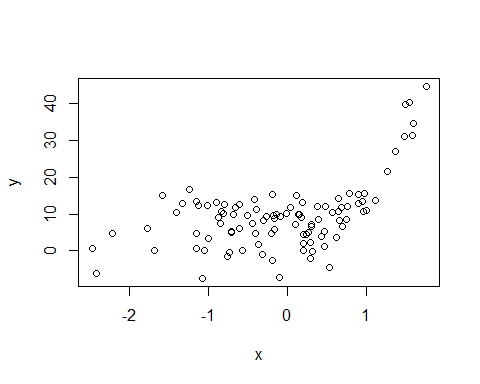
HW6

Anish Mohan

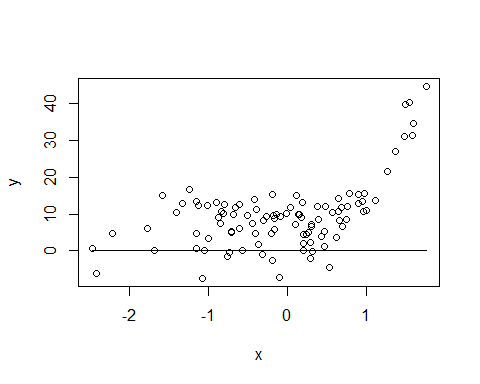
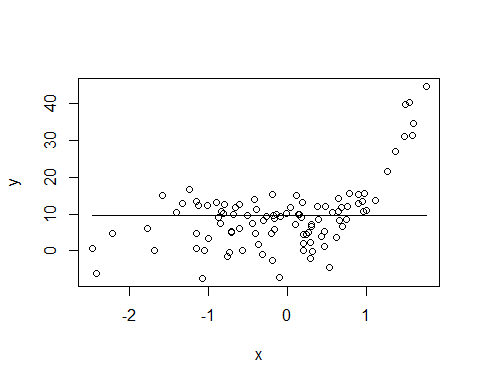
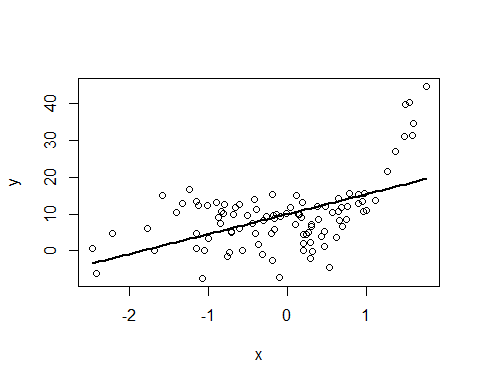
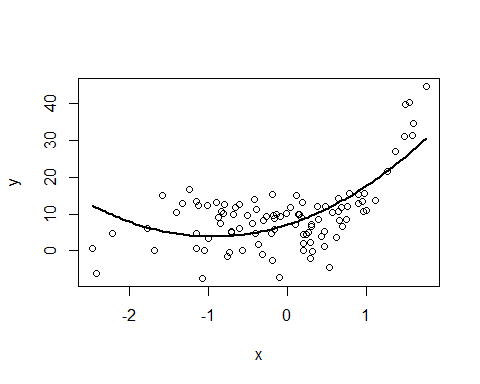
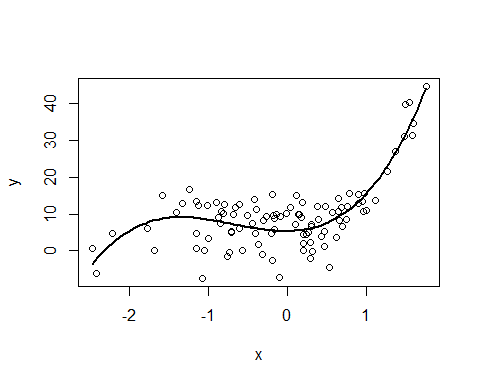
November 9, 2015

# Q1

x=rnorm(100)  
eps=rnorm(100)  
y=5+2\*x+7\*x\*x+3\*x\*x\*x+5\*eps  
plot(x,y)

