Binary Heap and Priority Queue

Dr. Animesh Chaturvedi

Assistant Professor: IIIT Dharwad

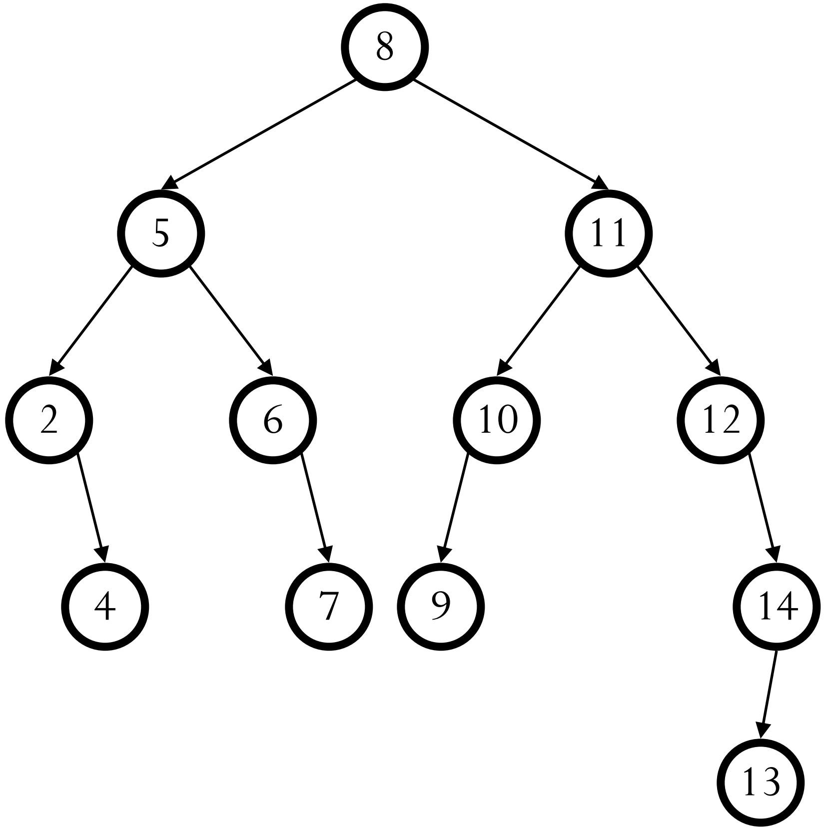
Young Researcher: Heidelberg Laureate Forum

Postdoc: King's College London & The Alan Turing Institute

PhD: IIT Indore MTech: IIITDM Jabalpur

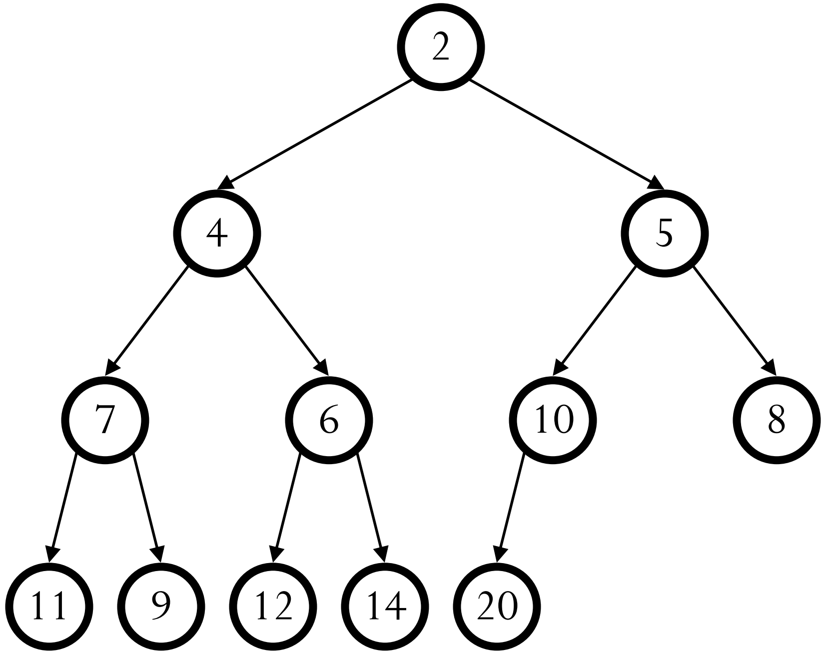
Binary Tree Data Structures

- **Unsorted list:**
  - ***insert:***
  - ***deleteMin:***
- Sorted list:
  - *insert:*
  - *deleteMin:*



Binary Heap - Data Structure

- Heap-order property
  - parent's key is less than children's keys
  - result: minimum is always at the top
- Structure property
  - complete tree with fringe nodes packed to the left
  - result: depth is always O(log n);
  - next open location always known



Binary Heap - Data Structure

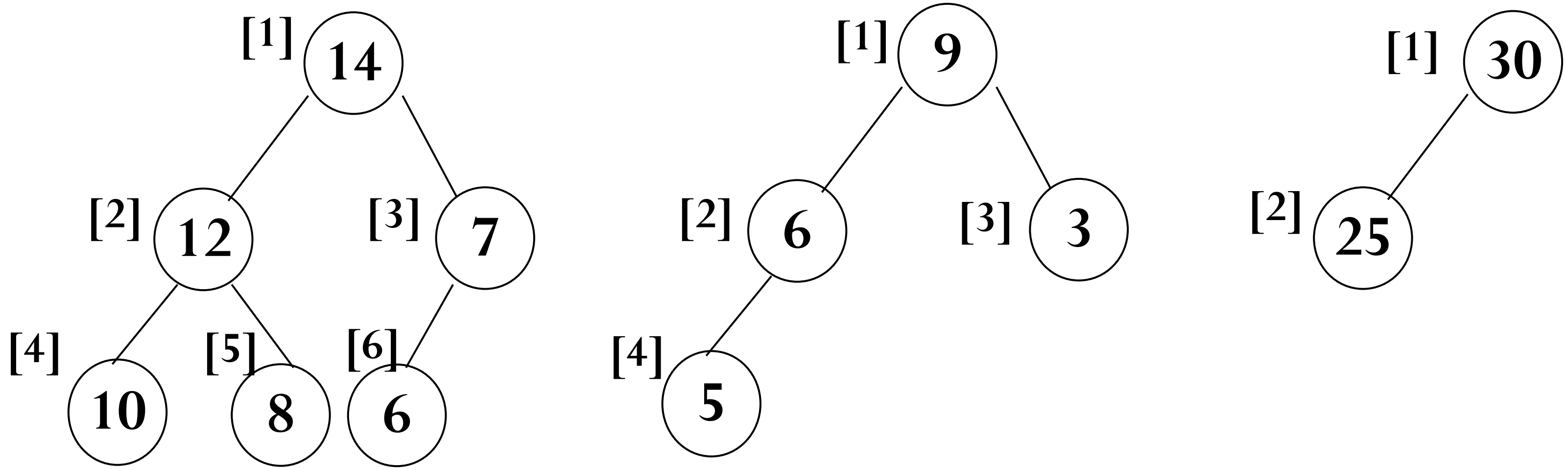
- A ***heap*** orders its node, but in a way different from a binary search tree
- A complete tree is a *heap* if
  - The value in the root is the smallest of the tree
  - Every subtree is also a heap
- Equivalently, a complete tree is a heap if
  - Node value $<$ child value, for each child of the node
- **Note:** This use of the word "heap" is entirely different from the heap that is the allocation area in Java

