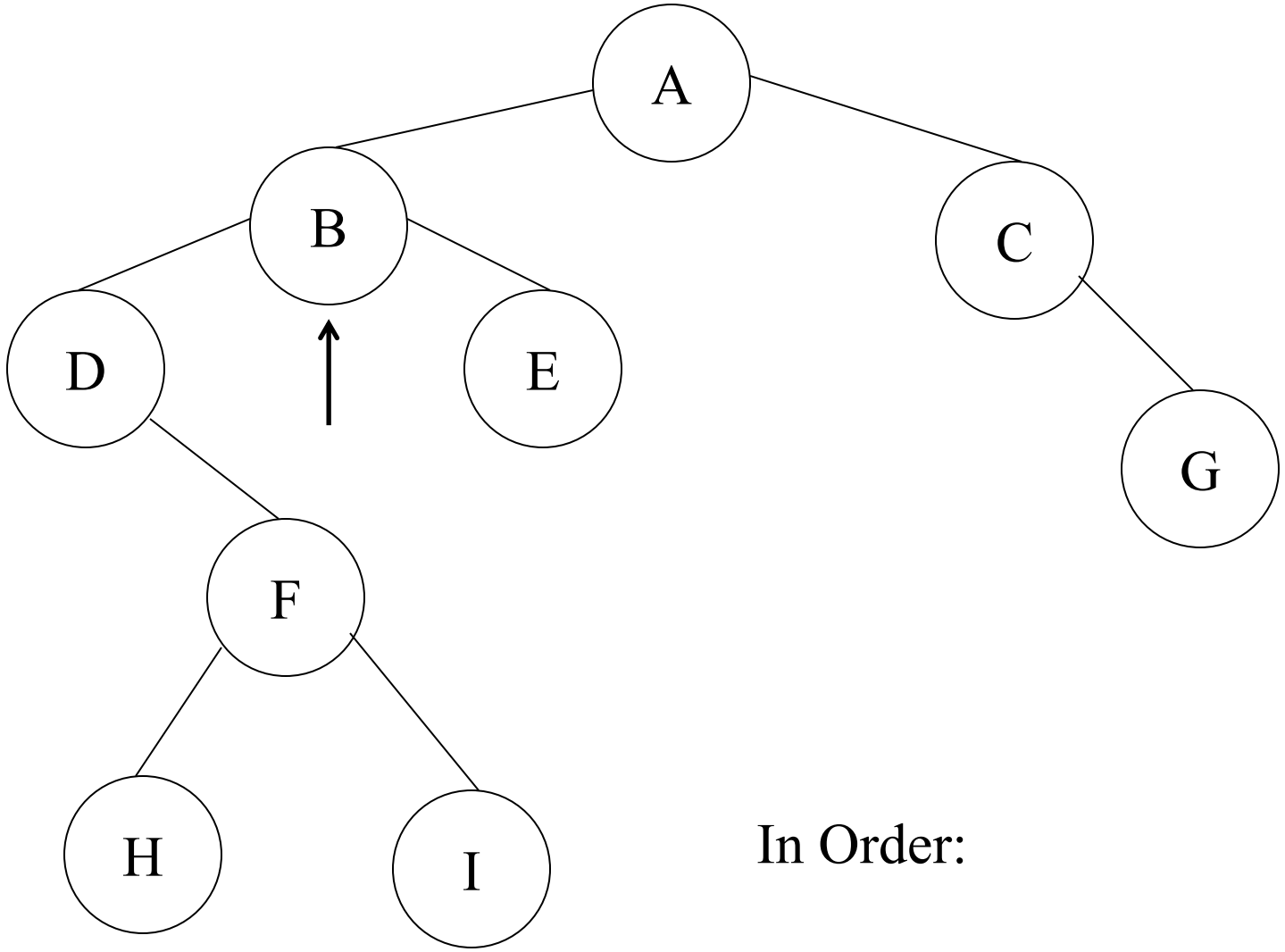
Postorder: HIFDEBGC**A**

Tree Traversals: Another Example



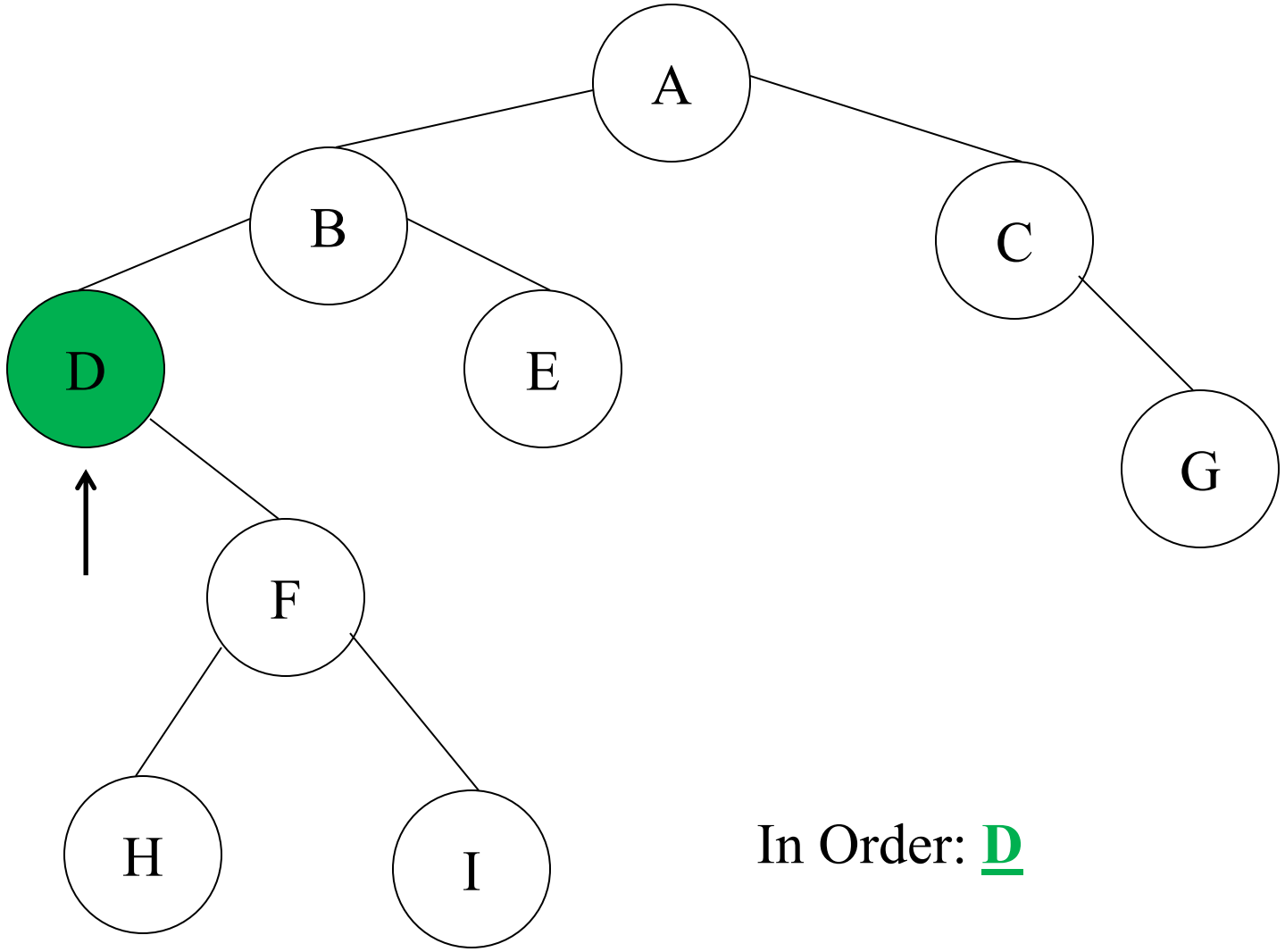
In Order:

Tree Traversals: Another Example



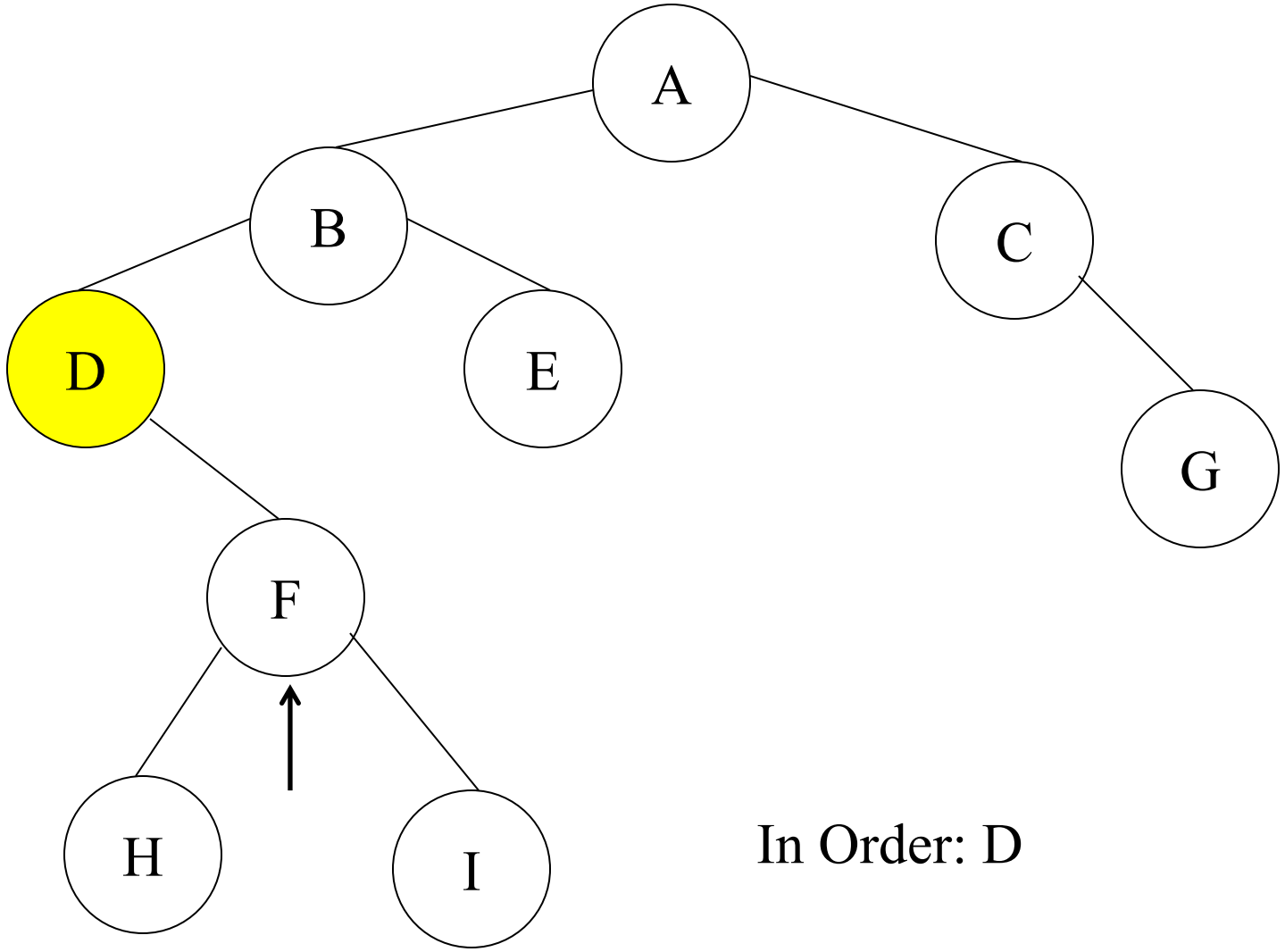
In Order:

