INPUT DC-DC CONVERTERS

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DC- DC CONVERTERS

3-1 Introduction

DC-DC converters are widely used in regulated switch-mode DC power supplies and in DC motor drive applications, [46]. As shown in Fig 3-1, the input to these converters is an unregulated DC voltage, and therefore it will fluctuate due to changes in the line-voltage magnitude. DC-DC converters are used to convert the unregulated DC input into a controlled DC output at a desired voltage level. PV power supply systems, often require that the maximum power point of the solar array is tracked, in spite of the variations in the load and the sunlight intensity, to make the most efficient use of the PV. PV power supply systems need to DC-DC converters to get the maximum power, or voltage regulation [43,44,47].

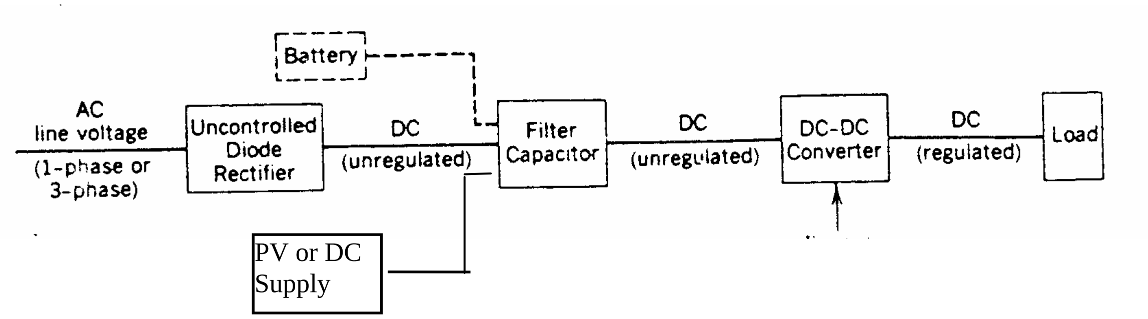
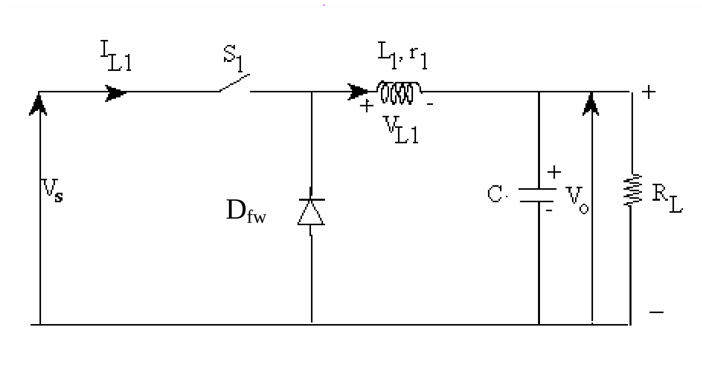


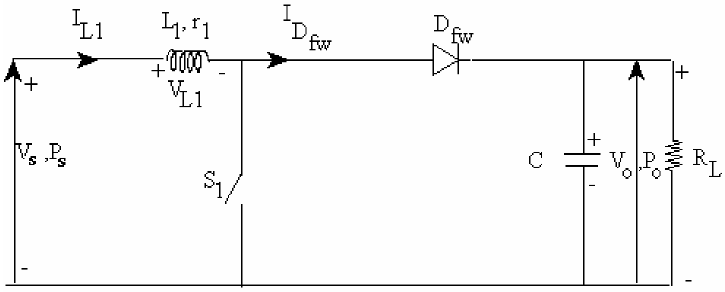
Fig.3-1, DC-DC Converter

3-2 The Development of Power Converters

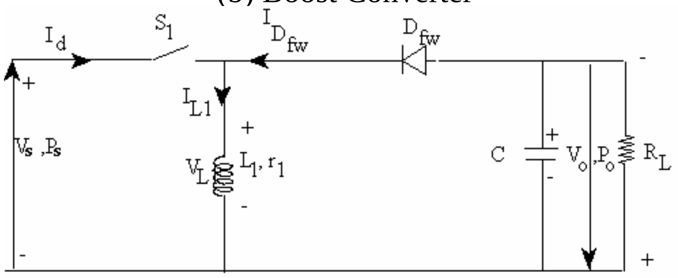
The heart of a switch-mode power supply (SMPS) is a DC-DC converter, which accepts a DC input and produces a controlled dc output. Semiconductor DC-DC converters have appeared in practical use since the 1960s, [48,49]. The three basic types of power converters are the Buck, the Boost, and the Buck-Boost converters. The Buck converter can work as step-down converter, while the Boost converter can work as step-up converter, and the Buck-Boost converter is working as step-up step-down. The circuits of the main three converters are shown in Fig. 3-2. ln each of the converter circuits, there is an electronic switch SW that is driven on and off at a high switching frequency (e.g., 5-500 kHz). It is the duty cycle of the electronic switch that controls the dc output voltage V_o. The output filtering capacitor in each circuit is used to smooth out the ripple component of the output voltage due to high-frequency switching. By adding a feedback circuit in a converter, the output voltage of the converter can be regulated.



(a) Buck Converter



(b) Boost Converter



(c) Buck-Boost Converter

Fig. 3-2, Basic square-wave converters.

Because of the square/ trapezoidal/ triangular shape of the current pulses in the switching device, these converters will generally be referred to as square-wave converters or simply converter. In each of the converter circuits, the energy storage inductance L_1 can be chosen to be so large that the current in it is substantially smoothed. The Buck converter, shown in Fig. 3-2-(a), is then characterized by a smoothed output current i_o, but a pulsating input current i_i. The Boost converter, shown in Fig. 3-2-(b), is characterized by a smoothed input current but a pulsating output current. The Buck-Boost converter, shown in Fig. 3-2-(c) has, on the other hand, both pulsating input and output currents. In 1977, CûK and Middlebrook introduced a new "optimum topology switching dc-to-dc converter" [22], which is now more commonly referred to as the CûK converter. Fig. 3-3 shows the basic circuit of the CûK converter, which functionally is a cascaded connection of a Boost converter followed by a Buck converter. Consequently, it works as step-up step-down converter. Various topologies of squarewave converters were also studied by CûK [50], and Landsman [51]. Recently: much attention has been focused on the design of high frequency converters with reduced weight and size [52,53], and the use of fast MOS or IGBT power transistors as switching elements, [54]. Since power converters and the SMPS are nonlinear circuits, computer aided analysis and design techniques for such circuits are highly desirable, [55, 56, & 57].

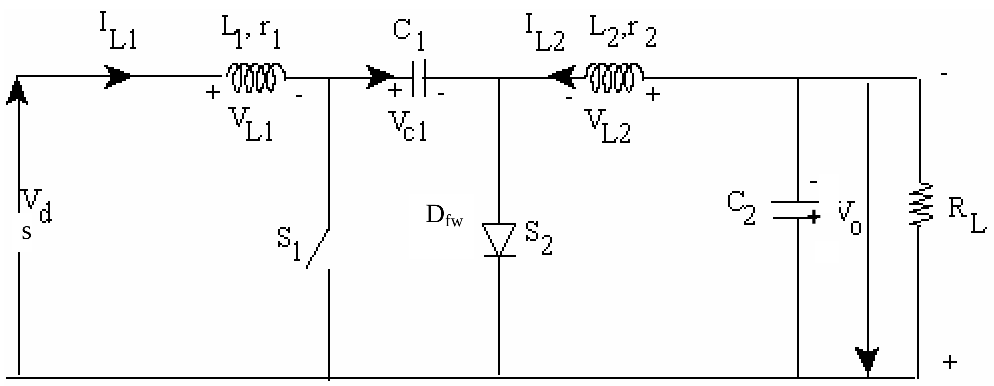


Fig. 3-3, CûK Converter.

In DC-DC converters, the average DC output voltage must be controlled to equal a desired level, though the input voltage and the output load may fluctuate. Switch-mode DC-DC converters utilize one or more switches to transform DC from one level to another. In a DC-DC converter with a given input voltage, the average output voltage is adjusted by controlling the electronic switch on and off durations (t_{on} and t_{off}).

The switch duty ratio can be expressed as

$$\mathbf{D} = \frac{\mathbf{t_{on}}}{\mathbf{T_s}} \tag{3-1}$$

3-3 Buck Converter

The basic Buck converter using a power switch transistor is shown in Fig. 3-2-(a). In a Buck converter, the average output voltage, V_a, is lower than its input voltage, V_s. The operation of the Buck converter can be divided into two modes, depending on the switching actions of its switching transistor. Depending on the continuity of the current flowing through the output inductor, the Buck converter can be operated either in the continuous inductor current or discontinuous inductor current.

Continuous Inductor Current

Mode 1 ($0<t\leq t_{on}$) "Transistor on"

At the beginning of a switching cycle (at t=0) during mode 1, the switching transistor, S_1, is switched on. The equivalent circuit for mode 1 is shown in Fig. 3-4. Since the input voltage, V_s, is greater than the average output voltage, V_a, the current in the inductor, $i_L(t)$, ramps upward during this interval. The voltage across the inductor, $v_L(t)$, is related to the rate of rise of its current and is given by

$$v_L(t) = L\frac{di}{dt} \tag{3-2}$$

For a large inductance value, $i_L(t)$ Increases linearly from I_1, to I_2 during t_{on}. Therefore,

$$\mathbf{V_s - V_a = L\frac{I_2 - I_1}{t_{on}} = L\frac{\Delta I}{t_{on}}} \tag{3-3}$$

And the duration of mode 1 is

$$\mathbf{t_{on} = L\frac{\Delta I}{V_s - V_a}} \tag{3-4}$$

Thus, mode 1 is characterized by inductor charging and the storage of electrical energy in magnetic form in the inductor.

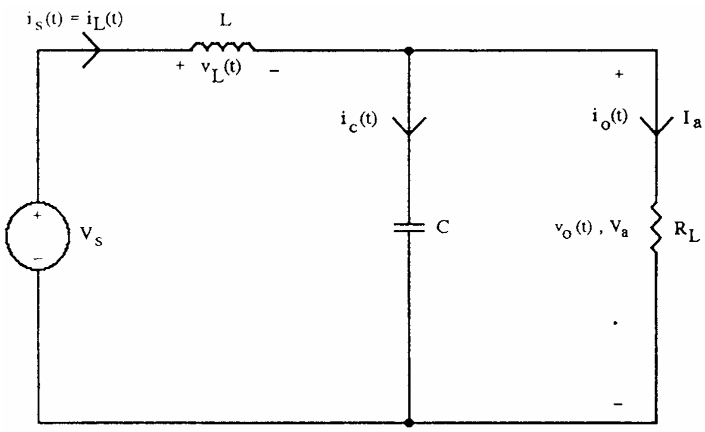


Fig. 3-4, Mode 1 equivalent circuit for the Buck converter ($o<t\leq t_{on}$).

