Minimum Spanning Tree

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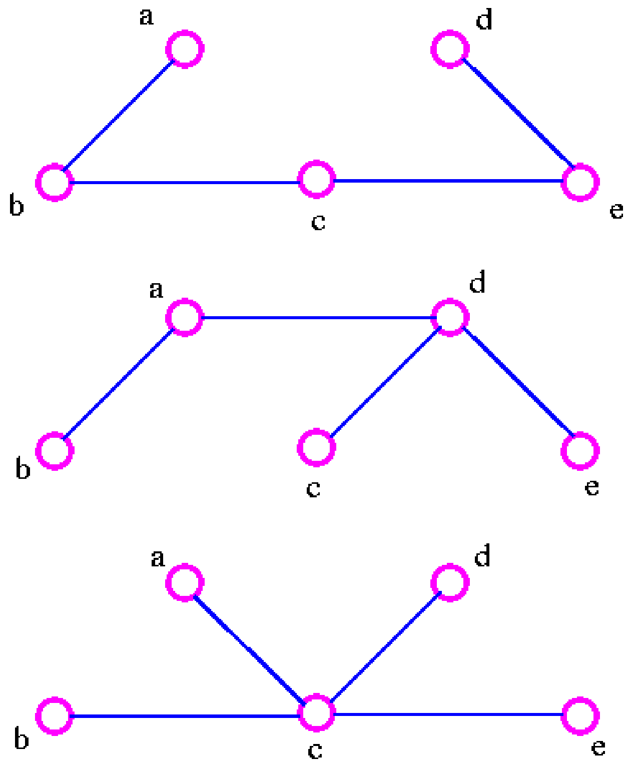
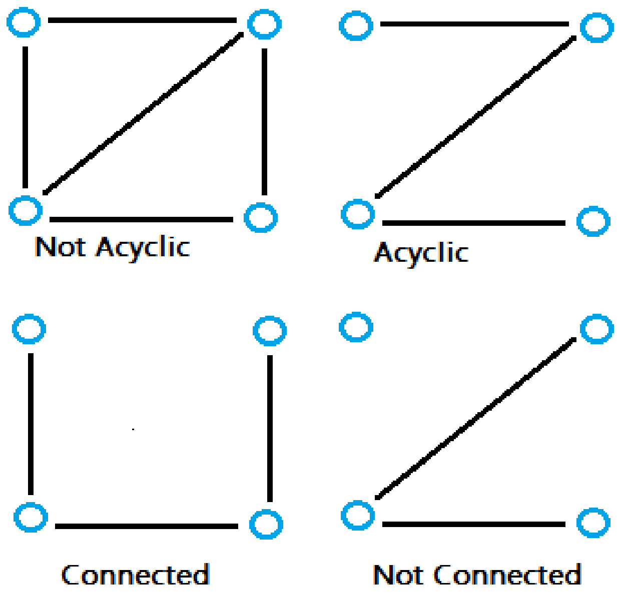
Spanning Tree

- A tree T is said to be a **spanning tree** of a connected graph G, if T is a subgraph of G and T contains all vertices of G.
- A **graph G is** said to be **connected** if there is at least one path between every pair of vertices in G. Otherwise, **G is disconnected**.
- A disconnected graph with **k components** has a spanning forest consist of **k spanning trees**.
- All spanning trees have exactly **|V| - 1 edges**.

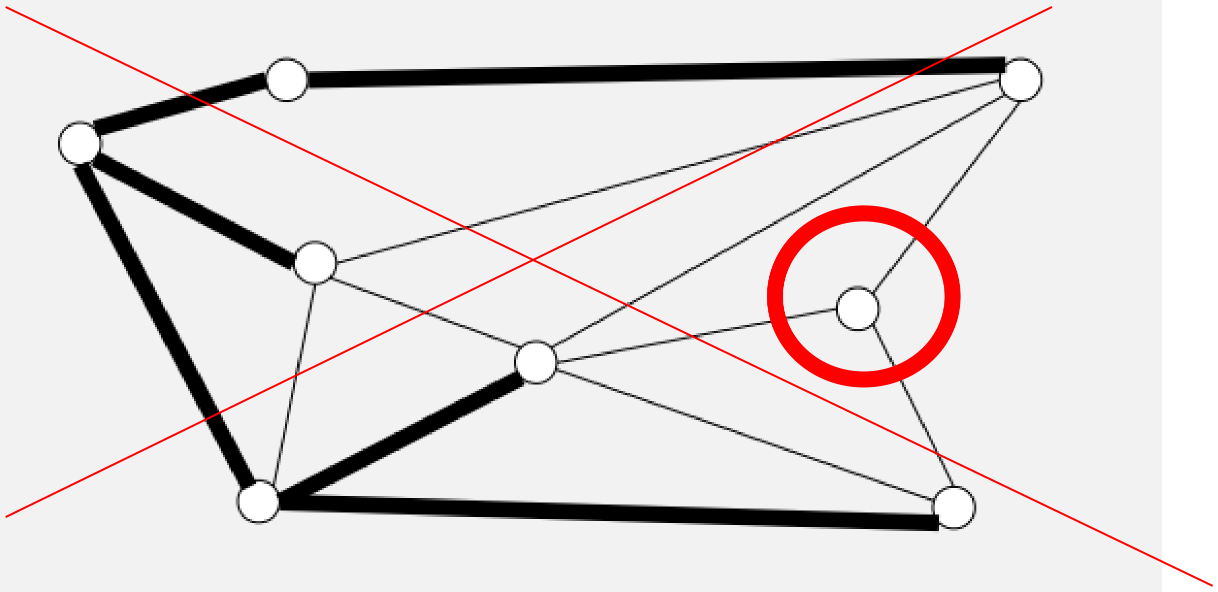




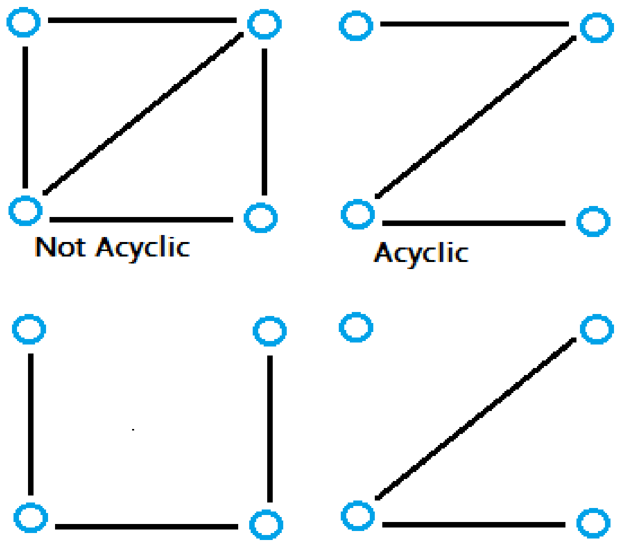
Spanning Tree

Def. A spanning tree of G is a subgraph T that is:

- Connected.
- Acyclic.
- Includes all of the vertices.



not spanning



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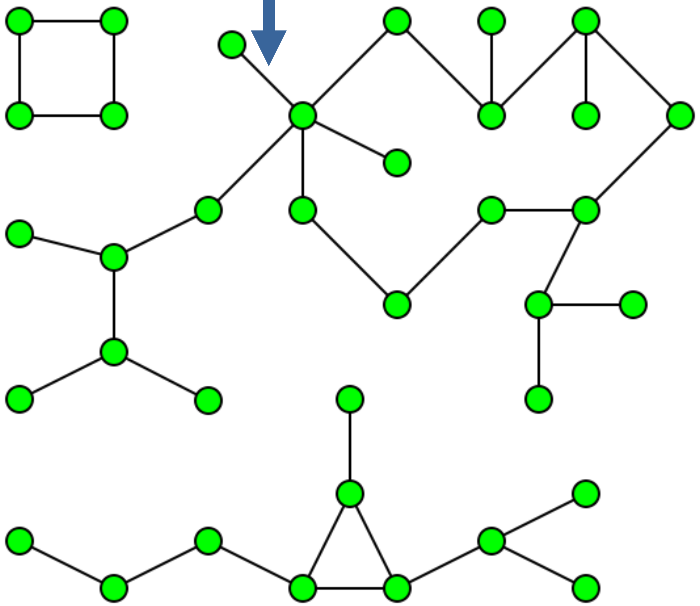
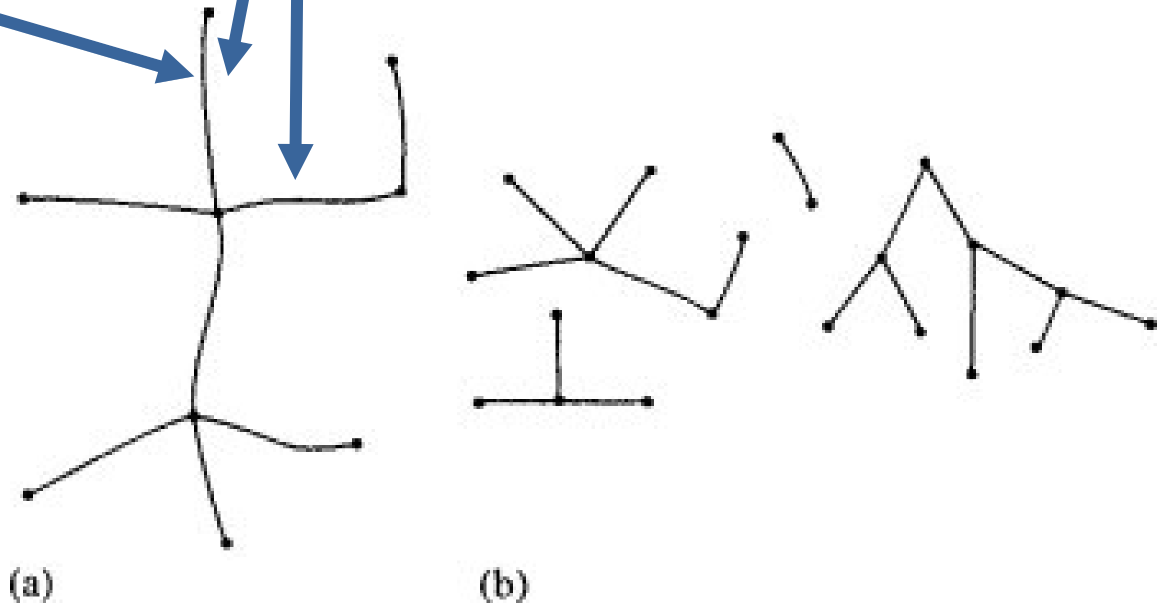
Spanning Tree Properties

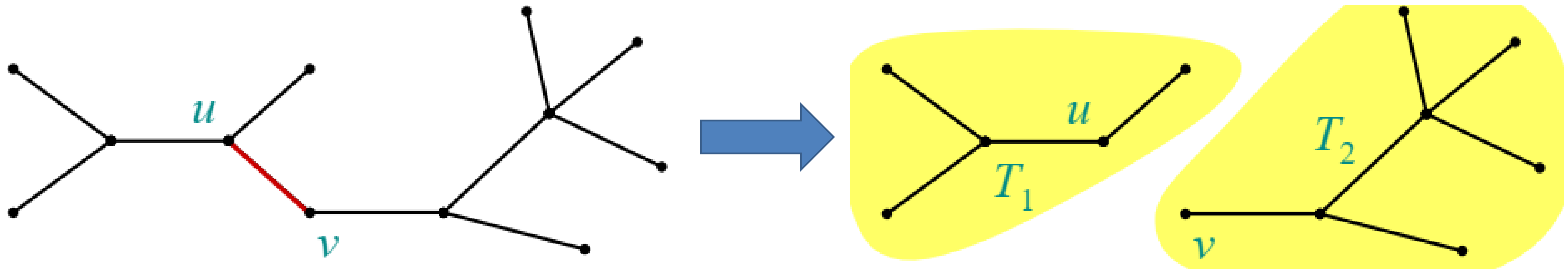
- Every connected graph has at least one spanning tree.
- **An edge** in a spanning tree T is called a **branch of T**.
- A **pendant edge** in a graph G is contained in every spanning tree of G.





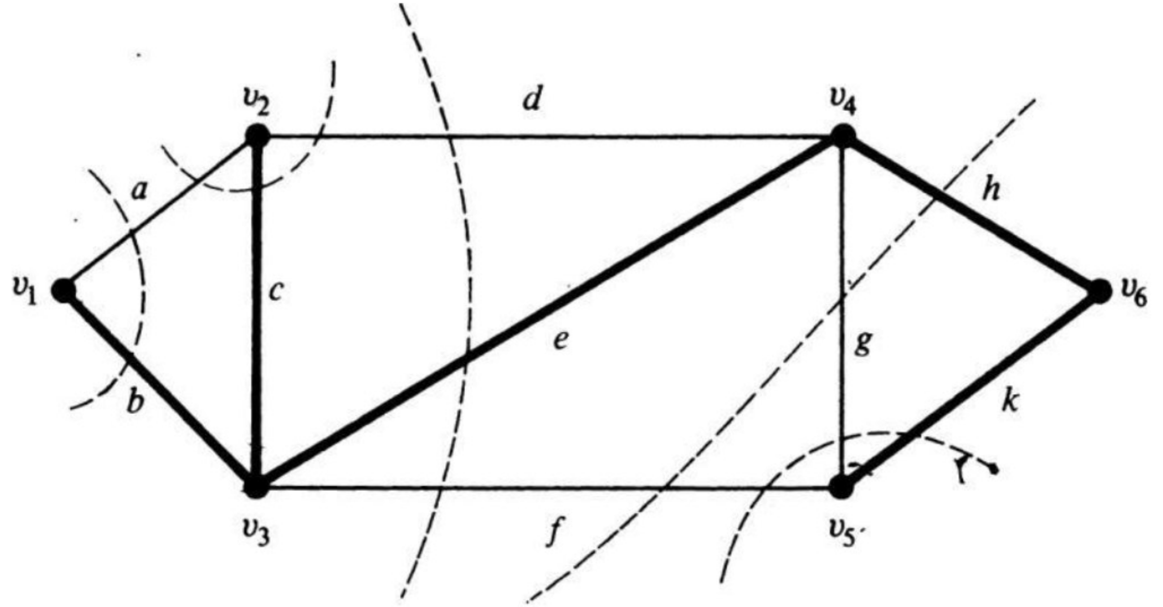
Spanning tree and Cut set

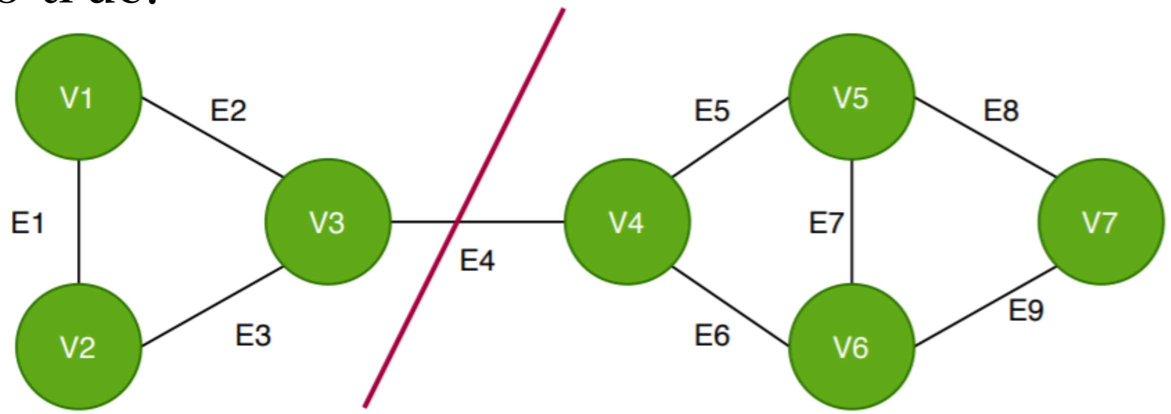
- In a connected graph G, a cut-set is a set of edges whose removal from G leaves G disconnected, provided removal of no proper subset of these edges disconnects G.
- Same is a true for a Spanning Tree as it is also a connected graph

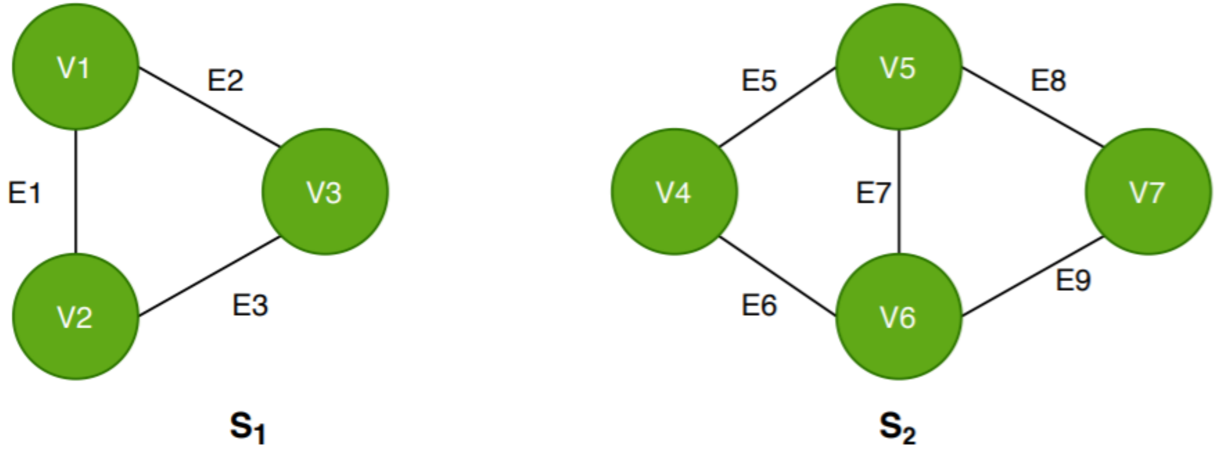


Spanning tree and Cut set

- Every cut-set in a connected graph G must contain at least one branch of every spanning tree of G, the converse is also true.
- In a connected graph G, any minimal set of edges containing at least one branch of every spanning tree of G is a cut-set.