Tree Traversals: Another Example



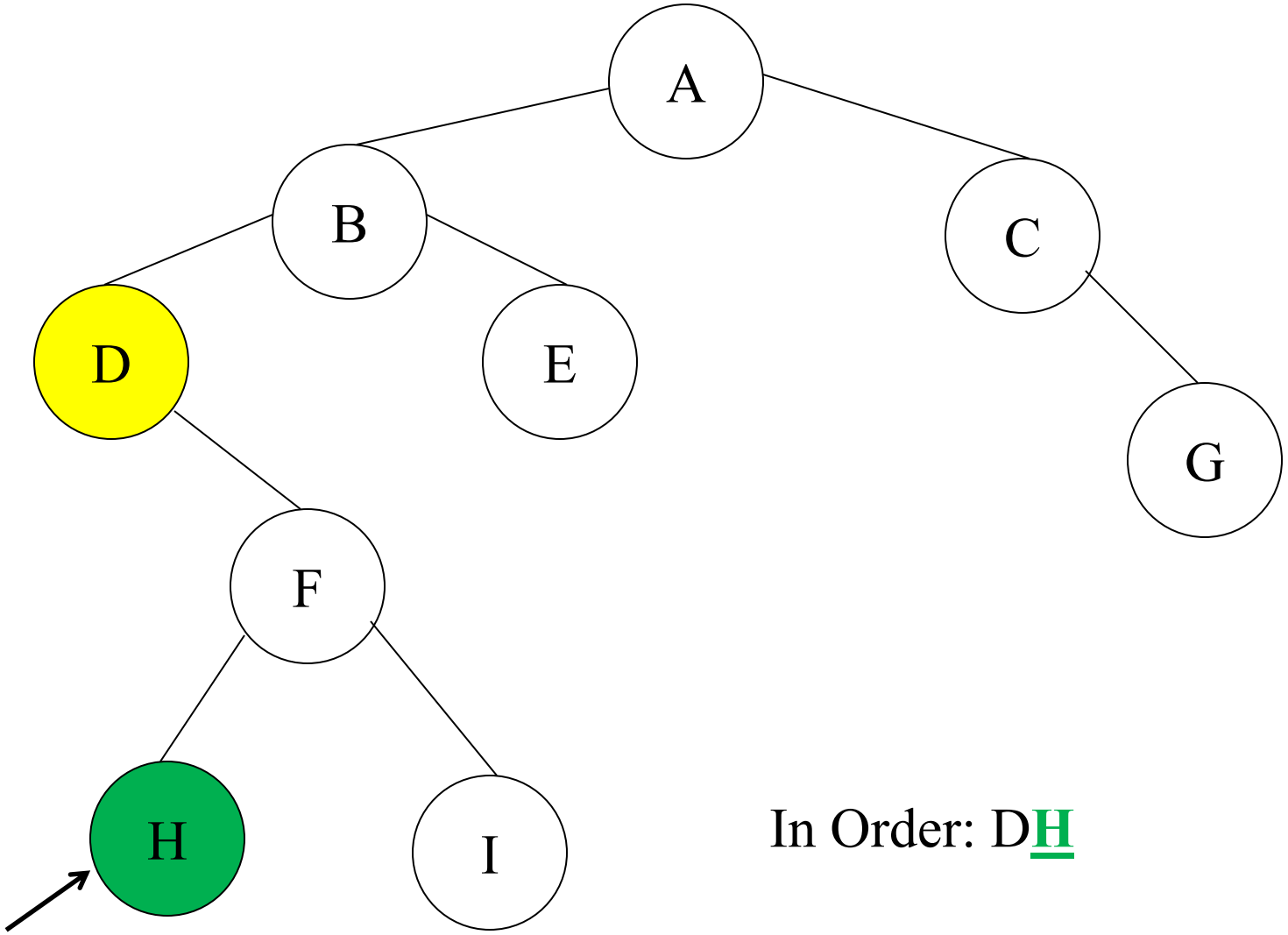
In Order: **D**

Tree Traversals: Another Example



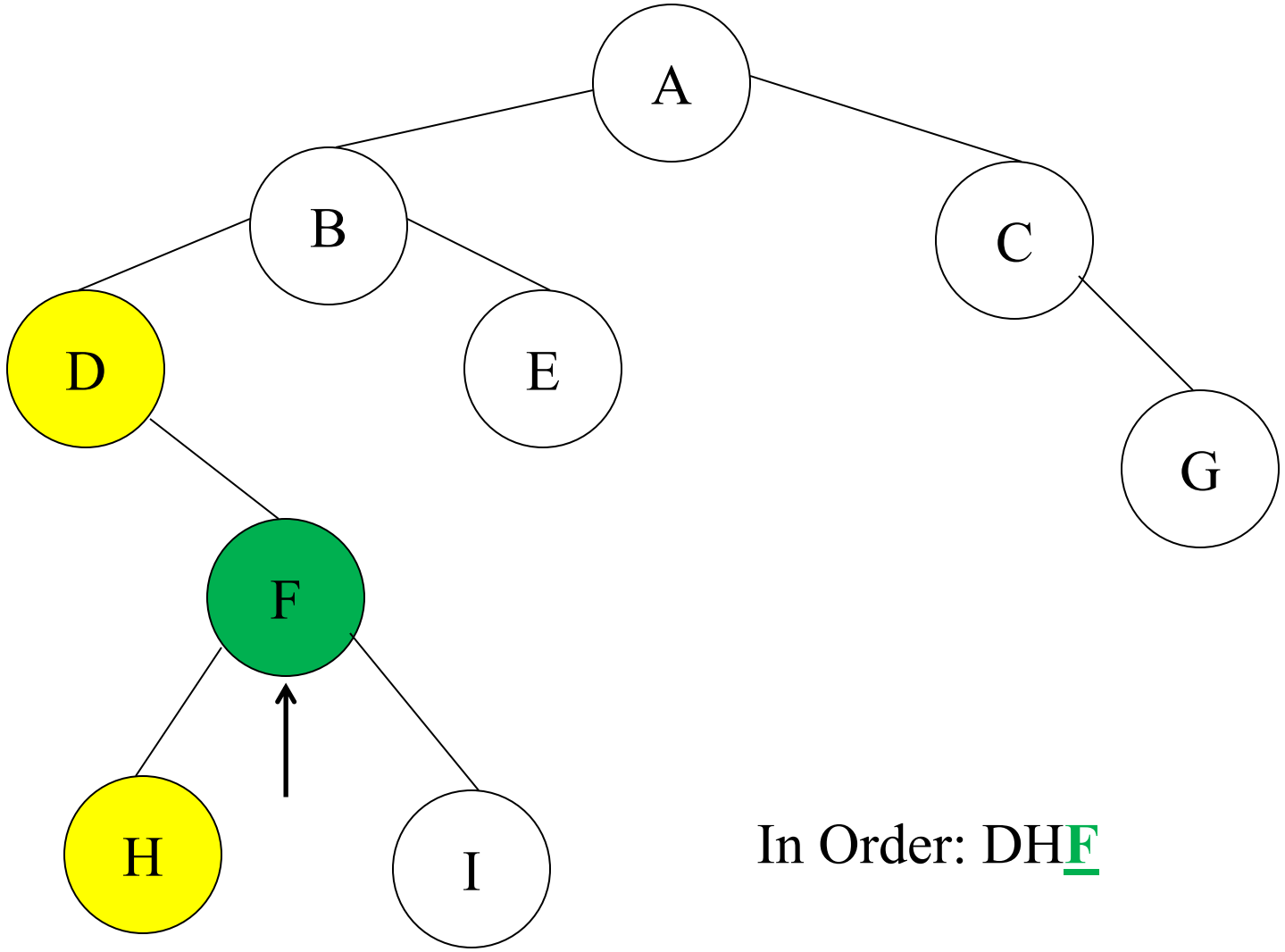
In Order: D

Tree Traversals: Another Example



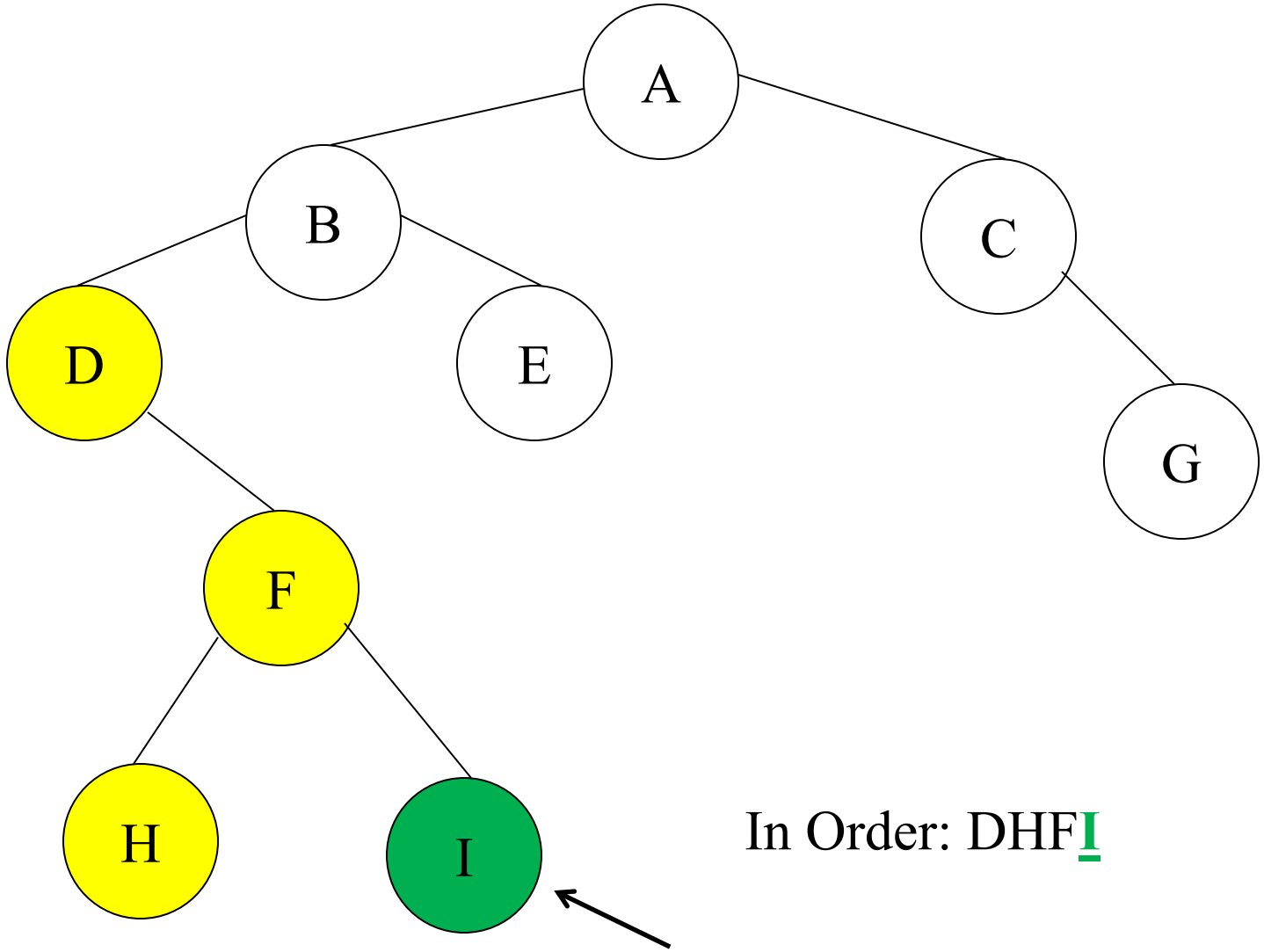
In Order: DH

Tree Traversals: Another Example



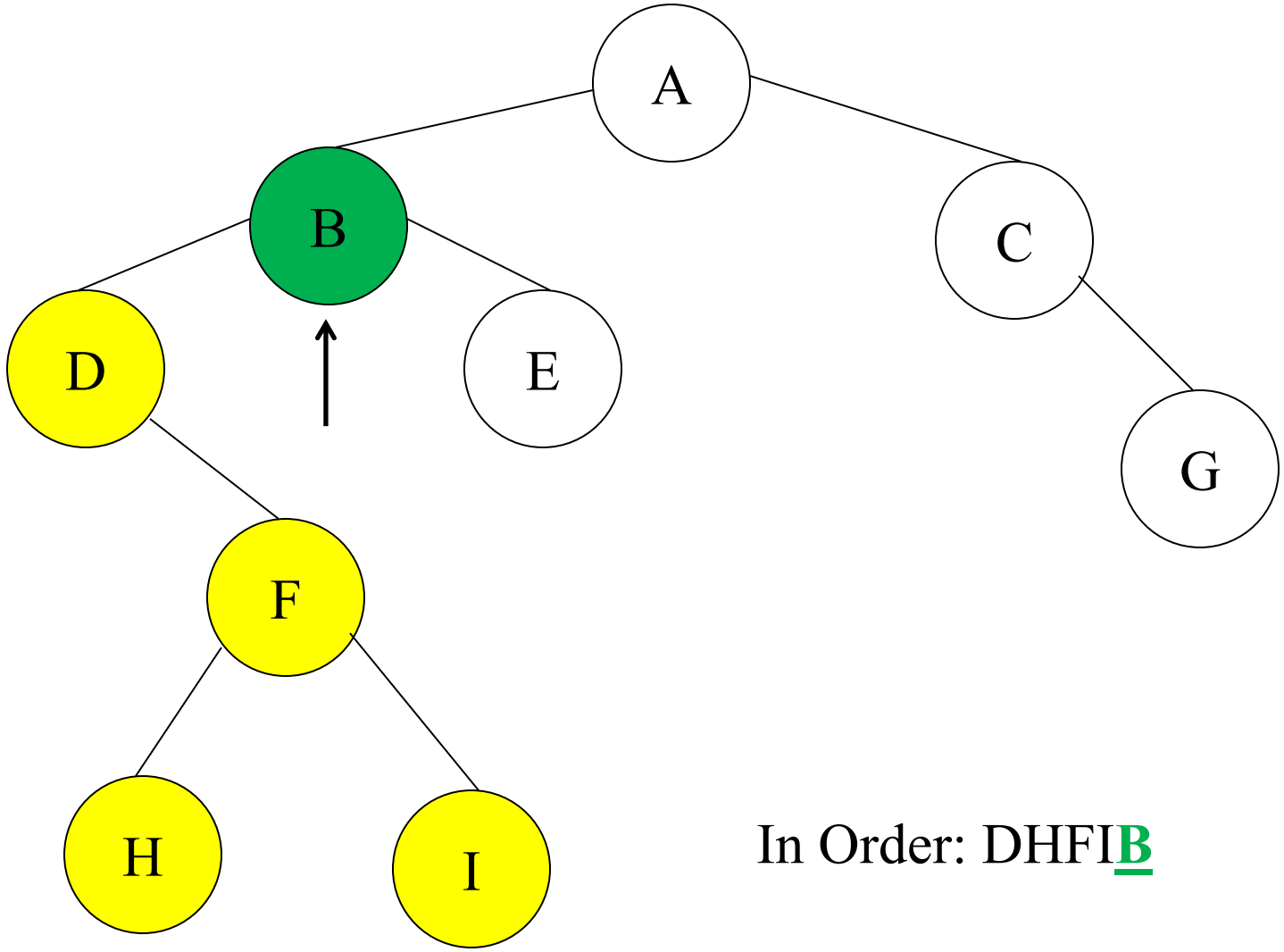
In Order: DHF

Tree Traversals: Another Example



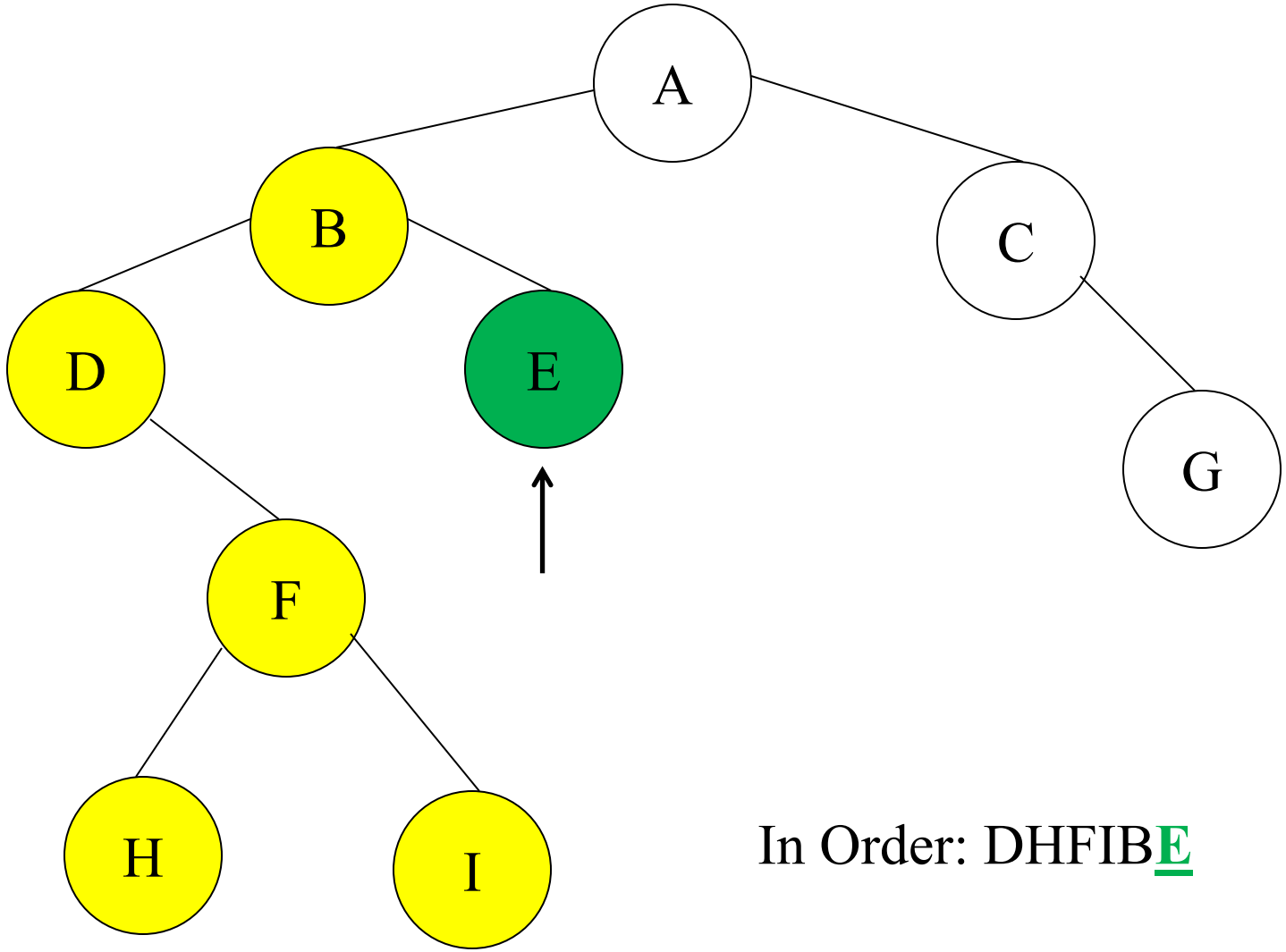
In Order: DHFI

Tree Traversals: Another Example



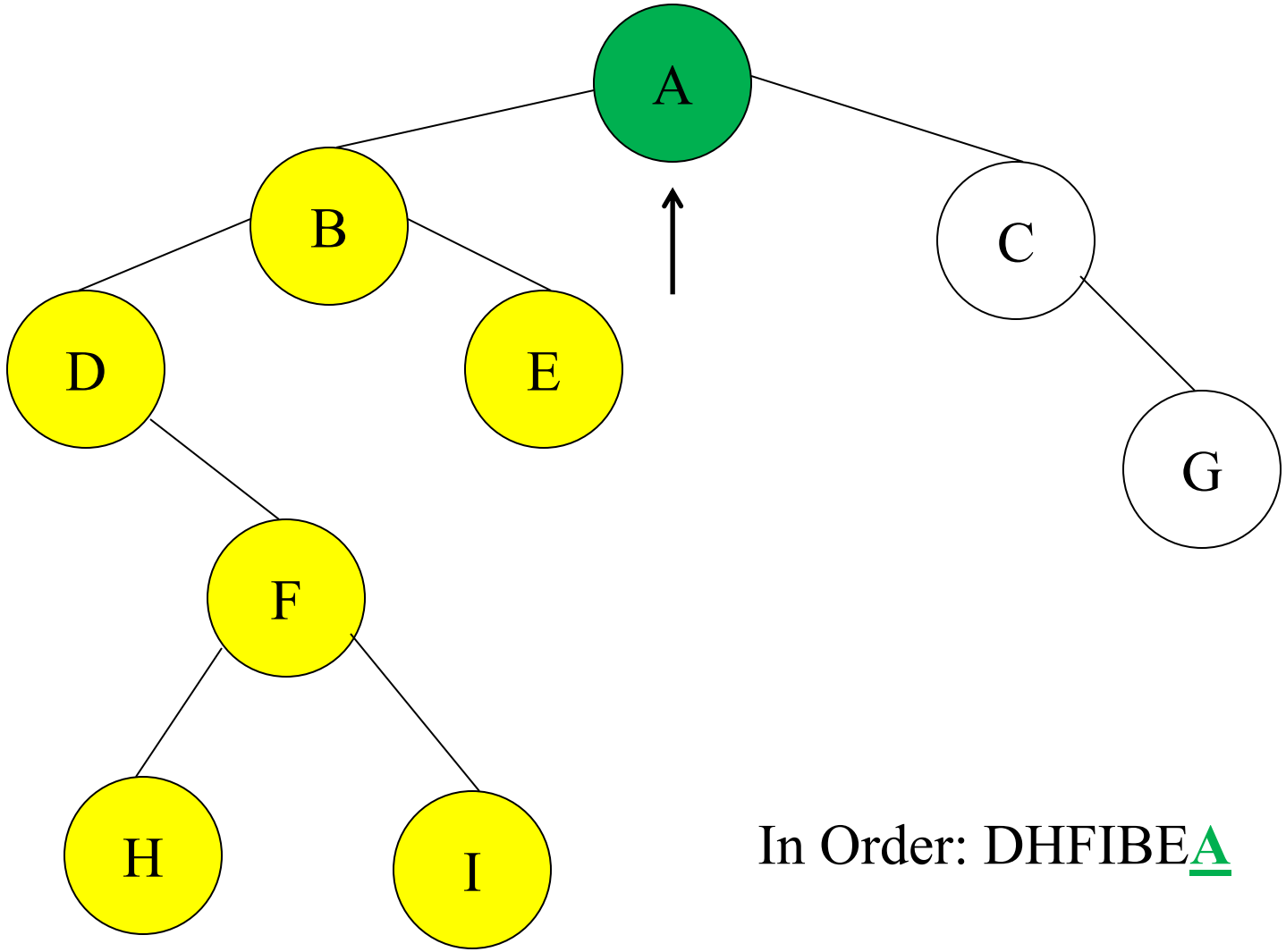
In Order: DHFI**B**

Tree Traversals: Another Example



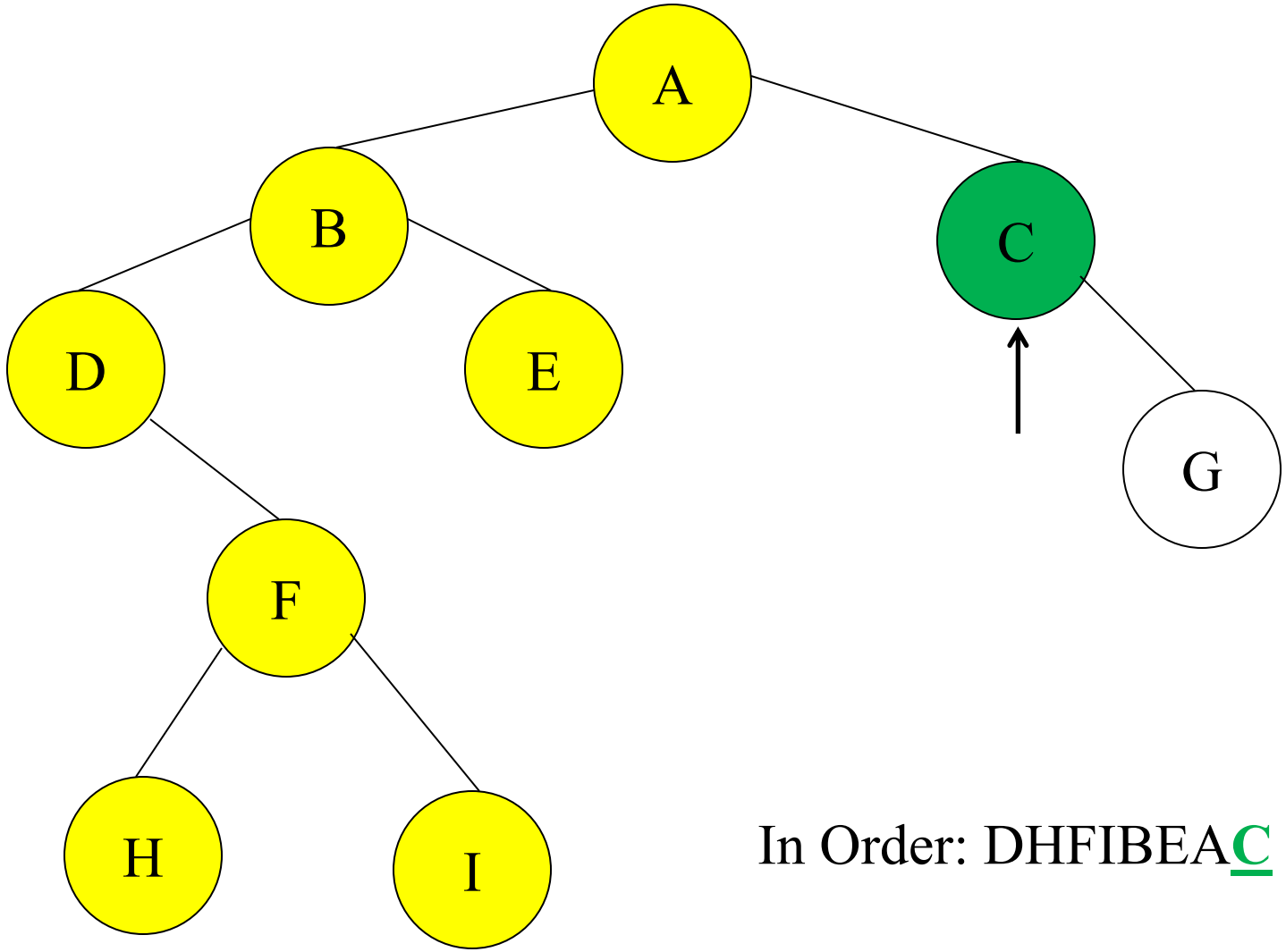
In Order: DHFIB**E**

Tree Traversals: Another Example



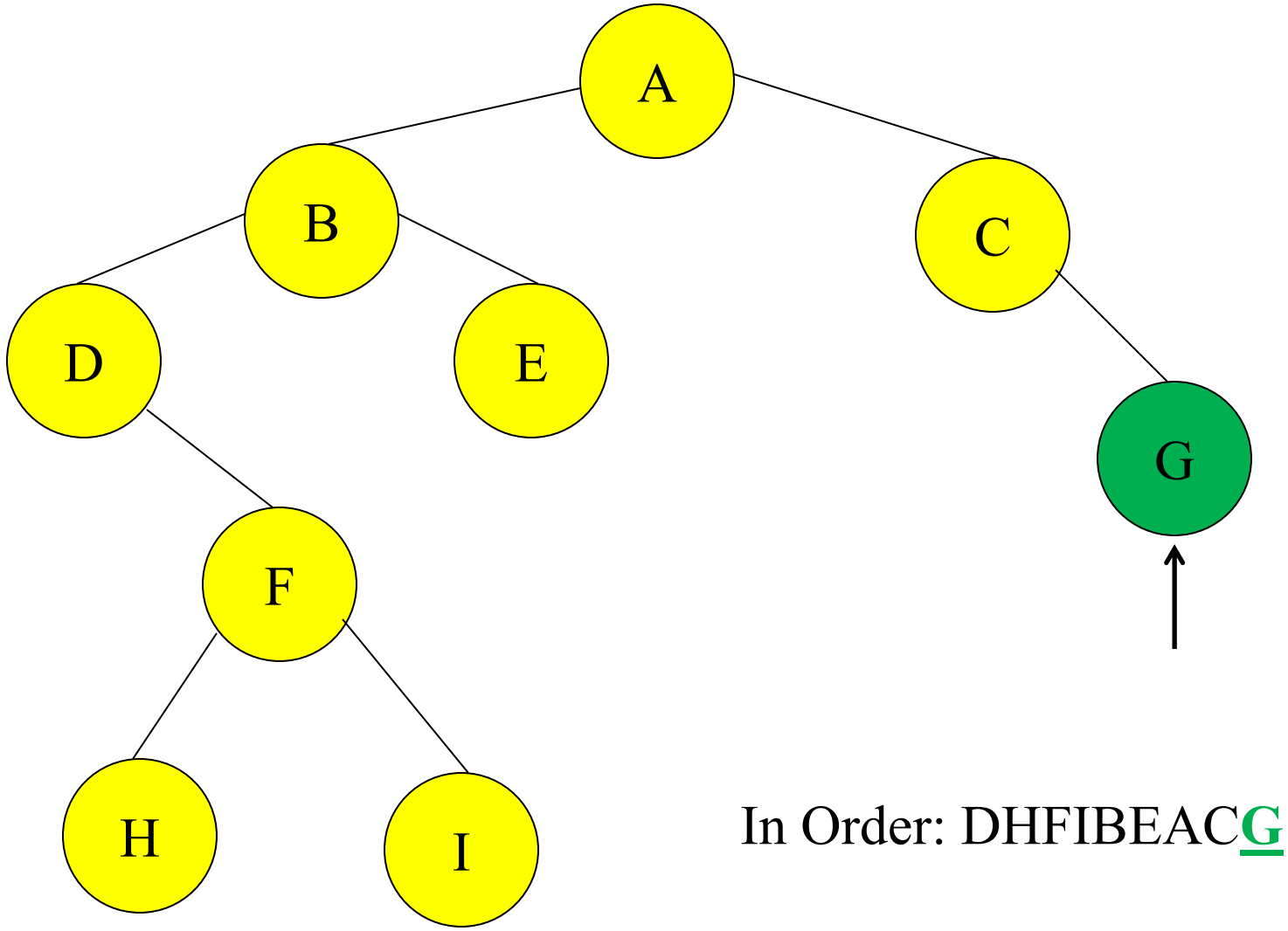
In Order: DHFIBE**A**

Tree Traversals: Another Example



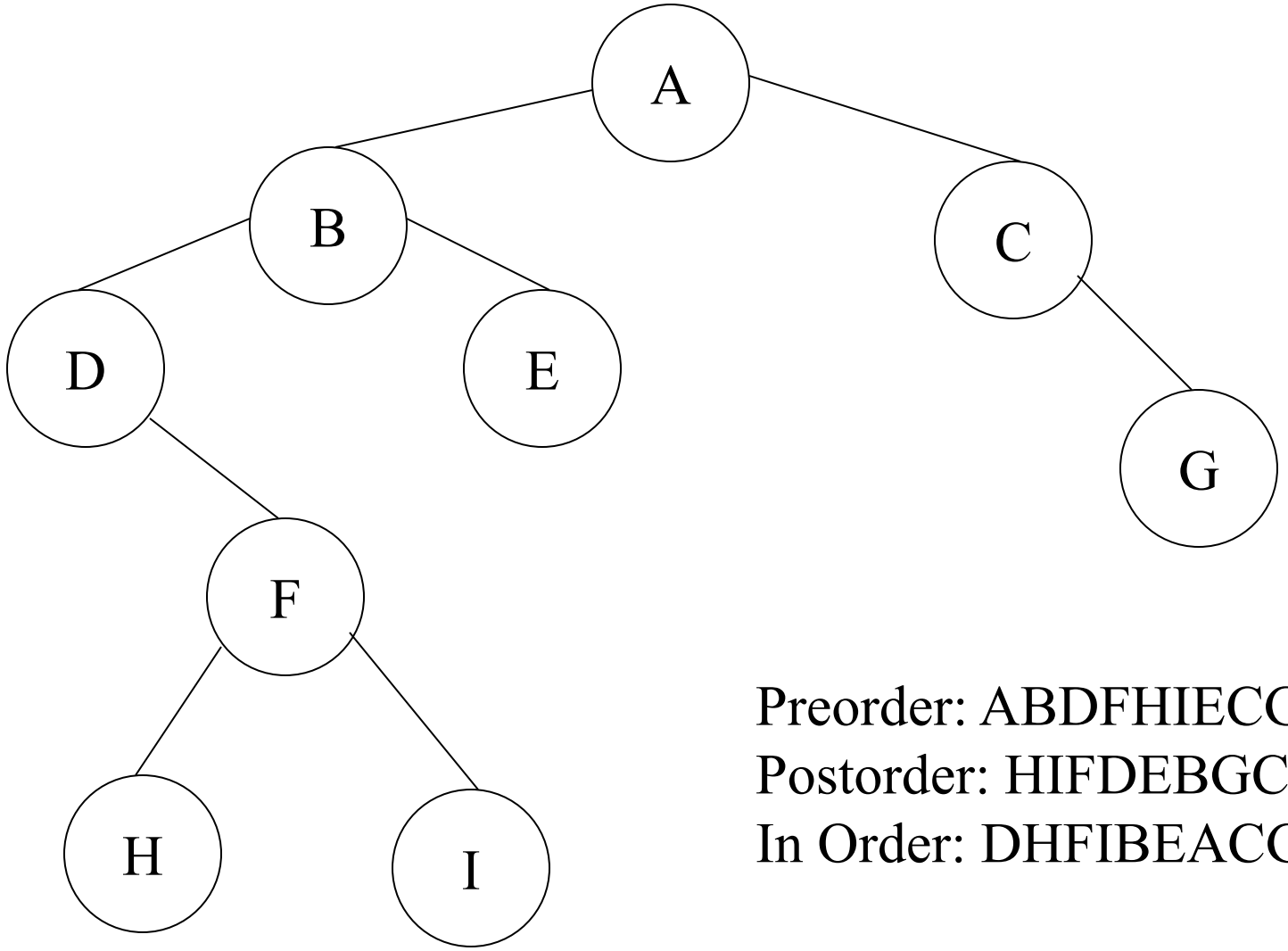
In Order: DHFIBEAC

Tree Traversals: Another Example



In Order: DHFIBEACG

Tree Traversals: Another Example



Preorder: ABDFHIECG
Postorder: HIFDEBGCA
In Order: DHFIBEACG

Binary Tree Implementation

Implementation of a Binary Tree

- Implement a binary tree in Java

```
public class Node {
  private int data;
  private Node left;
  private Node right;

public void setLeft(Node p)  {
  left = p;
}

public void setRight(Node p)  {
  right = p;
}

public int getData()  {
  return data;
}

public Node getLeft()  {
  return left;
}

  public Node getRight()  {
  return right;
}

  public void setData(int x)  {
  data = x;
  }
}
```

```
public class Tree {
    private Node root;

    // tree() - The default constructor - Starts
    //          the tree as empty

    public Tree() {
        root = null;
    }

    // Tree() - An initializing constructor that
    //          creates a node and places in it
    //          the initial value
```

```
public Tree(int x) {