Mode 2 ($t_{on}<t\leq T$) "Transistor off"

Mode 2 begins when the switching transistor, S_1, is switched off at t =t_{on}. Its equivalent circuit is shown in Fig. 3-5. Since it is not possible to change the current flowing through the inductor instantaneously, the voltage polarity across the inductor immediately reverses in an attempt to maintain the same current I_2 which was flowing just prior to S_1, being switched off. The freewheeling diode, D_{fw}, conducts since it is forward biased just as the inductor voltage reverses its polarity. The inductor current decreases as the load expends the energy stored in the inductor. The voltage across the inductor, $v_L(t)$, is now $-V_a$, and the inductor current decreases linearly from I_2 to I_1 in time t_{off}.

$$\mathbf{V_a = L\frac{I_2 - I_1}{t_{off}} = L\frac{\Delta I}{t_{off}}} \qquad (3\text{-}5)$$

The peak-to-peak ripple current ΔI is the same during $0<t\le t_{on}$ and $t_{on}<t\le T$ for steady-state operation, and is

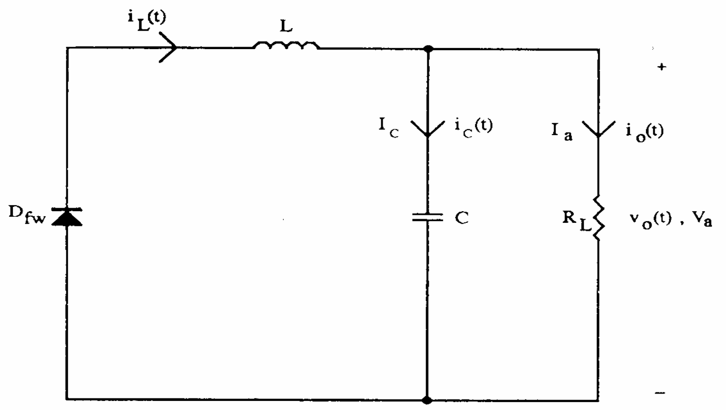


Fig. 3-5, Mode 2 equivalent circuit for the Buck converter ($t_{on}<t\le T$).

$$\mathbf{\Delta I = \frac{(V_s - V_a)t_{on}}{L} = \frac{V_a t_{off}}{L}} \qquad (3\text{-}6)$$

Substituting t_{on}=DT and t_{off}=(1-D)T into equation (3-6) gives

$$\mathbf{V_a = \frac{V_s DT}{T} = V_s D} \qquad (3\text{-}7)$$

The peak-to-peak ripple current in the inductor, ΔI, can then be expressed as

$$\mathbf{\Delta I = \frac{V_a (V_s - V_a)T}{LV_s} = \frac{DV_s(1-D)}{Lf_s}} \qquad (3\text{-}8)$$

Thus, the peak-to-peak ripple current in the inductor is inversely proportional to the inductance value and switching frequency, f_s.

Hence, to decrease the peak-to-peak output ripple voltage, the product LC should be large and the switching frequency should be high. Switching waveforms for the Buck converter operating in the continuous inductor current are shown in Fig. 3-6.

The input current is discontinuous, and a smoothing input filter, consisting of a series inductor and a shunt capacitor, is normally required to product the smoothing output current. The output inductor current is continuous due to the freewheeling action of the freewheeling diode. During the on time, the output capacitor is initially discharged because the output inductor current is smaller than the required load current. However, as the output inductor current increases beyond the current required by the load, the output capacitor is charged. The maximum capacitor charging current, I_2-I_a, occurs at the end of the on time, i.e., DT. The maximum capacitor discharging current, I_1-I_a, occurs at the end of the switching cycle. It should be noted that the capacitor ripple voltage lags the current by 90°. The advantages of the Buck converter are its ability to easily control output voltages and currents during turn-on and turn-off and under fault conditions.

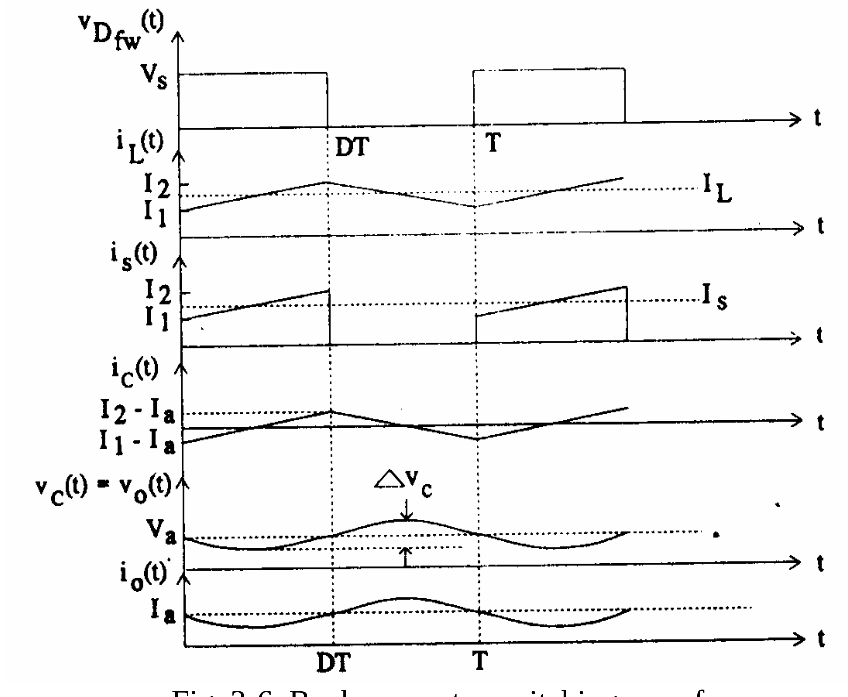


Fig. 3-6, Buck converter switching waveforms

3-4 Boost Converter

The Boost converter (see Fig.3-2(b)) is capable of providing an output voltage or which is greater than the input voltage. It is also known as a step-up converter. A Boost converter uses a switch transistor. Its switching waveforms are shown in Fig. 3-7. The operation of the Boost converter can also be divided into two modes, depending on the switching actions of its switching transistor. Similar to the Buck converter, the

Boost converter can be operated in either continuous or discontinuous inductor current model.

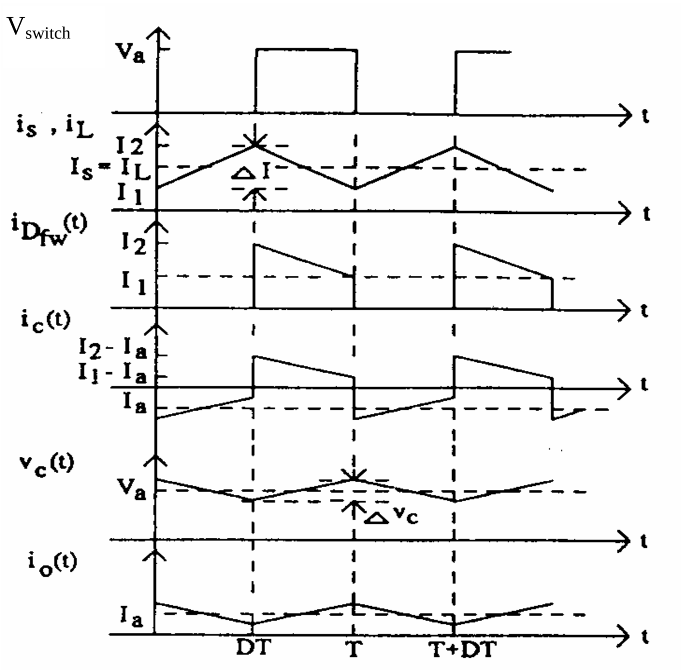


Fig. 3-7, Waveforms for the Boost converter.

Continuous Inductor Current

Mode 1 ($0<t\leq t_{on}$) "Transistor on"

Mode 1 begins when the switching transistor is switched on t=0 and it terminates at $t=t_{on}$. The equivalent circuit for mode 1 is shown in Fig. 3-8. The voltage across diode D_{fw} is reversed since the voltage drop across the switching transistor is smaller than the output voltage. The inductor current, $i_L(t)$, ramps up linearly from I_1, to I_2 in time t_{on} so that

$$\mathbf{V_s = L\frac{I_2 - I_1}{t_{on}} = L\frac{\Delta I}{t_{on}}} \tag{3-9}$$

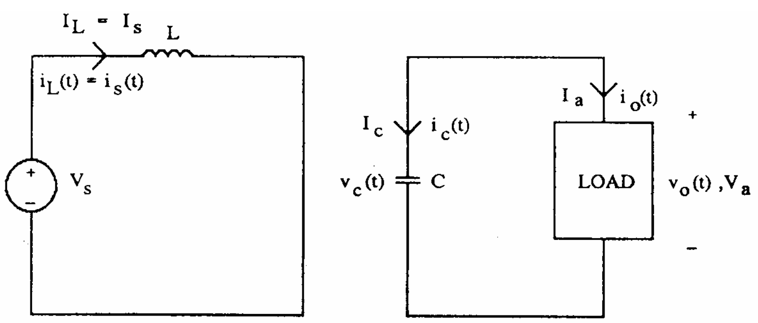


Fig. 3-8, Mode 1 equivalent circuit for the Boost converter ($o<t\leq t_{on}$).

The output current during this interval is supplied entirely from the output capacitor, C, which is chosen large enough to supply the load current during t_{on} with a minimum specified drop in output current.