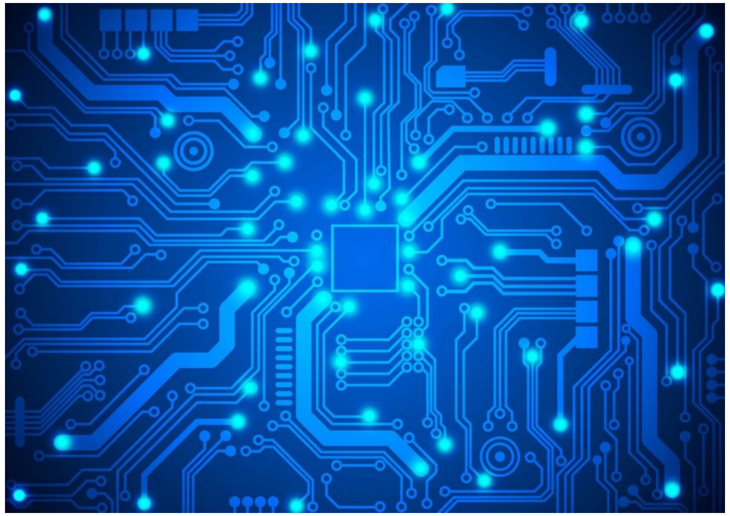






Minimum Spanning Tree

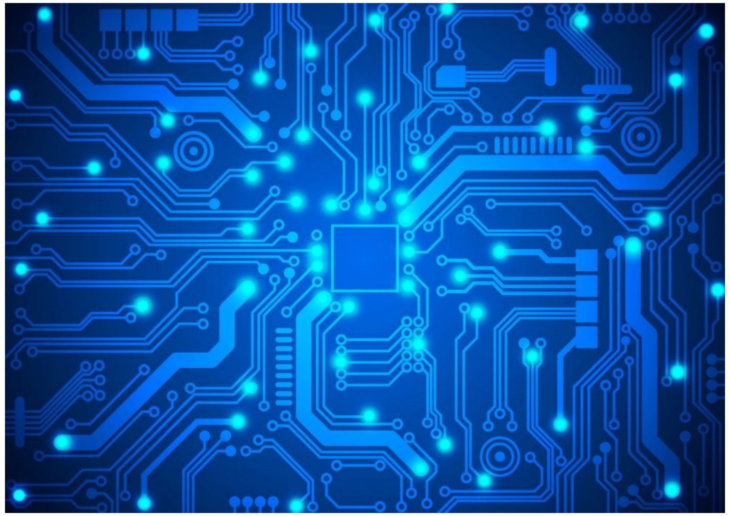
Design of Electronic circuitry



- In the **Design of Electronic circuitry**,
  pins of several components electrically equivalent
  by wiring them together.
- To interconnect a set of n pins,
  - we can use an arrangement of n - 1 wires,
  - each connecting two pins
- The one that uses the least amount of wire is usually the most desirable.

Cormen, T. H., Leiserson, C. E., Rivest, R. L., & Stein, C. (2009). *Introduction to Algorithms* (Vol. 3, pp. 624-642). Cambridge: MIT press.

Design of Electronic circuitry



Model this wiring problem with a connected, undirected graph $G = (V, E)$,

where V is the set of pins,
E is the set of possible interconnections between pairs of pins.

Find an acyclic subset that connects all the vertices and whose total weight is minimized the cost (amount of wire needed).

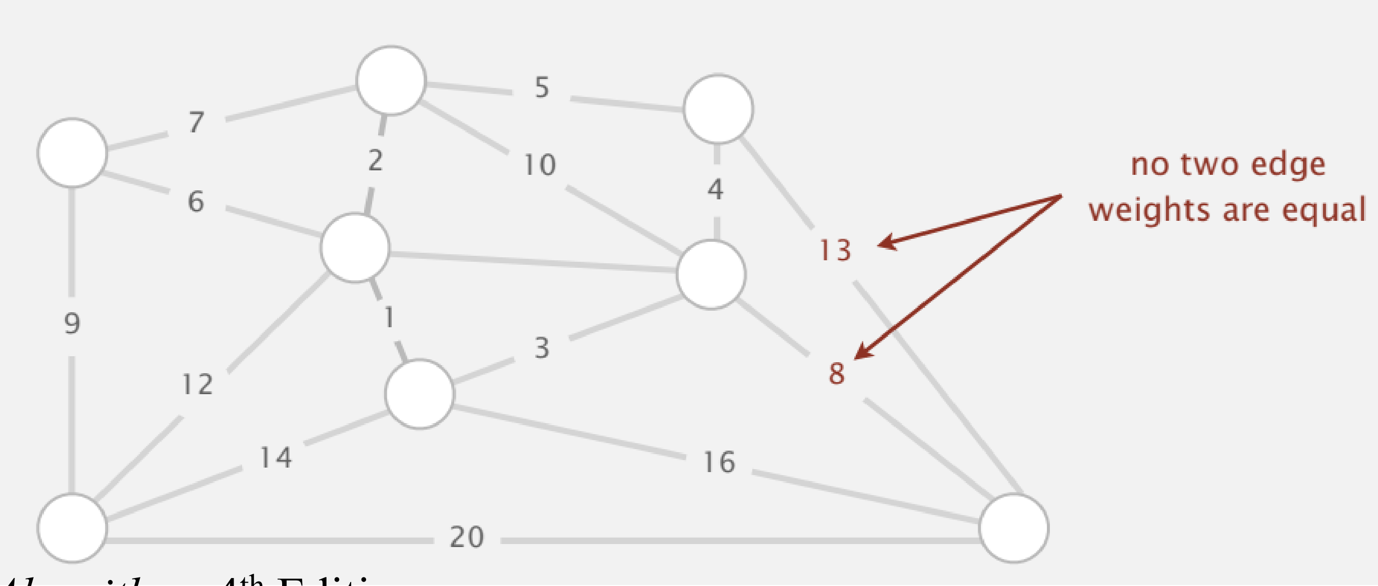
This forms a tree T as a acyclic and connects all of the vertices, which we call the ***minimum-spanning-tree***

Cormen, T. H., Leiserson, C. E., Rivest, R. L., & Stein, C. (2009). *Introduction to Algorithms* (Vol. 3, pp. 624-642). Cambridge: MIT press.

Minimum Spanning Tree

- **“minimum spanning tree”** is a shortened form of the phrase **“minimum-weight spanning tree.”**
- Minimizing the number of edges in *T*,
- Graph is connected and Edge weights are distinct.
- Then, MST exists and is unique.
- All spanning trees have exactly

  |V| - 1 edges



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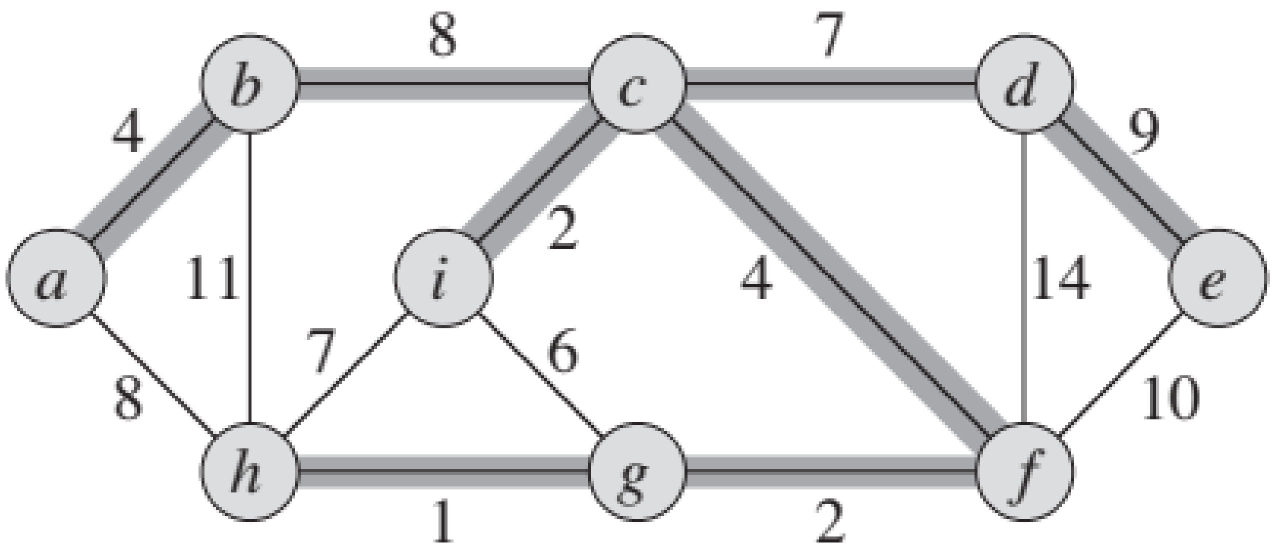
Minimum Spanning Tree

- The weights on edges and the edges in a minimum spanning tree are shaded. The total weight of the tree is 37.
- This minimum spanning tree is not unique: removing the edge (b, c) and replacing it with the edge (a, h) yields another spanning tree with weight 37.



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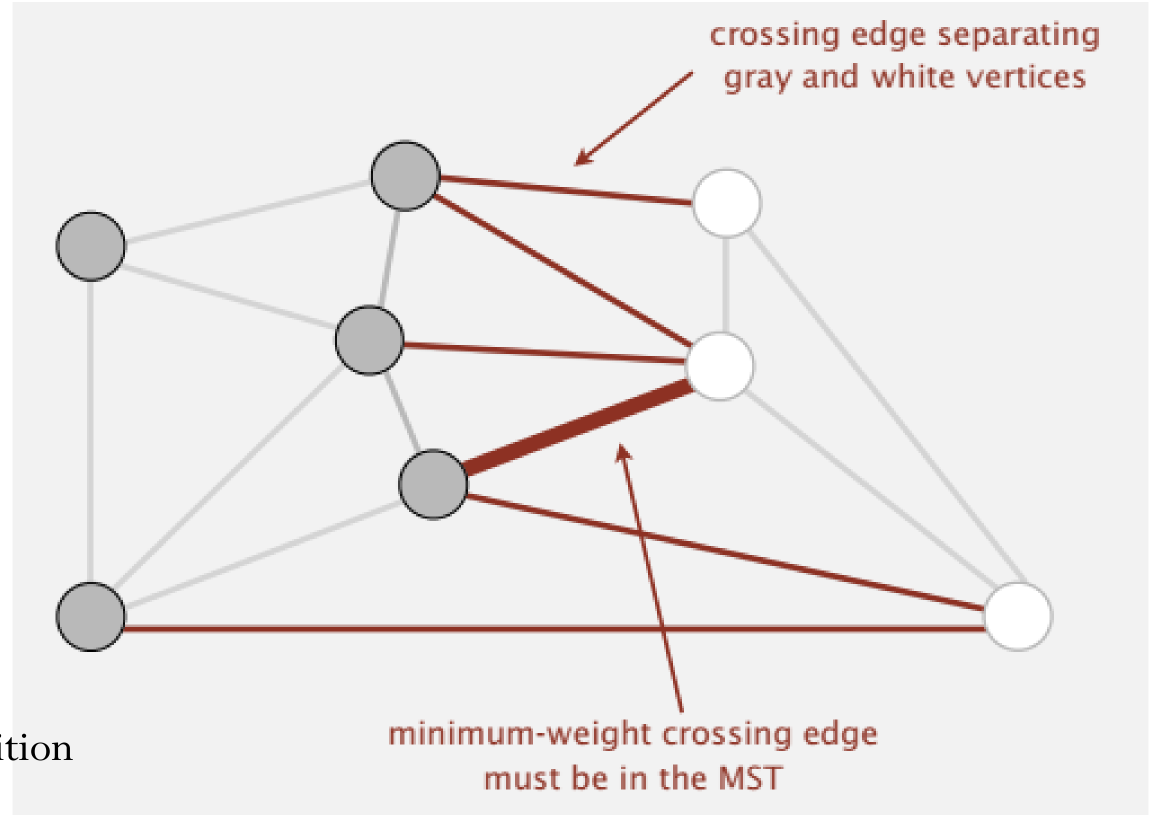
Minimum Spanning Tree and Cut-sets

- A cut in a graph is a partition of its vertices into two (nonempty) sets.
- A crossing edge connects a vertex in one set with a vertex in the other.
- Cut property. Given any cut, the crossing edge of min weight is in the MST.