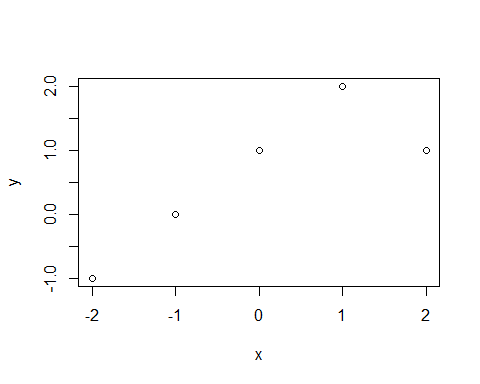


u=lm(y~-1)  
u0=lm(y~1)  
u1=lm(y~poly(x,1))  
u2=lm(y~poly(x,2))  
u3=lm(y~poly(x,3))  
u4=lm(y~poly(x,4))  
u5=lm(y~poly(x,5))  
xlim=range(x)  
xgrid=seq(from=xlim[1],to=xlim[2], length.out = 100)

* 1a. , m=0
* Since , to minimize the equation, we need be zero or tend towards zero. This is possible with a function g(x)=0
* plot(x,y)  
  tempval=rep(0,100)  
  lines(xgrid,tempval)
* 
* 1b. , m=1
* Since , to minimize the equation, we need to be lowest or tend towards zero. This is only possible if is a constant function i.e g(x)=
* plot(x,y)  
  val=u0$coefficients[1]  
  tempval=rep(val,100)  
  lines(xgrid,tempval)
* 
* 1c. , m=2
* Since , to minimize the equation, we need to be lowest or tend towards zero. This is possible if is constant i.e is a linear function with constant slope. Hence the function would be g(x)=.
* plot(x,y)  
  xlim=range(x)  
  xgrid=seq(from=xlim[1],to=xlim[2], length.out = 100)  
  pred=predict(u1,newdata=list(x=xgrid),se=T)  
  lines(xgrid, pred$fit, lwd=2)
* 
* 1d. , m=3
* Since , to minimize the equation, we need to be lowest or tend towards zero. This is possible if is constant. This implied is uniformly increasing or has constant slope i.e the function would be incresing at a constant rate. This is possible with a function g(x)=
* plot(x,y)  
  xlim=range(x)  
  xgrid=seq(from=xlim[1],to=xlim[2], length.out = 100)  
  pred=predict(u2,newdata=list(x=xgrid),se=T)  
  lines(xgrid, pred$fit, lwd=2)
* 
  + 1e. , m=3
* Since , to minimize the equation, we need residual erro to be lowest. This is possible with a function that minimizes the RSS which in this case is g(x)= a shown by low p value from the summary results.
* plot(x,y)  
  xlim=range(x)  
  xgrid=seq(from=xlim[1],to=xlim[2], length.out = 100)  
  pred=predict(u3,newdata=list(x=xgrid),se=T)  
  lines(xgrid, pred$fit, lwd=2)
* 
* summary(u4)
* ##   
  ## Call:  
  ## lm(formula = y ~ poly(x, 4))  
  ##   
  ## Residuals:  
  ## Min 1Q Median 3Q Max   
  ## -15.6386 -3.1319 0.6367 3.8068 9.1043   
  ##   
  ## Coefficients:  
  ## Estimate Std. Error t value Pr(>|t|)   
  ## (Intercept) 9.492 0.517 18.362 < 2e-16 \*\*\*  
  ## poly(x, 4)1 48.803 5.170 9.440 2.60e-15 \*\*\*  
  ## poly(x, 4)2 39.694 5.170 7.678 1.42e-11 \*\*\*  
  ## poly(x, 4)3 43.296 5.170 8.375 4.85e-13 \*\*\*  
  ## poly(x, 4)4 6.683 5.170 1.293 0.199   
  ## ---  
  ## Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1  
  ##   
  ## Residual standard error: 5.17 on 95 degrees of freedom  
  ## Multiple R-squared: 0.6983, Adjusted R-squared: 0.6856   
  ## F-statistic: 54.97 on 4 and 95 DF, p-value: < 2.2e-16

# Q2

x=-2:2  
 beta0=1  
 beta1=1  
 beta2=-2  
 y=rep(NA,5)  
   
 for(i in 1:5 ) {  
 if(x[i]<1){  
 y[i]=beta0+x[i]\*beta1  
 } else {  
 y[i]=beta0+beta1\*x[i]+beta2\*(x[i]-1)^2  
 }  
 }  
 plot(x,y)



# Q3

* 3a. Functions will have a smaller training error as it is higher order polynomial and has higher order penalty function. It will fit the training data well
* 3b. Functions could have a smaller test error as the higher order function will tend to overfit the noise.
* 3c. Functions and are same when , hence they will have same training and test error.