Mode 2 (t_{on}<t≤T) "Transistor off"

Mode 2 begins when the switching transistor is switched off at $t=t_{on}$ The equivalent circuit for this mode is shown in Fig. 3-9

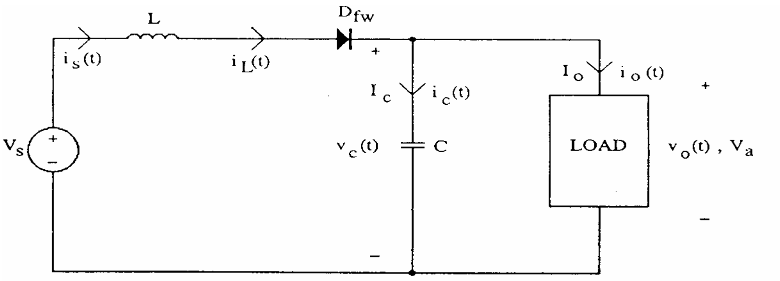


Fig. 3-9, Mode 2 equivalent circuit for the Boost converter (t_{on}<t≤T)

Since the current in the inductor cannot change instantaneously, the voltage in the inductor reverses its polarity in an attempt to maintain a constant current. The current that was flowing through the switching transistor will now flow through L, C, diode D_{fw}, and the load. The inductor current decreases until the switching transistor is turned on again during the next cycle. The inductor delivers its stored energy to the output capacitor C, and charges it up via D_{fw} to a higher voltage than the input voltage. This energy supplies the current and replenishes the charge drained away from the output capacitor while it alone was supplying the load current during the on time. The voltage across the inductor is V_s-V_a, and the inductor current decreases linearly from I_2 to I_1 in time t_{off}:

$$\mathbf{V_s - V_a = L\frac{I_1 - I_2}{t_{off}}} \tag{3-10 a}$$

or

$$\mathbf{V_a - V_s = L\frac{\Delta I}{t_{off}}} \tag{3-10 b}$$

Since the change in the peak-to-peak inductor ripple current, ΔI, is the same during t_{on} and t_{off} for steady-state operation, it can be shown from equations (3-9) and (3-10b) that

$$\Delta I = \frac{V_s t_{on}}{L} = \frac{(V_a - V_s) t_{off}}{L} \tag{3-11}$$

Substituting t_{on}=DT and t_{off}=(1-D)T into equation (3-11), and Simplifying,

$$V_a = \frac{V_s}{1-D} \tag{3-12}$$

The average input current I_s, can be expressed as

$$I_s = \frac{I_a}{1-D} \tag{3-13}$$

Note that the average output current, I_a, is reduced by a factor of 1-D from the average input current since the output power can only be, at best, equal to the input power. The peak-to-peak ripple current in the inductor, ΔI, is

$$\Delta I = \frac{V_s D}{L f_s} \tag{3-14}$$

3-5 Buck-Boost Converter

The Buck-Boost converter (see Fig.3-2(c)) is a special cascade combination of a Buck converter and a Boost converter which provides an output voltage that may be less than or greater than the input voltage, with a polarity opposite to that of the input voltage. As such, it is also known as an inverting converter. Its switching waveforms are shown in Fig. 3-10.

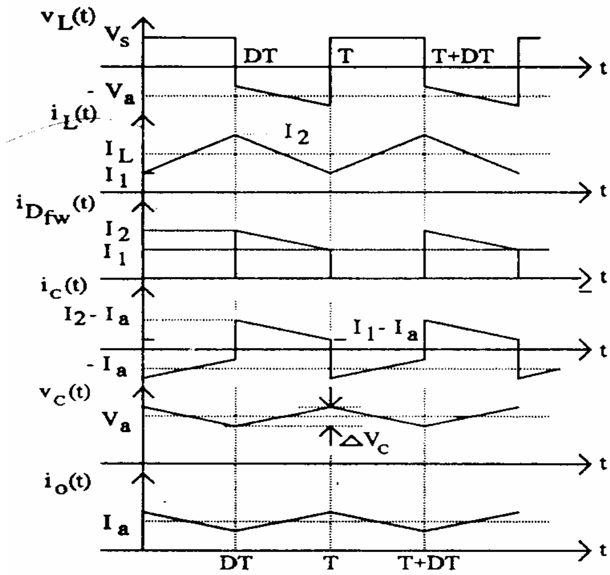


Fig. 3-10, Buck-Boost converter waveforms.

The operation of this converter can also be divided into two modes, depending on the switching actions of its switching transistor. Depending on the continuity of the current flowing through the inductor, the operating mode of the Buck-Boost converter can be classified as either continuous or discontinuous inductor current.

Continuous Inductor Current

Mode 1 ($0<t\leq t_{on}$) "Transistor on"

Mode I begins when the switching transistor is turned on at t=0. Its equivalent circuit is shown in Fig. 3-11. The freewheeling diode is reversing biased since the voltage across the inductor is near the input voltage assuming that the output voltage is of negative polarity.

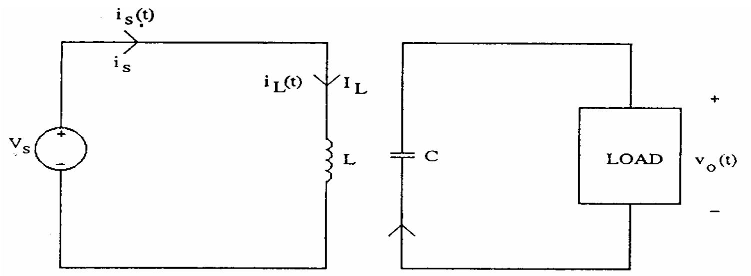


Fig. 3-11, Mode 1 equivalent circuit of the Buck-Boost converter ($0<t\leq t_{on}$)

The inductor current increases linearly from I_1, to I_2 in time t_{on}:

$$\mathbf{V_s = L\frac{I_2 - I_1}{t_{on}} = L\frac{\Delta I}{t_{on}}} \tag{3-15}$$

Mode 2 ($t_{on}<t\leq T$) "Transistor off"

Mode 2 begins when the switching transistor is switched off at t_{on}. Its equivalent circuit is shown in Fig. 3-12. The polarity of the voltage across the inductor reverses in an attempt to keep the current from changing direction. Thus, at the instant of turn off, the current that was flowing through the inductor now flows through L, C, D_{fw}, and the load. This current, flowing upward through the output capacitor, charges the top end of the capacitor to a negative voltage as previously assumed.

The energy stored in inductor L is transferred to the load and the inductor current decreases until the switching transistor is switched on again in the next cycle. Assuming the inductor current decreases linearly from I_2 to I_1 in time t_{off}:

$$\mathbf{V_a = -L\frac{\Delta I}{t_{off}}} \tag{3-16}$$

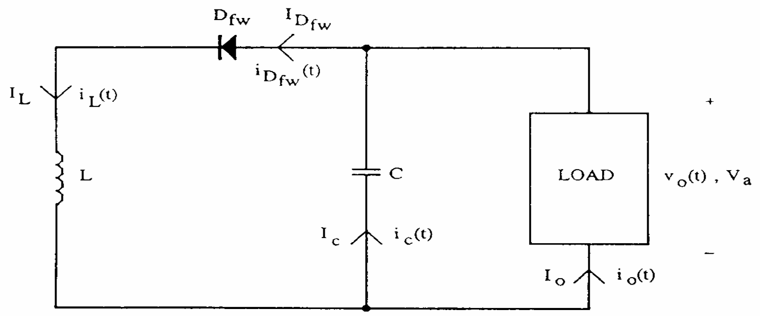


Fig. 3-12, Mode 2 equivalent circuit for the Buck-Boost converter ($t_{on}<t\leq T$).

Since the peak-to-peak inductor ripple currents present during the t_{on}, and t_{off} intervals are the same as for steady-state operation, it can be shown from equations (3-15) and (3-16) that

$$\Delta I = \frac{V_s t_{on}}{L} = -\frac{V_a t_{off}}{L} \tag{3-17}$$

Substituting t_{on}=DT and t_{off}=(I-D)T into the above equation. We get the average output voltage,

$$V_a = -\frac{V_s D}{1-D} \tag{3-18}$$

As can be seen, V_a, has a polarity opposite to the input voltage, V_s. The D in the numerator is the output conversion factor of a Buck converter, while the denominator 1-D is the output conversion factor for a Boost converter. It should be emphasized that simply cascading a Boost converter with a Buck converter cannot mathematical form Buck-Boost converter. Neglecting the losses Buck-Boost converter, the input power is equal to the output power. The average input current can be expressed as

$$I_s = -\frac{I_a D}{1-D} \tag{3-19}$$

The peak-to-peak inductor ripple current, Δl, with simplifying

$$\Delta I = \frac{V_s D}{f_s L_c} \tag{3-20}$$

The peak-to-peak inductor ripple current is, therefore, similar to that of the Boost converter. When the switching transistor is switched on, the output capacitor supplies the load current for the entire on-time interval. The average discharging current of the capacitor is equal to I_a. During the off-time interval, the output capacitor is charged by the stored energy in the inductor. The capacitor charging current decreases linearly from I_2-I_a, to I_1-I_a during this interval. For steady-state operation, the average capacitor charging current-time product during the off-time interval must be equal to the average capacitor discharging current-time product during the on-time interval.

3-6 Cûk Converter

The CûK converter (see Fig.3-3) is effectively a cascaded connection of a Boost converter. Fig. 3-13 shows how a CûK converter may be decomposed into a Boost converter and a Buck converter. The two converters shown share the same electronic switch and freewheeling diode D_{fw}. While the energy storage capacitor C is used as the output filtering capacitor for the Boost converter, it is also effectively used as a battery providing power to the Buck converter.

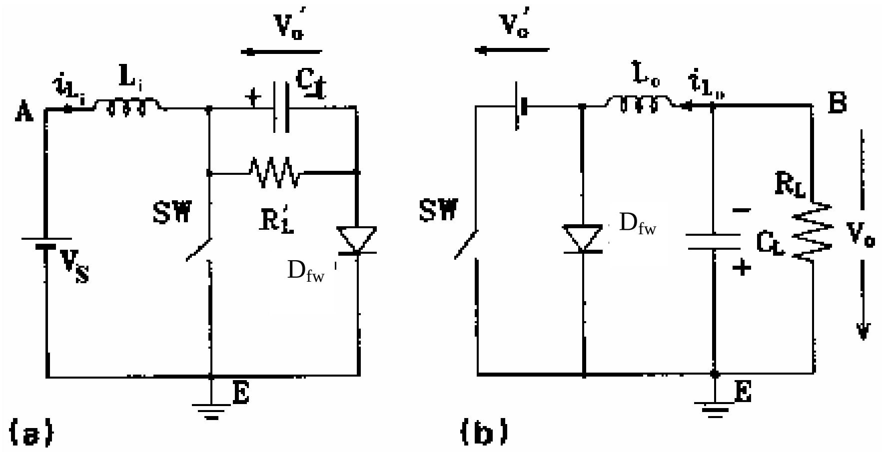


Fig. 3-13, Decomposition of a CûK converter into (a) a Boost converter and (b) a Buck converter.