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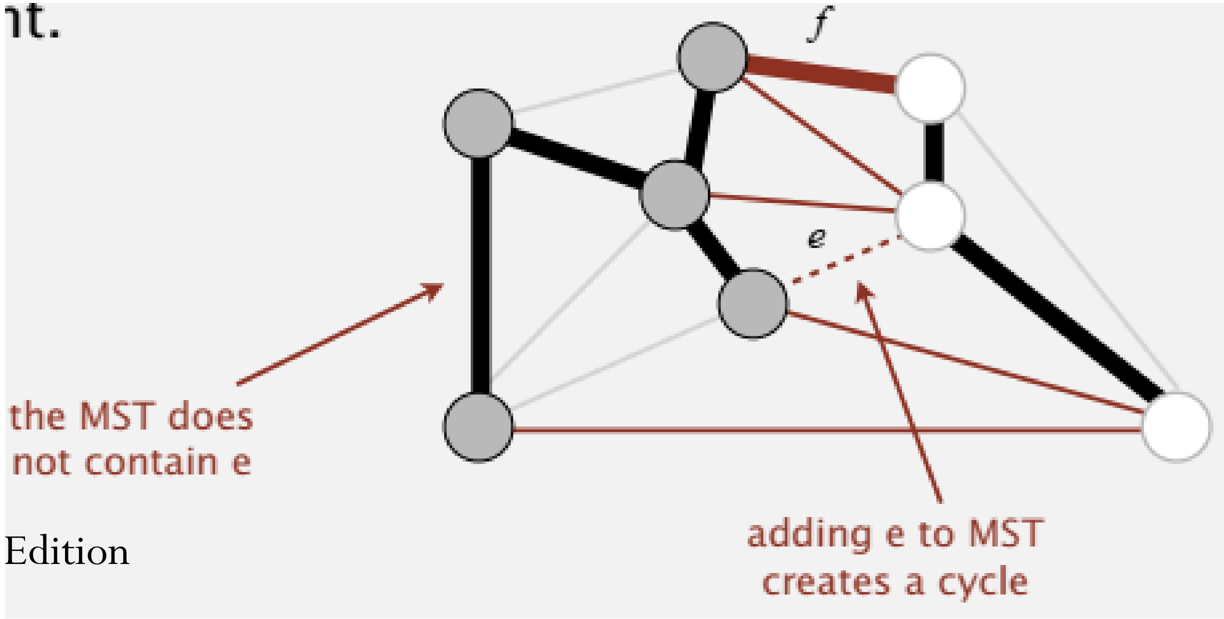
Minimum Spanning Tree and Cut-sets

- Suppose min-weight crossing edge e is not in the MST.
- Adding e to the MST creates a cycle.
- Some other edge f in cycle must be a crossing edge.
- Removing f and adding e is also a spanning tree.
- Since weight of e is less than the weight of f, that spanning tree is lower weight.
- Contradiction



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Generic-MST

```
GENERIC-MST(G, w)
1  A = Ø
2  while A does not form a spanning tree
3      find an edge (u, v) that is safe for A
4      A = A ∪ {(u, v)}
5  return A
```

- ***Safe edge*** for ***A,*** since it can be safely added to A while maintaining the invariant (MST do not form loop or cycle).
- **Initialization:** line 1 the set ***A*** trivially satisfies the invariant.
- **Maintenance:** The loop in lines 2-4 maintains the invariant by adding only safe edges
- **Termination:** All edges (|V|-1) added to ***A* are** in a MST**,** and so the set ***A*** is returned in line 5 must be a minimum spanning **tree.**

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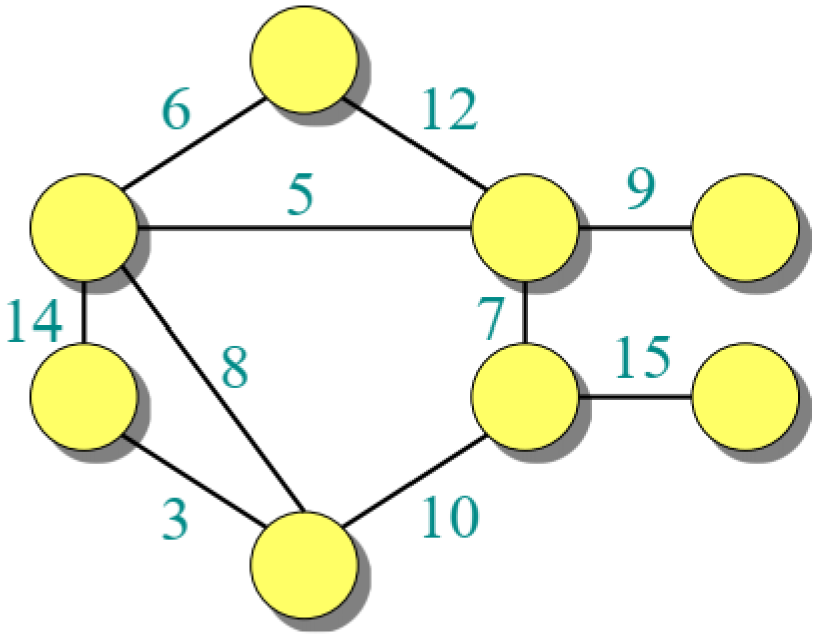
Minimum Spanning Tree



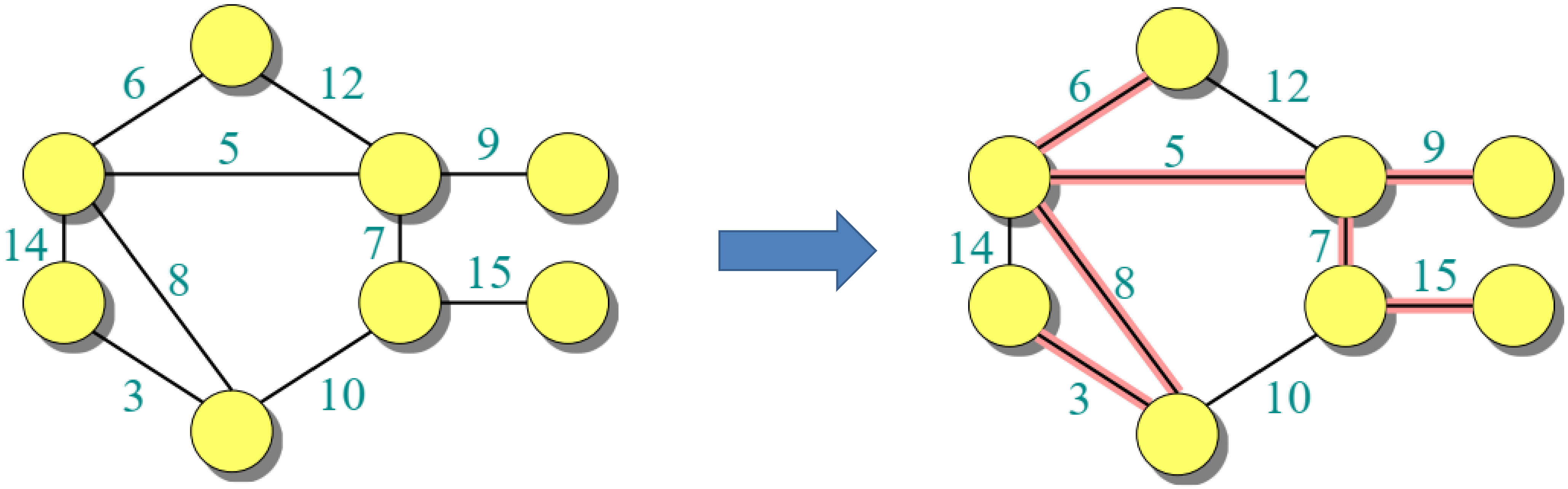
9 + 12 + 6 + 14 + 3 + 10 + 15 = 69

9 + 12 + 6 + 8 + 3 + 10 + 15 = 63

9 + 12 + 6 + 8 + 3 + 7 + 15 = 60

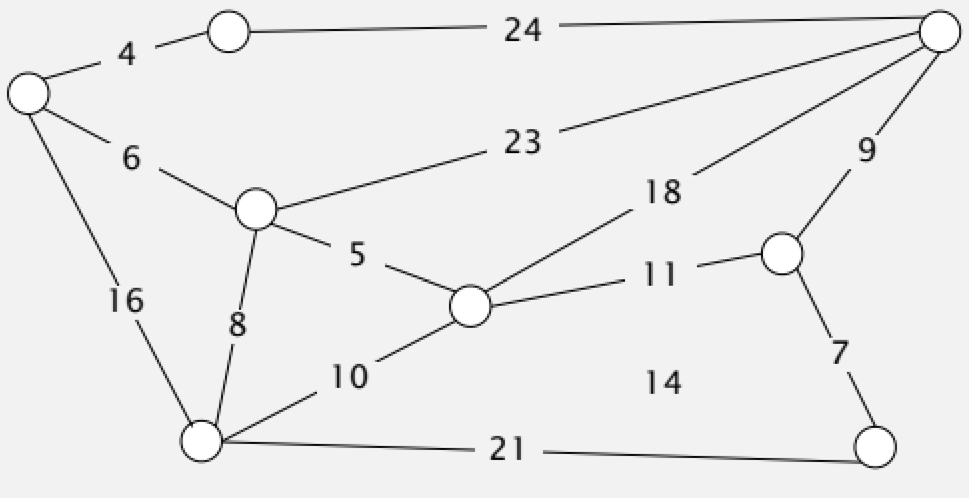
and many more

Minimum Spanning Tree



9 + 5 + 6 + 8 + 3 + 7 + 15 = 53

Minimum Spanning Tree



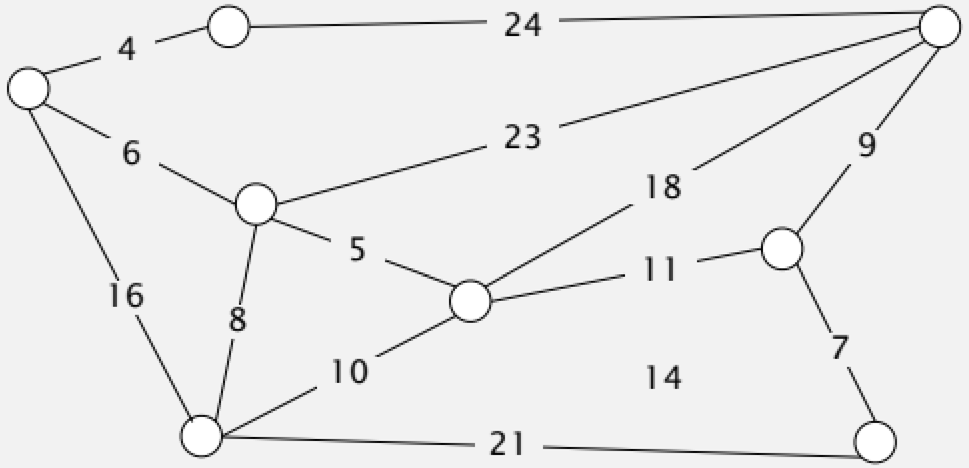
edge-weighted graph G

24 + 4 + 6 + 8 + 10 + 11 + 7 = 70

9 + 4 + 6 + 8 + 10 + 11 + 7 = 55

and many more

Minimum Spanning Tree



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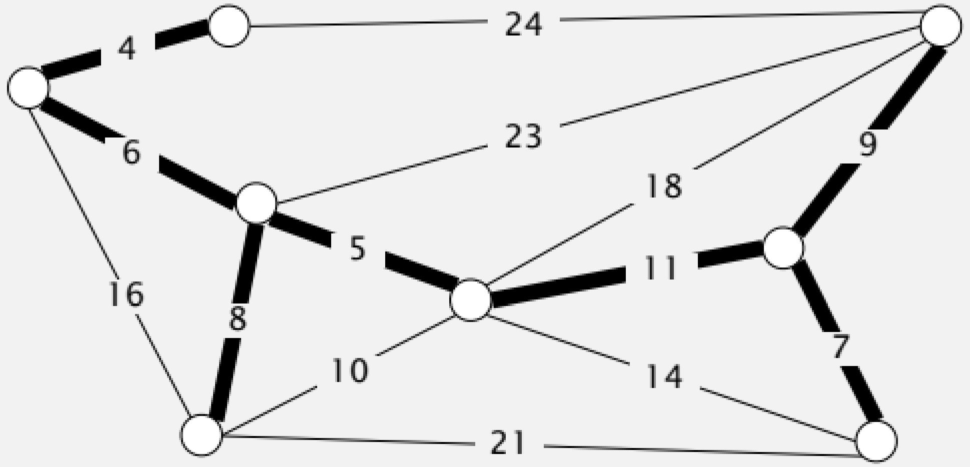

minimum spanning tree T
(cost = 50 = 4 + 6 + 8 + 5 + 11 + 9 + 7)

Minimum Spanning Tree Applications

- Cluster analysis.
- Max bottleneck paths.
- Real-time face verification.
- Find road networks in satellite and aerial imagery.
- Reducing data storage in sequencing amino acids in a protein.
- Model locality of particle interactions in turbulent fluid flows.
- Autoconfig protocol for Ethernet bridging to avoid cycles in a network.
- Approximation algorithms for NP-hard problems (e.g., TSP).
- Network design (communication, electrical, hydraulic, computer, road).

Kruskal’s algorithm and Prim’s algorithm

Kruskal’s algorithm and Prim’s algorithm

- Two algorithms for solving the minimum spanning-tree problem:
  - Kruskal’s algorithm
  - Prim’s algorithm
- The two algorithms are
  - greedy algorithms

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Kruskal's algorithm and Prim's algorithm

- At each step of an algorithm, one of several possible choices. Then,
- **Greedy strategy:** make the choice that is the best at the moment
- Not generally guaranteed to find globally optimal solutions to problems.
- Certain greedy do yield a spanning tree with minimum weight.
- Both algorithms elaborate the generic algorithm because they uses a specific rule to determine a safe edge in line 3 of GENERIC-MST.

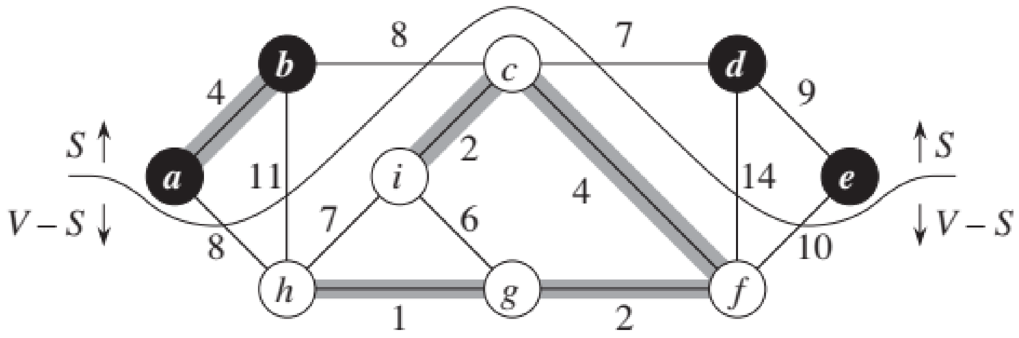
GENERIC-MST(G, w)

1 $A = \emptyset$
2 **while** A does not form a spanning tree
3   find an edge (u, v) that is safe for A
4     $A = A \cup \{(u, v)\}$
5 **return** A

Cormen, T. H., Leiserson, C. E., Rivest, R. L., & Stein, C. (2009). *Introduction to Algorithms* (Vol. 3, pp. 624-642). Cambridge: MIT press.

Kruskal's algorithm and Prim's algorithm

- An edge is a ***light edge*** crossing a cut if its weight is the minimum of any edge crossing the cut.
- Light edge = minimum-weight crossing edge
- ***Light edge*** satisfy MST properties, and its weight is the minimum of any other edges satisfying the MST properties.
- more than one light edge crossing a cut in the case of ties.



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Kruskal's algorithm

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return A

- **In Kruskal's algorithm**,
  - the set A is a forest.
  - the safe edge added to A is always a least-weight edge in the graph that connects two distinct components.
- Greedy algorithm because it adds an edge of least possible weight to the forest.