# Q4

library(ISLR)

## Warning: package 'ISLR' was built under R version 3.2.2

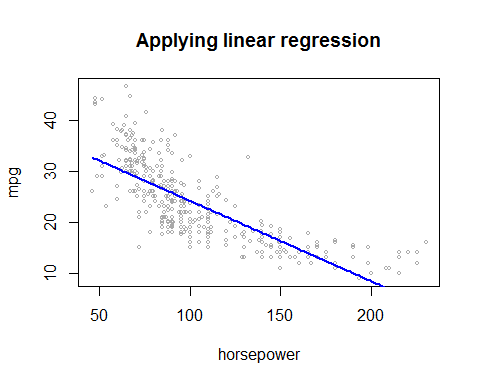
library(boot)

## Warning: package 'boot' was built under R version 3.2.2

attach(Auto)  
   
 set.seed(1)  
 #10-Fold validation comparing a linear and cubic model  
   
 fit.lm=glm(mpg~horsepower, data=Auto)  
 cv.error=cv.glm(Auto,fit.lm,K=10)  
 cv.error$delta

## [1] 24.10716 24.09865

hplims=range(Auto$horsepower)  
 hp.grid=seq(from=hplims[1],to=hplims[2])  
   
 plot(horsepower, mpg, xlim=hplims, cex=0.5, col="darkgrey")  
 preds=predict(fit.lm,newdata=list(horsepower=hp.grid), se=T)  
 lines(hp.grid, preds$fit, lwd=2, col="blue")  
 title("Applying linear regression")



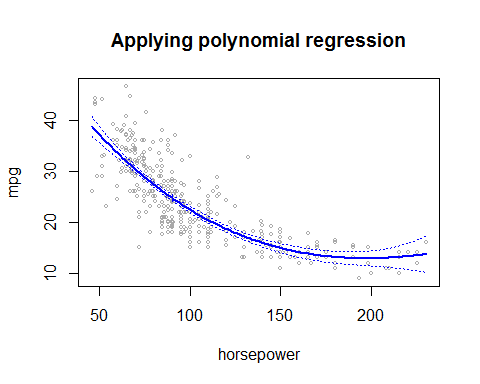
fit.poly=glm(mpg~poly(horsepower,3),data=Auto)  
 cv.error=cv.glm(Auto,fit.poly,K=10)  
 cv.error$delta

## [1] 19.37347 19.35072

print(paste0("Error from polynomial regression is smaller than linear regression" ))

## [1] "Error from polynomial regression is smaller than linear regression"

#Using Polynomial regression  
  
 preds=predict(fit.poly,newdata=list(horsepower=hp.grid), se=T)  
 se.bands=cbind(preds$fit+2\*preds$se,preds$fit-2\*preds$se.fit)  
   
 plot(horsepower, mpg, xlim=hplims, cex=0.5, col="darkgrey")  
 title("Applying polynomial regression")  
 lines(hp.grid, preds$fit, lwd=2, col="blue")  
 matlines(hp.grid, se.bands, lwd=1, col="blue", lty=3)



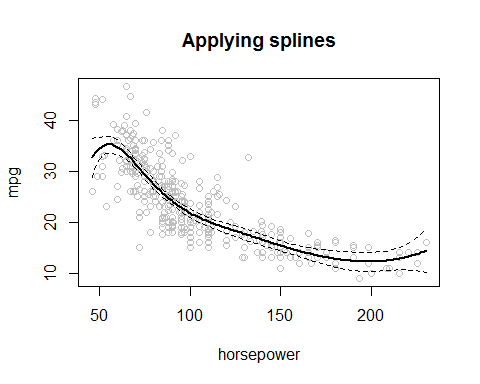
#using ANOVA to analyze variance.  
 fit.1=lm(mpg~horsepower, data=Auto)  
 fit.2=lm(mpg~poly(horsepower,2), data=Auto)  
 fit.3=lm(mpg~poly(horsepower,3), data=Auto)  
 fit.4=lm(mpg~poly(horsepower,4), data=Auto)  
   
 anova(fit.1,fit.2,fit.3,fit.4)