The load resistance R_L' shown in the Boost converter is used to represent the loading effect of the Buck converter. The main advantage of the CûK converter is the possibility of designing it to give a very smooth input current and output current. In other words, it combines the merits of a Boost converter, which has a smoothed input current, and a Buck converter, which has a smoothed output current. However, it should be noted that, in order to achieve this advantage, both the Boost and Buck parts of the CûK converter need to operate in the continuous mode operation (i.e., a current is maintained in the freewheeling diode during the entire period of $DT<t<T$, when the switch SW is turned off). Its switching waveforms are shown in Fig. 3-14. The continuous mode of operation of the CûK converter is discussed below.

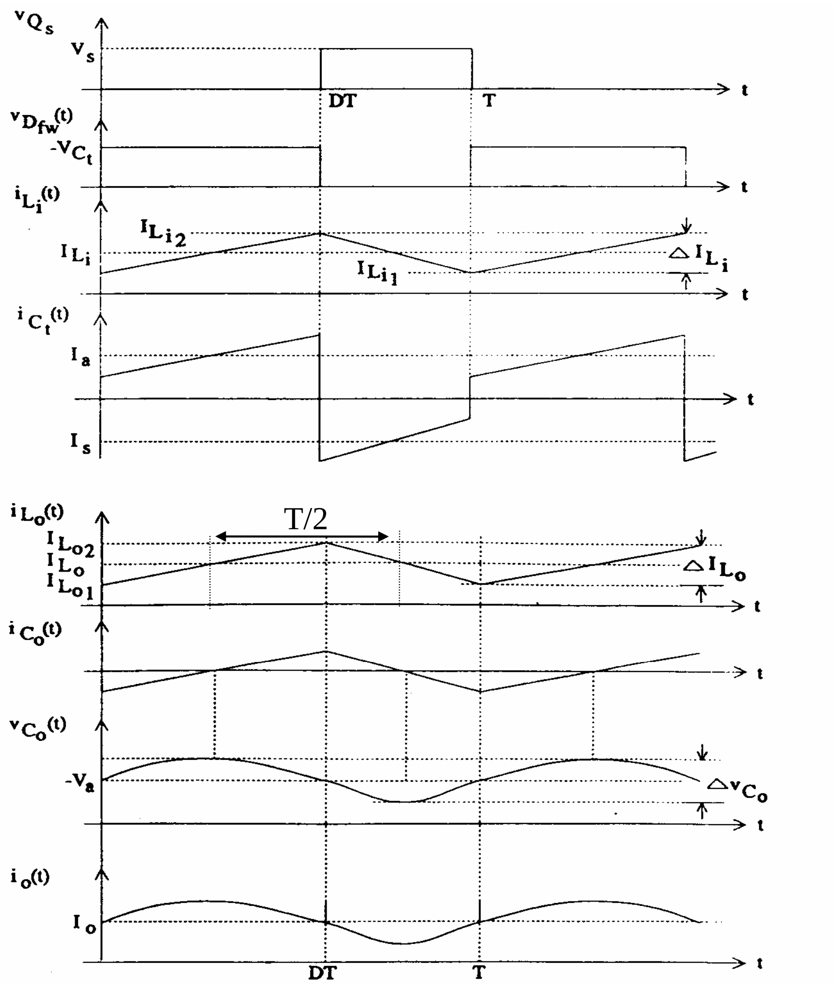


Fig. 3-14, CûK converter switching waveforms.

Mode 1 (0<t≤t_{on}) "Transistor on"

Mode 1 begins when the switching transistor is switched on at t=0. Its equivalent circuit is shown in Fig. 3-15. The current flowing through the input inductor, $i_{Li}(t)$, increases. At the same time, the voltage across the energy-transfer capacitor reverse-biases diode and turns it off. The energy-transfer capacitor, C_t, discharges its energy into the circuit formed by C_t, C_o, L_o, and the load. Assuming that the current of the input inductor increases linearly from I_{Li1} to I_{Li2} in time t_{on}, we obtain

$$\mathbf{V_s = L_i \frac{I_{L_{i2}} - I_{L_{i1}}}{t_{on}} = L_i \frac{\Delta I_{L_i}}{t_{on}}} \tag{3-21}$$

Due to the discharge of the energy-transfer capacitor the current flowing through the output inductor, $i_{Li}(t)$, increases linearly from I_{Lo1} to I_{Lo2} in time t_{on} so that

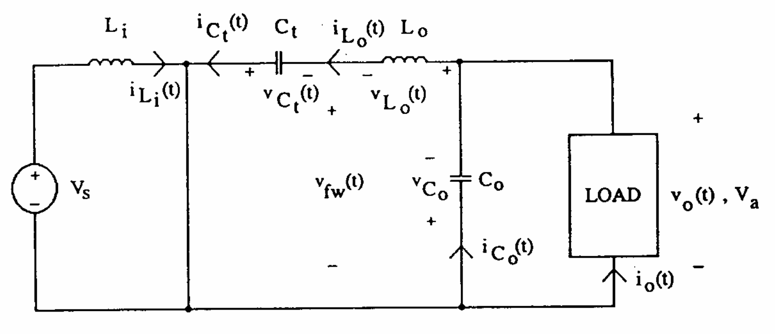


Fig. 3-15, Mode 1 equivalent circuit for the CûK converter.

$$V_{ct} + V_{a} = \frac{L_{o}(I_{L_{o2}} - I_{L_{o1}})}{t_{on}} \tag{3-22}$$

Mode 2 (t_{on} < t ≤ T) "Transistor off"

Mode 2 begins when the switching transistor is switched off at t=t_{on}. The voltage across the input inductor reverses its polarity in order to maintain its current uninterrupted. The diode D_{fw} is forward biased since the anode is at a higher potential than its cathode. The converter's equivalent circuit is shown in Fig. 3-16. The energy-transfer capacitor is charged by the input source, V_s, and the energy stored in the input inductor. The load current, $i_o(t)$, is now supplied by the energy stored in the output inductor, L_o, and the output capacitor, C_o.

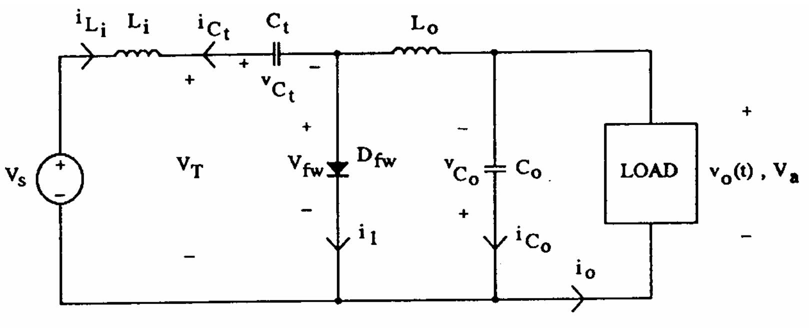


Fig. 3-16, Mode 2 equivalent circuit for the CûK converter.

The current flowing through the input Inductor, $i_{Li}(t)$, decreases linearly from I_{Li1} to I_{Li2} in time t_{off}

$$V_{s} - V_{ct} = -\frac{L_{i}\Delta I_{L_{i}}}{t_{off}} \tag{3-23}$$

where V_{Ct} is the average voltage of C_t. At the same time, the current flowing through the output inductor, $i_{Lo}(t)$, decreases linearly from I_{Lo2} to I_{Lo1} in time t_{off}. Therefore,

$$\mathbf{V_a = -\frac{L_o \Delta I_{L_o}}{t_{off}}} \tag{3-24}$$

The peak-to-peak ripple current in the input inductor,can be found from equations (3-21) and (3-23):

$$\mathbf{\Delta I_{L_i} = \frac{V_s t_{on}}{L_i} = \frac{-(V_s - V_{ct})t_{off}}{L_i}} \tag{3-25}$$

Substituting t_{on}=DT and t_{off}=(1-D)T into the above equation and solving for V_{ct} gives

$$\mathbf{V_{c_t} = \frac{V_s}{1-D}} \tag{3-26}$$

The peak-to-peak ripple current in the output inductor, can be found from equations (3-22) and (3-24):

$$\mathbf{\Delta I_{L_o} = \frac{(V_{c_t} + V_a)t_{on}}{L_o} = -\frac{V_a t_{off}}{L_o}} \tag{3-27}$$

Substituting t_{on}=DT and t_{off} = (1-D)T into the above equation, we obtain

$$\mathbf{V_{c_t} = -\frac{V_a}{D}} \tag{3-28}$$

Equating equations (3-26) and (3-28) then the average output voltage is

$$\mathbf{V_a = -\frac{DV_s}{1-D}} \tag{3-29}$$

This is similar to the average output voltage of the Buck-Boost converter. Neglecting the losses CûK converter, the input power is equal to the output power. Then, the average input current is

$$\mathbf{I_s = -\frac{DI_a}{1-D}} \tag{3-30}$$

Thus, the peak-to-peak ripple current in the output inductor is also inversely proportional to the switching frequency, f_s and its inductance value, L_o. When the switching transistor is switched on, the energy-transfer capacitor discharges its stored energy to L_o, C_o, and the load circuit. The discharging current increases from I_{Lo1} to I_{Lo2}. The average discharging current I_{Ct} in C_t is equal to the average output current, I_a.

