THE STATISTICAL SOMMELIER

An Introduction to Linear Regression

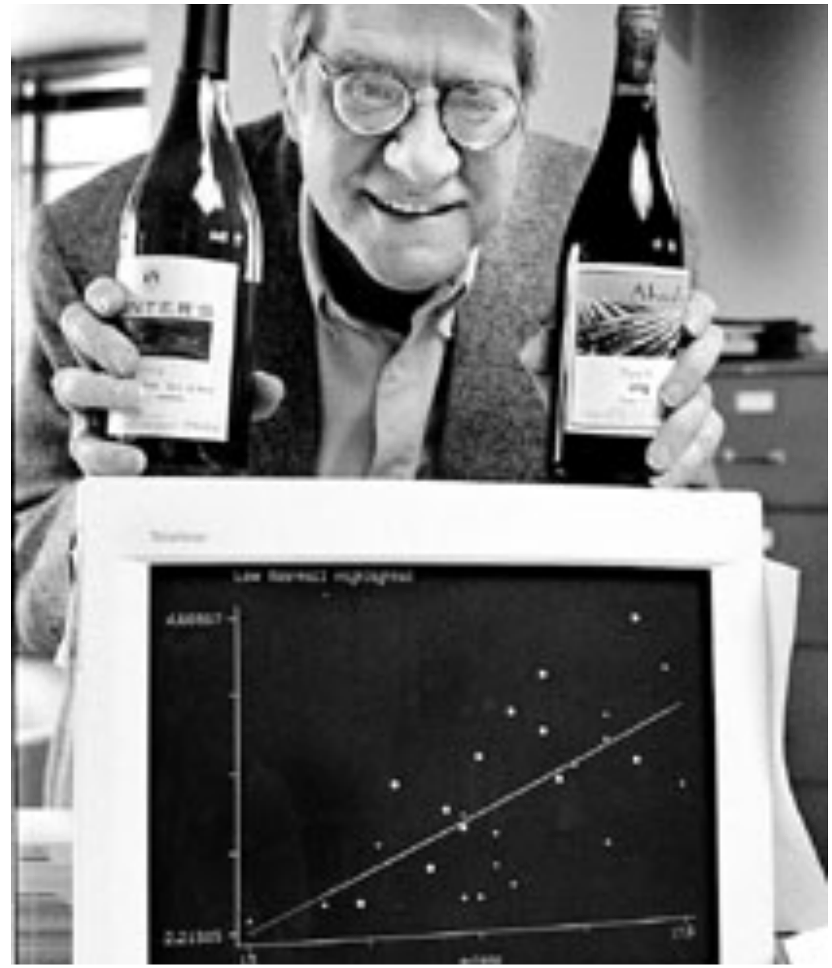
15.071 – The Analytics Edge

Bordeaux Wine

- Large differences in price and quality between years, although wine is produced in a similar way
- Meant to be aged, so hard to tell if wine will be good when it is on the market
- Expert tasters predict which ones will be good
- Can analytics be used to come up with a different system for judging wine?

Predicting the Quality of Wine

- March 1990 - Orley Ashenfelter, a Princeton economics professor, claims he can predict wine quality without tasting the wine