## Analysis of Variance Table  
##   
## Model 1: mpg ~ horsepower  
## Model 2: mpg ~ poly(horsepower, 2)  
## Model 3: mpg ~ poly(horsepower, 3)  
## Model 4: mpg ~ poly(horsepower, 4)  
## Res.Df RSS Df Sum of Sq F Pr(>F)   
## 1 390 9385.9   
## 2 389 7442.0 1 1943.89 101.6666 <2e-16 \*\*\*  
## 3 388 7426.4 1 15.59 0.8155 0.3670   
## 4 387 7399.5 1 26.91 1.4076 0.2362   
## ---  
## Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

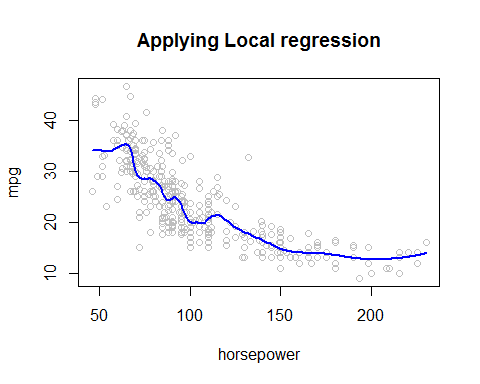
print(paste0("Quadratic model seems to be a good fit"))

## [1] "Quadratic model seems to be a good fit"

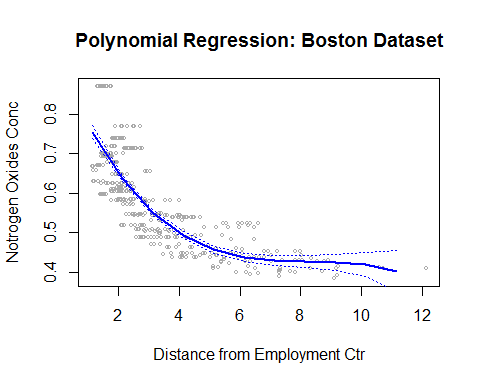
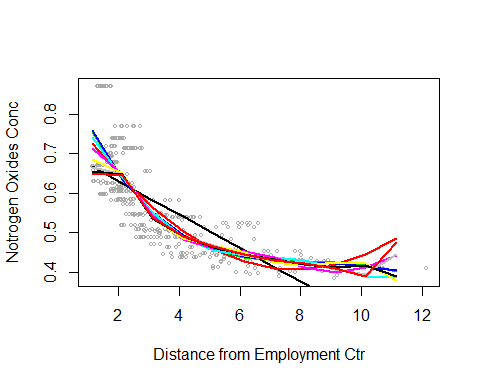
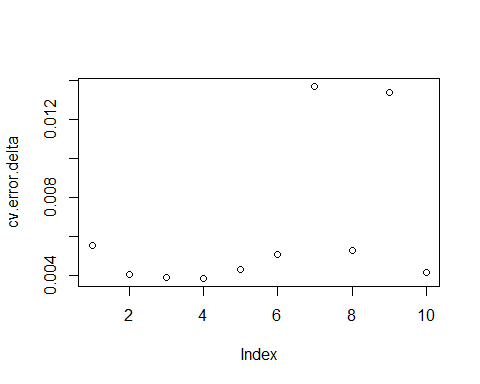
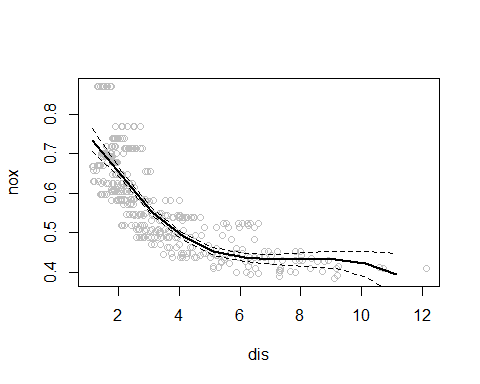
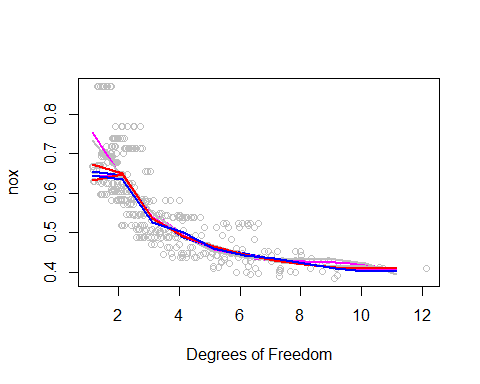
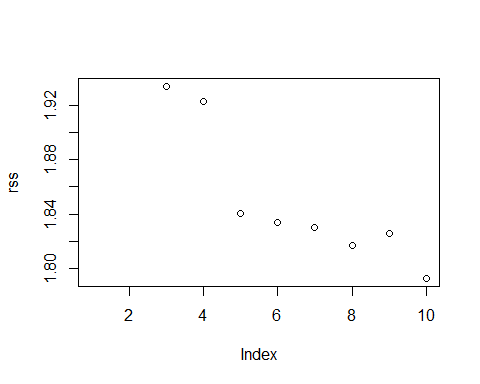
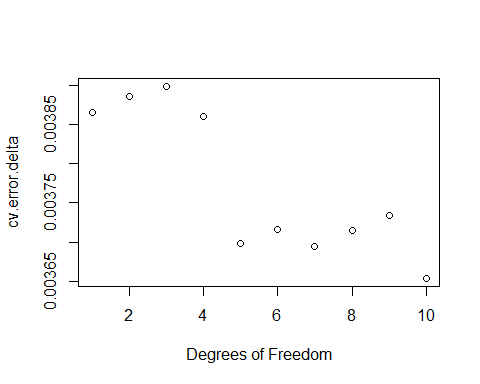
#Applying spline model  
 library(splines)  
 fit=lm(mpg~bs(horsepower, df=6),data=Auto)  
 pred=predict(fit,newdata=list(horsepower=hp.grid),se=T)  
 plot(horsepower,mpg, col="gray")  
 title('Applying splines')  
 lines(hp.grid, pred$fit,lwd=2)  
 lines(hp.grid, pred$fit+2\*pred$se,lty="dashed")  
 lines(hp.grid, pred$fit-2\*pred$se,lty="dashed")



