CS 535 Design and Analysis of Algorithms **Spring Semester, 2019**

Homework 1 version 1.1

Assigned: Jan 24 *Due: Feb. 7*

Remote students: use blackboard to submit your solutions. In-class students: hard coppies are required. Blackboard is optional (and useful).

Problem 1 Suppose that you have a "black-box" worst-case linear-time median subroutine. Give a simple, linear-time algorithm that, given an array $A[1..n]$ and a positive integer $i \leq n$ finds the i^{th} smallest element of A. Present pseudocode using procedures from the textbook or notes; give complete specifications and state the running time of the procedure in terms of its parameters.

Problem 2 Problem 9-2 (Weighted Median) from the textbook. It has the same number in the second edition of the textbook.

Problem 3 Consider the following recursive algorithm for computing minimum spanning trees. Given as input a complete graph $G = (V, E)$, randomly partition the set V of vertices into two sets V_1 and V_2 such that $|V_2| = 1$. Let E_1 be the set of edges that are incident only on vertices in V_1. Recursively solve a minimum spanning tree problem on the subgraphs $G_1 = (V_1, E_1)$. Finally, select the minimum-weight edge in e that crosses the cut (V_1, V_2), and use this edge to unite the resulting two minimum spanning trees into a single spanning tree.

1. Give an example (a weighted graph!) showing that the algorithm can fail to produce a minimum spanning tree (if the partition is done in worst-case manner). Explain what the algorithms does and why the output is not optimum.

2. Argue that the algorithm will produce a minimum spanning tree, if (with a lot of luck) the partition is always done in best-case manner.

Problem 4 Let $G = (V, E, c)$ be a weigted undirected graph where all the costs c_e, for $e \in E$, are strictly positive and distinct. Let T be a minimum spanning tree in $G = (V, E, c)$. Now suppose we replace the cost of each edge $e \in E$ by $c'_e = c_e^2$, creating the instance $G' = (V, E, c')$. Prove or disprove: T is a minimum spanning tree in G'.

Now assume $s, t \in V$ are also given, and P is a shortest $s - t$ path in G. Prove or disprove: P is the shortest $s - t$ path in G'.