Binary Heap

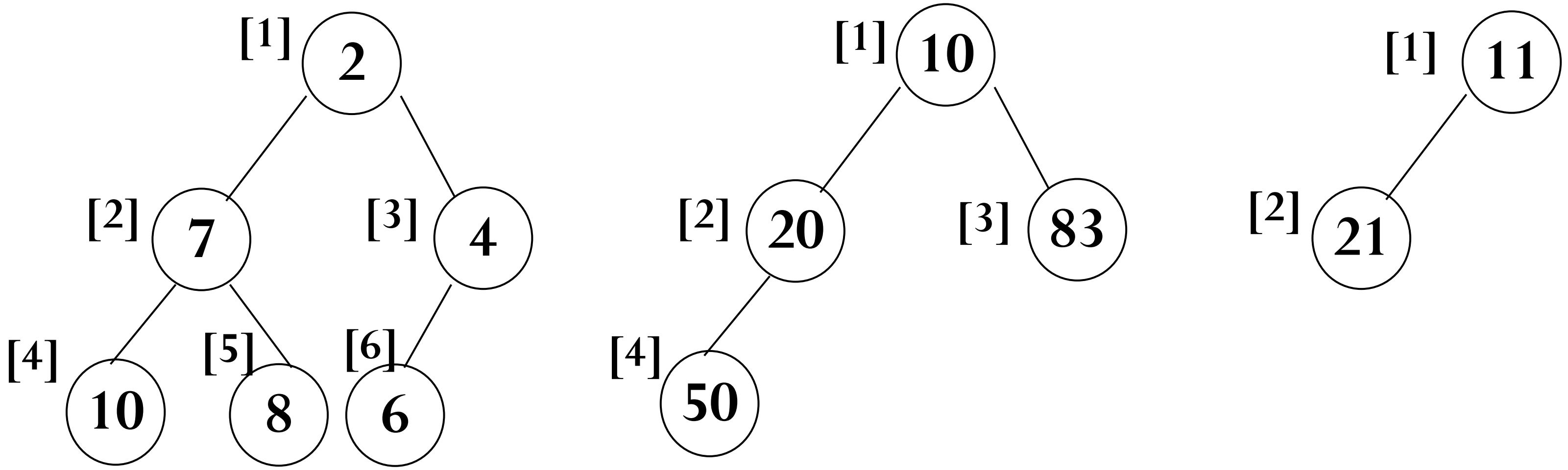
- *Max tree*: the key value in each node is no smaller than the key values in its children.
  - *Max heap* is a complete binary tree that is also a max tree.
- *Min tree*: the key value in each node is no larger than the key values in its children.
  - *Min heap* is a complete binary tree that is also a min tree.
- Operations on heaps
  - creation of an empty heap
  - insertion of a new element into the heap;
  - deletion of the largest element from the heap

Binary Heap

- The root of max heap contains the largest element.

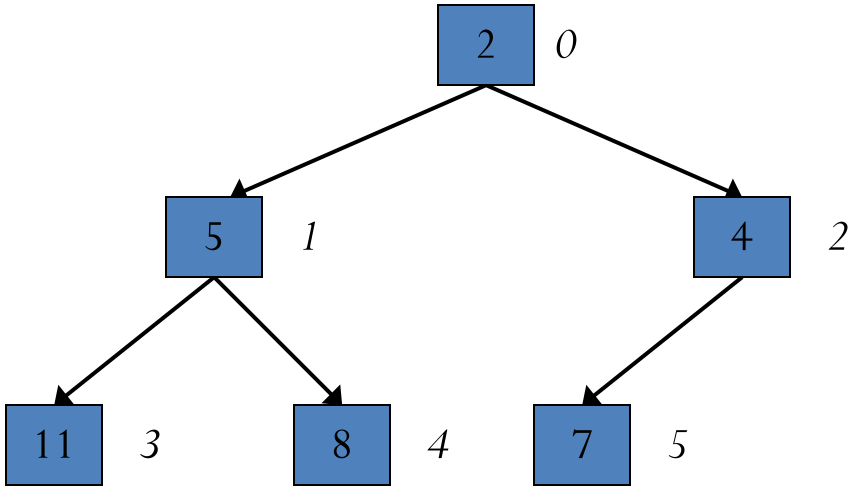


- The root of min heap contains the smallest element.



Implementing a Heap

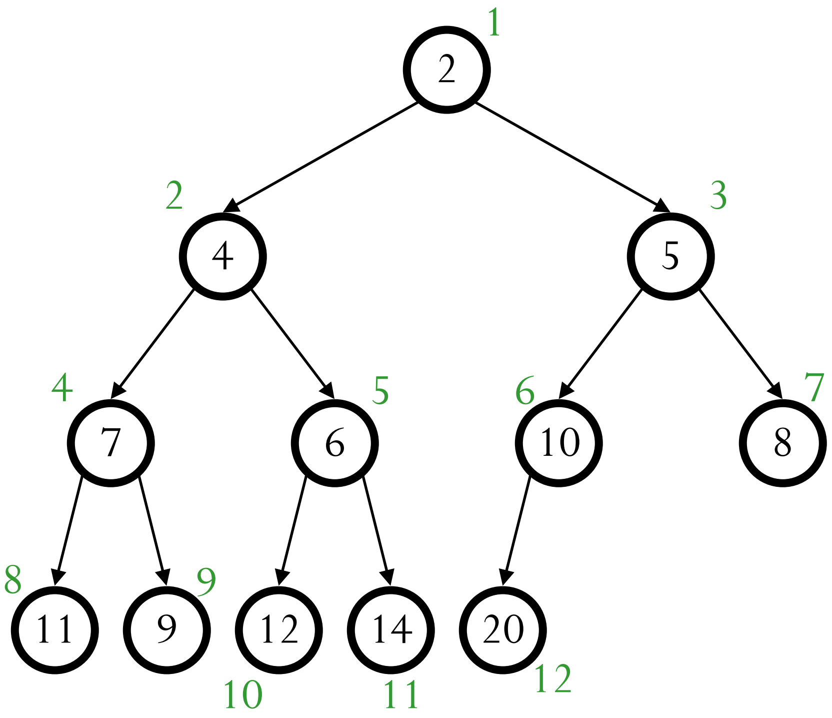
- Recall: a heap is a ***complete binary tree***
  - (plus the heap ordering property)
- A complete binary tree fits nicely in an ***array:***
  - The root is at index 0
  - Children of node at index i are at indices 2i+1, 2i+2



0	1	2	3	4	5
2	5	4	11	8	7

Storage (Min Heap)

- Calculations:
  - child:
  - parent:
  - root:
  - next free:



0	1	2	3	4	5	6	7	8	9	10	11	12	
12	2	4	5	7	6	10	8	11	9	12	14	20	

Removing an Item from a Heap

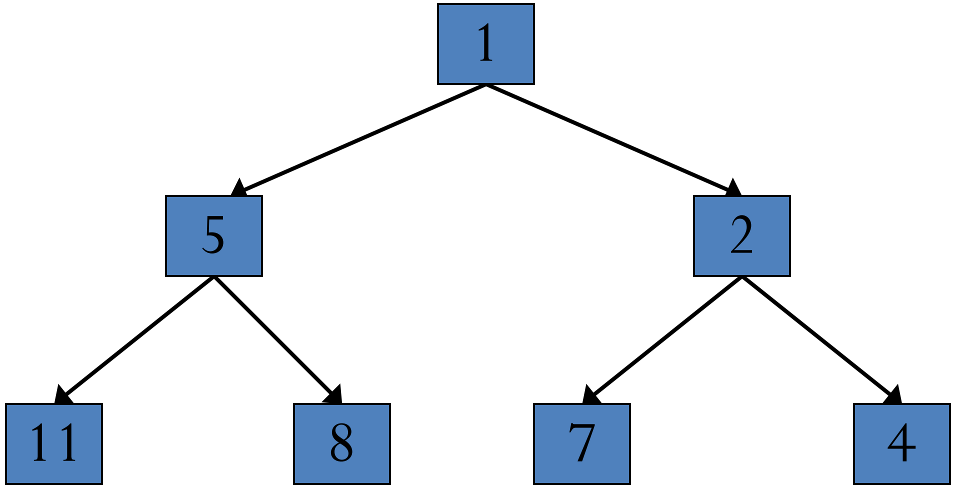
- Removing an item is always from the *top:*
  - Remove the *root* (minimum element):
    - Leaves a “hole”:
  - Fill the “hole” with the last item (lower right-hand) L
    - *Preserve completeness*
  - Swap L with smallest child, as necessary
    - *Restore “heap-ness”*