#Applying local regression  
 fit=loess(mpg~horsepower, span=0.2, data=Auto)  
 pred=predict(fit, newdata=data.frame(horsepower=hp.grid))  
 plot(horsepower,mpg, col="gray")  
 title('Applying Local regression')  
 lines(hp.grid, pred,col="blue", lwd=2)

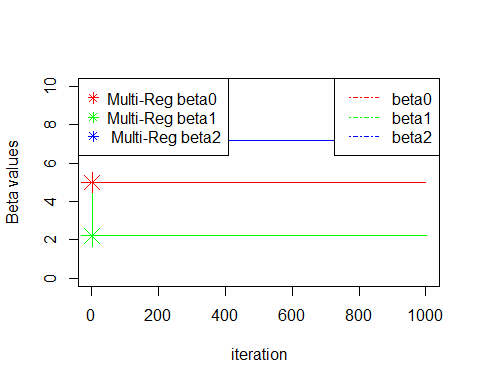


# Q5

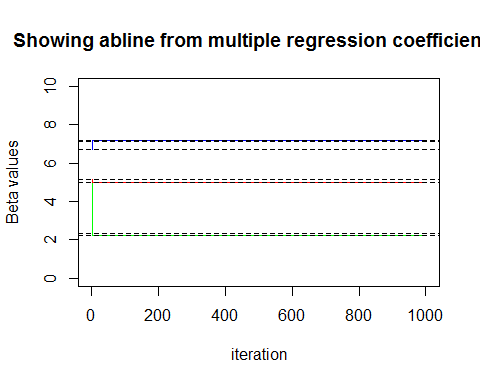
* 5a.
* library(MASS)  
   attach(Boston)  
    