    root = new Node();
    root.setData(x);
    root.setLeft(null);
    root.setRight(null);
}

public Node getRoot() {
    return root;
}

// newNode() -  Creates a new node with a
//              zero as data by default
public Node newNode() {
    Node p = new Node();
    p.setData(0);
    p.setLeft(null);
    p.setRight(null);
    return(p);
}
```

```
// newNode() -  Creates a new node with the
//              parameter x as its value
public Node newNode(int x)  {
    Node p = new Node();
    p.setData(x);
    p.setLeft(null);
    p.setRight(null);
    return(p);
}

//travTree() - initializes recursive
//   traversal of tree
 public void  travTree() {
    if (root != null)
      travSubtree(root);
    System.out.println();
}
//travSubtree() - recursive method used to
 //  traverse a binary tree (inorder)
public void  travSubtree(Node p) {
    if (p != null)  {
        travSubtree(p.getLeft());
        System.out.print(p.getData() + "\t");
        travSubtree(p.getRight());
    }
}
```

```
// addLeft() -  Inserts a new node containing
//              x as the left child of p
public void  addLeft(Node p, int x) {
    Node   q = newNode(x);
    p.setLeft(q);
}

// addRight() -  Inserts a new node containing
//               x as the right child of p
public void  addRight(Node p, int x) {
    Node   q = newNode(x);
    p.setRight(q);
}
}
```

Binary Tree Traversal: Recursion

- **preorder(n)**:
  - Visit the root,
  - traverse the left subtree (preorder(n)) and
  - then traverse the right subtree (preorder(n))
- **postorder(n)**:
  - Traverse the left subtree (postorder(n)),
  - traverse the right subtree (postorder(n)) and
  - then visit the root.
- **inorder**:
  - Traverse the left subtree (inorder(n)),
  - visit the root and
  - the traverse the right subtree (inorder(n)).