Example Removing From a Heap

Remove: returns 1

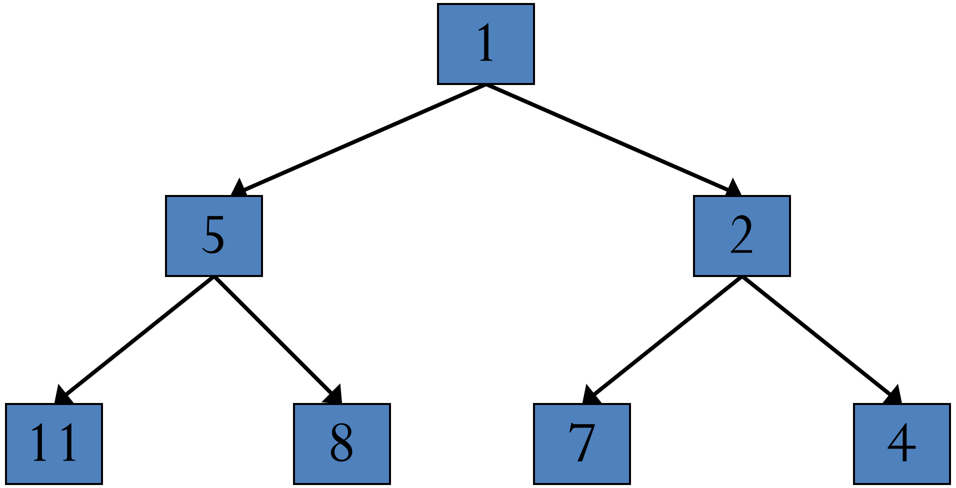
Move 4 to root

Swap down



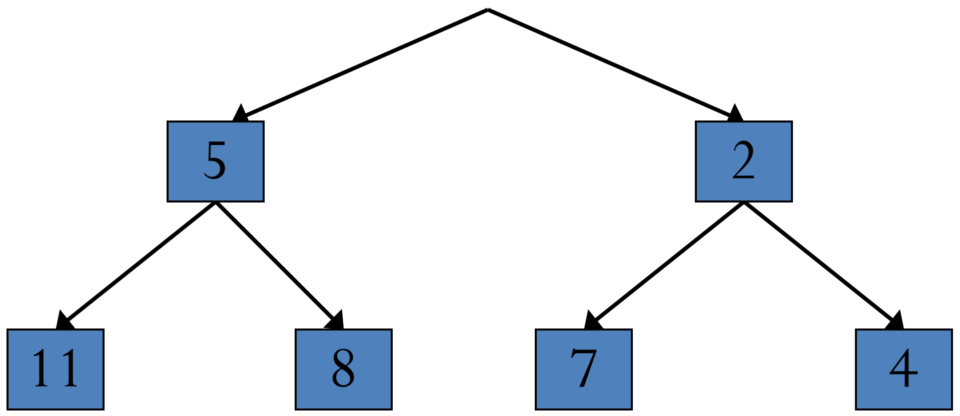
Example Removing From a Heap

Remove: returns 1



Example Removing From a Heap

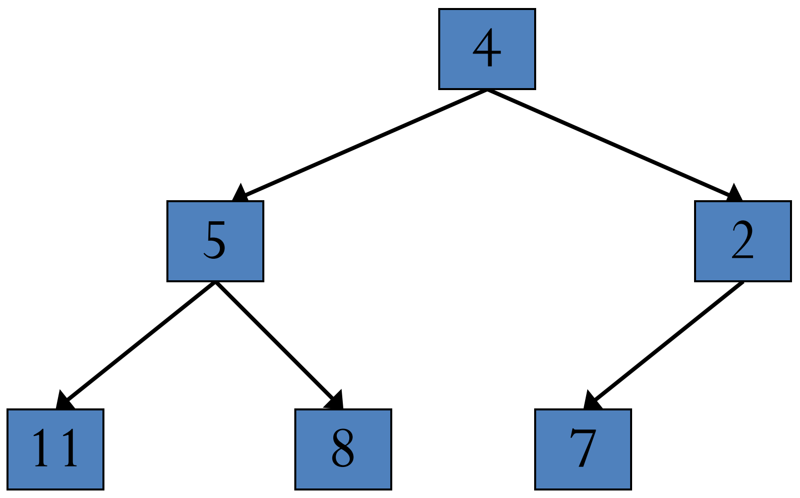
Remove: returns 1



Example Removing From a Heap

Remove: returns 1

Move 4 to root

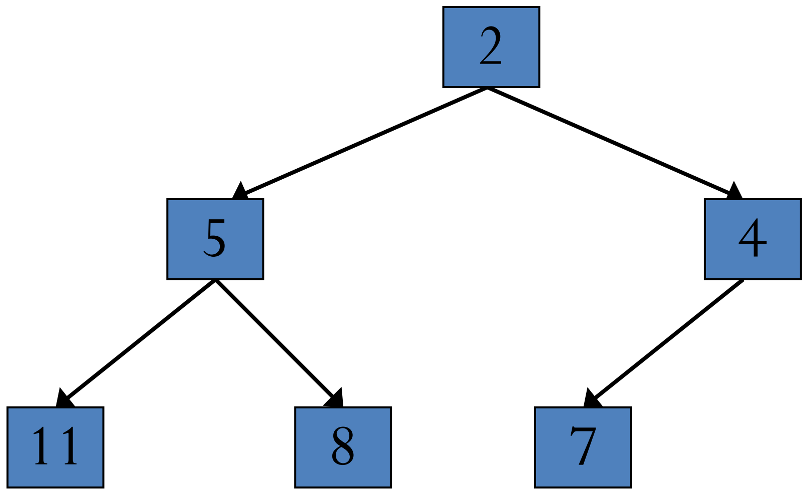


Example Removing From a Heap

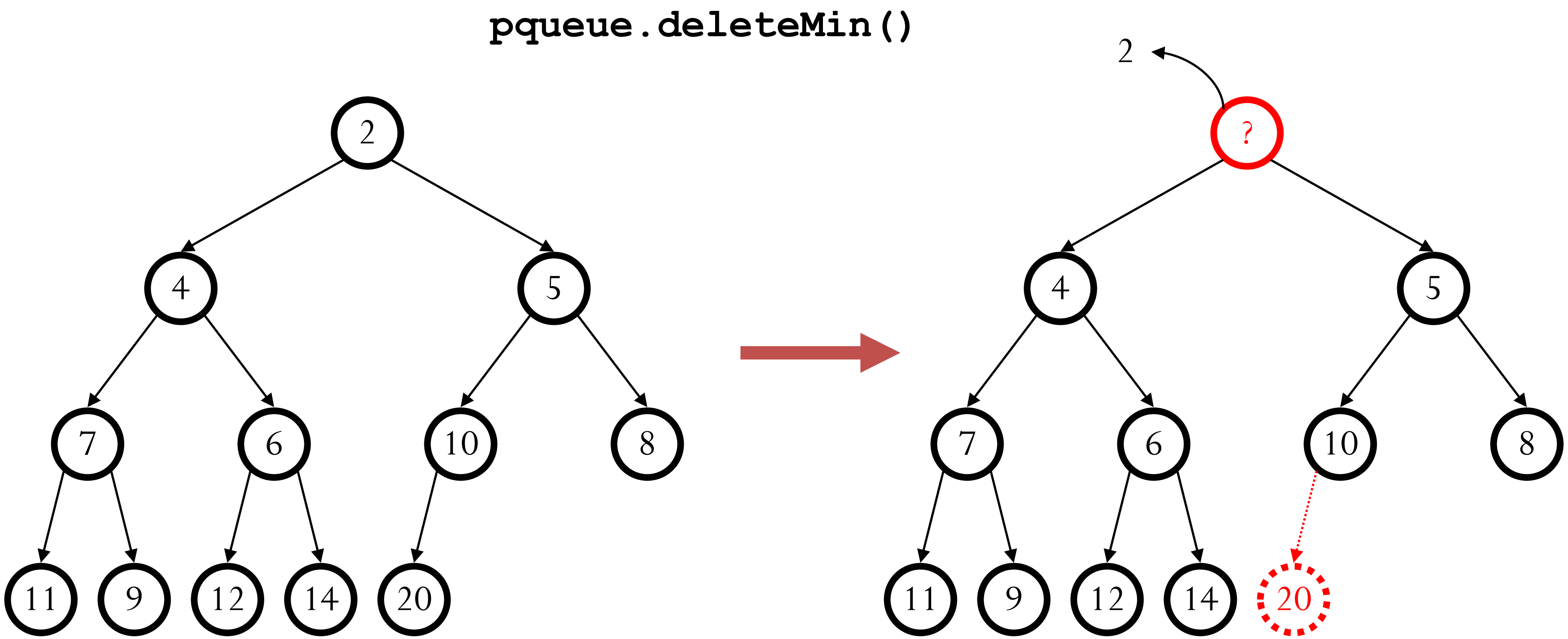