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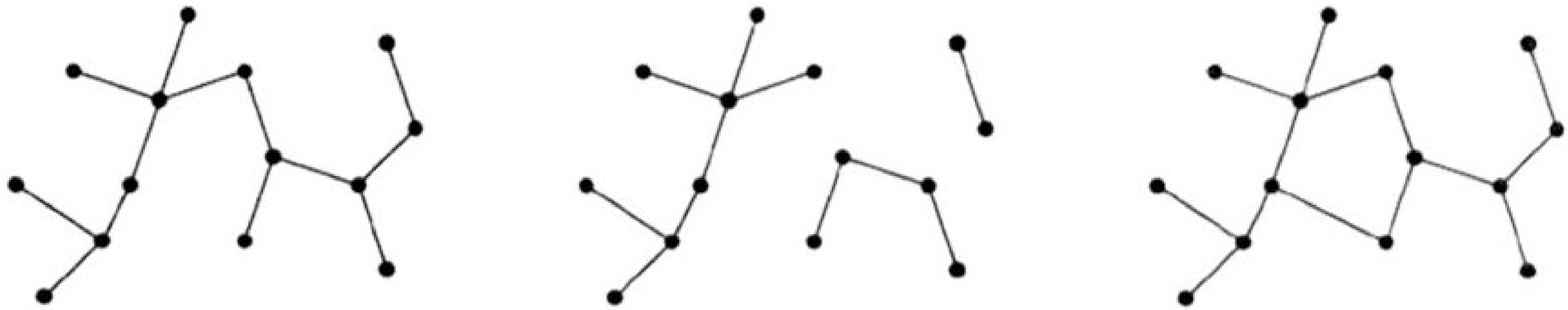
return A

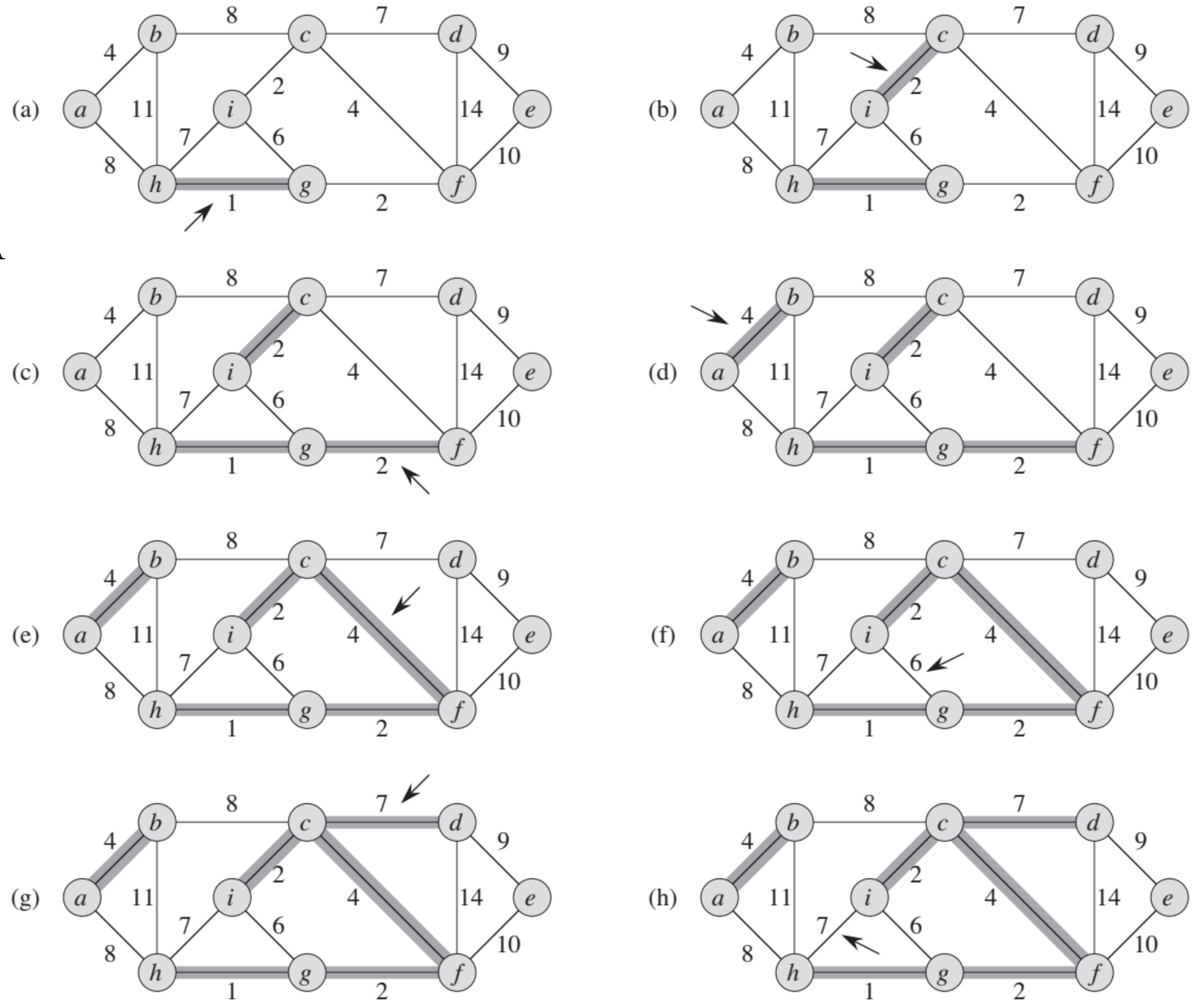
- **In Kruskal's algorithm**,
  - the set A is a **forest**.
  - the safe edge added to A is always a least-weight edge in the graph that connects two distinct components.
- A tree is a connected, acyclic, undirected graph.
- A **forest** is a set of trees (non necessarily connected)



Tree Forest Graph with Cycle

Kruskal’s algorithm

- Shaded edges belong to the forest A being grown.
- The edges are considered by the algorithm in sorted order by weight.
- An arrow points to the edge under consideration at each step of the algorithm.
- If the edge joins two distinct trees in the forest, it is added to the forest, thereby merging the two trees.



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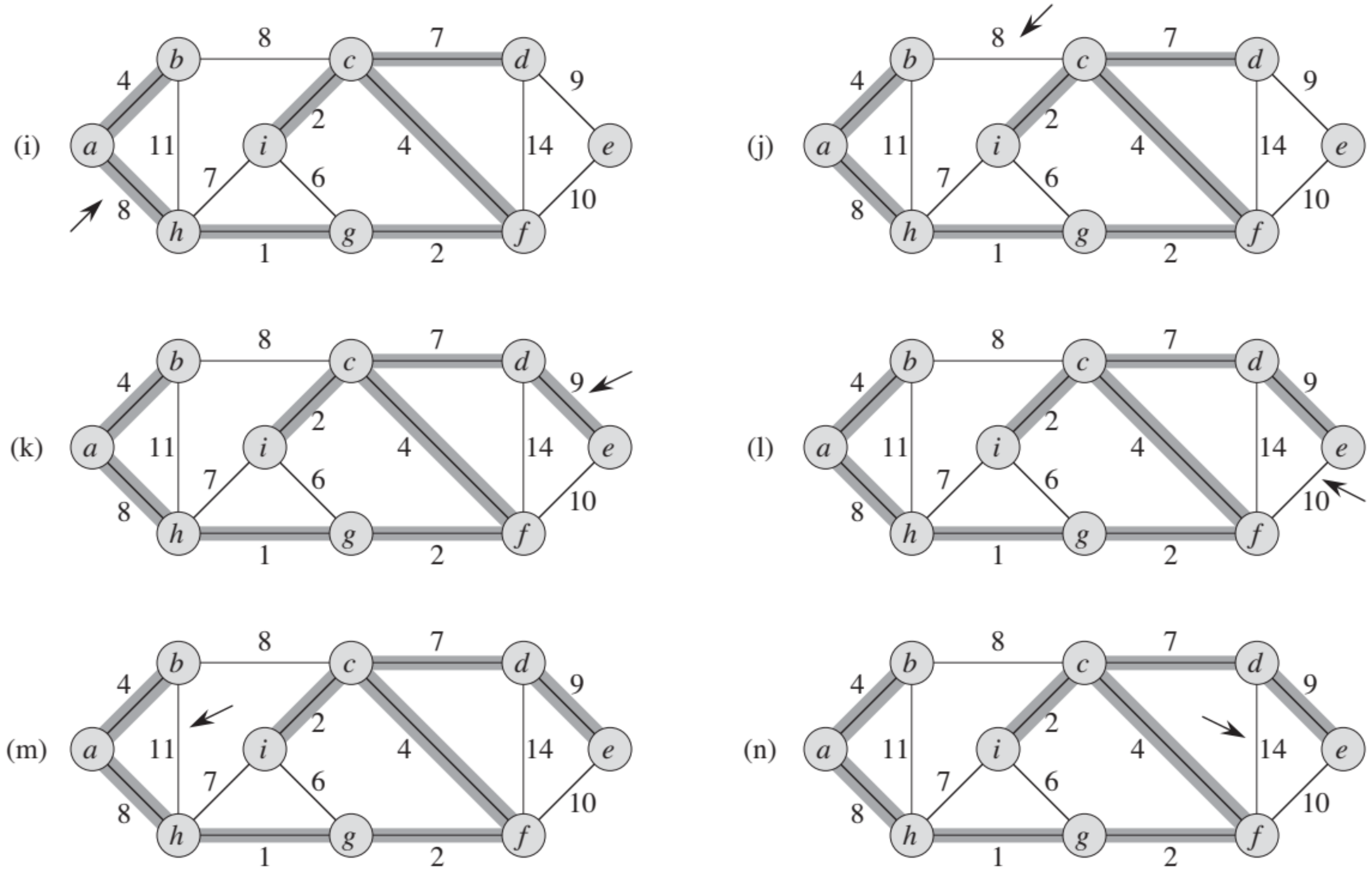

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Kruskal’s algorithm

- It uses a **disjoint-set** data structure to maintain several disjoint sets of elements.
- A **disjoint-set** data structure keeps track of a set of elements partitioned into a number of disjoint (non-overlapping) subsets.
- A **union-find** data structure performs three useful operations
- **Making** a new set containing a new element.
- **Find**: Determine which subset a particular element. This can be used for determining whether two elements are in the same subset.
- **Union**: Join two subsets into a single set.

https://en.wikipedia.org/wiki/Disjoint-set_data_structure
Cormen, T. H., Leiserson, C. E., Rivest, R. L., & Stein, C. (2009). *Introduction to Algorithms* (Vol. 3, pp. 624-642). Cambridge: MIT press.

Kruskal's algorithm

```
MST-KRUSKAL(G, w)
A = ∅
for each vertex v ∈ G.V
    MAKE-SET(v)
sort the edges of G.E into nondecreasing order by weight w
for each edge (u, v) ∈ G.E, taken in nondecreasing order by weight
    if FIND-SET(u) ≠ FIND-SET(v)
        A = A ∪ {(u, v)}
        UNION(u, v)
return A
```

- Each set contains the vertices in **a tree** of the current forest. The operation **FIND-SET(u)** returns a representative element from the set that contains ***u*.**
- **Thus,** we can determine whether two vertices ***u*** and **v** belong to the same tree by testing whether **FIND-SET(u)** equals **FIND-SET(v).**
- The combining of trees is accomplished by the **UNION** procedure.

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5  for each edge (u, v) ∈ G.E, taken in nondecreasing order by weight
6      if FIND-SET(u) ≠ FIND-SET(v)
7          A = A ∪ {(u, v)}
8          UNION(u, v)
9  return A
```

- Lines **1-3:** initialize the set ***A*** to the empty set and create trees, one containing each vertex.
- Line **4:** The edges in ***E*** are sorted into non-decreasing order by weight**.**

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Kruskal's algorithm

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6      if FIND-SET(u) ≠ FIND-SET(v)
7          A = A ∪ {(u, v)}
8          UNION(u, v)
9  return A
```

- The **for** loop in lines ***5-8*** checks, for each edge ***(u, v),*** whether the endpoints u and *v* belong to the same tree.
  - If they do, then the edge (u, v) cannot be added to the forest because it create a cycle, thus the edge is discarded.
  - Otherwise, the two vertices belong to different trees.
- In this case, the edge ***(u, v)*** is added to ***A*** in line 7, and the vertices in the two trees are merged in line **8.**

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Kruskal's algorithm

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6      if FIND-SET(u) ≠ FIND-SET(v)
7          A = A ∪ {(u, v)}
8          UNION(u, v)
9  return A
```

- Running time depends on the implementation of the disjoint set data structure
- the set A in line 1 takes O(1) time,
- MAKE-SET operations in the **for** loop of lines 2–3 takes O(V),
- the **time to sort the edges in line 4** is **O(E lg E),**
- the **for** loop of lines 5–8 performs O(E) FIND-SET and UNION operations on the disjoint-set forest.
- Observing that $|E| < |V|^2$, we have $\mathbf{lg}\,|E| = \mathbf{O(lg\,V)}$, and so we can restate the running time of Kruskal's algorithm as **O(E lg V)**.

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GENERIC-MST(G, w)

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1  A = Ø
2  while A does not form a spanning tree
3      find an edge (u, v) that is safe for A
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5  return A
```

Prim's algorithm

- Both **Kruskal's and Prim's algorithm**
  - elaborate the generic algorithm because they uses a specific rule to determine a safe edge in line 3 of GENERIC-MST.
- **In Prim's algorithm**,
  - the set A forms a **single tree**.
  - the safe edge added to A is always a least-weight edge connecting the tree to a vertex not in the tree.
- Greedy algorithm because the tree is augmented at each step with an edge that contributes the minimum amount possible to the tree's weight.

Prim's Algorithm

- Prim's algorithm operates much like Dijkstra's algorithm for finding shortest paths in a graph.
- The edges in the set A always form a single tree.
- The tree starts from an arbitrary vertex and grows until the tree spans all the vertices in V.
- At each step, a light edge is added to the tree A, that connects A to an isolated vertex.

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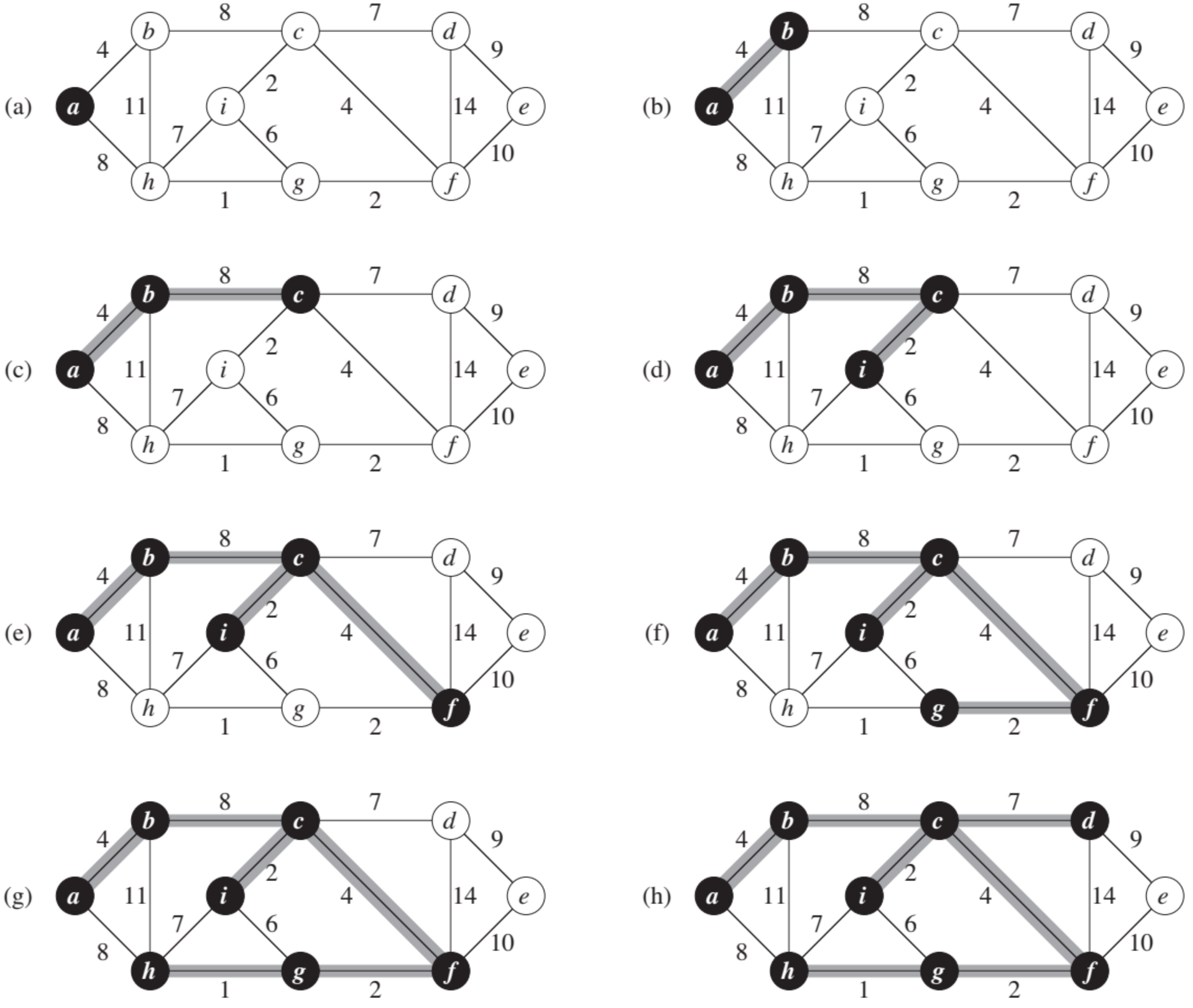
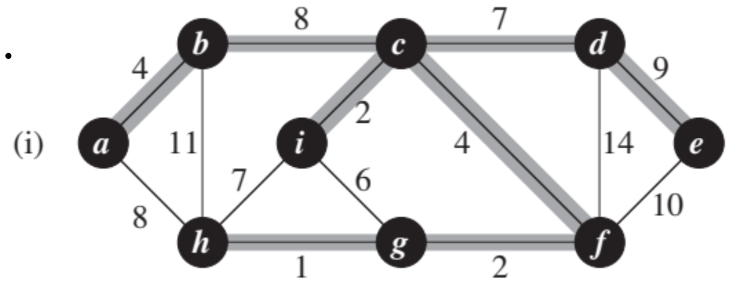
Prim’s Algorithm



- The root vertex is *a*. Shaded edges are in the tree being grown, and the vertices in the tree are shown in black.
- The vertices in the tree determine a cut of the graph, and a light edge crossing the cut is added to the tree.
- the algorithm has a choice of adding either edge (b, c) or edge (a, h) to the tree since both are light edges crossing the cut.