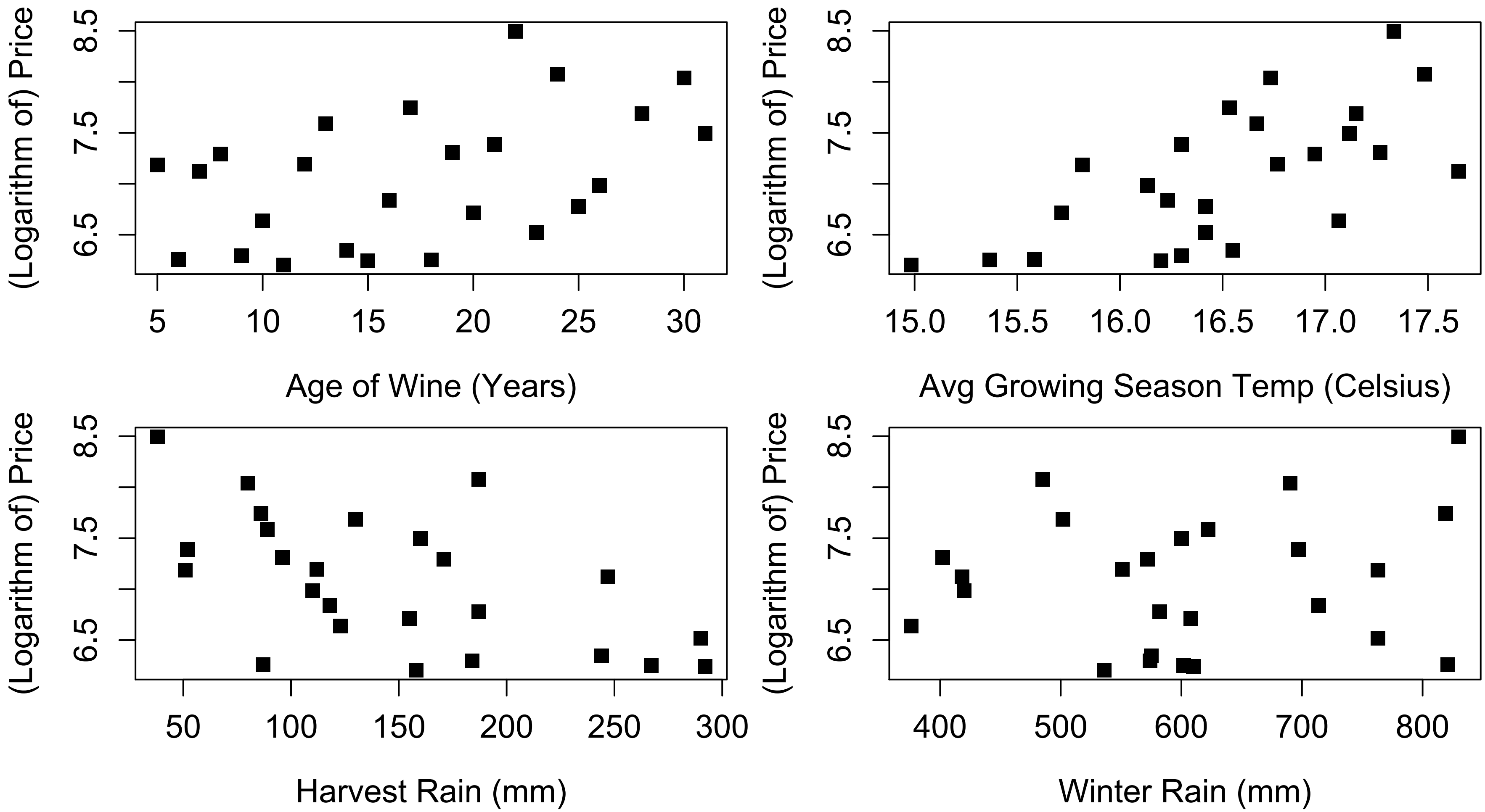
Building a Model

- Ashenfelter used a method called **linear regression**
  - Predicts an outcome variable, or *dependent variable*
  - Predicts using a set of *independent variables*

- Dependent variable: typical price in 1990-1991 wine auctions (approximates quality)

- Independent variables:
  - Age – older wines are more expensive
  - Weather
    - Average Growing Season Temperature
    - Harvest Rain
    - Winter Rain

The Data (1952 – 1978)