Remove: returns 1

Move 4 to root

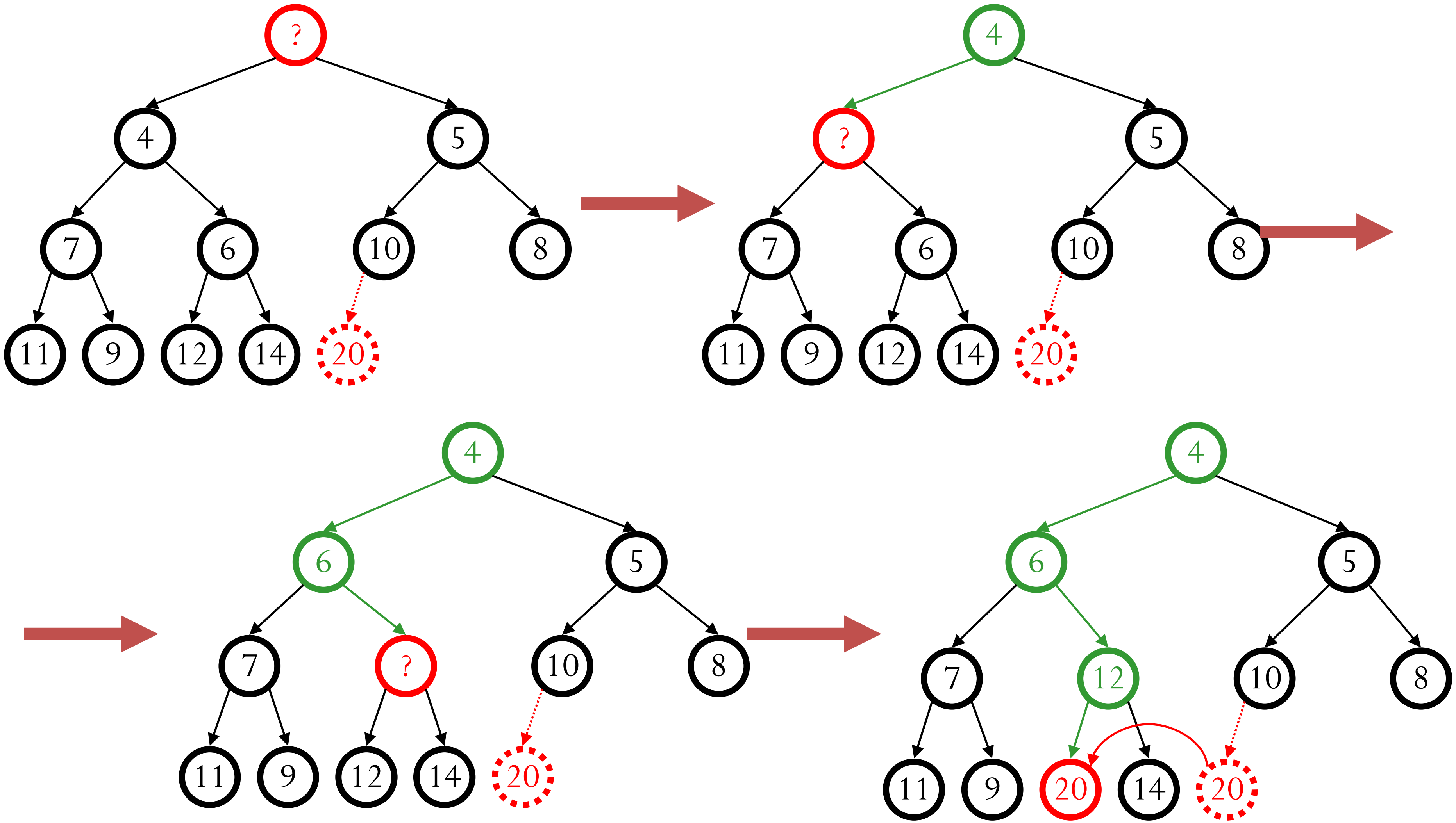
Swap down



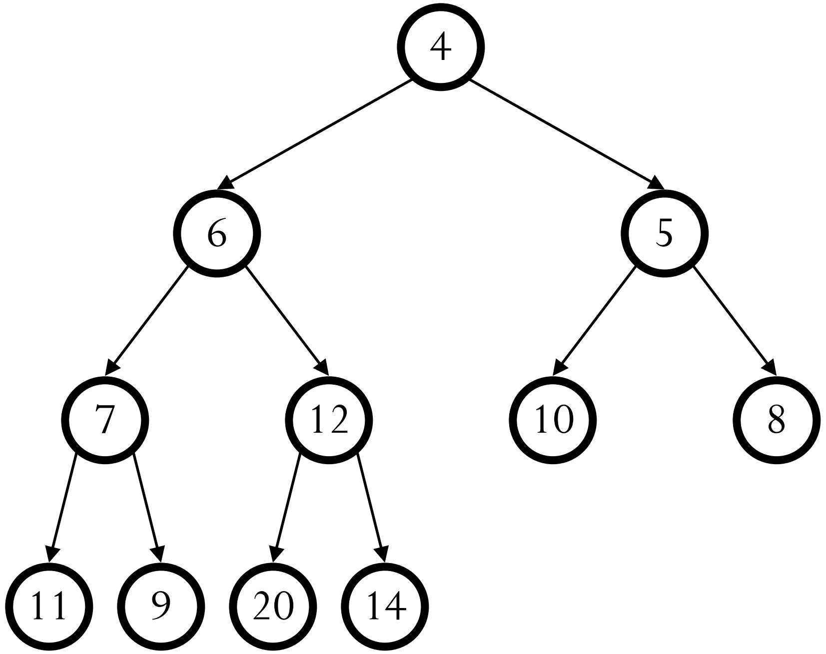
DeleteMin



Percolate Down



Finally…

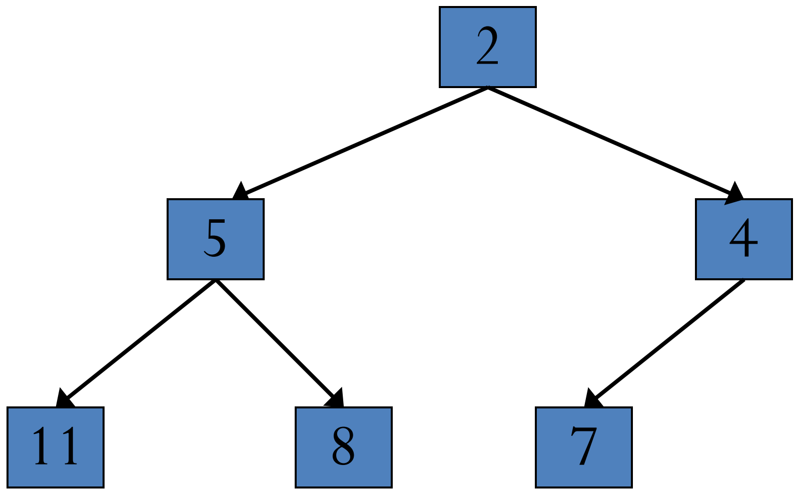


Inserting an Item into a Heap

1. Insert the item in the next position across the bottom of the complete tree: *preserve completeness*
2. *Restore "heap-ness":*
    1. **while** new item not root and < parent
    2. swap new item with parent