During the off-time, the energy-transfer capacitor is charged by the energy stored in the input inductor and the input source. The charging current decreases from I_{Li2} to I_{Li1}. Therefore, the average charging current, I_{Ct} is the same as the average input current, I_s. The charging and discharging current-time products of the energy-transfer capacitor, for steady-state operation, must be equal. The peak-to-peak ripple voltage of the energy-transfer capacitor is

$$\Delta \mathbf{v}_{c_t} = \frac{\mathbf{I_s t_{off}}}{\mathbf{C_t}} = \frac{\mathbf{I_s (1-D)}}{\mathbf{f_s C_t}} \tag{3-31}$$

Thus, the peak-to-peak ripple voltage across the energy-transfer capacitor is directly proportional to the average input current and inversely proportional to the product of the switching frequency and the capacitance value. Assuming that the peak-to-peak load ripple current is negligible, then $\Delta i_{C_o} = \Delta i_{L_o}$. The average charging current of the output capacitor, which flows for time T/2, is $I_{C_o} = \Delta I_{L_o}/4$, (see Fig.3-14). Therefore,

$$\Delta \mathbf{v}_{c_o} = \frac{\mathbf{T \Delta I_{L_o}}}{\mathbf{8C_o}} = \frac{\mathbf{\Delta I_{L_o}}}{\mathbf{8 f_s C_o}} \tag{3-32}$$

The peak-to-peak output ripple voltage is equal to the peak-to-peak output capacitor ripple voltage since the output capacitor is connected directly across the load. The most effective way to reduce the output ripple voltage is to increase the switching frequency since the output ripple voltage is inversely proportional to the square of the switching frequency. The output ripple voltage and ripple current of the CûK converter are much smaller than those of the Buck-Boost converter. A substantial weight and size reduction of the CûK converter over the Buck-Boost converter is the result of a smaller output filter inductor, L_o, and a smaller energy-transfer capacitor, C_t, because the capacitive energy storage in a CûK converter is more efficient than the inductive energy storage in a conventional Buck-Boost converter. The switching transistor and the freewheeling diode in the CûK converter has to carry the currents flowing through both the input and output inductors. In fact, the CûK converter has been shown to have an overall higher efficiency when compared to the conventional Buck-Boost converter with the same component values and output requirements. A comparison of the voltage conversion ratio for the four types of switching converters is given in Fig. 3-17, [59].

As shown previously, a linear relationship between the input and output voltages is a characteristic of the Buck switching converter. The rapidly changing output voltage of the Boost Buck-Boost and CûK switching converters, when operating at a duty cycle of greater than 50%, presents some challenging stability problems in the design of these switching converters.

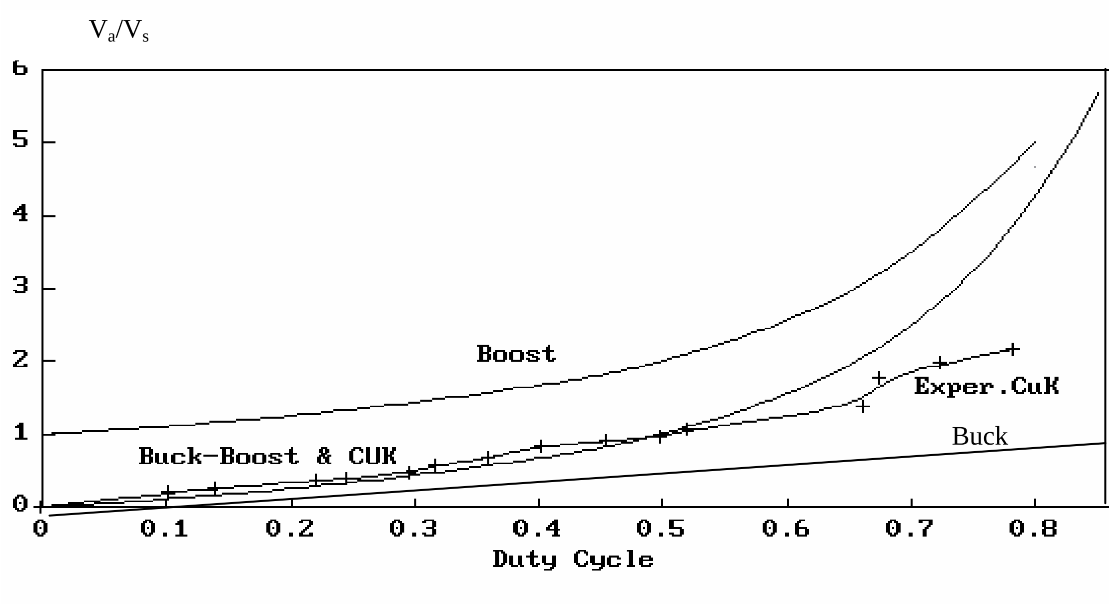


Fig. 3-17, Comparisons of the voltage conversion ratios of switching converters.

3-7 Dynamic Analysis of Switching Converters

The dynamic characteristic of the switching converter can be used to predict the (a)margin of stability of the switching converter, (b) input supply ripple rejection and the transient response due to input supply perturbation, and (c) output impedance and the transient response due to load perturbation. Dynamic analysis of switching converters will be performed using the state-space averaging technique. Transfer functions of switching converters are readily obtained from results of the state-space averaging. These transfer functions are useful in dynamic analysis of switching converters, [59]. The state-space averaging method is easy to use in simulation than the Pspice software. Hence, the state space averaging mothod is used, as it also converts the nonlinear system (on/off the switch of converter) to linear system.

3-7-1 State-Space Averaging

State-space averaging is an approximation technique that approximates the switching converter as a continuous linear system. State-space averaging requires that the effective output filter corner frequency, f_c, be much smaller than the switching frequency, f_s, or f_c/f_s<<1. This is similar to the requirement for a low output switching ripple. Final results of the state-space averaging can be either a mathematical or an equivalent circuit model. The mathematical model permits the designer to determine voltages, currents, and small-signal transfer functions of the switching converter. However, this model does not enable the designer to physically visualize electrical processes occurring in the switching converter. The equivalent circuit model provides the designer with a better understanding of the physical operation of the switching converter. In general, both the mathematical and equivalent circuit models are necessary and recommended in the design of practical switching converters. There are two drawbacks of the state-space-averaging technique. The major one is that it does not result in a general linearized model of a switching converter model that is independent of the switching converter configuration, operating mode, and controlled quantity. The other drawback is that it requires extensions and modifications if the controlled quantity is other than the duty cycle. Procedures for state-space averaging are as follows:

Step 1. Identify switched models over a switching cycle. Draw the linear switched circuit model for each state of the switching converter.

Step 2. Identify state variables of the switching converter. Write state equations for each switched circuit model using Kirchhoff's voltage and current laws.

Step 3. Perform state-space averaging using the duty cycle as weighting factor and combine state equations into a single averaged state equation. The state-space-average equation is [60]:

$$\dot{X} = [A_1 D + A_2(1-D)]X + [B_1 D + B_2(1-D)]U \tag{3-33}$$

where: X is the state vector;
A is the state coefficient matrix;
U is the source vector;
B is the source coefficient matrix

3-7-2 State-Space-Averaged Model for an Ideal Buck Converter

We now illustrate the state-space averaging method for an ideal Buck converter operating in continuous mode shown in Fig. 3-18. State variables for this Buck converter are chosen as the inductor current, $I_L \equiv X_1$, and the capacitor voltage, $V_c \equiv X_2$. With the assumption of ideal switching devices, two switched models are shown in Figs. 3-19a and 3-19b, respectively. From Kirchhoff's voltage law in Fig. 3-19a, the state equation is

$$U_1 = L\dot{X}_1 + X_2 \tag{3-34}$$

and from Kirchhoff's current law, the state equation is

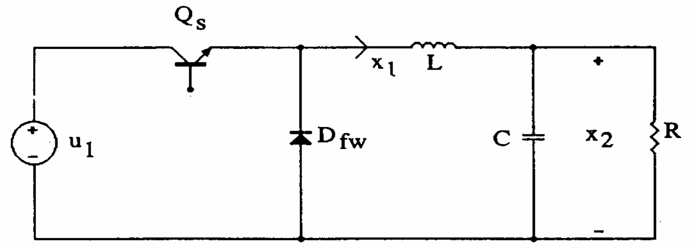


Fig. 3-18, Schematic circuit of an ideal Buck converter with the state and source variables indicated.

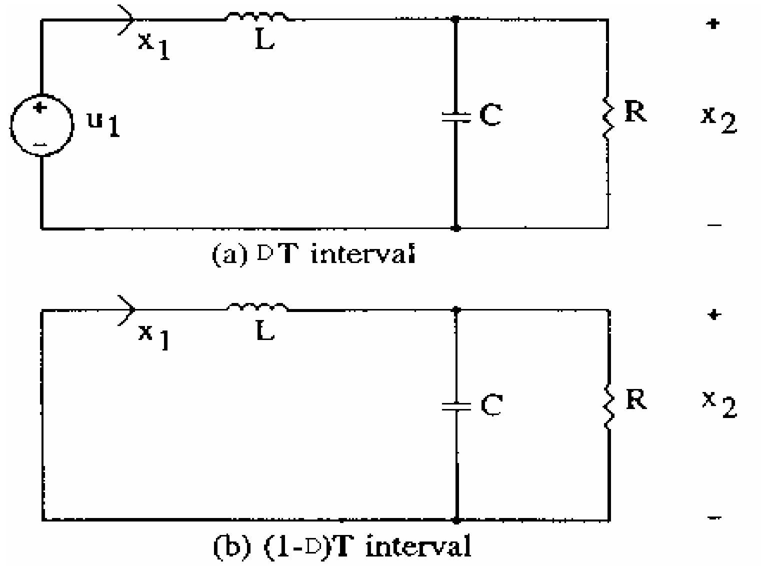


Fig. 3-19, Switched models for the ideal Buck converter.

$$X_1 = C\dot{X}_2 + \frac{X_2}{R} \tag{3-35}$$

For the interval when the switching transistor is switched on, i.e.,DT. Similarly, applying Kirchhoffs voltage law to the switched model shown in Fig. 3-19b, the state equation is

$$0 = L\dot{X}_1 + X_2 \tag{3-36}$$

Applying Kirchhoff's current law, the state equation is

$$X_1 = C\dot{X}_2 + \frac{X_2}{R} \tag{3-37}$$

For the interval when the switching transistor is switched off, i.e., (1-D)T. Equations (3-34) to (3-37) can be written in matrix form as

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} \begin{bmatrix} U_1 \end{bmatrix} \tag{3-38}$$

for the DT interval and

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} \begin{bmatrix} U_1 \end{bmatrix} \tag{3-39}$$

for the (1-D)T interval, respectively. The state-space-averaged state coefficient matrix is

$$A = \begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix} D + \begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix} (1-D) = \begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix} \tag{3-40}$$

The state-space-averaged source coefficient matrix is

$$B = \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} D + \begin{bmatrix} 0 \\ 0 \end{bmatrix} (1-D) = \begin{bmatrix} \frac{D}{L} \\ 0 \end{bmatrix} \tag{3-41}$$

State-space-averaged equations for the Buck converter in matrix form are

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} \frac{D}{L} \\ 0 \end{bmatrix} \begin{bmatrix} U_1 \end{bmatrix} \tag{3-42}$$

Fig. 3-20 shows, the simulation blocks diagram of the Buck converter analysis by MATLAB-SIMULINK. The Dynamic performance of Buck converter with step change in duty cycle is shown in Fig. 3-21. The input voltage starts at 0.01sec. While, the duty cycle changes from 0.55 to 0.3 at time 0.3sec and constant input voltage.