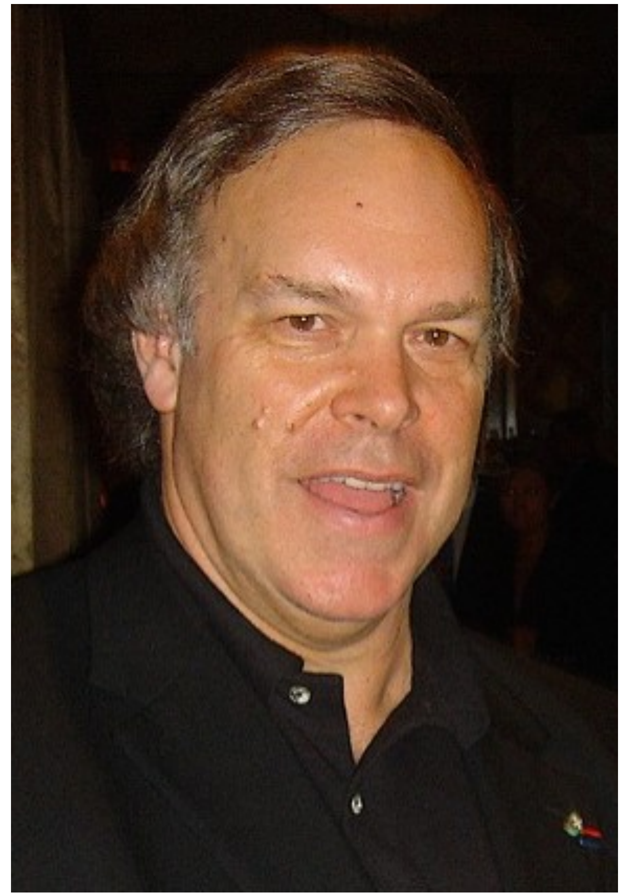


The Expert's Reaction

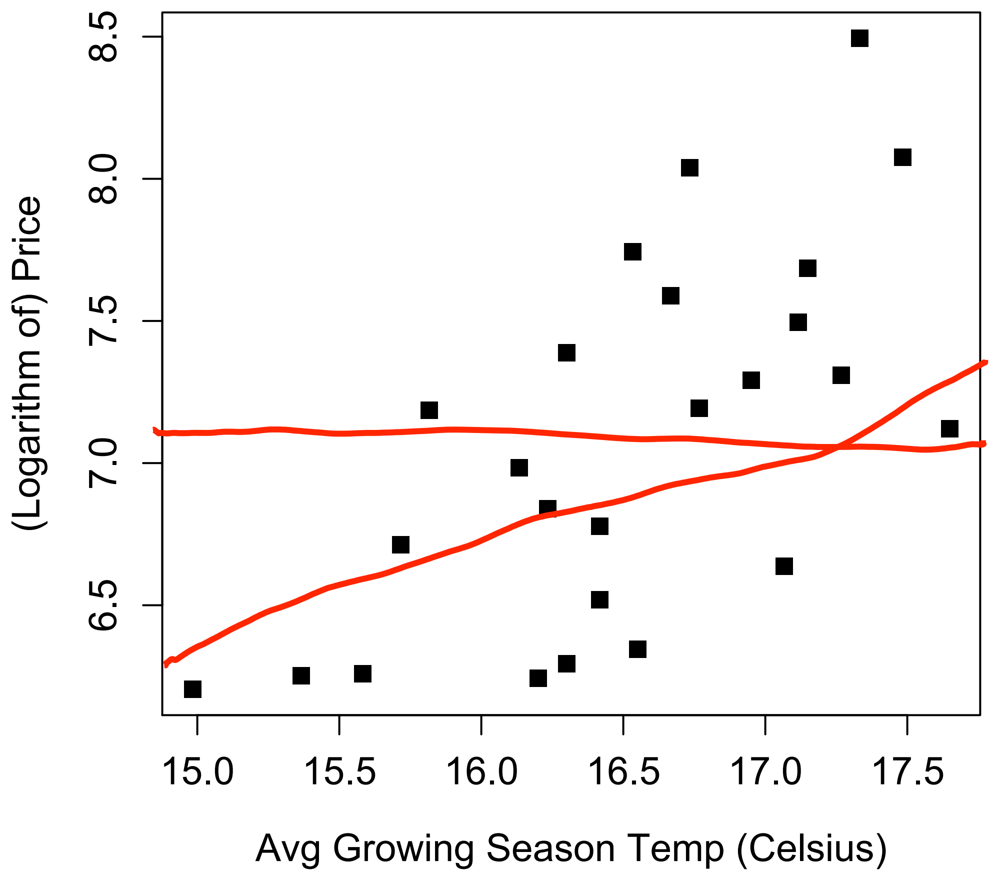
Robert Parker, the world's most influential wine expert:

> **"Ashenfelter is an absolute total sham"**

> "rather like a movie critic who never goes to see the movie but tells you how good it is based on the actors and the director"