Example Inserting into a Heap

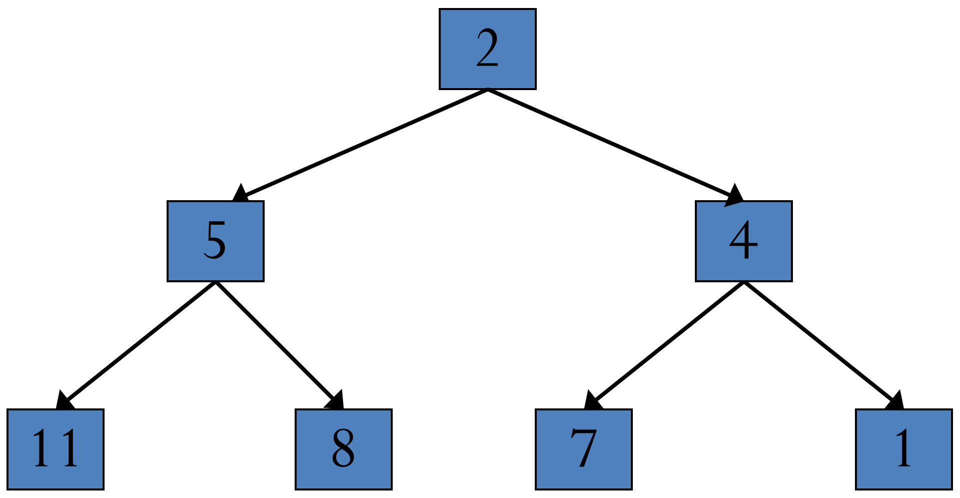
Insert 1



Example Inserting into a Heap

Insert 1

Add as leaf

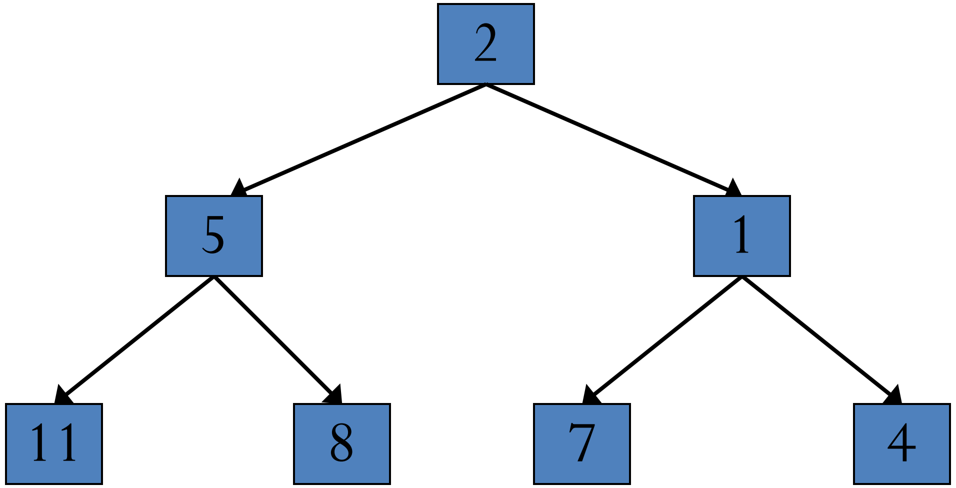


Example Inserting into a Heap

Insert 1

Add as leaf

Swap up



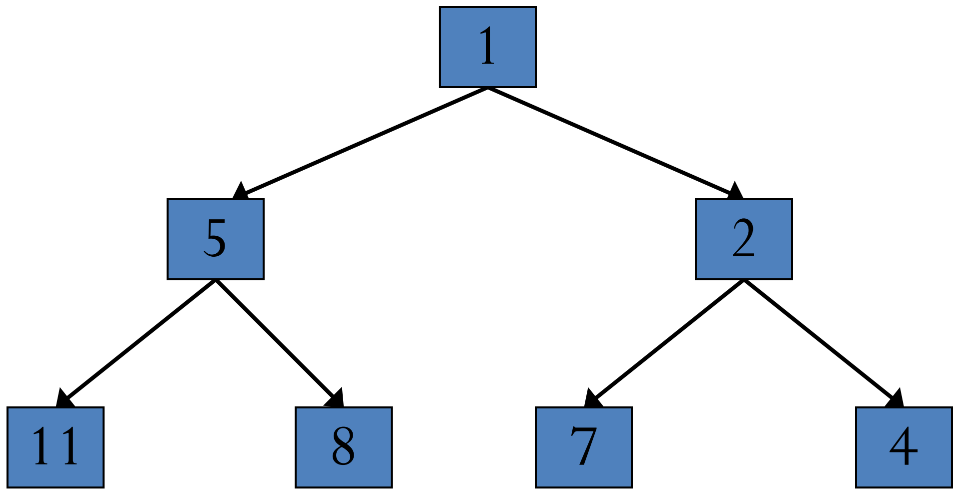
Example Inserting into a Heap