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Prim's Algorithm

Lines **1-5** set the **key of each vertex to** ∞ (except for the **root** ***r***, whose **key is set to 0** so that it will be the first vertex processed).

Set the parent of each vertex to **NIL**, and initialize the min-priority queue ***Q*** to contain all the vertices.

- *v.key* is the minimum weight of any edge connecting v to a vertex in the tree
- *v.*∏ is the names the parent of v in the tree

```
MST-PRIM(G, w, r)
1  for each u ∈ G.V
2      u.key = ∞
3      u.π = NIL
4  r.key = 0
5  Q = G.V
6  while Q ≠ ∅
7      u = EXTRACT-MIN(Q)
8      for each v ∈ G.Adj[u]
9          if v ∈ Q and w(u, v) < v.key
10             v.π = u
11             v.key = w(u, v)
```

Cormen, T. H., Leiserson, C. E., Rivest, R. L., & Stein, C. (2009). *Introduction to Algorithms* (Vol. 3, pp. 624-642). Cambridge: MIT press.

Prim's Algorithm

Prior to each iteration of the **while** loop of lines 6–11,

1. $A = \{(v, v.\pi) : v \in V - \{r\} - Q\}$.
2. The vertices already placed into the minimum spanning tree are those in $V - Q$.
3. For all vertices $v \in Q$, if $v.\pi \neq \text{NIL}$, then $v.key < \infty$ and $v.key$ is the weight of a light edge $(v, v.\pi)$ connecting v to some vertex already placed into the minimum spanning tree.

MST-PRIM(G, w, r)

1 **for** each $u \in G.V$
2   $u.key = \infty$
3   $u.\pi = \text{NIL}$
4 $r.key = 0$
5 $Q = G.V$
6 **while** $Q \neq \emptyset$
7   $u = \text{EXTRACT-MIN}(Q)$
8   **for** each $v \in G.Adj[u]$
9   **if** $v \in Q$ and $w(u, v) < v.key$
10       $v.\pi = u$
11       $v.key = w(u, v)$

Cormen, T. H., Leiserson, C. E., Rivest, R. L., & Stein, C. (2009). *Introduction to Algorithms* (Vol. 3, pp. 624-642). Cambridge: MIT press.

Prim’s Algorithm

The for loop of lines 8-11 update the *key* and ∏ fields of every vertex ***v*** adjacent to ***u*** but not in the tree. The updating maintains the third part of the loop invariant.

```
MST-PRIM(G, w, r)
1  for each u ∈ G.V
2      u.key = ∞
3      u.π = NIL
4  r.key = 0
5  Q = G.V
6  while Q ≠ ∅
7      u = EXTRACT-MIN(Q)
8      for each v ∈ G.Adj[u]
9          if v ∈ Q and w(u, v) < v.key
10             v.π = u
11             v.key = w(u, v)
```

Cormen, T. H., Leiserson, C. E., Rivest, R. L., & Stein, C. (2009). *Introduction to Algorithms* (Vol. 3, pp. 624-642). Cambridge: MIT press.

Prim's Algorithm

- the **BUILD-MIN-HEAP** procedure to perform lines 1–5 in **O(V) time**.
- **while** loop executes **|V|** times, and each **EXTRACT-MIN** operation takes **O(lg V) time**, the total time is **O(V lg V)**.
- The **for** loop in lines 8–11 executes **O(E)** times, and line 11 involves in **O(lg V)** time, the total time is **O(E lg V)**.
- The total time is **O(V lg V + E lg V) = O(E lg V)**, which is asymptotically the same as for Kruskal's algorithm.

MST-PRIM(G, w, r)

```
1   for each u ∈ G.V
2       u.key = ∞
3       u.π = NIL
4   r.key = 0
5   Q = G.V
6   while Q ≠ ∅
7       u = EXTRACT-MIN(Q)
8       for each v ∈ G.Adj[u]
9           if v ∈ Q and w(u, v) < v.key
10              v.π = u
11              v.key = w(u, v)
```

Cormen, T. H., Leiserson, C. E., Rivest, R. L., & Stein, C. (2009). *Introduction to Algorithms* (Vol. 3, pp. 624-642). Cambridge: MIT press.

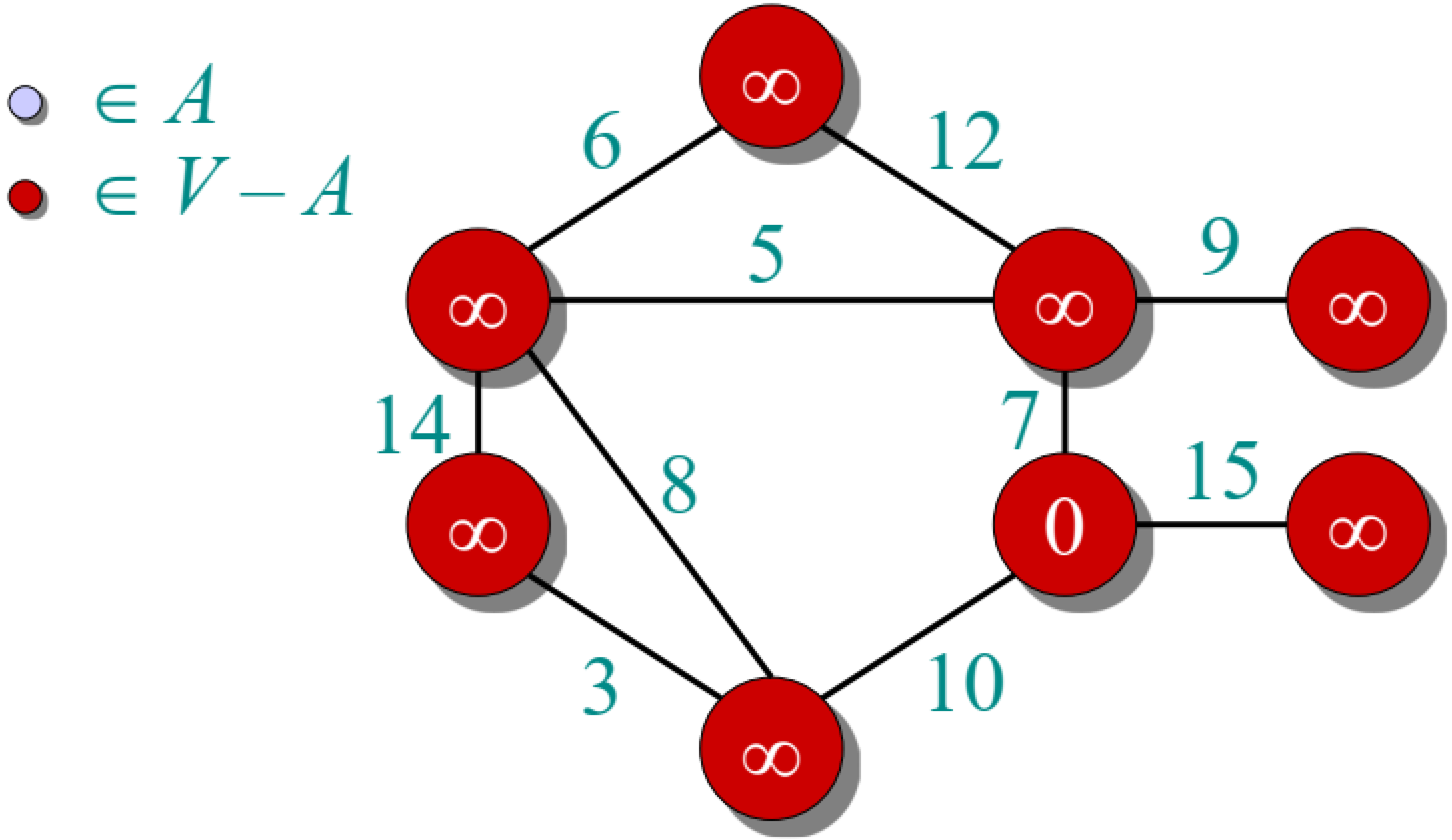
Prim's Algorithm

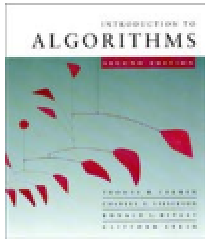
- The running time of Prim's algorithm depends on how min-priority queue Q is implemented. Implement Q as a **binary min-heap**,
- **while** loop executes |**V**| times, and each EXTRACT-MIN operation takes **O(lg V)**, the total time is **O(V lg V)**.
- The **for** loop in lines 8–11
  - executes **O(E)** times,
- line 11 takes **O(1)**,
  - the total time is **O(E)**.
- the running time of Prim's algorithm improves to **O(E + V lg V)**.

```
MST-PRIM(G, w, r)
1   for each u ∈ G.V
2       u.key = ∞
3       u.π = NIL
4   r.key = 0
5   Q = G.V
6   while Q ≠ ∅
7       u = EXTRACT-MIN(Q)
8       for each v ∈ G.Adj[u]
9           if v ∈ Q and w(u, v) < v.key
10              v.π = u
11              v.key = w(u, v)
```

Cormen, T. H., Leiserson, C. E., Rivest, R. L., & Stein, C. (2009). *Introduction to Algorithms* (Vol. 3, pp. 624-642). Cambridge: MIT press.