One-Variable Linear Regression



$y = 7.07$

$y = 0.5(AGST) - 1.25$

The Regression Model

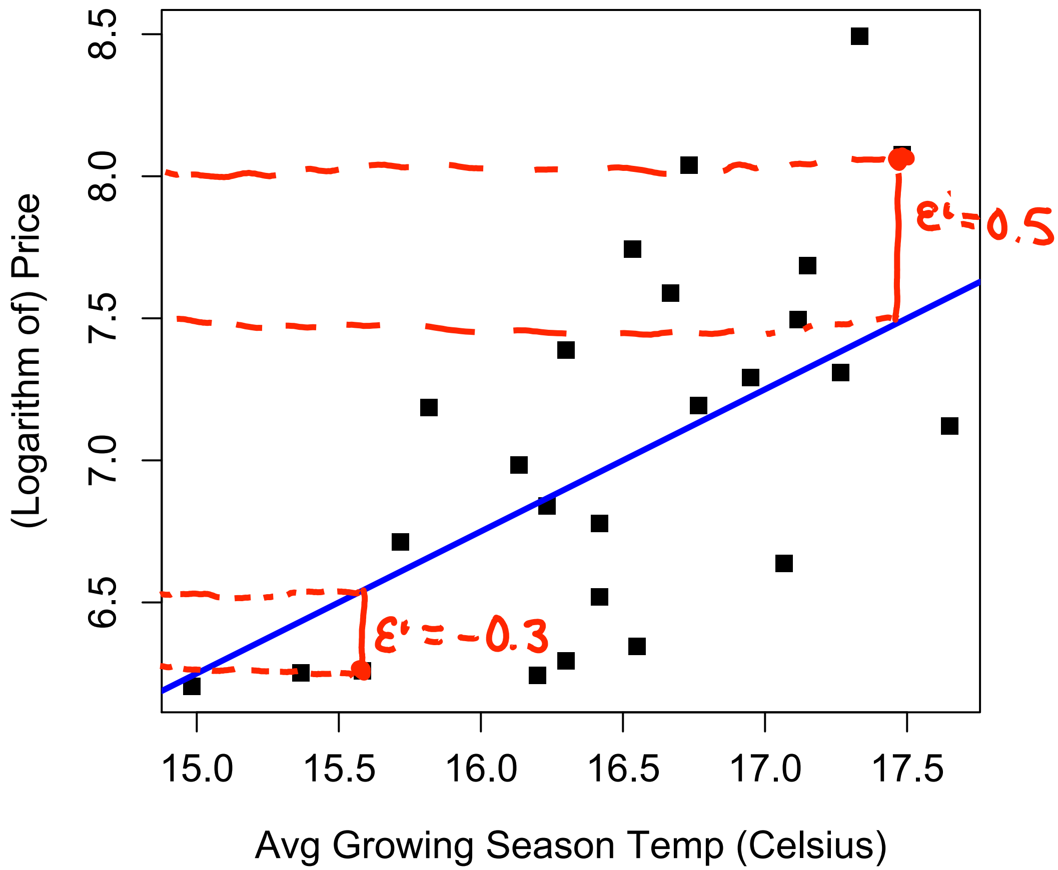
- One-variable regression model

$$y^i = \beta_0 + \beta_1 x^i + \epsilon^i$$

y^i = dependent variable (wine price) for the i^{th} observation
x^i = independent variable (temperature) for the i^{th} observation
ϵ^i = error term for the i^{th} observation
β_0 = intercept coefficient
β_1 = regression coefficient for the independent variable