   fit.poly=glm(nox~poly(dis,3))  
   print(paste0("Coefficient of the polynomial eqn"))
* ## [1] "Coefficient of the polynomial eqn"
* coef(fit.poly)
* ## (Intercept) poly(dis, 3)1 poly(dis, 3)2 poly(dis, 3)3   
  ## 0.5546951 -2.0030959 0.8563300 -0.3180490
* summary(fit.poly)
* ##   
  ## Call:  
  ## glm(formula = nox ~ poly(dis, 3))  
  ##   
  ## Deviance Residuals:   
  ## Min 1Q Median 3Q Max   
  ## -0.121130 -0.040619 -0.009738 0.023385 0.194904   
  ##   
  ## Coefficients:  
  ## Estimate Std. Error t value Pr(>|t|)   
  ## (Intercept) 0.554695 0.002759 201.021 < 2e-16 \*\*\*  
  ## poly(dis, 3)1 -2.003096 0.062071 -32.271 < 2e-16 \*\*\*  
  ## poly(dis, 3)2 0.856330 0.062071 13.796 < 2e-16 \*\*\*  
  ## poly(dis, 3)3 -0.318049 0.062071 -5.124 4.27e-07 \*\*\*  
  ## ---  
  ## Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1  
  ##   
  ## (Dispersion parameter for gaussian family taken to be 0.003852802)  
  ##   
  ## Null deviance: 6.7810 on 505 degrees of freedom  
  ## Residual deviance: 1.9341 on 502 degrees of freedom  
  ## AIC: -1370.9  
  ##   
  ## Number of Fisher Scoring iterations: 2
* dislims=range(Boston$dis)  
   dis.grid=seq(from=dislims[1],to=dislims[2])  
   preds=predict(fit.poly,newdata=list(dis=dis.grid), se=T)  
   se.bands=cbind(preds$fit+2\*preds$se,preds$fit-2\*preds$se.fit)  
    
   plot(dis,nox, xlim=dislims,xlab="Distance from Employment Ctr", ylab="Notrogen Oxides Conc", cex=0.5, col="darkgrey")  
   title("Polynomial Regression: Boston Dataset")  
   lines(dis.grid, preds$fit, lwd=2, col="blue")  
   matlines(dis.grid, se.bands, lwd=1, col="blue", lty=3)
* 
* 5b.
* fit=matrix(0,nrow=10,ncol=1)  
  plot(dis,nox, xlim=dislims,xlab="Distance from Employment Ctr", ylab="Notrogen Oxides Conc", cex=0.5, col="darkgrey")  
  for (i in 1:10){  
   lm.fit=glm(nox~poly(dis,i),data=Boston)  
   fit[i]=sum(lm.fit$residuals^2)  
   preds=predict(lm.fit,newdata=list(dis=dis.grid), se=T)  
   lines(dis.grid, preds$fit, lwd=2, col=i)  
  }
* 
* fit
* ## [,1]  
  ## [1,] 2.768563  
  ## [2,] 2.035262  
  ## [3,] 1.934107  
  ## [4,] 1.932981  
  ## [5,] 1.915290  
  ## [6,] 1.878257  
  ## [7,] 1.849484  
  ## [8,] 1.835630  
  ## [9,] 1.833331  
  ## [10,] 1.832171
* 5c.
* set.seed(1)  
   cv.error.delta=rep(NA,10)  
   for(i in 1:10){  
   fit.poly=glm(nox~poly(dis,i),data=Boston)  
   cv.error=cv.glm(Boston,fit.poly,K=10)  
   cv.error.delta[i]=cv.error$delta[1]  
   }  
   plot(cv.error.delta)
* 
* Degree 4 polynomial is the simplest model with lowest Cross-Validation error.
* A liner model does not fit the data, hence it has higher error. The quadratic and cubic models are comparable as the difference in error between them on a 10 fold CV set is small. Degree 4 polynomial shows the smallest error.
* For polynomials>4, the residual errors are high implying that the model fits to the training data but gives poor results on the CV test data.
* 5d.
* library(splines)  
  fit=lm(nox~bs(dis,df=4),data=Boston)  
  dislims=range(Boston$dis)  
  dis.grid=seq(from=dislims[1],to=dislims[2])  
  summary(fit)
* ##   
  ## Call:  
  ## lm(formula = nox ~ bs(dis, df = 4), data = Boston)  
  ##   
  ## Residuals:  
  ## Min 1Q Median 3Q Max   
  ## -0.124622 -0.039259 -0.008514 0.020850 0.193891   
  ##   
  ## Coefficients:  
  ## Estimate Std. Error t value Pr(>|t|)   
  ## (Intercept) 0.73447 0.01460 50.306 < 2e-16 \*\*\*  
  ## bs(dis, df = 4)1 -0.05810 0.02186 -2.658 0.00812 \*\*   
  ## bs(dis, df = 4)2 -0.46356 0.02366 -19.596 < 2e-16 \*\*\*  
  ## bs(dis, df = 4)3 -0.19979 0.04311 -4.634 4.58e-06 \*\*\*  
  ## bs(dis, df = 4)4 -0.38881 0.04551 -8.544 < 2e-16 \*\*\*  
  ## ---  
  ## Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1  
  ##   
  ## Residual standard error: 0.06195 on 501 degrees of freedom  
  ## Multiple R-squared: 0.7164, Adjusted R-squared: 0.7142   
  ## F-statistic: 316.5 on 4 and 501 DF, p-value: < 2.2e-16
* pred=predict(fit,newdata=list(dis=dis.grid), se=T)  
  plot(dis,nox, col="gray")  
  lines(dis.grid, pred$fit, lwd=2)  
  lines(dis.grid, pred$fit+2\*pred$se, lty="dashed")  
  lines(dis.grid, pred$fit-2\*pred$se, lty="dashed")
* 
* Knots were chosen by specifying the df attribute that chose the knots at uniform quantile of data. In this case, the knots were at 25th, 50th and 75th quantiles
* 5e.
* plot(dis,nox, col="gray", xlab="Degrees of Freedom")  
  rss=rep(NA,10)  
  for(i in 3:10){  
   lm.fit=lm(nox~bs(dis,df=i),data=Boston)  
   pred=predict(lm.fit,newdata=list(dis=dis.grid), se=T)  
   lines(dis.grid, pred$fit, lwd=2, col=i\*10)  
   rss[i]=sum(lm.fit$residuals^2)  
  }
* 
* plot(rss)
* 
* RSS decreases monotonically as we increase degrees of freedom.Lowest RSS is with df=10
* 5f
* set.seed(1)  
  cv.error.delta=rep(NA,10)  
  for(i in 1:10){  
   fit.spline=glm(nox~bs(dis,df=i),data=Boston)  
   cv.error=cv.glm(Boston,fit.spline,K=10)  
   cv.error.delta[i]=cv.error$delta[1]  
  }  
  plot(cv.error.delta, xlab="Degrees of Freedom")
* 
  + Lowest error obtained through cross validation is for df=10.
  + As we increase the degrees of freedome, initially, till we reach df=3, there is increase in the RSS. However, error starts decreasing after that till df=5. Errors fluctuate a bit, before getting the lowest error at df=10.