Insert 1

Add as leaf

Swap up

Swap up



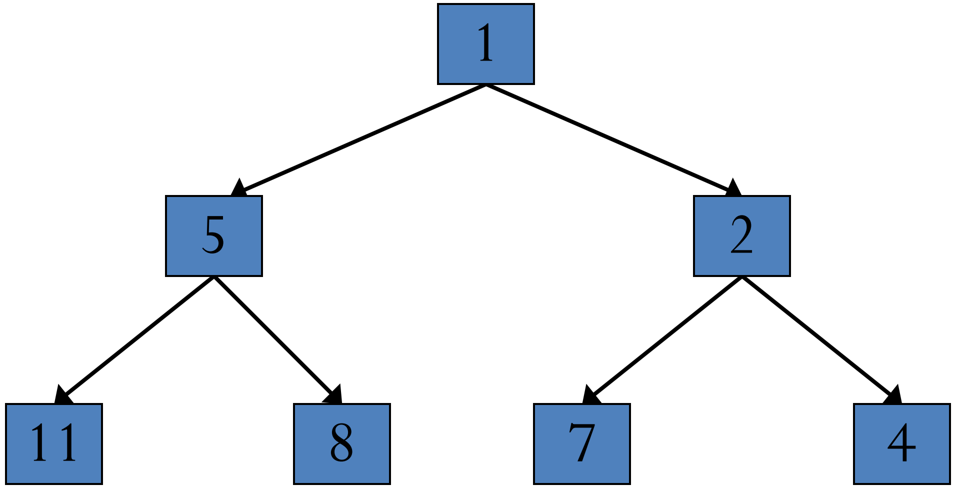
Example Inserting into a Heap

Insert 1

Add as leaf

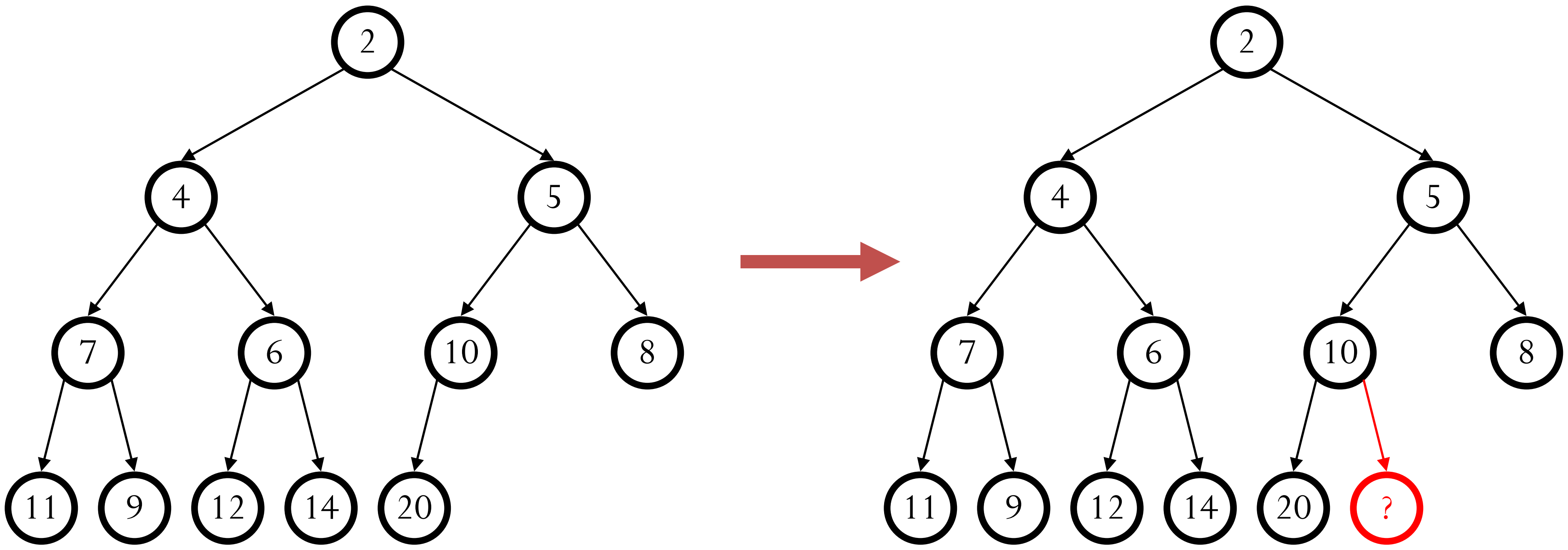
Swap up

Swap up

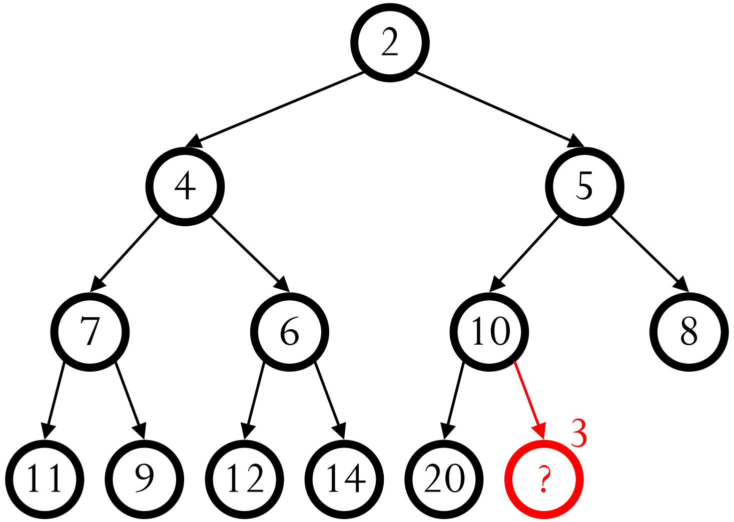


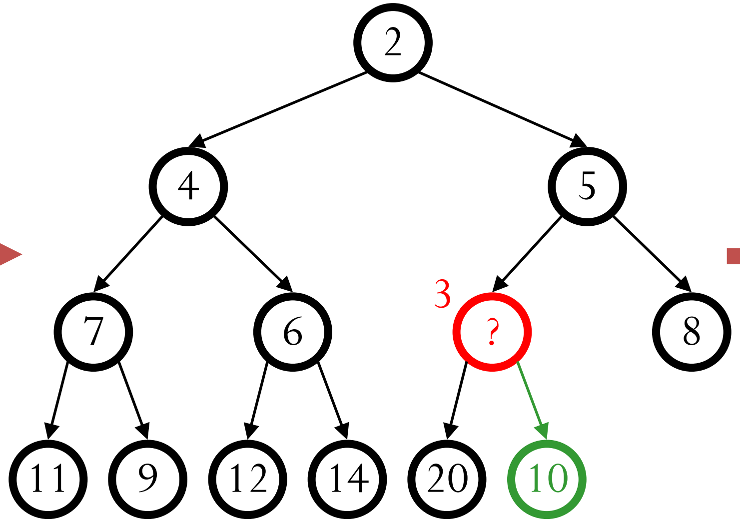
Insert