- The best model (choice of coefficients) has the smallest error terms

Selecting the Best Model

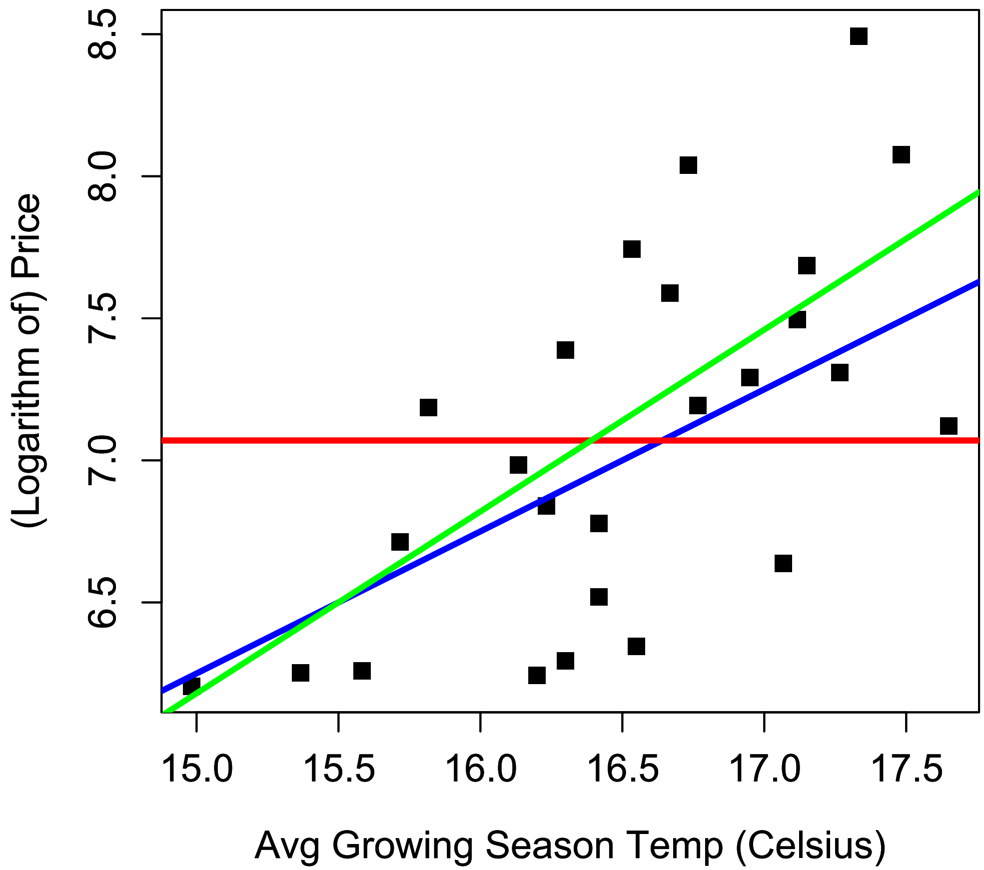


SSE

$= (\varepsilon^1)^2 + (\varepsilon^2)^2 + \ldots + (\varepsilon^N)^2$

N = # data points

Selecting the Best Model



SSE = 10.15

SSE = 6.03

SSE = 5.73

Other Error Measures

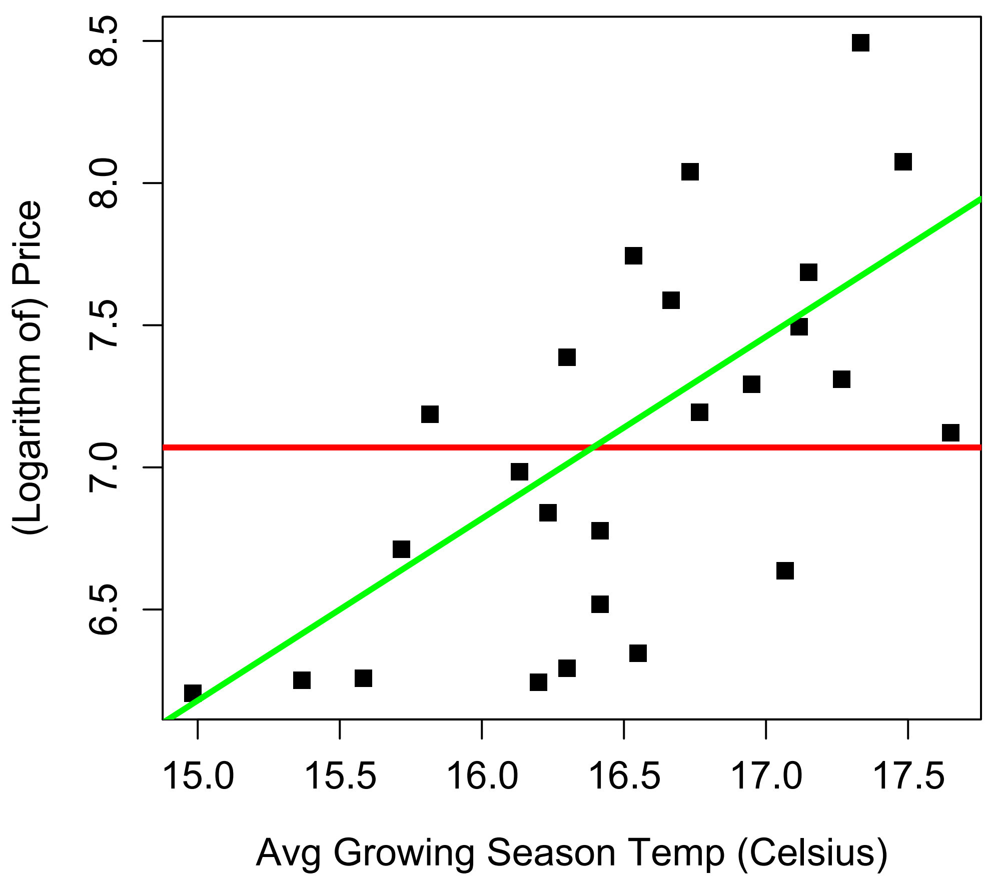
- SSE can be hard to interpret
  - Depends on N
  - Units are hard to understand

- Root-Mean-Square Error (RMSE)

$$RMSE = \sqrt{\frac{SSE}{N}}$$

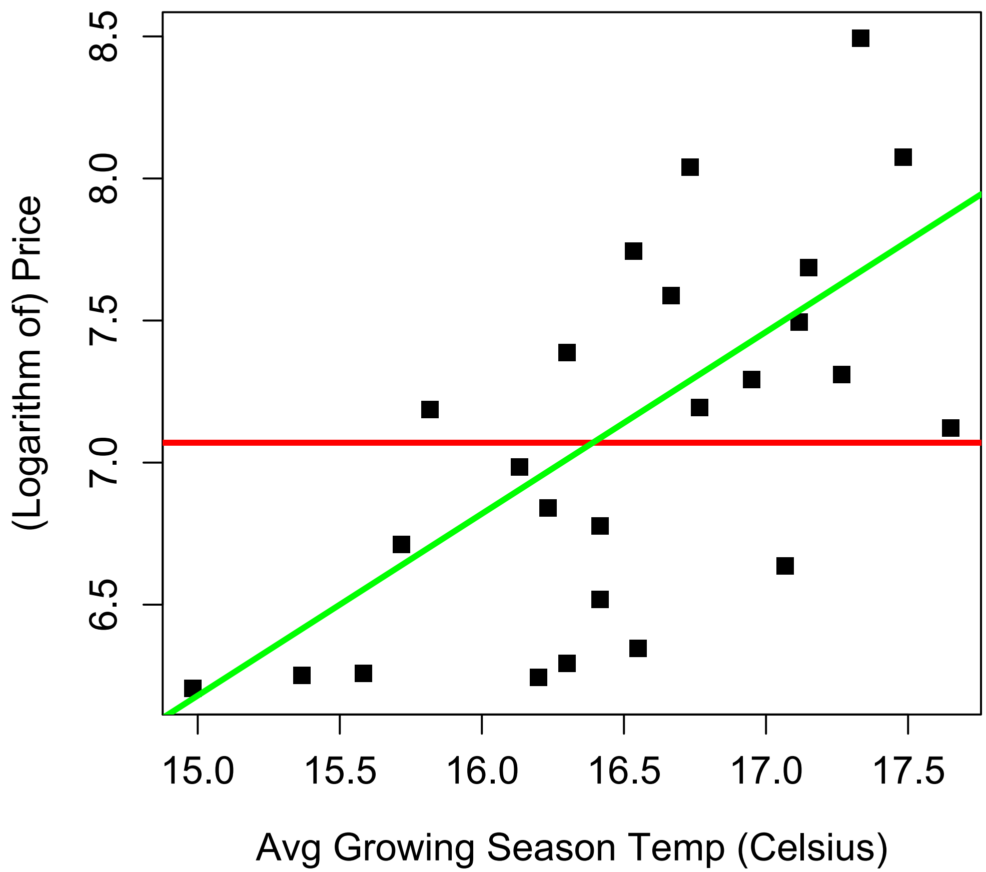
- Normalized by N, units of dependent variable