Example of Prim's algorithm

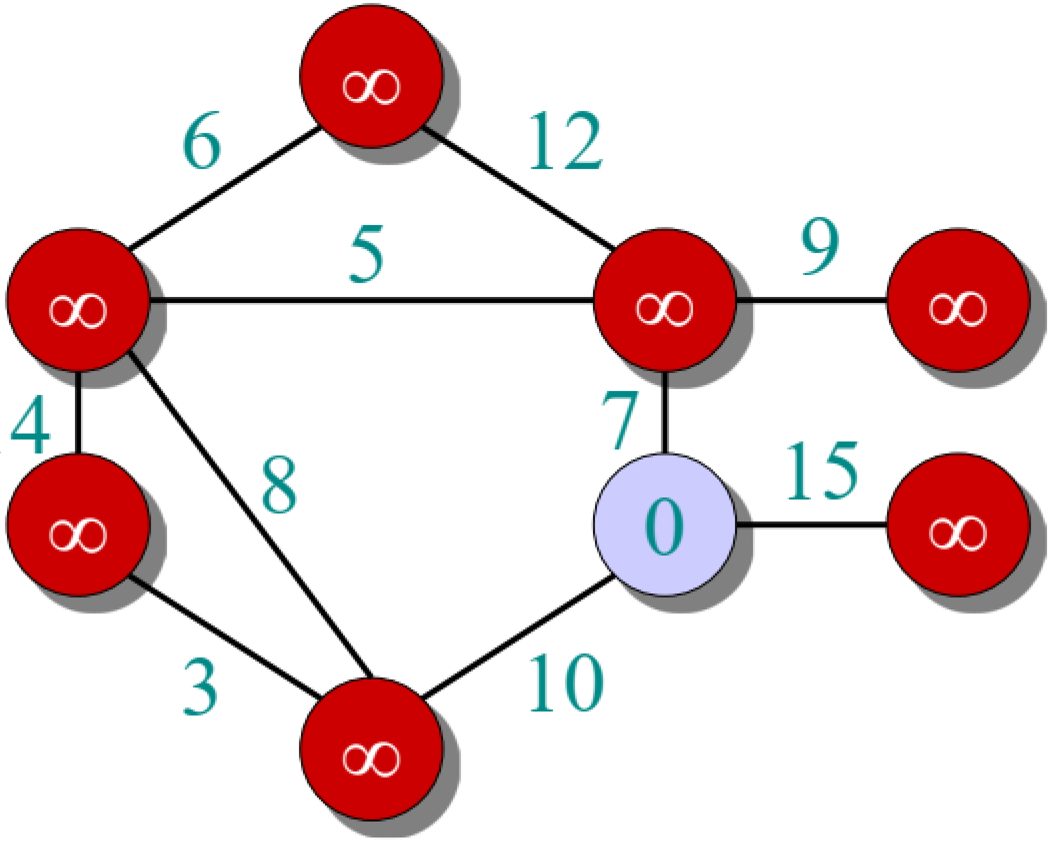


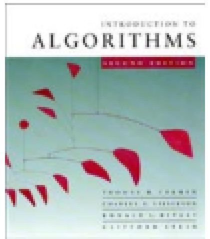


Example of Prim’s algorithm

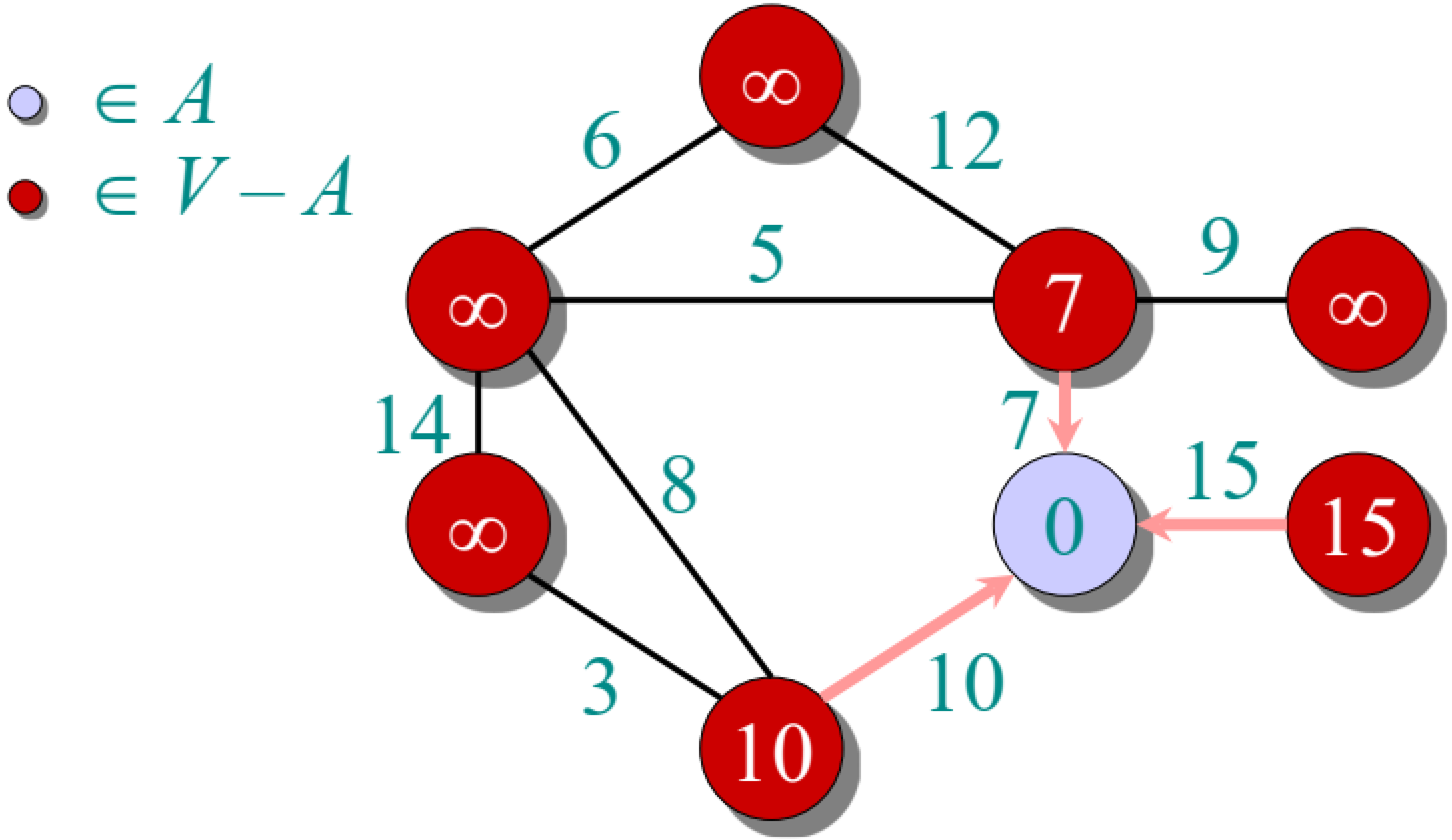
○ $\in A$

● $\in V - A$

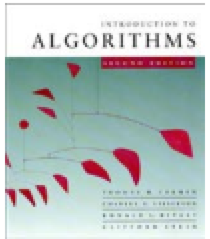


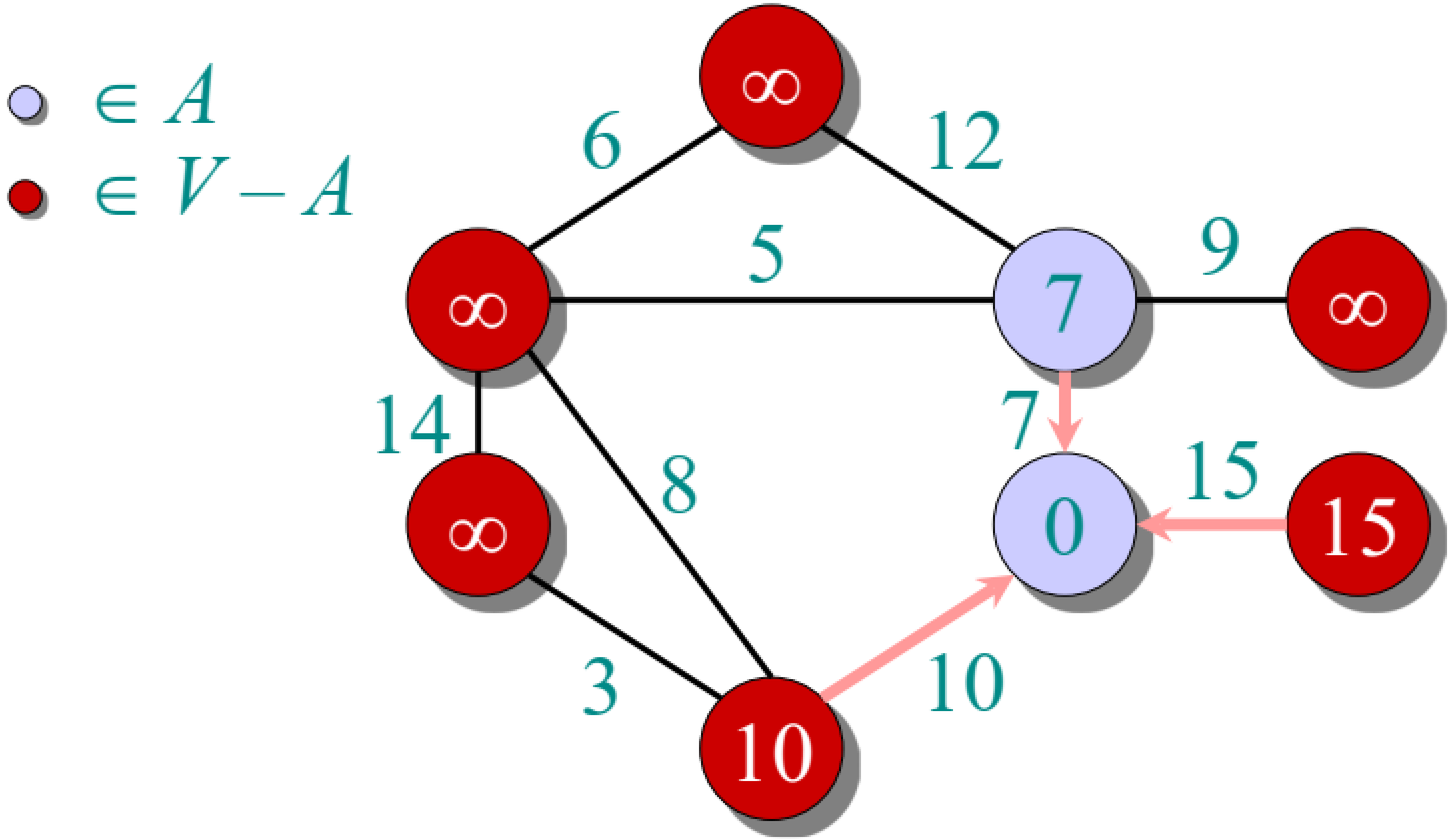


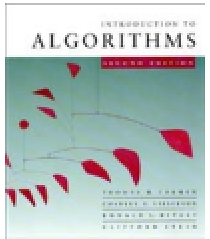
Example of Prim's algorithm



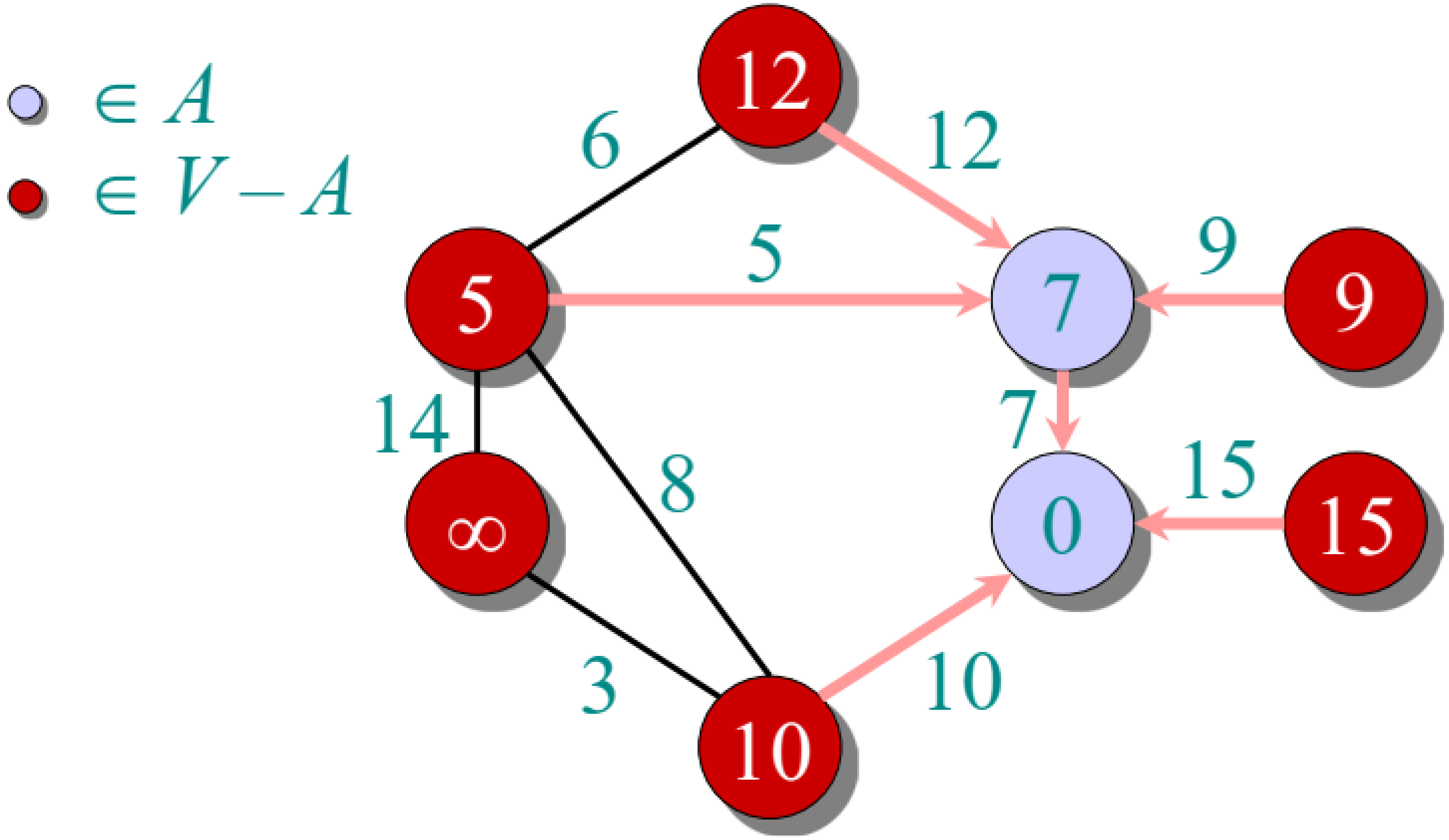
Example of Prim's algorithm

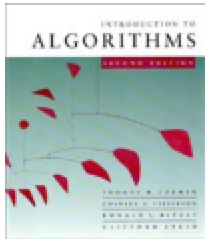






Example of Prim's algorithm





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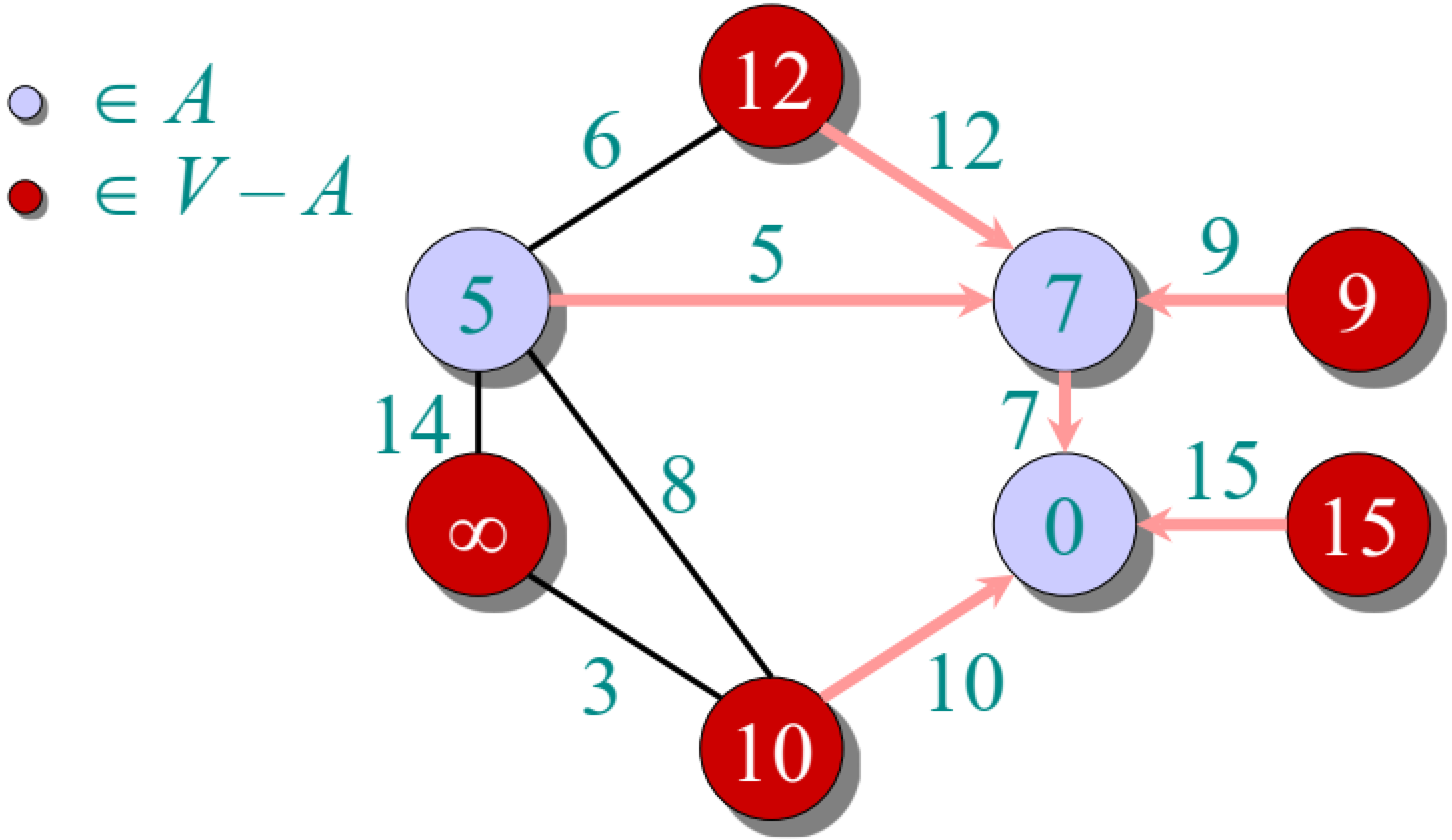
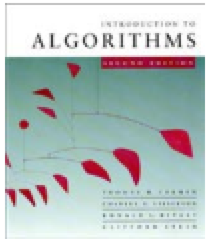
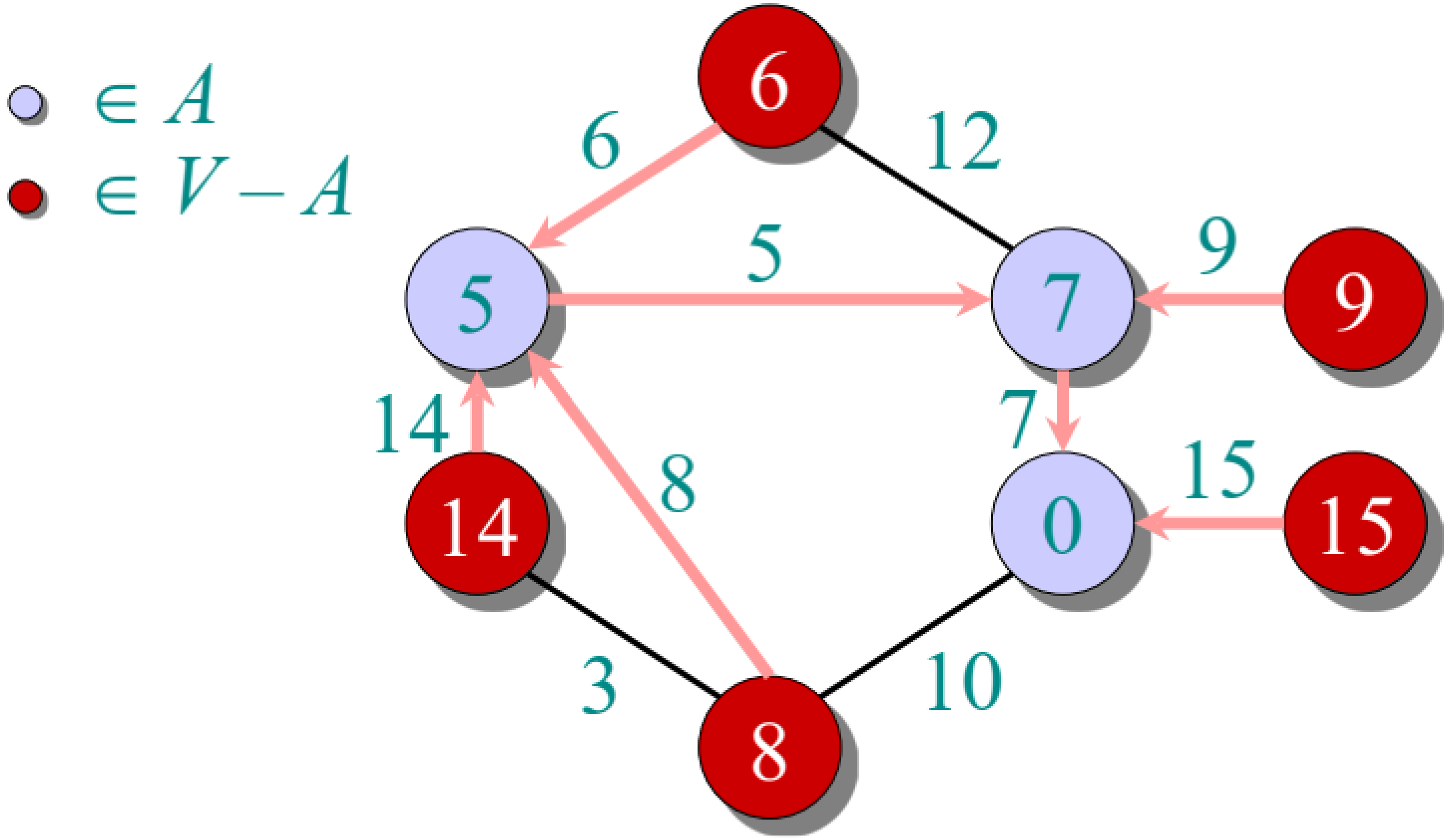
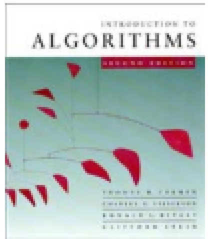


