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Eigenvalue analysis enhanced framework to determine the onset of instabilities in finite strain elasticity problems using deal.II

Fall 2020 Research Report

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1 INTRODUCTION

Soft matter can undergo large deformation under the influence of external stimuli such as mechanical forces, change in temperature and magnetic/electric fields. An interesting characteristic of these materials is that they exhibit a variety of instabilities such as buckling, wrinkling and creasing due to their low elastic modulus and high susceptibility to external stimuli. Traditionally, instabilities were considered as behaviours that limit the performance of the structures. However, recent research on soft matter has demonstrated numerous techniques to exploit these instabilities to our own advantage, see for instance [1, 2]. Although, the importance of modeling the morphological instabilities in soft matter has grown over the years, the advancement in this field is hindered by a number of mechanical and mathematical complexities. The purpose of this work is to establish an eigenvalue analysis enhanced algorithm [3] to determine the onset of instabilities in finite strain elasticity problems using deal.II.

The rest of the report is outlined as follows. Section 2 gives an overview of the computational framework used to capture the instabilities. In Section 3, we consider two representative examples dealing with the instabilities in finite strain elasticity problems. Section 4 draws the conclusions from our study.

2 COMPUTATIONAL FRAMEWORK

In this study, we perform a non-linear finite element analysis with displacement load increments. We apply the full Newton-Raphson method to iteratively solve for the displacements in the system at each increment. To solve the linear system of equations, we use direct solver as opposed to iterative solver. This is crucial to account for ill-conditioning of the system of linear equations near the instability point. We model the material using a Neo-Hookean type hyperelastic constitutive law. The primary characteristic of this non-linear elastic constitutive law is that the stress-strain relationship is derived from the strain-energy function as given in Eq. (1)

$$\psi(\mathbf{C}) = \frac{\Lambda}{4}(J^2 - 1) - (\mu + \frac{\Lambda}{2})lnJ + \frac{\mu}{2}(tr(\mathbf{C}) - 3) \tag{1}$$

where ψ is the free energy function, $\mathbf{C}$ is the right Cauchy-Green deformation tensor, $J^2 = det(\mathbf{C})$, Λ and μ are material parameters. For more theoretical details see [3].

We design the algorithm such that at each increment, it checks the smallest eigenvalue (λ_{min}) of the tangent matrix after the convergence of Newton-Raphson method. If λ_{min} is positive, the simulation moves to the next increment. As soon as the algorithm detects a negative λ_{min}, it uses bisection method to adjust the increment till the smallest eigenvalue becomes approximately equal to zero. The displacement field at that instant gives the critical strain in the system. The associated eigenvector identifies the instability mode in the system. We perform the eigenvalue analysis in shift-invert mode using SLEPc [4], a software library for the solution of large scale sparse eigenvalue problems on parallel computers. We use the code [5] as the basis for our work.

3 REPRESENTATIVE EXAMPLES AND RESULTS

3.1 BUCKLING OF A SLENDER BEAM

In this example, we determine the critical strain due to the buckling of a slender beam under compression. Fig. 1 illustrates the geometry and the boundary conditions of the system. We study the mesh sensitivity of bi-quadratic element type by varying the number of elements along the length of the beam. We take the beam to be nearly incompressible ($\nu = 0.49$). The results are shown in Fig. 2

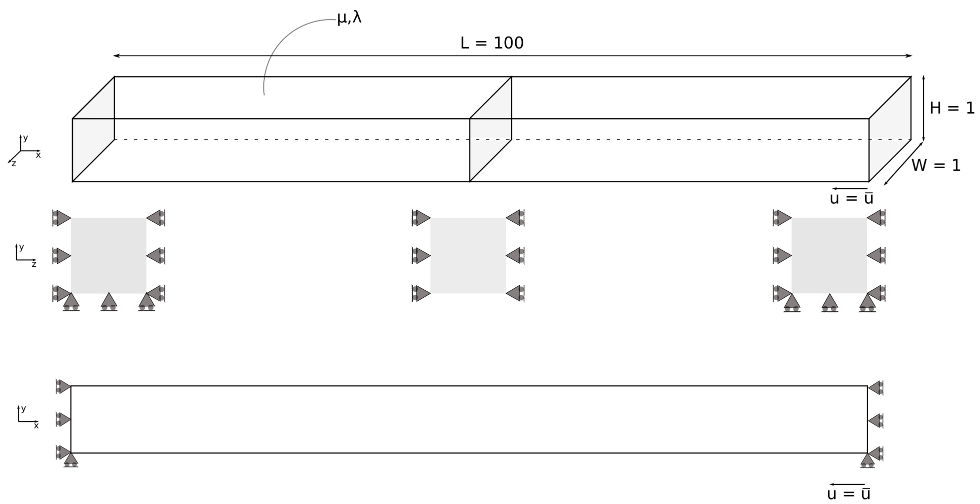

Figure 1
Geometry and boundary conditions of a slender beam with compressive displacement $\bar{u}$