Preorder traversal

- In preorder, the root is visited *first*
- Here's a preorder traversal to print out all the elements in the binary tree:

```
public void preorderPrint(BinaryTree bt) {
    if (bt == null) return;
    System.out.println(bt.value);
    preorderPrint(bt.leftChild);
    preorderPrint(bt.rightChild);
}
```

Inorder traversal

- In inorder, the root is visited *in the middle*
- Here's an inorder traversal to print out all the elements in the binary tree:

```
public void inorderPrint(BinaryTree bt) {
    if (bt == null) return;
    inorderPrint(bt.leftChild);
    System.out.println(bt.value);
    inorderPrint(bt.rightChild);
}
```

Postorder traversal

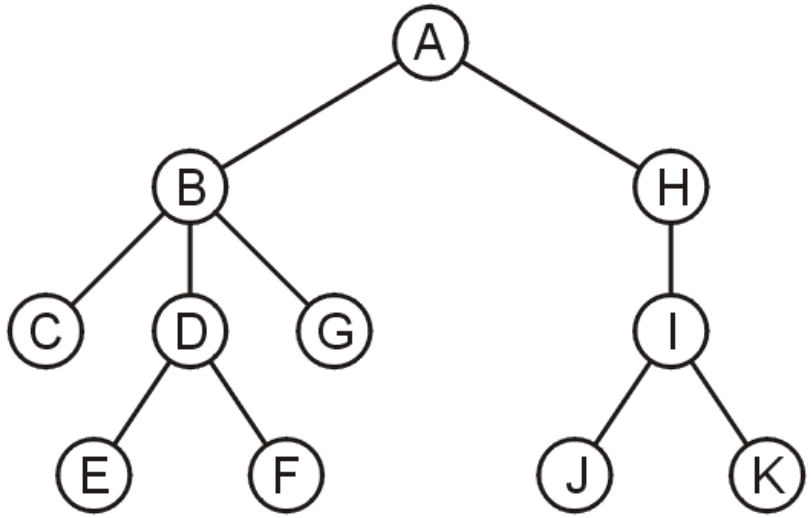
- In postorder, the root is visited *last*
- Here's a postorder traversal to print out all the elements in the binary tree:

```
public void postorderPrint(BinaryTree bt) {
    if (bt == null) return;
    postorderPrint(bt.leftChild);
    postorderPrint(bt.rightChild);
    System.out.println(bt.value);
}
```

Breadth First Search using Queue

Application

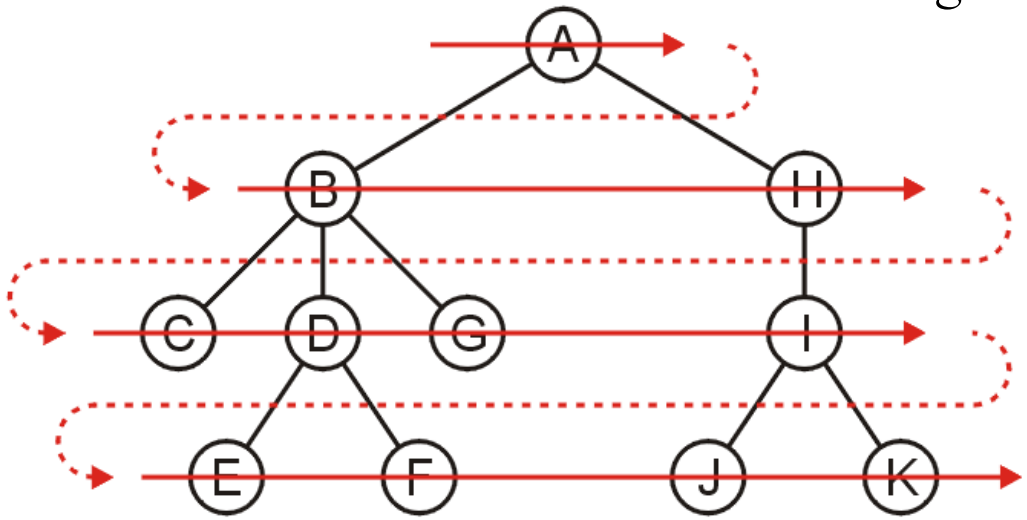
- Another application is performing a breadth-first traversal of a directory tree
  - Consider searching the directory structure



Application

- We would rather search the more shallow directories first then plunge deep into searching one sub-directory and all of its contents

- One such search is called a *breadth-first traversal*
  - Search all the directories at one level before descending a level



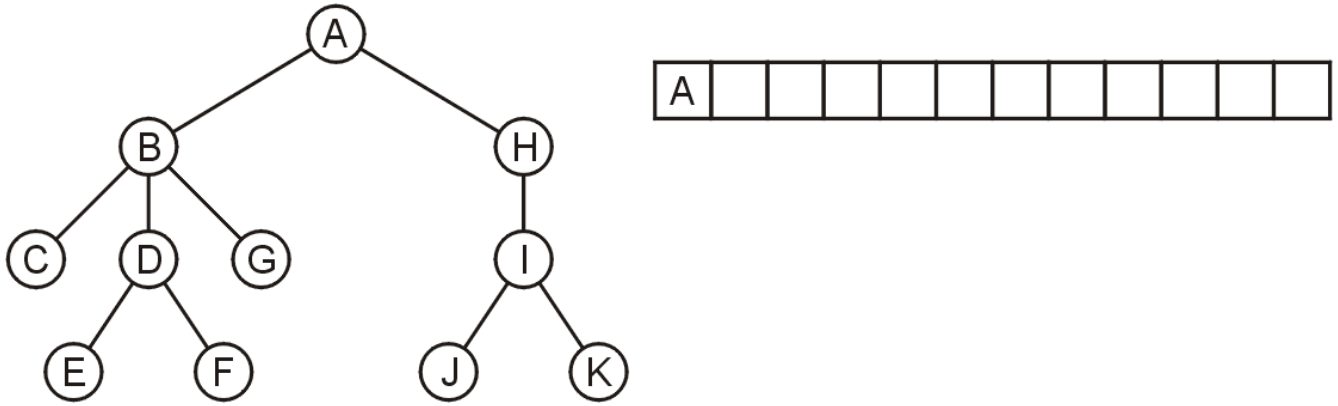
Application

- The easiest implementation is:
  - Place the root directory into a queue
  - While the queue is not empty:
    - Pop the directory at the front of the queue
    - Push all of its sub-directories into the queue

- The order in which the directories come out of the queue will be in breadth-first order