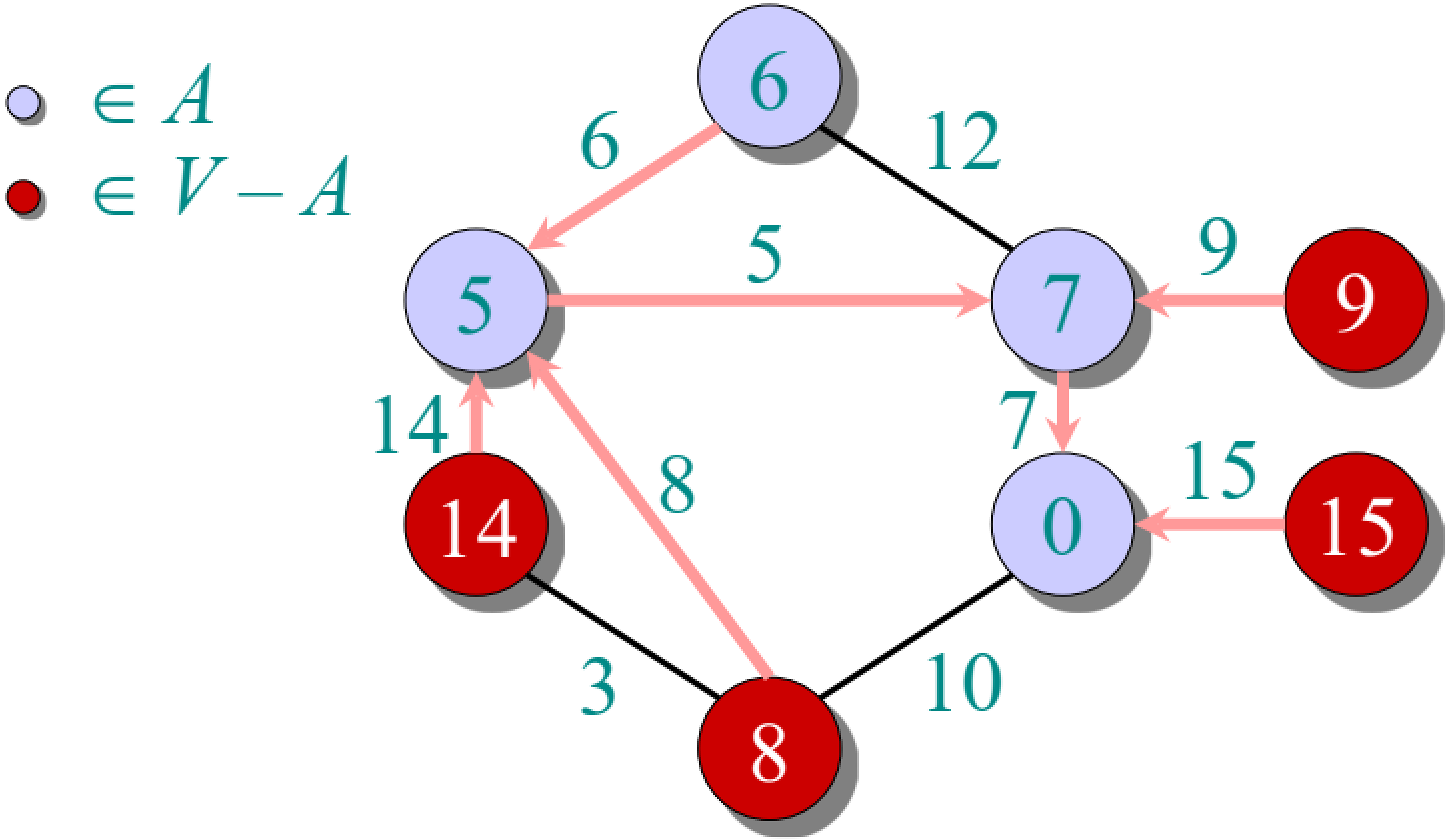


Example of Prim's algorithm

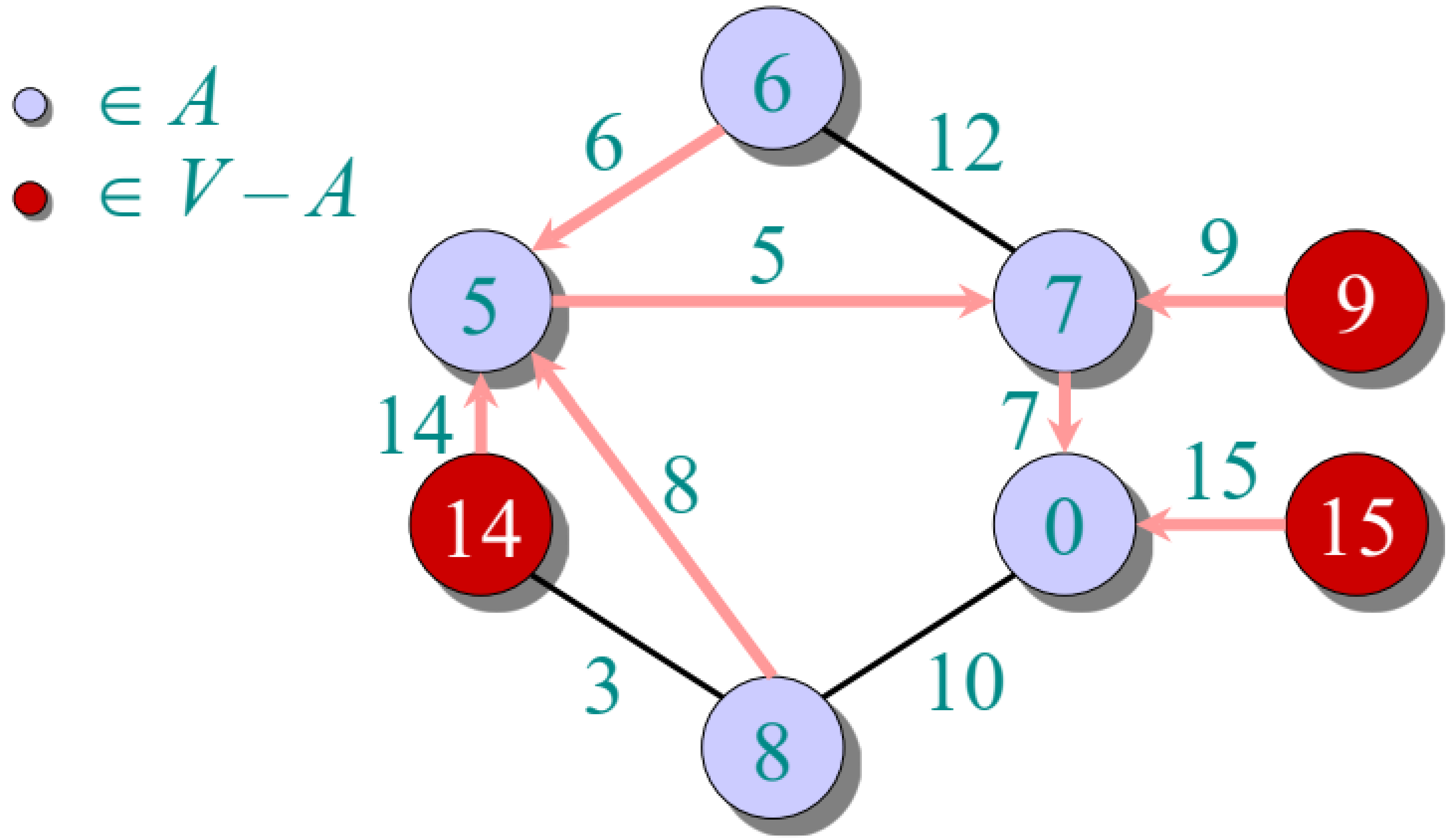


Example of Prim's algorithm





Example of Prim's algorithm



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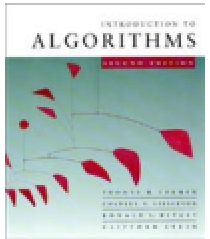

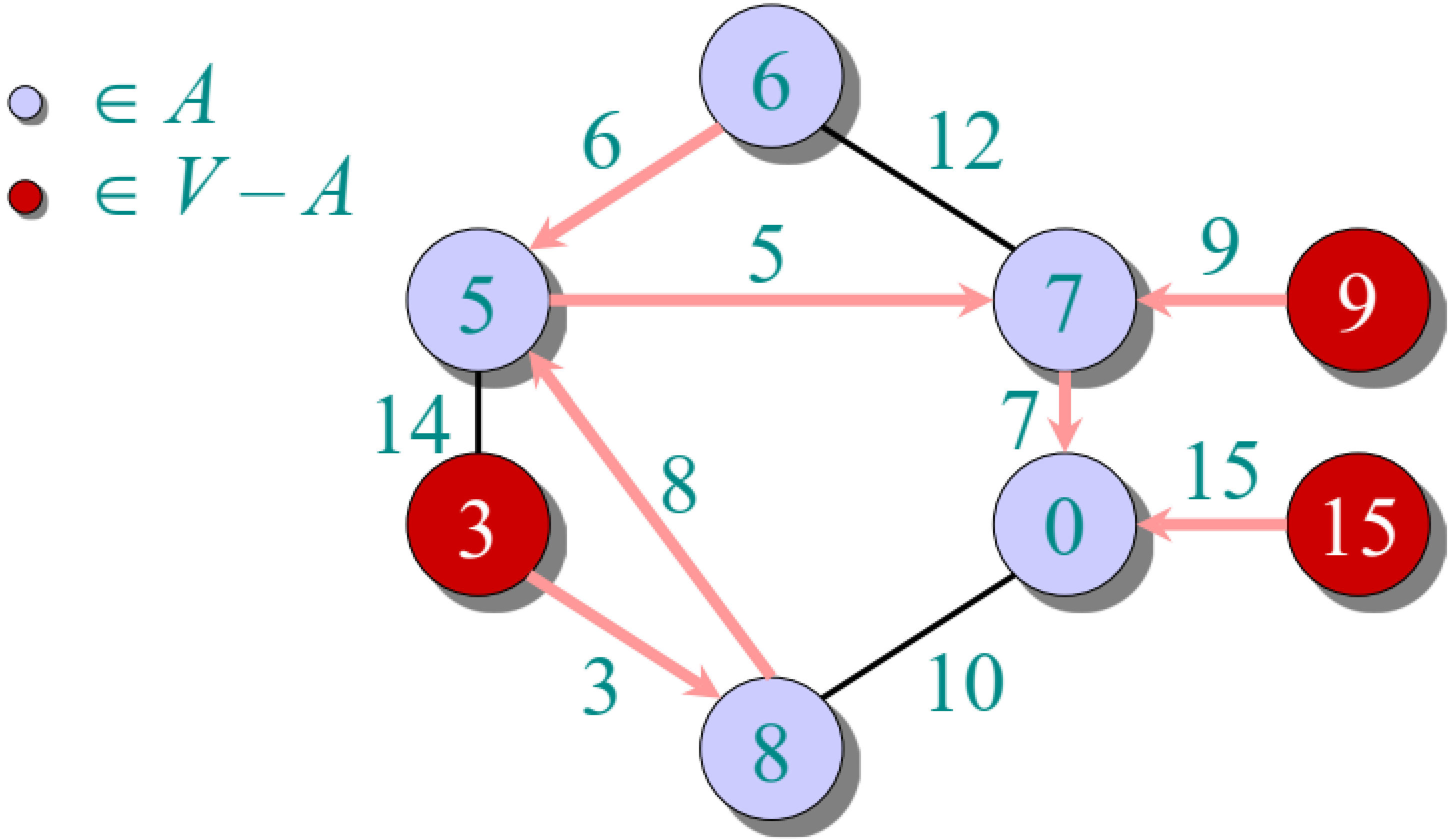

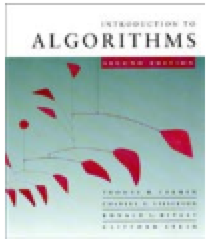

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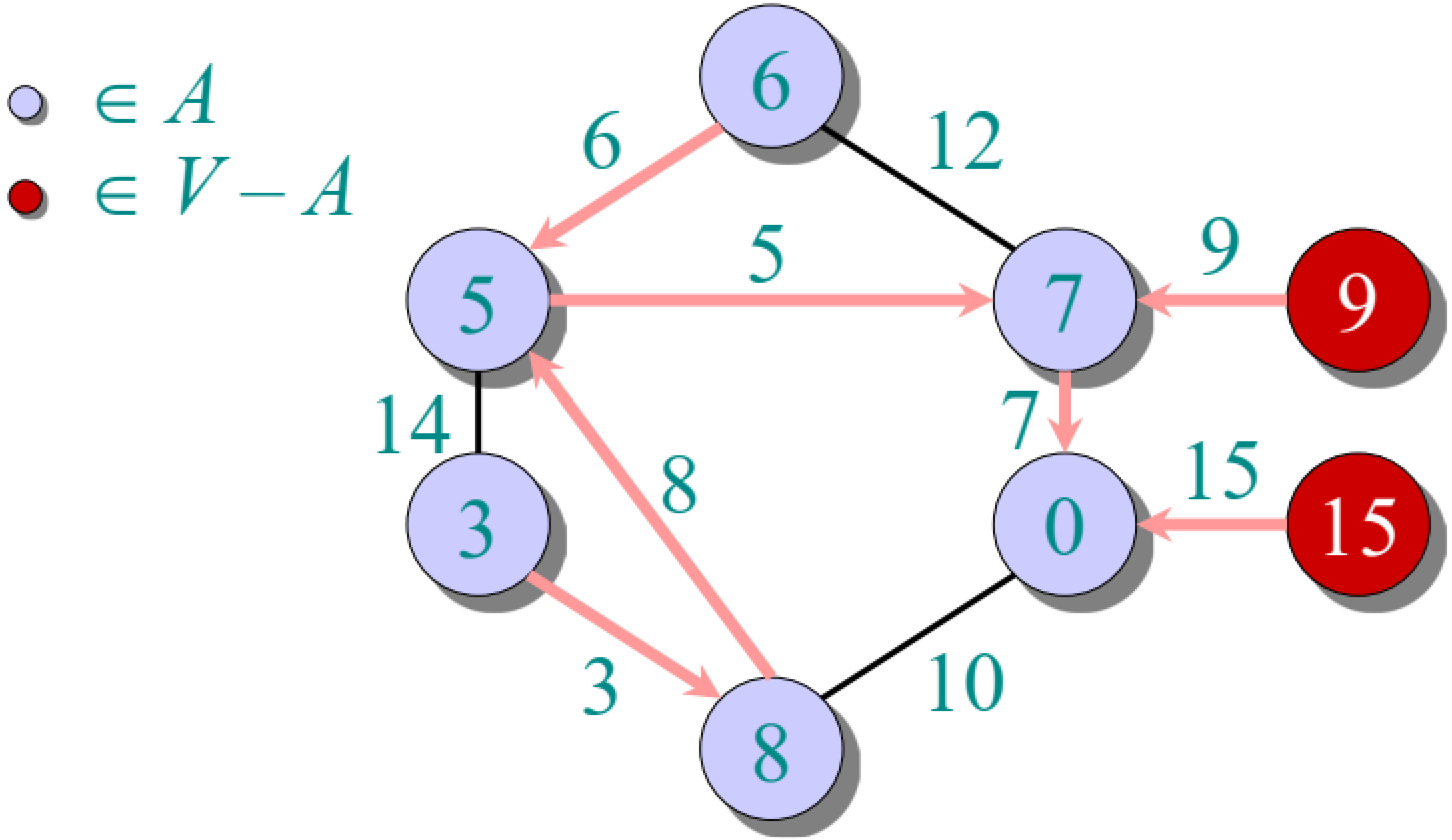