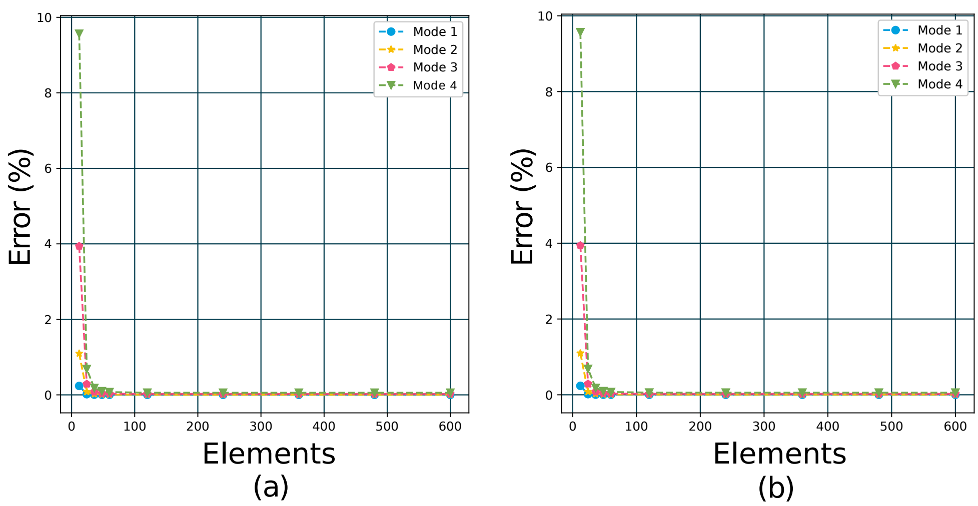

Figure 2
Error (%) vs No. of elements along the length of the beam: (a) One element along the width (b) Four elements along the width

3.2 WRINKLING OF A THIN FILM ATTACHED TO A SOFT SUBSTRATE

In this example, we calculate the critical strain due to wrinkling of a thin film attached to a soft substrate. The geometry and the boundary conditions of the system are as shown in Fig. 3. The poisson's ratio of the film (ν_f) and the substrate (ν_s) is 0.49. The stiffness ratio is defined as the ratio between the shear modulus of film (μ_f) and that of substrate (μ_s). We vary the stiffness ratio ($\frac{\mu_f}{\mu_s}$) from 5 to 1000 and calculate the corresponding critical strain values. The results are illustrated in Fig. 4

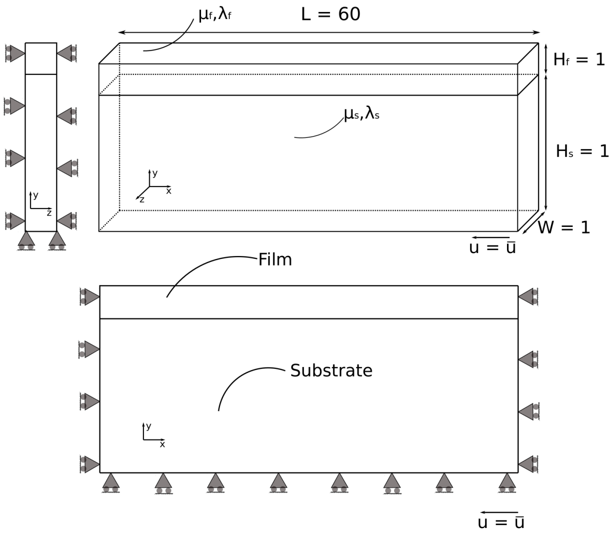

Figure 3
Geometry and boundary conditions of a thin film attached to a soft substrate with compressive displacement $\bar{u}$

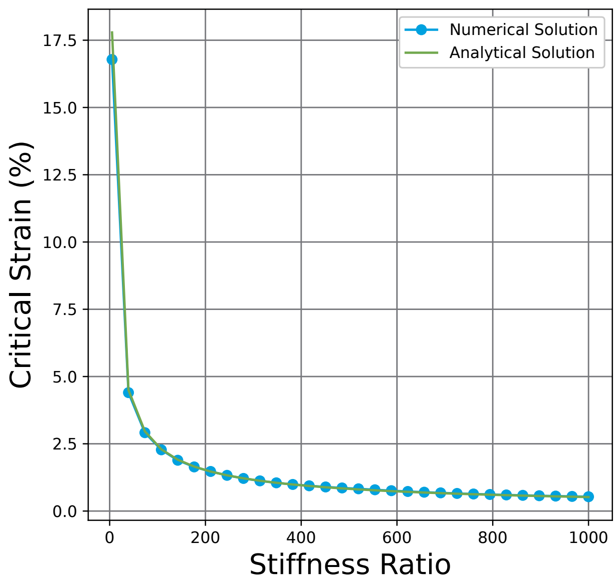

Figure 4
Variation of critical strain with stiffness ratio of the film over the substrate

4 CONCLUSION

The results from the eigenvalue analysis in beam buckling and film wrinkling problems using SLEPc are consistent with the analytical results from [6] as demonstrated in Fig. 2 and Fig. 4

References

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