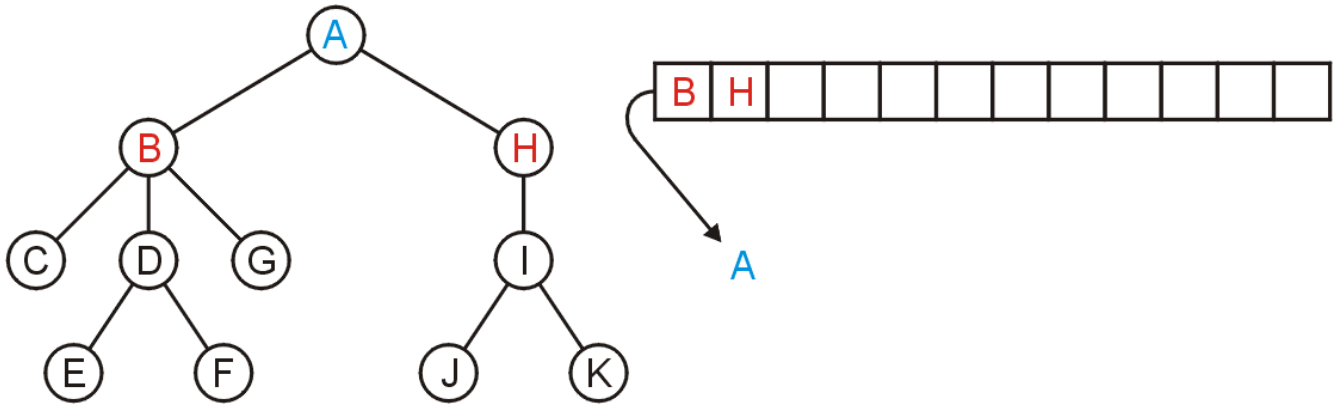
Application

- Push the root directory A



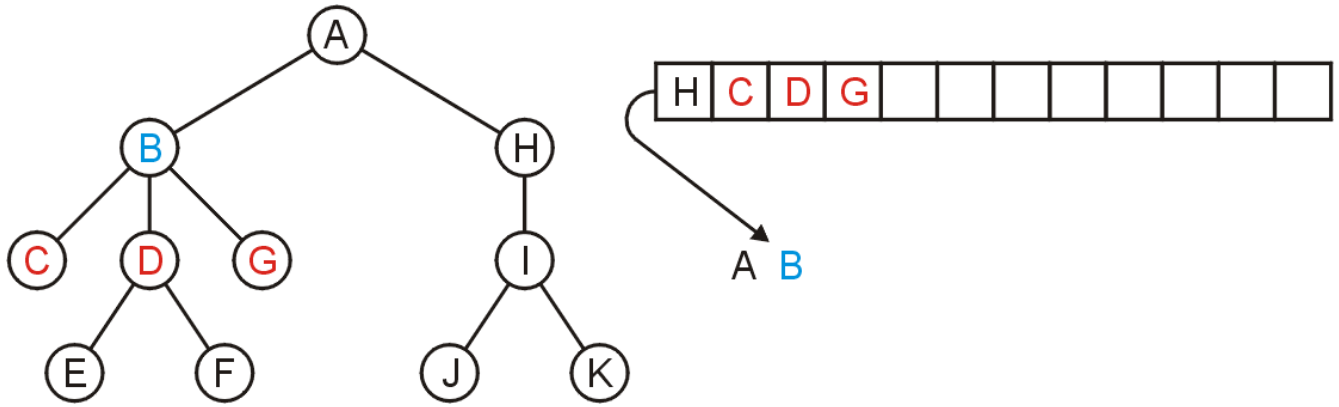
Application

- Pop A and push its two sub-directories: B and H



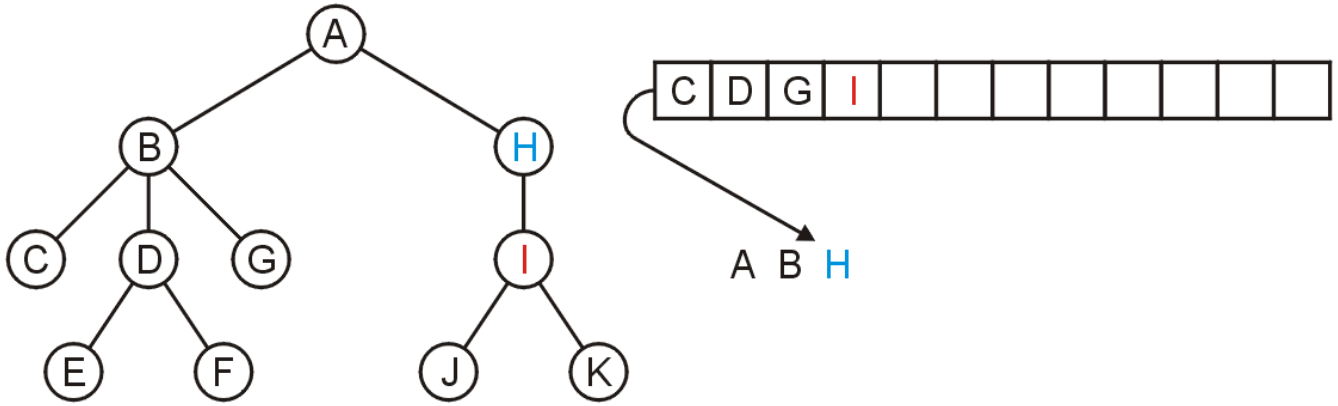
Application

- Pop B and push C, D, and G



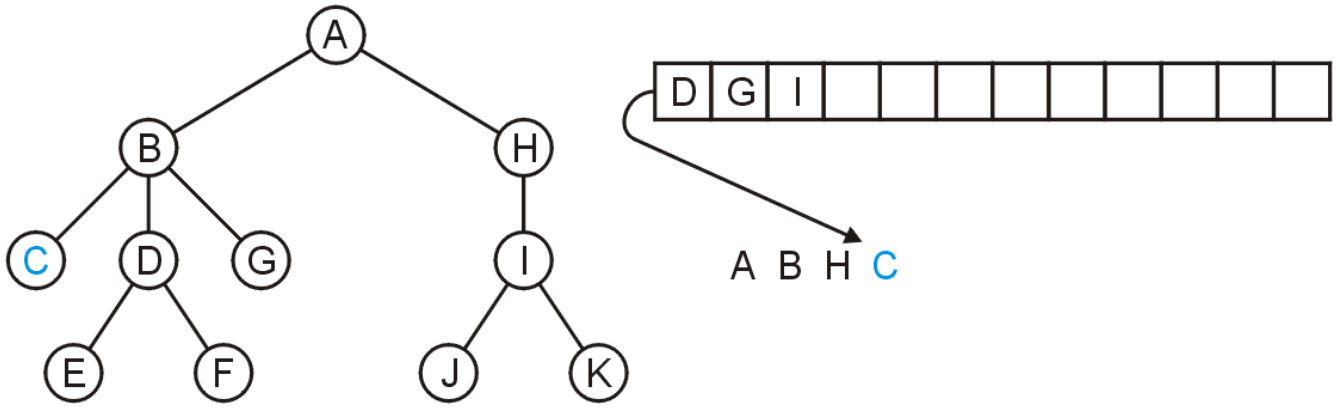
Application

- Pop H and push its one sub-directory I



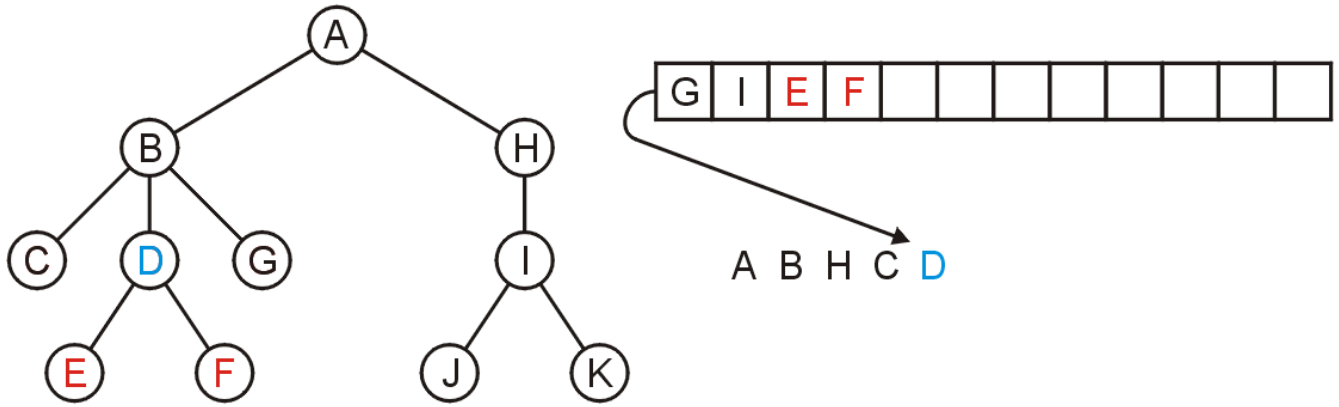
Application

- Pop C: no sub-directories



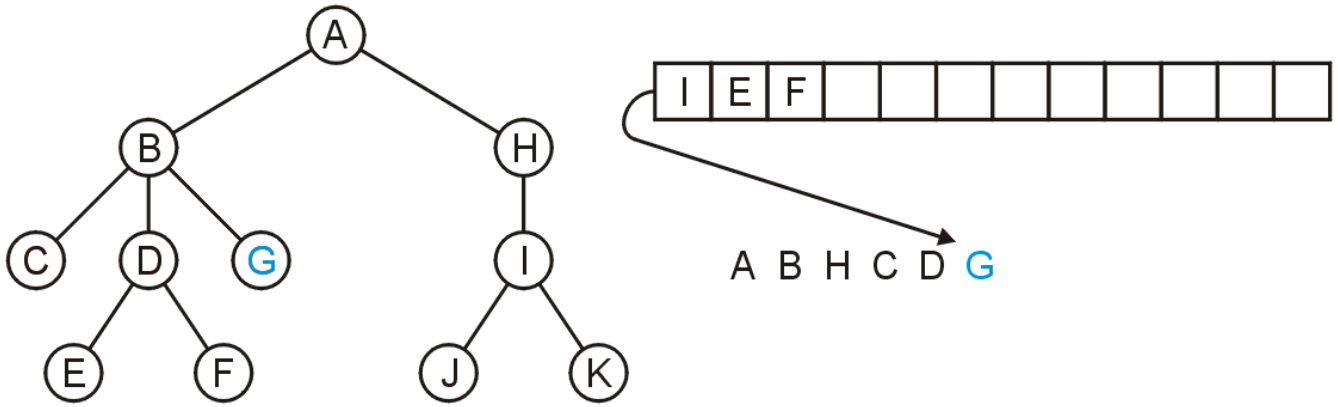
Application

- Pop D and push E and F



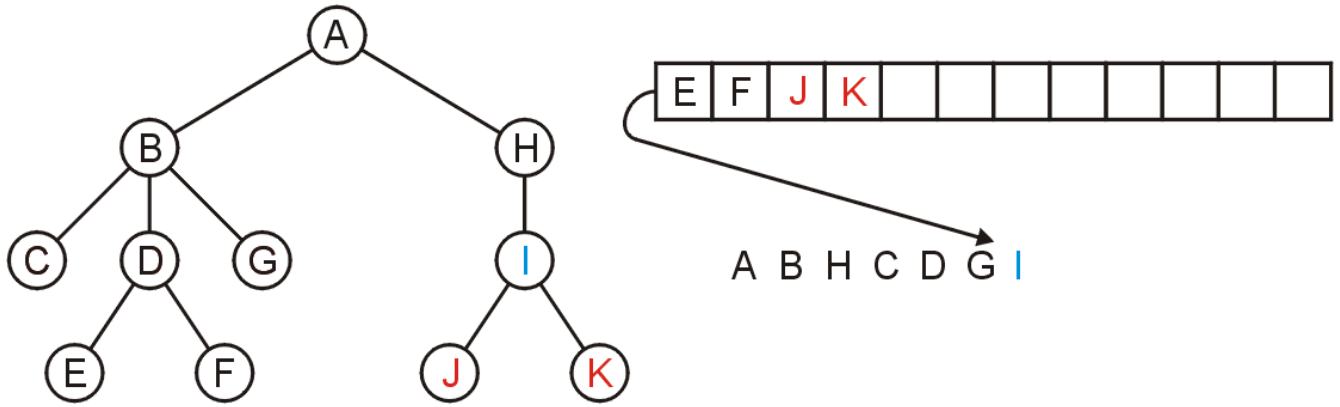
Application

- Pop G



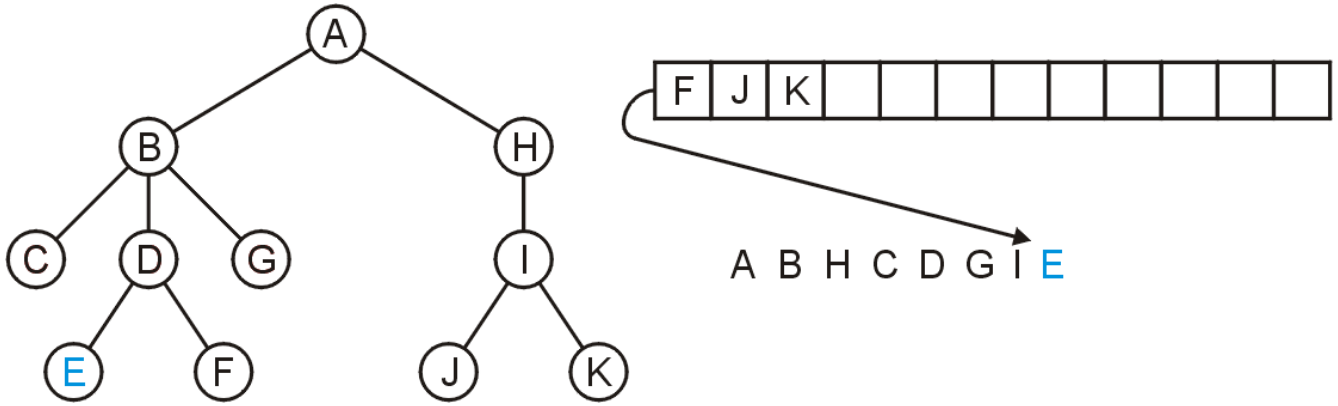
Application

- Pop I and push J and K



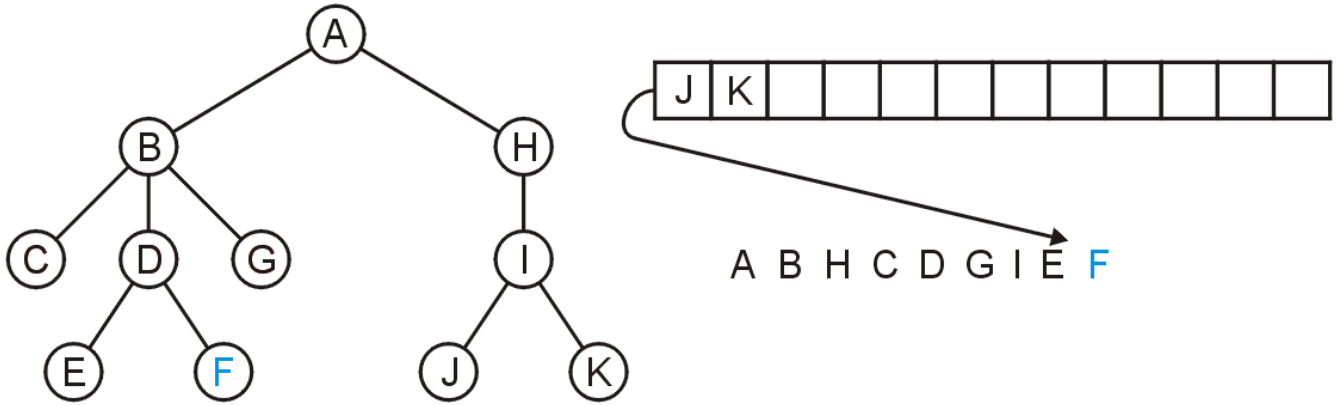
Application

- Pop E



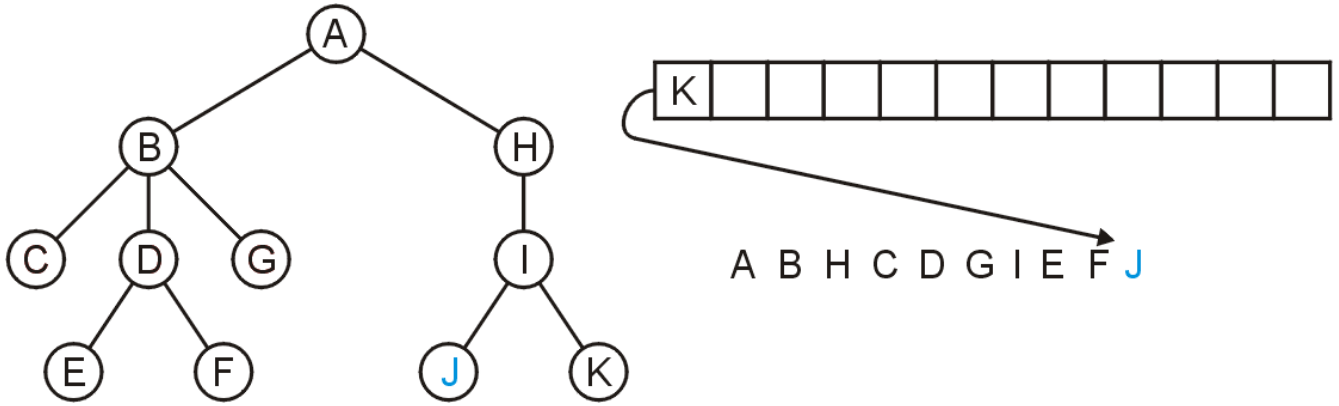
Application

- Pop F



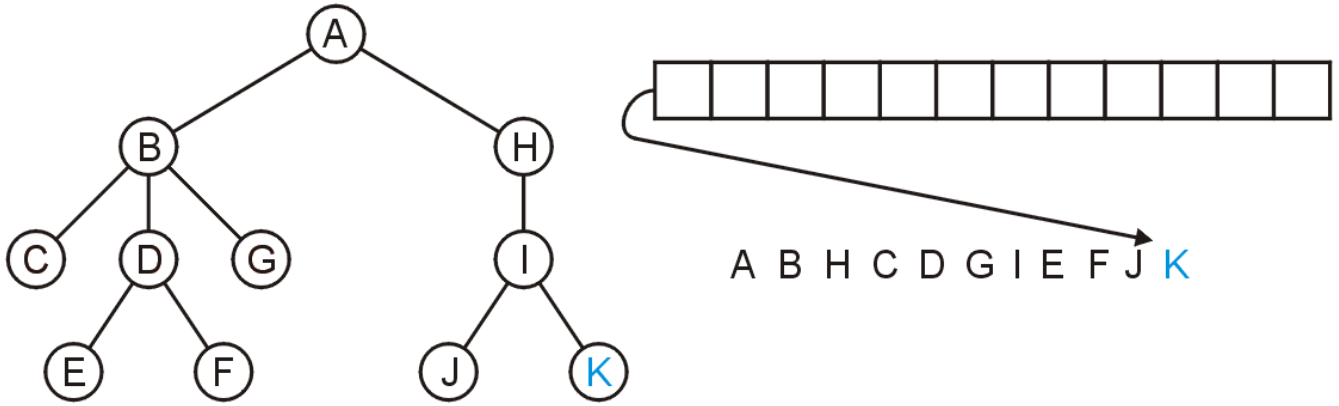
Application

- Pop J



Application

- Pop K and the queue is empty

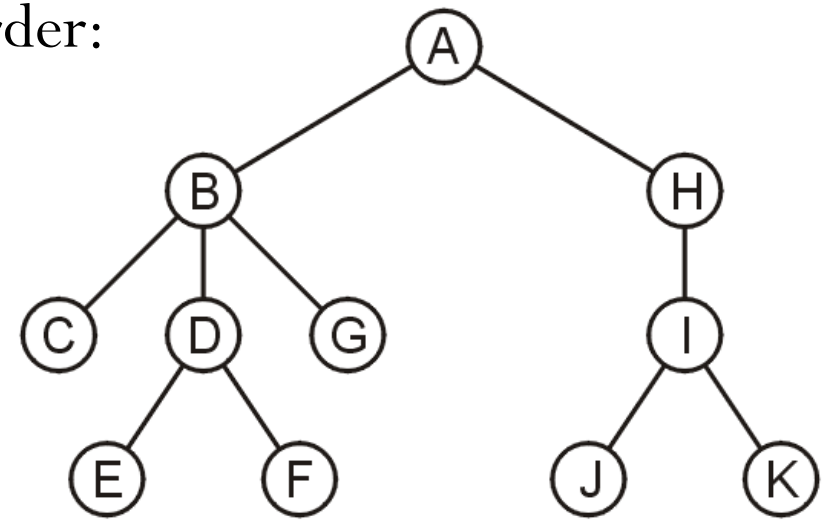


Application

- The resulting order

  A B H C D G I E F J K

- is in breadth-first order:

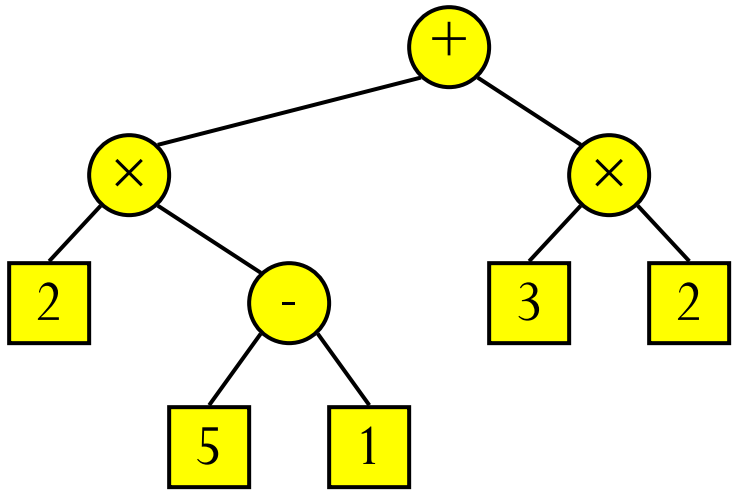


Arithmetic Expression Tree

Arithmetic Expression Tree

- Binary tree for an arithmetic expression
  - internal nodes: operators
  - leaves: operands
- Example: arithmetic expression tree for the expression

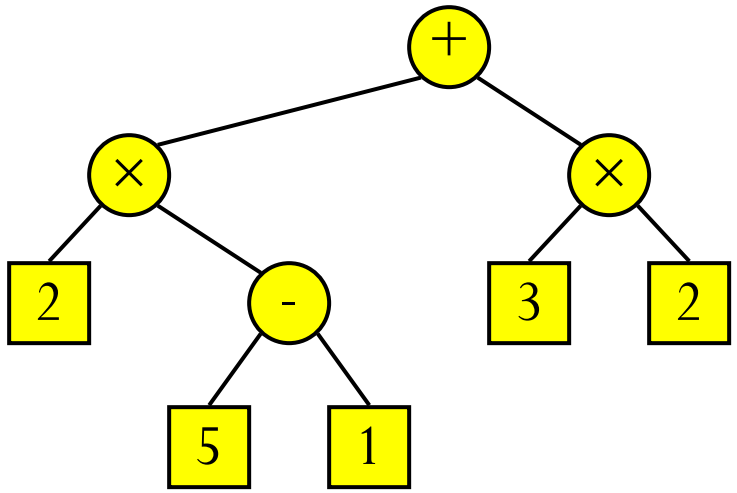
$$((2 \times (5 - 1)) + (3 \times 2))$$



Print Arithmetic Expressions

- inorder traversal:
  - print "(" before traversing left subtree
  - print operand or operator when visiting node
  - print ")" after traversing right subtree

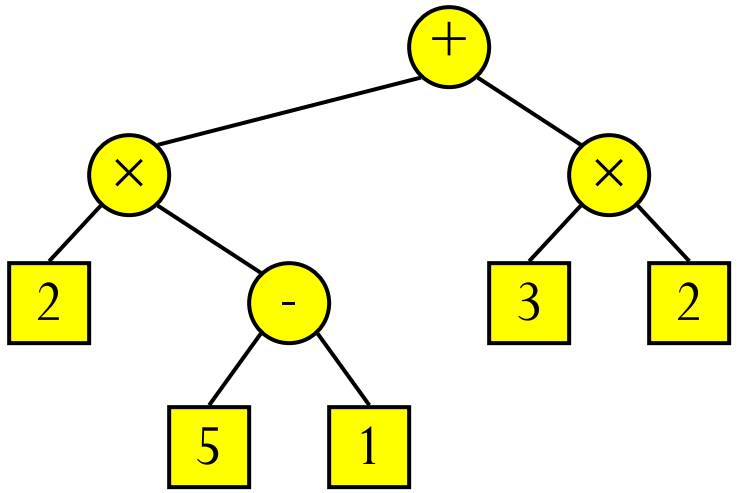
```
void printTree(t)
//binary operands only
   if (t.left != null)
      print("(");
      printTree (t.left);
   print(t.element );
   if (t.right != null)
      printTree (t.right);
      print (")");
```



((2 × (5 - 1)) + (3 × 2))

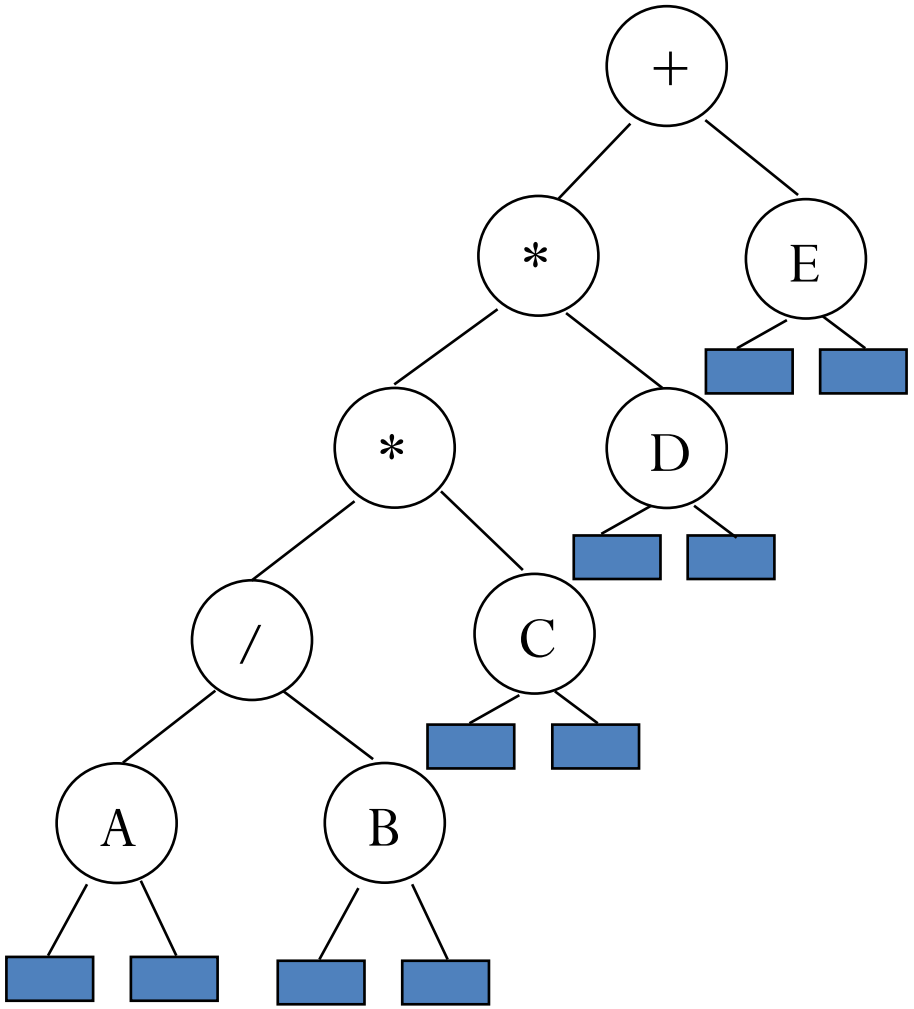
Evaluate Arithmetic Expressions

- postorder traversal
  - Recursively evaluate subtrees
  - Apply the operator after subtrees are evaluated




```
int evaluate (t)
//binary operators only
    if (t.left == null)
      //external node
      return t.element;
    else  //internal node
      x = evaluate (t.left);
      y = evaluate (t.right);
      let o be the operator
        t.element
      z = apply o to x and y
      return z;
```