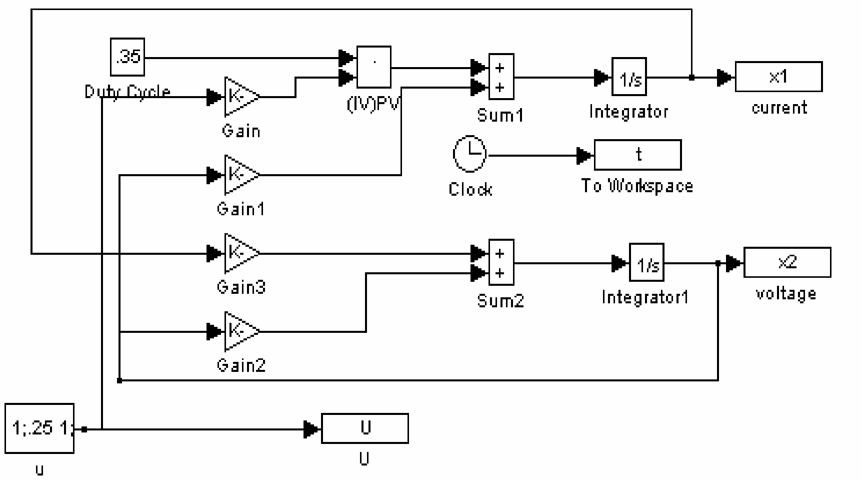


Fig. 3-20, Simulation blocks diagram of the Buck converter analysis $V_{i/p}$ and $V_{o/p}$

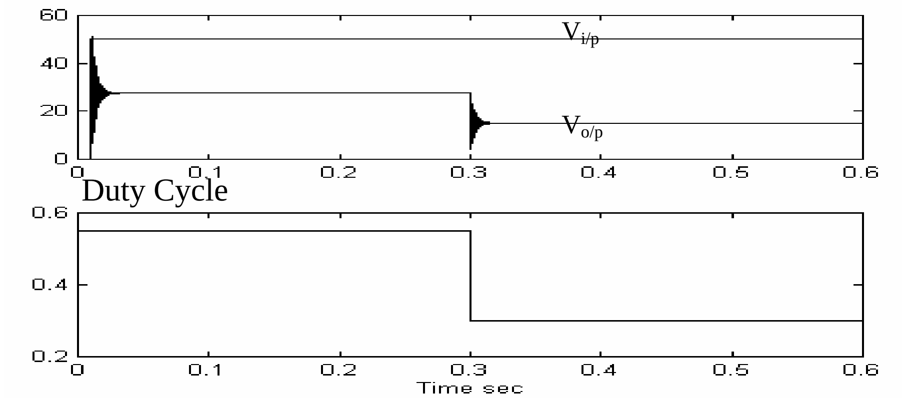


Fig. 3-21, Dynamic performance of Buck converter with step change in duty cycle.

3-7-3 State-Space-Averaged Model for an Ideal Boost Converter

An ideal Boost converter with a variable source U_2 is used to simulate the load current modulation. With the assumption of ideal circuit elements, two switched models are shown in Fig. 3-22. State variables for this Boost converter are chosen as the inductor current, $I_L \equiv X_1$, and the capacitor voltage, $V_c \equiv X_2$. State-space-averaged equations in matrix form is given as follows.

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} = \begin{bmatrix} 0 & -\frac{(1-D)}{L} \\ \frac{1-D}{C} & -\frac{1}{RC} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} \frac{1}{L} & 0 \\ 0 & \frac{1}{C} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \end{bmatrix} \tag{3-43}$$

The Dynamic performance of Boost converter with step change in duty cycle shows in Fig. 3-23.

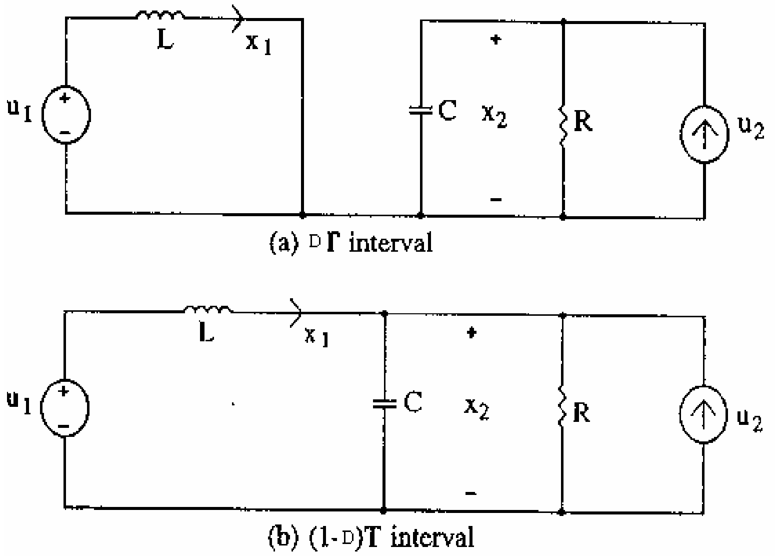


Fig. 3-22, Switched models for the ideal Boost converter.

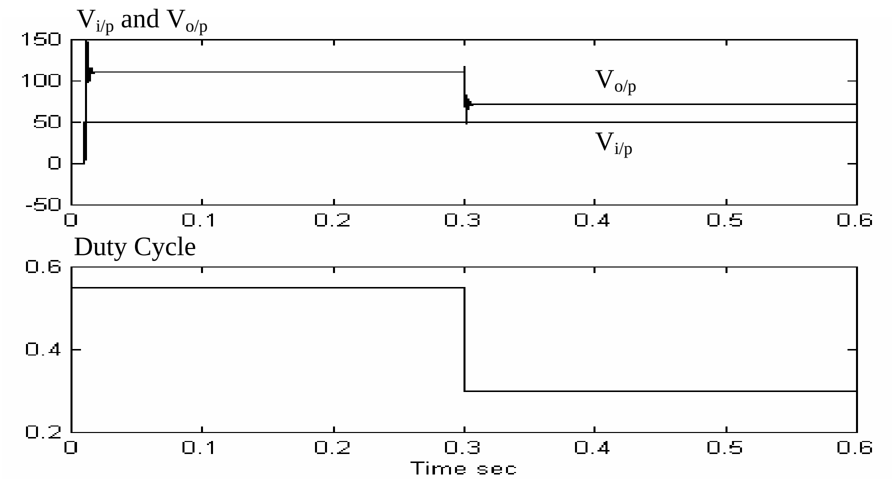


Fig. 3-23, Dynamic performance of Boost converter with step change in duty cycle.

3-7-4 State-Space-Averaged Model for an Ideal Buck-Boost Converter

An ideal Buck-Boost converter to simulate the load current modulation. With the assumption of ideal circuit elements, two switched models are shown in Fig. 3-24. State variables for this Buck-Boost converter are chosen as the inductor current, $I_L \equiv X_1$, and the capacitor voltage, $V_c \equiv X_2$. State-space-averaged equations in matrix form are

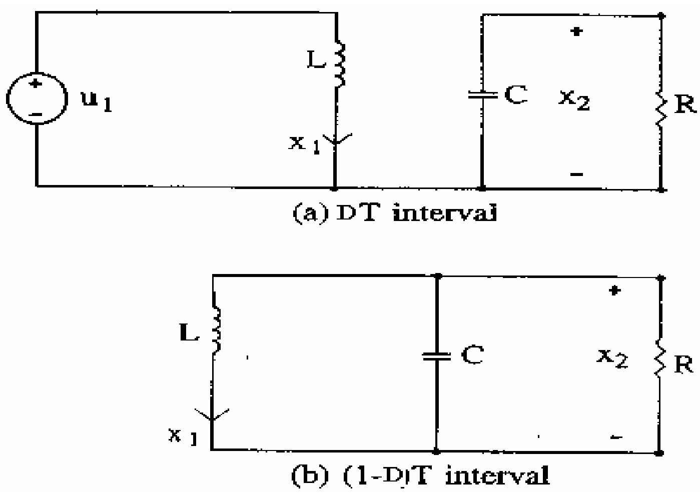


Fig. 3-24, Switched models for the ideal Buck-Boost converter.

$$\begin{bmatrix} \dot{\mathbf{X}}_1 \\ \dot{\mathbf{X}}_2 \end{bmatrix} = \begin{bmatrix} 0 & \dfrac{(1-\mathbf{D})}{\mathbf{L}} \\ -\dfrac{(1-\mathbf{D})}{\mathbf{C}} & -\dfrac{1}{\mathbf{RC}} \end{bmatrix} \begin{bmatrix} \mathbf{X}_1 \\ \mathbf{X}_2 \end{bmatrix} + \begin{bmatrix} \dfrac{\mathbf{D}}{\mathbf{L}} \\ 0 \end{bmatrix} \begin{bmatrix} \mathbf{U}_1 \end{bmatrix} \tag{3-44}$$

The Dynamic performance of Buck-Boost converter with step change in duty cycle is shown in Fig. 3-25.

