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TECHNOLOGY

Recursion and Tower of Hanoi

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Recursion

- A process by which a function calls itself repeatedly.
- Such a function is called a recursive function
- Recursion may be direct or cyclically in a chain
- Direct recursion.
- When a function $f(\dots)$ calls $f(\dots)$.
- Cyclically in a chain recursion.
 - $f_1(\dots)$ calls $f_2(\dots)$, $f_2(\dots)$ calls $f_3(\dots)$. . . $f_i(\dots)$ calls $f_1(\dots)$

Some Examples of Recursion

- Example 1: Factorial calculation

$$n! = n \times (n-1) \times (n-2) \times \dots \times 3 \times 2 \times 1$$

$$n! = n \times (n-1)!$$

$$\text{factorial}(n) = n \times \text{factorial}(n-1)$$

- Example 2: Fibonacci number sequence

$$1, 1, 2, 3, 5, 8, 13, 21, \dots$$

$$n_i = n_{i-1} + n_{i-2}$$

$$\text{fibonacci}(n) = \text{fibonacci}(n-1) + \text{fibonacci}(n-2)$$

Some Examples of Recursion

- Example 3: GCD of two positive integers

$$\text{gcd}(10, 15) = 5, \text{gcd}(11, 13) = 1$$

$$\text{gcd}(m, n) = \text{gcd}(m-n, n), \text{ if } m > n \text{ else } \text{gcd}(m, n-m)$$