R^2



- Compares the best model to a "baseline" model
- The baseline model does not use any variables
  - Predicts same outcome (price) regardless of the independent variable (temperature)

R^2



SSE = 5.73

SST = 10.15

$$R^2 = 1 - \frac{SSE}{SST}$$

$$= 1 - \frac{5.73}{10.15}$$

$$= 0.44$$

Interpreting R^2

$$R^2 = 1 - \frac{SSE}{SST}$$

$0 \leq SSE \leq SST$

$0 \leq SST$

- R^2 captures value added from using a model
  - $R^2 = 0$ means no improvement over baseline
  - $R^2 = 1$ means a perfect predictive model
- Unitless and universally interpretable
  - Can still be hard to compare between problems
  - Good models for easy problems will have $R^2 \approx 1$
  - Good models for hard problems can still have $R^2 \approx 0$

Available Independent Variables

- So far, we have only used the Average Growing Season Temperature to predict wine prices

- Many different independent variables could be used
  - Average Growing Season Temperature
  - Harvest Rain
  - Winter Rain
  - Age of Wine (in 1990)
  - Population of France

Multiple Linear Regression

- Using each variable on its own:
  - $R^2 = 0.44$ using Average Growing Season Temperature
  - $R^2 = 0.32$ using Harvest Rain
  - $R^2 = 0.22$ using France Population
  - $R^2 = 0.20$ using Age
  - $R^2 = 0.02$ using Winter Rain
- Multiple linear regression allows us to use all of these variables to improve our predictive ability

The Regression Model

- Multiple linear regression model with k variables

$$y^i = \beta_0 + \beta_1 x_1^i + \beta_2 x_2^i + \ldots + \beta_k x_k^i + \epsilon^i$$

y^i = dependent variable (wine price) for the i^{th} observation

x_j^i = j^{th} independent variable for the i^{th} observation

ϵ^i = error term for the i^{th} observation

β_0 = intercept coefficient

β_j = regression coefficient for the j^{th} independent variable

- Best model coefficients selected to minimize SSE

Adding Variables

Variables	R^2
Average Growing Season Temperature (AGST)	0.44
AGST, Harvest Rain	0.71
AGST, Harvest Rain, Age	0.79
AGST, Harvest Rain, Age, Winter Rain	0.83
AGST, Harvest Rain, Age, Winter Rain, Population	0.83

- Adding more variables can improve the model
- Diminishing returns as more variables are added

Selecting Variables

- Not all available variables should be used
  - Each new variable requires more data
  - Causes *overfitting:* high R^2 on data used to create model, but bad performance on unseen data