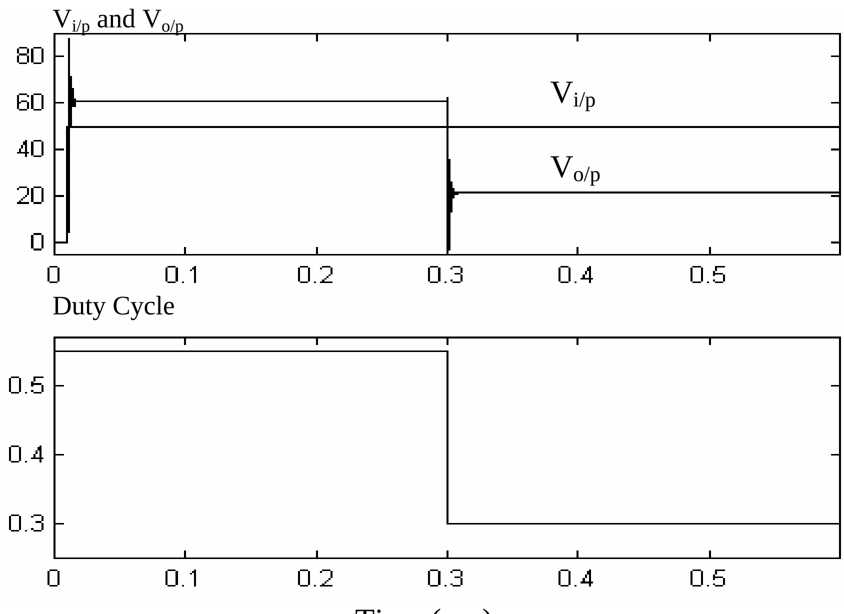
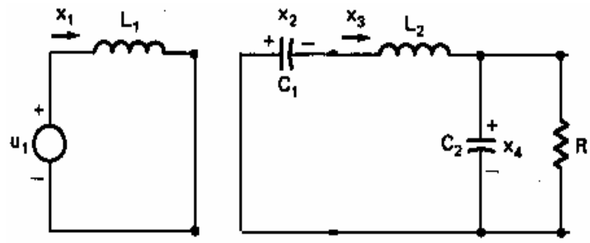


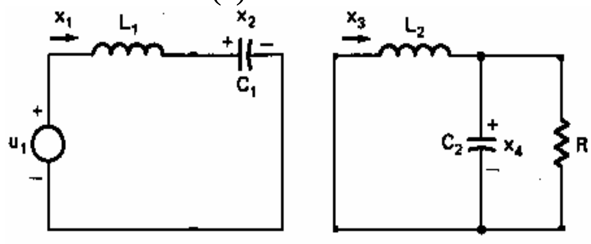
Fig. 3-25, Dynamic performance of Buck-Boost converter with step change in duty cycle.

3-7-5 State-Space-Averaged Model for an Ideal CûK Converter

An ideal CûK converter is used to simulate the load current modulation. With the assumption of ideal circuit elements, the first step is to write the four linear differential equations corresponding to the state variables for each of the two switched models are shown in Fig. 3-26, State variables for this CûK converter are chosen as the inductor current $I_L \equiv X_1$, the capacitor voltage $V_c \equiv X_2$, the inductor current $I_{L2} \equiv X_3$, and the load voltage, $V_L \equiv X4$. In the DT interval, state equations (S_1 ON/S_2 OFF) are



(a) DT interval



(b) (1-D)T interval

Fig. 3-26, Switched models for the ideal CûK converter.

$$\mathbf{U_1 = L_1 \dot{X}_1} \tag{3-45}$$

$$\mathbf{X_2} + \mathbf{L_2}\dot{\mathbf{X}}_3 + \mathbf{X_4} = \mathbf{0} \tag{3-46}$$

According to Kirchhoff’s voltage law, and

$$\mathbf{X_3} = \mathbf{C_1}\dot{\mathbf{X}}_2 \tag{3-47}$$

$$\mathbf{X_3} = \mathbf{C_2}\dot{\mathbf{X}}_4 + \frac{\mathbf{X_4}}{\mathbf{R}} \tag{3-48}$$

According to Kirchhoff’s current law. State equations in matrix form are

$$\begin{bmatrix} \dot{\mathbf{X}}_1 \\ \dot{\mathbf{X}}_2 \\ \dot{\mathbf{X}}_3 \\ \dot{\mathbf{X}}_4 \end{bmatrix} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \frac{\mathbf{1}}{\mathbf{C_1}} & \mathbf{0} \\ \mathbf{0} & -\frac{\mathbf{1}}{\mathbf{L_2}} & \mathbf{0} & -\frac{\mathbf{1}}{\mathbf{L_2}} \\ \mathbf{0} & \mathbf{0} & \frac{\mathbf{1}}{\mathbf{C_2}} & -\frac{\mathbf{1}}{\mathbf{RC_2}} \end{bmatrix} \begin{bmatrix} \mathbf{X_1} \\ \mathbf{X_2} \\ \mathbf{X_3} \\ \mathbf{X_4} \end{bmatrix} + \begin{bmatrix} \frac{\mathbf{1}}{\mathbf{L_1}} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{U_1} \end{bmatrix} \tag{3-49}$$

During the (1-D)T interval, the state equation using Kirchhoff’s voltage law is

$$\mathbf{U_1} = \mathbf{L_1}\dot{\mathbf{X}}_1 + \mathbf{X_2} \tag{3-50}$$

$$\mathbf{L_2}\dot{\mathbf{X}}_3 + \mathbf{X_4} = \mathbf{0} \tag{3-51}$$

and the state equation using Kirchhoff’s current law is

$$\mathbf{X_1} = \mathbf{C_1}\dot{\mathbf{X}}_2 \tag{3-52}$$

$$\mathbf{X_3} = \mathbf{C_2}\dot{\mathbf{X}}_4 + \frac{\mathbf{X_4}}{\mathbf{R}} \tag{3-53}$$

These state equations can be expressed in matrix form as

$$\begin{bmatrix} \dot{\mathbf{X}}_1 \\ \dot{\mathbf{X}}_2 \\ \dot{\mathbf{X}}_3 \\ \dot{\mathbf{X}}_4 \end{bmatrix} = \begin{bmatrix} \mathbf{0} & -\frac{\mathbf{1}}{\mathbf{L_1}} & \mathbf{0} & \mathbf{0} \\ \frac{\mathbf{1}}{\mathbf{C_1}} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & -\frac{\mathbf{1}}{\mathbf{L_2}} \\ \mathbf{0} & \mathbf{0} & \frac{\mathbf{1}}{\mathbf{C_2}} & -\frac{\mathbf{1}}{\mathbf{RC_2}} \end{bmatrix} \begin{bmatrix} \mathbf{X_1} \\ \mathbf{X_2} \\ \mathbf{X_3} \\ \mathbf{X_4} \end{bmatrix} + \begin{bmatrix} \frac{\mathbf{1}}{\mathbf{L_1}} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{U_1} \end{bmatrix} \tag{3-54}$$

The state-space-averaged state coefficient matrix is

$$\mathbf{A} = \begin{bmatrix} 0 & -\frac{(1-\mathbf{D})}{\mathbf{L}_1} & 0 & 0 \\ \frac{(1-\mathbf{D})}{\mathbf{C}_1} & 0 & \frac{\mathbf{D}}{\mathbf{C}_1} & 0 \\ 0 & -\frac{\mathbf{D}}{\mathbf{L}_2} & 0 & -\frac{1}{\mathbf{L}_2} \\ 0 & 0 & \frac{1}{\mathbf{C}_2} & -\frac{1}{\mathbf{RC}_2} \end{bmatrix} \tag{3-55}$$

The state-space-averaged source coefficient matrix is

$$\mathbf{B} = \begin{bmatrix} \frac{1}{\mathbf{L}_1} \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{3-56}$$

State-space-averaged equations in matrix form are

$$\begin{bmatrix} \dot{\mathbf{X}}_1 \\ \dot{\mathbf{X}}_2 \\ \dot{\mathbf{X}}_3 \\ \dot{\mathbf{X}}_4 \end{bmatrix} = \begin{bmatrix} 0 & -\frac{(1-\mathbf{D})}{\mathbf{L}_1} & 0 & 0 \\ \frac{(1-\mathbf{D})}{\mathbf{C}_1} & 0 & \frac{\mathbf{D}}{\mathbf{C}_1} & 0 \\ 0 & -\frac{\mathbf{D}}{\mathbf{L}_2} & 0 & -\frac{1}{\mathbf{L}_2} \\ 0 & 0 & \frac{1}{\mathbf{C}_2} & -\frac{1}{\mathbf{RC}_2} \end{bmatrix} \begin{bmatrix} \mathbf{X}_1 \\ \mathbf{X}_2 \\ \mathbf{X}_3 \\ \mathbf{X}_4 \end{bmatrix} + \begin{bmatrix} \frac{1}{\mathbf{L}_1} \\ 0 \\ 0 \\ 0 \end{bmatrix} [\mathbf{U}_1] \tag{3-57}$$

Fig. 3-27 shows, the simulation blocks diagram of the Cûk converter analysis by MATLAB-SIMULINK. The Dynamic performance of Cûk converter with step change in duty cycle is shown in Fig. 3-28. It is noties that there is high oscillation at output voltage.

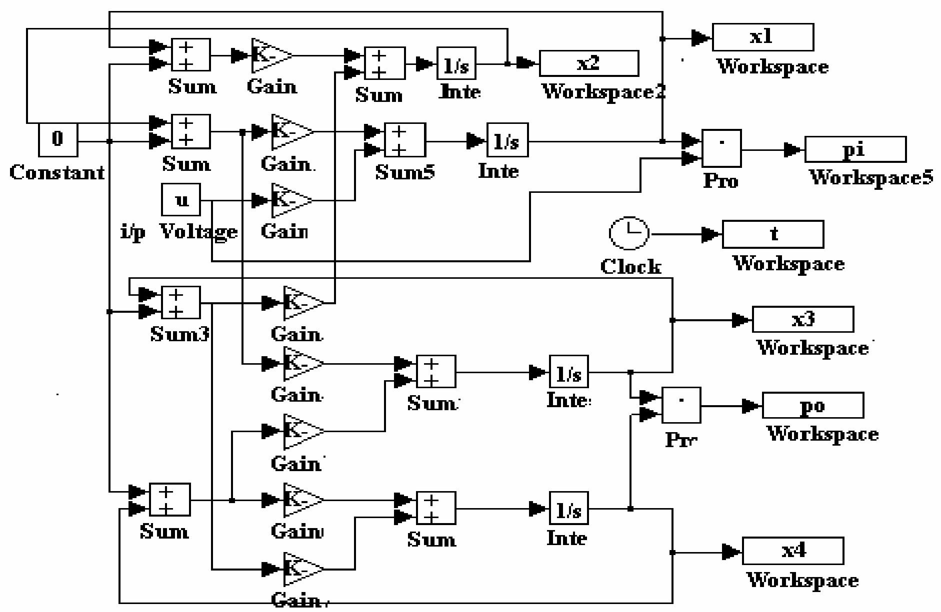


Fig.3-27, Simulation blocks diagram of the Cûk converter analysis

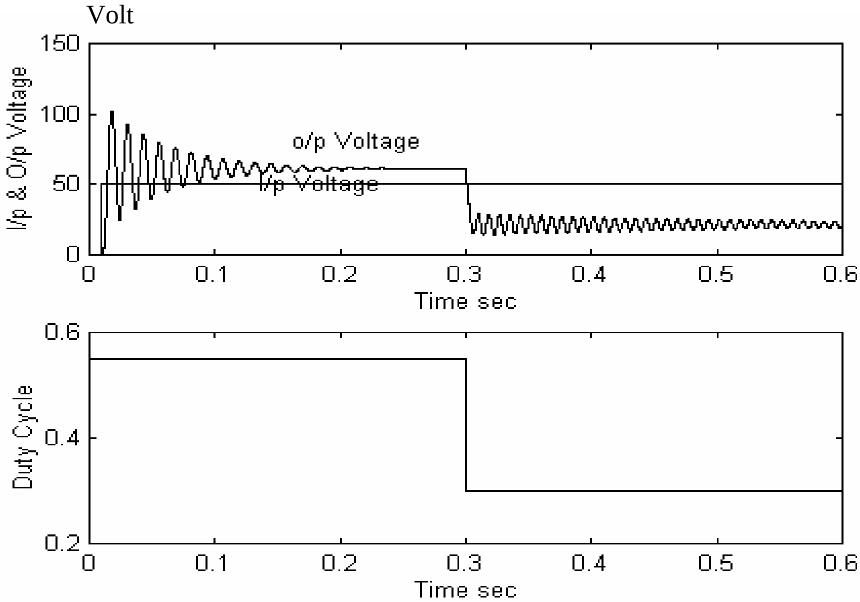


Fig. 3-28, Dynamic performance of Cûk converter with step change in duty cycle.