# Q6

x1=rnorm(100)  
 x2=rnorm(100)  
 eps=rnorm(100)   
 y=5+2\*x1+7\*x2+eps  
   
 beta0=rep(NA,1000)  
 beta1=rep(NA,1000)  
 beta2=rep(NA,1000)  
   
 beta1[1]=5  
   
 for(i in 1:1000){  
 a=y-beta1[i]\*x1  
 beta2[i]=lm(a~x2)$coef[2]  
 beta0[i]=lm(a~x2)$coef[1]  
   
 a=y-beta2[i]\*x2  
 beta1[i+1]=lm(a~x1)$coef[2]  
   
 }  
   
 plot(1:1001,beta1, col="green", type='l', ylim=c(0,10), xlab="iteration", ylab="Beta values")  
 lines(1:1000,beta0, col="red")  
 lines(1:1000,beta2, col="blue")  
   
   
 u=lm(y~x1+x2)  
 points(coef(u)[1],col="red", pch=8, cex=2)  
 points(coef(u)[2],col="green", pch=8, cex=2)  
 points(coef(u)[3],col="blue", pch=8, cex=2)  
 legend("topright",c("beta0","beta1","beta2"), lty=4,col = c("red","green","blue"))  
 legend("topleft",c("Multi-Reg beta0","Multi-Reg beta1"," Multi-Reg beta2"), pch=8,col = c("red","green","blue"))



plot(1:1001,beta1, col="green", type='l', ylim=c(0,10), xlab="iteration", ylab="Beta values")  
 title("Showing abline from multiple regression coefficients.")  
 lines(1:1000,beta0, col="red")  
 lines(1:1000,beta2, col="blue")  
 abline(h=beta0, lty="dashed")  
 abline(h=beta1, lty="dashed")  
 abline(h=beta2, lty="dashed")



print(paste0("Within 2nd approximations the backfitting answers were good estimates"))

## [1] "Within 2nd approximations the backfitting answers were good estimates"

beta0[1:5]

## [1] 5.149512 5.014518 5.009577 5.009396 5.009389

beta1[1:5]

## [1] 5.000000 2.319657 2.221546 2.217955 2.217824

beta2[1:5]

## [1] 6.693492 7.145315 7.161853 7.162459 7.162481