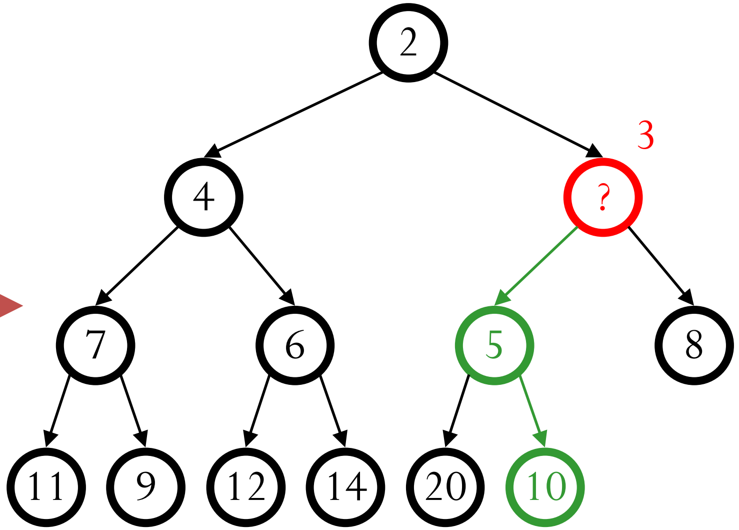
`pqueue.insert(3)`

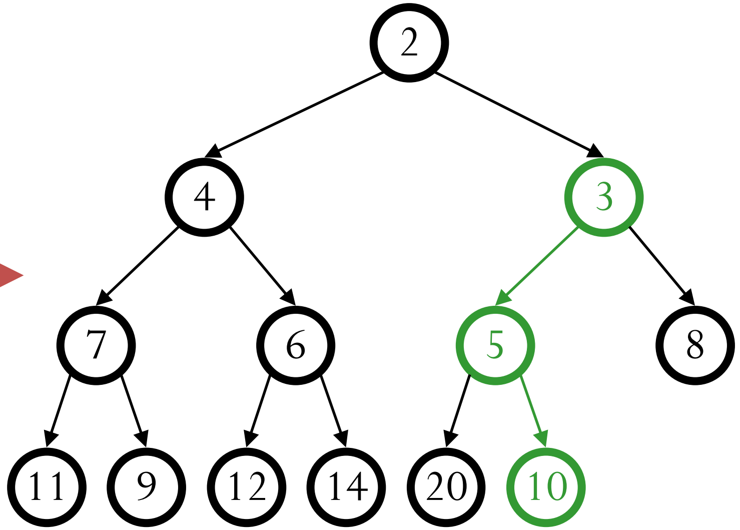


Percolate Up







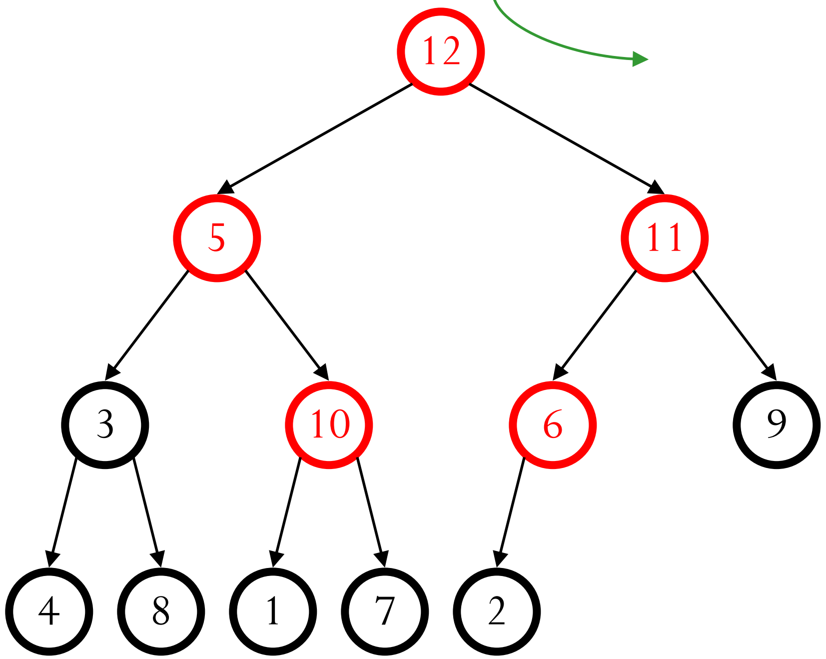


Build Heap

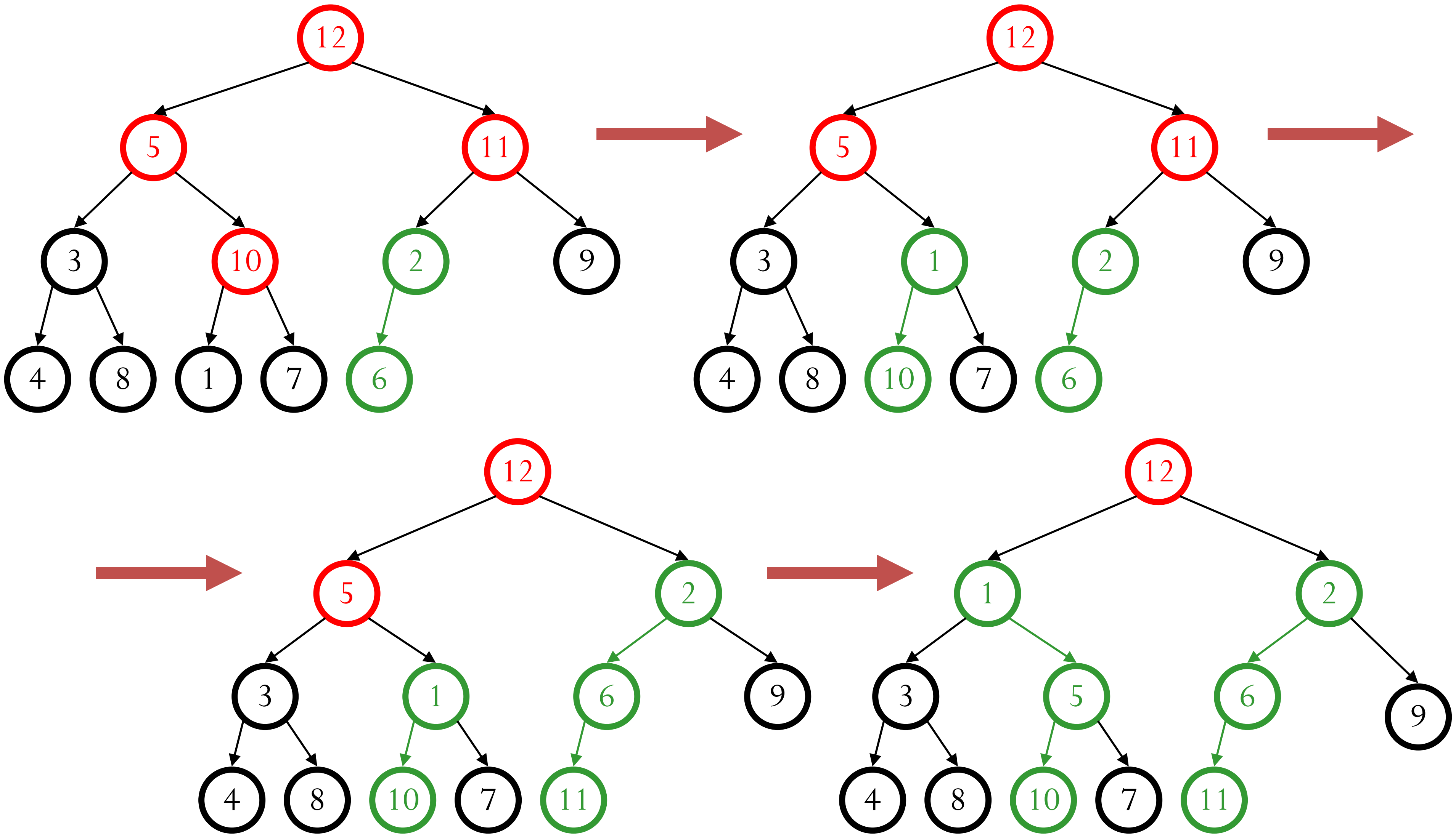
Floyd's Method.

12	5	11	3	10	6	9	4	8	1	7	2

pretend it's a heap and fix the heap-order property!