- Example 4: Recursion formula

$$T(n) = n + 2 \times T(n-1)$$

$$T(100) = ?$$

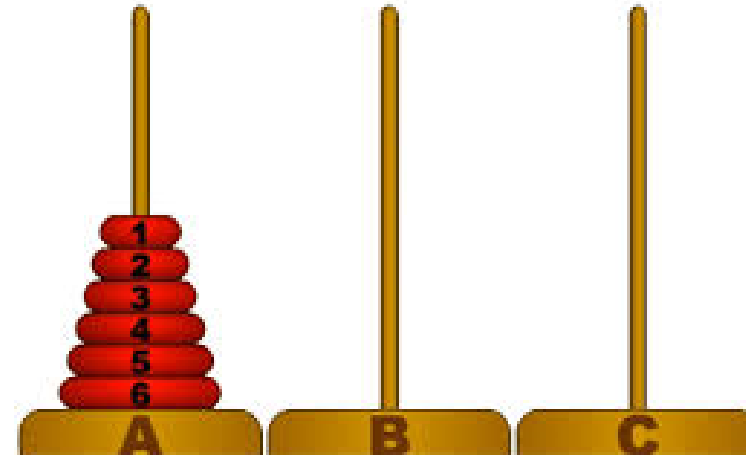
- Example 5: Tower of Hanoi

Move n disks from A C

= move $(n-1)$ disks from A to B

+ move the disk from A to C

+ move $(n-1)$ disk from B to C



Some Examples of Recursion

- Example 6: Sorting

Merge Sort, Quick Sort,

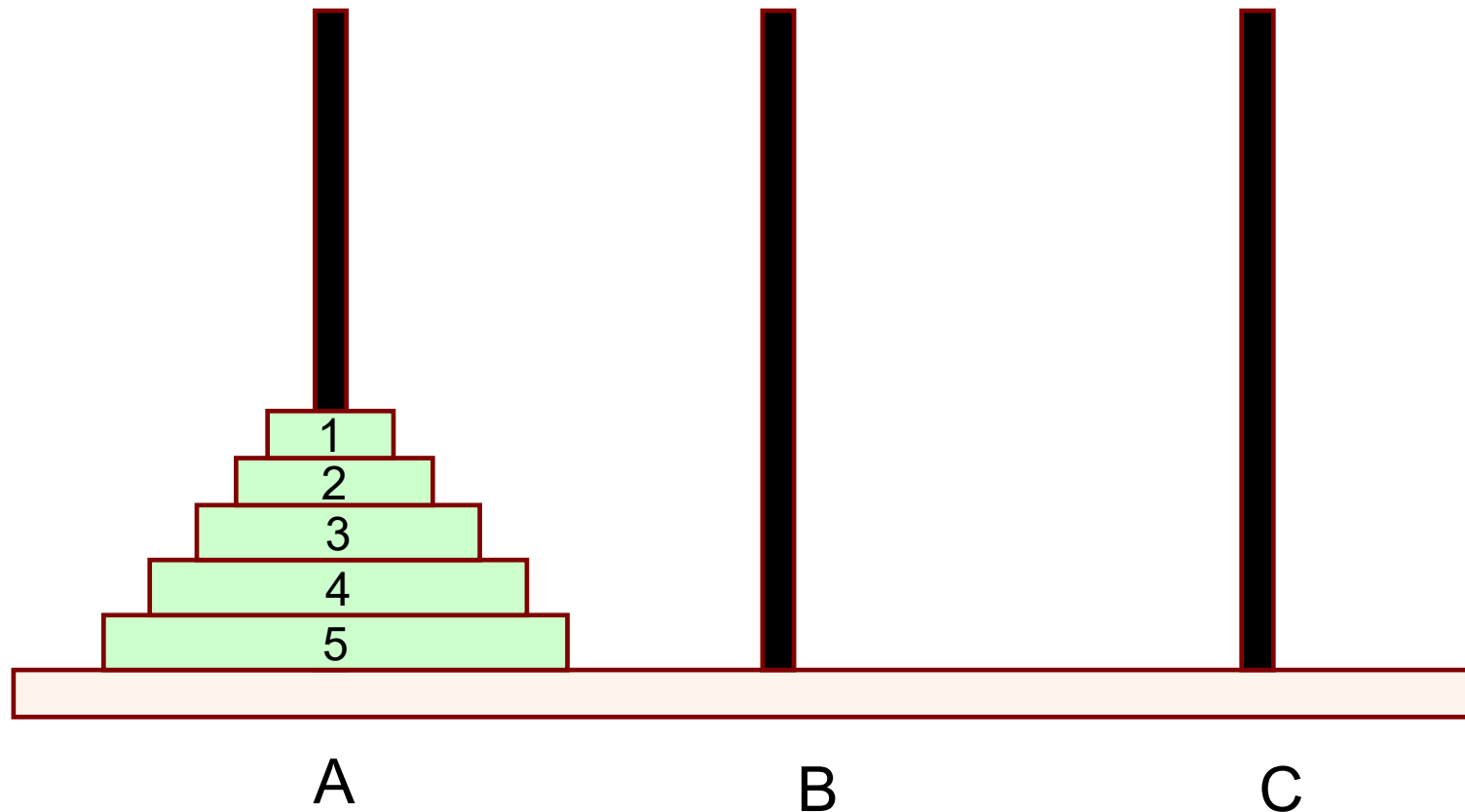
- Example 7: Traversal of Binary Tree

Pre-order, Inorder, Postorder Traversal of Binary Tree

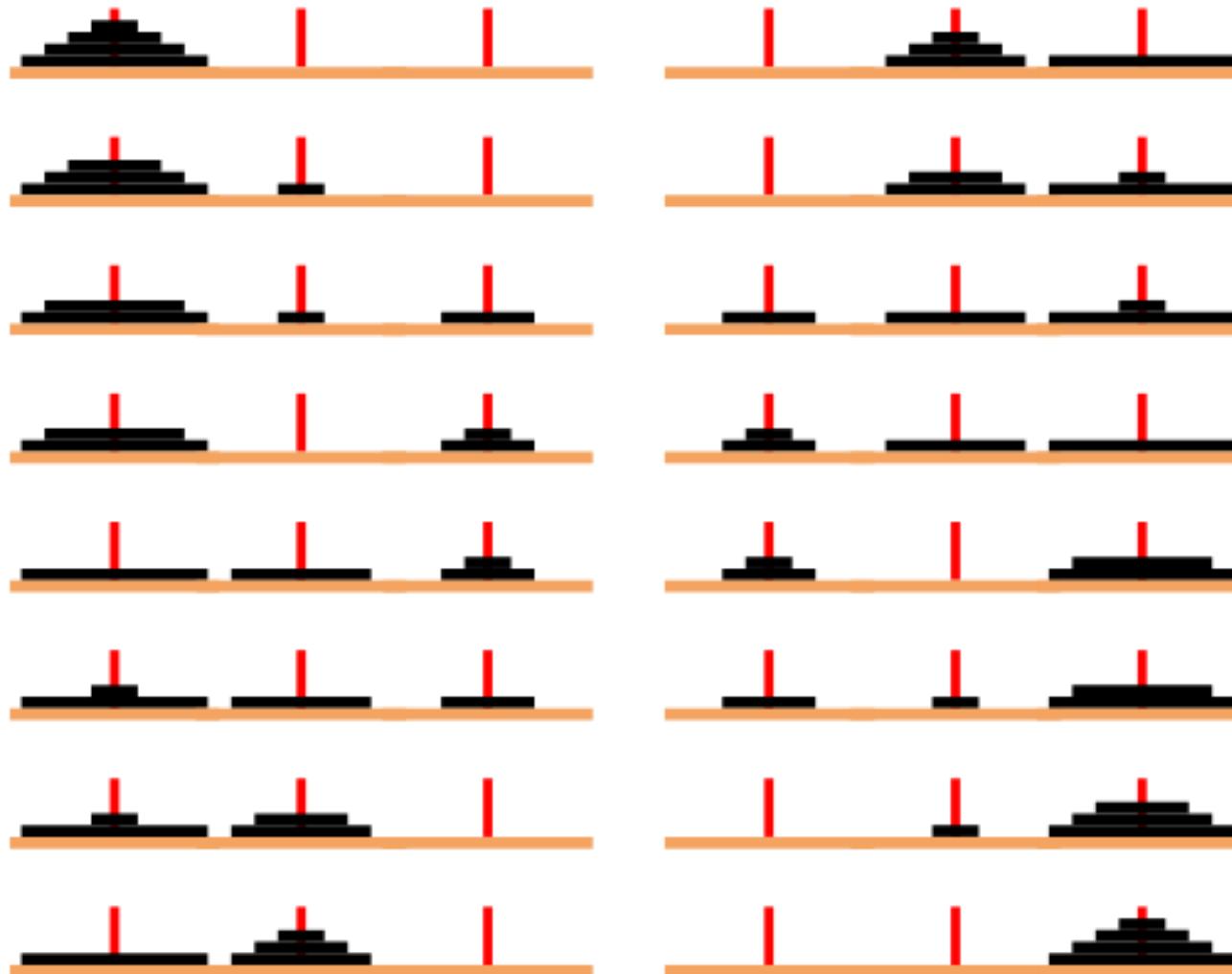
Base case and Recursive case

- For a function to be written in recursive form, two conditions are to be satisfied:
- Condition 1: **Base case**
 - The problem statement must include a stopping condition. A simple occurrence that can be answered directly.
- Condition 2: **Recursive case**
 - It should be possible to express the problem in recursive form.
 - A more complex occurrence of the problem that cannot be directly answered, but can instead be described in terms of smaller occurrences of the same problem.