When, we apply small-signal in input voltage (U=U+Δu) to equation (3-57), the input and output voltage are shown in Fig. 3-29. While, applying small-signal in duty cycle (D=D+Δd) to the same equation, the duty cycle and output voltage are shown in Fig. 3-30.

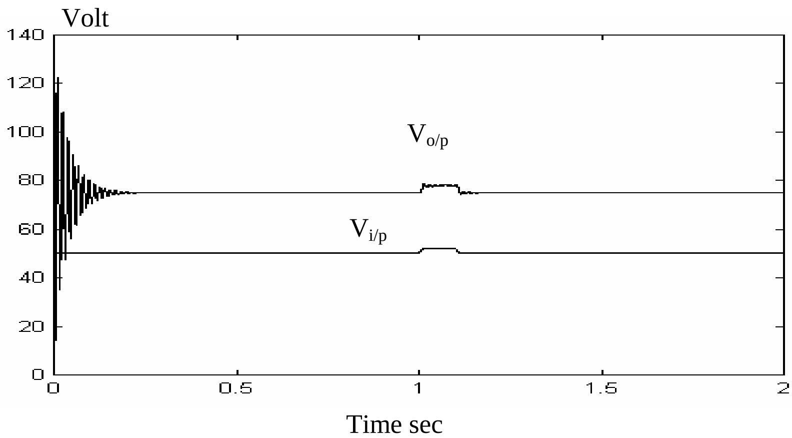


Fig.3-29, Dynamic performance of Cûk converter with small signal Δu.

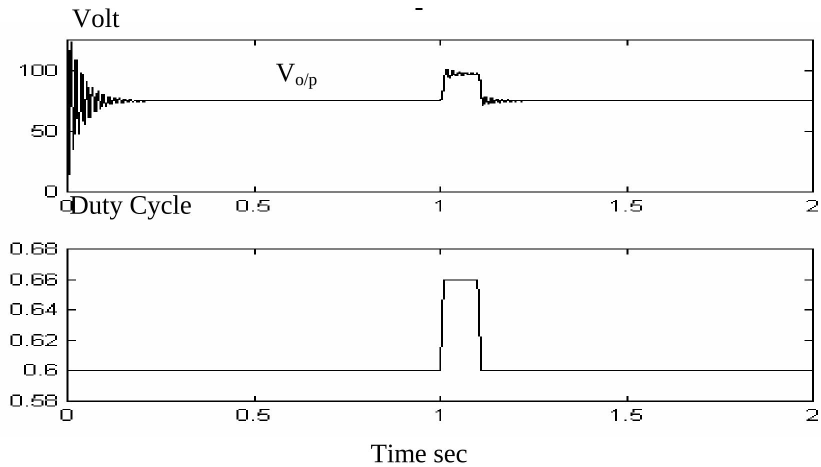


Fig. 3-30, Dynamic performance of Cûk converter with small signal Δd.

The input and output voltage and current are shown in Fig. 3-31. When, we apply small-signal (X=X+Δx) to equation (3-57). The results of the small signal are oscillated in the output voltage.

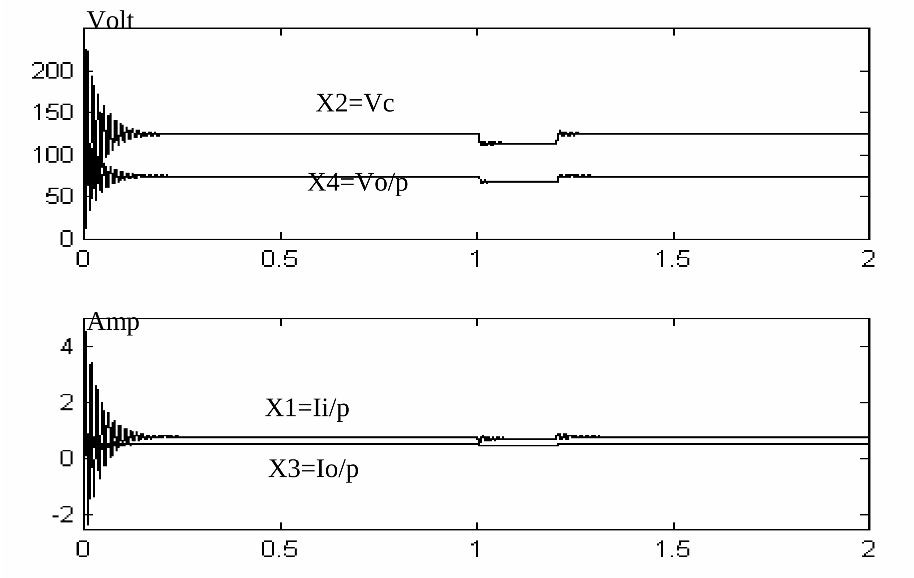


Fig. 3-31, Dynamic performance of Cûk converter with small signal Δx.

3-8 Switch Utilization in Converter:

In this section a study of the utilization of switch in a converter is presented, [58]

$$\text{Switch utilization ratio} = \frac{(V_o I_o)_{max}}{q V_T I_T} \tag{3-58}$$

In practice, the switch utilization ratio would be much smaller for the following reasons:

1- Switch ratings are chosen conservatively to provide safety margins.

2- The ripple in the output current would influence the switch current rating.

The differential equation of converter

$$\dot{\mathbf{X}} = \mathbf{AX} + \mathbf{BU}$$

at maximum Utilization, $\dot{\mathbf{X}} = 0$

$$\mathbf{0} = \mathbf{AX} + \mathbf{BU} \tag{3-57}$$

1- Switch utilization for Buck-Boost converter

From equation (3-44)

$$0 = \frac{1-\mathbf{D}}{\mathbf{L}}\mathbf{X}_2 + \frac{\mathbf{D}}{\mathbf{L}}\mathbf{U} \qquad \Rightarrow \qquad \mathbf{X}_2 = \mathbf{V_o} = -\frac{\mathbf{D}}{1-\mathbf{D}}\mathbf{U} \tag{3-60}$$

$$0 = -\frac{1-\mathbf{D}}{\mathbf{C}}\mathbf{X}_1 + \frac{1}{\mathbf{R_L C}}\mathbf{X}_2 \quad \Rightarrow \quad \mathbf{X}_1 = \mathbf{I_T} = \frac{\mathbf{D}}{\mathbf{R_L}(1-\mathbf{D})^2}\mathbf{U} \qquad (3\text{-}61)$$

$$\mathbf{I_o} = \frac{\mathbf{X}_2}{\mathbf{R_L}} = -\frac{\mathbf{D}}{\mathbf{R_L}(1-\mathbf{D})}\mathbf{U} \qquad (3\text{-}62)$$

$$\mathbf{V_{T_{max}}} = \mathbf{V_o} - \mathbf{V_i} = -\frac{1}{1-\mathbf{D}}\mathbf{U} \qquad (3\text{-}63)$$

$$\text{Switch Utilization ratio} = \frac{(\mathbf{V_o I_o})_{\mathbf{max}}}{\mathbf{qV_T I_T}} = \mathbf{D}(1-\mathbf{D}) \qquad (3\text{-}64)$$

2- Switch utilization for CûK converter:

From equation (3-57)

$$0 = -\frac{1-\mathbf{D}}{\mathbf{L}_1}\mathbf{X}_2 + \frac{1}{\mathbf{L}_1}\mathbf{U} \quad \Rightarrow \quad \mathbf{X}_2 = \mathbf{V_T} = \frac{1}{1-\mathbf{D}}\mathbf{U} \qquad (3\text{-}65)$$

$$0 = -\frac{\mathbf{D}}{\mathbf{L}_2}\mathbf{X}_2 - \frac{1}{\mathbf{L}_2}\mathbf{X}_4 \quad \Rightarrow \quad \mathbf{X}_4 = \mathbf{V_o} = -\frac{\mathbf{D}}{1-\mathbf{D}}\mathbf{U} \qquad (3\text{-}66)$$

$$0 = -\frac{1}{\mathbf{C}_2}\mathbf{X}_3 - \frac{1}{\mathbf{R_L C}_2}\mathbf{X}_4 \quad \Rightarrow \quad \mathbf{X}_3 = \mathbf{I_o} = -\frac{\mathbf{D}^2}{\mathbf{R_L}(1-\mathbf{D})}\mathbf{U} \qquad (3\text{-}67)$$

$$0 = \frac{1-\mathbf{D}}{\mathbf{C}_1}\mathbf{X}_1 + \frac{\mathbf{D}}{\mathbf{C}_1}\mathbf{X}_3 \quad \Rightarrow \quad \mathbf{X}_1 = \mathbf{I_T} = \frac{\mathbf{D}^2}{\mathbf{R_L}(1-\mathbf{D})^2}\mathbf{U} \qquad (3\text{-}68)$$

Switch Utilization ratio **=(1-D)** (3-69)

From equations (3-64) & (3-69) the maximum switch utilization ratio for CûK converter is greater than Switch Utilization ratio for Buck-Boost Converter (D<1).

The electronic switch that I have used :

$$\text{Switch Utilization ratio} = \frac{(\mathbf{V_o I_o})_{\mathbf{max}}}{\mathbf{qV_T I_T}} = \frac{\mathbf{225}}{\mathbf{600 * 21}} = \mathbf{0.0179}$$