Arithmetic Expression Using BT



inorder traversal

A / B * C * D + E

infix expression

preorder traversal

+ * * / A B C D E

prefix expression

postorder traversal

A B / C * D * E +

postfix expression

level order traversal

+ * E * D / C A B

Inorder Traversal (recursive version)

```
void inorder(tree_pointer ptr)
/* inorder tree traversal */
{
     if (ptr) {
          inorder(ptr->left_child);
          printf("%d", ptr->data);
          indorder(ptr->right_child);
     }
}
```

A / B * C * D + E

Preorder Traversal (recursive version)

```
void preorder(tree_pointer ptr)
/* preorder tree traversal */
{
     if (ptr) {
          printf("%d", ptr->data);
          preorder(ptr->left_child);
          predorder(ptr->right_child);
     }
}
```

Postorder Traversal (recursive version)

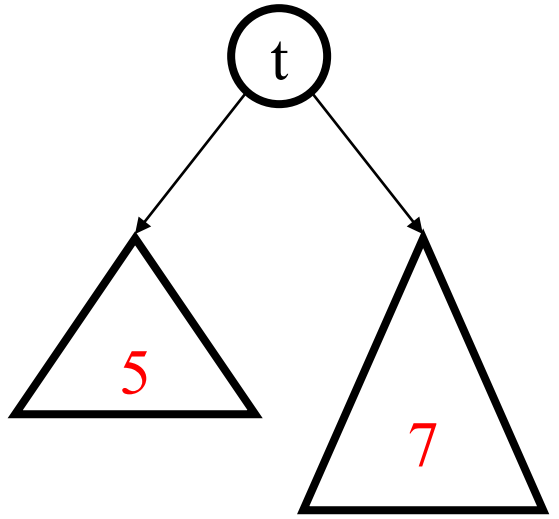
```
void postorder(tree_pointer ptr)
/* postorder tree traversal */
{
    if (ptr) {
        postorder(ptr->left_child);
        postdorder(ptr->right_child);
        printf("%d", ptr->data);
    }
}
```

A B / C * D * E +

Height Balanced Trees

Height Balanced Trees

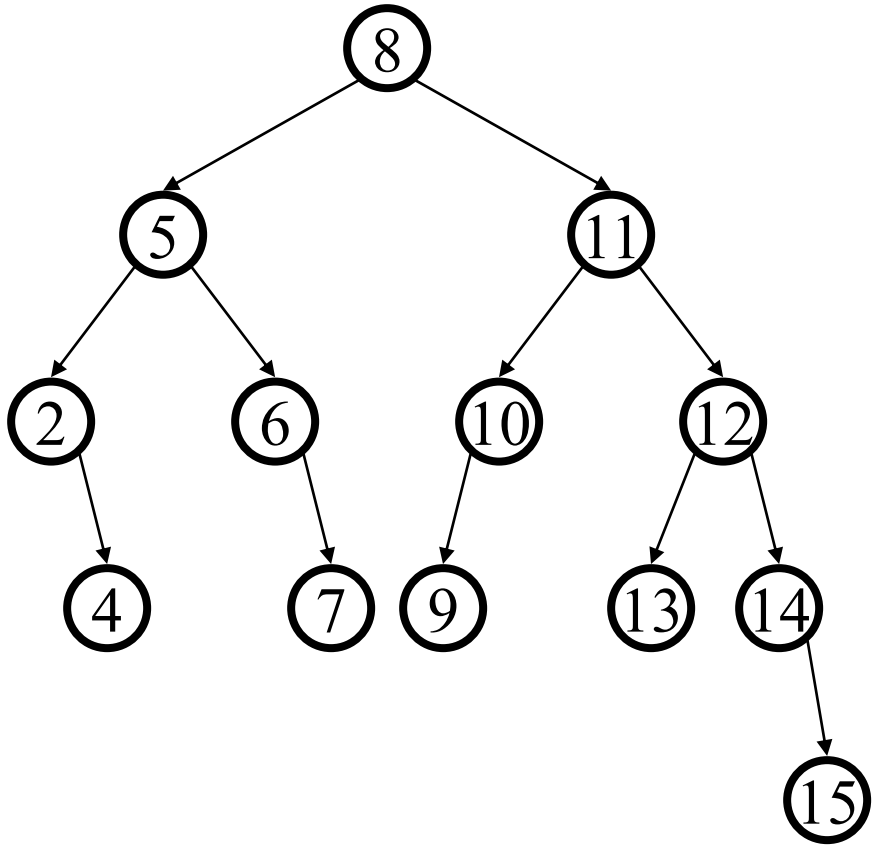
- also known as Amazingly Vexing Letters (AVL) Trees
- Balance == height(left subtree) - height(right subtree)
  - zero everywhere ⇒ perfectly balanced
  - small everywhere ⇒ balanced enough
- Balance between -1 and 1 everywhere
- maximum height of ~1.44 log n



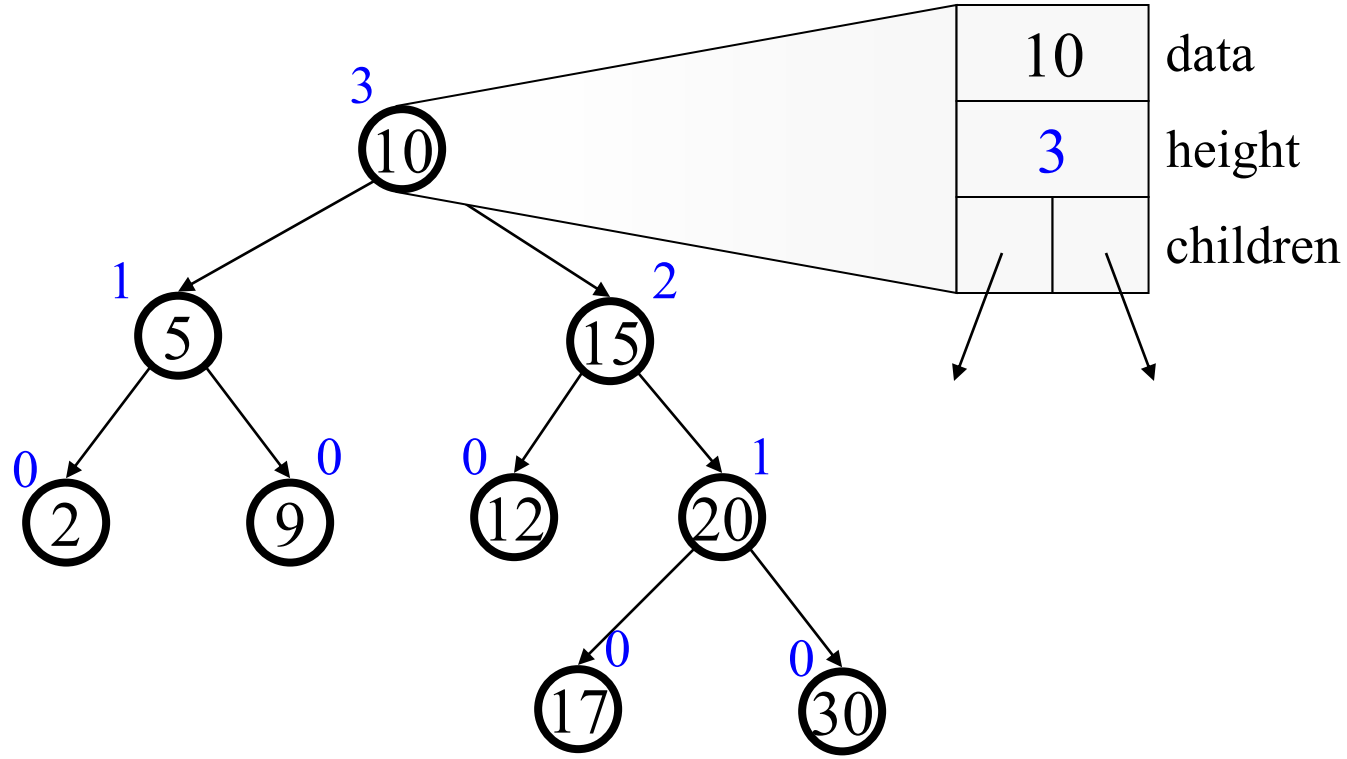
AVL Tree: Dictionary Data Structure

- Binary search tree properties
  - binary tree property
  - search tree property
- Balance property
  - balance of every node is:

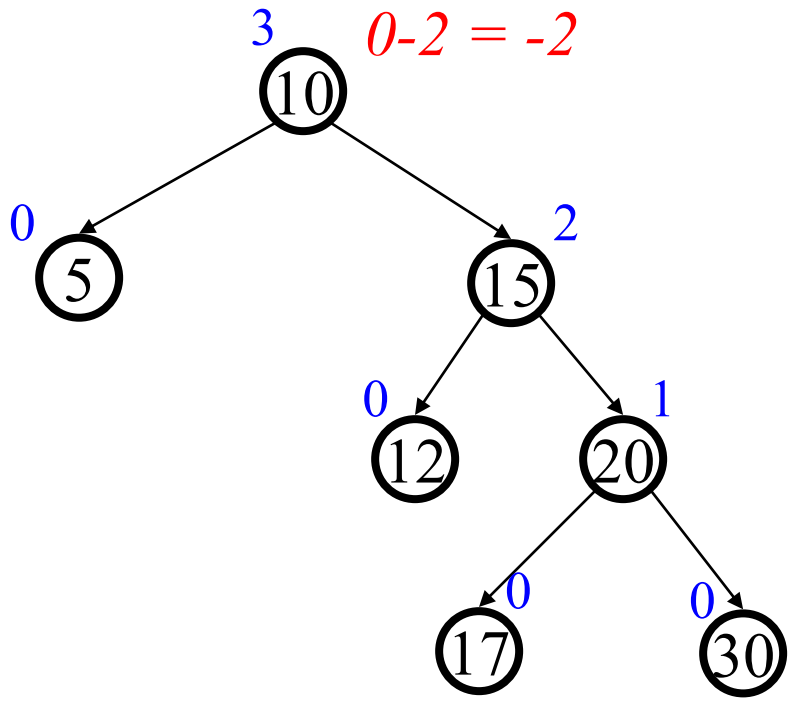
    $-1 \leq b \leq 1$
  - result:
    - depth is $\Theta(\log n)$

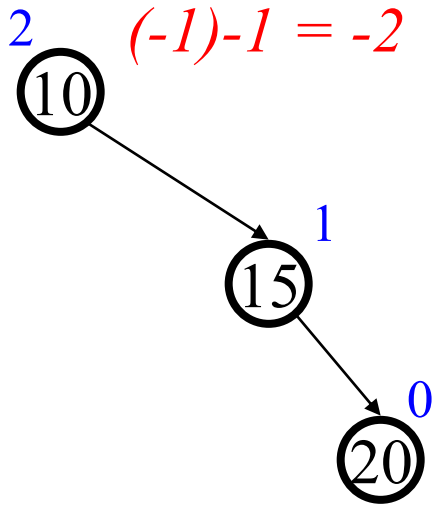


An AVL Tree



Not AVL Trees





Note: height(empty tree) == -1

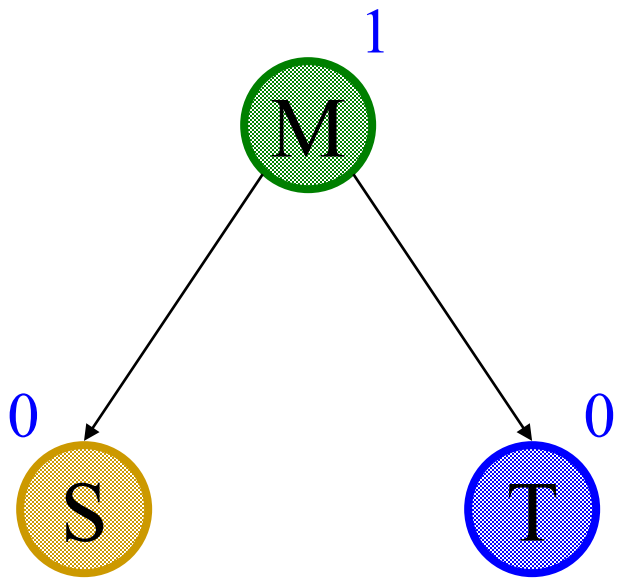
Good Insert Case: Balance Preserved

Good case: insert middle, then small,then tall

Insert(middle)

Insert(small)

Insert(tall)

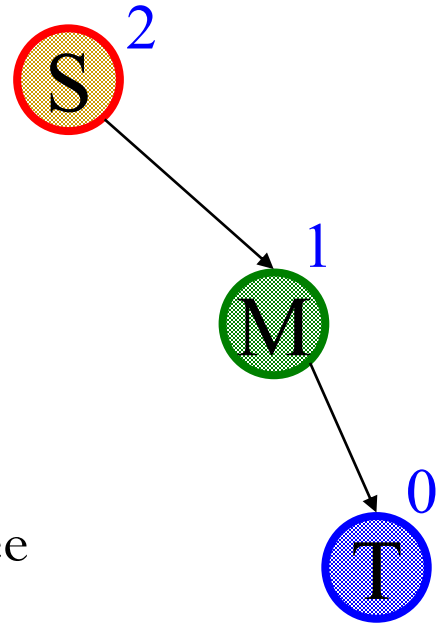


Bad Insert Case #1: Left-Left or Right-Right Imbalance

Insert(small)

Insert(middle)

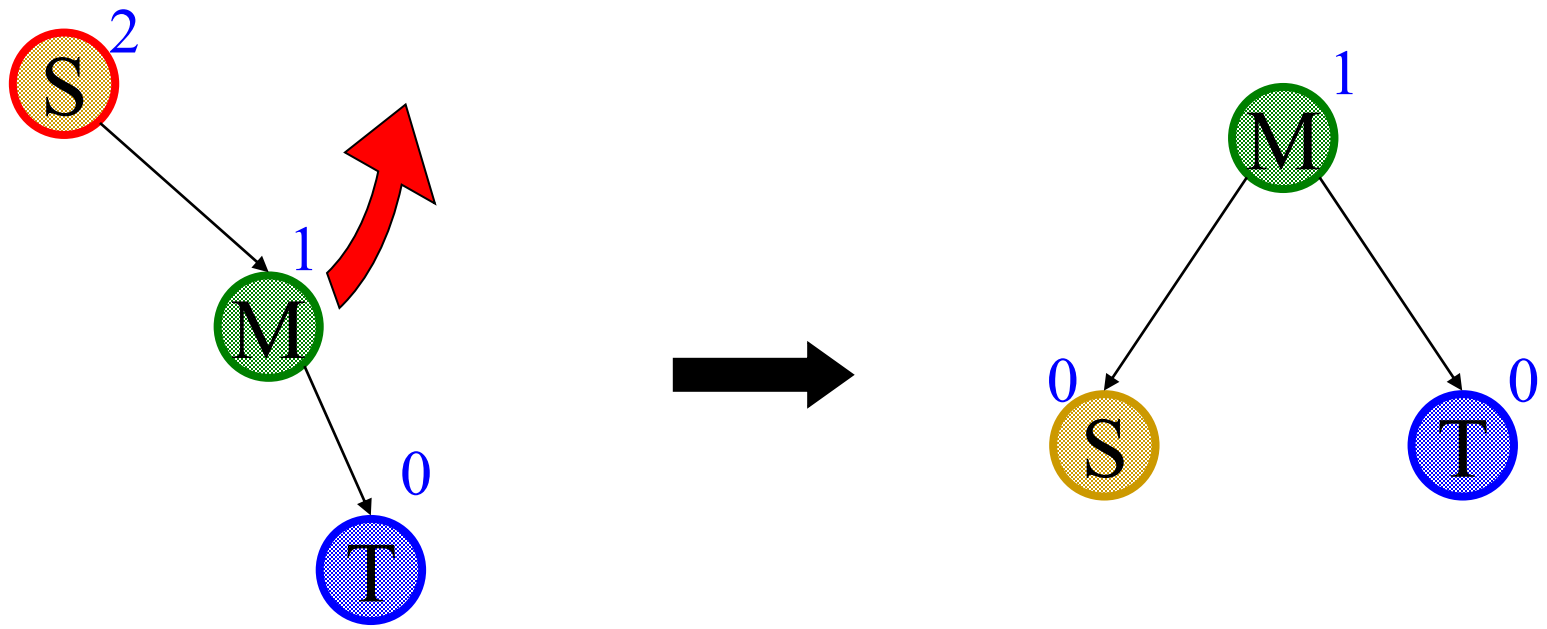
Insert(tall)