Towers of Hanoi Problem

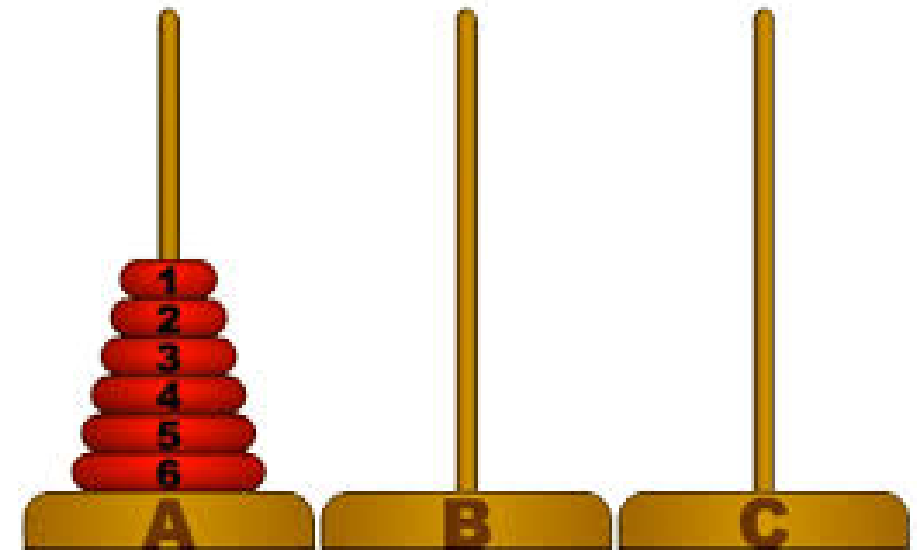


Towers of Hanoi Problem



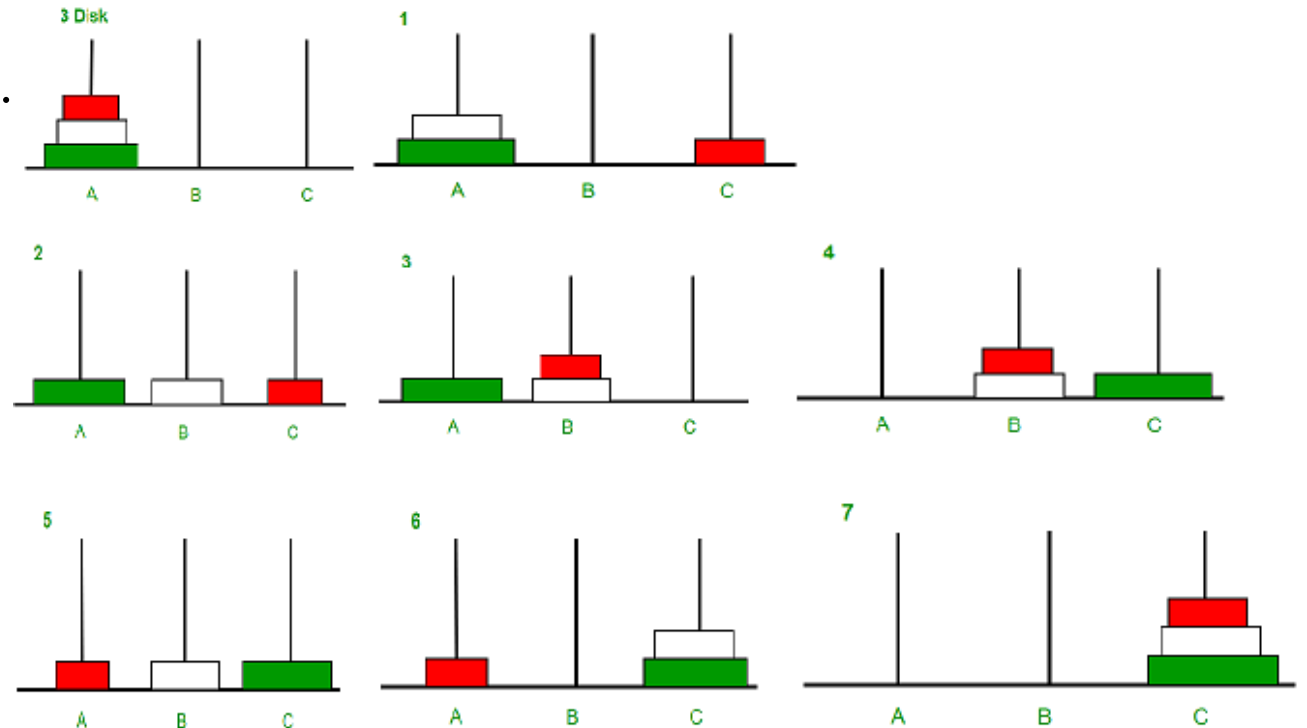
Towers of Hanoi Problem

- The problem statement
- Initially all the disks are stacked on the A pole.
- Required to transfer all the disks to the C pole.
 - Only one disk can be moved at a time.
 - A larger disk cannot be placed on a smaller disk.
- C pole is used for temporary storage of disks.



Towers of Hanoi Problem

- Recursive statement of the general problem of n disks
- Step 1:
 - Move the top $(n-1)$ disks from A to B
- Step 2:
 - Move the largest disk from A to C.
- Step 3:
 - Move the $(n-1)$ disks from B to C.



Towers of Hanoi Problem

```
#include <stdio.h>
```

```
void move(int n, char A, char B, char C);
```

```
int main()
```

```
{ int n;          /* Number of disks */  
  scanf ("%d", &n);  
  move (n, 'A', 'B', 'C');  
  return 0;  
}
```

```
void move (int n, char A, char B, char C)
```

```
{ if (n > 0) {  
    move (n-1, A, C, B);  
    printf ("Move disk %d from %c to %c \n", n, A, C);  
    move (n-1, B, C, A);  
  }
```

```
return;
```

```
}
```

Towers of Hanoi – Execution

3

Move disk 1 from A to C
Move disk 2 from A to B
Move disk 1 from C to B
Move disk 3 from A to C
Move disk 1 from B to A
Move disk 2 from B to C
Move disk 1 from A to C

4

Move disk 1 from A to B
Move disk 2 from A to C
Move disk 1 from B to B
Move disk 3 from A to B
Move disk 1 from C to L
Move disk 2 from C to B