- We will see later how to appropriately choose variables to remove

Understanding the Model and Coefficients

$\frac{\text{Estimate}}{\text{Std. Error}}$

```
Coefficients:
                        Estimate Std. Error t value Pr(>|t|)
(Intercept)           -4.504e-01  1.019e+01  -0.044 0.965202
AvgGrowingSeasonTemp   6.012e-01  1.030e-01   5.836 1.27e-05 ***
HarvestRain           -3.958e-03  8.751e-04  -4.523 0.000233 ***
Age                    5.847e-04  7.900e-02   0.007 0.994172
WinterRain             1.043e-03  5.310e-04   1.963 0.064416 .
FrancePopulation      -4.953e-05  1.667e-04  -0.297 0.769578
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
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Correlation

A measure of the linear relationship between variables

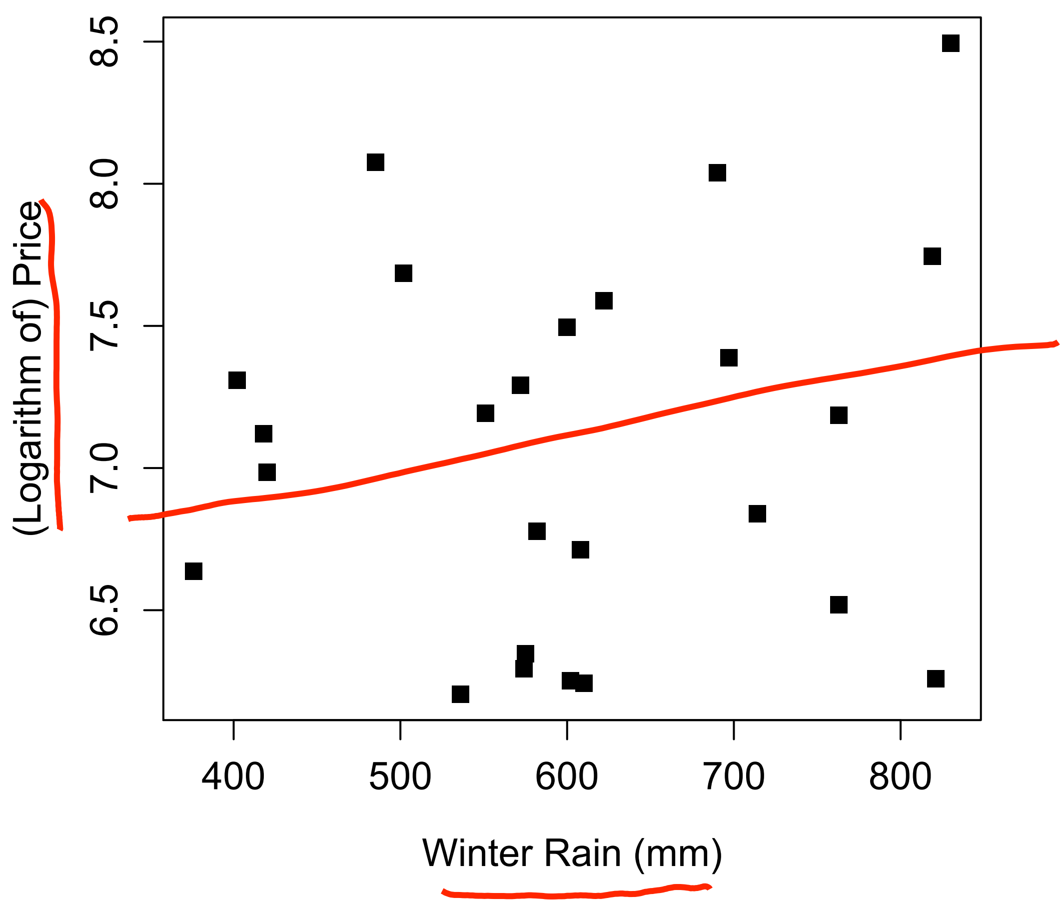
+1 = perfect positive linear relationship

0 = no linear relationship

-1 = perfect negative linear relationship

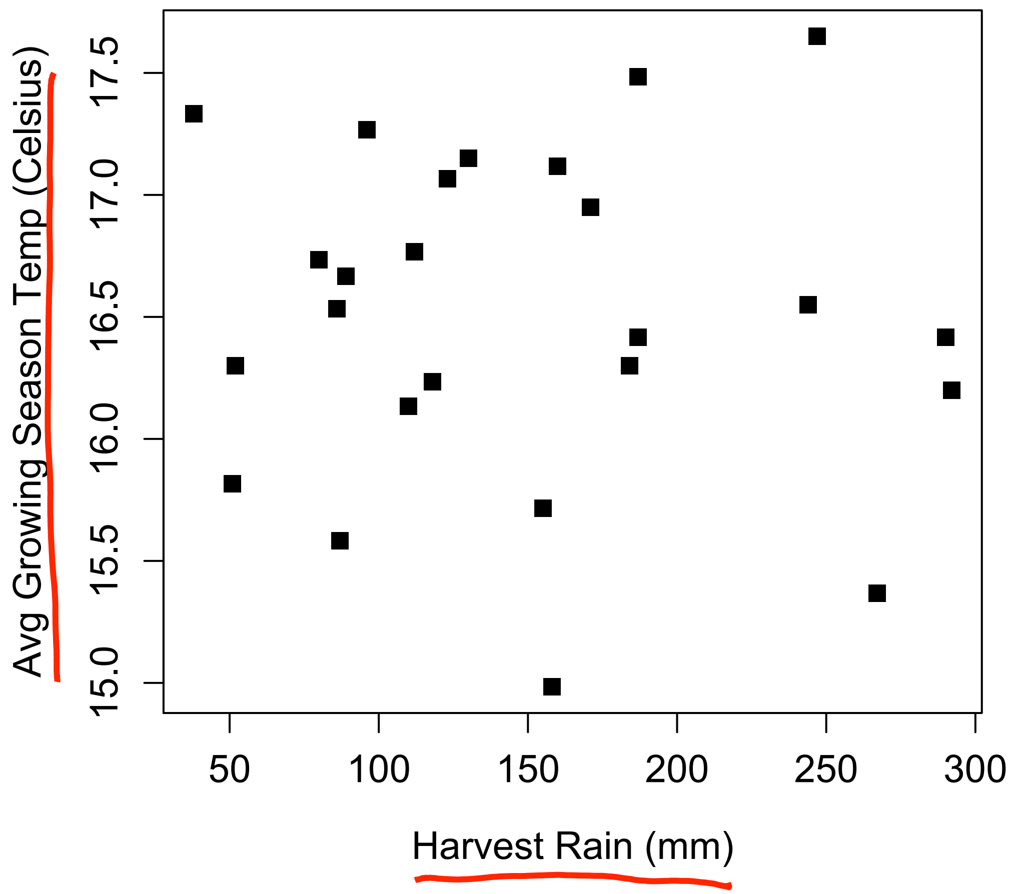
Examples of Correlation

cor
= 0.14



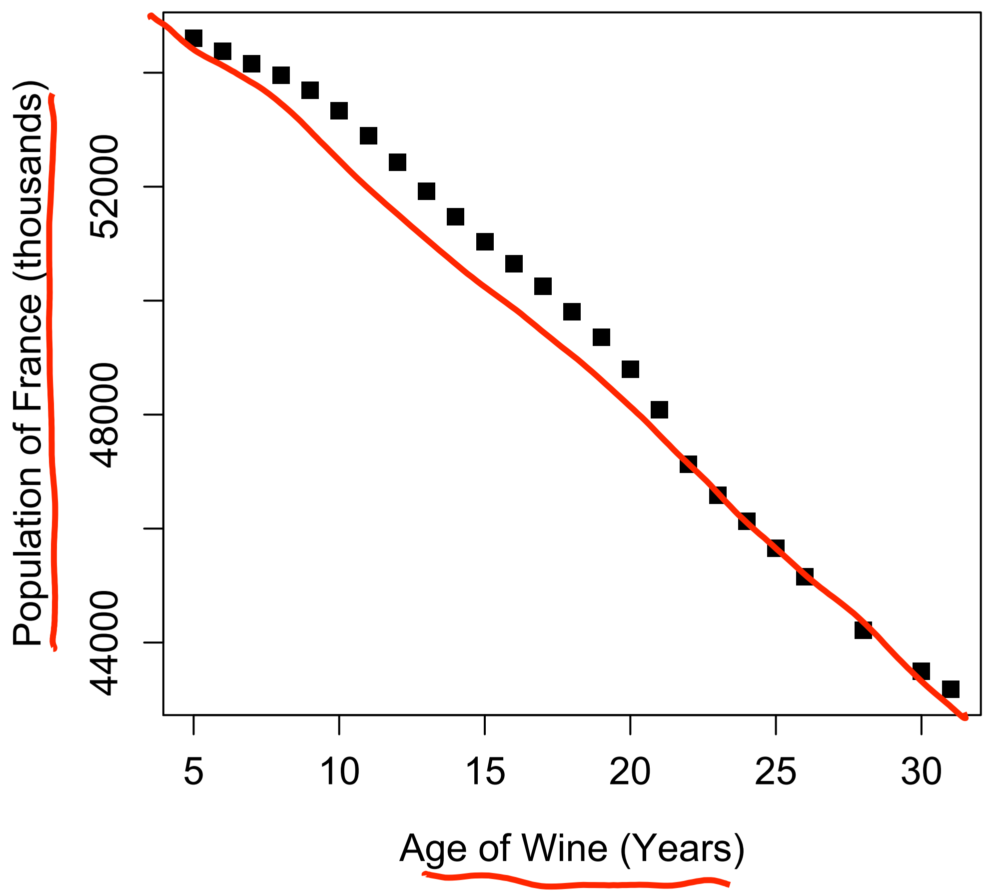
Examples of Correlation

cor
= -0.06



Examples of Correlation

cor
= −0.99



Predictive Ability

- Our wine model had a value of $R^2 = 0.83$
- Tells us our accuracy on the data that we used to build the model