BC#1 Imbalance caused by either:

1. Insert into **left** child's **left** subtree
2. Insert into **right** child's **right** subtree

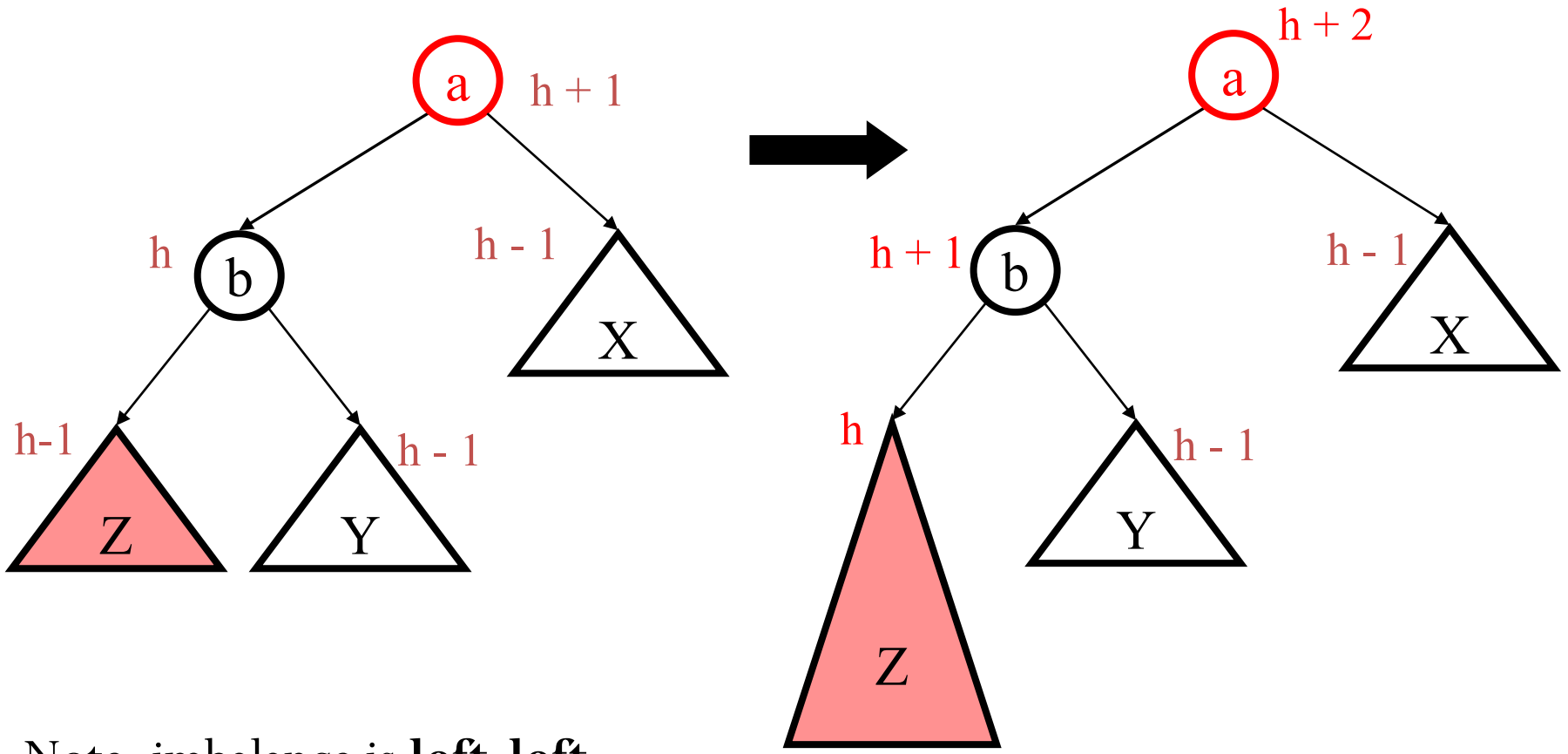
Single Rotation



Basic operation used in AVL trees:

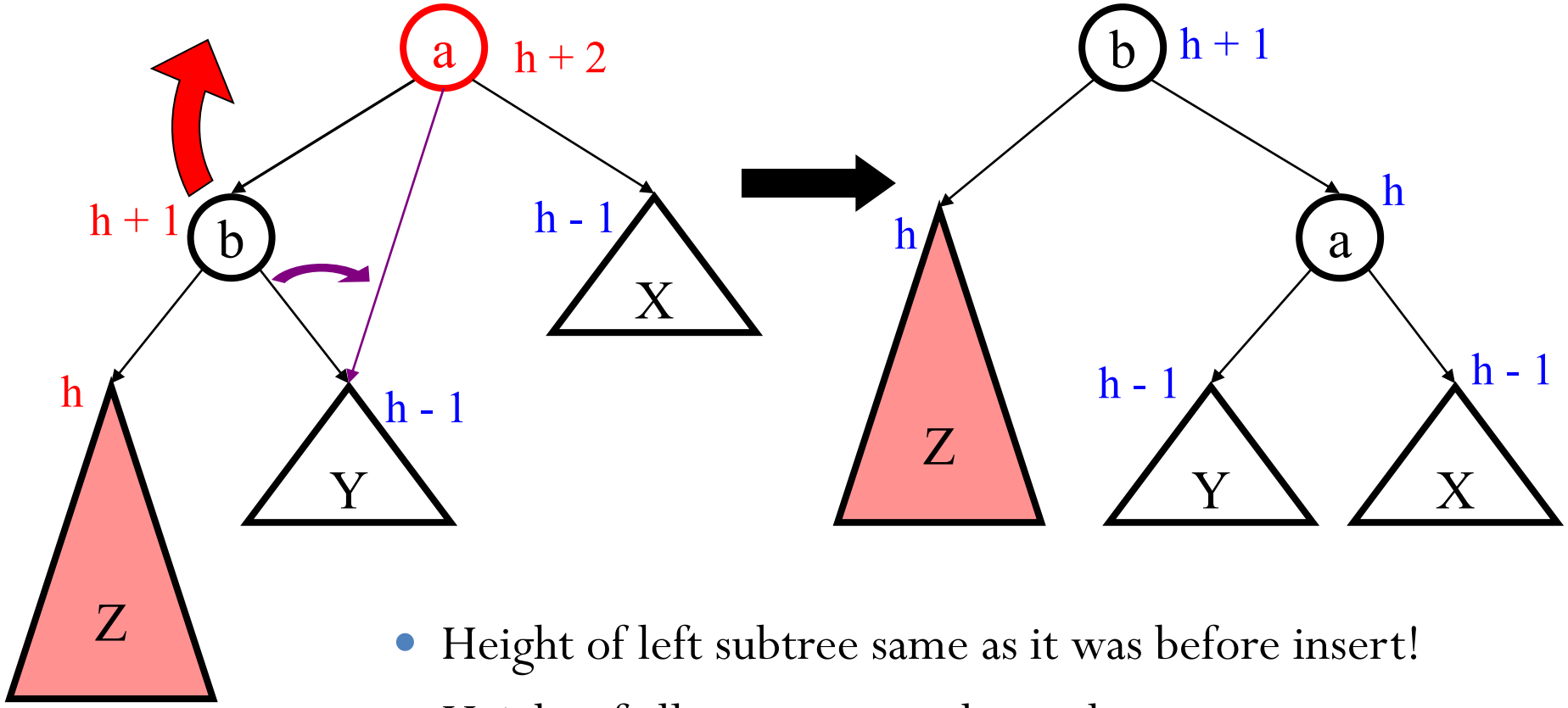
A right child could legally have its parent as its left child.

General Bad Case #1



Note: imbalance is **left-left**

Single Rotation Fixes Case #1 Imbalance



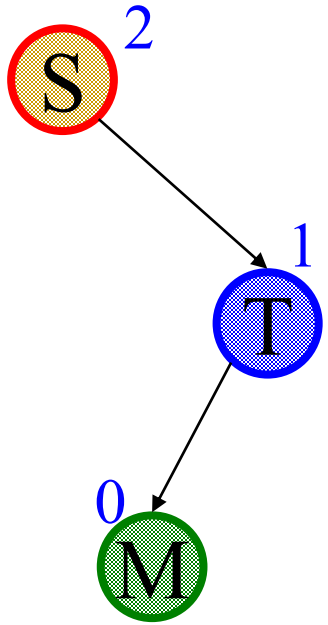
- Height of left subtree same as it was before insert!
- Height of all ancestors unchanged
  - *We can stop here!*

Bad Insert Case #2: Left-Right or Right-Left Imbalance

Insert(small)

Insert(tall)

Insert(middle)

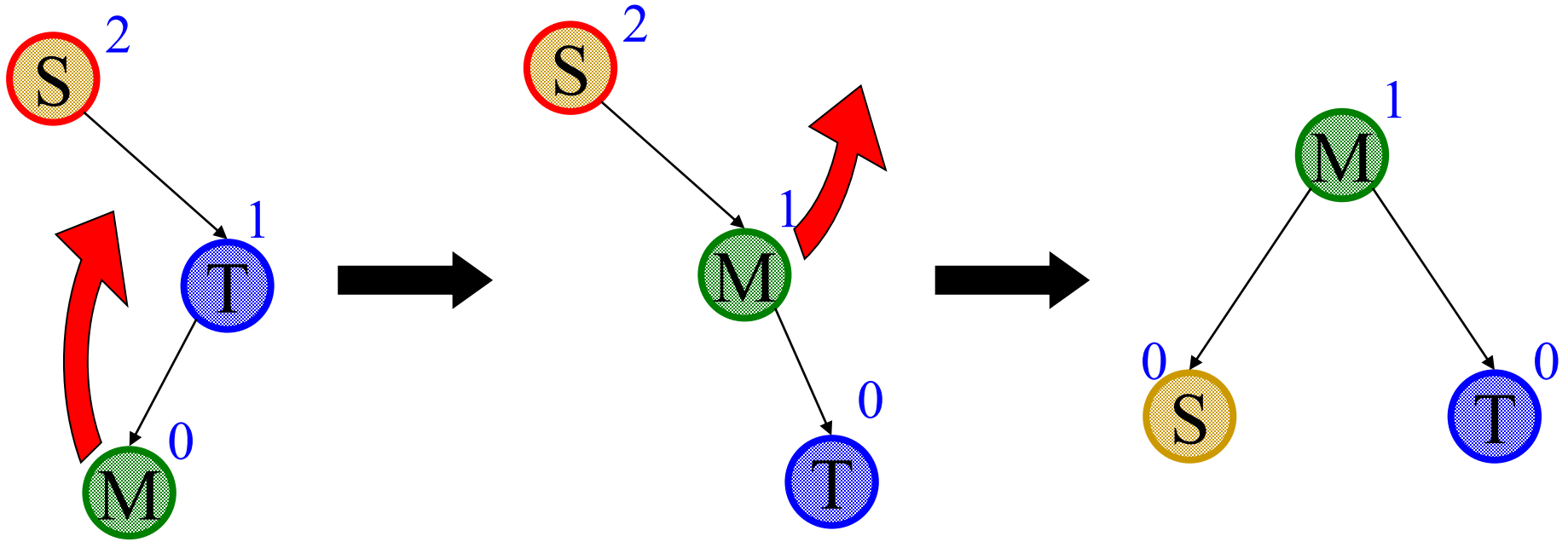


BC#2 Imbalance caused by either:

1. Insert into **left** child's **right** subtree
2. Insert into **right** child's **left** subtree

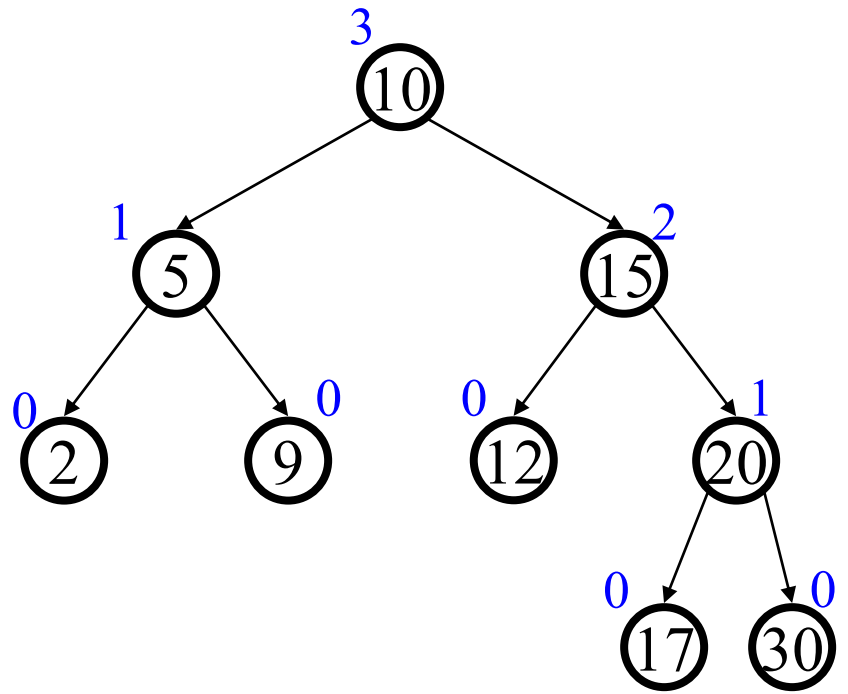
Will a single rotation fix this?

Double Rotation



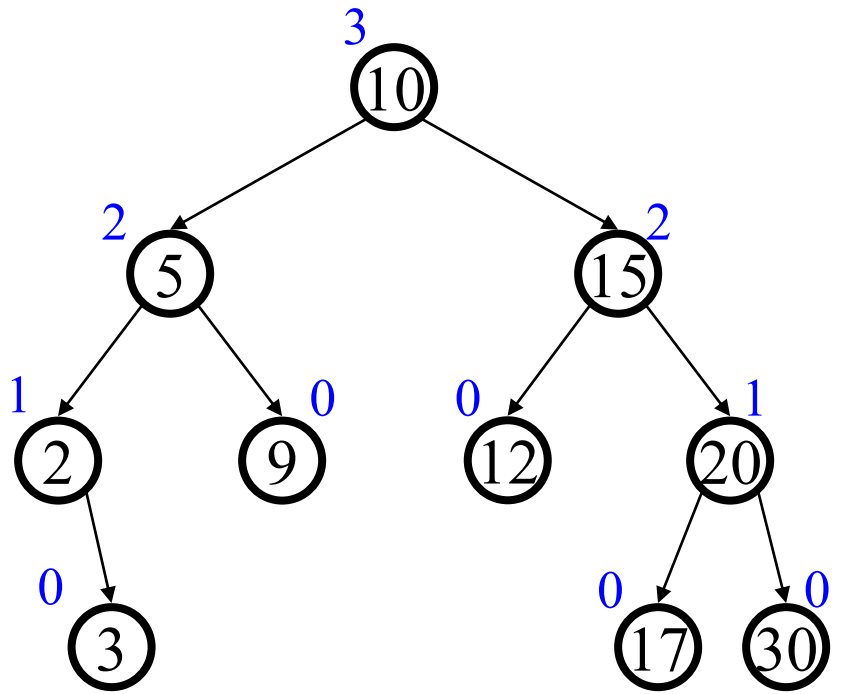
Easy Insert

Insert(3)

