Homework 1

STAT 697STA Spring 2023

Due February 15, 2023, 9:40am on Gradescope

1 Reading

- Read through Chapter 2.6 (p. 31) of the text.

2 Questions

1. HRW 2.1

   (a) Complete parts (a) through (e) as in the book.

   (f) Based on the exploration in this problem, make a statement about when alternate (non-penalized) spline bases will give the same mathematical fit.

   (g) Apply the `vcov()` R function to the fits in part (e) to compare the covariances of the coefficient estimates for the two fits. Which has lower covariance, the linear basis or the linear B-spline basis? Why is lower covariance desirable?

   (h) Within the interval $[0, 1]$, verify the following properties of the linear B-spline basis (features of B-spline bases in general):

      - The sum of the basis functions at any point is equal to 1.
      - Each basis function has support on only a finite portion of the range of x (here, this does not apply to B_2 because of the small range of $[0, 1]$ compared to the support of each basis function).

2. HRW 2.2

   Note: for part (e), instead of using `lm()`, I used the formula following (2.7) directly. In this expression, the fit is left-multiplied by the design matrix C to get the fitted values. The fitted coefficients are given by this expression without the first C. You can use the resulting fit to create the plots as in the previous section. This link is helpful for matrix operations in `R`: http://www.philender.com/courses/multivariate/notes/matr.html

3. (EXTRA CREDIT points) Page 17/section 2.2 asserts that spline basis for the form $\{(x - k_k)_+\}^p$ has continuous derivatives up to $(p - 1)$. Show that this is true either in general or for $p = 3$.

4. (EXTRA CREDIT points) Look around on the internet for images of quadratic and cubic B-spline bases. Include copies of these plots in your write-up (cite the website where you found them). Convince yourself that the quadratic ones have continuous first derivative and the cubic ones have continuous first and second derivative. Briefly, describe why you conclude this.

3 Online Questions

For this week's online questions, please complete the poll on your background and interests here if you have not already. It is also linked from Moodle.