CS 535 Design and Analysis of Algorithms	**Spring Semester, 2019**
Homework 5	
Assigned: April 2	*Due: April 16*

Problem 1 Let $G = (V, E, c, s, t)$ be a flow network with integer capacities, and f be a feasible flow with integer values. Suppose there is a edge e with head s and such that $f(e) = 1$. Describe an $O(|V| + |E|)$ algorithm (An English explanation may be enough) that produces another feasible flow f' with $|f| \leq |f'|$ and such that $f'(e) = 0$.

Be precise though: if you use an algorithm from the texbook, explain which graph is the input of the algorithm. Justify the overall running time and correctness.

Problem 2 A **multiple source-sink network** is a tuple $G = (V, E, c, S, T)$, where V is a set of vertices, E is a set of directed edges (parallel edges are allowed), $S \subset V$ is the set of **sources**, and $T \subset V$ is the set of **sinks**, c is a **capacity** function: $c : E \to Z_+$. Also, $S \cap T = \emptyset$. That is, sources are distinct from sinks.

A function $f : E \to R_+$ is called a *flow* if the following three conditions are satisfied:

1. *conservation of flow at interior vertices:* for all vertices u not in $S \cup T$,
$$\sum_{e \in \delta^-(u)} f(e) = \sum_{e \in \delta^+(u)} f(e) \ ;$$
2. *capacity constraints:* $f \leq c$ pointwise: i.e. for all $e \in E$,
$$f(e) \leq c(e) \ .$$

Assume that non-negative quantities p_s, for $s \in S$, and q_t, for $t \in T$, are given. The goal of this problem is to determine if a valid flow exists such that for all $s \in S$:
$$\sum_{e \in \delta^+(s)} f(e) - \sum_{e \in \delta^-(s)} f(e) = p_s$$
and such that for all $t \in T$:
$$\sum_{e \in \delta^-(t)} f(e) - \sum_{e \in \delta^+(t)} f(e) = q_t.$$

Use Network Flows to give a polynomial-time algorithm for this **decision** problem (the answer is YES or NO). Hint: read chapter 26.1 of the textbook.

Problem 3 The edge connectivity of an undirected multigraph is the minimum number k of edges that must be removed to disconnect the graph. For example, the edge connectivity of a tree is 1, and the edge connectivity of a cyclic chain of vertices is 2. Show how to determine the edge connectivity of an undirected multigraph $G = (V, E)$ by running a maximum-flow algorithm on at most $|V|$ flow networks, each having $O(|V|)$ vertices and $O(|E|)$ edges.

Argue correctness.