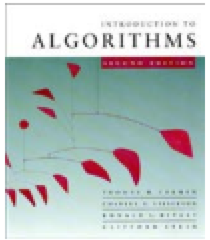

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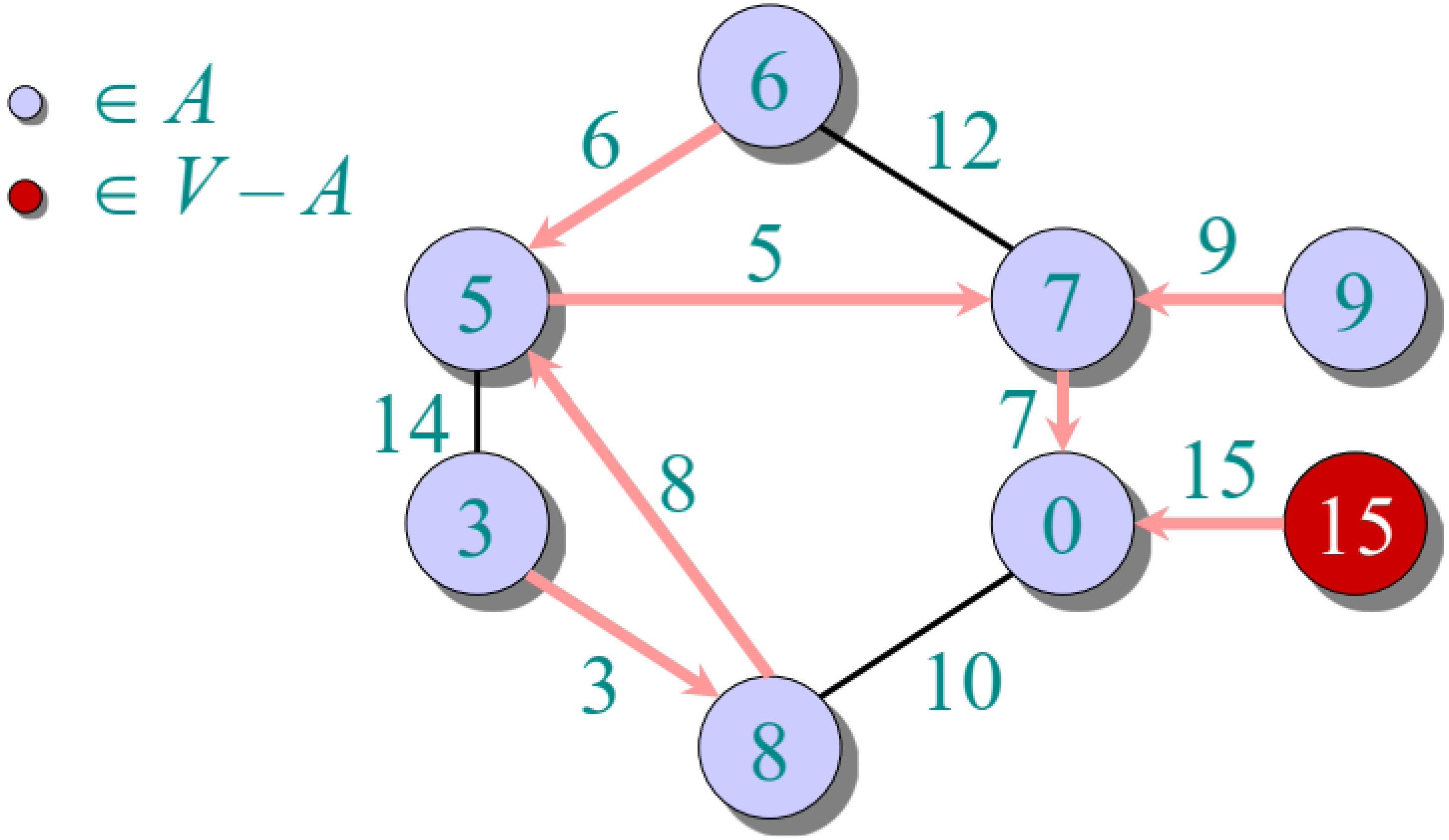

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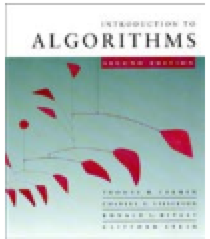

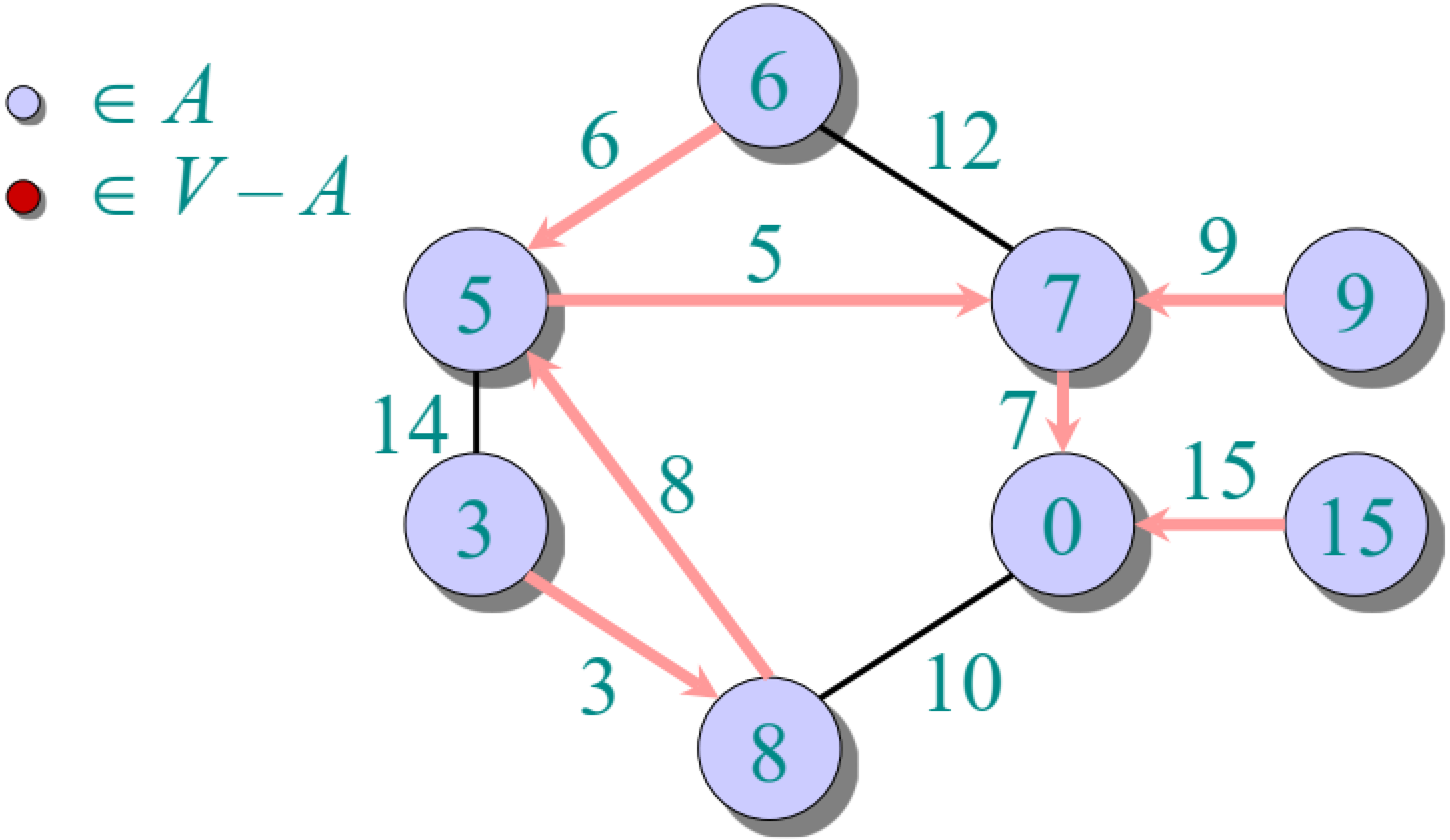

Applications

application	item	connection
map	intersection	road
web content	page	link
circuit	device	wire
schedule	job	constraint
commerce	customer	transaction
matching	student	application
computer network	site	connection
software	method	call
social network	person	friendship

Java code Implementation of Kruskal’s algorithm and Prim’s algorithm

Java Implementation

```
public class EdgeWeightedGraph
{
   private final int V;
   private final Bag<Edge>[] adj;

   public EdgeWeightedGraph(int V)
   {
      this.V = V;
      adj = (Bag<Edge>[]) new Bag[V];
      for (int v = 0; v < V; v++)
         adj[v] = new Bag<Edge>();
   }

   public void addEdge(Edge e)
   {
      int v = e.either(), w = e.other(v);
      adj[v].add(e);
      adj[w].add(e);
   }

   public Iterable<Edge> adj(int v)
   {  return adj[v];  }
}
```

same as Graph, but adjacency lists of Edges instead of integers

constructor

add edge to both adjacency lists

```
public static void main(String[] args)
{
   In in = new In(args[0]);
   EdgeWeightedGraph G = new EdgeWeightedGraph(in);
   MST mst = new MST(G);
   for (Edge e : mst.edges())
      StdOut.println(e);
   StdOut.printf("%.2f\n", mst.weight());
}
```

Java Implementation

```
public class Edge implements Comparable<Edge>
{
   private final int v, w;
   private final double weight;

   public Edge(int v, int w, double weight)
   {
      this.v = v;
      this.w = w;
      this.weight = weight;
   }

   public int either()
   {  return v;  }

   public int other(int vertex)
   {
      if (vertex == v) return w;
      else return v;
   }

   public int compareTo(Edge that)
   {
      if      (this.weight < that.weight) return -1;
      else if (this.weight > that.weight) return +1;
      else                                return  0;
   }
}
```

constructor

either endpoint

other endpoint

compare edges by weight

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Java Implementation – Kruskal's Algorithm

```java
public static void main(String[] args) {
    In in = new In(args[0]);
    EdgeWeightedGraph G = new EdgeWeightedGraph(in);
    KruskalMST mst = new KruskalMST(G);
    for (Edge e : mst.edges()) {
        StdOut.println(e);
    }
    StdOut.printf("%.5f\n", mst.weight());
}
```

MST-KRUSKAL(G, w)

```
A = ∅
for each vertex v ∈ G.V
    MAKE-SET(v)
sort the edges of G.E into nondecreasing order by weight w
for each edge (u, v) ∈ G.E, taken in nondecreasing order by weight
    if FIND-SET(u) ≠ FIND-SET(v)
        A = A ∪ {(u, v)}
        UNION(u, v)
return A
```

```java
public KruskalMST(EdgeWeightedGraph G) {

    // create array of edges, sorted by weight
    Edge[] edges = new Edge[G.E()];
    int t = 0;
    for (Edge e: G.edges()) {
        edges[t++] = e;
    }
    Arrays.sort(edges);

    // run greedy algorithm
    UF uf = new UF(G.V());
    for (int i = 0; i < G.E() && mst.size() < G.V() - 1; i++) {
        Edge e = edges[i];
        int v = e.either();
        int w = e.other(v);

        // v-w does not create a cycle
        if (uf.find(v) != uf.find(w)) {
            uf.union(v, w);     // merge v and w components
            mst.enqueue(e);     // add edge e to mst
            weight += e.weight();
        }
    }

    // check optimality conditions
    assert check(G);
}
```

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Java Implementation – Prim's Algorithm

MST-PRIM(G, w, r)

```
for each u ∈ G.V
    u.key = ∞
    u.π = NIL
r.key = 0
Q = G.V
while Q ≠ Ø
    u = EXTRACT-MIN(Q)
    for each v ∈ G.Adj[u]
        if v ∈ Q and w(u, v) < v.key
            v.π = u
            v.key = w(u, v)
```

```java
public static void main(String[] args) {
    In in = new In(args[0]);
    EdgeWeightedGraph G = new EdgeWeightedGraph(in);
    PrimMST mst = new PrimMST(G);
    for (Edge e : mst.edges()) {
        StdOut.println(e);
    }
    StdOut.printf("%.5f\n", mst.weight());
}
```

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Java Implementation – Prim’s Algorithm

```
MST-PRIM(G, w, r)
for each u ∈ G.V
    u.key = ∞
    u.π = NIL
r.key = 0
Q = G.V
while Q ≠ Ø
    u = EXTRACT-MIN(Q)
    for each v ∈ G.Adj[u]
        if v ∈ Q and w(u, v) < v.key
            v.π = u
            v.key = w(u, v)
```

```java
public PrimMST(EdgeWeightedGraph G) {
    edgeTo = new Edge[G.V()];
    distTo = new double[G.V()];
    marked = new boolean[G.V()];
    pq = new IndexMinPQ<Double>(G.V());
    for (int v = 0; v < G.V(); v++)
        distTo[v] = Double.POSITIVE_INFINITY;

    for (int v = 0; v < G.V(); v++)      // run from each vertex to find
        if (!marked[v]) prim(G, v);      // minimum spanning forest

    // check optimality conditions
    assert check(G);
}

// run Prim's algorithm in graph G, starting from vertex s
private void prim(EdgeWeightedGraph G, int s) {
    distTo[s] = 0.0;
    pq.insert(s, distTo[s]);
    while (!pq.isEmpty()) {
        int v = pq.delMin();
        scan(G, v);
    }
}
```

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Java Implementation – Prim's Algorithm

```
MST-PRIM(G, w, r)
for each u ∈ G.V
    u.key = ∞
    u.π = NIL
r.key = 0
Q = G.V
while Q ≠ ∅
    u = EXTRACT-MIN(Q)
    for each v ∈ G.Adj[u]
        if v ∈ Q and w(u, v) < v.key
            v.π = u
            v.key = w(u, v)
```

```java
public double weight() {
    double weight = 0.0;
    for (Edge e : edges())
        weight += e.weight();
    return weight;
}
```

```java
// scan vertex v
private void scan(EdgeWeightedGraph G, int v) {
    marked[v] = true;
    for (Edge e : G.adj(v)) {
        int w = e.other(v);
        if (marked[w]) continue;         // v-w is obsolete edge
        if (e.weight() < distTo[w]) {
            distTo[w] = e.weight();
            edgeTo[w] = e;
            if (pq.contains(w)) pq.decreaseKey(w, distTo[w]);
            else                pq.insert(w, distTo[w]);
        }
    }
}
```

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ขอบคุณ

Thai

Grazie

Italian

תודה רבה

Hebrew

ধন্যবাদ

Bangla

धन्यवादः

Sanskrit

Ευχαριστώ

Greek

Thank You

English

ಧನ್ಯವಾದಗಳು

Kannada

Спасибо

Russian

Gracias

Spanish

Obrigado

Portuguese

شكرا

Arabic

https://sites.google.com/site/animeshchaturvedi07

Merci

French

多謝

Traditional Chinese

धन्यवाद

Hindi

Danke

German

多谢

Simplified Chinese

நன்றி

Tamil

Tamil

ありがとうございました

Japanese

감사합니다

Korean