Answer 3

2.9.1 $E(Y|X=x) = \beta_0 + \beta_1 x$

$E(Y|Z=z) = \gamma_0 + \gamma_1 z$

$Z = aX + b$

$E(Y|Z=z) = \gamma_0 + \gamma_1 (ax + b)$

$E(Y|Z=z) = \gamma_0 + \gamma_1 b + \gamma_1 a x$

comparing

$\beta_0 = \gamma_0 + \gamma_1 b$

$\beta_1 = \gamma_1 a$

$$\boxed{\gamma_1 = \beta_1 / a \quad , \quad \gamma_0 = \beta_0 - \beta_1 b / a}$$

$\hat{\sigma}^2 = \frac{RSS}{n-2}$ $\quad var(\beta_1 | X) = \hat{\sigma}^2 \frac{1}{SXX}$

$var(\gamma_1 | Z) = \hat{\sigma}^2 \cdot \frac{1}{SZZ}$

This will remain constant because the response variable hasn't changed, hence the residual sum of square and df will remain constant

$$S_{xx} = \sum (x_i - \bar{x})^2$$

$$S_{zz} = \sum (z_i - \bar{z})^2 = \sum (a x_i + b - a\bar{x} - b)^2$$

$$S_{zz} = a^2 S_{xx}$$

$$Var(\hat{\beta}_{1z}) = \frac{var(\beta_1 | X)}{a^2} = \sigma^2 \frac{1}{a^2 S_{xx}}$$

Similarly for

$$Var(\beta_0 | x) = \sigma^2 \left(\frac{1}{n} + \frac{\bar{x}^2}{S_{xx}} \right)$$

$$Var(Y_0 | x) = \sigma^2 \left(\frac{1}{n} + \frac{(ax+b)^2}{a^2 S_{xx}} \right)$$

$$= \sigma^2 \left(\frac{1}{n} + \frac{x}{S_{xx}} + \frac{2axb}{a S_{xx}} + \frac{b^2}{a^2 S_{xx}} \right)$$

2.9.2 $\quad E(Y|X) = \beta_0 + \beta_1 x$

$$d E(Y|X) = d\beta_0 + d\beta_1 x$$

$$E(dY|x) = d\beta_0 + \beta_1 x$$

$$E(dY|x) = d\beta_0 + d\beta_1 x$$

$$\delta_0 = d\beta_0 \qquad \delta_1 = d\beta_1$$

estimate of variance remains ~~constant~~
constant because n prediction doesn't
change

The t-test for slope
i.e

$$\beta_1 = \frac{\sigma^2}{\sqrt{Sxx}}$$

2.9.1 $Szz = a^2 Sxx$

so $\beta_1 = \frac{\sigma^2}{\sqrt{Sxx}}$, $\gamma_1 = \frac{S^2}{\sqrt{a^2 Sxx}}$

$$= \frac{\sigma^2}{a\sqrt{Sxx}}$$

$$= a\beta_1$$

so t test for slope for β_1 & γ_1 are in a ratio of $\underline{a}$

2.9.2 t test for both the β_1 & δ_1
will remain constant because
Sxx will be constant &
σ^2 will be constant