Move disk 1 from A to B
Move disk 4 from A to C
Move disk 1 from B to C
Move disk 2 from B to A
Move disk 1 from C to A
Move disk 3 from B to C
Move disk 1 from A to B
Move disk 2 from A to C
Move disk 1 from B to C

5

Move disk 1 from A to C
Move disk 2 from A to B
Move disk 1 from C to B
Move disk 3 from A to C
Move disk 1 from B to A
Move disk 2 from B to C
Move disk 1 from A to C
Move disk 4 from A to B
Move disk 1 from C to B
Move disk 2 from C to A
Move disk 1 from B to A
Move disk 3 from C to B
Move disk 1 from A to C
Move disk 2 from A to B
Move disk 1 from C to B
Move disk 5 from A to C
Move disk 1 from B to A
Move disk 2 from B to C
Move disk 1 from A to C
Move disk 3 from B to A
Move disk 1 from C to B
Move disk 2 from C to A
Move disk 1 from B to A
Move disk 4 from B to C
Move disk 1 from A to C
Move disk 2 from A to B
Move disk 1 from C to B
Move disk 3 from A to C
Move disk 1 from B to A
Move disk 2 from B to C
Move disk 1 from A to C

How many moves are
required for n disks?

תודה רבה

Hebrew

Ευχαριστώ

Greek

Спасибо

Russian

Danke

German

Merci

French

धन्यवादः

Sanskrit

நன்றி

Tamil

شكراً

Arabic

ಧನ್ಯವಾದಗಳು

Kannada

Thank You

English

നന്നി

Malayalam

Grazie

Italian

ధన్యవాదాలు

Telugu

આભાર

Gujarati

多謝

Traditional Chinese

Gracias

Spanish

ਧੰਨਵਾਦ

Punjabi

धन्यवाद

Hindi & Marathi

多谢

Simplified Chinese

<https://sites.google.com/site/animeshchaturvedi07>

Obrigado

Portuguese

ありがとうございました

Japanese

ขอบคุณ

Thai

감사합니다

Korean