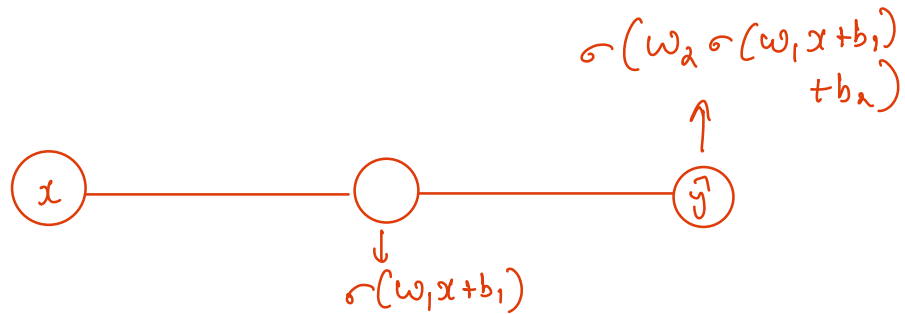


Ans #1 :-



$$\sigma(x) = \frac{1}{1 + e^{-x}}$$

$$L(\theta) = \frac{1}{2} \left( \sigma(w_2 \sigma(w_1 x + b_1) + b_2) - y \right)^2$$

$$\nabla_{\theta} L = \begin{bmatrix} \frac{\partial L}{\partial w_1} \\ \frac{\partial L}{\partial w_2} \\ \frac{\partial L}{\partial b_1} \\ \frac{\partial L}{\partial b_2} \end{bmatrix}$$

$$\textcircled{1} \quad \text{Let } g(w_1, b_1) = \sigma(w_1 x + b_1)$$

$$f(w_2, g, b_2) = w_2 \sigma(g(w_1, b_1)) + b_2$$

$$L(\theta) = \frac{1}{2} \left( \sigma(f(w_2, g, b_2)) - y \right)^2$$

$$\frac{\partial L}{\partial w_1} = (\sigma(f) - y) \left( \frac{\partial \sigma}{\partial f} \frac{\partial f}{\partial g} \frac{\partial g}{\partial w_1} \right)$$

$$\text{where } \frac{\partial \sigma}{\partial f} = \sigma(f)(1 - \sigma(f))$$

$$\frac{\partial f}{\partial g} = w_2 [\sigma(g)(1 - \sigma(g))]$$

$$\frac{\partial g}{\partial w_1} = x (\sigma(w_1 x + b_1)) [1 - \sigma(w_1 x + b_1)]$$

$$\textcircled{2} \quad \frac{\partial L}{\partial w_2} = (\sigma(f) - y) \left( \frac{\partial \sigma}{\partial f} \frac{\partial f}{\partial w_2} \right)$$

$$\frac{\partial \sigma}{\partial f} = \frac{\partial \sigma}{\partial f} = \sigma(f)(1 - \sigma(f))$$

$$\frac{\partial f}{\partial w_2} = \sigma(g)$$

$$\textcircled{3} \quad \frac{\partial L}{\partial b_1} = (\sigma(f) - y) \left( \frac{\partial \sigma}{\partial f} \frac{\partial f}{\partial g} \frac{\partial g}{\partial b_1} \right)$$

$$\frac{\partial \sigma}{\partial f} = \sigma(f)(1 - \sigma(f))$$

$$\frac{\partial f}{\partial g} = w_2 [\sigma(g)(1 - \sigma(g))]$$

$$\frac{\partial g}{\partial b_1} = (\sigma(w_1 x + b_1)) [1 - \sigma(w_1 x + b_1)]$$

$$\textcircled{4} \quad \frac{\partial L}{\partial b_2} = (\sigma(f) - y) \left( \frac{\partial \sigma}{\partial f} \frac{\partial f}{\partial b_2} \right)$$

where:-

$$\frac{\partial \sigma}{\partial f} = \sigma(f)(1 - \sigma(f))$$

$$\frac{\partial f}{\partial b_2} = 1$$