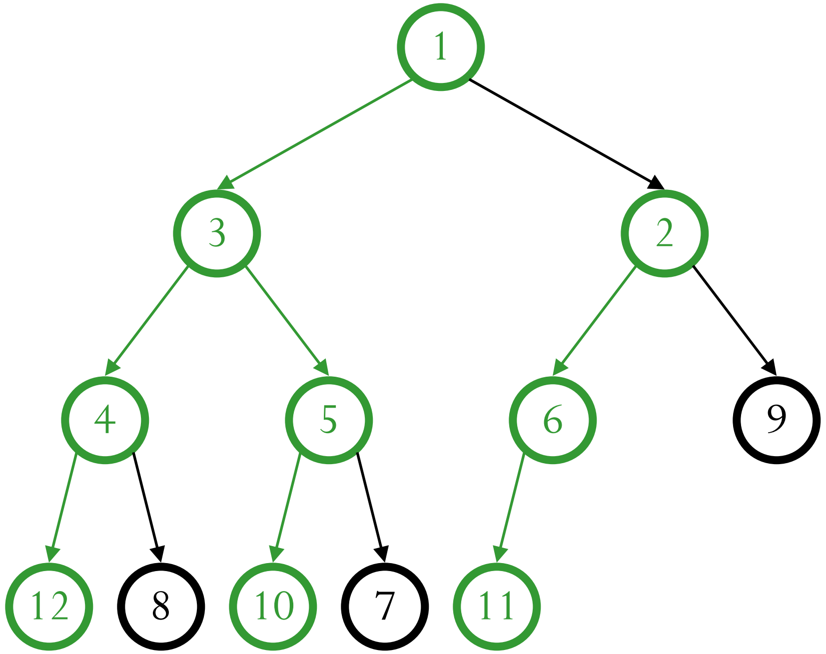


Build Heap



12
5
11
3
10
2
9
4
8
1
7
6
12
5
11
3
1
2
9
4
8
10
7
6
12
5
2
3
1
6
9
4
8
10
7
11
12
1
2
3
5
6
9
4
8
10
7
11

Build Heap

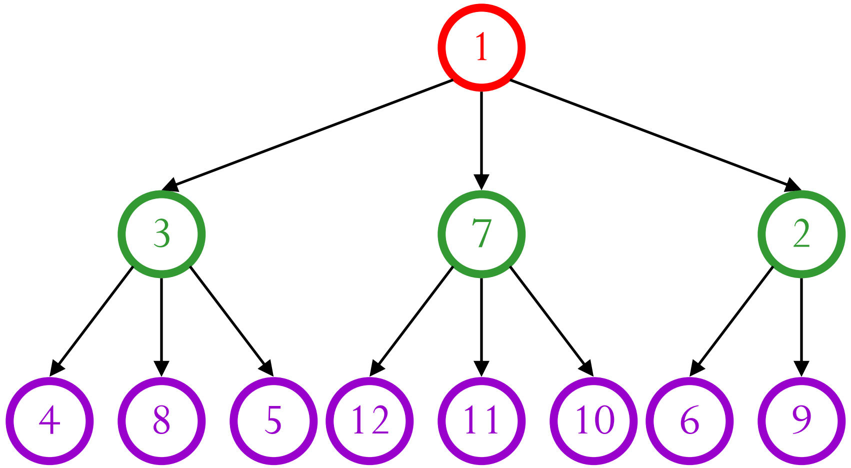


Thinking about Heaps

- Observations
  - finding a child/parent index is a multiply/divide by two
  - operations jump widely through the heap
  - each operation looks at only two new nodes
  - inserts are at least as common as deleteMins
- Realities
  - division and multiplication by powers of two are **fast**
  - looking at one new piece of data sucks in a cache line
  - with **huge** data sets, disk accesses dominate

Solution: d-Heaps

- Each node has *d* children
- Still representable by array
- Good choices for *d*:
  - optimize performance based on # of inserts/removes
  - choose a power of two for efficiency
  - fit one set of children in a cache line
  - fit one set of children on a memory page/disk block

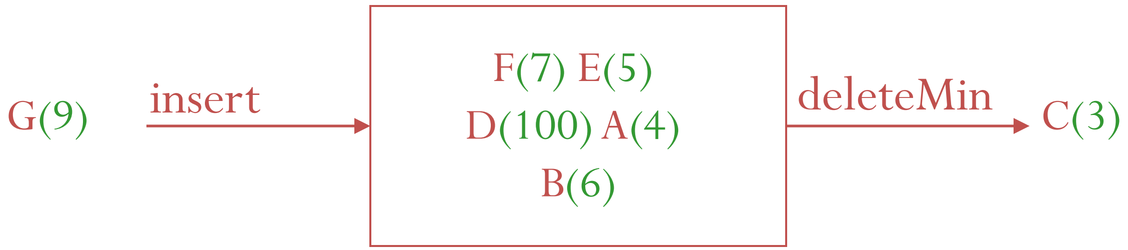


12	1	3	7	2	4	8	5	12	11	10	6	9

Priority Queue

Priority Queue ADT

- Priority Queue operations
  - create
  - destroy
  - insert
  - deleteMin
  - is_empty



- Priority Queue property: for two elements in the queue, *x* and *y*, if *x* has a lower priority value than *y*, *x* will be deleted before *y*

Changing Priorities

- In many applications the priority of an object in a priority queue may change over time
  - if a job has been sitting in the printer queue for a long time increase its priority
  - unix "renice"

- Must have some (separate) way of find the position in the queue of the object to change (*e.g.* a hash table)

Other Priority Queue Operations

- buildHeap
  - given a set of items, build a heap
- decreaseKey
  - given the position of an object in the queue, reduce its priority value
- increaseKey
  - given the position of an object in the queue, increase its priority value
- remove
  - given the position of an object in the queue, remove it

Applications of the Priority Q

- Hold jobs for a printer in order of length
- Store packets on network routers in order of urgency
- Simulate events
- Select symbols for compression
- Sort numbers
- Anything *greedy*

References

- Donald E. Knuth, The Art of Computer Programming, Volume 1: Fundamental Algorithms, 3rd Ed., Addison Wesley, 1997, §2.2.1, p.238.
- Weiss, Data Structures and Algorithm Analysis in C++, 3rd Ed., Addison Wesley, §3.3.1, p.75.
- Donald E. Knuth, The Art of Computer Programming, Volume 1: Fundamental Algorithms, 3rd Ed., Addison Wesley, 1997, §2.2.1, p.238.
- Cormen, Leiserson, and Rivest, Introduction to Algorithms, McGraw Hill, 1990, §11.1, p.200.
- Weiss, Data Structures and Algorithm Analysis in C++, 3rd Ed., Addison Wesley, §3.3.1, p.75.
- Koffman and Wolfgang, "Objects, Abstraction, Data Structures and Design using C++", John Wiley & Sons, Inc., Ch. 6.
- Wikipedia, http://en.wikipedia.org/wiki/Double-ended_queue
- CSE326: Data Structure, Department of Computer Science and Engineering, University of Washington https://courses.cs.washington.edu/courses/cse326
- Mike Scott, CS 307 Fundamentals of Computer Science, https://www.cs.utexas.edu/~scottm/cs307/
- Debasis Samanta, Computer Science & Engineering, Indian Institute of Technology Kharagpur, Spring-2017, Programming and Data Structures. https://cse.iitkgp.ac.in/~dsamanta/courses/pds/index.html

ขอบคุณ
Thai

Grazie
Italian

תודה רבה
Hebrew

धन्यवादः
Sanskrit

ಧನ್ಯವಾದಗಳು
Kannada

Ευχαριστώ
Greek

Thank You
English

Gracias
Spanish

Спасибо
Russian

Obrigado
Portuguese

شكراً
Arabic

https://sites.google.com/site/animeshchaturvedi07

Merci
French

多謝
Traditional Chinese

धन्यवाद
Hindi

Danke
German

多谢
Simplified Chinese

நன்றி
Tamil
Tamil

ありがとうございました
Japanese

감사합니다
Korean