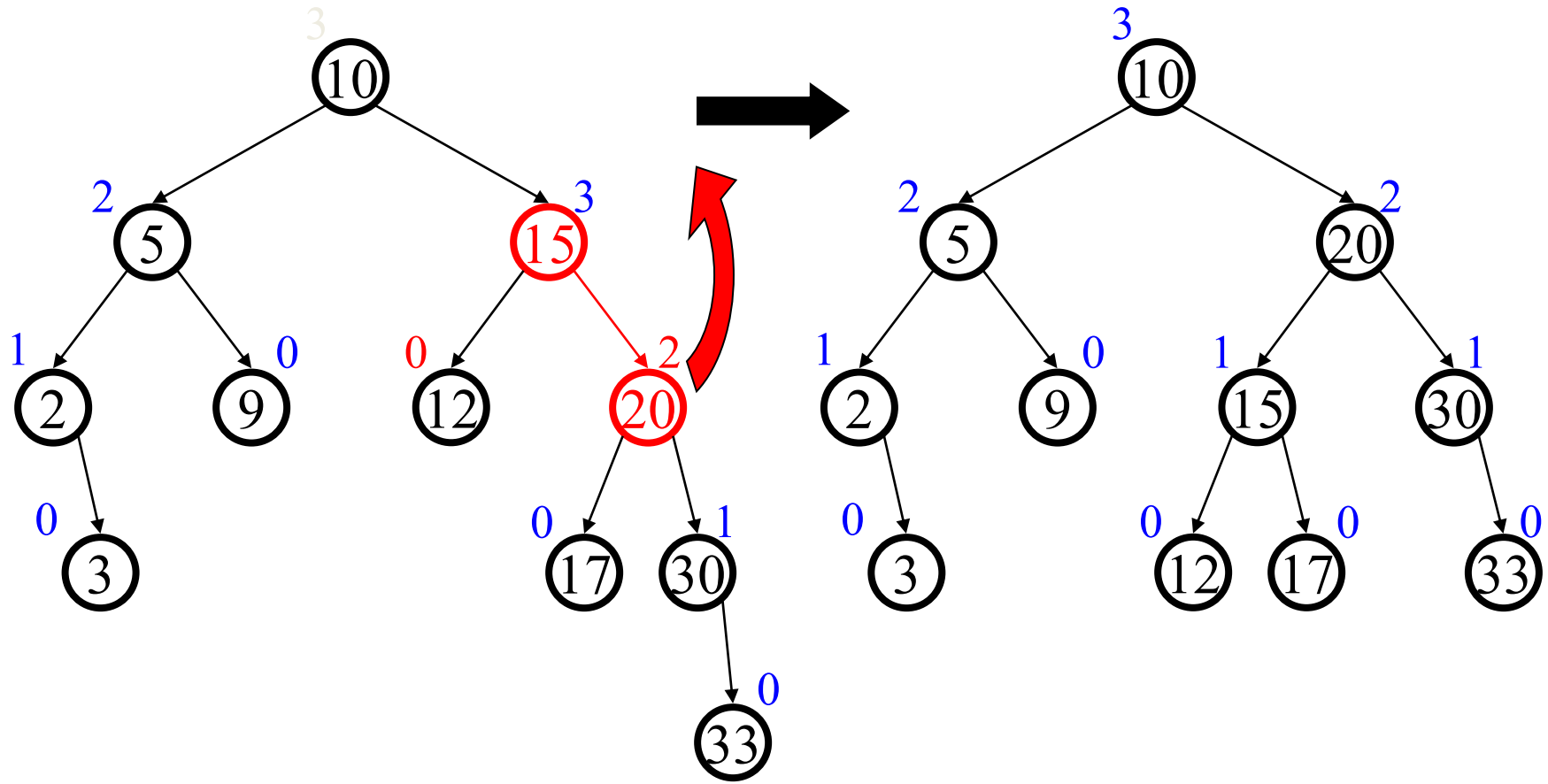


Hard Insert (Bad Case #1)

Insert(33)

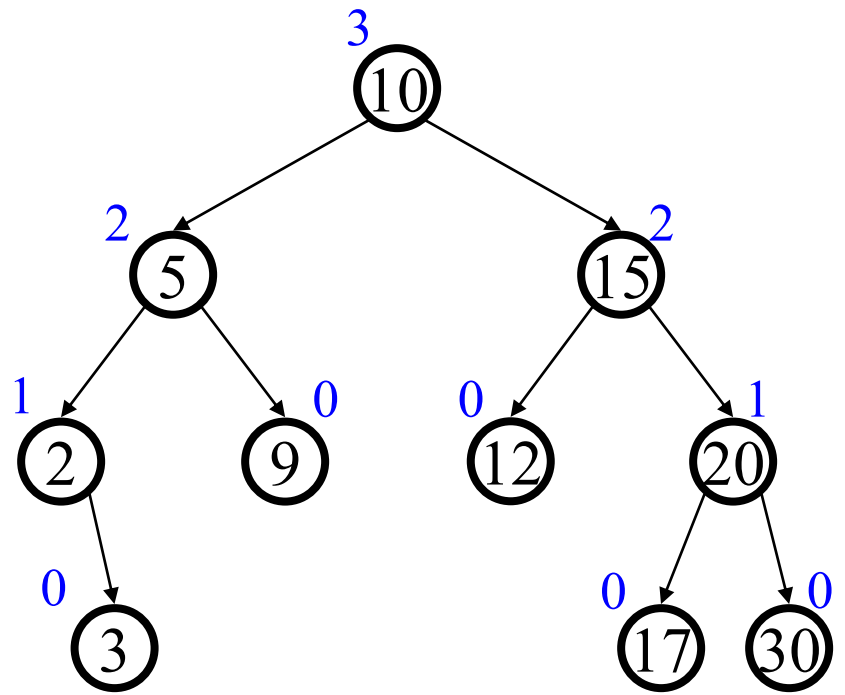


Single Rotation

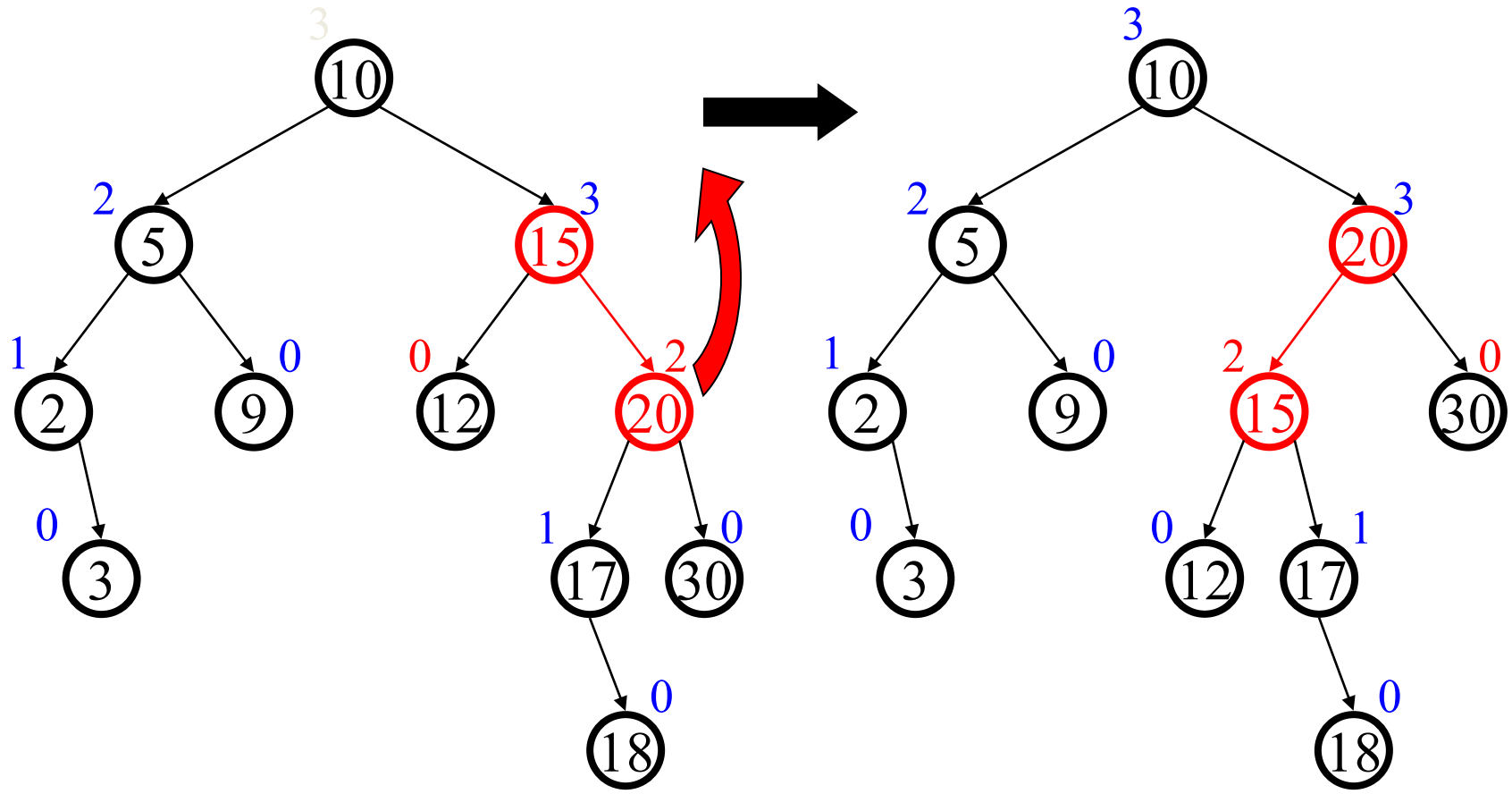


Hard Insert (Bad Case #2)

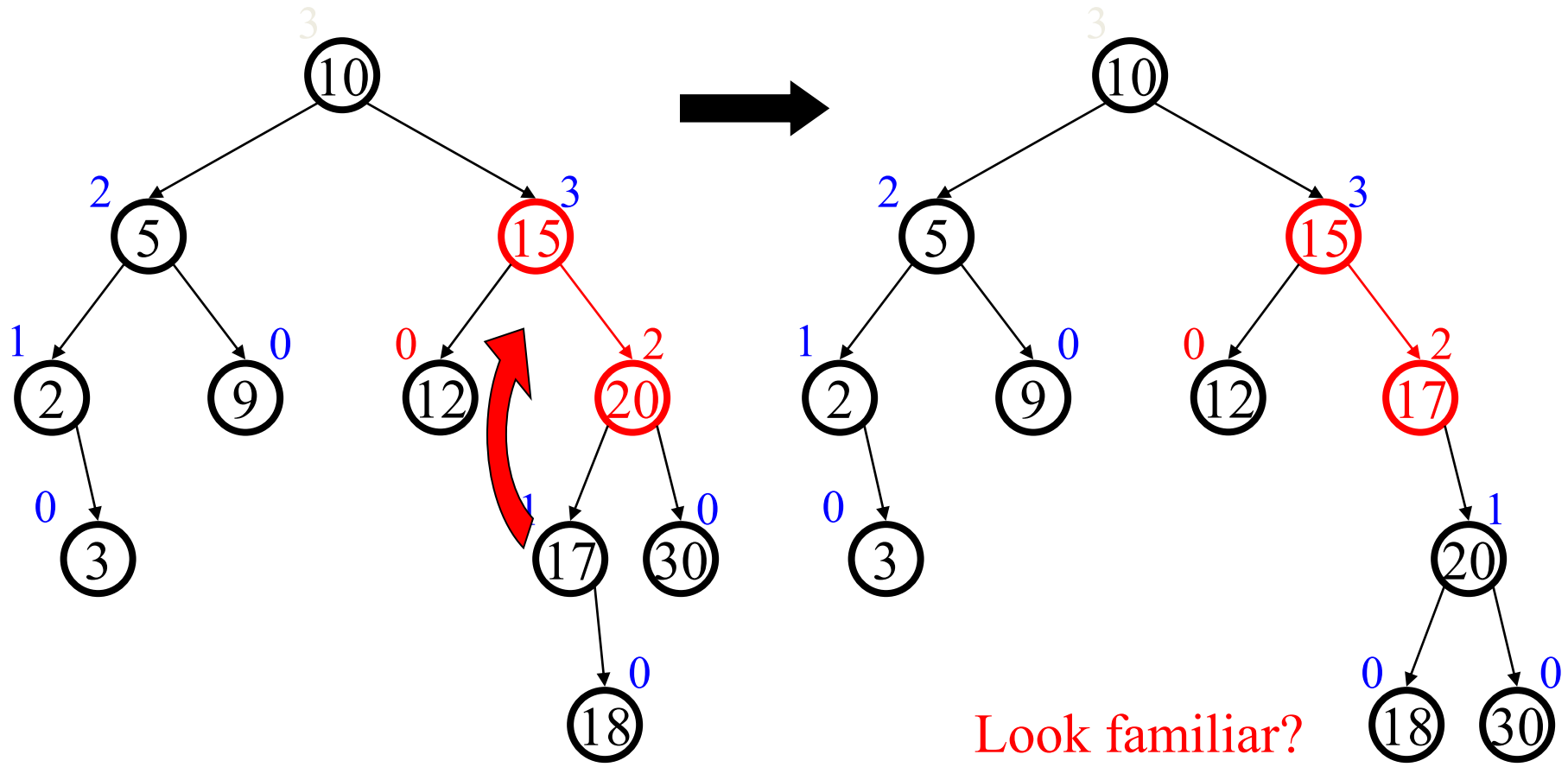
Insert(18)



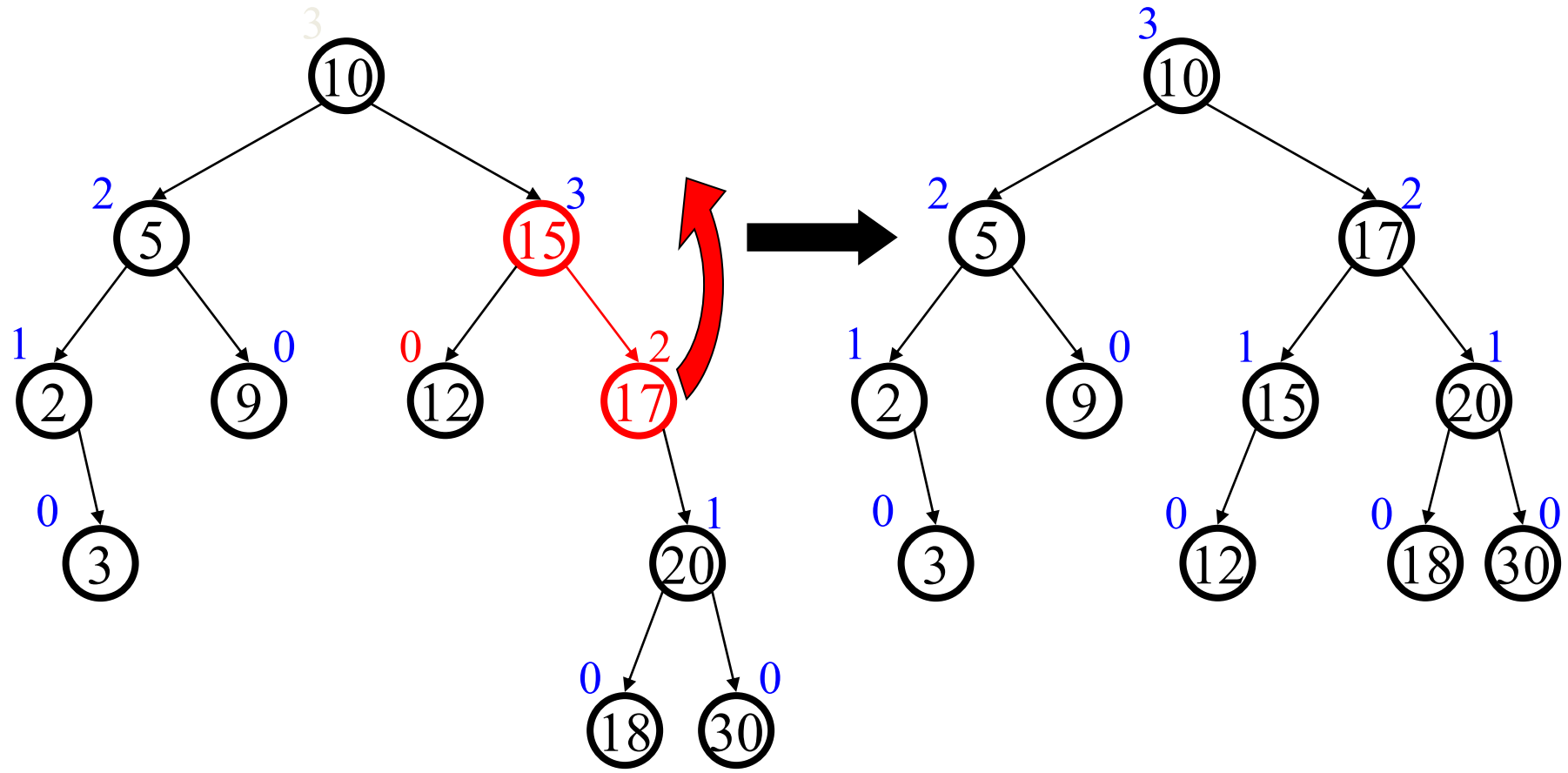
Single Rotation (oops!)



Double Rotation (Step #1)



Double Rotation (Step #2)



Summary

- Terminology used for the tree data structure: root node, leaf node, parent node, children, and siblings, ordered trees, paths, depth, and height, ancestors, descendants, and subtrees
- XHTML and CSS Tree
- Binary Tree
- Arithmetic Expression Tree
- Breadth First Traversal
- Height Balanced Tress

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תודה רבה
Hebrew

Ευχαριστώ
Greek

Danke
German

Спасибо
Russian

धन्यवादः
Sanskrit

Merci
French

ধন্যবাদ
Bangla

நன்றி
Tamil

شكراً
Arabic

ಧನ್ಯವಾದಗಳು
Kannada

Thank You
English

നന്ദി
Malayalam

Grazie
Italian

多謝
Traditional Chinese

ధన్యవాదాలు
Telugu

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