  training

  test
- But how well does the model perform on new data?
  - Bordeaux wine buyers profit from being able to predict the quality of a wine years before it matures

Out-of-Sample R^2

Variables	Model R^2	Test R^2
AGST	0.44	0.79
AGST, Harvest Rain	0.71	-0.08
AGST, Harvest Rain, Age	0.79	0.53
AGST, Harvest Rain, Age, Winter Rain	0.83	0.79
AGST, Harvest Rain, Age, Winter Rain, Population	0.83	0.76

- Better model R^2 does not necessarily mean better test set R^2
- Need more data to be conclusive
- Out-of-sample R^2 can be negative!

The Results

- **Parker:**
  - 1986 is "very good to sometimes exceptional"
- **Ashenfelter:**
  - 1986 is mediocre
  - 1989 will be "the wine of the century" and 1990 will be even better!

- In wine auctions,
  - 1989 sold for more than twice the price of 1986
  - 1990 sold for even higher prices!

- Later, Ashenfelter predicted 2000 and 2003 would be great
- Parker has stated that "2000 is the greatest vintage Bordeaux has ever produced"

The Analytics Edge

- A linear regression model with only a few variables can predict wine prices well
- In many cases, outperforms wine experts' opinions
- A quantitative approach to a traditionally qualitative problem